NS5-branes and line bundles in Heterotic/M-/F-Theory duality

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Physics & Geometry of F-Theory
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Based on [1803.xxxxx] with Andreas Braun, Callum Brodie, Andre Lukas

Motivation

- Duality: Heterotic spectral cover bundles ⇔ F-Theory studied since the early days **[Friedman, Morgan, Witten`97]**
- Duals of heterotic with bundles that don't have a (useful) spectral cover description?

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- Duality: Heterotic spectral cover bundles ⇔ F-Theory studied since the early days **[Friedman, Morgan, Witten`97]**
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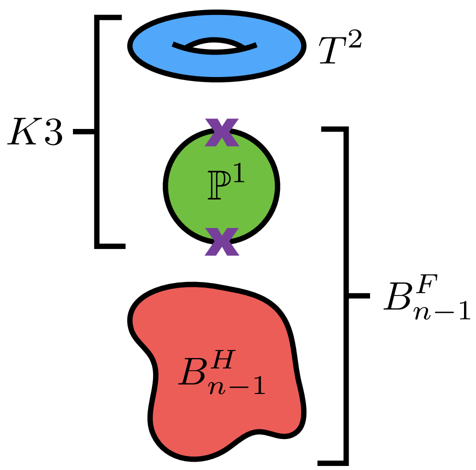
- Heterotic NS5 branes in Horava-Witten M-Theory ⇒ tensionless strings from M2 branes between M5 or M5-E8 branes **[Strominger`95][Ganor, Hanany`96; Bershadsky, Johansen`96]**
- Relation to SCFTs **[Heckman,Morrison,Vafa`13, …, many talks here]**
- How do the NS5/M5 branes in Het M-Theory map to F-Theory?

Outline

- Het/M/F-Theory duality
  - Stable degeneration
  - Horizontal / vertical NS5 brane duals
- F-Theory duals of line bundles with NS5 branes
  - Anomalies
  - Stability
  - Matter
- NS5 branes
  - Matching NS5 brane spectra in Heterotic / F-Theory
  - Transversely intersecting & coincident NS5 branes
- Conclusion

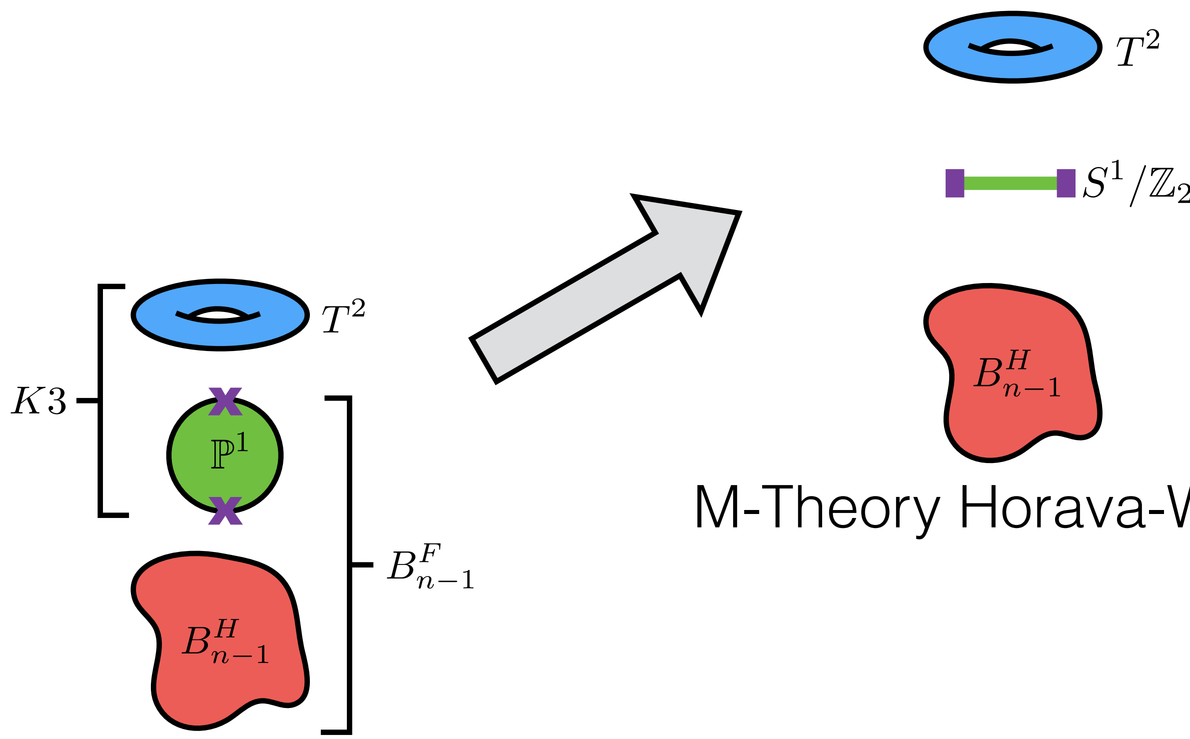
Recap Het / M / F-Theory duality

F/M-Theory duality



F-Theory on CY$_n$ Y_n

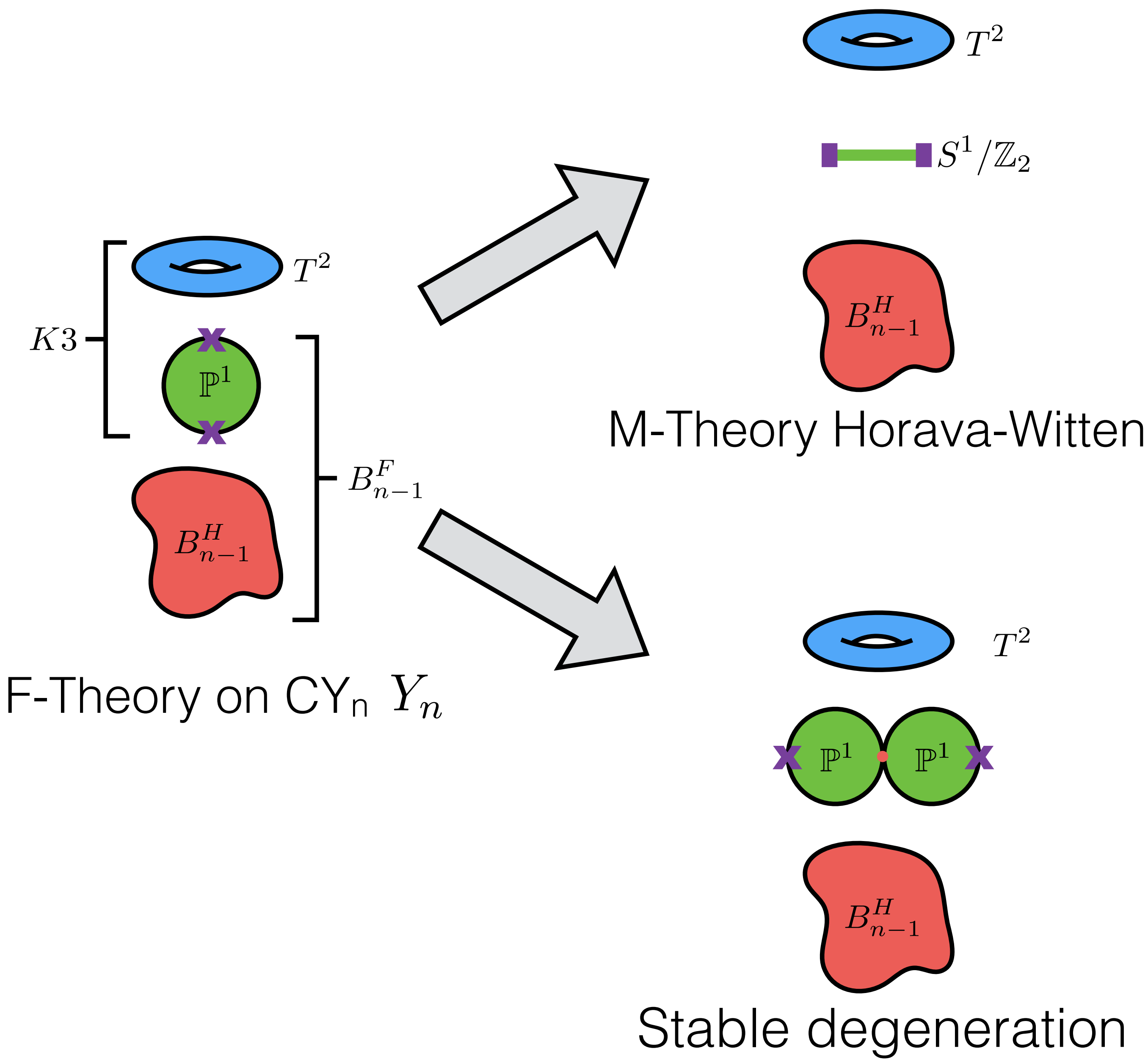
F/M-Theory duality



M-Theory Horava-Witten

F-Theory on CYn Y_n

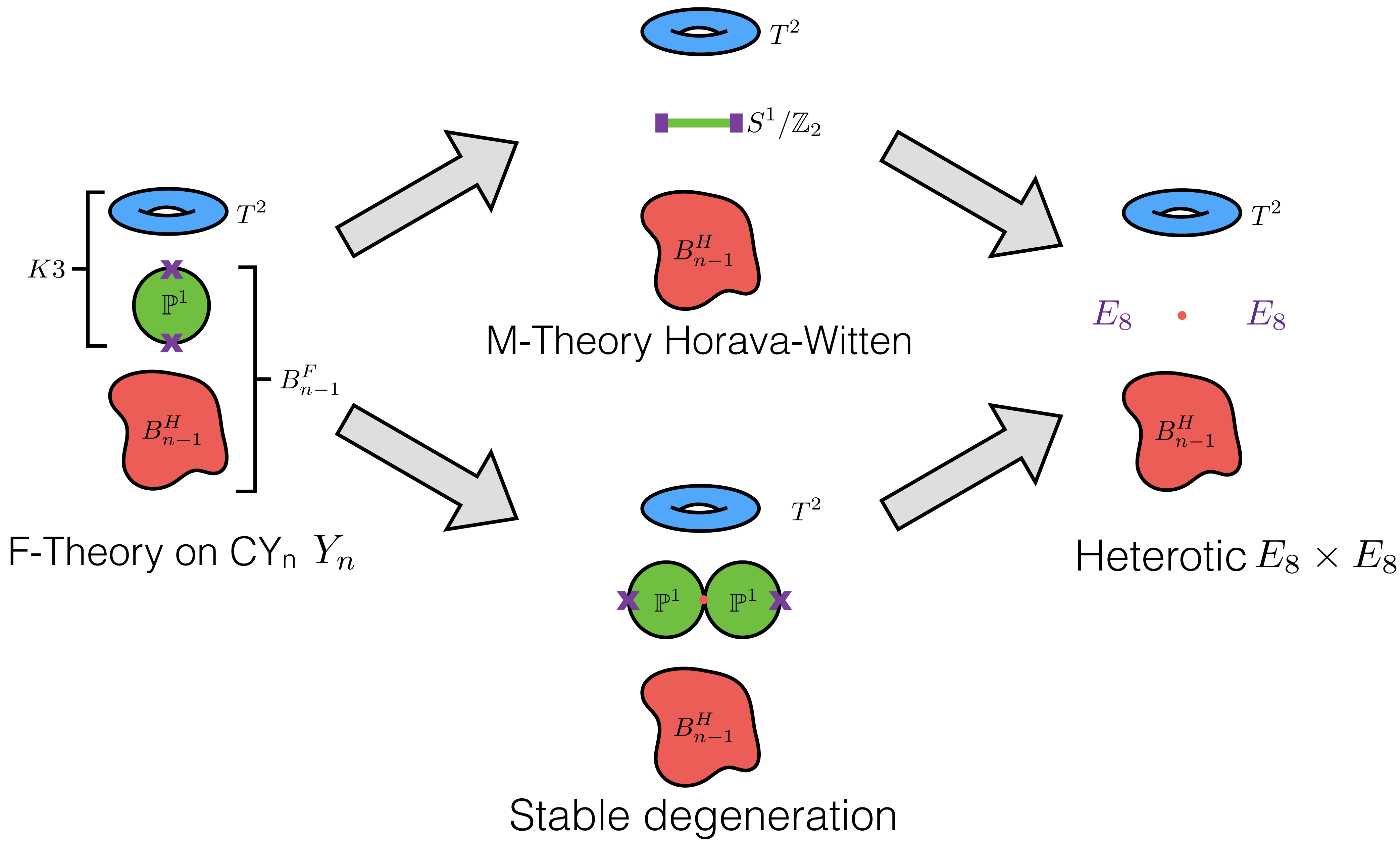
F/M-Theory duality



Stable degeneration limit: $\mathrm{vol}(B^H_{n-1}) \gg \mathrm{vol}(T^2) \to \infty$

$\mathrm{vol}(\mathbb{P}^1) \to 0$

F/M-Theory duality



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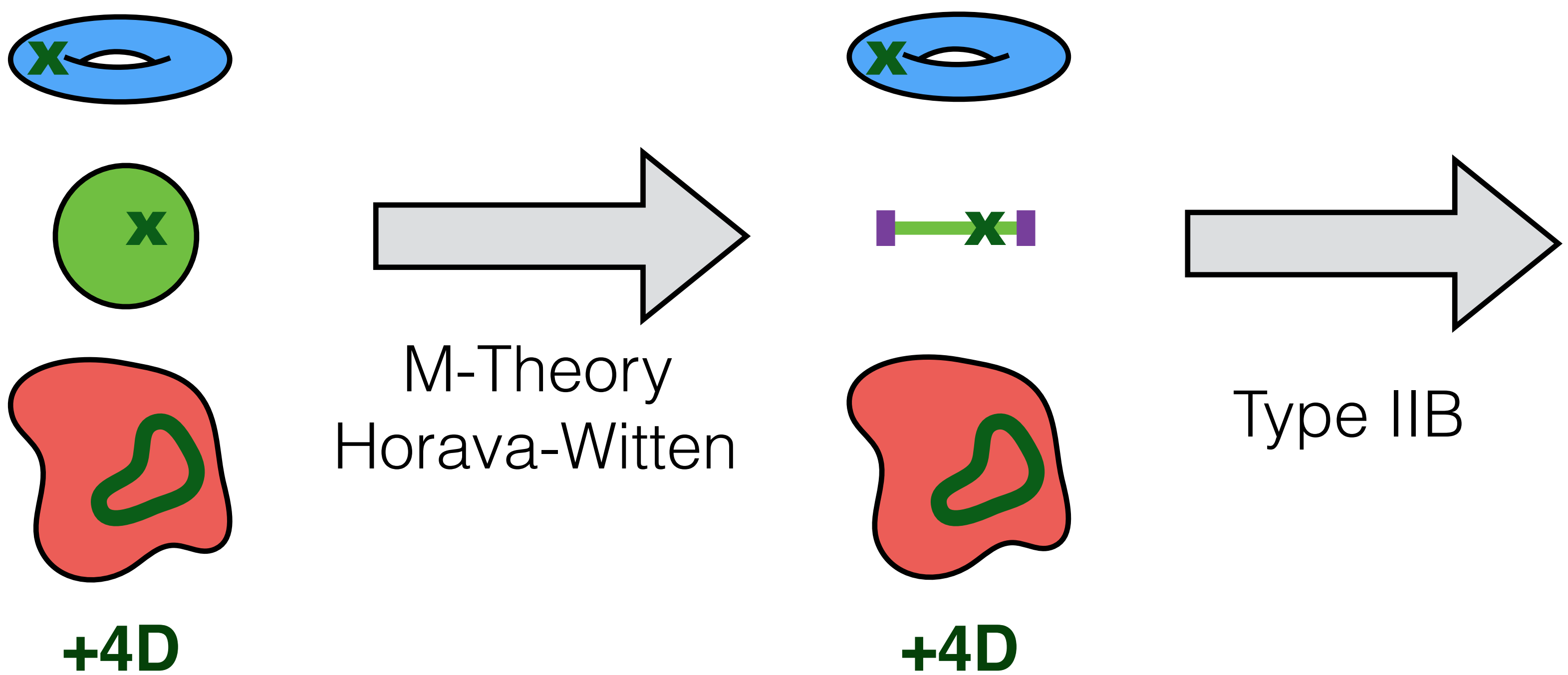
NS5 branes

Horizontal **NS5** branes



Geometry of CY 3-fold:
Blowups in the base
along the **curve** wrapped
by the NS5 brane

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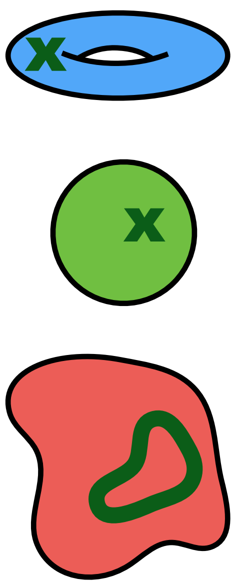
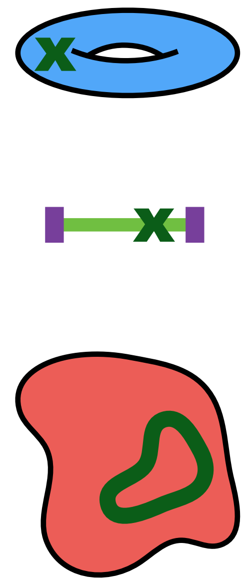


+4D

M-Theory
Horava-Witten



+4D

Type IIB

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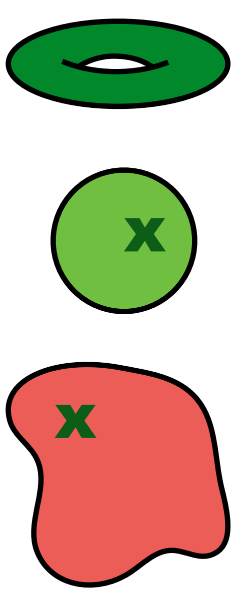

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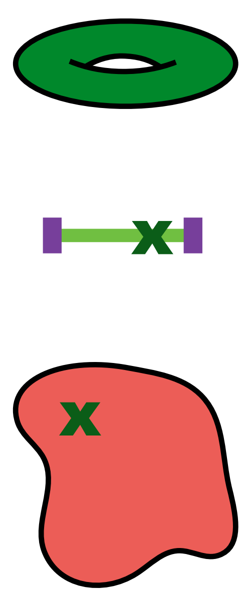

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D3 branes

[Diaconescu,Rajesh`99]

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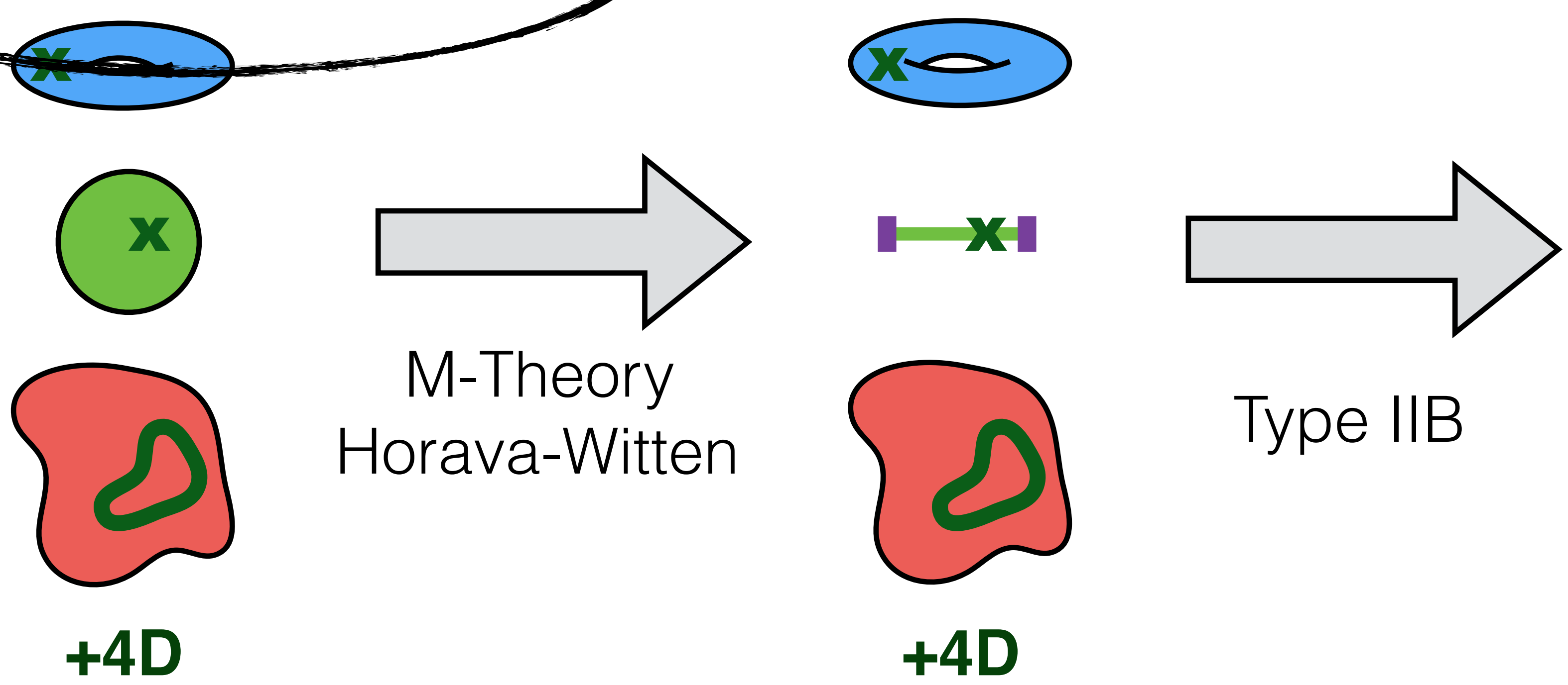

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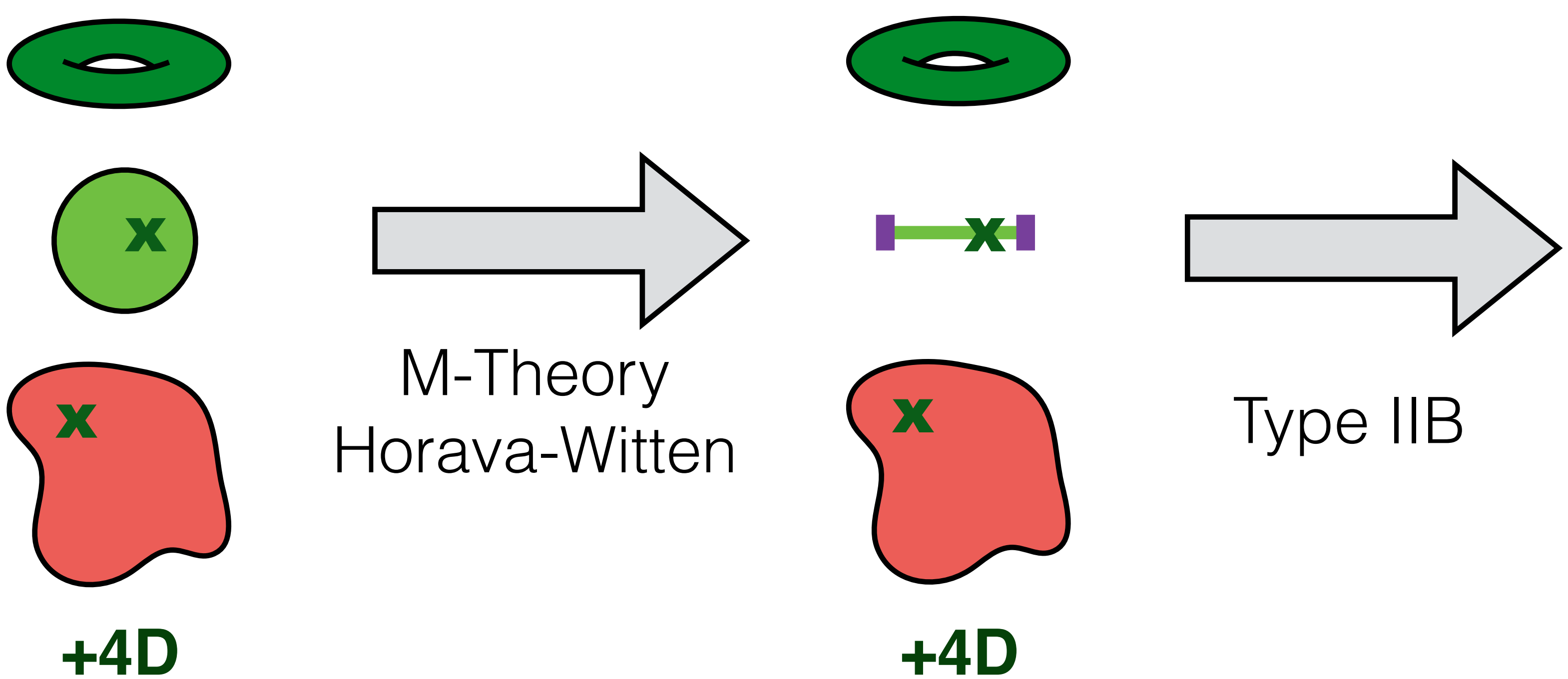

D3 branes

[Diaconescu,Rajesh`99]

F-Theory duals of line bundles with NS5 branes

Line bundles in F-Theory

- GG in F-Theory: Construct elliptic fibration with ADE singularity in codimension 1
- GG in Heterotic: Construct bundle with structure group H s.t. $G \times H \subset E_8 \times E_8$

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- F-Theory spectral cover: Het. bundle V w/ structure group H mapped onto (f, g) s.t. Kodaira singularity corresponding to G occurs in Weierstrass model
- For line bundles $V = \bigoplus \mathcal{L}_a,\ c_1(V) = 0$ spectral sheet trivial $\Rightarrow$ all info in spectral sheaf

Line bundles in F-Theory

- Start w/ $E_8 \times E_8$ singularity in F-Theory and break w/ G_4 (flux) rather than tuning ADE singularity (geometry)
    - 2 E_8 GUT surfaces in Y_4 , each diff. to het. base B_2

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- Write $V = k_a^I D_I = k_a^0 D_0 + k_a^i D_i \quad c_1(\mathcal{L}_a) = \mathcal{O}_{X_3}(k_a^0 \ldots, k_a^{h_{11}-1})$

(1,1) form dual to fiber curve

(1,1) form dual to base curves

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- Corresponding flux in IIB: Flux on the two E_8 7-brane stacks

Anomalies

- Het. Bianchi Identities: $ch_2(V) - ch_2(X_3) = W$

effective curve class
wrapped by 5-brane

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$$\int_{B_2^H} c_2(B_2^H) + 11c_1(B_2^H)^2 + \frac{1}{2}d_{0ij}k_a^i k_a^j \overset{!}{=} 0$$

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$$\frac{\chi(Y_4)}{24} = [\text{lots of cancellations}] = \frac{1}{2}\int_{B_3^F} c_1(B_3^F) c_2(B_3^F) + 10 \int_{B_2^H} c_1(B_2^H)^2$$

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$$\underbrace{\frac{1}{2} \int_{B_3^F} c_1(B_3^F) c_2(B_3^F)}_{= \chi_{hol}(B_3^F) = \int_{B_3^F} \mathrm{td}(B_3^F)\big|_3 = 1} \stackrel{?}{=} \underbrace{\int_{B_2^H} c_2(B_2^H) + c_1(B_2^H)^2}_{= \chi_{hol}(B_2^H) = \int_{B_2^H} \mathrm{td}(B_2^H)\big|_2 = 1}$$

Stability (D-flatness)

- Het. bundle stability: $0 = \int_{X_3} J \wedge J \wedge k_a^i D_i = d_{IJK} t^I t^J k_a^K$

[Donaldson`85; Uhlenbeck,Yau`86]

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- Insert $k_a^0 = 0, \;\; d_{ijk} = 0, \;\; \ldots$:

$$0 = t^0 (\underbrace{t^i k_a^j d_{0ij}}_{= \int_{B_2^H} J_{B_2^H} \wedge k_a^i D_i} - \underbrace{2 t^0 d_{00j} k_a^j}_{\propto \, \mathrm{vol}(T^2) \ll \mathrm{vol}(B_2^H)})$$

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- Insert $k_a^0 = 0, \;\; d_{ijk} = 0, \;\; \dots$:

$$0 = \int_{B_2^H} J_{B_2^H} \wedge k_a^i D_i$$

- F-Theory stability on fluxed 7-brane: $0 = J \wedge G_4$

$$0 = \int_S J \wedge k_a^i D_i$$

Spectrum

- Chiral index of bundle:

$$\chi(X_3, V) = \int_{X_3} \mathrm{ch}(V)\mathrm{td}(X) = \sum_a d_{IJK}\Big[-\frac{1}{3}k_a^I k_a^J k_a^K + \frac{1}{12}k_a^I[c_1(B_2^H)^2 + c_2(B_2^H)]\Big]$$

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- Insert $k_a^0 = 0, \;\; d_{ijk} = 0, \;\; \ldots : \chi(X_3, V) = 0$
- Matching the homology groups:
  - Heterotic: Use Leray spectral sequence, Serre duality, absence of sections of stable bundle

$$n_R = h^1(B_2^H, V) + \underbrace{h^2(B_2^H, V^*)}, \;\; n_{R^*} = h^1(B_2, V^*) + \underbrace{h^2(B_2^H, V)}$$

$= 0$ for effective $-K_{B_2^H}$

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  - F-Theory: Multiplet count from (twisted) theory on fluxed 7-brane **[Beasley,Heckman,Vafa`08]**

$$n_R=h^0(S,V^*)+h^1(S,V)+h^2(S,V^*)\,,\ n_{R^*}=h^0(S,V)+h^1(S,V^*)+h^2(S,V)$$

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[Donagi,Wijnholt`08]

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$= 0$ for effective S and $h^{2,0}(S) = 0$

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NS5 branes

Matching NS5 brane spectra

Heterotic

1. Chiral multiplets from CY+ dilaton: $h^{1,1}(X_3) + h^{2,1}(X_3) + 1$
2. Each NS5 brane N_i on genus g curve contributes 1 chiral and g vectors
3. Each N_i contributes $n_{\rm def}(N_i)$ chiral deformation moduli

[Lukas,Ovrut,Waldram`98]

$$n_{\rm chiral} = h^{1,1}(X_3) + h^{2,1}(X_3) + 1 + n(N_i) + \sum_i n_{\rm def}(N_i)$$

$$n_{\rm vector} = \sum_i g(N_i)$$

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$$n_{\text{vector}} = \sum_i g(N_i)$$

F-Theory

1. Chiral multiplets in 4d from fourfold:
$$n_{\text{chiral}} = h^{1,1}(Y_4) + h^{3,1}(Y_4) + (h^{2,1}(Y_4) - h^{2,1}(B_3^F))$$
2. Vector multiplets in 4d from fourfold:
$$n_{\text{vector}} = h^{2,1}(B_3^F) + h^{1,1}(Y_4) - h^{1,1}(B_3^F) - 1$$
[Mohri`97;Curio,Lust`98;Grimm`10;Grimm,Taylor`12]
3. Number of extra sections (U(1)s): $h^{1,1}(Y_4) - h^{1,1}(B_3^F) - 1$

$$n_{\text{chiral}} = h^{1,1}(Y_4) + h^{3,1}(Y_4) + (h^{2,1}(Y_4) - h^{2,1}(B_3^F))$$
$$n_{\text{vector}} = h^{2,1}(B_3^F)$$

Matching NS5 brane spectra - Vectors

$$\sum g(N_i) \stackrel{?}{=} h^{2,1}(B_3^F)$$

- Kunneth and $h^{1,0}(B_2^H) = 0 \Rightarrow h^{2,1}(B_2^H \times \mathbb{P}^1) = 0$
- Before blowup: $\chi(B_3^F) = 2 + 2(h^{1,1}(B_3^F) - h^{2,1}(B_3^F))$
- Blowup genus g curve: $\Delta\chi(B_3^F) = 2 - 2g$
- Since $\Delta h^{1,1}(B_3^F) = 1 \Rightarrow \Delta h^{2,1}(B_3^F) = g$

Matching NS5 brane spectra - Chirals

$$\sum n_{\rm def}(N_i) - 12c_1(B_2^H)^2 + n(N_i) \stackrel{?}{=} 2h^{2,1}(Y_4) - \sum g(N_i)$$

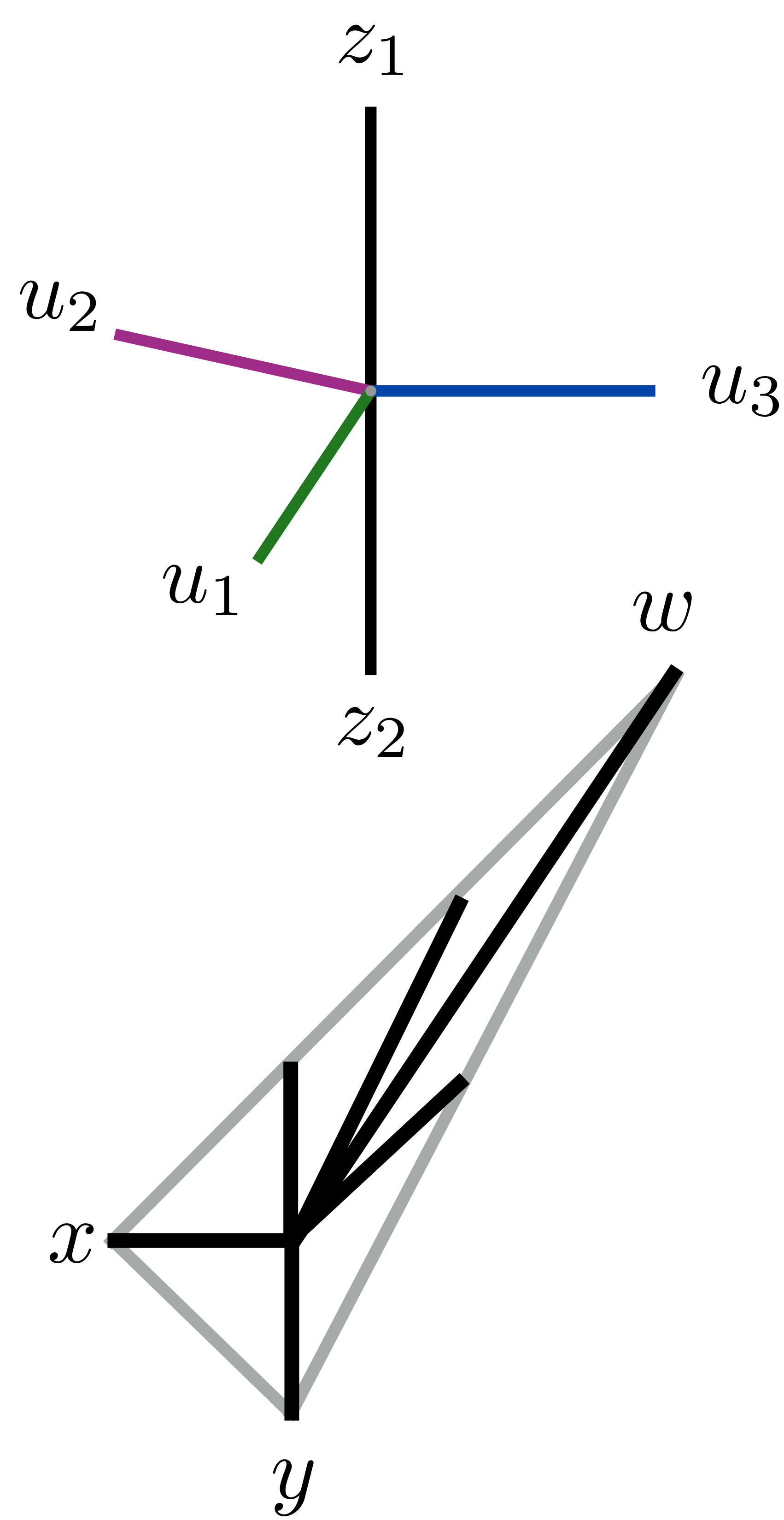
- Assume $-K_{B_2^H} \cdot C_i \neq 0 \; \forall \; i \quad \Rightarrow \quad h^{2,1}(Y_4) = h^{2,1}(B_3^F)$ (otherwise there are subtleties with the brane moving in fiber direction $\Rightarrow$ see paper)
- From vectors: $h^{2,1}(B_3^F) = \sum g(N_i)$
- $n_{\rm def}(N_i) = -K_{B_2^H} \cdot C_i - 1 + g(N_i)$
- Blowup a total of $-12K_{B_2^H}$
- Get $\sum n_{\rm def}(N_i) = 12c_1(B_2^H) - n(N_i) + \sum g(N_i)$

Toric description of blowups

- Have (4,6,12) curve of sing. in codim 2 where $E_8 \cap \Delta_{\mathrm{res}}$
- Tune curve of singularities to toric loci
- Blowup above/below the rays of B_2^H
  - Toric Blowups $\leftrightarrow$ $\mathbb{P}^1 s$
  - NEF partitions $\leftrightarrow$ more general degree n curves

$\mathrm{vol}(C) \,\widehat{=}$ M5 brane position in Horava-Witten $S^1/\mathbb{Z}_2$

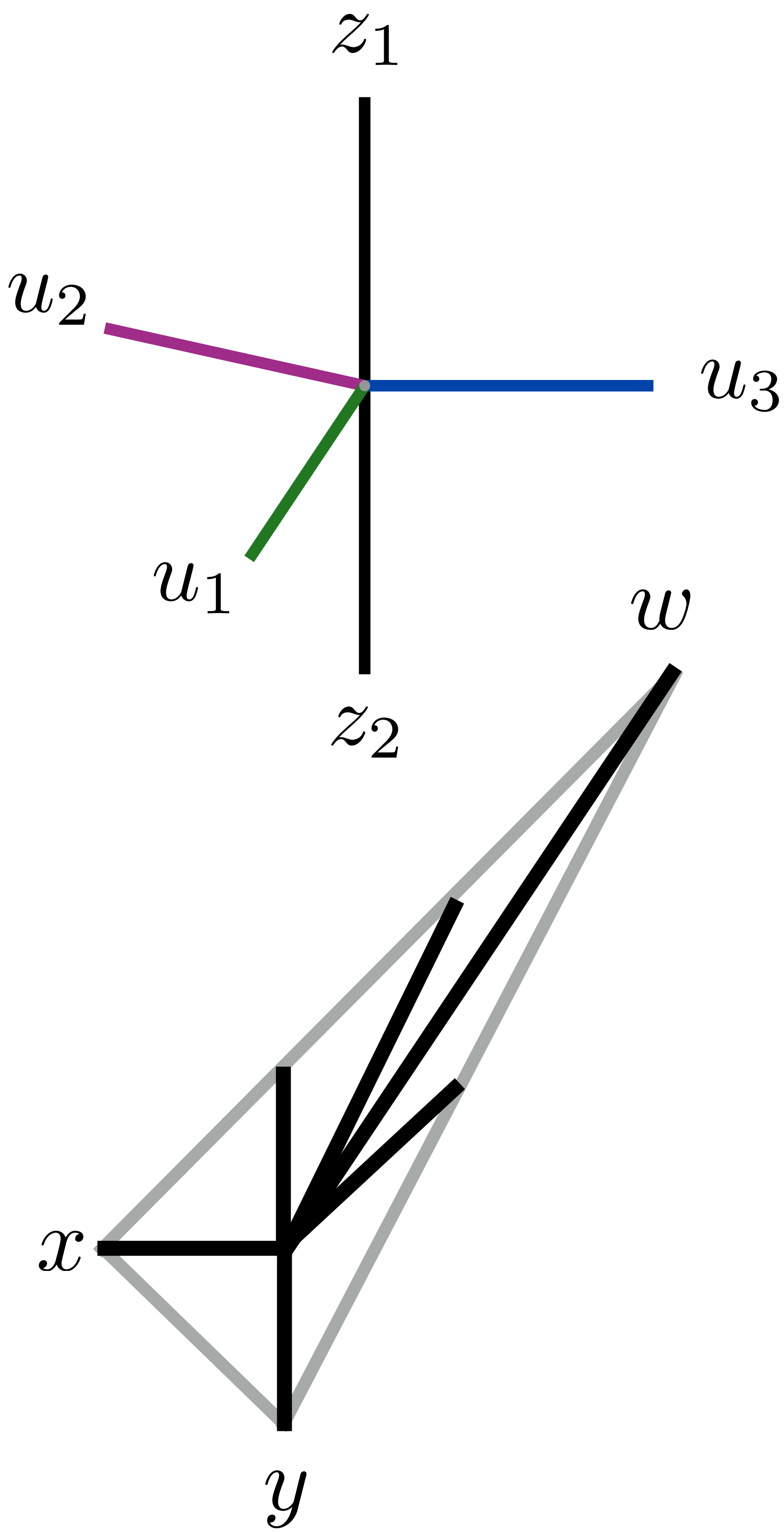
Example: $B_3^F = \mathbb{P}^2 \times \mathbb{P}^1$



$$
\begin{aligned}
u_1 &= (-1,\ 0,\ 0,\ 2,\ 3)\\
u_2 &= (0,\ -1,\ 0,\ 2,\ 3)\\
u_3 &= (1,\ 1,\ 0,\ 2,\ 3)\\
z_1 &= (0,\ 0,\ 1,\ 2,\ 3)\\
z_2 &= (0,\ 0,\ -1,\ 2,\ 3)\\
x &= (0,\ 0,\ 0,\ -1,\ 0)\\
y &= (0,\ 0,\ 0,\ 0,\ -1)\\
w &= (0,\ 0,\ 0,\ 2,\ 3)
\end{aligned}
$$

$\mathbb{P}^2$

Example: $B_3^F = \mathbb{P}^2 \times \mathbb{P}^1$



$$
\begin{aligned}
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y &= (\ \ 0,\ \ 0,\ \ 0,\ \ 0,-1)\\
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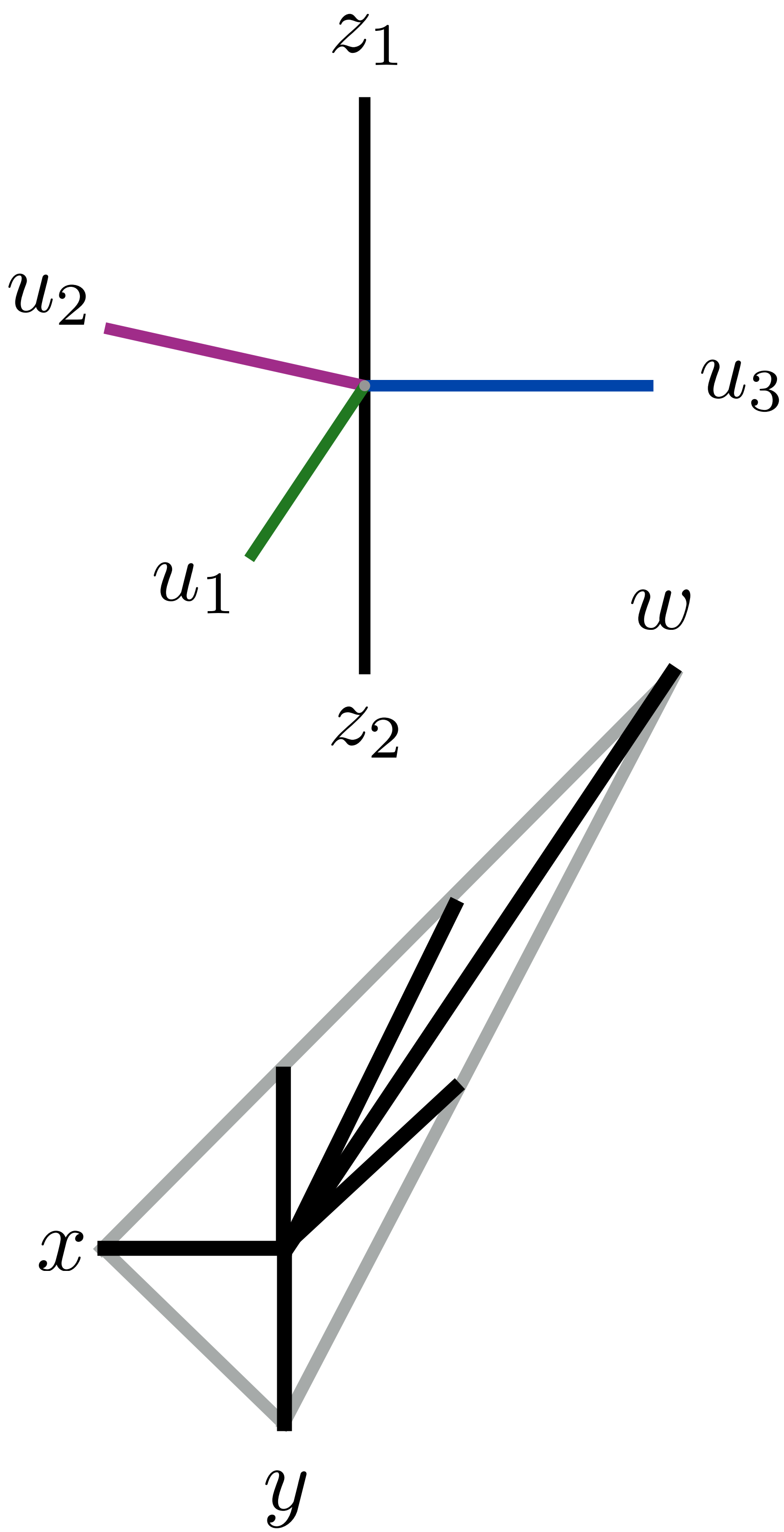
\end{aligned}
$$

$\mathbb{P}^1$

Example: $B_3^F = \mathbb{P}^2 \times \mathbb{P}^1$



$$
\begin{array}{rl}
u_1 = & (-1,\ 0,\ 0,\ 2,\ 3) \\
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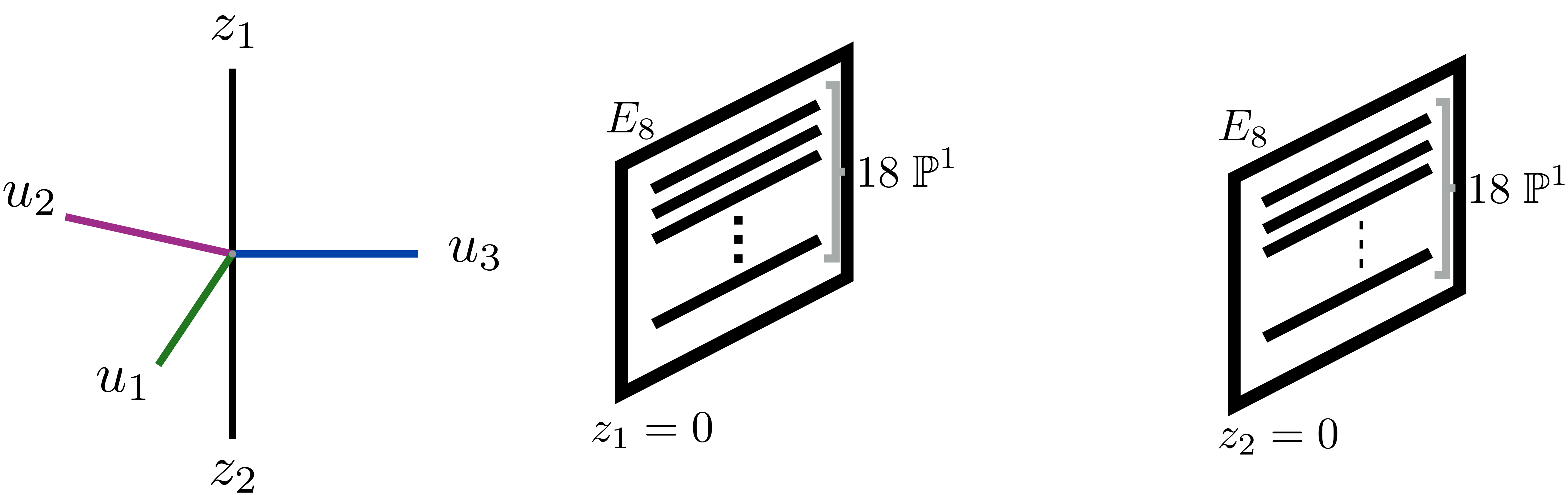
x = & (\ 0,\ 0,\ 0,-1,\ 0) \\
y = & (\ 0,\ 0,\ 0,\ 0,-1) \\
w = & (\ 0,\ 0,\ 0,\ 2,\ 3)
\end{array}
$$

T^2

Example: $B_3^F = \mathbb{P}^2 \times \mathbb{P}^1$

- Need a total of $-12K_{B_2^H}$ for class of blowup curves (for $B_2^H = \mathbb{P}^2$: $36\ H$)



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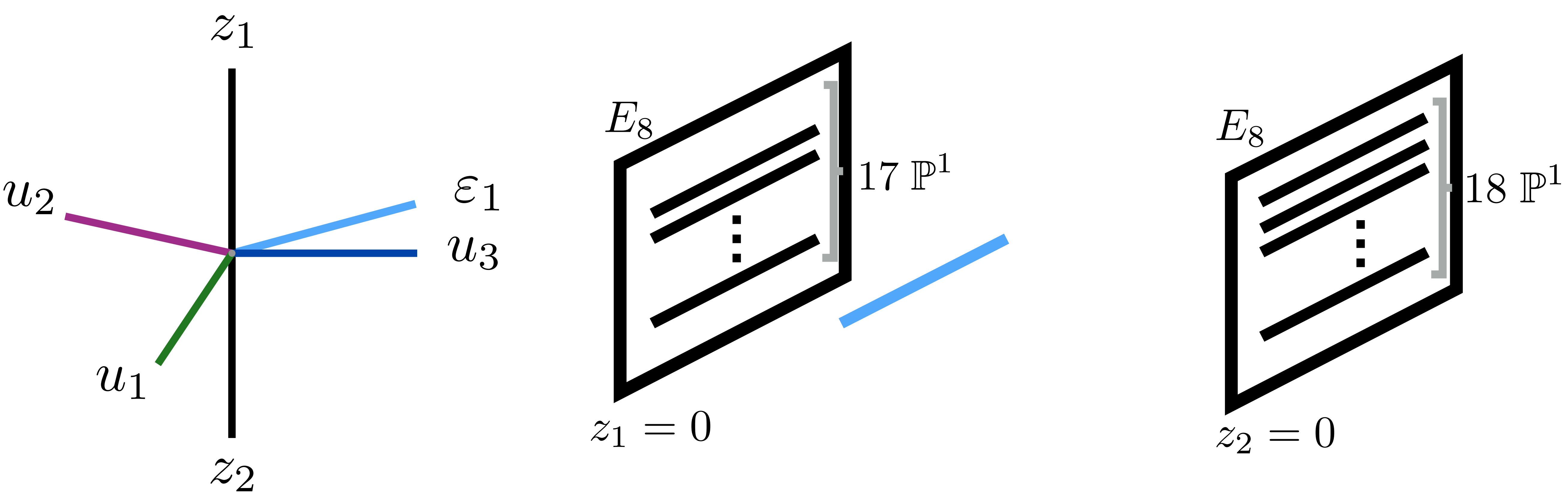

$$\varepsilon_1 = (\ 1,\ 1,\ 1,\ 2,\ 3)$$

Example: $B_3^F = \mathbb{P}^2 \times \mathbb{P}^1$

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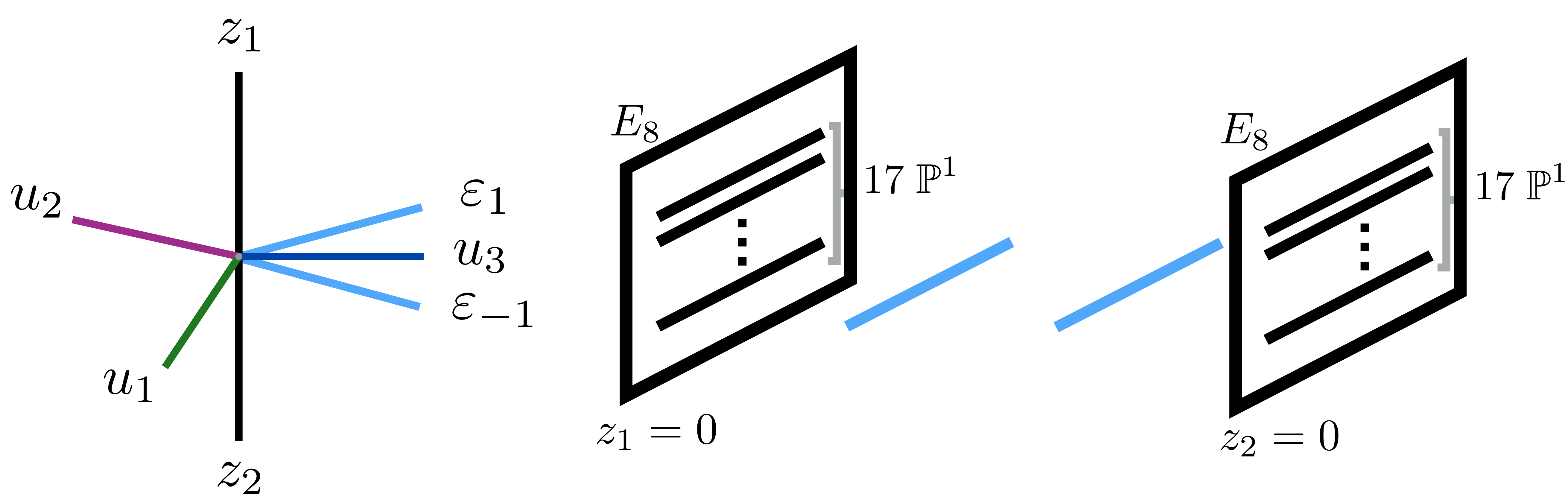

$$\varepsilon_1 = (\ 1,\ 1,\ 1,\ 2,\ 3)$$
$$\varepsilon_{-1} = (\ 1,\ 1, -1,\ 2,\ 3)$$

Example: $B_3^F = \mathbb{P}^2 \times \mathbb{P}^1$

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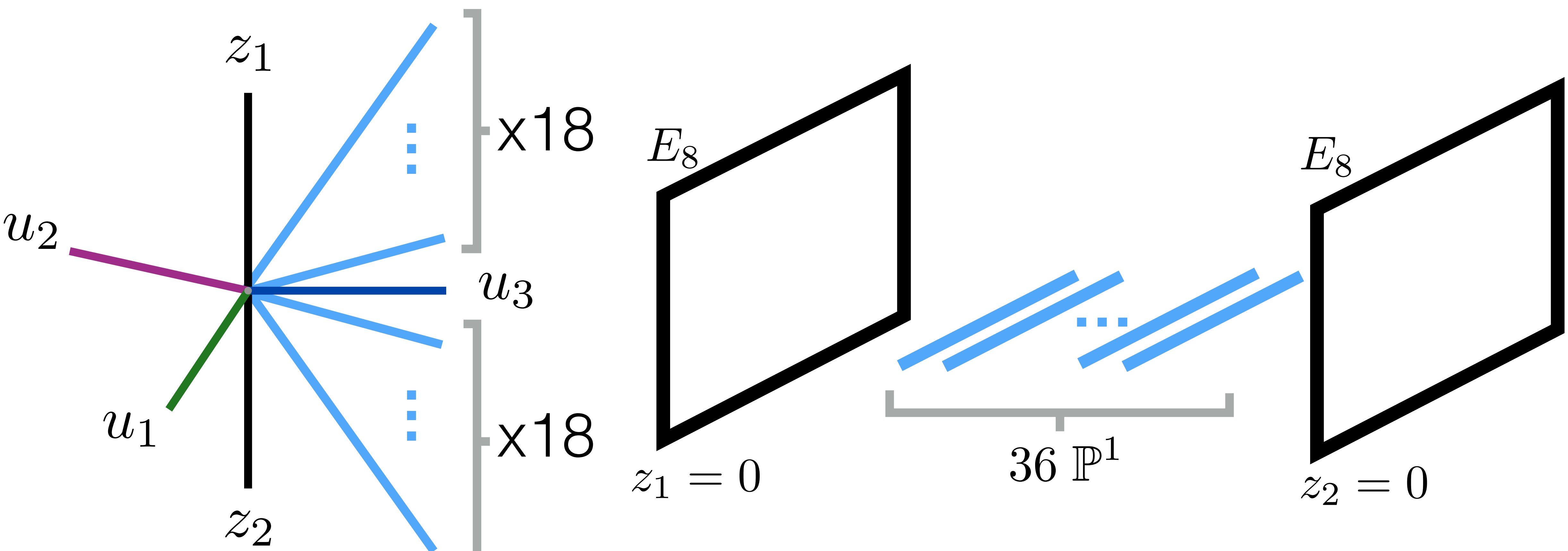
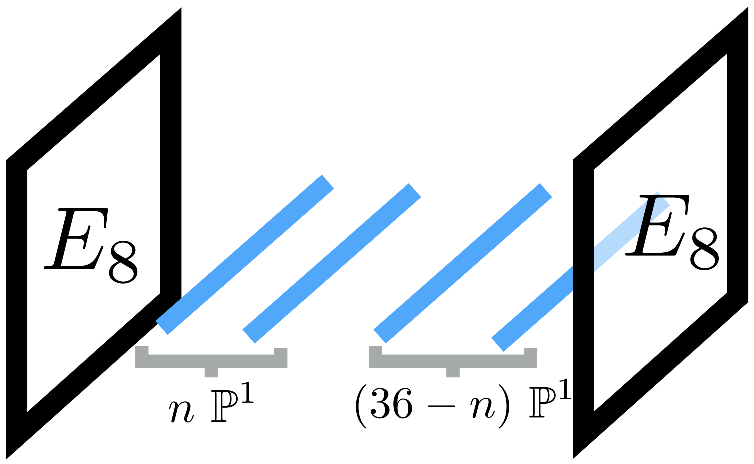


$$\varepsilon_i = (\ \ 1,\ \ 1,\ \ i,\ \ 2,\ \ 3)$$

Example: $B_3^F = \mathbb{P}^2 \times \mathbb{P}^1$

Toric blowups



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\end{aligned}
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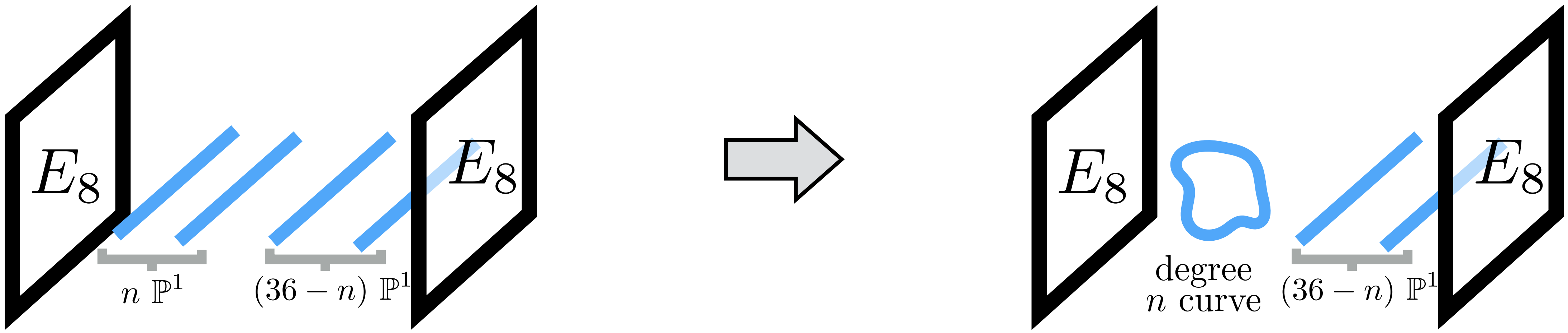
Example: $B_3^F = \mathbb{P}^2 \times \mathbb{P}^1$

Toric blowups

NEF Partitions



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u_3 &= (\ \ 1, \quad 1, \quad 0, \quad 2, \quad 3) \\
z_1 &= (\ \ 0, \quad 0, \quad 1, \quad 2, \quad 3) \\
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x &= (\ \ 0, \quad 0, \quad 0, -1, \quad 0) \\
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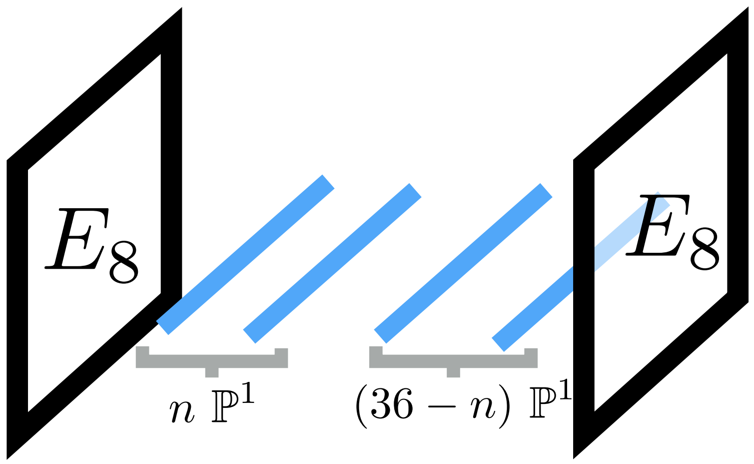
Example: $B_3^F = \mathbb{P}^2 \times \mathbb{P}^1$

Toric blowups



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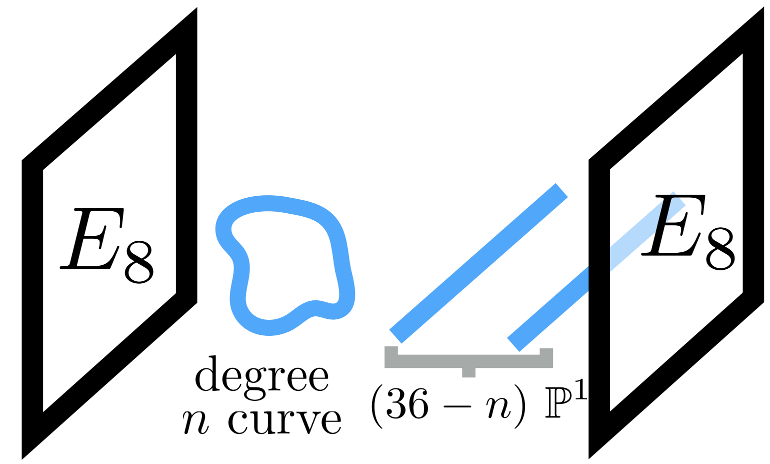
NEF Partitions

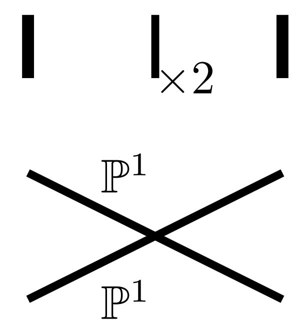


1. Add new coordinate & eqn
   $\xi = (\ 0,\ 0,\ 0,\ 0,\ 0,\ 1)$
   $\xi = p(u_1, u_2, u_3 \prod \varepsilon_i)$
2. Assign $-n$ to new direction for u_2, 0 for all others
3. Blowup $\zeta = 2x + 3y + \xi + z_1$

$$W = 0\ ,\ \xi\zeta = p(u_1, u_2, u_3 \prod \varepsilon_i)$$

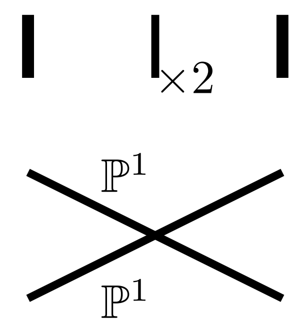
Transversely intersecting M5 branes

M-Theory

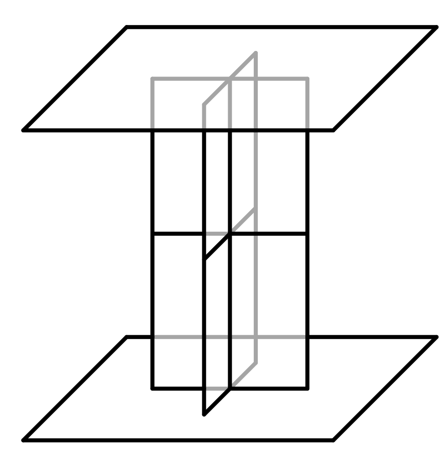


Transversely intersecting M5 branes

M-Theory



F-Theory



Transversely intersecting M5 branes

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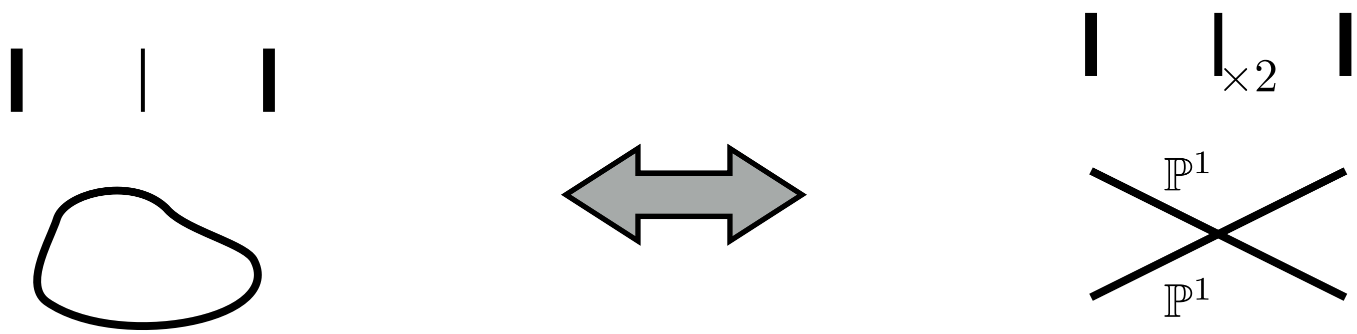

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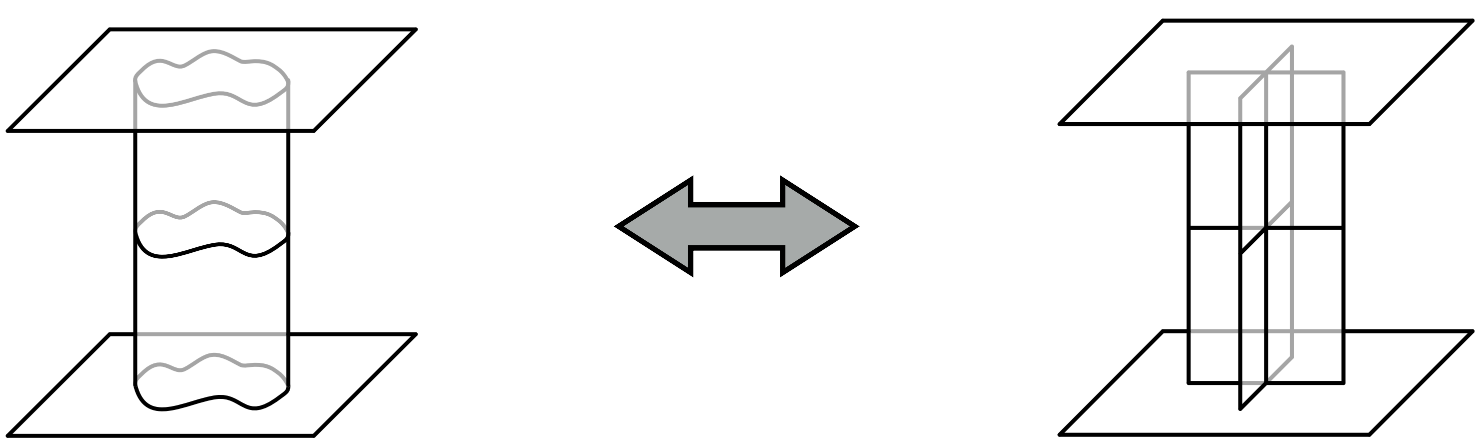

Transversely intersecting M5 branes

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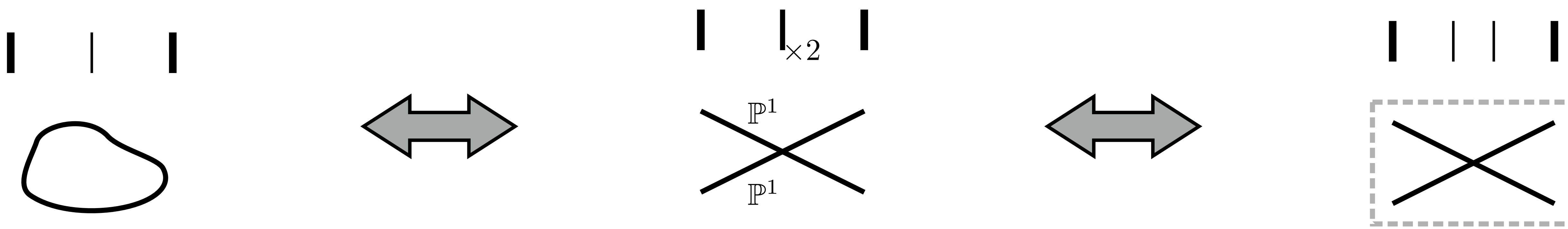

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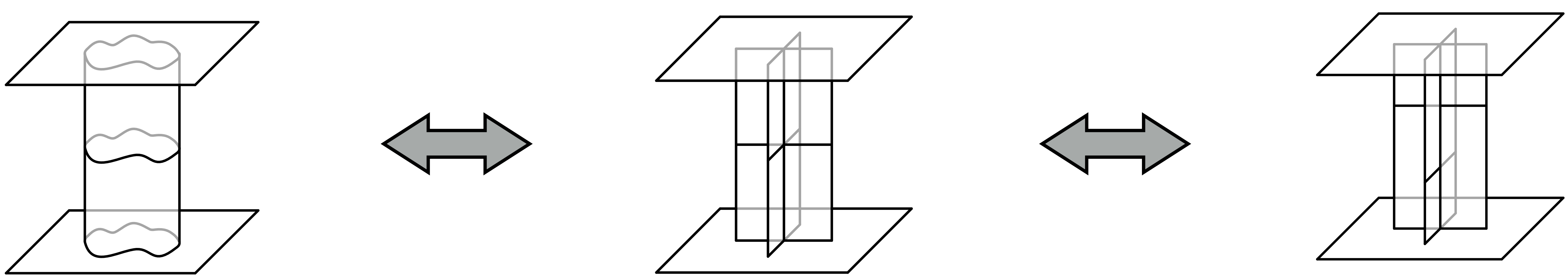
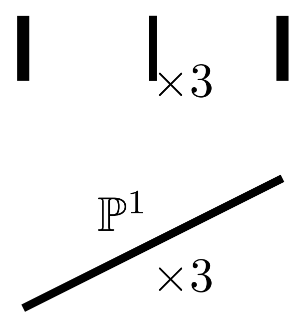


Coincident M5 branes

M-Theory



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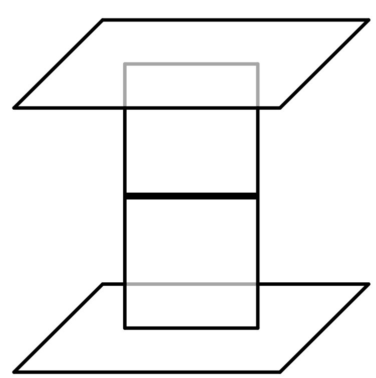

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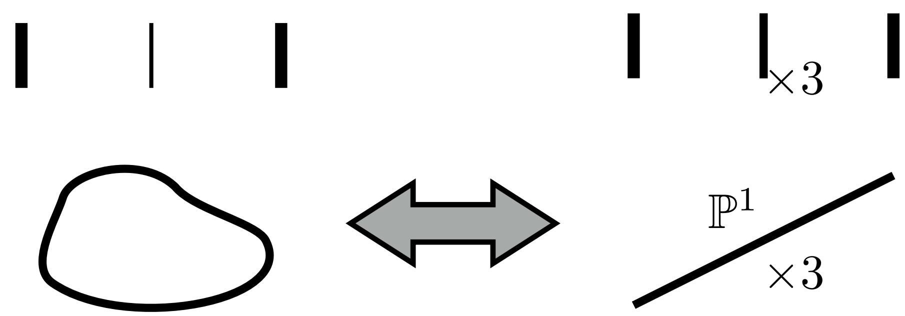

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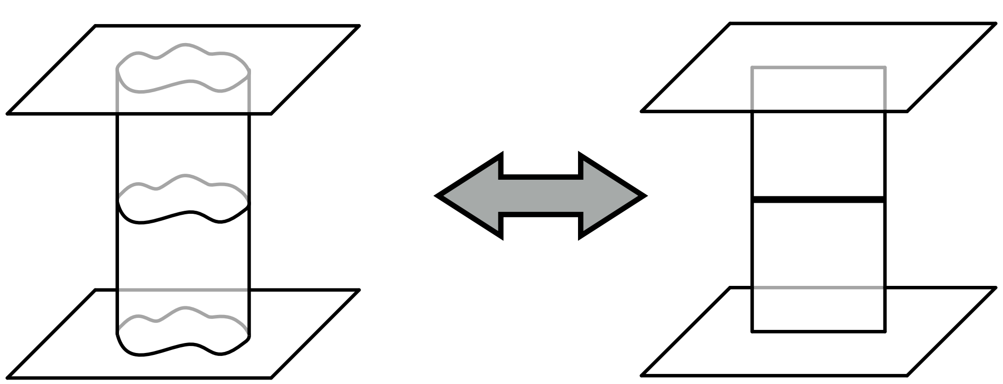

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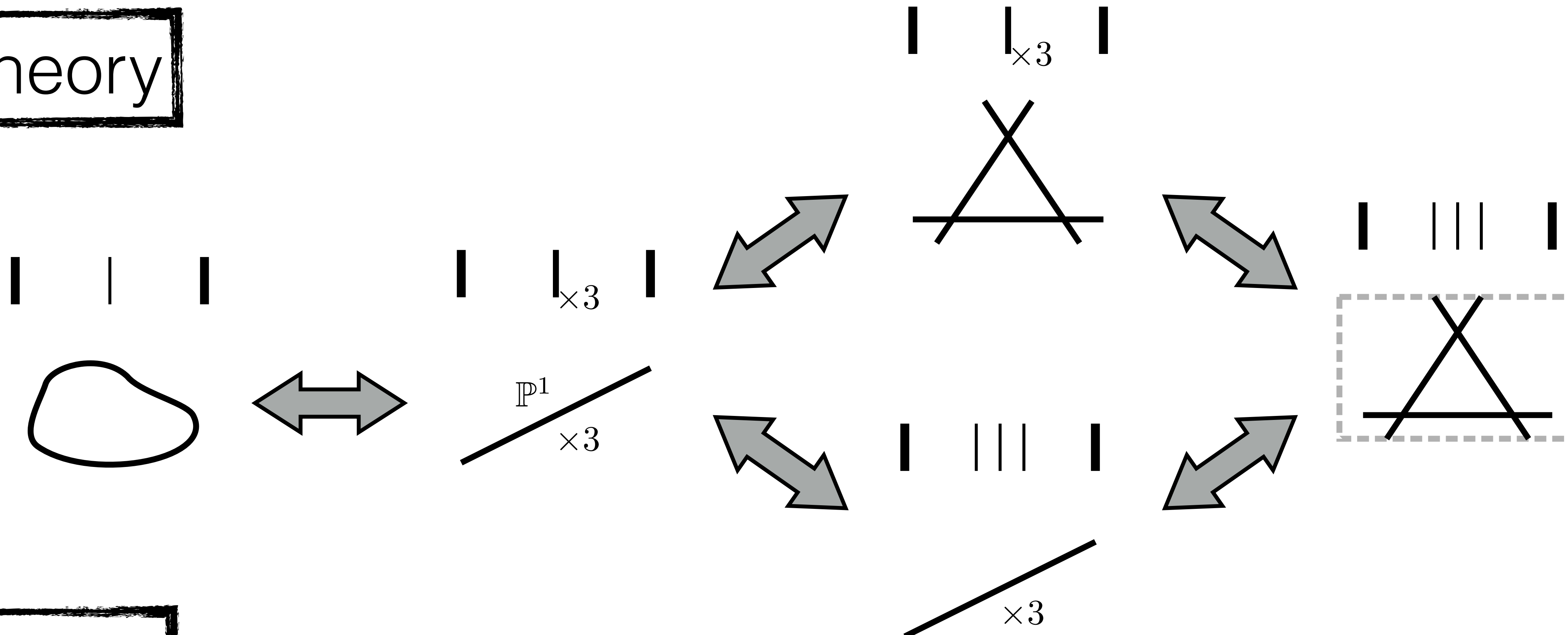

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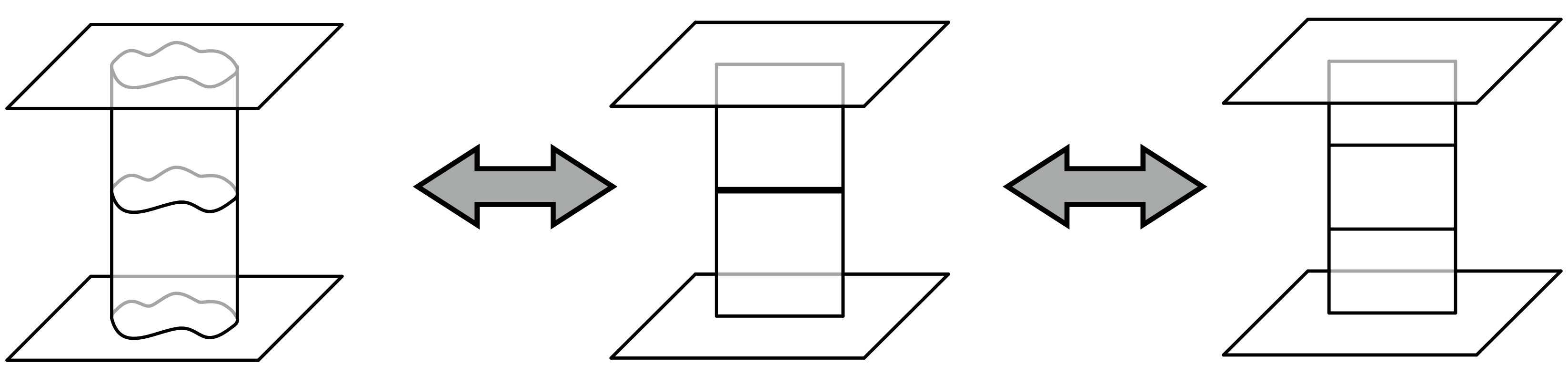

Coincident M5 branes

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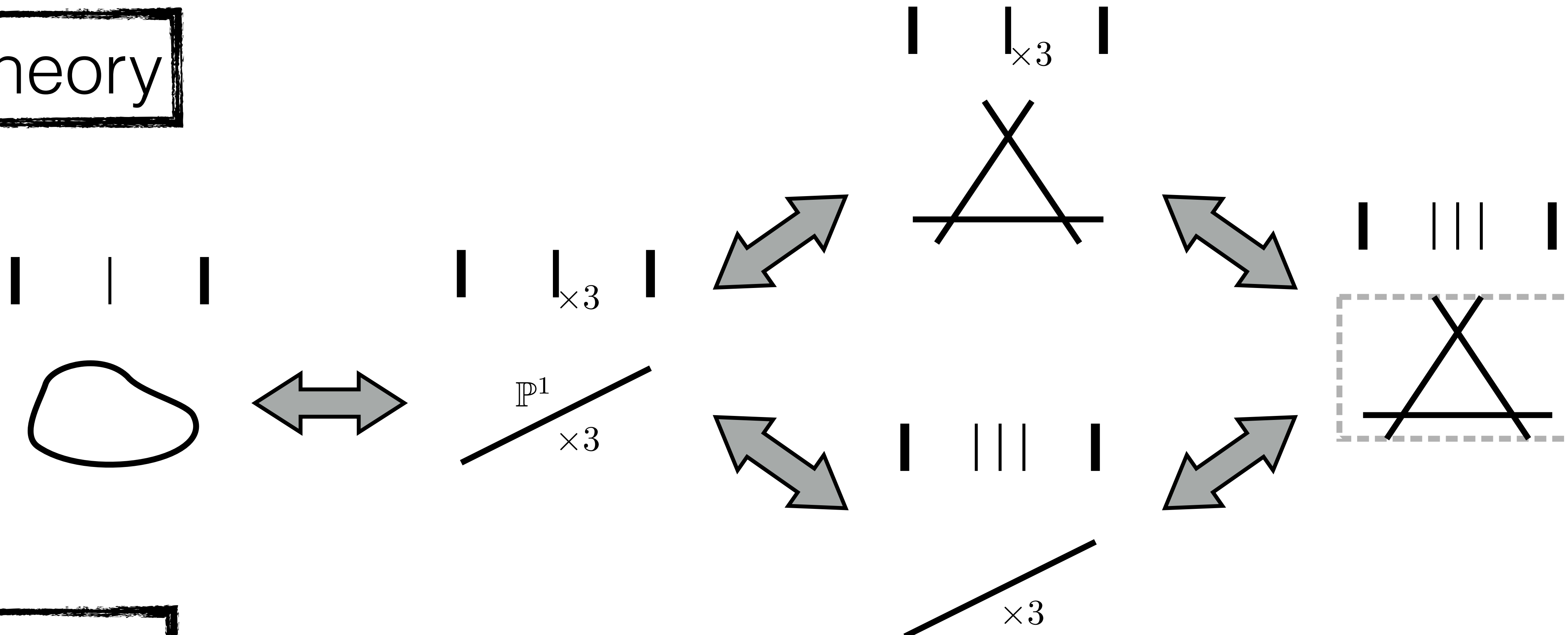

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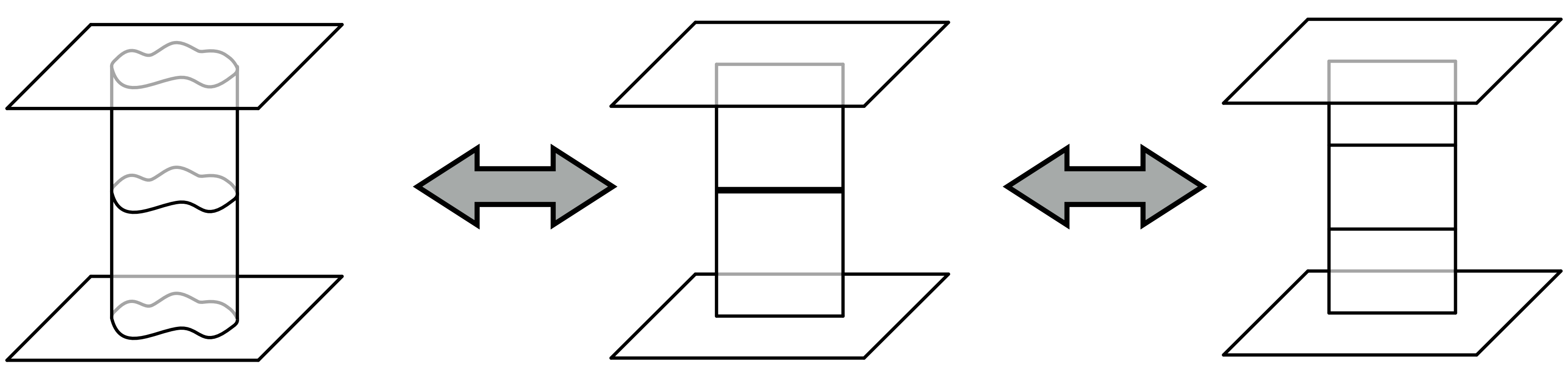


$U(1)^{g_{\rm def}}$ $\mathrm{SU}(N) + \mathrm{CFT?}$ $U(1)^{Ng}$

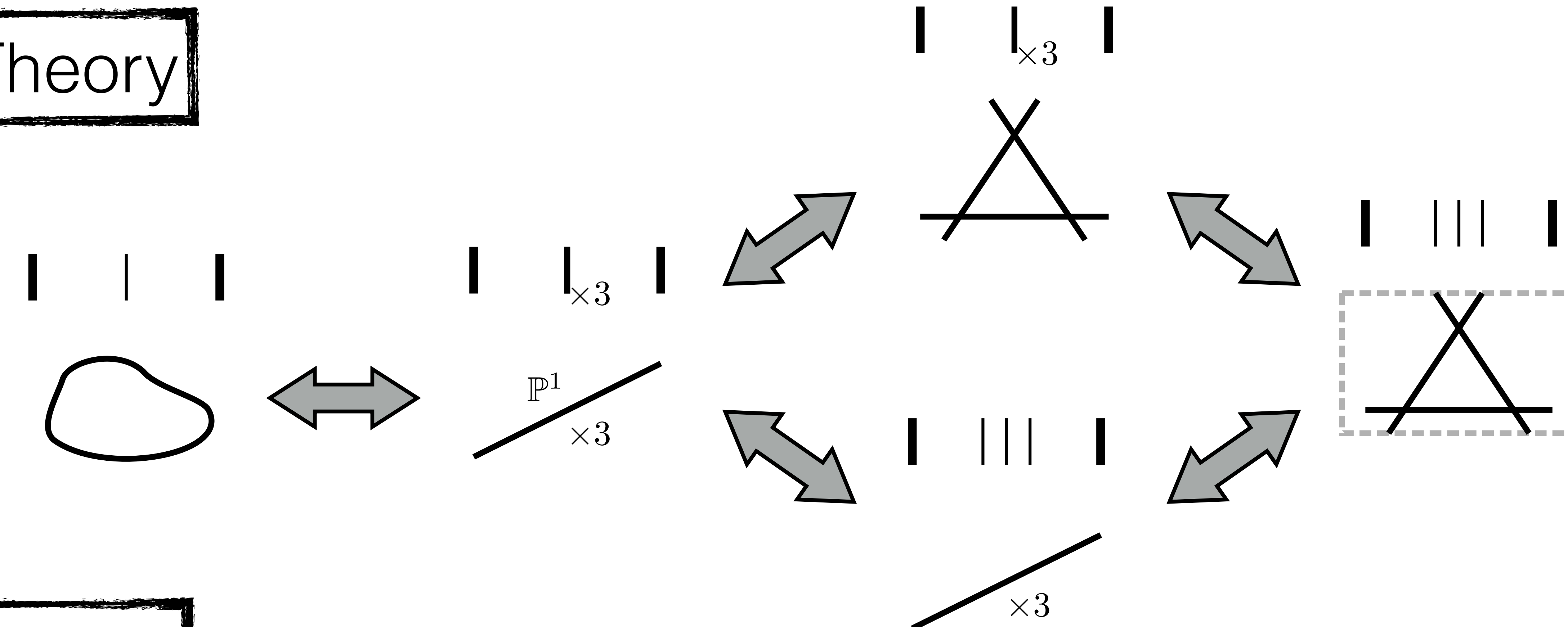
Match spectra for M-theory on Y_4 with surface of sing.

[Jockers,Katz, Morrison,Plesser`16]

$\Rightarrow$ paper

Coincident M5 branes

M-Theory



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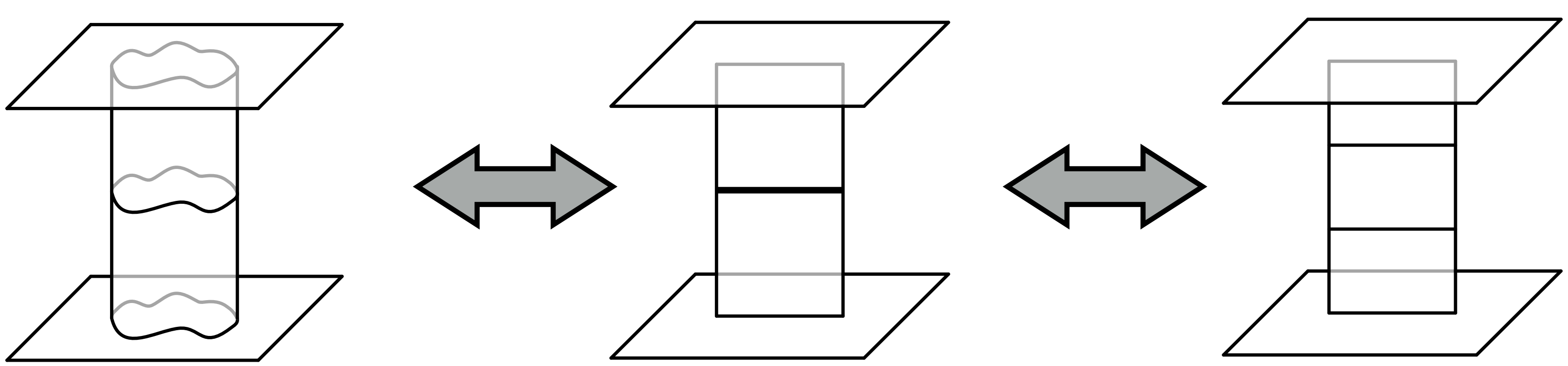
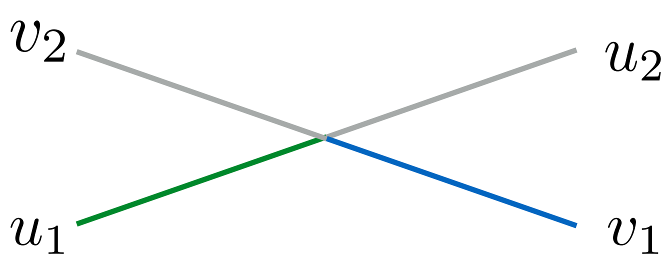
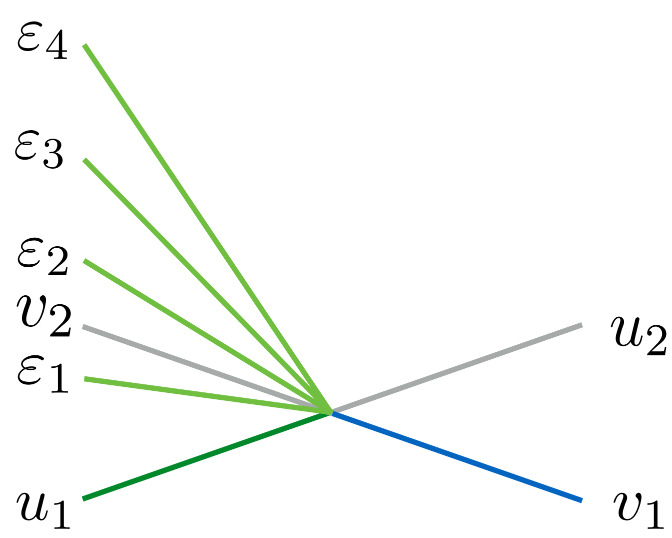


Where does information about position in the interval go in F-Theory?

M5 brane resolutions/deformations



M5 brane resolutions/deformations



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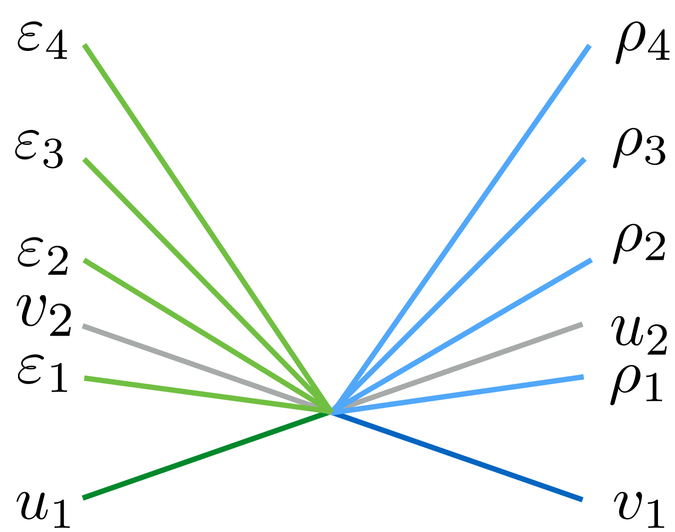

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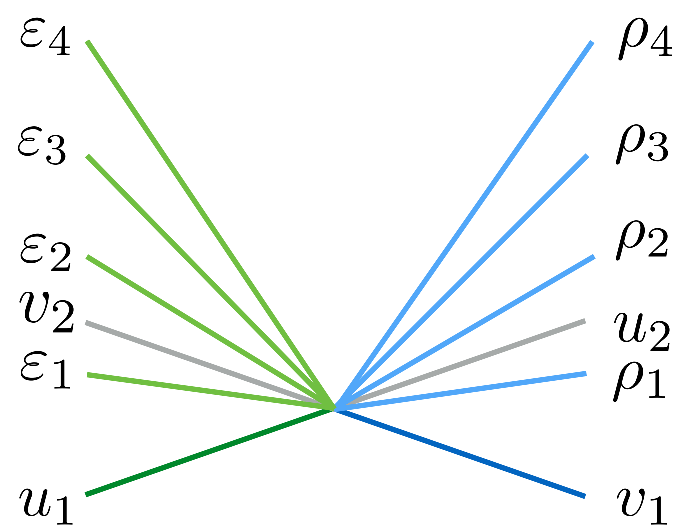
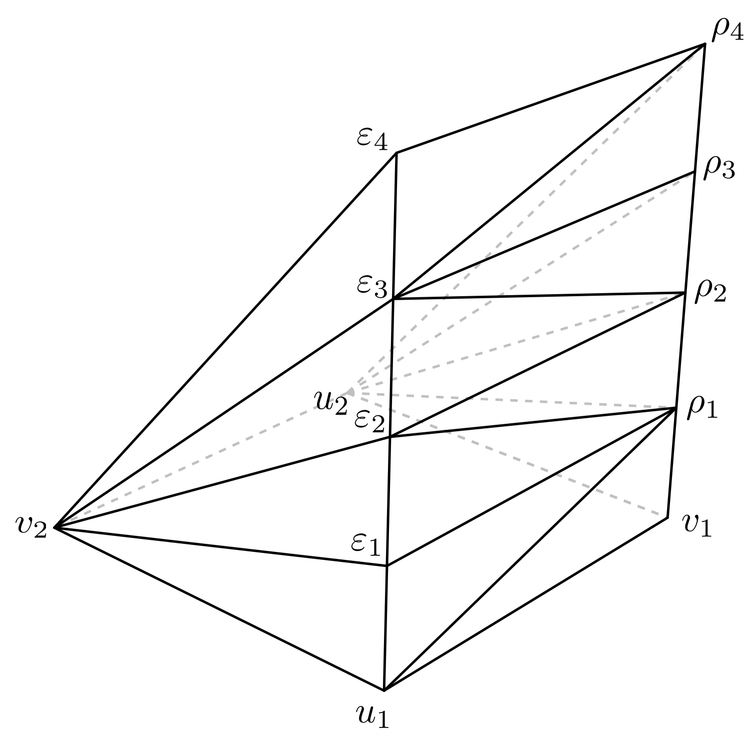
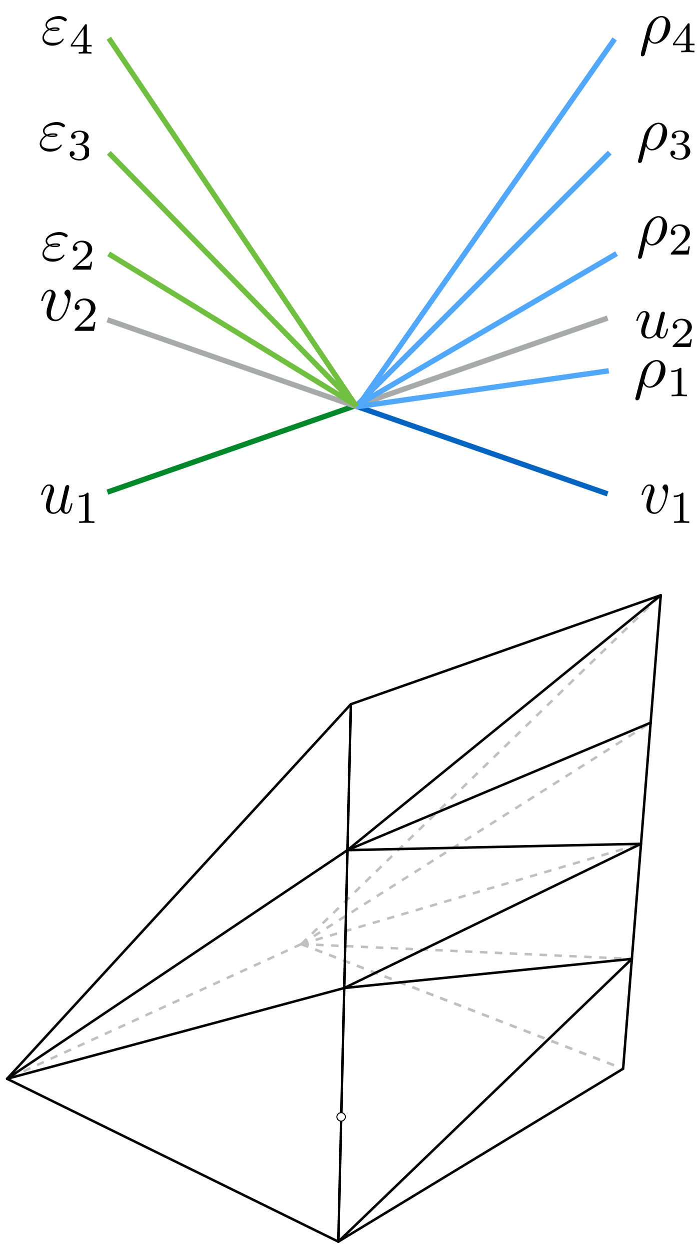


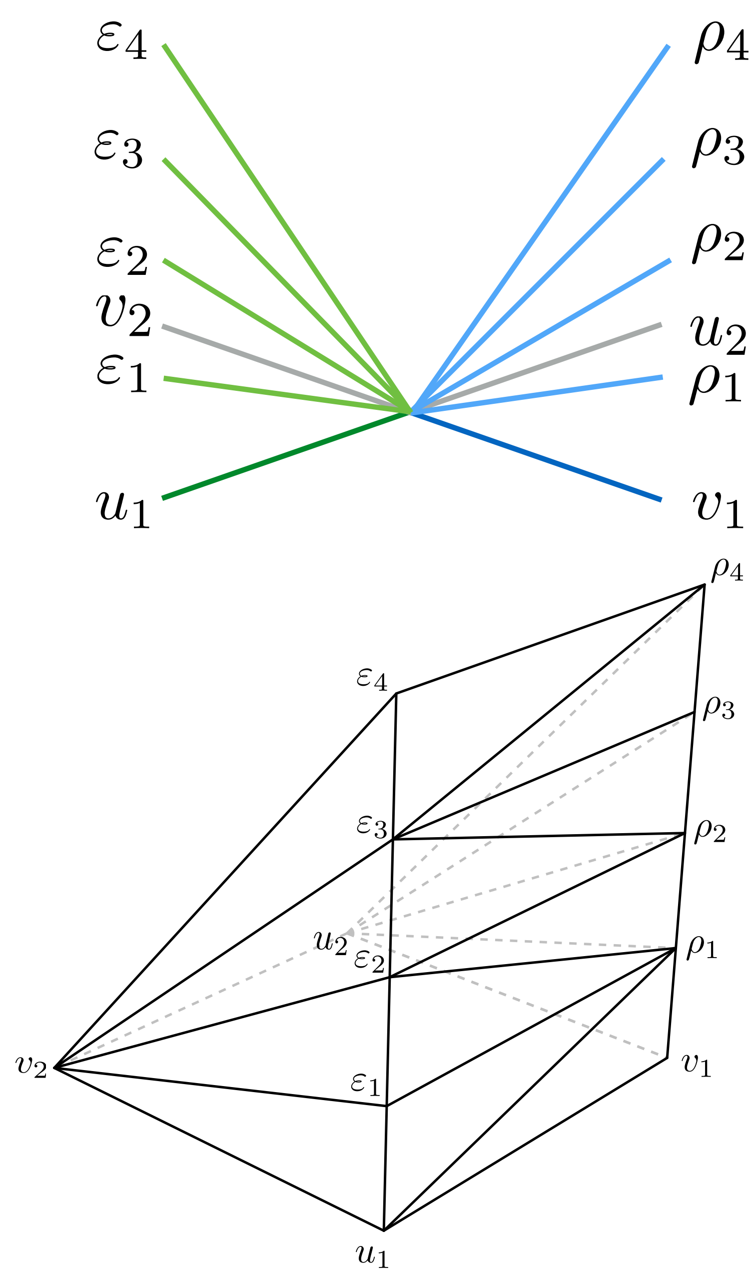


some M5 brane configuration

M5 brane resolutions/deformations



Coincident M5 brane



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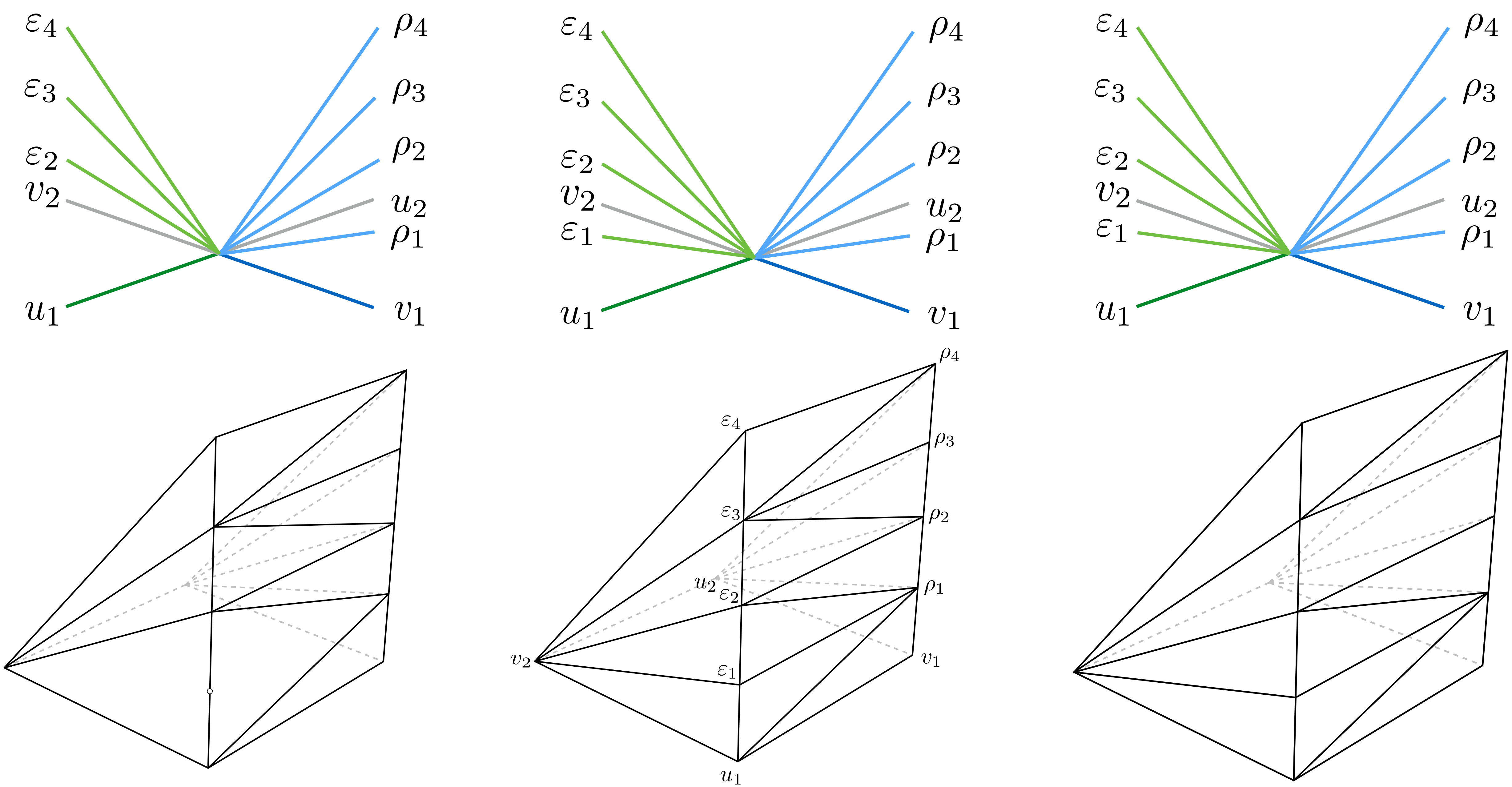

Coincident M5 brane

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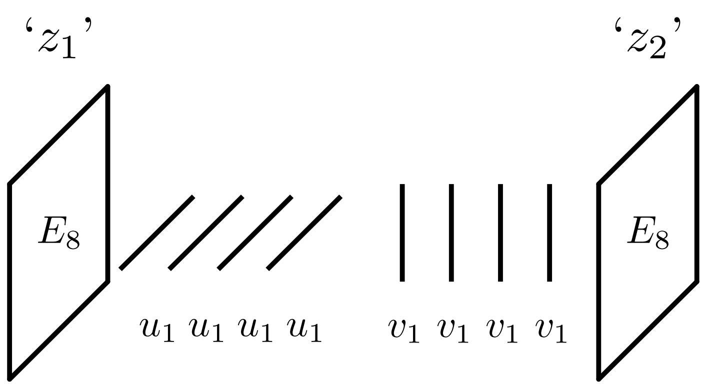
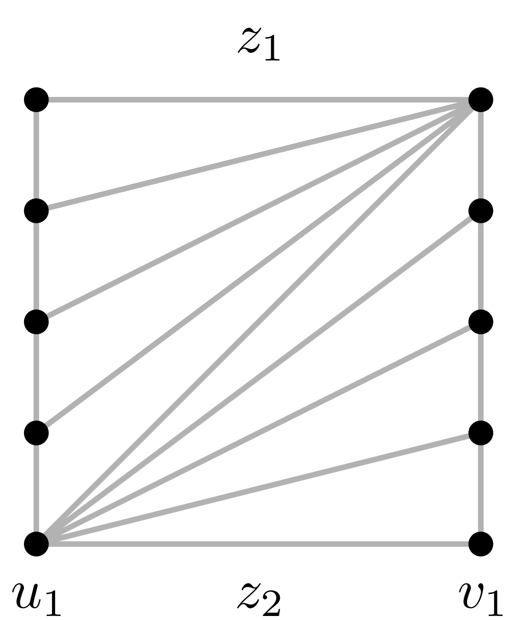
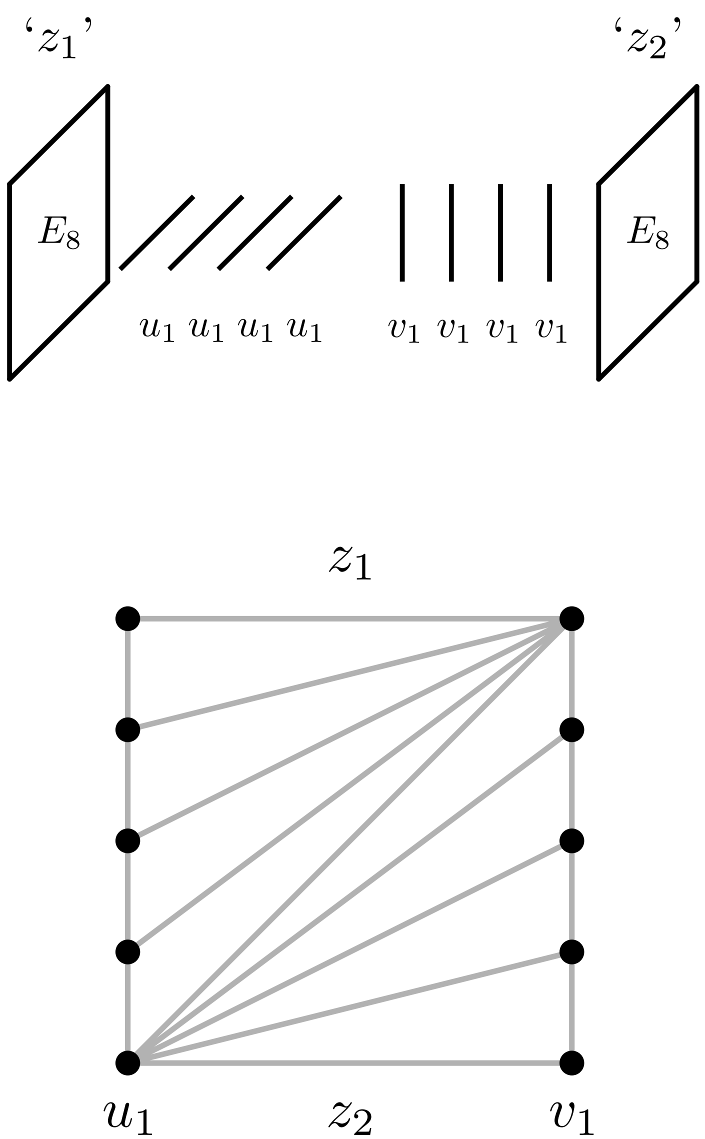
Transversely inter-secting NS5 branes

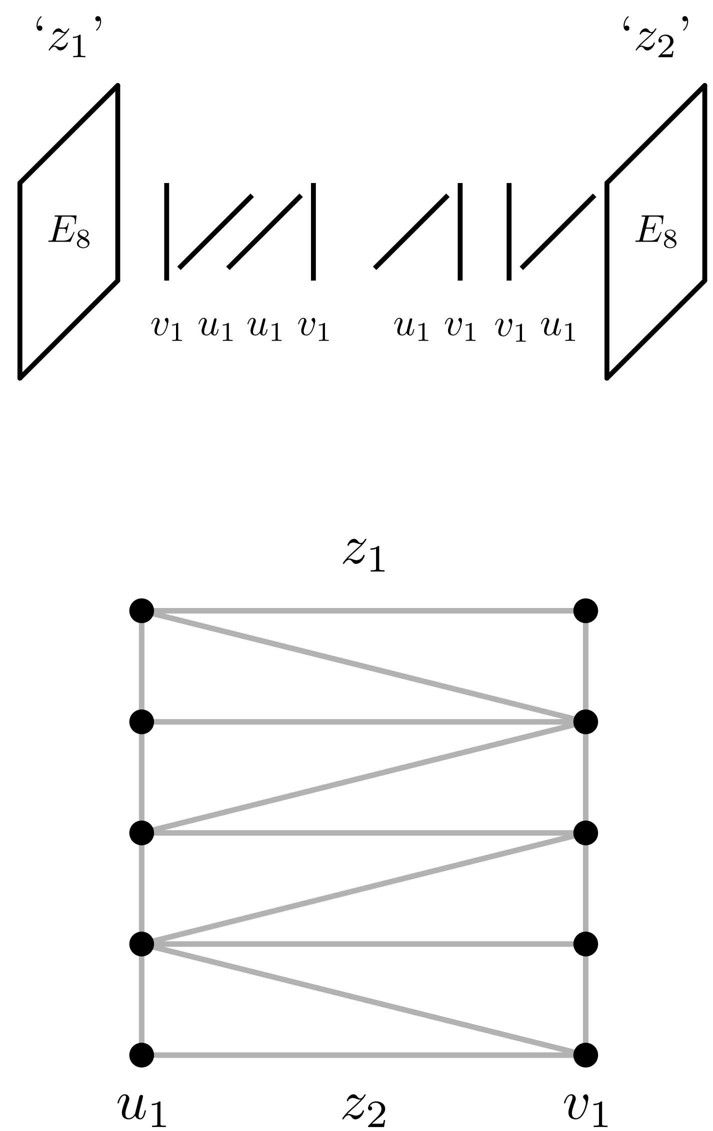
M5 brane resolutions/deformations



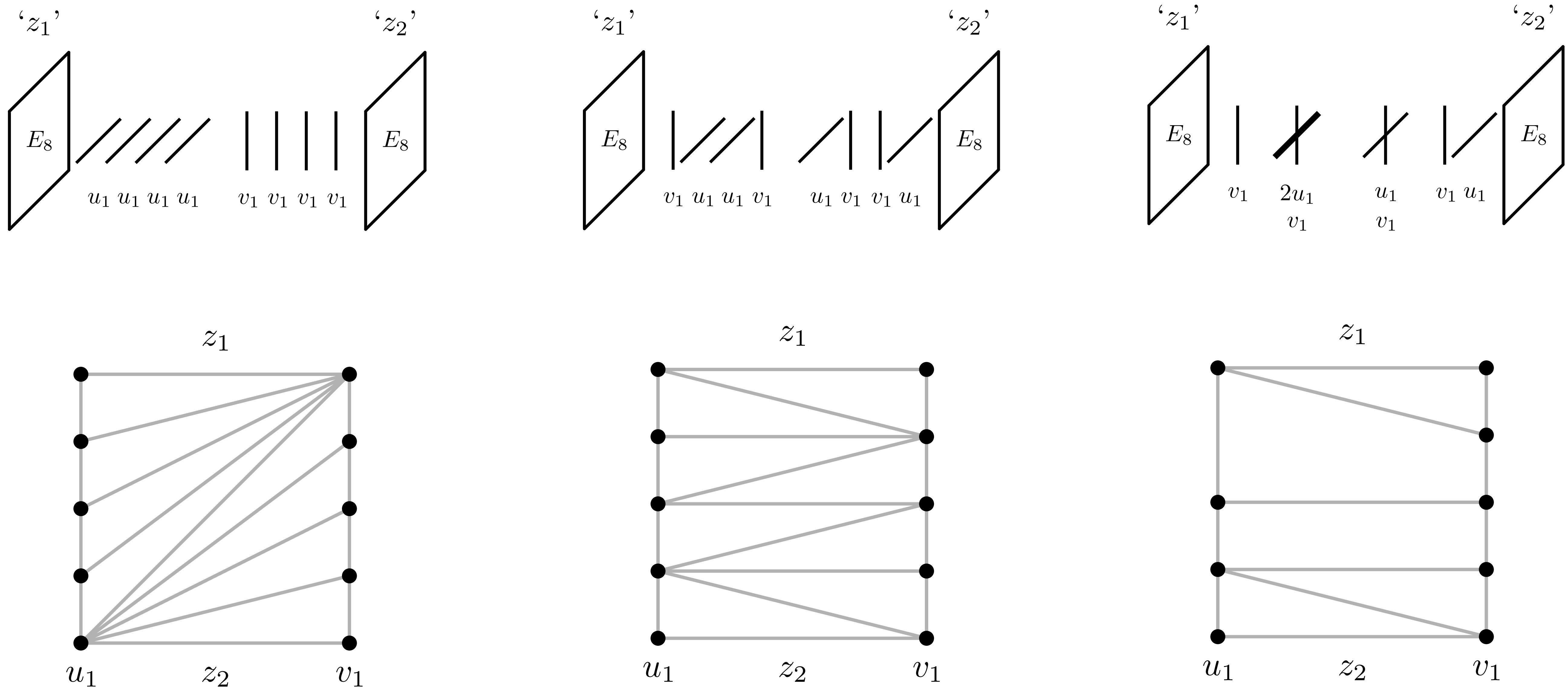


M5 brane resolutions/deformations





M5 brane resolutions/deformations



Conclusions

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- Het/F-Theory duality with line bundles
  - Gauge group broken by flux rather than geometry in spectral cover
  - Flat on fiber:
    - Non-chiral models
    - Need horizontal NS5 branes
  - Match tadpole, spectra, stability

Conclusion

- Het/F-Theory duality with line bundles
  - Gauge group broken by flux rather than geometry in spectral cover
  - Flat on fiber:
    - Non-chiral models
    - Need horizontal NS5 branes
  - Match tadpole, spectra, stability
- NS5 branes
  - Describe by toric geometry / NEF partitions
  - Match M-/F-Theory picture of NS5 branes
    - Explained map of chirals and vectors to geometry
    - M5 brane position in bulk $\leftrightarrow$ triangulation

Thank you for your attention!