Probing high energy effects in multijet production

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based on

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LHC Working Group on Forward Physics and Diffraction

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Outline

High energy limit

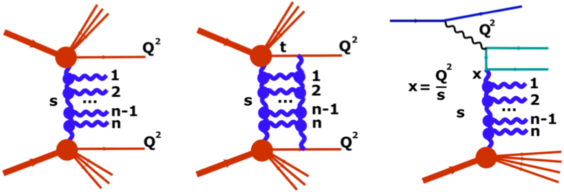
The high energy limit studies a limited part of the phase space, but allow us to compute things otherwise impractical

Purely theoretical

- Spin Chains, Integrability
- Scattering amplitudes, bootstrap
- CFT's
- AdS/CFT
- Special Functions

High energy limit

Phenomenology

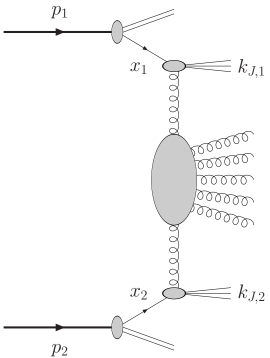


Mueller-Navelet jets / Muellet-Tang jets / DIS at small x

With the advent of LHC we have access to higher energies:

- Test **pQCD in the high-energy limit**.
- Explore applicability region of **BFKL resummation**.

Mueller–Navelet jets



Picture from
[D. Colferai, F. Schwennsen, L. Szymanowski, S. Wallon (2010)]

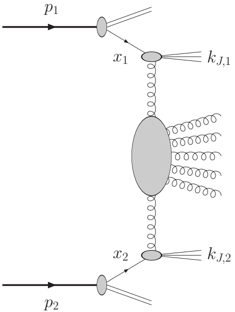
Big hit joining theory and experiment in the past years.
Originally proposed in

[A. H. Mueller, H. Navelet (1987)]

- At Y=0, no minijet radiation in the rapidity interval. Exact correlation $d\sigma \sim \delta^2(\vec{k}_{J,1} + \vec{k}_{J,2})$
- At large Y the BFKL approach predicts decorrelations (minijets)

Key observable: correlation in the azimuthal angle of the 2 tagged jets.

Mueller–Navelet jets



Picture from
[D. Colferai, F. Schwennsen, L. Szymanowski, S. Wallon (2010)]

Large jet transverse momenta:
$\vec{k}_{J,1}^{\,2} \sim \vec{k}_{J,2}^{\,2} \gg \Lambda_{\rm QCD}^2$
DGLAP evolution. pQCD applicable.

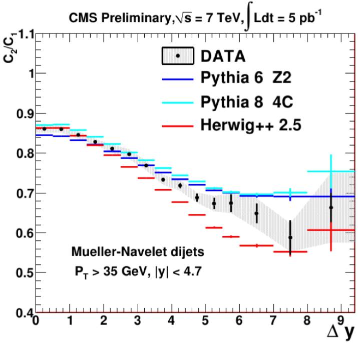
Large rapidity interval between jets:
$Y = \ln \frac{x_1 x_2 s}{|\vec{k}_{J,1}||\vec{k}_{J,2}|}$
BFKL resummation effects $\alpha Y \sim 1$

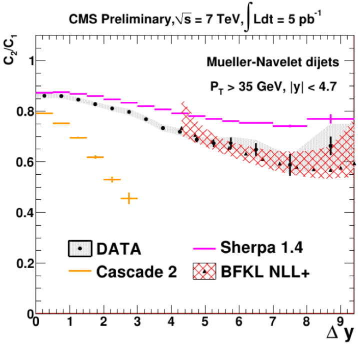
- LLA BFKL: $(\alpha_s \log \frac{s}{Q^2})^q$ terms
- NLLA BFKL: $\alpha_s(\alpha_s \log \frac{s}{Q^2})^q$ terms

All orders result in perturbation thery!

$$\frac{d\sigma}{dx_1 dx_2\, d|\vec{k}_{J,1}|\, d|\vec{k}_{J,2}| d\theta_1 d\theta_2} = \frac{1}{(2\pi)^2}\left[\mathcal{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\theta)\, \mathcal{C}_n\right]$$

Mueller–Navelet jets: Experiment





NLLA predictions against LHC data are quite successful for large rapidities.

[B. Ducloué, L. Szymanowski, S. Wallon (2014)]

[CMS Collaboration (2016)]

Mueller–Navelet jets: Experiment

The observed sensitivity to the implementation of the colour-coherence effects in the DGLAP MC generators and the reasonable data-theory agreement shown by the NLL BFKL analytical calculations at large Δy, may be considered as indications that the kinematical domain of the present study lies in between the regions described by the DGLAP and BFKL approaches. Possible manifestations of BFKL signatures are expected to be more pronounced at increasing collision energies.

But at the studied energy, the data can be described within both BFKL and DGLAP approaches. We do not have a clear BFKL signature.

[CMS Collaboration (2016)]

Mueller–Navelet jets: Lessons

- Big dependence on high order corrections in $\mathcal{C}_0$ due to collinear contamination, better to define ratios.

  [A. Sabio Vera, F. Schwennsen (2007)]

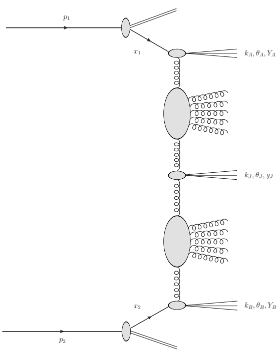
- Focusing in azimuthal angle correlations is more fruitful than the usual "growth with energy" behaviour.
- We need to go to higher energies or to explore more fine tuned regions of phase space to observe a clear signature.

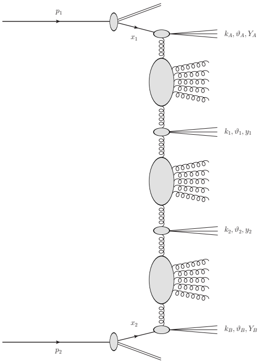
Motivation

- BFKL shows a factorization between longitudinal Y and transverse $\vec{k}$ d.o.f. If this is the case we will learn something new about QCD.
- Exploring the "growth with energy" behaviour mainly explores the longitudinal degree of freedom.
- The 2-jet correlations explore one of the components of the transverse d.o.f., the azimuthal angles.
- We would like to investigate observables for which the transverse momentum modulus enters into the game.
- Including more jets allow us to study azimuthal correlations and its dependence on transverse momentum. Less inclusive observables!

Multi-jet production!

Three- and four-jet production

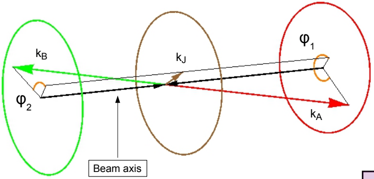




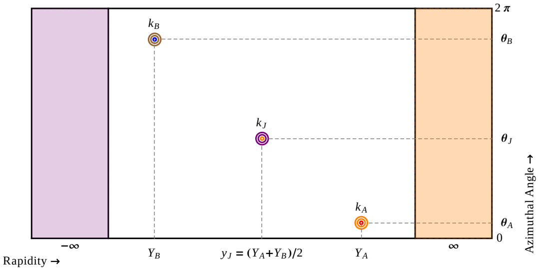
[F. Caporale, G. Chachamis, B. Murdaca, A. Sabio Vera (2015)]
[F. Caporale, F.G. Celiberto, G. Chachamis, D.G.G., A. Sabio Vera (2016) (2017)]

[F. Caporale, F.G. Celiberto., G. Chachamis, A. Sabio Vera (2016)]
[F. Caporale, F.G. Celiberto, G. Chachamis, D. G.G., A. Sabio Vera (2016)]

An event with three tagged jets



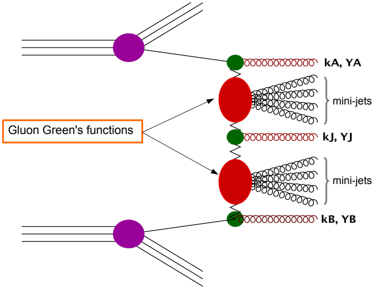
$$Y_B < y_J < Y_A$$



The three-jet partonic cross section

Starting point: differential partonic cross-section (no PDFs)

$$\frac{d^3\hat{\sigma}^{3-\text{jet}}}{dk_J d\theta_J dy_J} = \frac{\bar{\alpha}_s}{\pi k_J}\int d^2\vec{p}_A \int d^2\vec{p}_B\, \delta^{(2)}\left(\vec{p}_A + \vec{k}_J - \vec{p}_B\right) \times$$
$$\times \quad \varphi\left(\vec{k}_A, \vec{p}_A, Y_A - y_J\right)\varphi\left(\vec{p}_B, \vec{k}_B, y_J - Y_B\right)$$



- *Multi-Regge kinematics* rapidity ordering: $Y_B < y_J < Y_A$
- k_J lie above the experimental resolution scale
- φ is the BFKL gluon Green function (LLA or NLLA)
- $\bar{\alpha}_s = \alpha_s N_c/\pi$

Generalized azimuthal correlations - partonic level

Prescription: integrate over all angles after using the projections on the two azimuthal angle differences indicated below...

→ ...to define:

$$\int_0^{2\pi} d\theta_A \int_0^{2\pi} d\theta_B \int_0^{2\pi} d\theta_J \cos\left(M\left(\theta_A-\theta_J-\pi\right)\right)\cos\left(N\left(\theta_J-\theta_B-\pi\right)\right)\frac{d^3\hat{\sigma}^{3-\mathrm{jet}}}{dk_J d\theta_J dy_J}$$

$$= \bar{\alpha}_s \sum_{L=0}^{N} \binom{N}{L} \left(k_J^2\right)^{\frac{L-1}{2}} \int_0^{\infty} dp^2 \left(p^2\right)^{\frac{N-L}{2}} \int_0^{2\pi} d\theta \frac{(-1)^{M+N}\cos\left(M\theta\right)\cos\left((N-L)\theta\right)}{\sqrt{\left(p^2+k_J^2+2\sqrt{p^2k_J^2}\cos\theta\right)^N}}$$

$$\times\, \phi_M\left(k_A^2, p^2, Y_A-y_J\right)\phi_N\left(p^2+k_J^2+2\sqrt{p^2k_J^2}\cos\theta, k_B^2, y_J-Y_B\right)$$

Main observables: **generalized azimuthal correlation ratios** (w/o the 0 component)

$$\mathcal{R}_{PQ}^{MN} = \frac{\mathcal{C}_{MN}}{\mathcal{C}_{PR}} = \frac{\langle\cos(M(\theta_A-\theta_J-\pi))\cos(N(\theta_J-\theta_B-\pi))\rangle}{\langle\cos(P(\theta_A-\theta_J-\pi))\cos(Q(\theta_J-\theta_B-\pi))\rangle}$$

Next step: hadronic level predictions

- Introduce PDFs and running of the strong coupling:

$$
\frac{d\sigma^{3-\text{jet}}}{dk_A\, dY_A\, d\theta_A\, dk_B\, dY_B\, d\theta_B\, dk_J\, dy_J d\theta_J} =
$$

$$
\frac{8\pi^3\, C_F\, \bar{\alpha}_s\,(\mu_R)^3}{N_C^3}\, \frac{x_{J_A}\, x_{J_B}}{k_A\, k_B\, k_J} \int d^2\vec{p}_A \int d^2\vec{p}_B\, \delta^{(2)}\left(\vec{p}_A + \vec{k}_J - \vec{p}_B\right)
$$

$$
\times \left(\frac{N_C}{C_F} f_g(x_{J_A}, \mu_F) + \sum_{r=q,\bar{q}} f_r(x_{J_A}, \mu_F) \right)
$$

$$
\times \left(\frac{N_C}{C_F} f_g(x_{J_B}, \mu_F) + \sum_{s=q,\bar{q}} f_s(x_{J_B}, \mu_F) \right)
$$

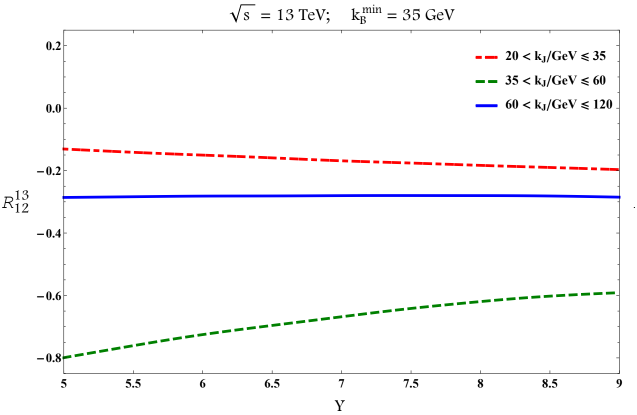
$$
\times\, \varphi\left(\vec{k}_A, \vec{p}_A, Y_A - y_J\right) \varphi\left(\vec{p}_B, \vec{k}_B, y_J - Y_B\right)
$$

- Match the LHC kinematical cuts (integrate $d\sigma^{3-\text{jet}}$ on k_T and rapidities Y_A, Y_B):

  - **1.** $35\text{ GeV} \le k_A \le 60\text{ GeV}$; $35\text{ GeV} \le k_B \le 60\text{ GeV}$; symmetric cuts
    2. $35\text{ GeV} \le k_A \le 60\text{ GeV}$; $50\text{ GeV} \le k_B \le 60\text{ GeV}$; asymmetric cuts
  - $Y = Y_A - Y_B$ fixed;
    $y_J = (Y_A + Y_B)/2$ .Maximize rapidity differences among tagged jets
  - $\sqrt{s} = 7, 13$ TeV

R_{12}^{13} vs Y for three different k_J bins. LLA.

$$k_A^{\min} = 35 \text{ GeV},\ k_B^{\min} = 35 \text{ GeV},\ k_A^{\max} = k_B^{\max} = 60 \text{ GeV (symmetric)}$$

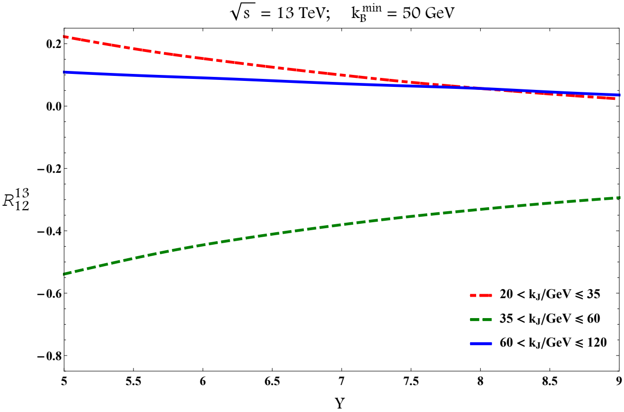


[F. Caporale, F.G. Celiberto, G. Chachamis, D.G.G., A. Sabio Vera (2016)]

Y is the rapidity difference between the most forward/backward jet; $y_J = \frac{Y_A+Y_B}{2}$.

R_{12}^{13} vs Y for three different k_J bins. LLA.

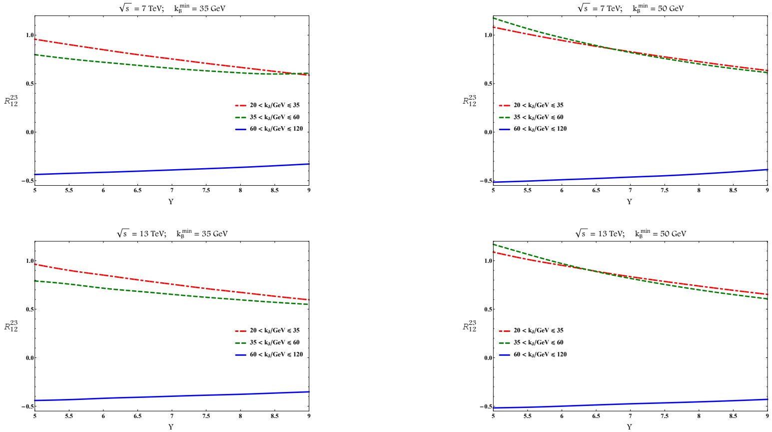
$$k_A^{\min} = 35 \text{ GeV},\ k_B^{\min} = 50 \text{ GeV},\ k_A^{\max} = k_B^{\max} = 60 \text{ GeV (asymmetric)}$$



[F. Caporale, F.G. Celiberto, G. Chachamis, D.G.G., A. Sabio Vera (2016)]

Y is the rapidity difference between the most forward/backward jet; $y_J = \frac{Y_A+Y_B}{2}$.

R_{12}^{23} vs $Y = Y_A - Y_B$, $\sqrt{s}$ and $k_B^{\min}$ for three k_J bins. LLA.



We have entered in an asymptotic regime.

[F. Caporale, F.G. Celiberto, G. Chachamis, D.G.G., A. Sabio Vera (2016)]

Three-jet at hadronic level. NLLA corrections

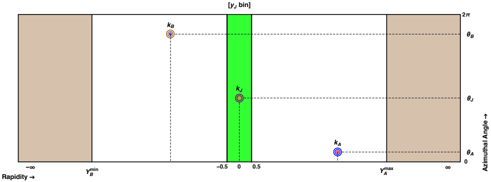
- We introduce higher order corrections to the GGF to check the stability of the perturbative expansion.
- BLM optimization procedure.

[S.J. Brodsky, G.P. Lepage, P.B. Mackenzie (1983)]
[B. Ducloué, L. Szymanowski, S. Wallon (2014)]
[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2015)]

- It allows us to estimate the theoretical uncertainty. Also checked using different PDF's sets.
- Impact factors at NLO are missing! Much more time-consuming but needed to compare with experiment (when data is available).

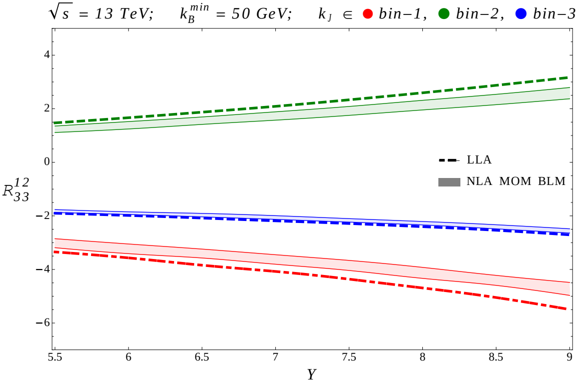
Kinematical cuts

- 35 GeV $\leq k_A \leq$ 60 GeV; 50 GeV $\leq k_B \leq$ 60 GeV (asymmetric cuts)
- $Y = Y_A - Y_B$ fixed Integrating $3 \leq |Y_{A,B}| \leq 4.7$; $y_J = (Y_A + Y_B)/2$ integrated over a rapidity bin of 1
- $\sqrt{s} = 7, 13$ TeV



R_{33}^{12} vs Y at 13 TeV - NLLA

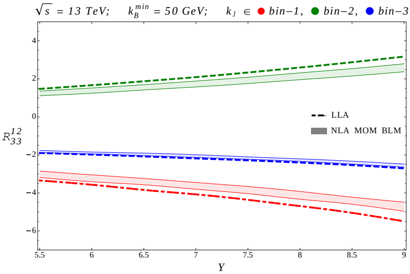
$$k_A^{\rm min} = 35 \text{ GeV}, \; k_B^{\rm min} = 50 \text{ GeV}, \; k_A^{\rm max} = k_B^{\rm max} = 60 \text{ GeV (asymmetric)}$$

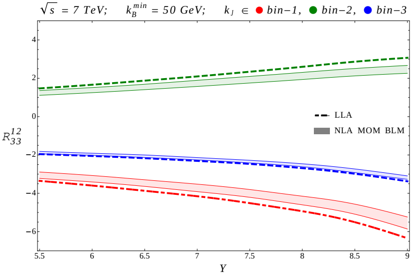


[F. Caporale, F.G. Celiberto, G. Chachamis, D.G.G., A. Sabio Vera (2017)]

Y is the rapidity difference between the most forward/backward jet; y_J in range $\frac{Y_A+Y_B}{2} \pm \frac{1}{2}$.

R_{33}^{12} vs Y at 13 and 7 TeV - NLLA

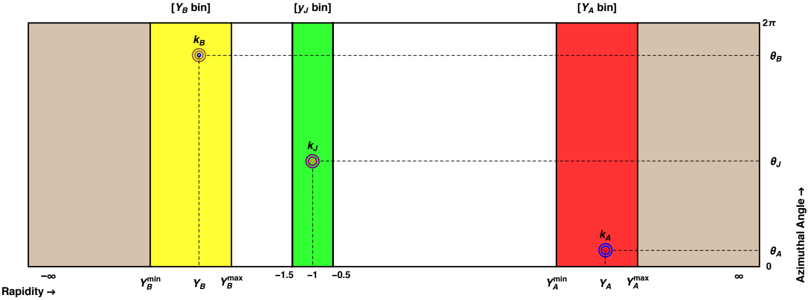




We have entered in an asymptotic regime.

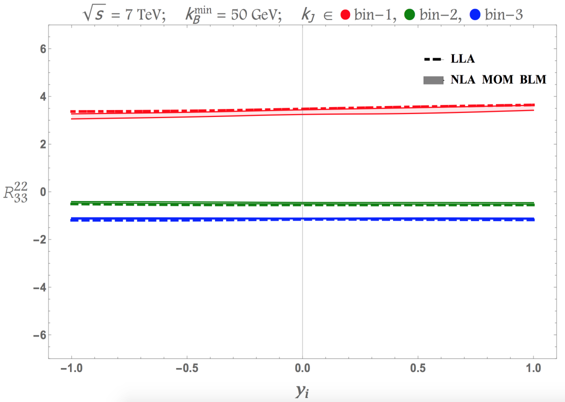
[F. Caporale, F.G. Celiberto, G. Chachamis, D.G.G., A. Sabio Vera (2017)]

Lego plot


[Y_B bin]
[y_J bin]
[Y_A bin]
k_B
k_J
k_A
2π
θ_B
θ_J
θ_A
0
Azimuthal Angle →
$-\infty$
Y_B^{min}
Y_B
Y_B^{max}
−1.5
−1
−0.5
Y_A^{min}
Y_A
Y_A^{max}
∞
Rapidity →

R_{33}^{22} vs y_i at 13 TeV - NLLA

$k_A^{\min} = 35$ GeV, $k_B^{\min} = 50$ GeV, $k_A^{\max} = k_B^{\max} = 60$ GeV (asymmetric)



[F. Caporale, F.G. Celiberto, G. Chachamis, D.G.G., A. Sabio Vera (2017)]

Y_A, Y_B, y_J integrated over bins. y_i is the center of the central jet bin of size 1.

Ongoing project: Veto

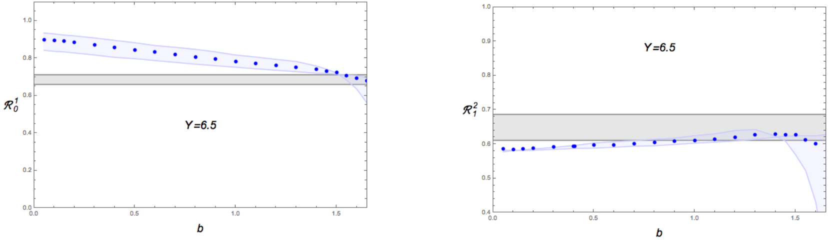
- Introduced by Lipatov and [B. Andersson, G. Gustafson, J. Samuelsson (1996)].

[Carl R. Schmidt (1999)]
[J.R. Forshaw, D.A. Ross, A. Sabio Vera (1999)]

- Gluon rapidities in the ladder constrained by $y_{i+1} - y_i \geq b$. It puts a cutoff in how close the emissions can be.
- Another way of controlling the higher order corrections (NLLA corrections are very large).
- Originally considered in the $Y >> 1$, $Y >> b$ limit. Is it relevant for phenomenology?
- Different implementations depending on the choice of NNLO terms.
- Test it in an observable which experimentalists point out could be between BFKL and DGLAP regimes: Mueller Navelet jets at 7 TeV. Data is already available.

NLLA NLO prediction and experiment at Y=6.5

$$k_A^{\rm min} = k_B^{\rm min} = 35 \text{ GeV},\ k_A^{\rm max} = k_B^{\rm max} = 60 \text{ GeV (symmetric)}$$



[F. Caporale, F.G. Celiberto, G. Chachamis, D.G.G., A. Sabio Vera (in progress)]

Conclusions

- Study of processes with **three** and **four** tagged jets to propose and **predict** new, more exclusive, BFKL observables: **generalized azimuthal correlation** with dependence on the transverse momenta of extra jets.
- The Mueller-Navelet analysis' data can be recycled.
- Ratios of correlation functions used to minimize the influence of higher order corrections.
- Very stable observables under NLLA corrections to the GGF.
- Comparison with other approaches such as fixed order calculations and Monte Carlo simulations are needed to determine if the observable is a genuine BFKL signal.
- Comparison with experimental data suggested and needed to know the window of applicability of the BFKL framework at LHC.

Outlook

- Three- and four-jets at full NLO accuracy (NLLA GGF+NLO IF) : improved kernel(s), scale optimization.
  [F. Caporale, F.G. Celiberto, G. Chachamis, D.G.G., A. Sabio Vera (in progress)]
- Comparison with Monte Carlo event generators.
- CASTOR kinematical cuts.
- Exploring the meaning and application of the veto.
  [F. Caporale, F.G. Celiberto, G. Chachamis, D.G.G., A. Sabio Vera (in progress)]
- Comparison with analyses where the four-jet predictions stem from two independent gluon ladders (double parton scattering).
  [R. Maciula, A. Szczurek (2014, 2015)]
  [K. Kutak, R. Maciula, M. Serino, A. Szczurek, A. van Hameren (2016, 2016)]
- Suggestions from experimentalists are welcome!

Thanks for your attention!!

BACKUP slides

BACKUP slides

How are the observables measured?

- The same data that was used for the MN analysis can be used here.
- Sort the events according to the rapidity difference of their outermost jets Y. We will bin that variable.
- Only events with AT LEAST one additional hard jet in the central region are considered. Apply kinematical cuts.
- For each event, measure the azimuthal angle difference between the forward-central jets, $\Delta\theta_1 \equiv \theta_A - \theta_J - \pi$, and the central-backward jets, $\Delta\theta_2 \equiv \theta_J - \theta_B - \pi$.
- For each event, calculate the relevant quantities $C_{M,N} = \cos\left(M\Delta\theta_1\right)\cos\left(N\Delta\theta_2\right)$.
- Calculate the average of this quantity for all the events in each rapidity bin, $\mathcal{C}_{MN}$.
- Finally, take ratios.

$$\mathcal{R}^{MN}_{PQ} = \frac{\mathcal{C}_{MN}}{\mathcal{C}_{PR}}$$

Motivation

So far, search for BFKL effects had these general drawbacks:

- too low $\sqrt{s}$ or rapidity intervals among tagged particels in the final state
- too inclusive observables, other approaches can fit them

Advent of LHC:

- → higher energies ↔ larger rapidity gaps
- → unique opportunity to **test pQCD in the high-energy limit**
- → disentangle applicability region of energy-log resummation (**BFKL approach**)

[V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975, 1976, 1977)]
[Y.Y. Balitskii, L.N. Lipatov (1978)]

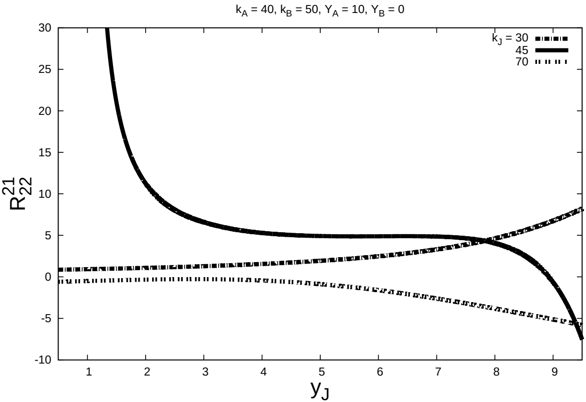
Last years:

hadroproduction of two jets featuring high transverse momenta and well separed in rapidity, so called **Mueller–Navelet jets**...

- ...possibility to define *infrared-safe* observables...
- ...and constrain the PDFs...
- ...theory vs experiment

[B. Ducloué, L. Szymanowski, S. Wallon (2014)]
[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014)]

Partonic prediction of $\mathcal{R}_{22}^{21}$ for $k_J = 30, 45, 70$ GeV

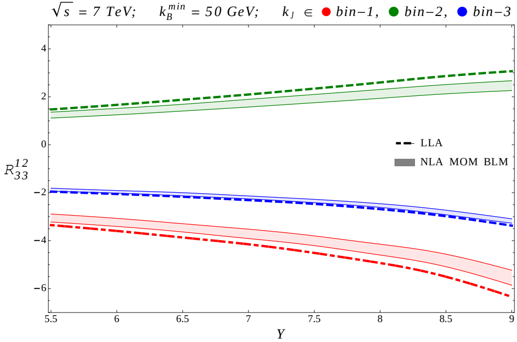


[F. Caporale, G. Chachamis, B. Murdaca, A. Sabio Vera (2015)]

$Y_A - Y_B$ is fixed to 10; y_J varies beetwen 0.5 and 9.5.

R_{33}^{12} vs Y at 7 TeV - NLLA preliminary results

$k_A^{\rm min} = 35$ GeV, $k_B^{\rm min} = 50$ GeV, $k_A^{\rm max} = k_B^{\rm max} = 60$ GeV (asymmetric)

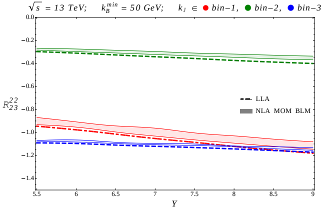


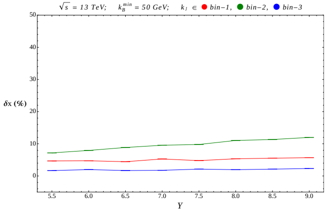
[F. Caporale, F.G. Celiberto, G. Chachamis, D. G.G., A. Sabio Vera (in progress)]

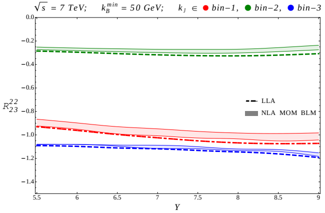
Y is the rapidity difference between the most forward/backward jet; $y_J = \frac{Y_A + Y_B}{2}$.

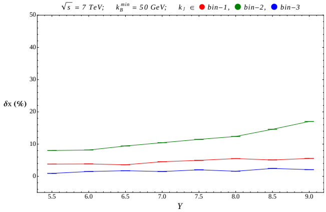
BACKUP slides

R_{23}^{22} vs Y at 13 and 7 TeV - NLLA preliminary results









[F. Caporale, F.G. Celiberto, G. Chachamis, D.G.G., A. Sabio Vera (in progress)]

Four-jets: generalized azimuthal coefficients - partonic level

$$\mathcal{C}_{MNL} = \int_0^{2\pi} d\vartheta_A \int_0^{2\pi} d\vartheta_B \int_0^{2\pi} d\vartheta_1 \int_0^{2\pi} d\vartheta_2 \;\cos\left(M\left(\vartheta_A - \vartheta_1 - \pi\right)\right)$$

$$\cos\left(N\left(\vartheta_1 - \vartheta_2 - \pi\right)\right)\cos\left(L\left(\vartheta_2 - \vartheta_B - \pi\right)\right) \frac{d^6\sigma^{4-\text{jet}}\left(\vec{k_A}, \vec{k_B}, Y_A - Y_B\right)}{dk_1\, dy_1\, d\vartheta_1\, dk_2\, d\vartheta_2\, dy_2}$$

$$= \frac{2\pi^2 \bar{\alpha}_s\left(\mu_R\right)^2}{k_1 k_2}\left(-1\right)^{M+N+L}\left(\tilde{\Omega}_{M,N,L} + \tilde{\Omega}_{M,N,-L} + \tilde{\Omega}_{M,-N,L}\right.$$

$$\left. + \tilde{\Omega}_{M,-N,-L} + \tilde{\Omega}_{-M,N,L} + \tilde{\Omega}_{-M,N,-L} + \tilde{\Omega}_{-M,-N,L} + \tilde{\Omega}_{-M,-N,-L}\right)$$

with

$$\tilde{\Omega}_{m,n,l} = \int_0^{+\infty} dp_A\, p_A \int_0^{+\infty} dp_B\, p_B \int_0^{2\pi} d\phi_A \int_0^{2\pi} d\phi_B$$

$$\frac{e^{-im\phi_A}\, e^{il\phi_B}\left(p_A e^{i\phi_A} + k_1\right)^n \left(p_B e^{-i\phi_B} - k_2\right)^n}{\sqrt{\left(p_A^2 + k_1^2 + 2p_A k_1 \cos\phi_A\right)^n}\sqrt{\left(p_B^2 + k_2^2 - 2p_B k_2 \cos\phi_B\right)^n}}$$

$$\varphi_m\left(|\vec{k_A}|, |\vec{p_A}|, Y_A - y_1\right)\varphi_l\left(|\vec{p_B}|, |\vec{k_B}|, y_2 - Y_B\right)$$

$$\varphi_n\left(\sqrt{p_A^2 + k_1^2 + 2p_A k_1 \cos\phi_A}, \sqrt{p_B^2 + k_2^2 - 2p_B k_2 \cos\phi_B}, y_1 - y_2\right)$$

Four-jets: generalized azimuthal coefficients - partonic level

$$\mathcal{C}_{MNL} = \int_0^{2\pi} d\vartheta_A \int_0^{2\pi} d\vartheta_B \int_0^{2\pi} d\vartheta_1 \int_0^{2\pi} d\vartheta_2 \;\cos\left(M\left(\vartheta_A - \vartheta_1 - \pi\right)\right)$$

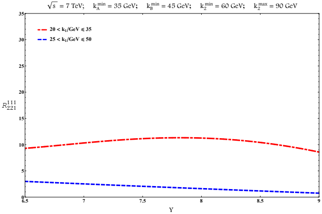
$$\cos\left(N\left(\vartheta_1 - \vartheta_2 - \pi\right)\right) \cos\left(L\left(\vartheta_2 - \vartheta_B - \pi\right)\right) \frac{d^6\sigma^{4-\text{jet}}\left(\vec{k_A}, \vec{k_B}, Y_A - Y_B\right)}{dk_1 dy_1 d\vartheta_1 dk_2 d\vartheta_2 dy_2}$$

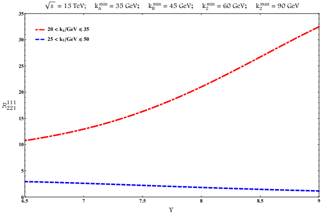
Main observables: **generalized azimuthal correlations**

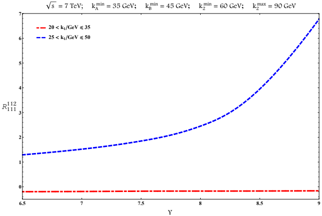
$$\mathcal{R}^{MNL}_{PQR} = \frac{C_{MNL}}{C_{PRQ}} = \frac{\langle \cos(M(\vartheta_A - \vartheta_1 - \pi)) \cos(N(\vartheta_1 - \vartheta_2 - \pi)) \cos(L(\vartheta_2 - \vartheta_B - \pi)) \rangle}{\langle \cos(P(\vartheta_A - \vartheta_1 - \pi)) \cos(Q(\vartheta_1 - \vartheta_2 - \pi)) \cos(R(\vartheta_2 - \vartheta_B - \pi)) \rangle}$$

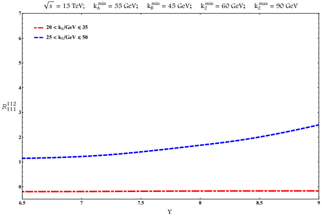
BACKUP slides

R^{111}_{221} and R^{112}_{111} vs $Y = Y_A - Y_B$ and $\sqrt{s}$ for two k_1 bins









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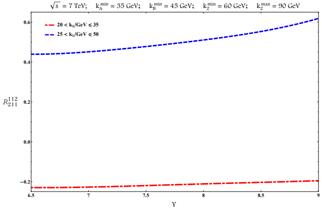
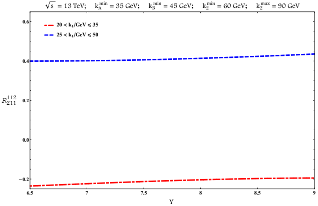
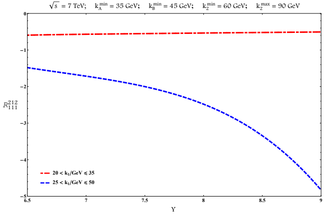
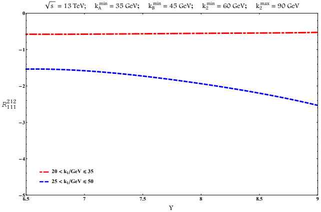
R^{112}_{211} and R^{212}_{111} vs $Y = Y_A - Y_B$ and $\sqrt{s}$ for two k_1 bins









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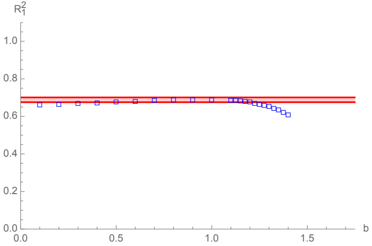
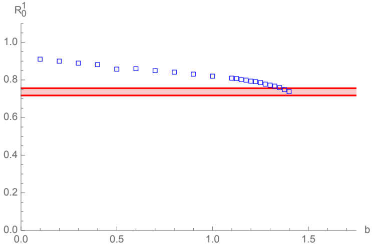
NLLA NLO and exp at Y=4.75

$$k_A^{\text{min}} = k_B^{\text{min}} = 35 \text{ GeV}, \; k_A^{\text{max}} = k_B^{\text{max}} = 60 \text{ GeV (symmetric)}$$





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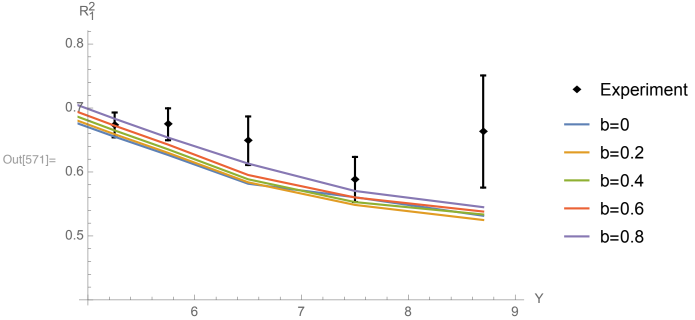
R_1^2 vs Y at 7 TeV - NLLA NLO

$$k_A^{\min} = k_B^{\min} = 35 \text{ GeV},\ k_A^{\max} = k_B^{\max} = 60 \text{ GeV (symmetric)}$$



[F. Caporale, F.G. Celiberto, G. Chachamis, D.G.G., A. Sabio Vera (in progress)]