# Reggeon dynamics from a Monte Carlo perspective

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In collaboration with A. Sabio Vera

**Phys. Rev. D94 (2016) no.3, 034019; arXiv:1801.04872 [hep-th]** and ongoing work

21 March 2018

LHC Forward Physics 2018, 20-23 March, IFT UAM-CSIC Madrid, Spain

# Outline

- Introduction
- The Reggeon
- The Pomeron - BFKL equation
- The Odderon - BKP equation
- Iterative numerical solutions
- Results
- Outlook

# Intro: Setting up the stage

- Perturbative QCD
- High energy scattering
- BFKL equation (Balitsky-Fadin-Kuraev-Lipatov)
- BKP equation (Bartels-Kwiecinski-Praszalowicz)
- Pomeron (Named after Pomeranchuk)
- Odderon (Lukaszuk & Nicolescu)
- Reggeon

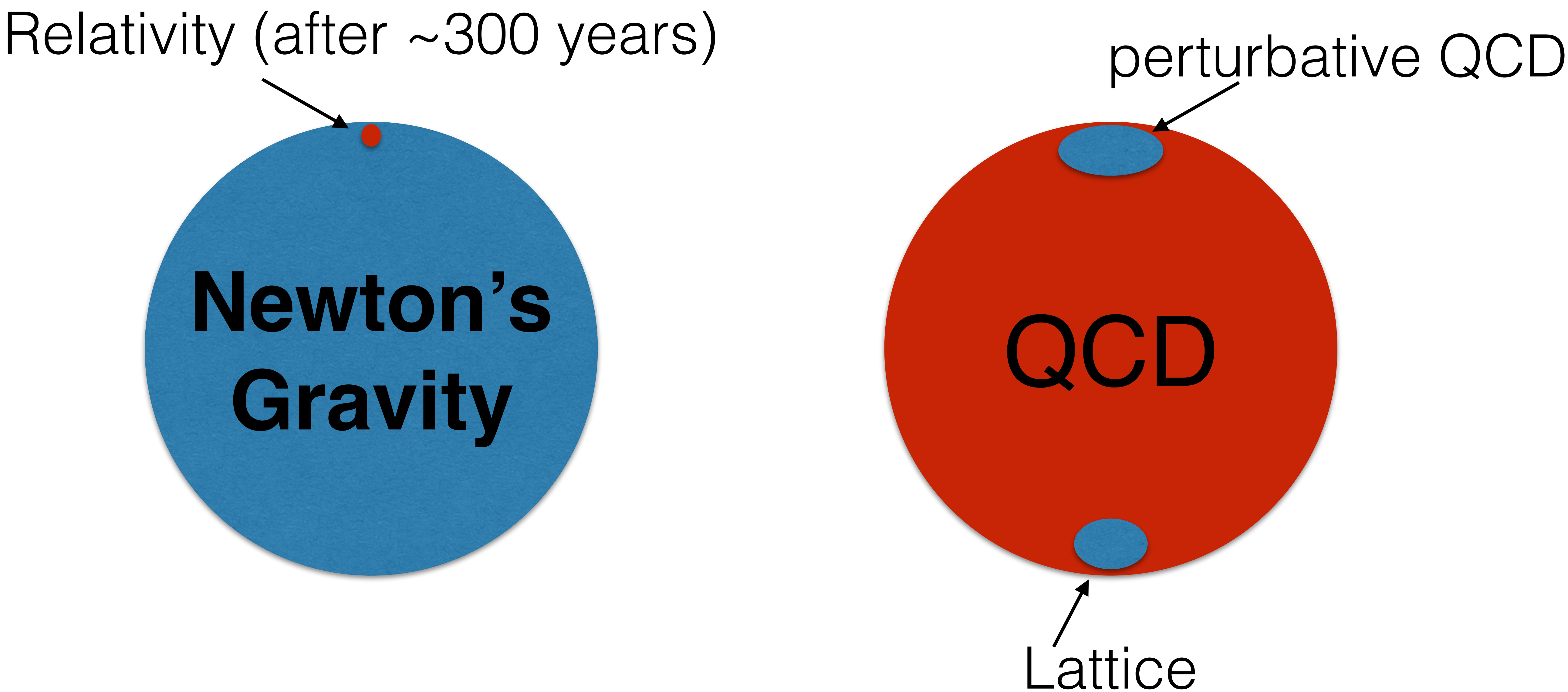
# Intro: Importance of high energy QCD

- High energy QCD studies only a part of the phase space, a certain limit, the limit of scattering at very high energies, however, there is a plethora or things we access from studying that limit:

  - Integrability
  - Gravity
  - AdS/CFT
  - BDS amplitudes
  - Factorization
  - Separation between transverse and longitudinal d.o.f.
  - Transition from hard to soft scale physics

- And this is only from the 'pure' theory point of view

# Because after all…

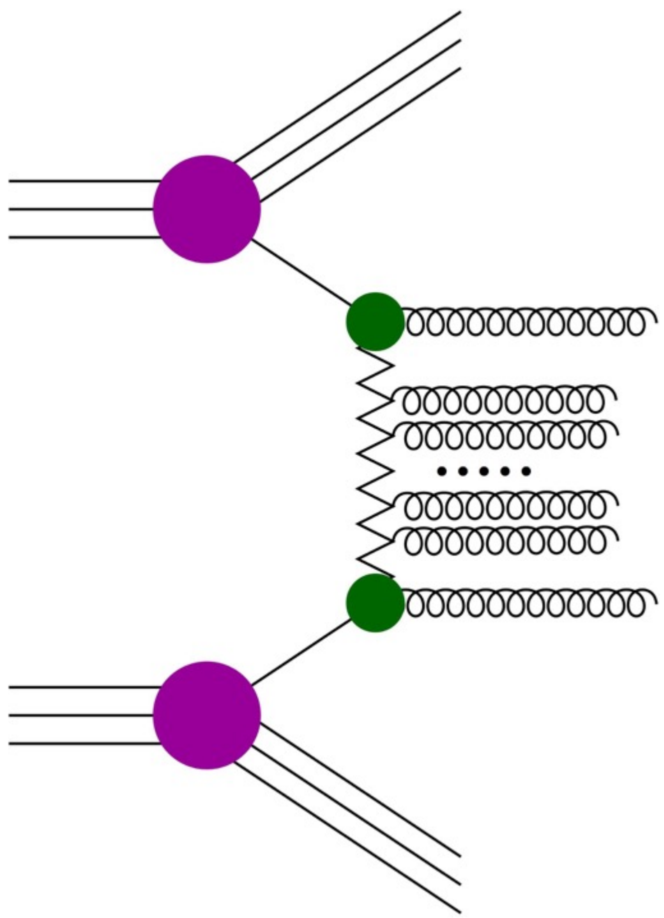



Blue phenomena we describe, red phenomena we do not

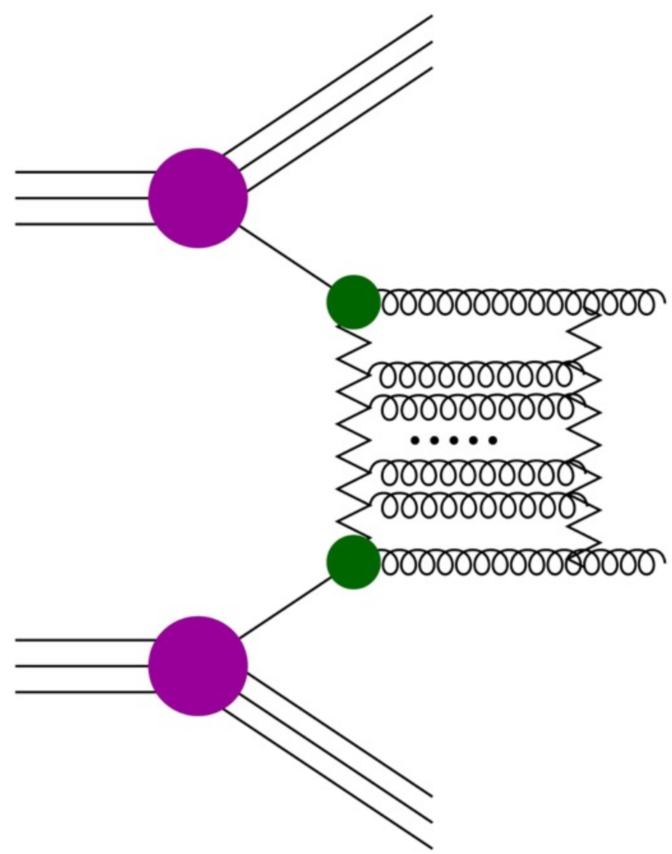
# Intro: Importance of high energy QCD

Rich phenomenology, e.g.

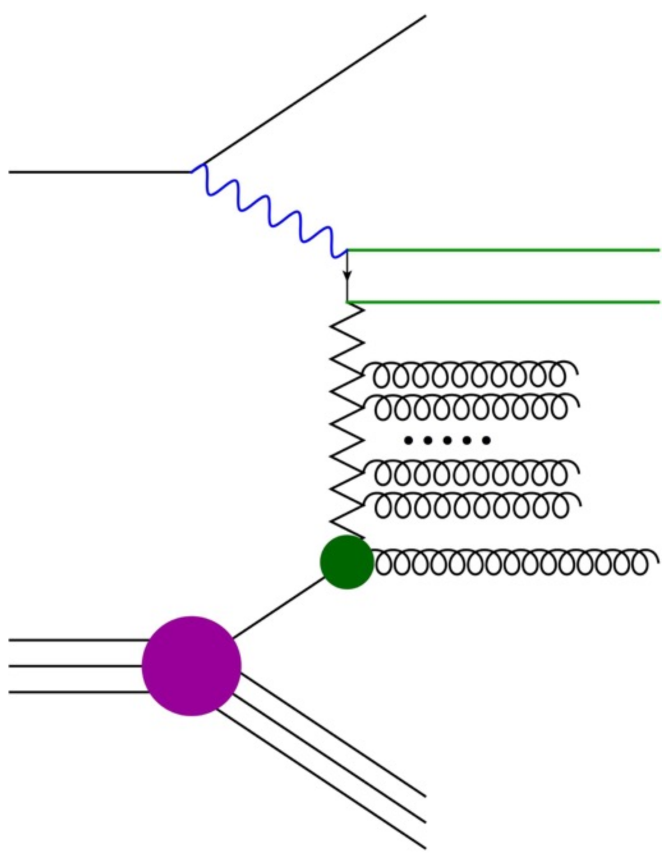
Multijets



rapidity gaps



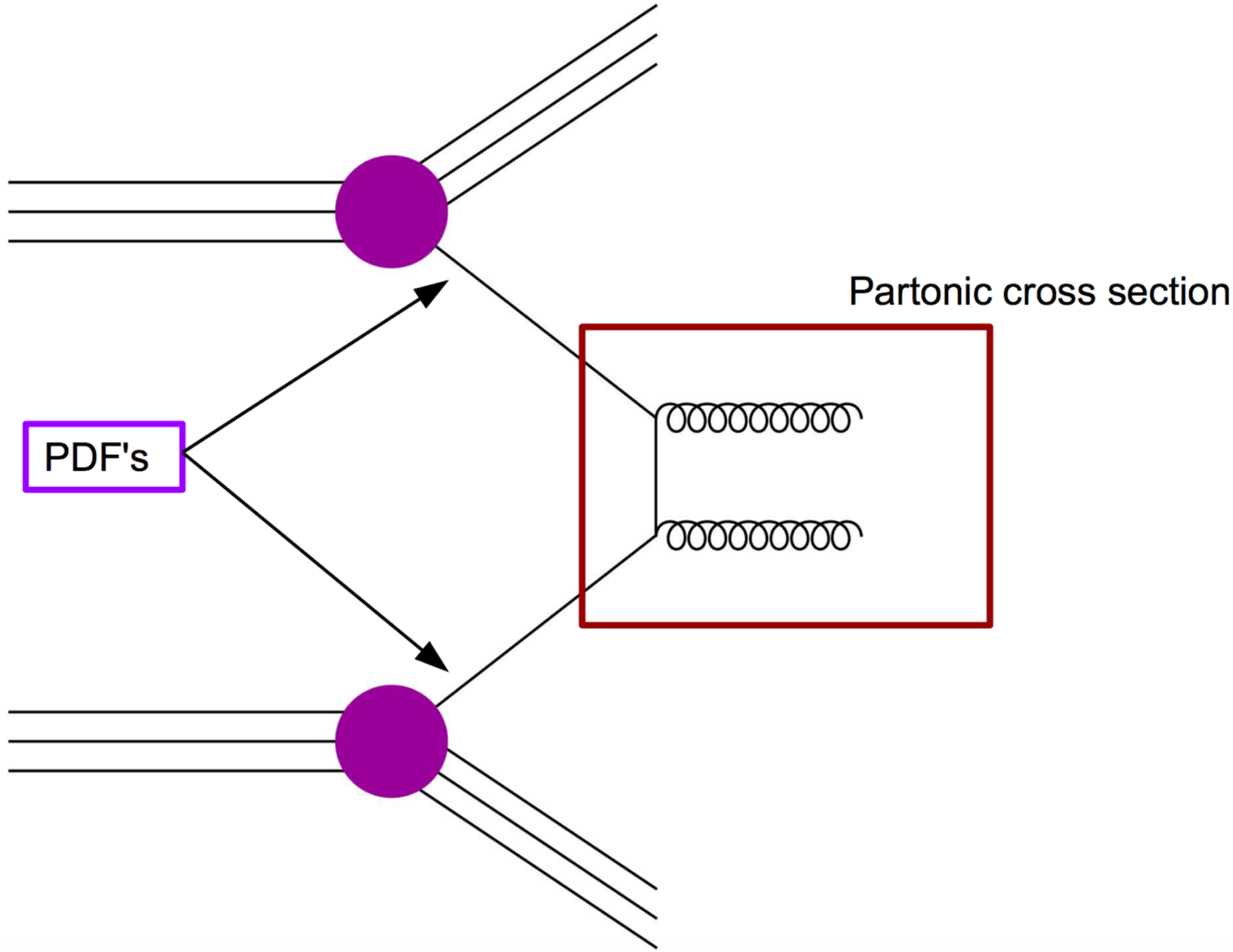
DIS



# Intro: Partonic vs Hadronic cross-section

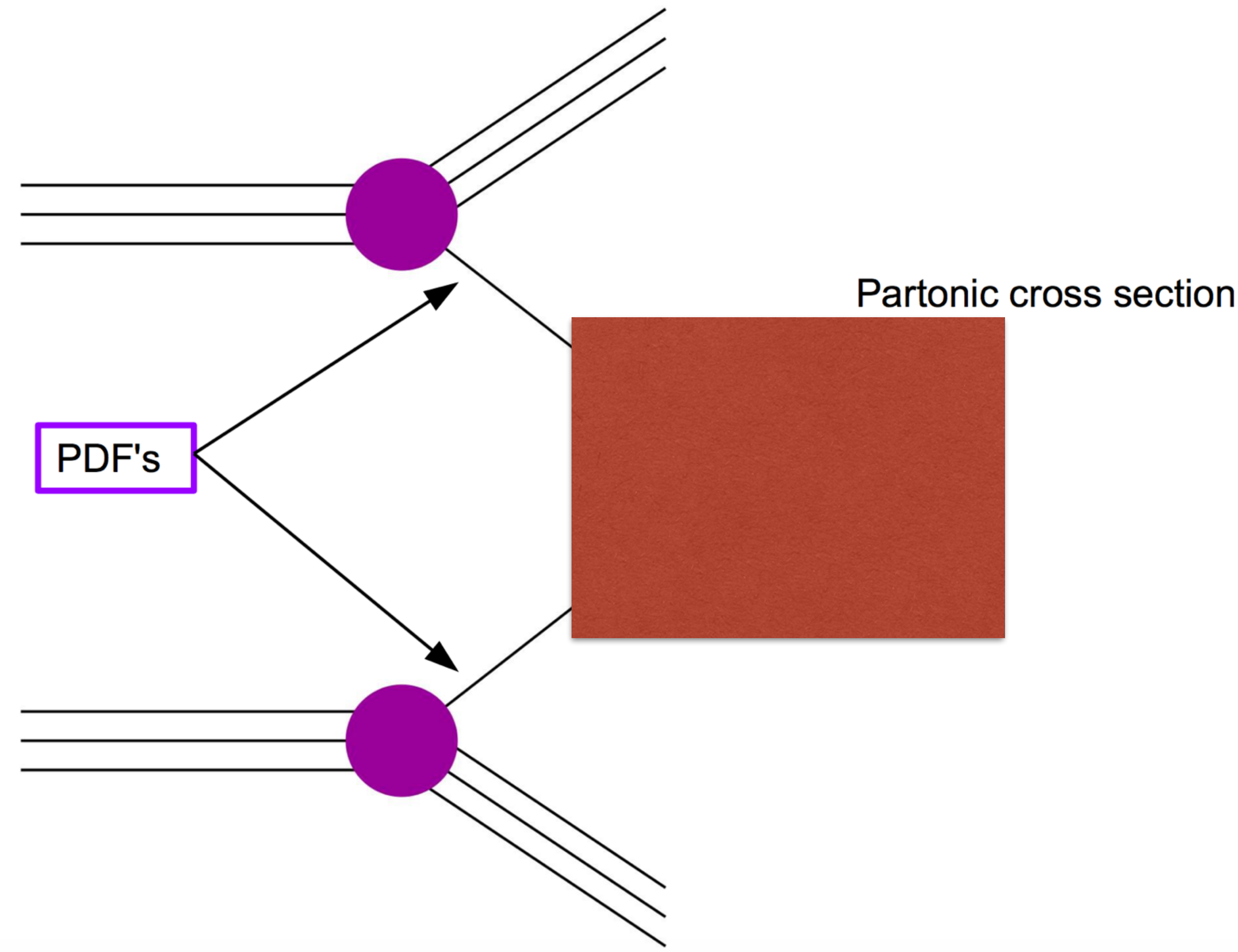
**Collinear factorization scheme**

In a hadronic collider things are complicated, one needs to consider the partonic cross-section and convolute that with the PDF's in order to produce theoretical estimates for an observable

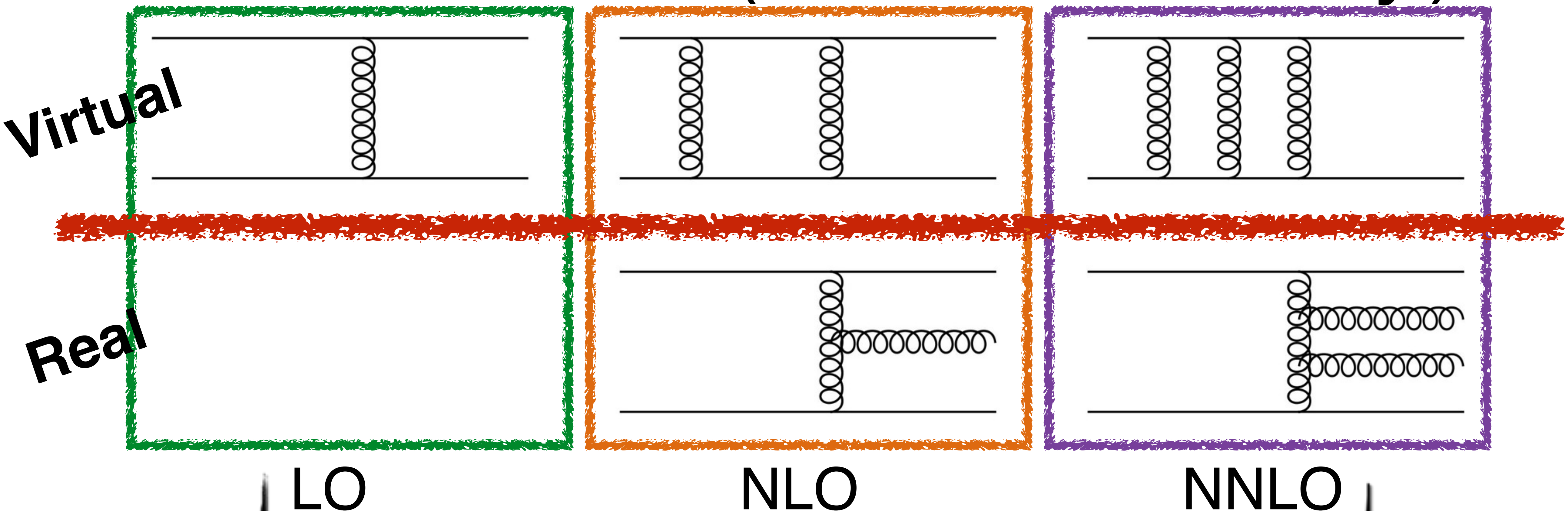


# Intro: Partonic vs Hadronic cross-section

This talk is focused on what is happening in the orange (which is the new black) box



# Intro: Higher order corrections (schematically)

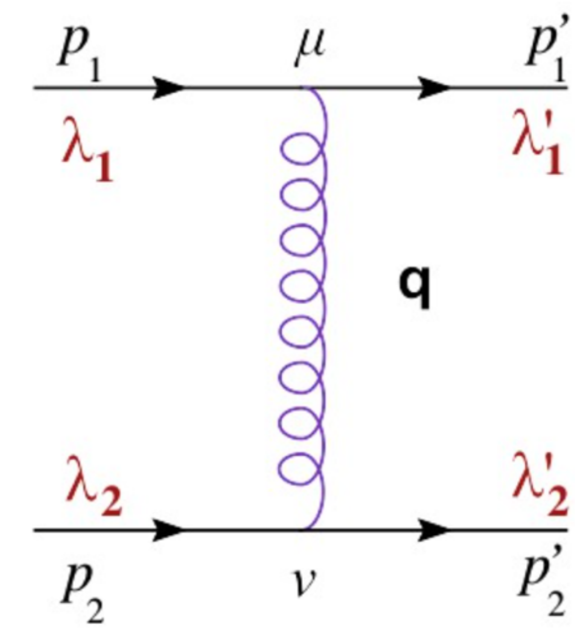



**In which case the perturbative expansion breaks down?**
**What happens in high energy scattering?**
**How is the partonic part of the cross section modified?**

FIND large logarithms in energy

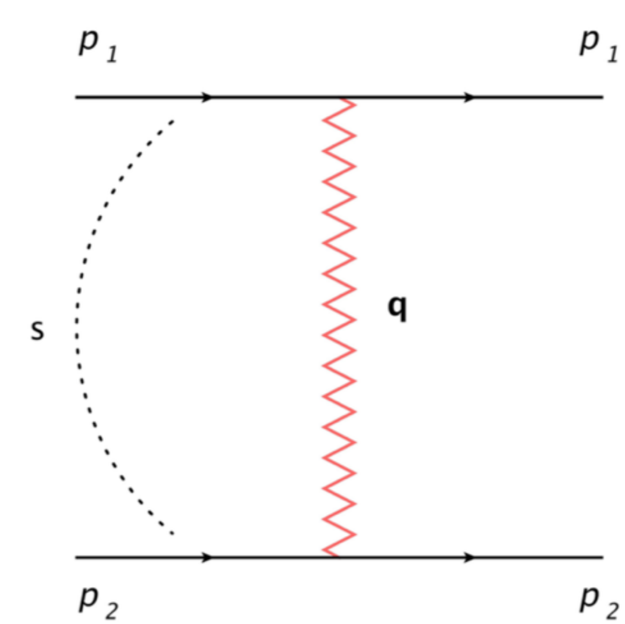
# Large logs from virtual corrections

A normal gluon propagator: $D_{\mu\nu}(s, q^2) = -i\dfrac{g_{\mu\nu}}{q^2}$




The reggeized gluon is a gluon with modified propagator:

$$D_{\mu\nu}(s, q^2) = -i\frac{g_{\mu\nu}}{q^2} \boxed{\left(\frac{s}{\mathbf{k}^2}\right)^{\omega(q^2)}}$$

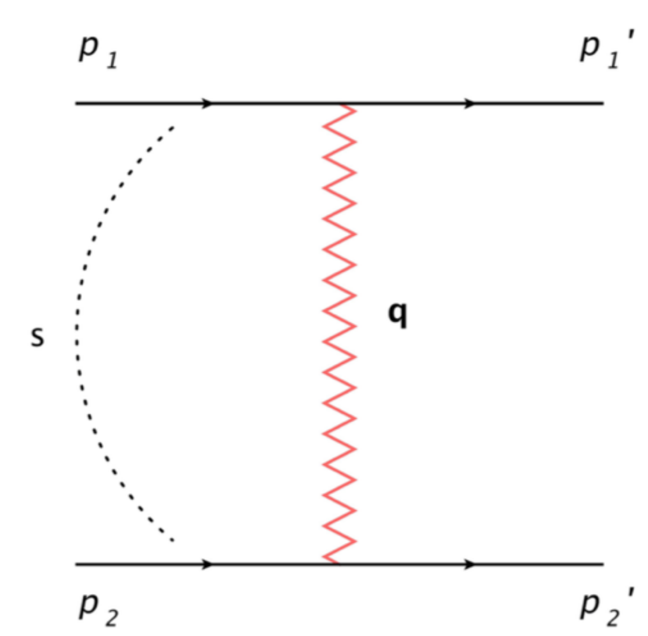


# The Reggeon

*All the <u>virtual corrections</u> that carry leading-logs in s are accounted for*

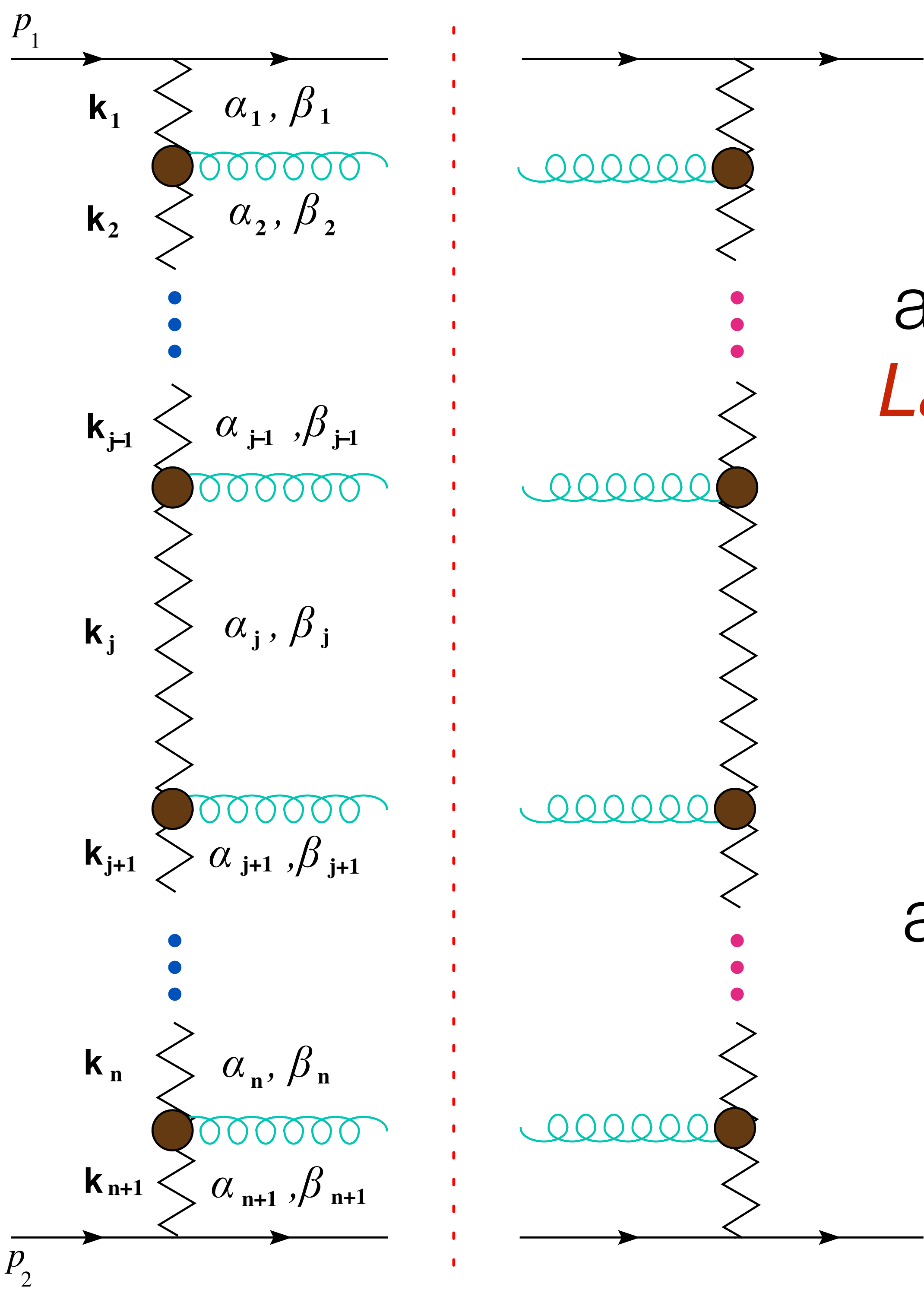
The reggeized gluon is a gluon with modified propagator:

$$D_{\mu\nu}(s,q^2) = -i\frac{g_{\mu\nu}}{q^2}\boxed{\left(\frac{s}{\mathbf{k}^2}\right)^{\omega(q^2)}}$$

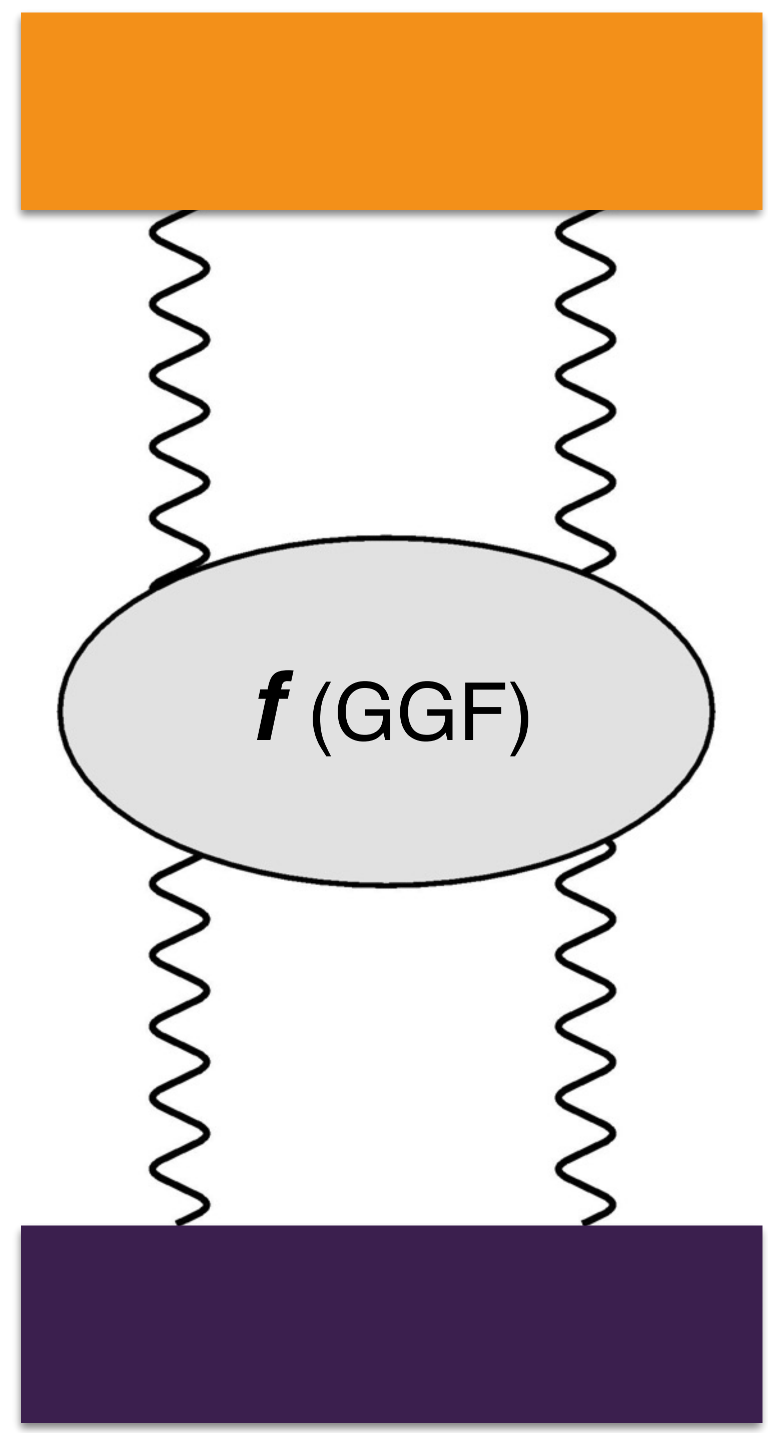



***<u>From now on, vertical propagators represent Reggeons</u>***

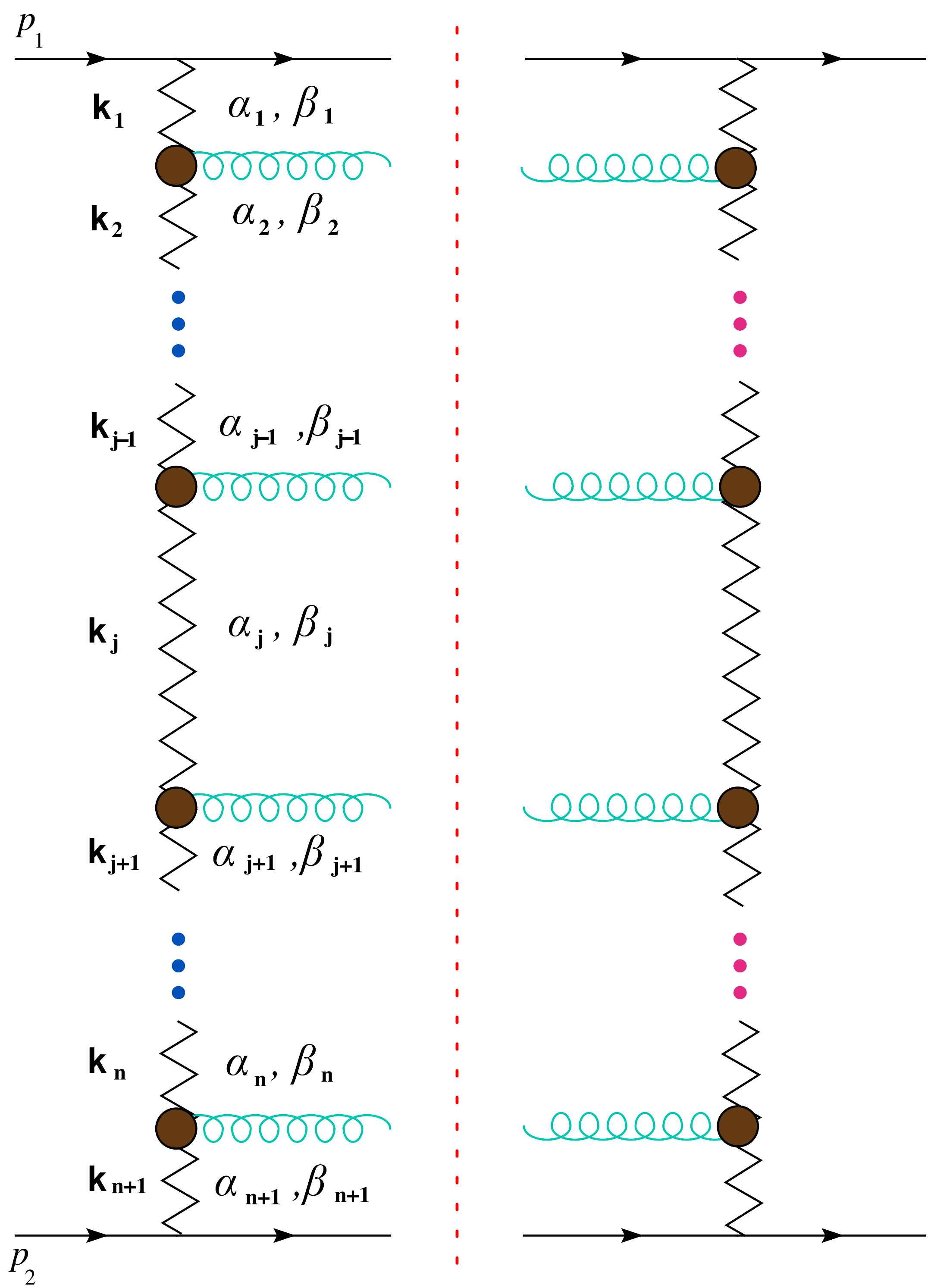
# Large logs from real emission corrections




To resum to all n, use BFKL. *Ladder diagrams* An important notion: KERNEL ITERATION Each iteration adds one **rung** to the **ladder** diagram



# Large logs from real emission corrections

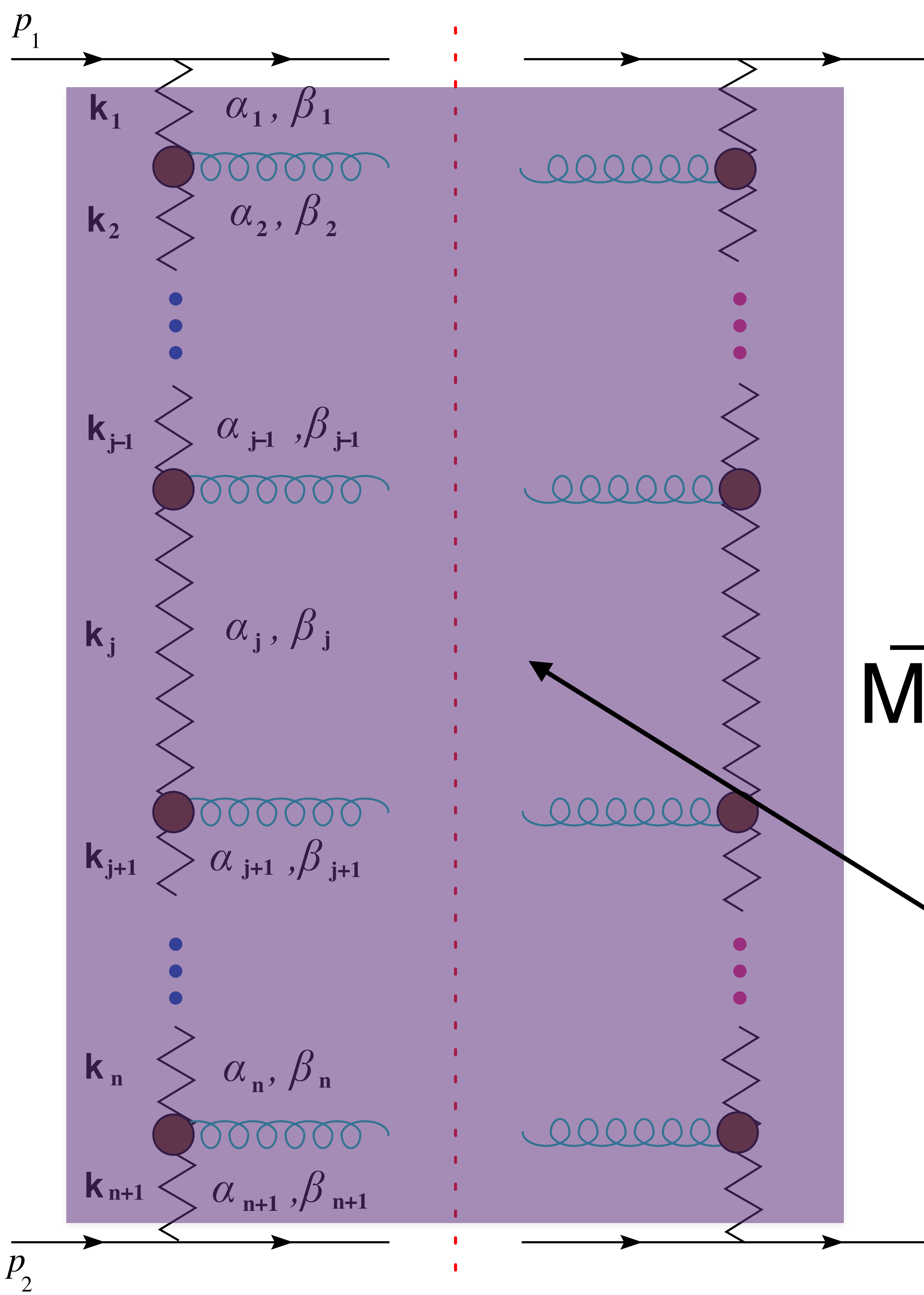



- Assume Reggeons in the t-channel
- Assume you have only one real emission
- Do the phase-space integration —> res1
- Now assume you have two real emissions
- Do the phase-space integration —> res2
- Add the results: RES = res1+res2
- Now assume you have three real emissions
- Do the phase-space integration —> res3
- Add the results: RES = RES + res3
- Repeat until you have N real emissions with resN so tiny compared to RES such that you are allowed to claim convergence

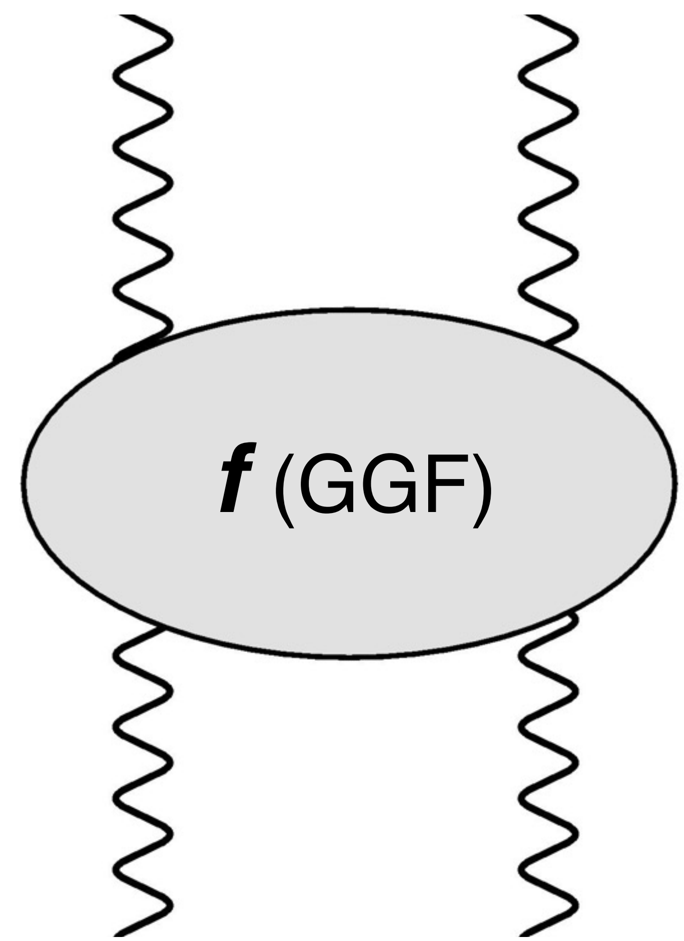
NOTE: The phase-space integration is over rapidity and transverse momenta.

# The Pomeron

**From now on, ‘rungs’ instead of real emissions**



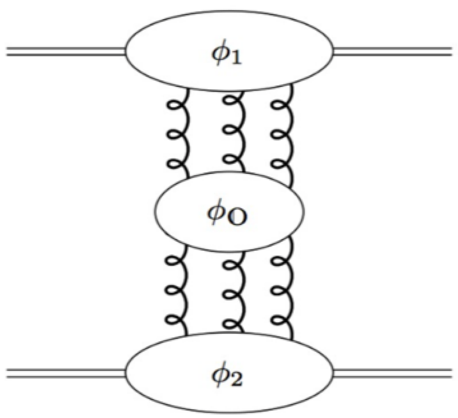

$$\xrightarrow[\text{Monte Carlo}]{\text{BFKL}}$$

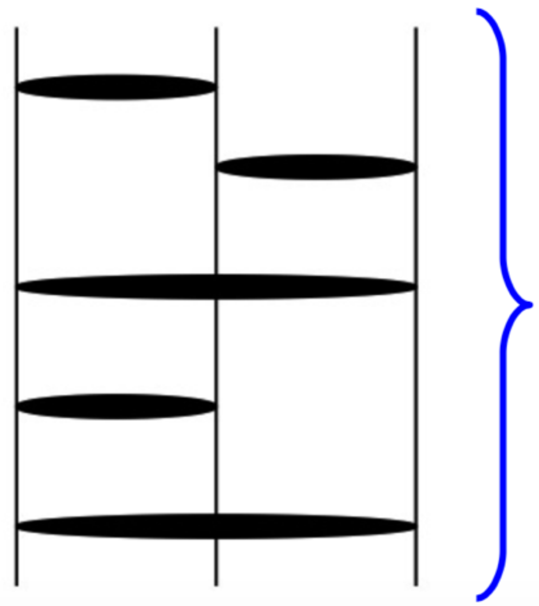



Many many
phase-space diagrams

# Pomeron vs Odderon

- Pomeron is the state of two interacting reggeized gluons in the t-channel in the color singlet. It has the quantum numbers of the vacuum
- Odderon is the state of three interacting gluons exchanged in the t-channel in the color singlet but with C =-1 and P=-1
- Any pair of two gluons in the Odderon forms symmetric color octet subsystems



Ladder structure of the Odderon. BKP resums term of the form $\alpha_s(\alpha_s \log s)^n$

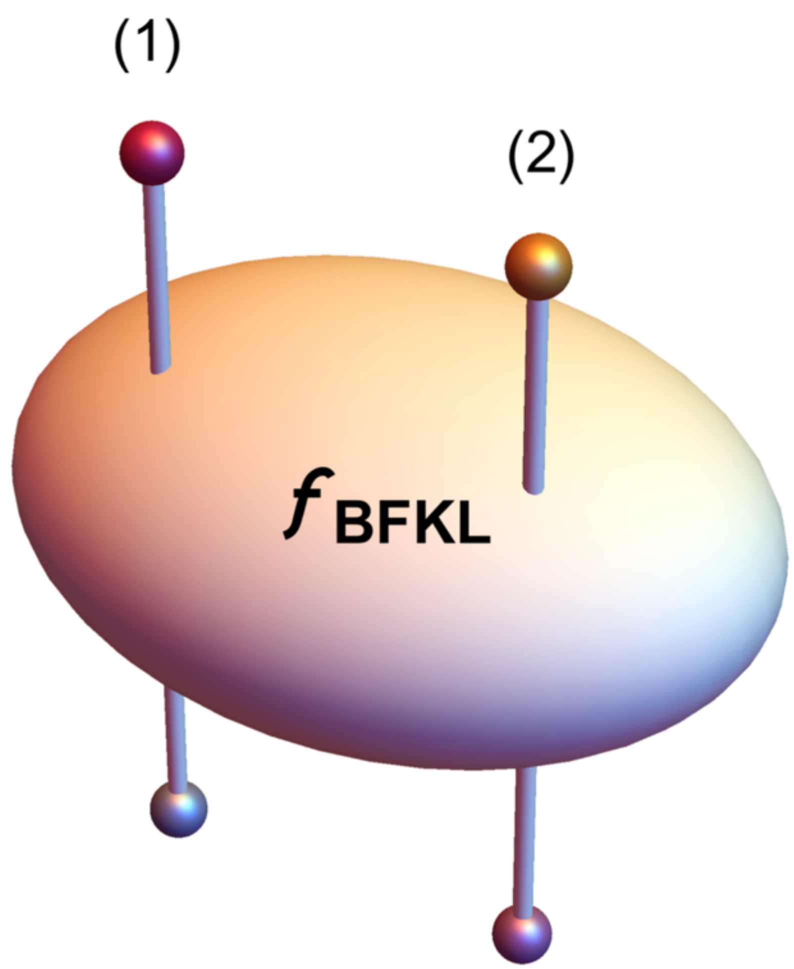
NLO corrections recently available

Bartels, Fadin, Lipatov, Vacca (2012)

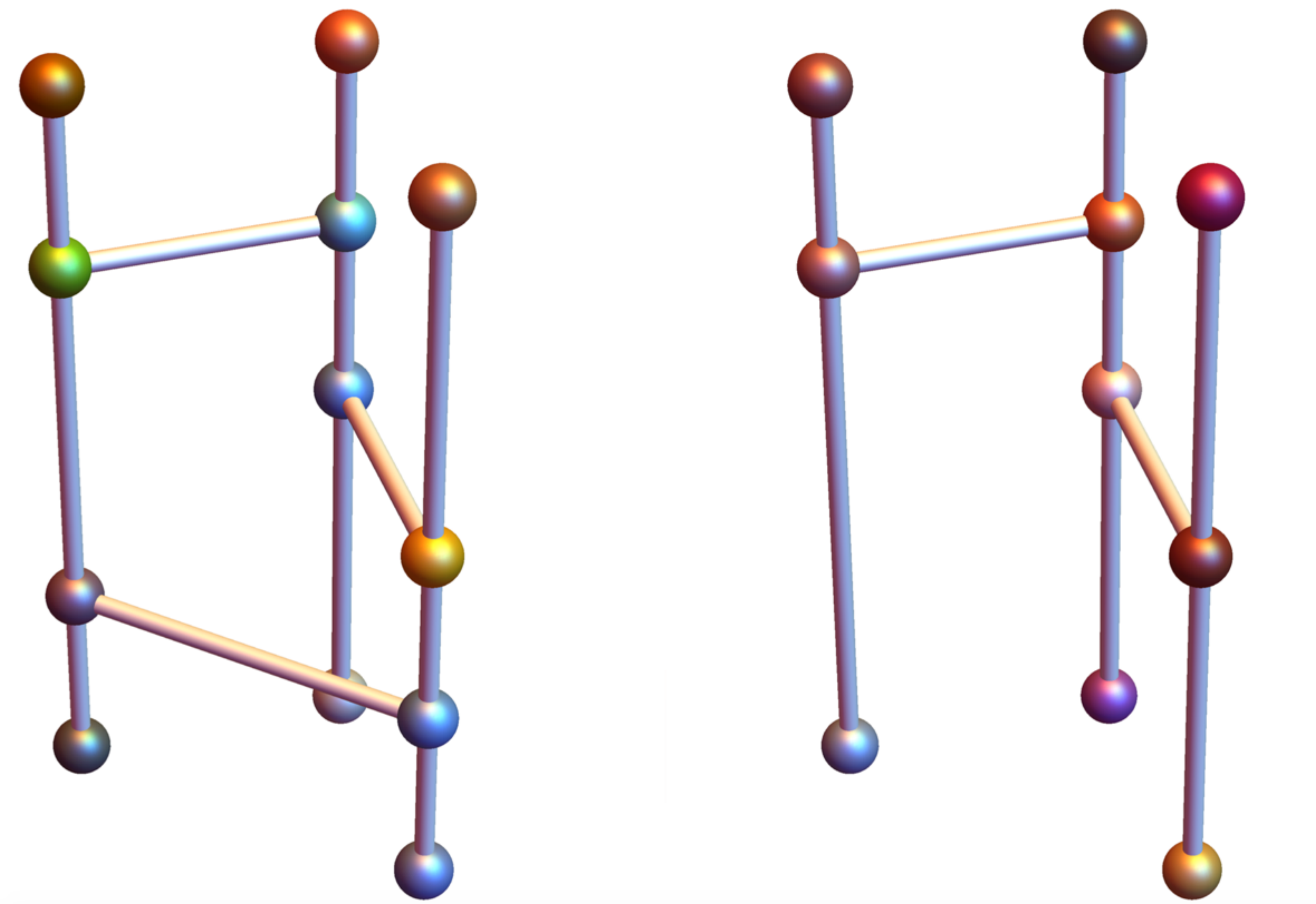
Has the Odderon been seen so far? *

* see talks of this Workshop (also Martynov, Nicolescu, Phys.Lett. B778 (2018) 414)

# From two to three Reggeons



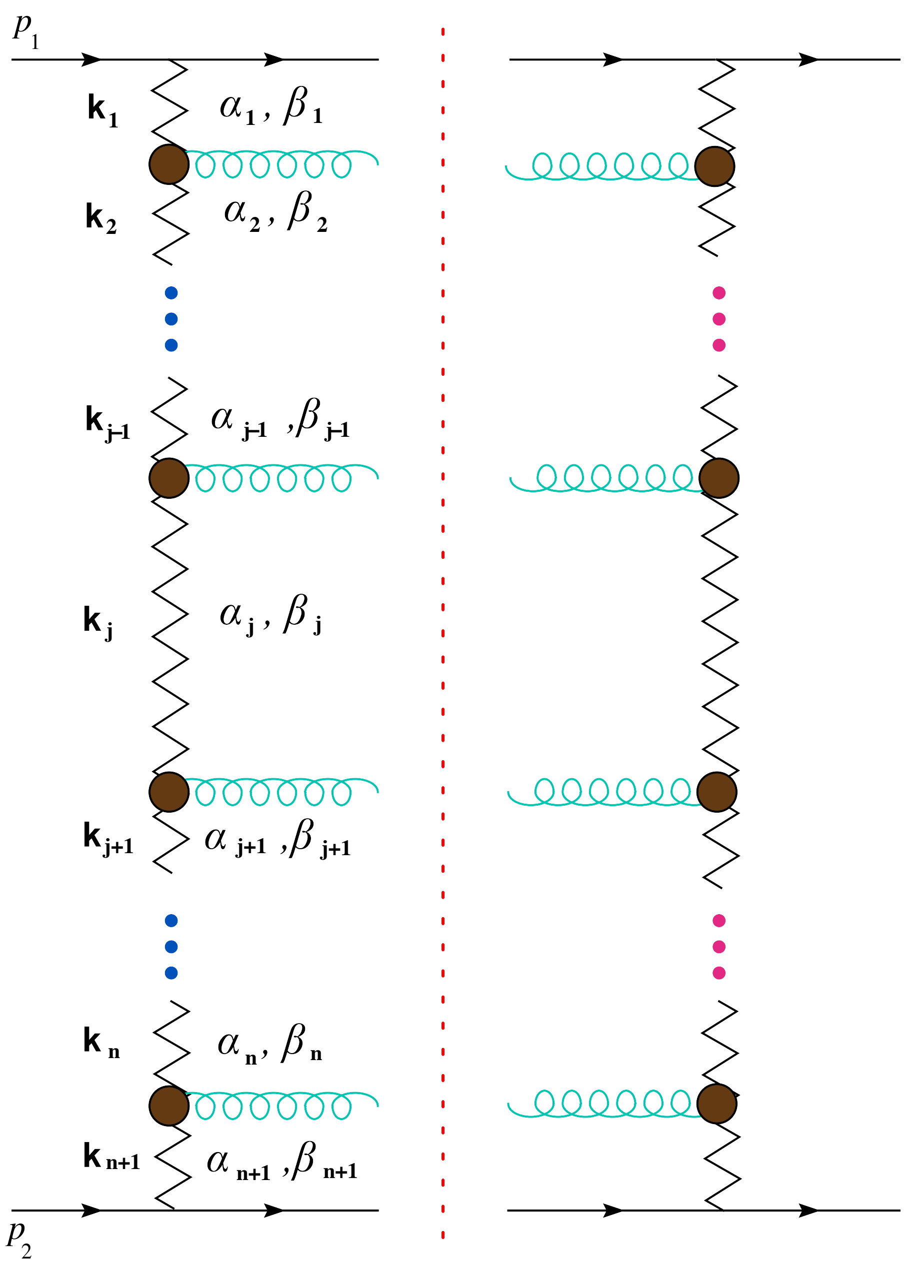
# Closed vs Open



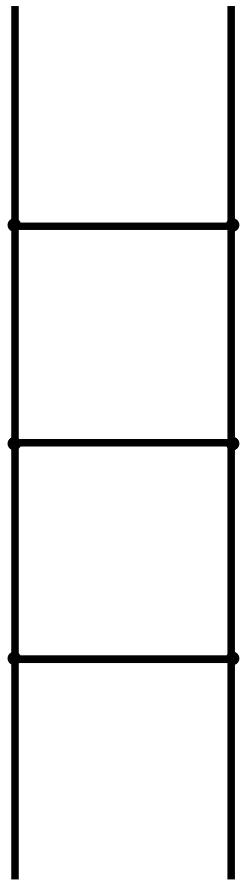
BKP was found to have a hidden integrability being equivalent to a periodic spin chain of a XXX Heisenberg ferromagnet. This was the first example of the existence of integrable systems in QCD

Lipatov (1986, 1990, 1993)

# Let us iterate

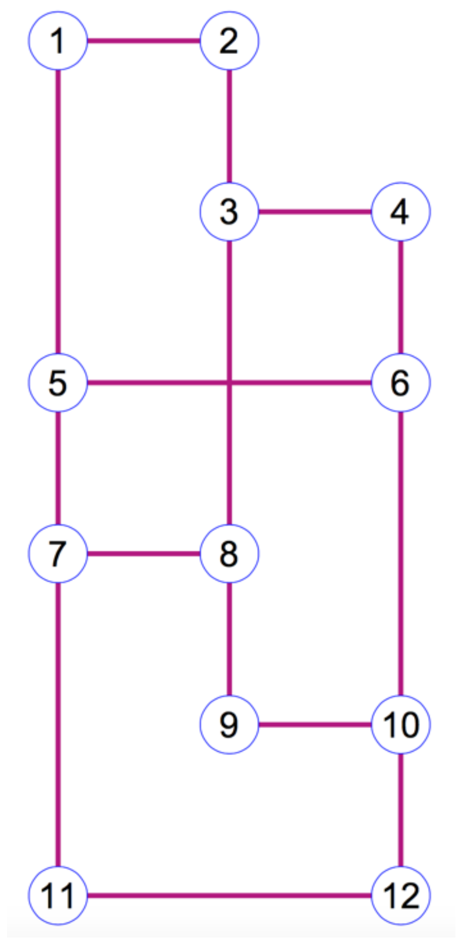



OK



BFKL

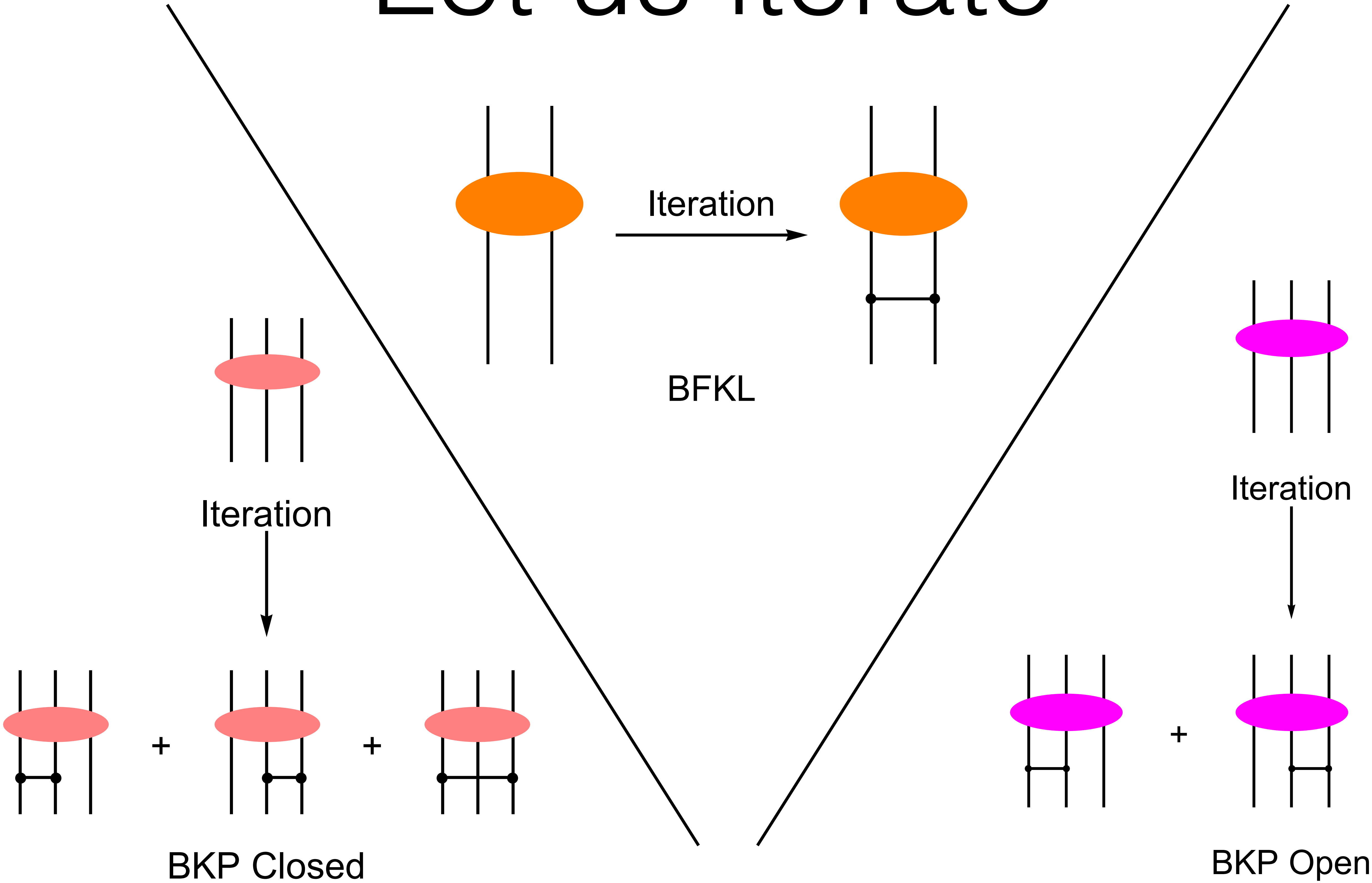
?




BKP

*Vertical lines are Reggeons*
*horizontal ones are gluons*

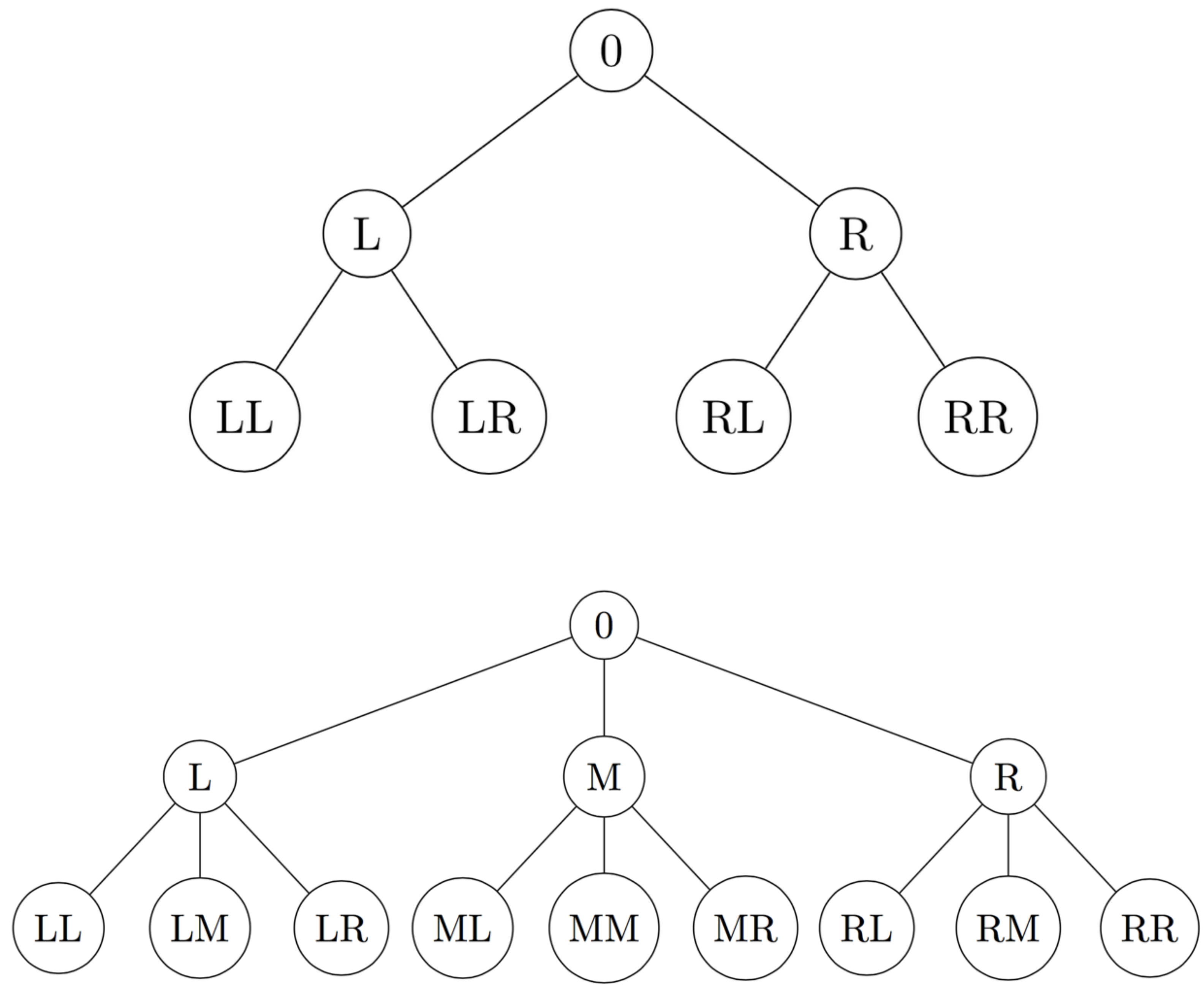
# Let us iterate



# Binary/ternary tree structure

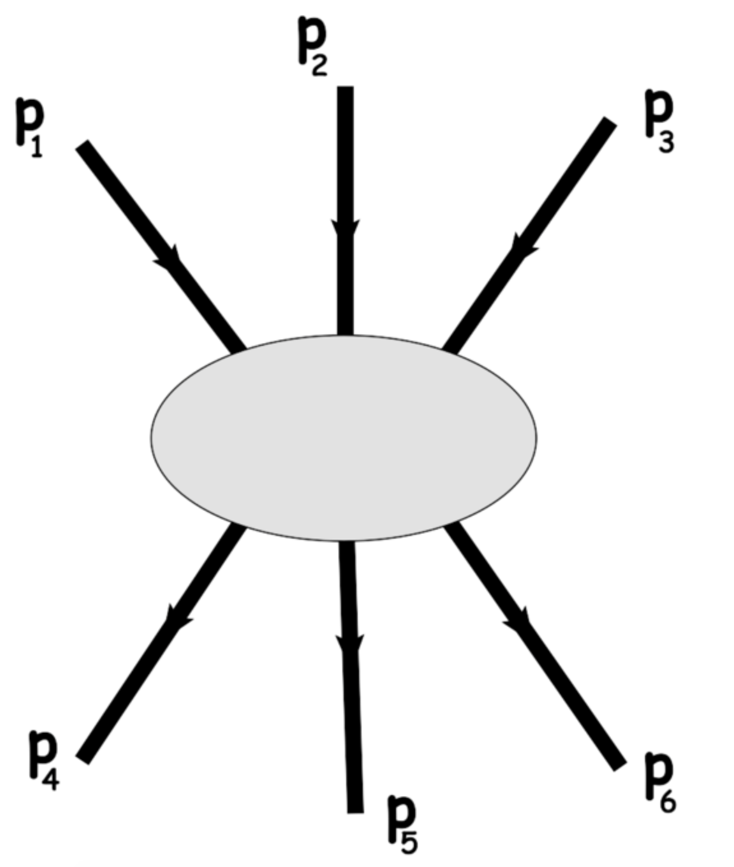
number of diagrams:
$2^n$ and $3^n$

| $n$ rungs | Number of diagrams |
|---|---|
| 2 | 9 |
| 3 | 27 |
| 4 | 81 |
| 5 | 243 |
| 6 | 729 |
| 7 | 2187 |
| 8 | 6561 |
| 9 | 19683 |
| 10 | 59049 |
| 11 | 177147 |
| 12 | 531441 |
| 13 | 1594323 |
| 14 | 4782969. |



# Results: kinematical configuration

All vectors live in the transverse momentum space



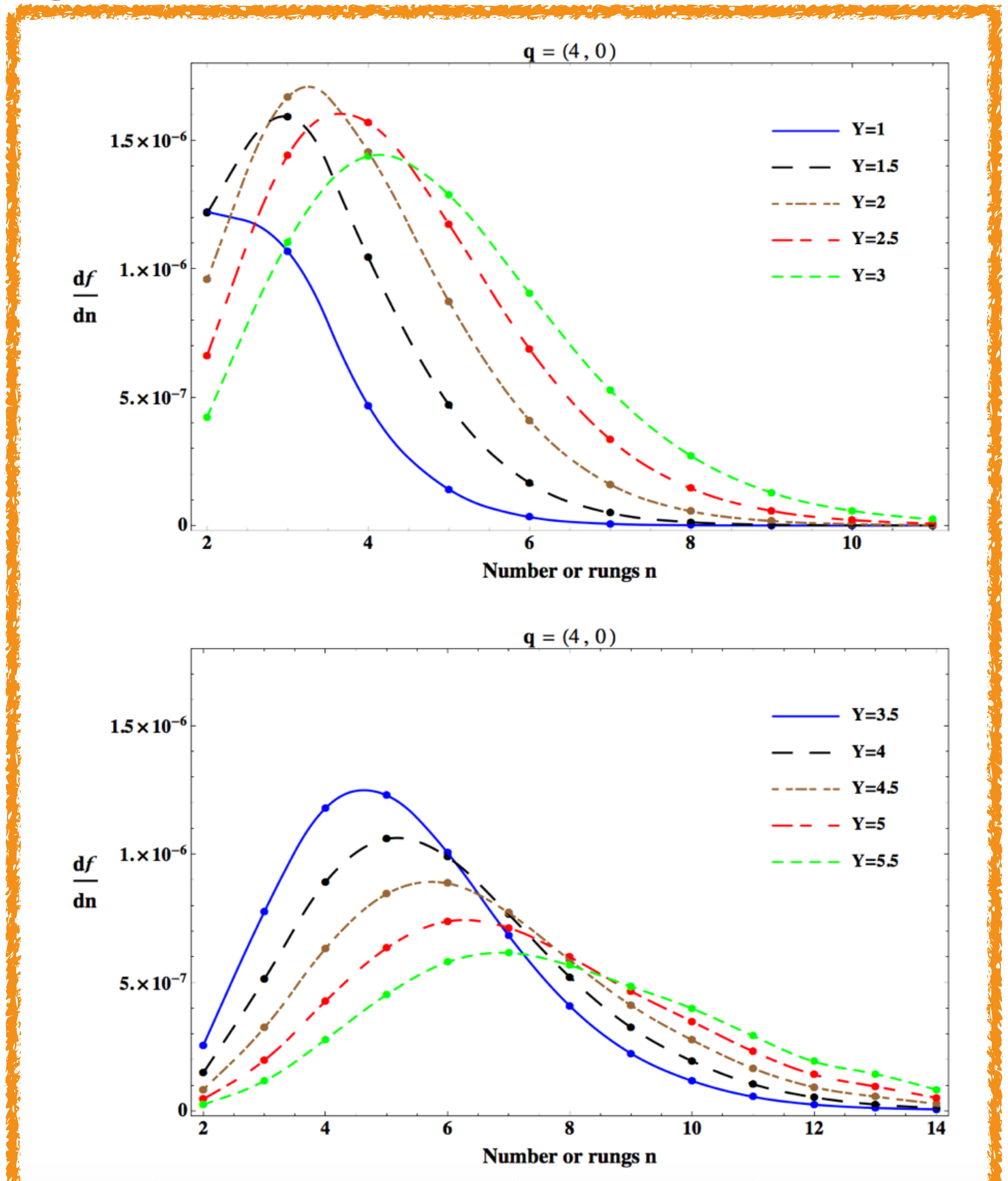

$$
\begin{aligned}
\mathbf{q} &= (4,0)\\
\mathbf{p}_1 &= (10,0)\\
\mathbf{p}_2 &= (20,\pi)\\
\mathbf{p}_3 &= (\mathbf{q}-\mathbf{p}_1)-\mathbf{p}_2 = (14,0)\\
\mathbf{p}_4 &= (20,0)\\
\mathbf{p}_5 &= (25,\pi)\\
\mathbf{p}_6 &= (\mathbf{q}-\mathbf{p}_4)-\mathbf{p}_5 = (9,0)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{q} &= (31,0)\\
\mathbf{p}_1 &= (10,0)\\
\mathbf{p}_2 &= (20,\pi)\\
\mathbf{p}_3 &= (\mathbf{q}-\mathbf{p}_1)-\mathbf{p}_2 = (41,0)\\
\mathbf{p}_4 &= (20,0)\\
\mathbf{p}_5 &= (25,\pi)\\
\mathbf{p}_6 &= (\mathbf{q}-\mathbf{p}_4)-\mathbf{p}_5 = (36,0)
\end{aligned}
$$

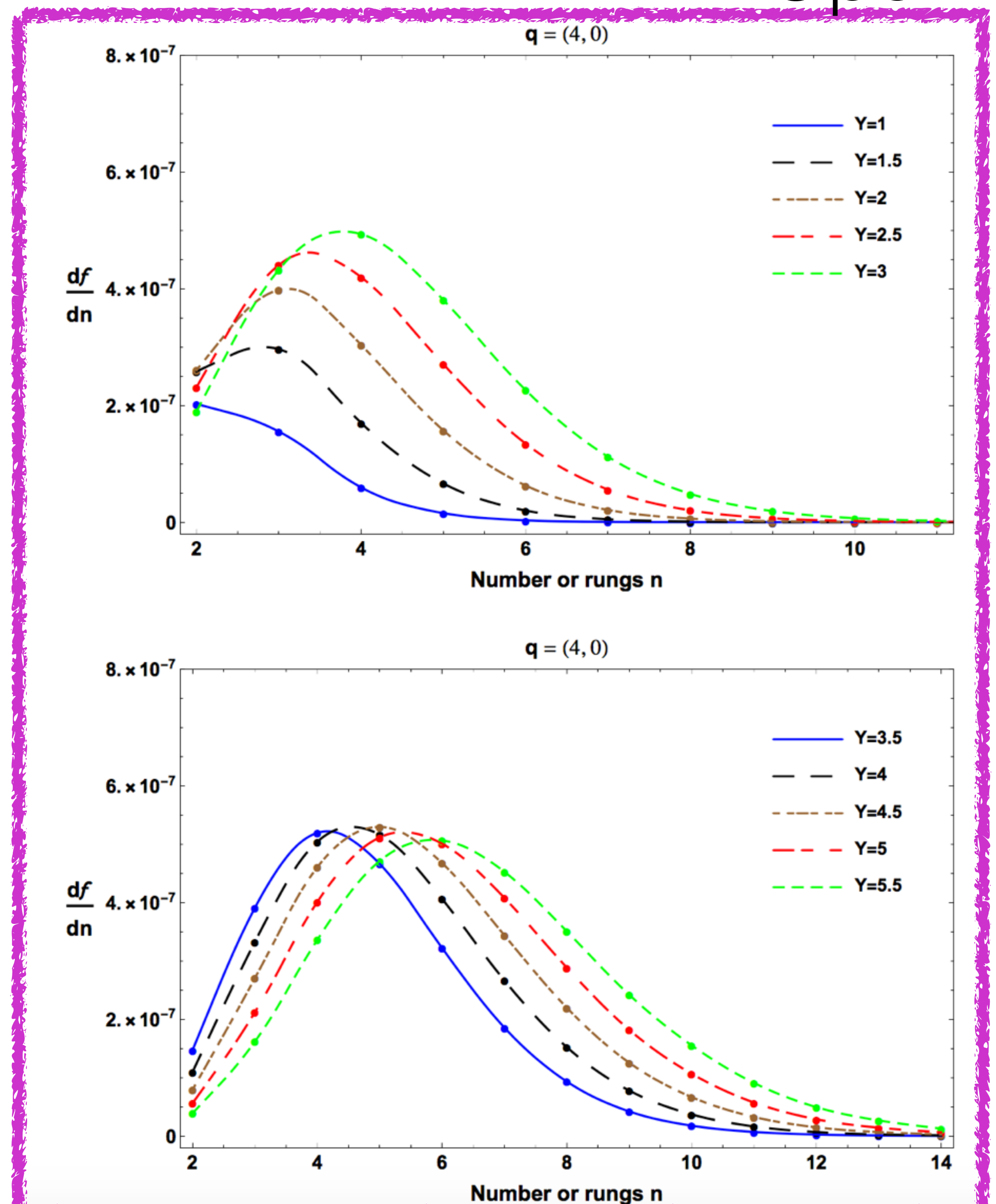
(r,θ): first component is in GeV, the second component in radians

# Results: "multiplicity"

Closed

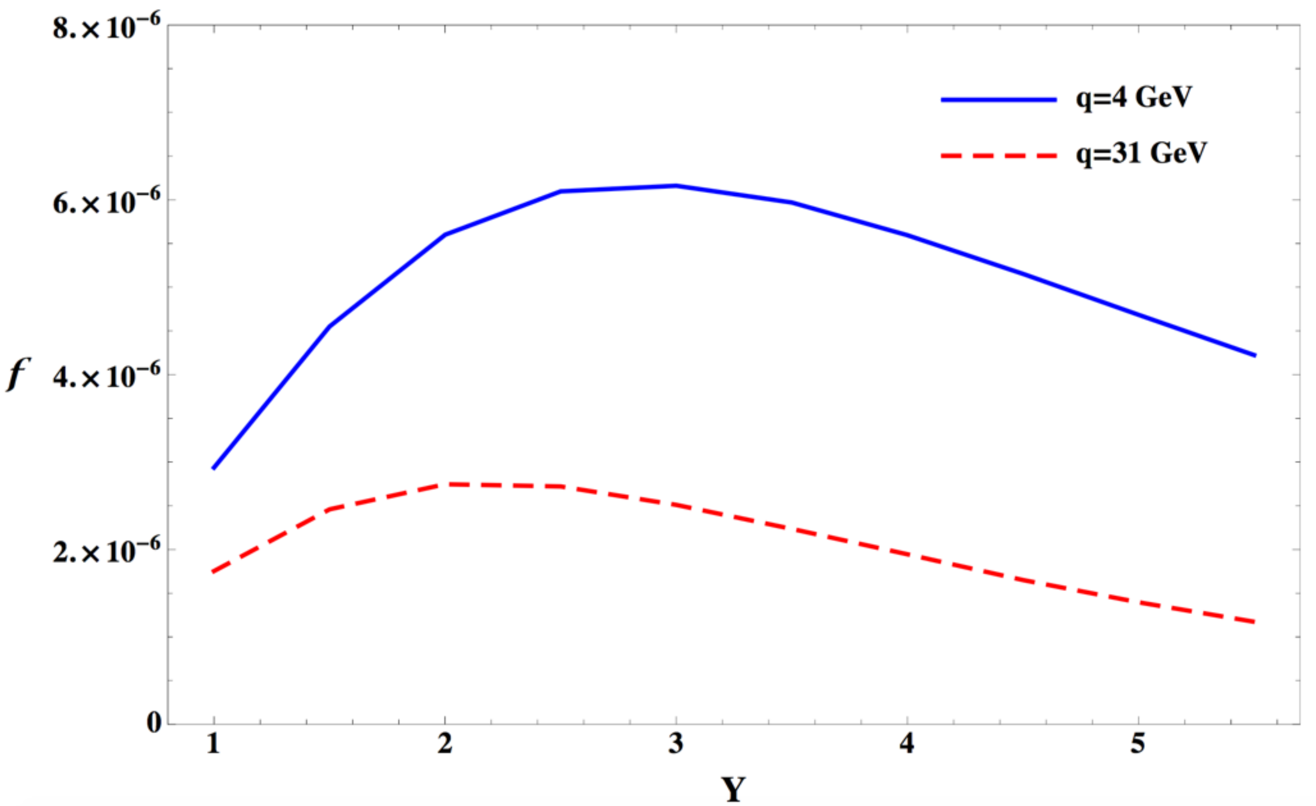



Open

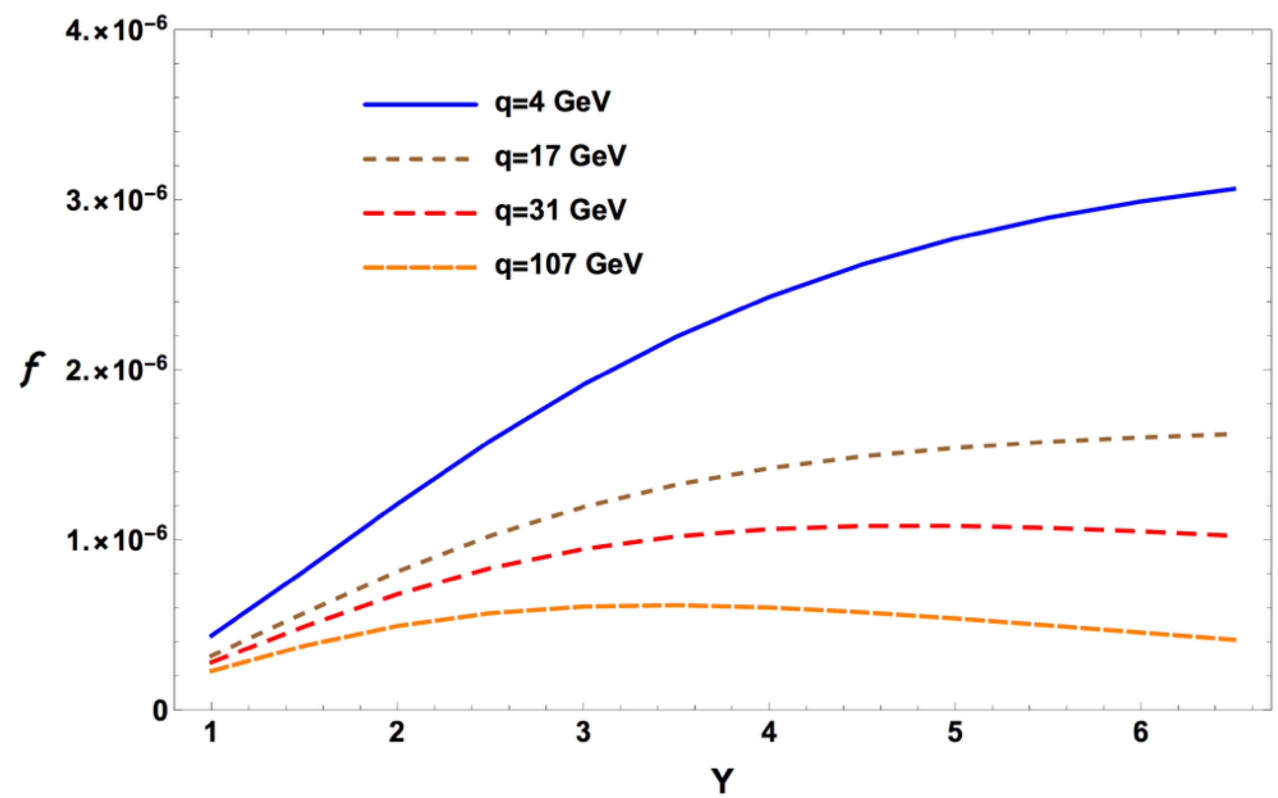


# Results: energy plots

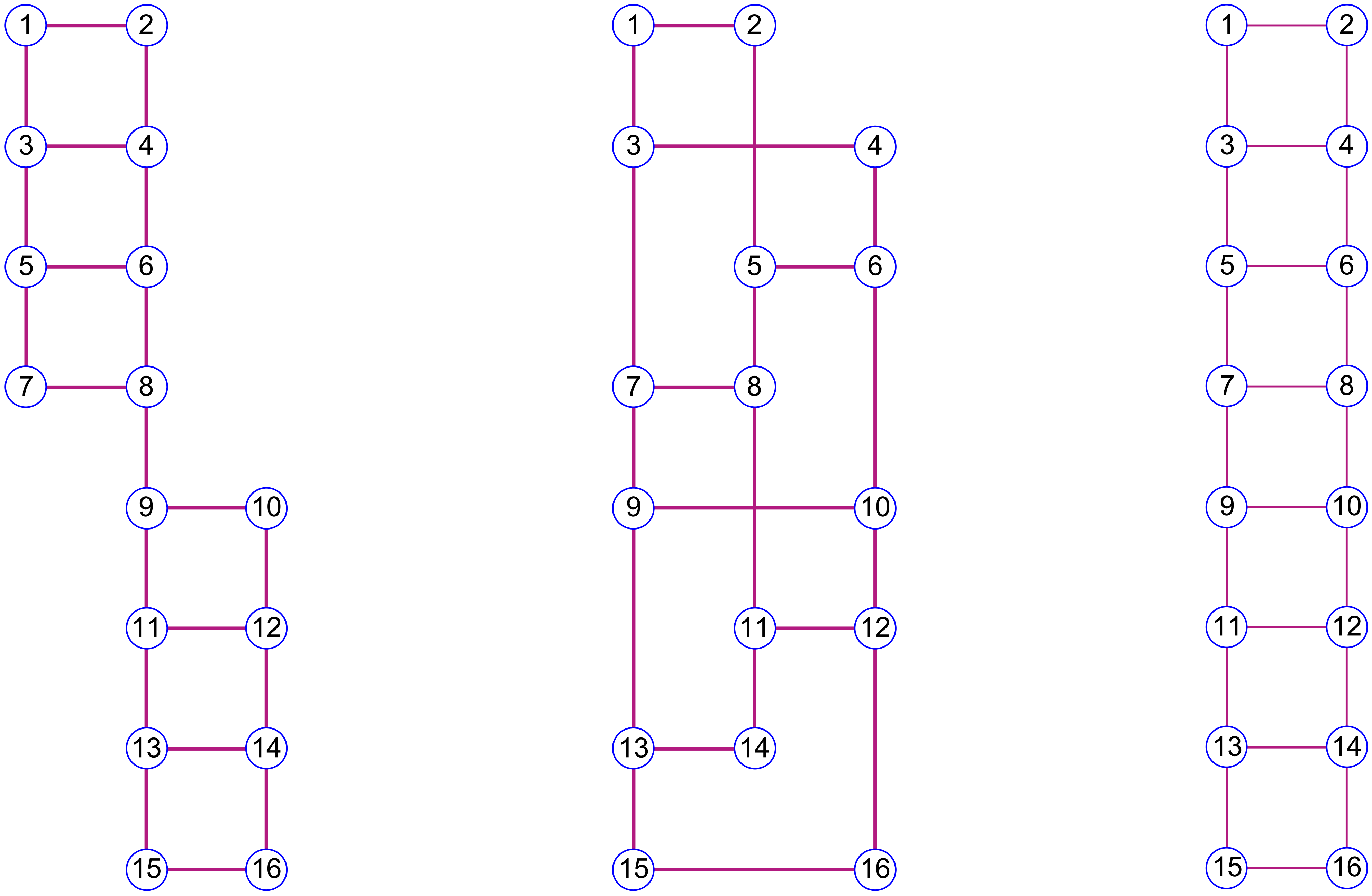
## Closed




## Open



# Topologies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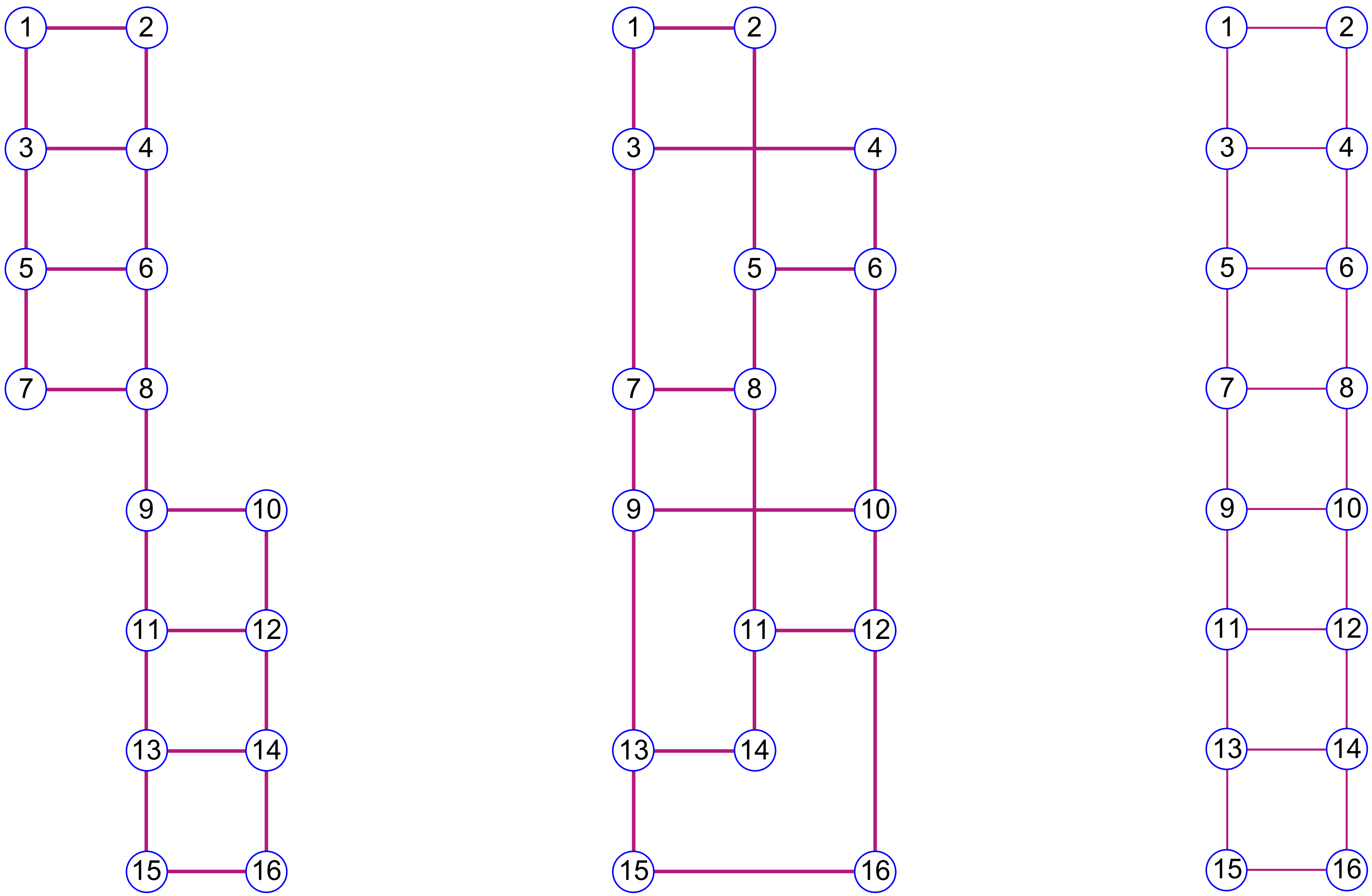
# Topologies & complexity




3136 10395 10864

# Graph Complexity

The matrix-tree theorem (Kirchhoff, 1847)

A spanning tree T of an undirected graph G is a subgraph that is a tree which includes all of the vertices of G, with minimum possible number of edges.

The complexity of an undirected connected graph corresponds to the number of all possible spanning trees of the graph.

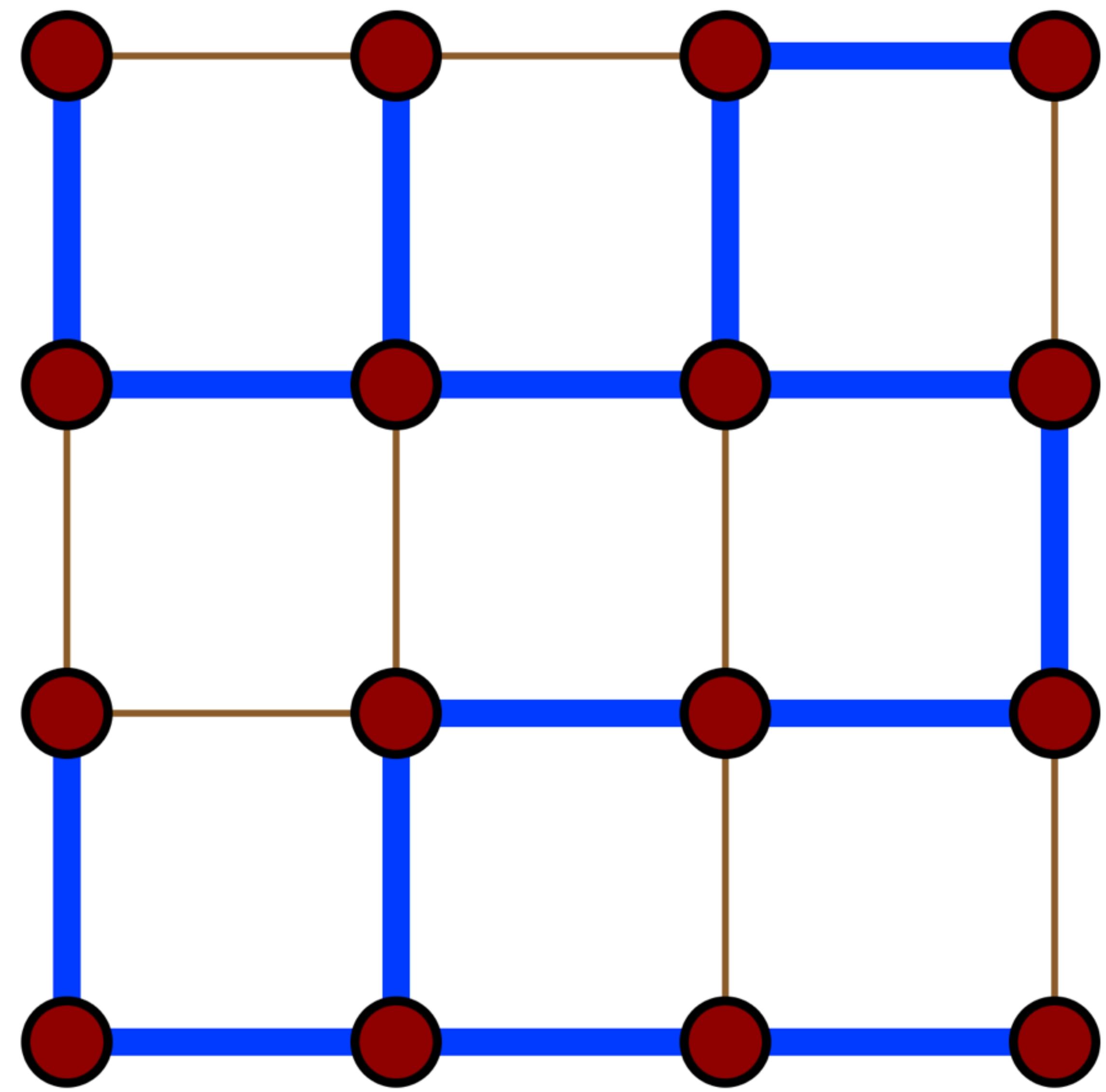
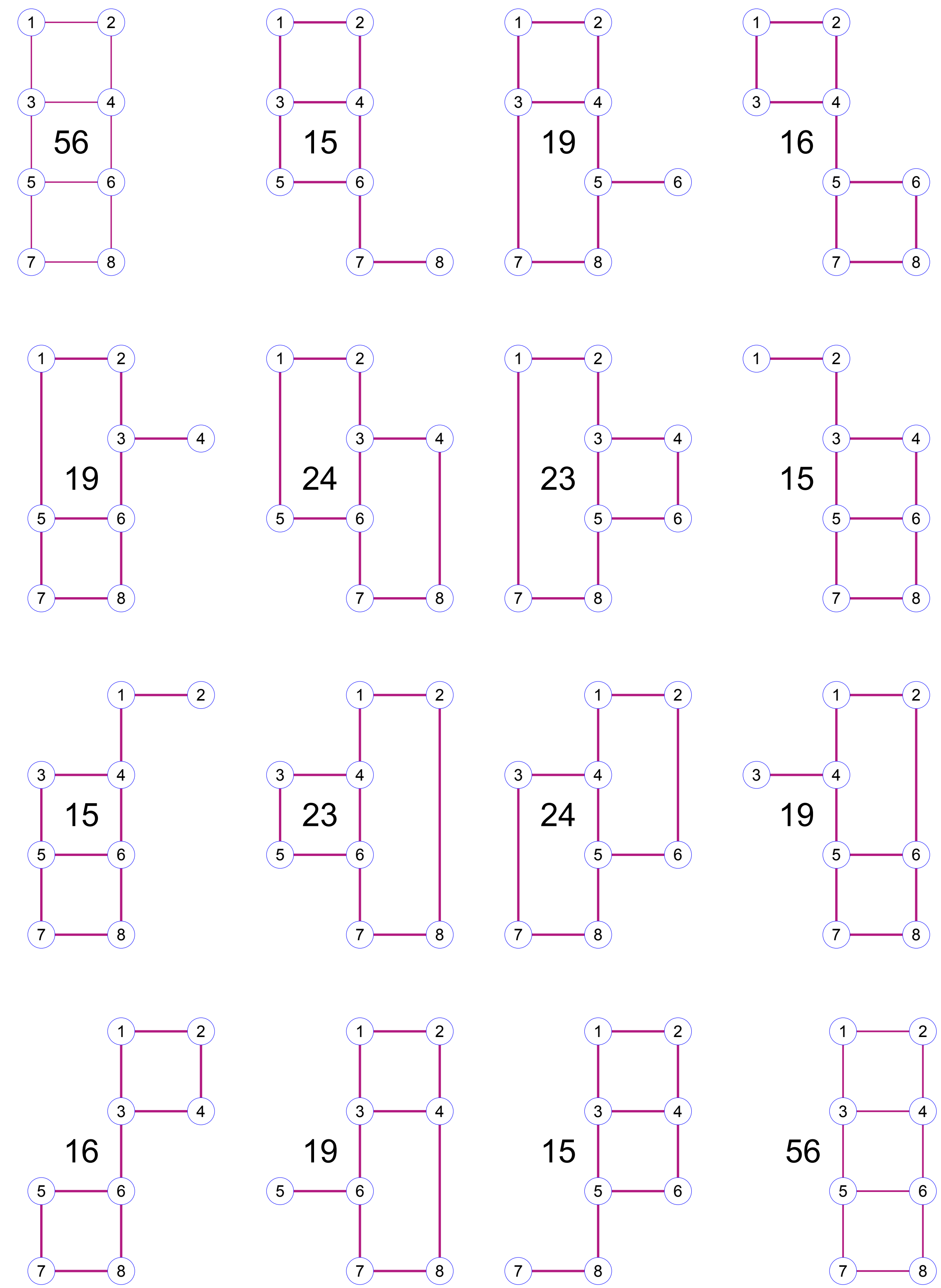
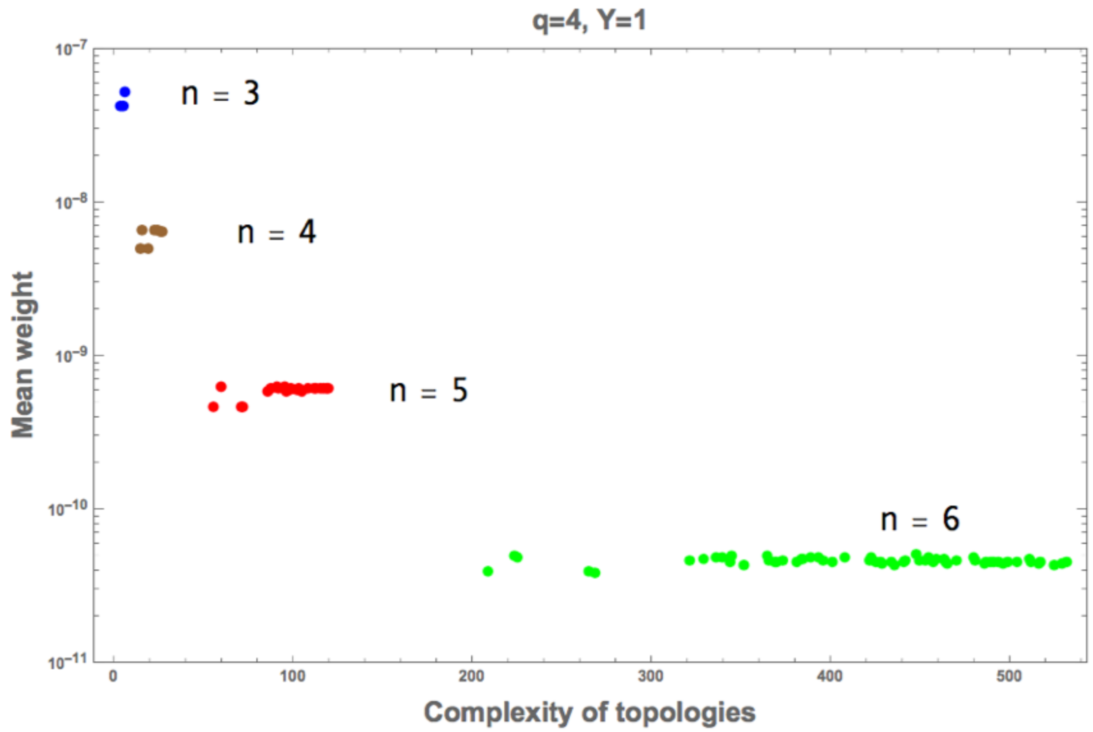


Figure from
https://en.wikipedia.org/wiki/Spanning_tree

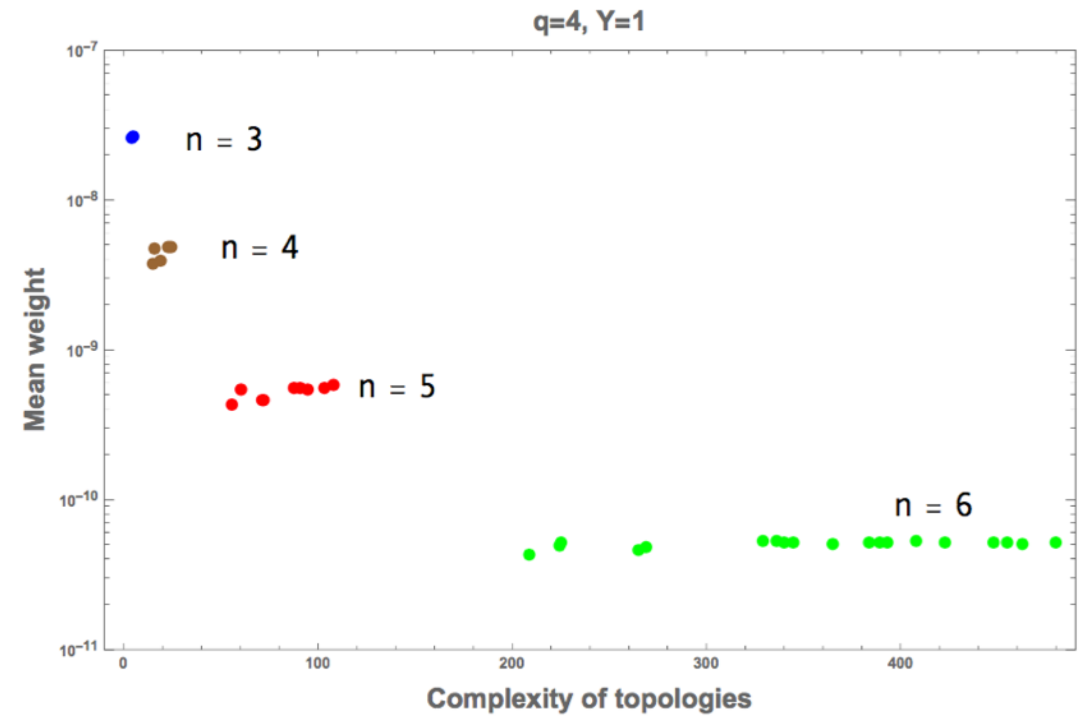
# Open 4-rung diagrams & complexities



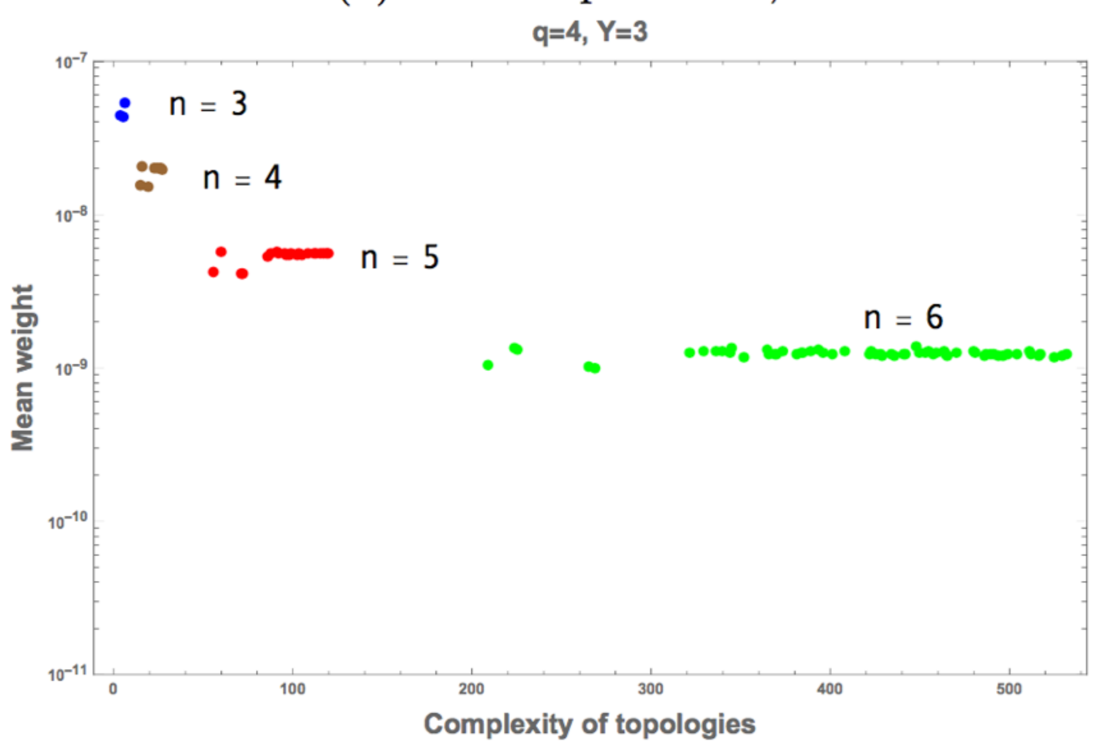
# Average weight per complexity



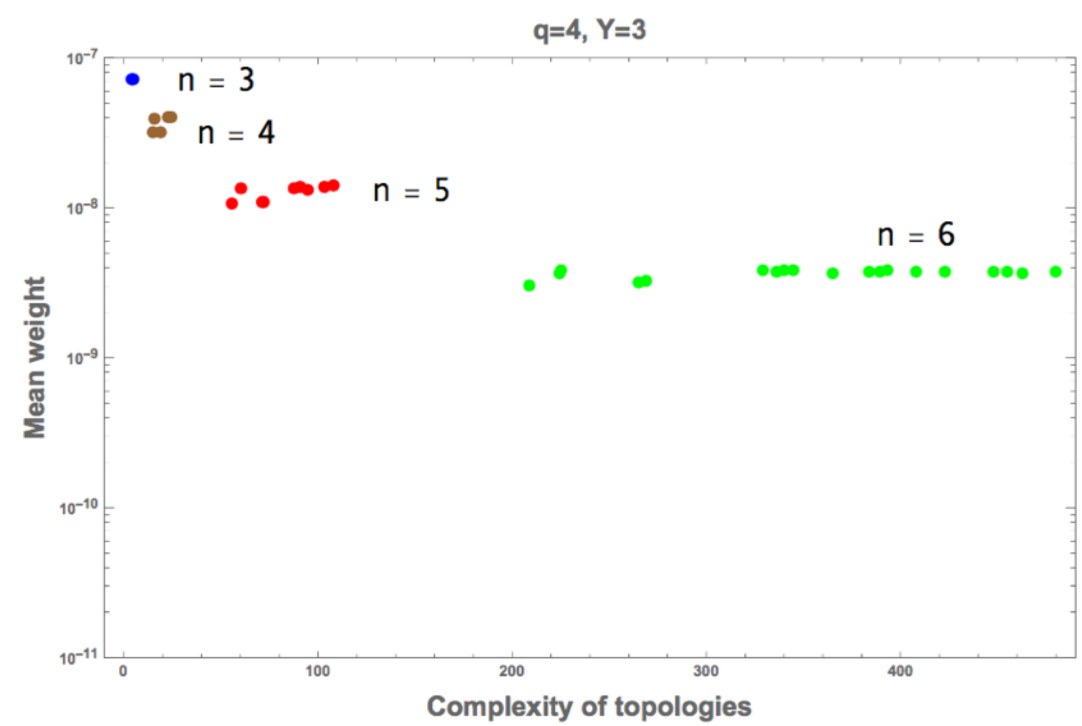

(a) Closed spin chain, $Y = 1$.




(a) Open spin chain, $Y = 1$.




(b) Closed spin chain, $Y = 3$.




(b) Open spin chain, $Y = 3$.

# Conclusions and Outlook

- We have used a Monte Carlo numerical integration of iterated integrals in transverse momentum and rapidity space to solve the BKP equation with three Reggeized gluons in the t-channel.
- Numerical convergence of the solution is achieved after applying the BKP kernel on the initial condition for a finite number of times at a given value of the strong coupling and the center-of-mass energy (in terms of rapidity, Y).
- The Green's function grows with Y for small values of this variable to then decrease at higher Y.
- The formalism can be applied to the BKP equation with a higher number of exchanged Reggeons. It can also be used beyond the leading logarithmic approximation and for cases with a total t-channel color projection not being in the singlet but in the adjoint representation. This is very important for the calculation of scattering amplitudes in N = 4 supersymmetric theories in the Regge limit. (G.C. and A. Sabio Vera, work in progress)
- Our approach also has obvious applications in the study of phenomenological cross sections devoted to the search of the elusive Odderon at hadron colliders.

# BACK UP

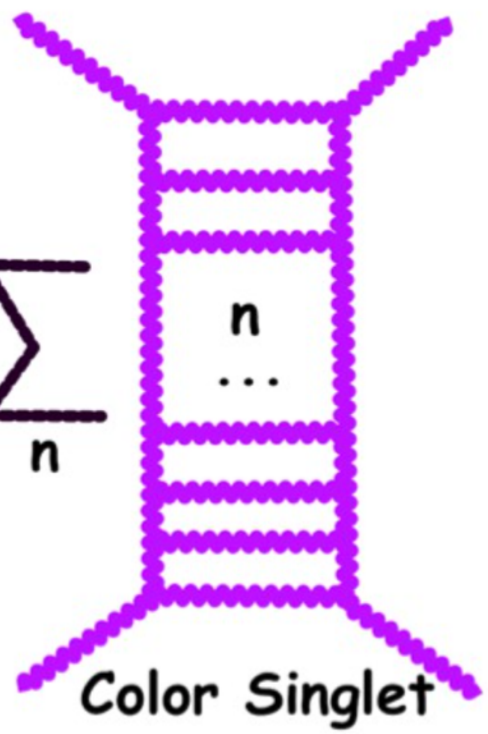
# Intro: Some considerations

- Q: What is the most relevant scale in high energy scattering?
- A: The center-of-mass energy squared s
- Q: In which functional form does s appear in the Feynman diagrams?
- A: $\alpha_s^m \ln(s)^n$
- Q: Can one isolate those Feynman diagrams that come with a numerically important [$\alpha_s^m \ln(s)^n \sim 1$] contribution?
- A: It depends (for this talk the answer is yes)
- Q: Can one resum all these diagrams with important $\alpha_s^m \ln(s)^n$ contributions to all orders in $\alpha_s$?
- A: It depends (for this talk the answer is yes)

# Ladder diagrams with two Reggeons

All-orders resummation of $\alpha_s(Q^2)\log\left(\frac{s}{Q^2}\right)$ terms: How? Ladder structure

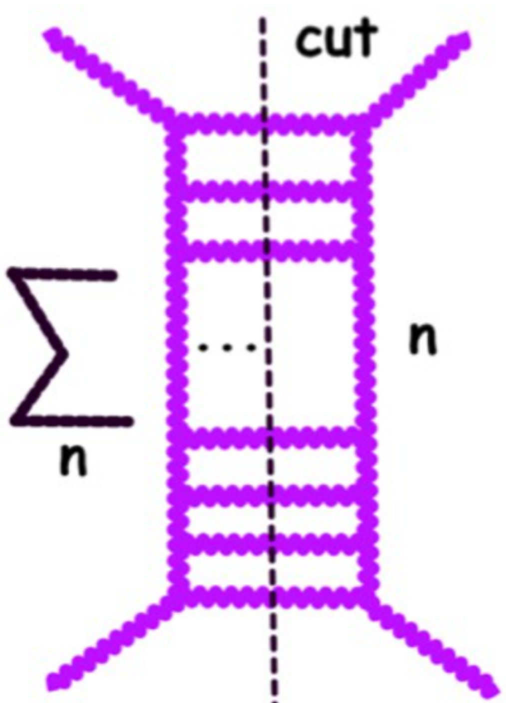
$A_{\text{elastic}}(s,t) = \sum_n$




*Loop diagrams can be seen as phase-space diagrams and the reverse*

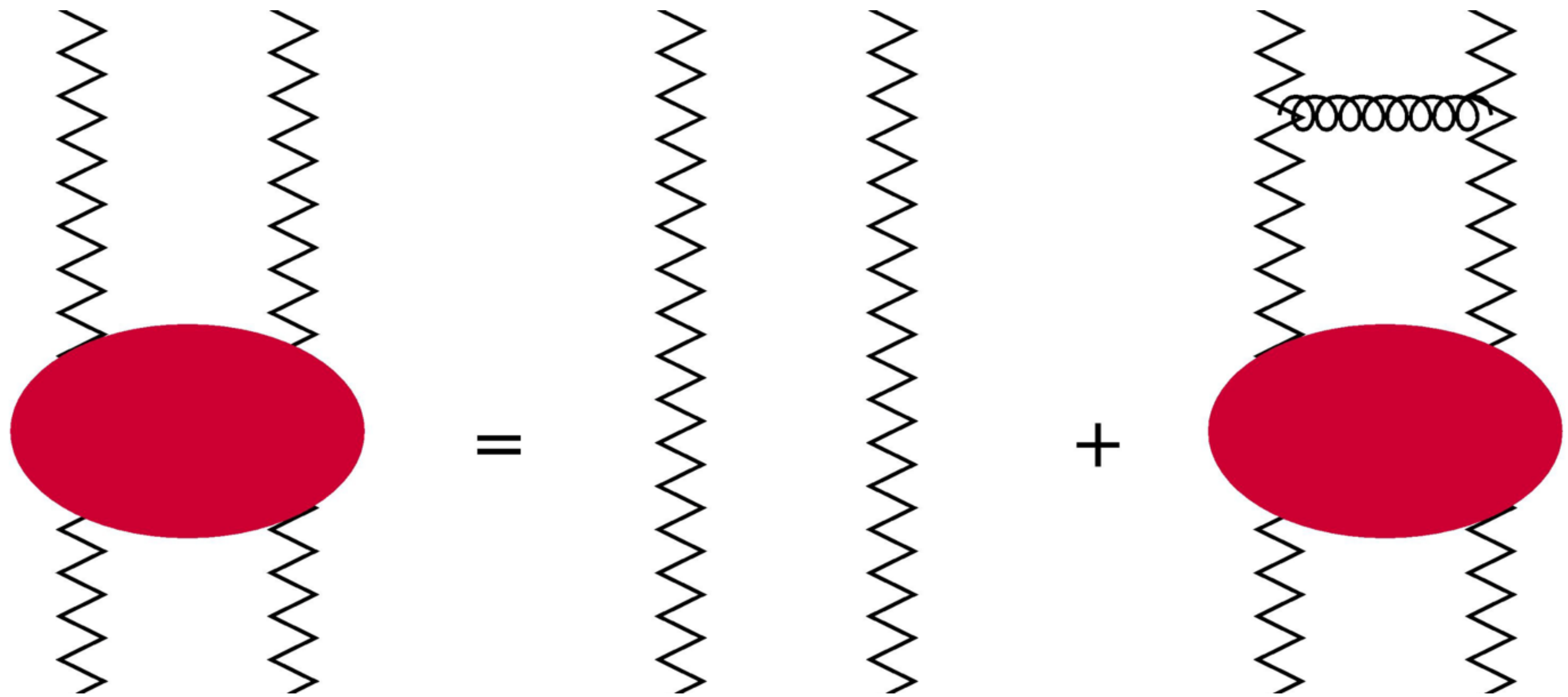
Optical Theorem :

$$\sigma_{\mathrm{TOT}} \simeq \frac{1}{s}\mathrm{Im}\mathcal{A}_{\text{elastic}}(s, t=0) = \frac{1}{s}\sum_n$$




$$= \frac{1}{s}\sum_n |A_n(s,t)|^2$$

# The gluon Green's function
# BFKL equation



What to keep from this figure:
Solving the BFKL equation iteratively amounts to adding one rung with each new iteration

# A few words on color

This -Nc is a color factor, assuming that the color state of the two gluons in the graph is the color singlet. If this is the case, then the kernel is IR finite!

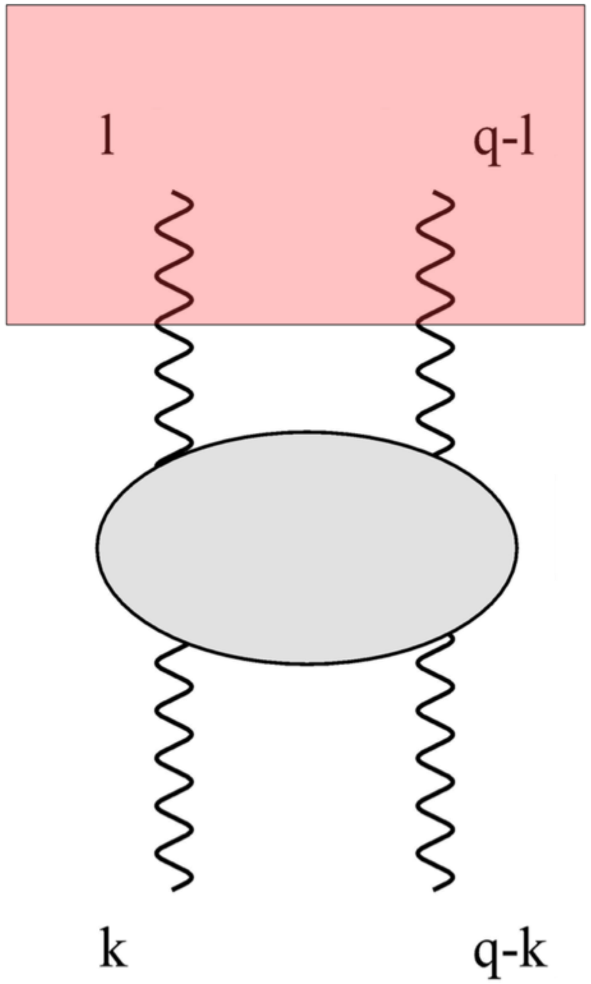
Color state of the two gluons

For QCD, the possible states are:

$$\mathbf{1},\ \mathbf{8_A},\ \mathbf{8_S},\ \mathbf{10}+\overline{\mathbf{10}},\ \mathbf{27}$$

with color factors:

$$-3,\ -\frac{3}{2},\ -\frac{3}{2},\ 0,\ 1$$




$$K_{\mathrm{BFKL}}(\mathbf{l},\mathbf{q}-\mathbf{l};\mathbf{k},\mathbf{q}-\mathbf{k}) = -N_c g^2\left[\mathbf{q}^2-\frac{\mathbf{k}^2(\mathbf{q}-\mathbf{l})^2}{(\mathbf{k}-\mathbf{l})^2}-\frac{(\mathbf{q}-\mathbf{k})^2\mathbf{l}^2}{(\mathbf{k}-\mathbf{l})^2}\right]$$
$$+(2\pi)^3\mathbf{k}^2(\mathbf{q}-\mathbf{k})^2\left[\omega(\mathbf{k})+\omega(\mathbf{q}-\mathbf{k})\right]\delta^{(2)}(\mathbf{k}-\mathbf{l})\,.$$

Where $\omega(\mathbf{k}^2) = -\frac{N_c}{2}g^2\int\frac{d^2\mathbf{l}}{(2\pi)^3}\frac{\mathbf{k}^2}{\mathbf{l}^2(\mathbf{l}-\mathbf{k})^2}$

# Color does matter

<u>Symmetric octet</u>

- It was in a generalized leading logarithmic approximation, and by iterating the BFKL kernel in the s-channel, where the Bartels-Kwiecinski-Praszalowicz (BKP) equation was proposed

Bartels (1980)

Kwiecinski, Praszalowicz (1980)

- BKP was found to have a hidden integrability being equivalent to a periodic spin chain of a XXX Heisenberg ferromagnet. This was the first example of the existence of integrable systems in QCD

Lipatov (1986, 1990, 1993)

- **It will be directly connected to any numerical solution of the BKP, if any such work is to be done with the aim to perform phenomenological studies for the Odderon**

<u>Antisymmetric octet</u>

- Corrections to the Bern-Dixon-Smirnov (BDS) iterative ansatz (Bern, Dixon, Smirnov, 2005) for the n-point maximally helicity violating (MHV) and planar amplitudes were found in MRK in the six-point amplitude at two loops

Bartels, Lipatov, Sabio Vera (2009, 2010)

in other words, it is a fundamental ingredient of the finite remainder of scattering amplitudes with arbitrary number of external legs and internal loops

# Kernel iteration in the Monte Carlo approach

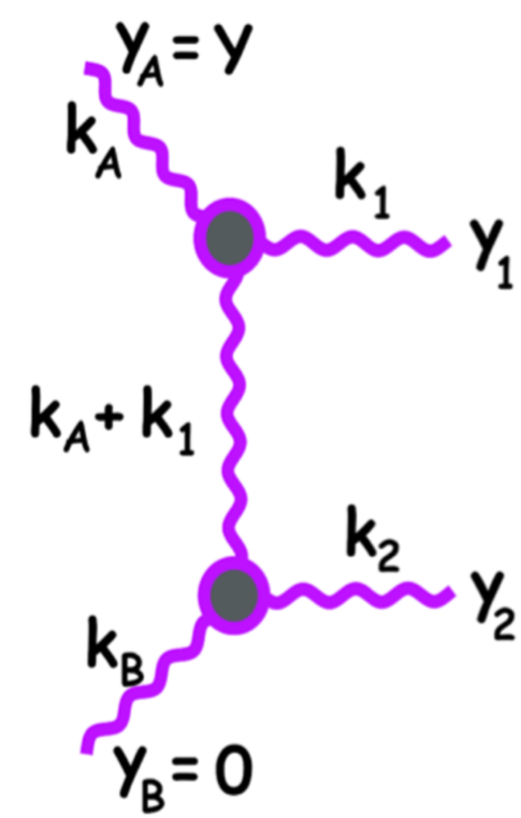
- Many people have worked on it, the origin goes back to the late 90's:

Kwiecinski, Lewis, Martin (1996), Schmidt (1996), Orr, Stirling (1998)

Effective Feynman Rules:
simplest case, $t = 0$, leading order

Gluon Regge trajectory:
$\omega\left(\vec{q}\right) = -\frac{\alpha_s N_c}{\pi} \log \frac{q^2}{\lambda^2}$

Modified $t$-channel propagators:
$\left(\frac{s_i}{s_0}\right)^{\omega(t_i)} = e^{\omega(t_i)(y_i - y_{i+1})}$




$$\left(\frac{\alpha_s N_c}{\pi}\right)^2 \int d^2\vec{k}_1 \frac{\theta\left(k_1^2 - \lambda^2\right)}{\pi k_1^2} \int d^2\vec{k}_2 \frac{\theta\left(k_2^2 - \lambda^2\right)}{\pi k_2^2} \delta^{(2)}\left(\vec{k}_A + \vec{k}_1 + \vec{k}_2 - \vec{k}_B\right)$$
$$\times \int_0^Y dy_1 \int_0^{y_1} dy_2 e^{\omega\left(\vec{k}_A\right)(Y - y_1)} e^{\omega\left(\vec{k}_A + \vec{k}_1\right)(y_1 - y_2)} e^{\omega\left(\vec{k}_A + \vec{k}_1 + \vec{k}_2\right) y_2}$$

Very simplified -pictorial- view, the main elements and ideas are here though