Anomaly conditions and an infinite swampland in 6D U(1) + SUGRA theories

Geometry and Physics of F-theory 2018, Madrid

March 7, 2018

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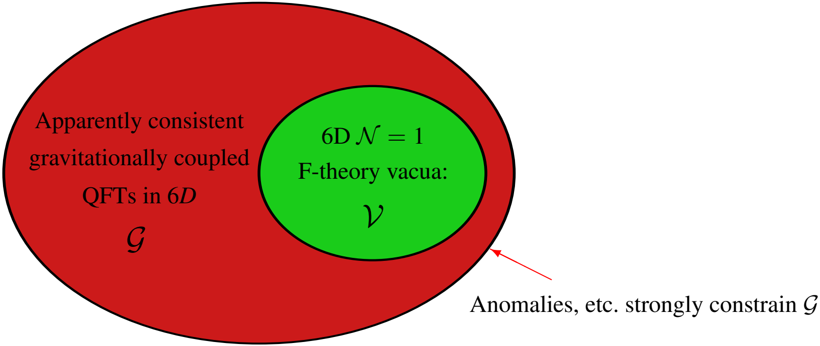
Based on:

arXiv: 1803.nnnnn	A. Turner, WT
arXiv: 1711.03210	N. Raghuram

Goal: Map the landscape and swampland for 6D $\mathcal{N} = 1$ SUGRA



Program: systematically analyze $\mathcal{G}$ for 6D $\mathcal{N} = 1$ SUGRA

Find "swampland" of apparently consistent theories w/o F-theory realization

If $x \in \mathcal{G} \setminus \mathcal{V}$, must indicate one of
$$\begin{cases} \text{a) new string construction: } \mathcal{V}_* \supset \mathcal{V} \\ \text{b) new low-E constraint: } \mathcal{G}_* \subset \mathcal{G} \\ \text{c) true stringy constraint } \mathcal{V}_* \subset \mathcal{G}_* \end{cases}$$

$\mathcal{G}_* = \mathcal{V}_* \Rightarrow$ "String universality" [cf. Garcia-Extebarria talk]

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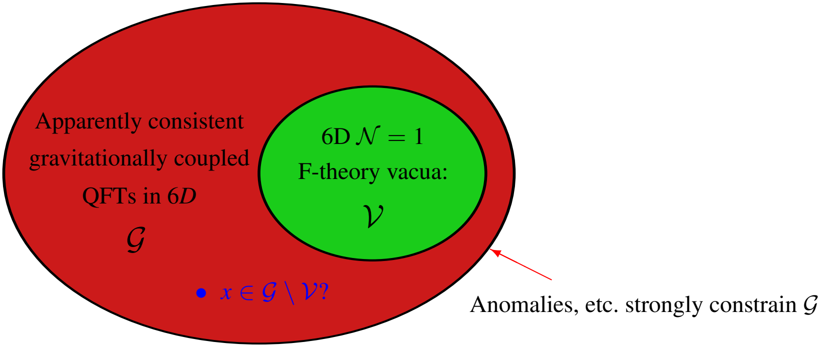

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Study U(1) models with charges $q_1, q_2, \ldots$

Compare, for nonabelian groups, $T < 9$:
— Finite number of anomaly-free spectra,
— Good fraction from F-theory, under relatively good control
— Some issues with exotic matter (e.g. [Klevers/Morrison/Raghuram/WT])

U(1) anomaly conditions ($a, \tilde{b}$ anomaly coefficients for BRR, BFF)

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\begin{aligned}
-a \cdot \tilde{b} &= \frac{1}{6} \sum q_i^2 \\
\tilde{b} \cdot \tilde{b} &= \frac{1}{3} \sum q_i^4
\end{aligned}
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For $T = 0$ models: very simple Diophantine equations

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Simple case: $T = 0, q = 1, 2$

U(1) models:

$$\tilde{b}\left(24 - \tilde{b}\right) \times (\pm\mathbf{1}) + \frac{1}{4}\tilde{b}\left(\tilde{b} - 6\right) \times (\pm\mathbf{2})$$

where $6 \leq \tilde{b} \leq 24, \tilde{b} \in 2\mathbb{Z}$

Compare SU(2) models with fundamentals, ≥ 1 adjoint

$$2b(12 - b) \times \square + \frac{1}{2}(b-1)(b-2) \times \square\square$$

- 1-1 match, SU(2) $\rightarrow$ U(1) by Higgsing, $\tilde{b} = 2b$
- All U(1) models from Morrison-Park
- All SU(2) models from simple Tate/UFD construction on degree b

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Matching for larger q?

No! Exist infinite families of anomaly-free solutions.

$$54 \times (\pm q) + 54 \times (\pm r) + 54 \times (\pm(q+r)), \quad \tilde{b} = 6\left(q^2 + qr + r^2\right), \quad q, r \in \mathbb{Z}$$

Another family:

$$54 \times (\pm a) + 54 \times (\pm b) + 54 \times (\pm c) + 54 \times (\pm d), \quad \tilde{b} = 12\left(m^2 - mn + n^2\right)^2$$

$$\begin{aligned} a &= m^2 - 2mn, \\ b &= 2mn - n^2, \\ c &= m^2 - n^2, \\ d &= 2\left(m^2 - mn + n^2\right). \end{aligned}$$

Asymptotics: $\tilde{b} \sim \sum q^2, \tilde{b}^2 \sim \sum q^4 \to \mathcal{O}(\tilde{b}^{(m-4)/2})$ w/ m distinct q's

Surprising: finite # from F-theory, finite nonabelian spectra

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Look at examples

Various models with $q = 1, 2, 3$ at $T = 0$ (245 spectra)

Which from F-theory?

— Many (199) unHiggsable to SU(2) w/ 3-symmetric matter (some DKRT)
— Some unHiggsable to SU(3), Higgs on 2 adjoints, $\tilde{b} = 6b$
— Some realized by non-UFD $q = 3$ construction
[Klevers/Pena/Oehlmann/Piragua/Reuter '14, Raghuram '17]

- Cannot definitely rule any out since don't know general F-theory form
- Similar for $q = 4$, only a few F-theory models known [Raghuram]
- Can get $q = 6$ in principle from Higgsing SU(6) on quartic

No specific model from the infinite set of anomaly-free solutions is known to be impossible in F-theory, though a cofinite set are impossible

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Some interesting constraints from UFD/Morrison-Park models

UFD F-theory construction of SU(2) models

$$\begin{aligned} f &= -\frac{1}{48}\phi^2 + f_1\sigma + f_2\sigma^2 \\ g &= \frac{1}{864}\phi^3 - \frac{1}{12}\phi f_1\sigma + g_2\sigma^2\,, \end{aligned}$$

since $[f_1] = -4a - b$, must have $b \leq -4a$

Compare anomaly constraint: $b \cdot b \leq -4a \cdot b$.

Similarly, Morrison-Park $\Rightarrow \tilde{b} \leq -8a$

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[w/ Raghuram, Turner, Wang]

Only clear bound for F-theory:

Kodaira bound for SU(N):

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(Follows from $\Delta \propto \sigma^N, \Delta \in -12K$)

Examples of U(1) + charges $\to$ SU(2) violate bound (Definite SU(2) Swamp!)

Example: $T = 1, x_1 = 0, x_2 = 150$ U(1) charges

UnHiggs to SU(2) with 76 adjoints, no fundamentals,
$b = (5, -20), -a = (2, 2)$

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Further $T > 0$ constraints: even $\tilde{b}$, positive cone

From Morrison-Park, Higgsing of SU(N), $\tilde{b}$ must be even

Less clear in low energy theory, but basically quantization of U(1) instantons

Monnier/Moore/Park: with mild assumptions: proved $\tilde{b} \in 2\Gamma$

Further issue: role of positivity cone in low energy theory

Example: $T = 1$, two possible unimodular lattices

$$\Omega_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \Omega_0 = U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Both arise in F-theory.

$\mathbb{F}_0 : \Omega = \Omega_0$, cone generators (1, 0), (0, 1)
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For Ω_0, could have $-a = (2, 2)$ or (4, 1); only (2, 2) in F-theory

Can we understand aspects of the positivity cone from low-energy?
E.g. -3 curve $\rightarrow$ SU(3)?

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Open Questions

- What combinations of U(1) charges q arise in F-theory? What is the maximum Q?
- Is there physical significance in low energy theory for UFD/MP models and constraints $b \leq -4a, \tilde{b} \leq -8a$
- Can we prove the Kodaira SU(N) bound $b \leq -12a/N$ from low-energy? Constraints on relationship between F^2, R^2 terms?
- Other low-energy conditions to limit q's, maximum Q?
- U(1)'s in presence of SU(N)'s? (cf. Lawrie/Schafer-Nameki/Wong, Cvetic/Lin)

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