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Universidad de las Américas Puebla

Recent results in low x phenomenology and theory

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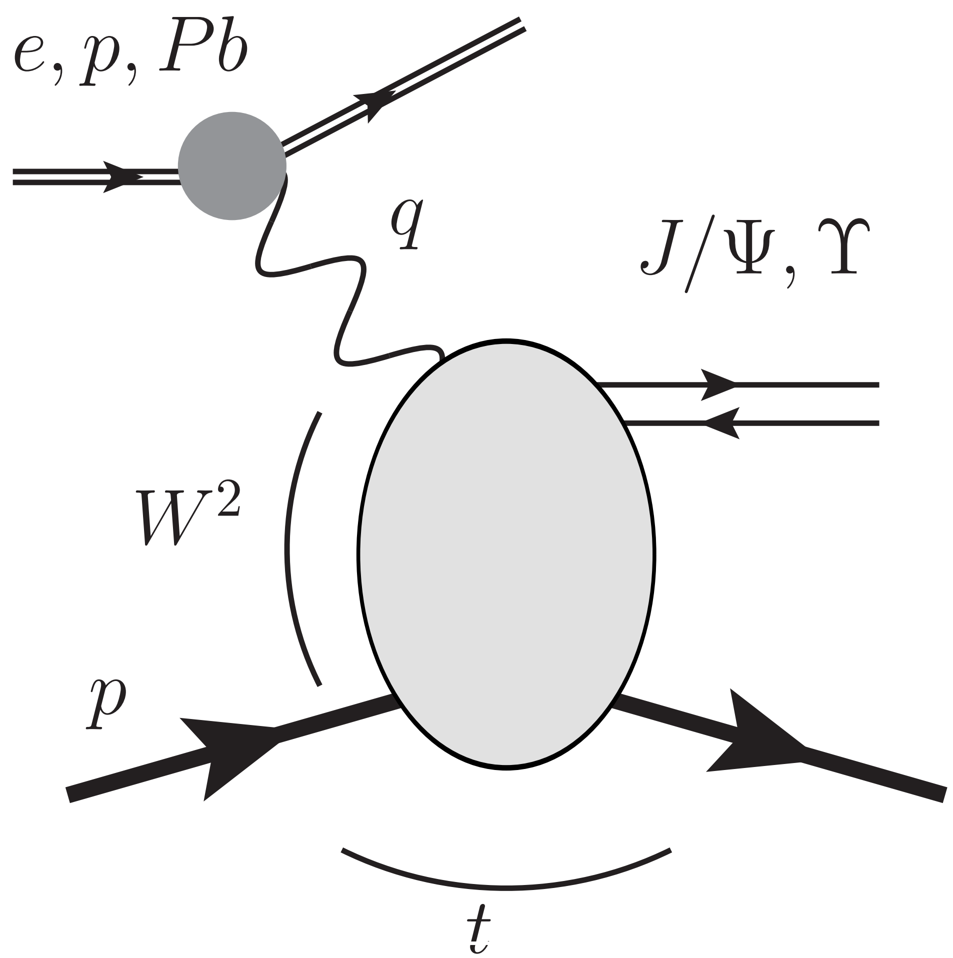
— Outline —

1. **Photo-production of vector mesons as a probe of low x evolution: the case of excited states**
   (with Alfredo Arroyo Garcia, UDLAP)

2. **Lipatov's high energy effective action and the Color Glass Condensate formalism**

3. **TMD splitting functions from kT factorization**
   (with Krzysztof Kutak, Aleksander Kusina, Cracow; Mirko Serino; Beer Sheva)

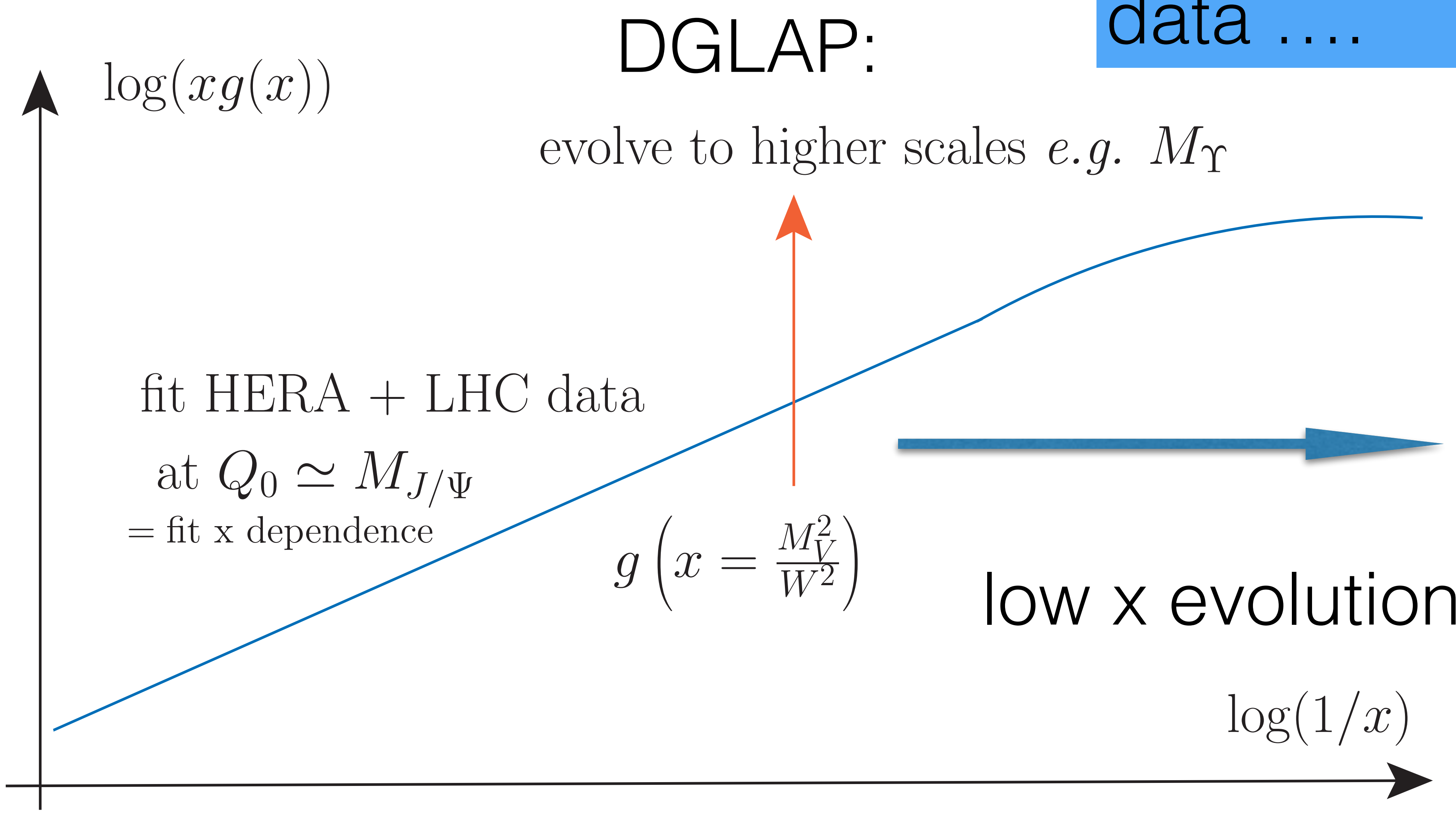
photo-production of J/Ψ and Υ: explore proton at ultra-small x



- measured at HERA (ep) and LHC (pp, ultra-peripheral pPb)
- charm and bottom mass provide hard scale ➡ pQCD
- exclusive process, but allows to relate to inclusive gluon

reach values down to $x = 4 \times 10^{-6} \rightarrow$ (unique ?) opportunity to explore the low x gluon

different ways to analyze data ....



- low x evolution with non-linear effects, dipole models: predict, compare to data, refit, …
- DGLAP: evolution from J/Ψ (2.4 GeV^2) to ϒ (22.4 GeV^2) → constrain pdfs at small x, not really a benchmark for saturation effects (effects die away fast, instability)
- Better: BFKL (linear low x evolution)

How to do that?
relate exclusive XSec. to inclusive gluon distribution
(imitate pdf studies)

procedure:

a) calculate diff. Xsec. at $t = 0$

→ *exclusive* scattering amplitude can be expressed through *inclusive* gluon distribution

b) parametrize t dependence $\frac{d\sigma(t)}{dt} = \frac{d\sigma(t=0)}{dt} \cdot e^{-|t|B_D(W)}$,

slope $B_D(W) = b_0 + 4\alpha' \ln \frac{W}{W_0}$ + fix parameters by (HERA) data
(here: values proposed by [Jones, Martin, Ryskin, Teubner; 1307.7099, 1312.6795])

→ cross-section: $\sigma^{\gamma p \to V p}(W) = \underbrace{\frac{1}{B_D(W)}}_{\text{phenomenological}} \underbrace{\left.\frac{d\sigma^{\gamma p \to V p}}{dt}\right|_{t=0}}_{\text{BFKL / theory}}$

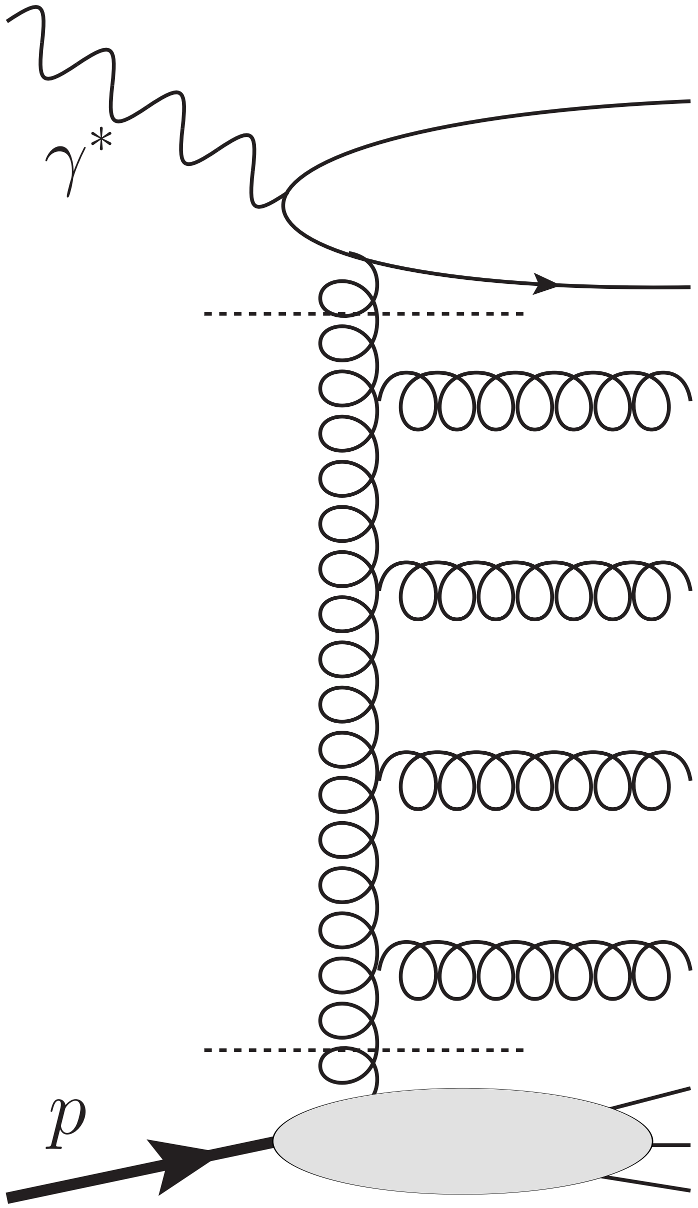
Studied so far: J/Ψ and Υ(1s)

[Bautista, Fernando Tellez, MH; 1607.05203]

Procedure in a nut-shell

- take light-cone wave function used for dipole/ saturation models (from literature) and calculate their transform to Mellin space
- combine with fit of NLO BFKL gluon
  [MH, Salas, Sabio Vera; 1209.1353; 1301.5283]
- improve the calculation of the real part of the scattering amplitude

The underlying NLO BFKL fit to DIS data



$$F_2(x,Q^2) = \int_0^\infty d\boldsymbol{k}^2 \int_0^\infty \frac{d\boldsymbol{q}^2}{\boldsymbol{q}^2} \Phi_2\left(\frac{\boldsymbol{k}^2}{Q^2}\right) \mathcal{F}^{\text{DIS}}_{\text{BFKL}}(x, \boldsymbol{k}^2, \boldsymbol{q}^2) \Phi_p\left(\frac{\boldsymbol{q}^2}{Q_0^2}\right)$$

virtual photon: quarks mass-less, $n_f = 4$ fixed

proton impact factor: $\Phi_p\left(\frac{\boldsymbol{q}^2}{Q_0^2}, \delta\right) = \frac{\mathcal{C}}{\pi\Gamma(\delta)}\left(\frac{\boldsymbol{q}^2}{Q_0^2}\right)^\delta e^{-\frac{\boldsymbol{q}^2}{Q_0^2}}$

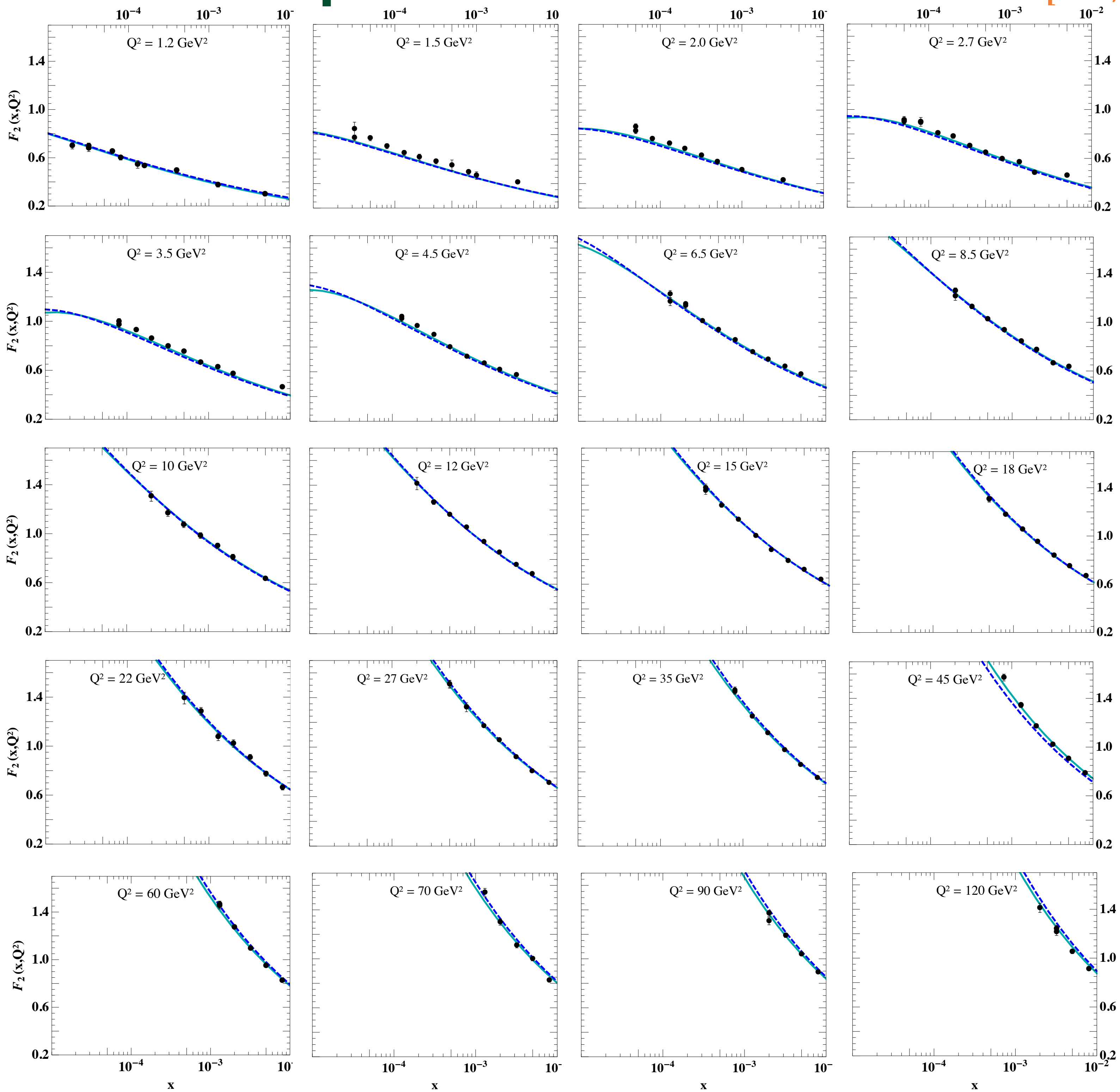
free parameters of proton impact factor from fit to combined HERA data [MH, Salas, Sabio Vera; 1209.1353; 1301.5283]

→ allows for definition of unintegrated gluon density [Chachamis, Deak, MH, Rodrigo, Sabio Vera; 1507.05778]

$$G(x, \boldsymbol{k}^2, Q_0^2) = \int \frac{d\boldsymbol{q}^2}{\boldsymbol{q}^2} \mathcal{F}^{\text{DIS}}_{\text{BFKL}}(x, \boldsymbol{k}^2, \boldsymbol{q}^2) \Phi_p\left(\frac{\boldsymbol{q}^2}{Q_0^2}\right)$$

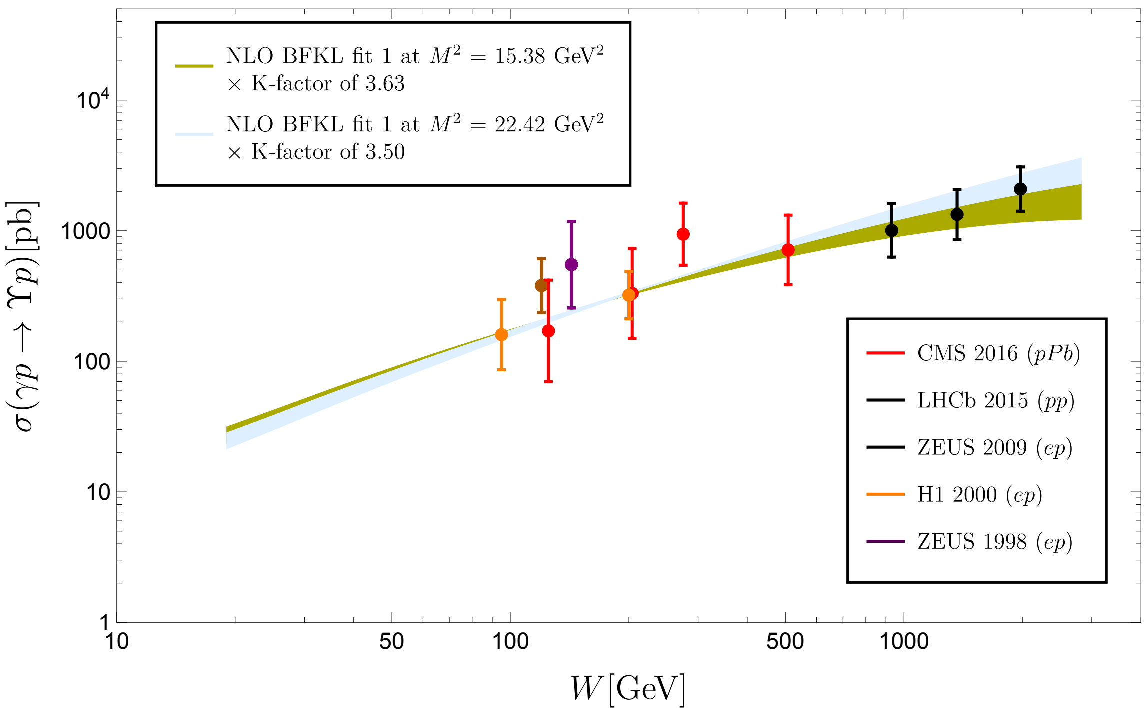
	virt. photon impact factor	Q_0/GeV	δ	$\mathcal{C}$	Λ_{QCD}/ GeV
fit 1	leading order (LO)	0.28	8.4	1.50	0.21
fit 2	LO with kinematic improvements	0.28	6.5	2.35	0.21

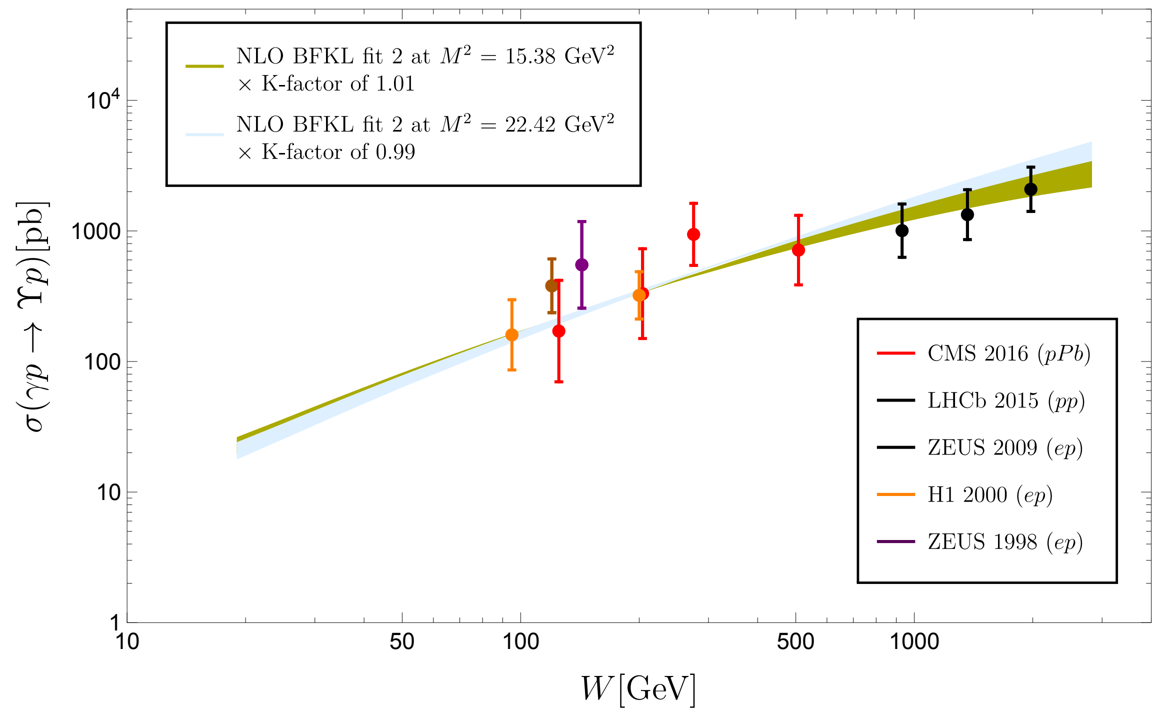
Good description of cominbed HERA [MH, Salas, Sabio Vera; 1209.1353; 1301.5283]

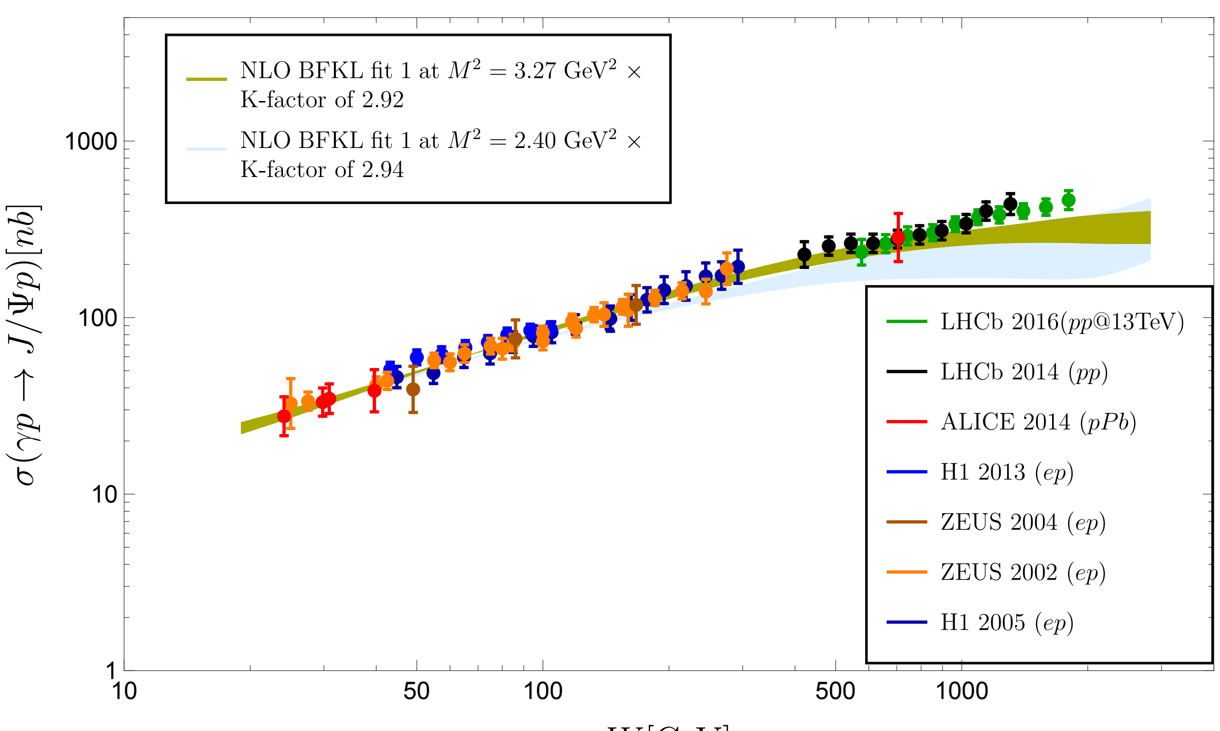


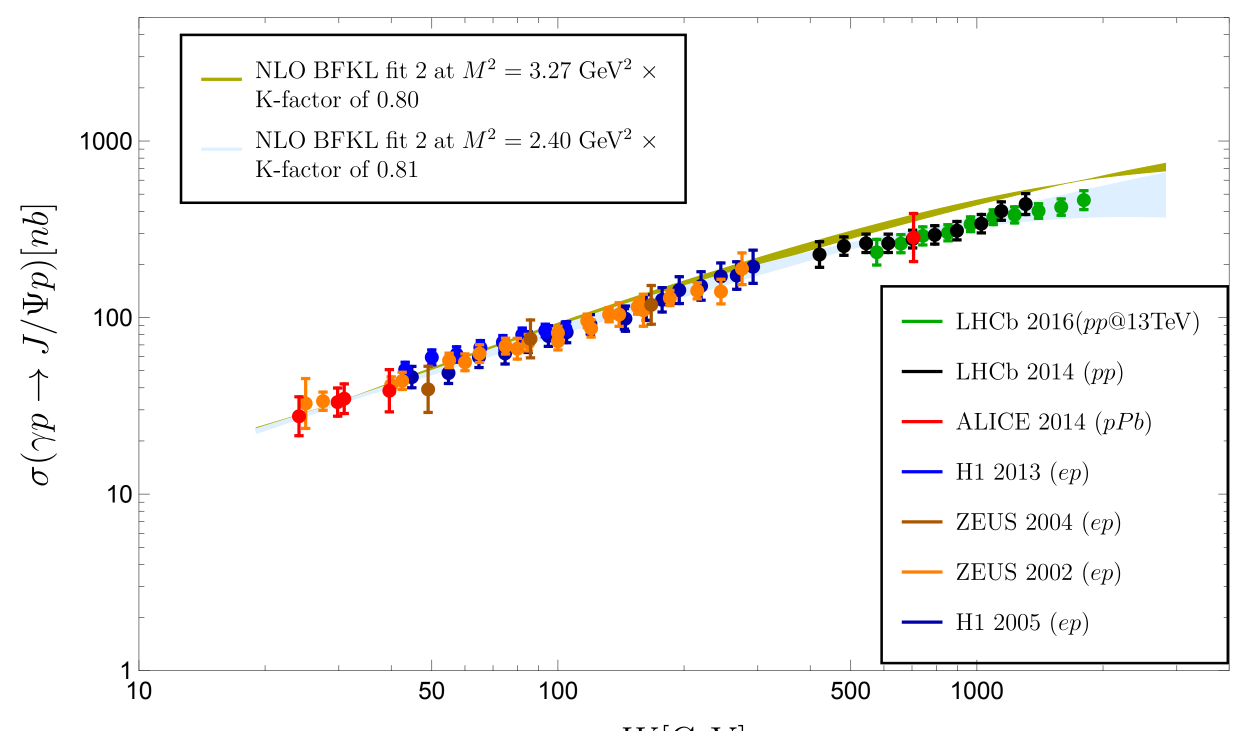
data: [H1 & ZEUS collab. 0911.0884]

and good description J/Ψ and Υ data





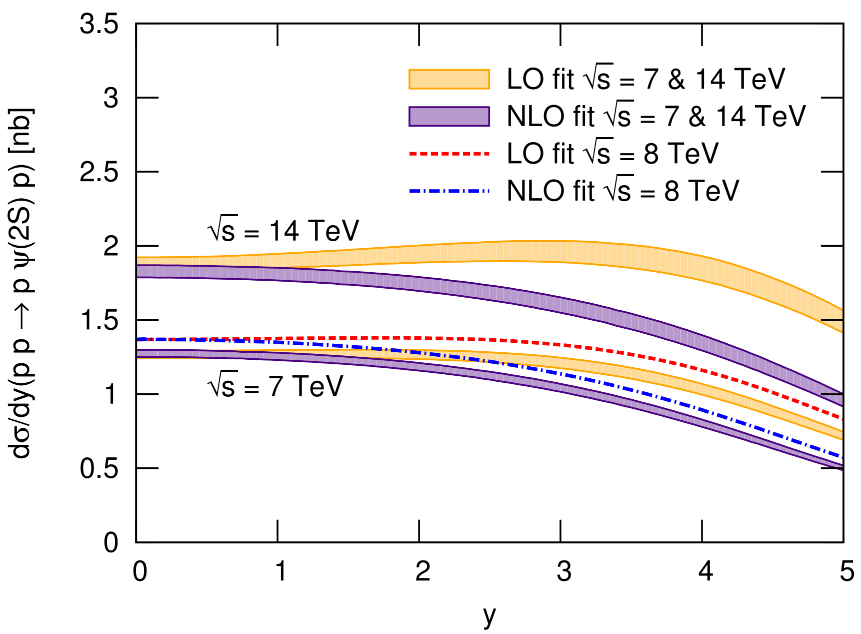




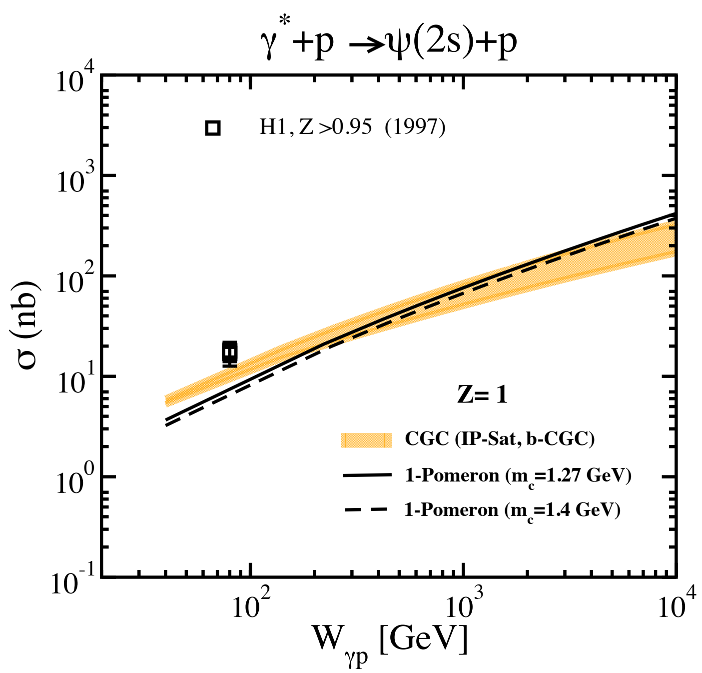
evolution from HERA to LHC: direct test of BFKL evolution

there are also excited states: Ψ(2s) and Υ(2s)

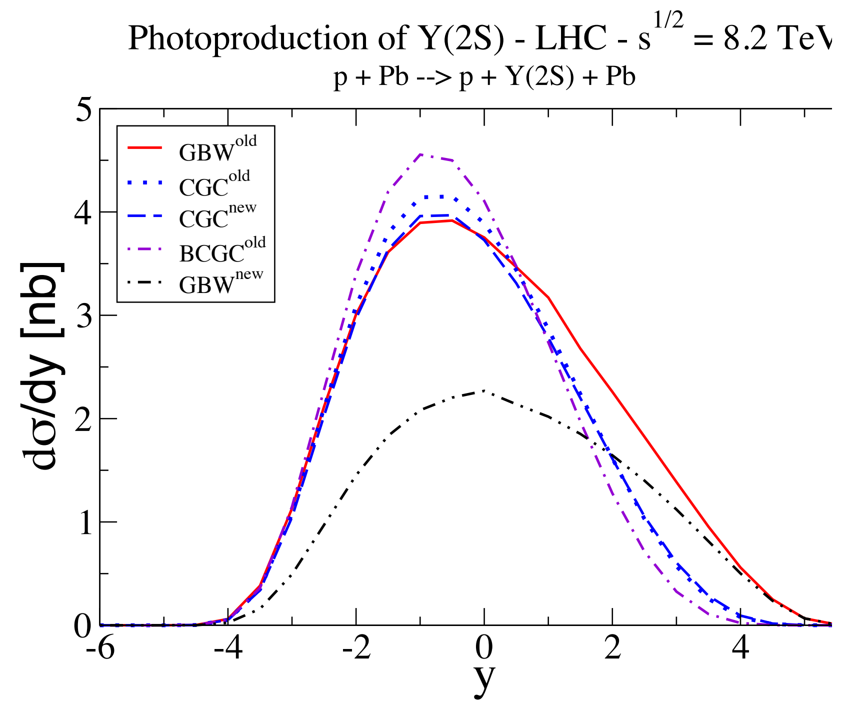
and theory predictions both based on DGLAP and saturation models



[Jones et. al.; 1312.6795]



[Nestor et. al.; 1402.4831]



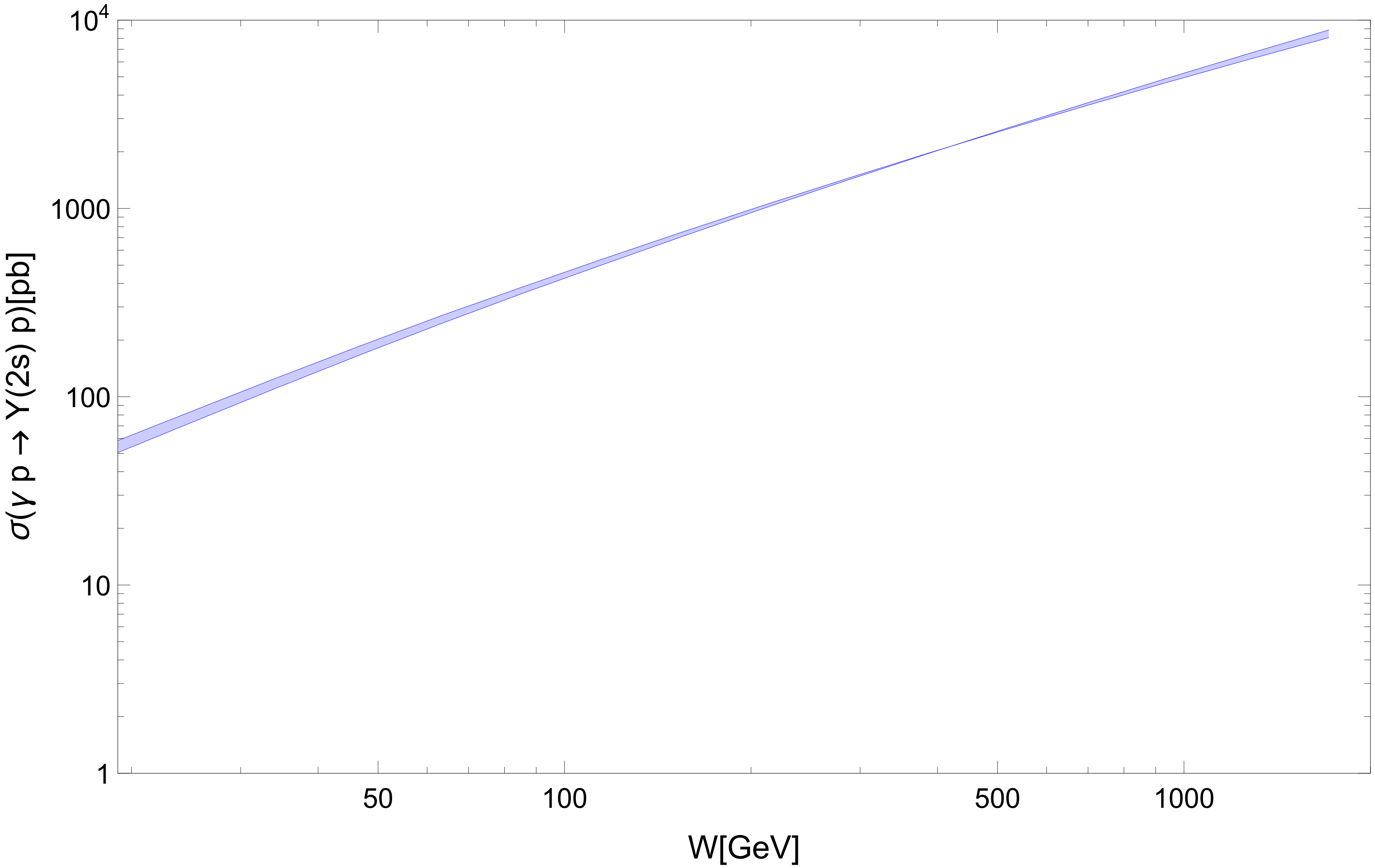
[Gay Ducati et. al.; 1610.06647]

to study it within the BFKL framework, follow the same path as before

= calculate the Mellin transform of the light-front wave function of excited states

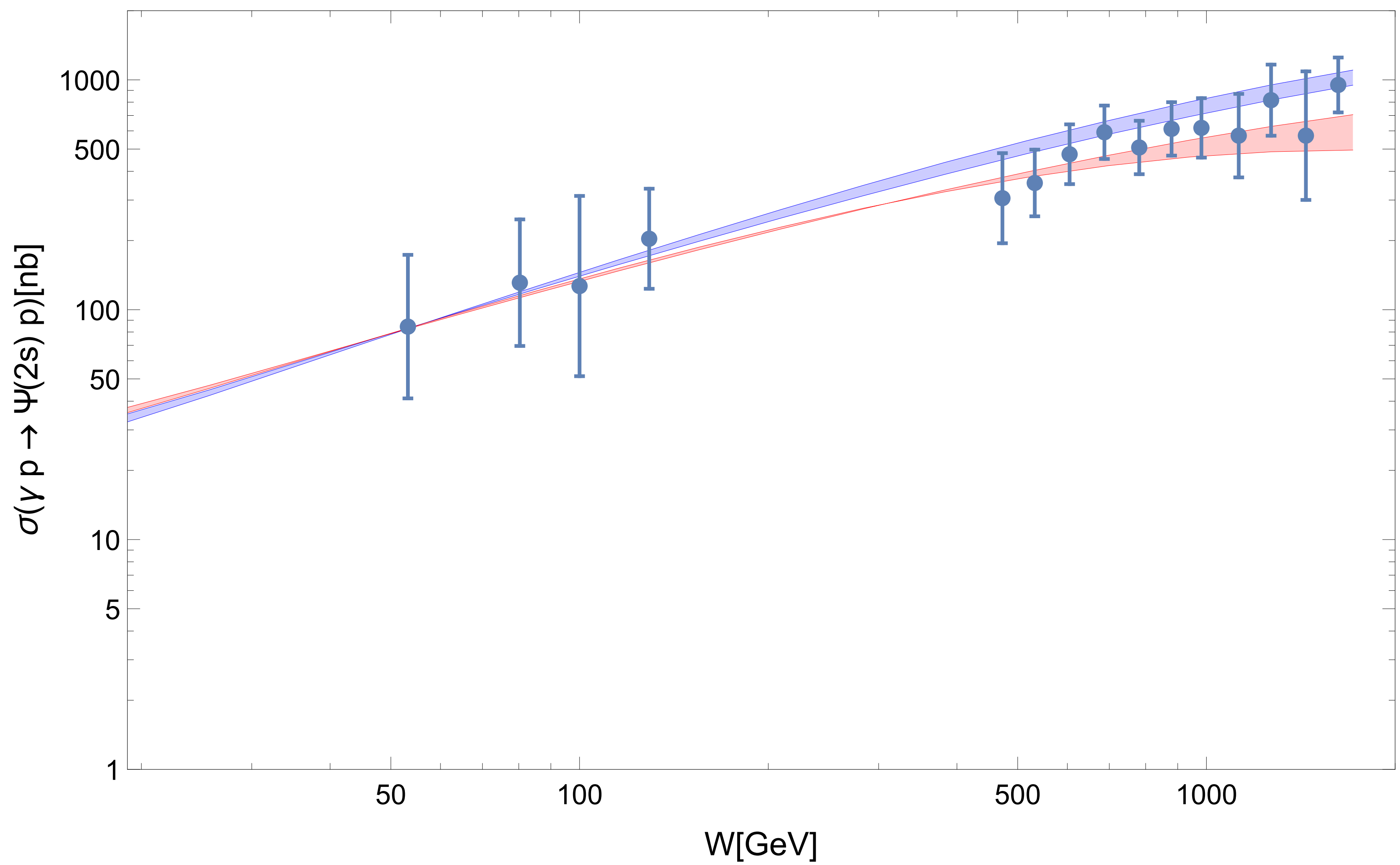
$$
\begin{aligned}
\Phi_{V,T}(\gamma,z,M) = 8\pi^2 e\hat{e}_f N_T \frac{\Gamma(\gamma)\Gamma(1-\gamma)}{m_f^2}\left(\frac{8z(1-z)}{M^2R_{2s}^2}\right)^{\gamma} e^{-\frac{m_f^2R_{2s}^2}{8z(1-z)}+\frac{m_f^2R_{2s}^2}{2}}\left(\frac{m_f^2R_{2s}^2}{8z(1-z)}\right)^2 \cdot \\
\cdot\left[\left(1+\alpha_{2s}\left(2+\frac{m_f^2R_{2s}^2}{4z(1-z)}-m_f^2R_{2s}^2\right)\right)U\left(2-\gamma,1,\frac{\epsilon^2R_{2s}^2}{8z(1-z)}\right)-...\right] \\
\left[...-2\left(2-\gamma\right)^2 U\left(3-\gamma,1,\frac{\epsilon^2R_{2s}^2}{8z(1-z)}\right)+...\right] \\
\left[+\left[z^2+(1-z)^2\right]\epsilon^2\left((2-\gamma)\left(1+\alpha_{2s}\left(\frac{m_f^2R_{2s}^2}{4z(1-z)}-m_f^2R_{2s}^2\right)\right)U\left(3-\gamma,2,\frac{\epsilon^2R_{2s}^2}{8z(1-z)}\right)\right)+\right] \\
\left[...+2\left(2-\gamma\right)^2(3-\gamma)\alpha_{2s}\cdot U\left(4-\gamma,2,\frac{\epsilon^2R_{2s}^2}{8z(1-z)}\right)\right]
\end{aligned}
\tag{7}
$$

Preliminary results: Υ(2s)



- vary renormalization scale to check stability
  → in general looks good
- don't trust normalization

Preliminary results: $\boldsymbol{\Psi}$(2s)



- data: H1 and LHCb; need to adjust normalization → problem already there for J/Ψ & Y: most likely correction to impact factor
- two choices of the hard scale are shown

Summary vector boson

- perturbative low x evolution (=BFKL) appears to describe also excited states of vector mesons (within errors)
- need to fix normalization constant (→ similar to J/Ψ and Υ(1s));
  here problem: low energy points with huge error bars
- normalization issue: virtual photon impact factor used in the underlying DIS fit, is kinematically improved → should do the same for vector bosons

2. Lipatov’s high energy effective action and the Color Glass Condensate formalism

theoretical descriptions in the high energy limit:
2 alternatives

- unintegrated gluon densities

  more formally: formalism based on reggeized gluons & effective production vertices
  — t-channel picture

- vs. dipole picture

  more formally: formalism based on propagators which resum strong background field
  — s-channel picture

- to relate both approaches: difficult at the level of the formalism, mainly done for evolution equations and/or observables

- examples: BFKL evolution, BKP evolution, triple Pomeron vertex from JIMWLK or BK evolution

[Bartels, Lipatov, Vacca, hep-ph/0404110]
[Chirilli, Szymanowski, Wallon,1010.0285]
[Ayala, Cazaroto, Hernandez, Jalilian-Marian; 1408.3080] …

- in general: very similar structure, but direct one-to-one correspondence not obvious

an action formalism for reggeized gluons: Lipatov's high energy effective action [Lipatov; hep-ph/9502308]

- idea: factorize QCD amplitudes in the high energy limit through introducing a new kind of field: the reggeized gluon

- the reggeized gluon is globally charged under $SU(N_C)$, but invariant under local gauge transformation → gauge invariant factorization

- took a while, now we know Lipton's action can be used for NLO calculation within the BFKL framework

[MH, Sabio Vera;1110.6741]

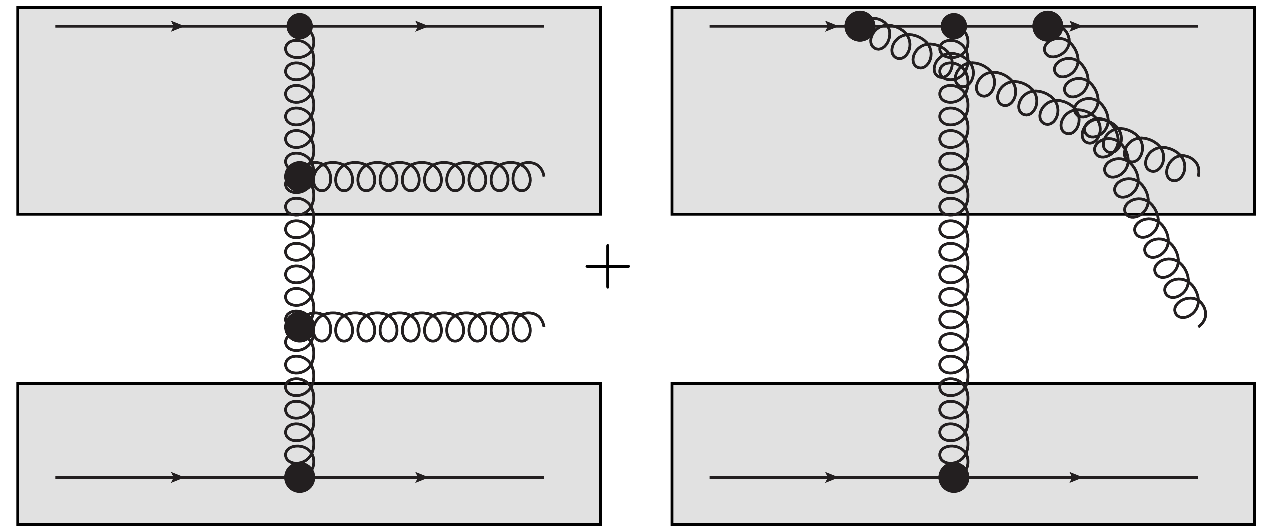
[Chachamis, MH, Madrigal, Sabio Vera; 1202.064, 1212.4992, 1307.2591]

[MH, Madrigal, Murdaca, Sabio Vera; 1404.2937, 1406.5625, 1409.6704]

[Bartels, Fadin, Lipatov,Vacca; 1210.0797] divide final state particles into clusters of particles "local in rapidity"

for each cluster

- integrate out specific details of fast $+/-$ fields
- dynamics in local cluster: QCD Lagrangian + universal eikonal factor
  (up to power suppressed corrections)



→ effective field theory for **each cluster** of particles local in rapidity

$$S_{\text{eff}} = S_{\text{QCD}} + S_{\text{ind.}}$$

non-local emissions from S_{ind}

$$S_{\text{ind.}} = \int \mathrm{d}^4 x \left\{ \mathrm{tr}\left[\left(W_-[v(x)] - A_-(x) \right) \partial_\perp^2 A_+(x) \right] + \mathrm{tr}\left[\left(W_+[v(x)] - A_+(x) \right) \partial_\perp^2 A_-(x) \right] \right\}.$$

eikonal

Lipatov's effective action & the CGC formalism

- numerous attempts to compare both formalisms, mainly on the level of effective Lagrangians

[Jalilian-Marian, Kovner, Leonidov, Weigert; NPB504, 415 (1997)]

[Hatta; hep-ph/0607126]

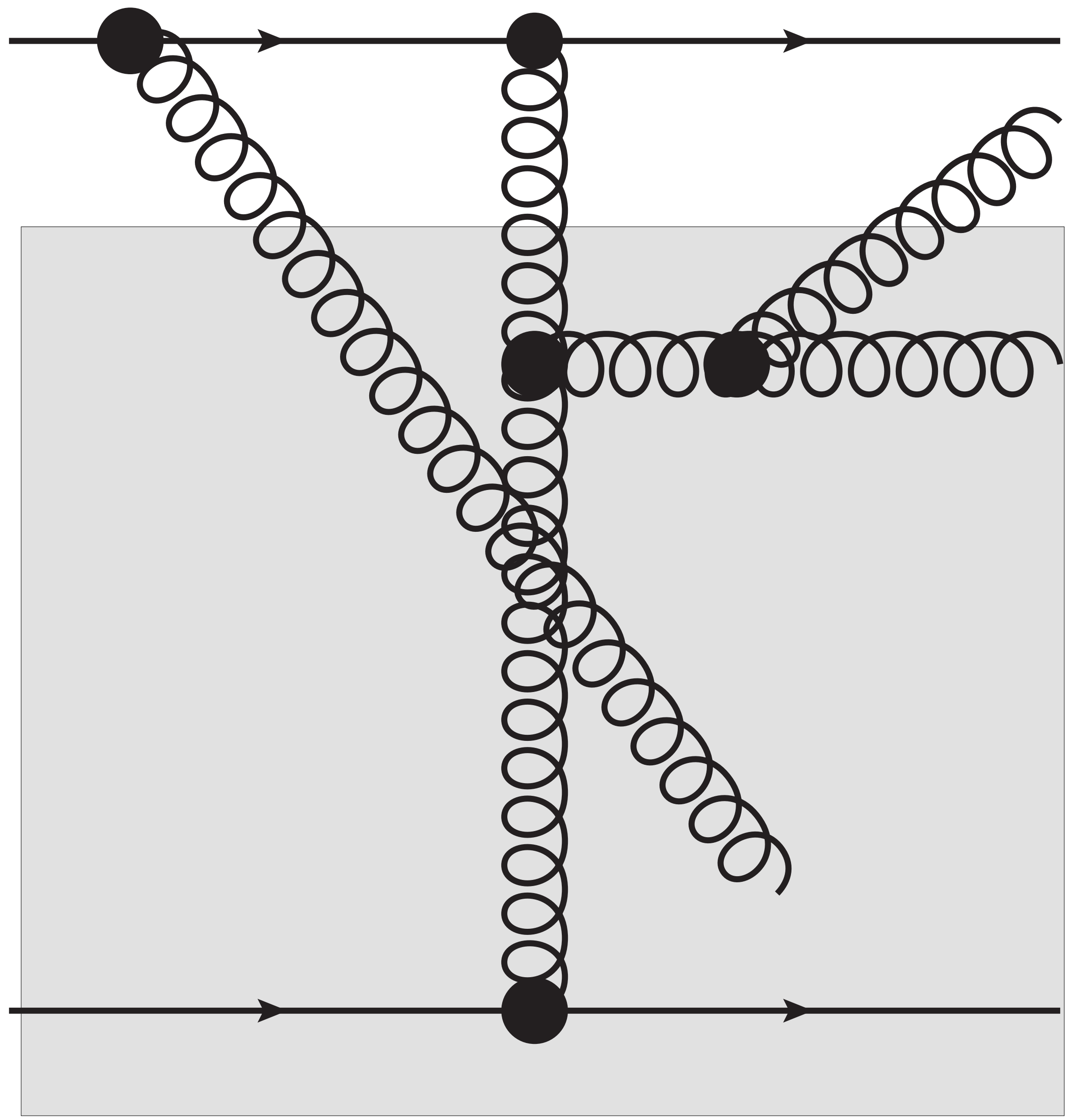
[Bondarenko, Lipatov,Pozdnyakov, Prygarin;1706.0027, 1708.05183]

[Bondarenko, Zubkov;1801.08066]

- here: pragmatic approach: compare results for scattering amplitudes & ***propagators***

[MH, 1802.06755]

- to start: quasi-elastic *i.e.* dilute/ dense scattering in presence of strong reggeized gluon field



- quasi-elastic scattering = integrate out fields only from one side
- corresponds to: scattering of dilute projectile in strong gluon field of target
- effective action: resum interaction of QCD fields with ∞ # of reggeized gluon fields (= transmit interaction with target)

quarks: relatively straightforward→ high energy kinematics allows to resum interaction into Wilson line

gluon: at first difficult ….

a trick proposed by Lipatov in 1995

$$V^\mu(x) = v^\mu(x) + \frac{1}{2}(n_-)^\mu B_+[v_-]$$

use a special parametrization of the gluon field

$$B_\pm[v_\mp] = U[v_\mp]A_\pm U^{-1}[v_\mp]$$

sort of: a gauge rotation of the reggeized gluon field $A_\pm$

Wilson line operator

$$U[v_\pm] = \frac{1}{1 + \frac{g}{\partial_\pm}v_\pm}$$

and its inverse …

$$U^{-1}[v_\pm] = 1 + \frac{g}{\partial_\pm}v_\pm$$

why of interest?

transformation properties

$$V^\mu(x) = v^\mu(x) + \frac{1}{2}(n_-)^\mu B_+[v_-]$$

shifted field transforms like gauge field → consistent transformation properties

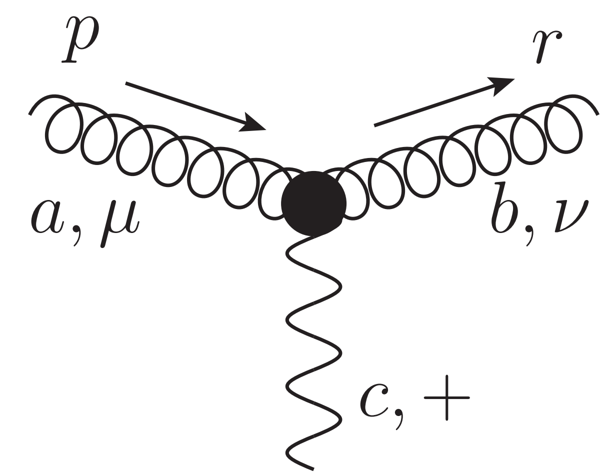
$$\delta V_\pm = [D_\pm, \chi] + [gB_\pm, \chi] = [D_\pm + gB_\pm, \chi]$$

this would NOT be true for $v_\pm \to V_\pm = v_\pm + A_\pm$
since

$$\delta_{\mathrm{L}} A_\pm = \frac{1}{g}[A_\pm, \chi_L] = 0$$

$$\delta_{\mathrm{L}} V_\mu = \frac{1}{g}[D_\mu, \chi_L],$$

a new gluon-gluon-reggeized gluon vertex



$$\Gamma_+^{\nu\mu}(r,p) = p^+ g^{\mu\nu} - (n^+)^\mu p^\nu - (n^+)^\nu r^\mu + \frac{r \cdot p}{p^+}(n^+)^\mu (n^+)^\nu$$

- already written down by Lipatov in 1995
- good properties: current conservation

$$r_\nu \cdot \Gamma_+^{\nu\mu}(r,p) = 0 = \Gamma_+^{\nu\mu}(r,p) \cdot p_\mu$$

- properties Lipatov didn't like: violates for individual Feynman diagrams Steinmann relations

argue: shifted version of a theory which respects Steinmann relations → OK for physical observables

another important property

$$n_\nu^+ \cdot \Gamma_+^{\nu\mu}(r,p) = 0 = \Gamma_+^{\nu\mu}(r,p) \cdot n_\mu^+$$

$$\Gamma_+^{\nu\alpha}(r,k) \cdot (-g_{\alpha\alpha'}) \cdot \Gamma_+^{\alpha'\mu}(k,p) = -p^+ \Gamma_+^{\nu\mu}(r,p)$$

- reggeization as defined by Bartels, Wüsthoff and Bartels, Ewerz → n reggeized gluons = 1 reggeized gluon × factor
- technical details aside: allows to sum up ∞ # of reggeized gluons into a Wilson line of reggeized gluons

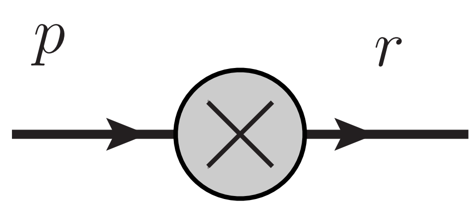
the reggeized gluon field as a shock wave

can argue:

$$A_+(x) = 2 \cdot \alpha(\boldsymbol{x})\delta(x^+)$$

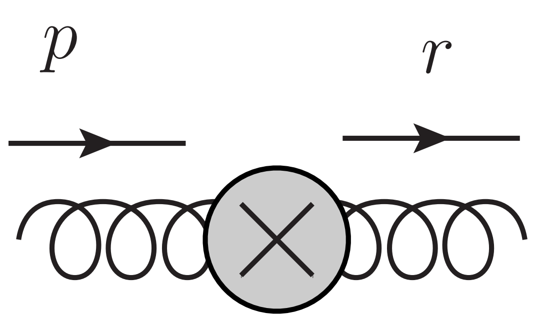
- used all the time in CGC calculation
- Lipatov's action: reggeized gluon field = classical field for given cluster
- dynamics: reggeized gluon propagator = connect clusters → imposes such a parametrization

vertices which resum interaction with an arbitrary # of reggeized gluon fields



$$= \tau_F(q,-r) = 2\pi\delta(p^+ - r^+)\not{n}^+ \int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{r})}$$

$$\cdot \left[\theta(p^+)\left[W(\boldsymbol{z}) - 1\right] - \theta(-p^+)\left[\left[W(\boldsymbol{z})\right]^\dagger - 1\right]\right].$$



$$= \tau^{ab}_{G,\nu\mu}(p,-r) = -4\pi\delta(p^+ - r^+)\Gamma_{\nu\mu}(r,p)\int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{r})}$$

$$\cdot \left[\theta(p^+)\left[U^{ba}(\boldsymbol{z}) - \delta^{ab}\right] - \theta(-p^+)\left[\left[U^{ba}(\boldsymbol{z})\right]^\dagger - \delta^{ab}\right]\right].$$

interaction resumed into Wilson lines

$$U^{ab}(\boldsymbol{z}) = \mathrm{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{\infty} dz^+ \tilde{A}_+\right) \qquad W(\boldsymbol{z}) = \mathrm{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{\infty} dz^+ A_+\right)$$

- vertices agree with CGC expressions for light-cone gauge →Lipatov's action: any gauge possible

- differs in content of Wilson line: reggeized gluon field vs. background field in light-cone gauge

- can show: $W[A](x) = e^{ig\alpha^a(\boldsymbol{x})t^a}$
  not possible for light-cone gauge background field

- for experts: induced vertices allow to reproduce the complete color structure (also anti-symmetric terms)

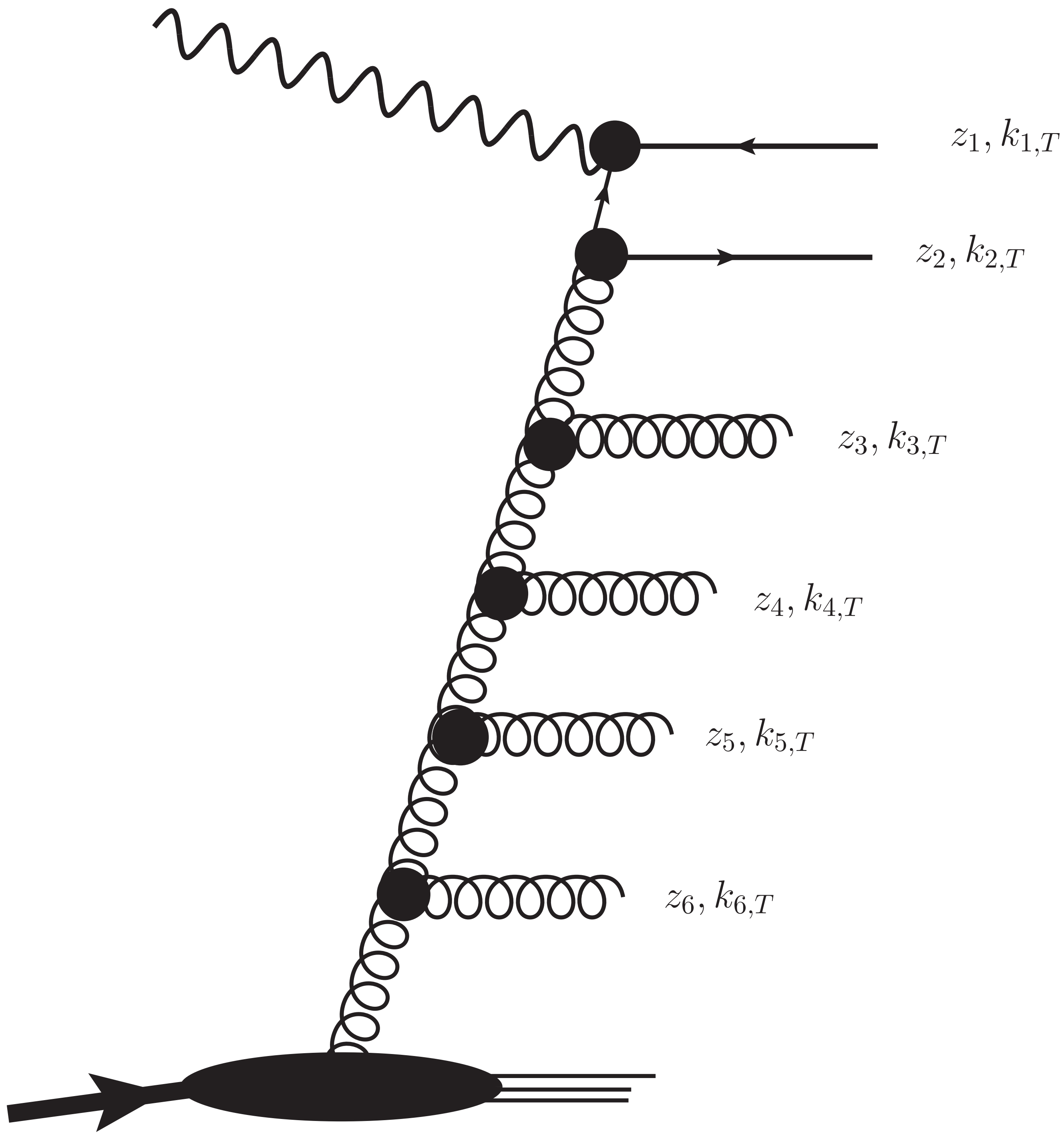
Can we re-obtain Balitsky-JIMWLK evolution form Lipatov's action? → Yes

- quantum fluctuations of Wilson lines within Lipatov's action →Balitsky-JIMWLK evolution (so far LL)

- effective action for central production processes → color decomposition imposed of effective action gives complication (similar problems in deriving the Triple Pomeron vertex **[MH, 0908.2576]**)

- essential take away point: both formalisms are 100% consistent; Lipatov's action provides an additional tool

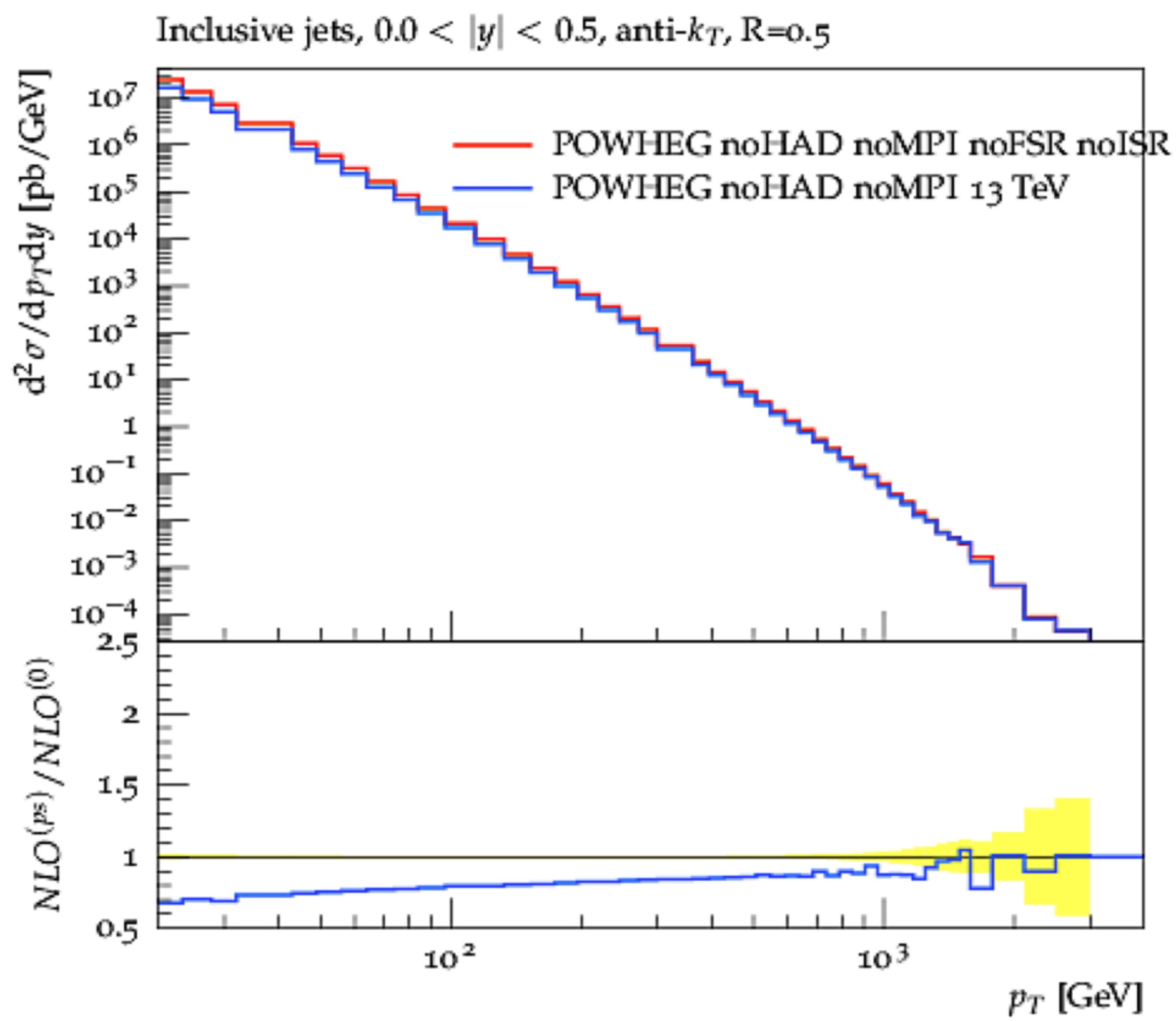
3. TMD splitting functions from kT factorization



2 versions of partonic evolution

- DGLAP: ordering in kT↔ kT not conserved
- BFKL: ordering in momentum fraction z → z/“energy” not conserved
- evolution which conserve both possible?

Why to try such a thing?



plot taken from Hannes Jung's talk at RBRC workshop, June 2017

$$K^{PS} = \frac{N^{(ps)}_{NLO-MC}}{N^{(0)}_{NLO-MC}}$$

- ratio: NLO with parton shower over NLO without parton shower
- theory: their the same, practice: not quite true
- message: kinematic effects are important

Why to try such a thing?

- practical need for low x phenomenologist: many (forward) observables require integration over gluon x → sensitivity to large x region
 (e.g. fragmentation function, not completely exclusive final state, applications to MPI …)

- need to model BFKL/BK gluon in large x region (error!) or introduce matching scheme (how?)

- BEST: low x pdf that works for all x

short history:

1. TMD P_{gq} by Catani-Hautmann (low resummed splitting kernels) [Catani, Hautmann, NPB427 (1994)]

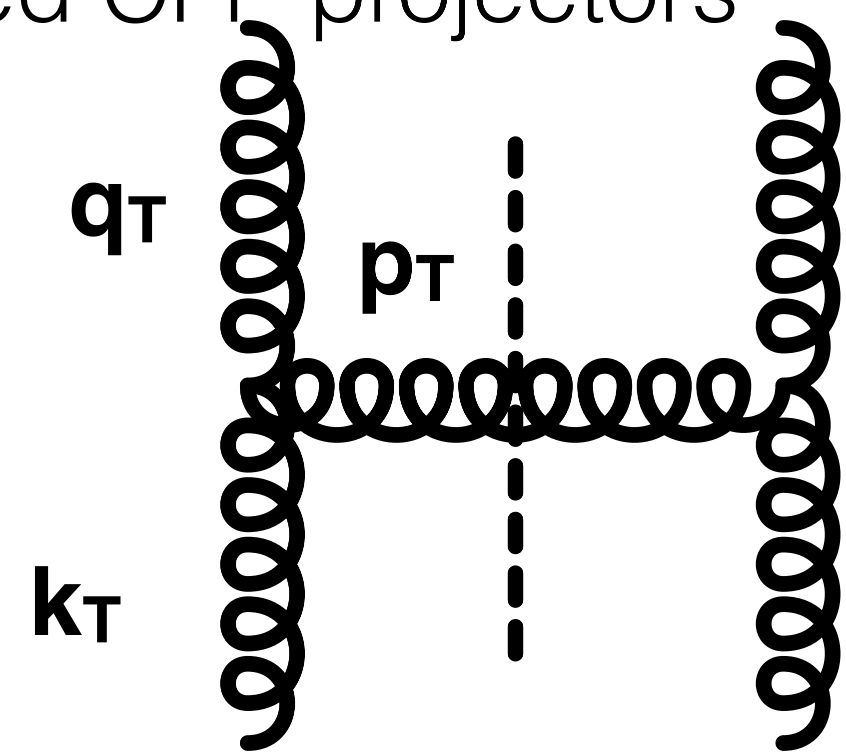
2. reproduced using effective vertices (reggeized quarks) adapted to finite momentum fraction
[Hautmann, MH, Jung; 1205.1759]

3. Curci-Furmanski-Petrozini formalism for DGLAP (light-cone gauge!) + gauge invariance in presence of off-shell initial reggeized quarks (generalized Lipatov vertices)→ quark splittings
[Gituliar, MH, Kutak, 1511.08439]

4. now: real part of TMD P_{gg} (gluon-to-gluon)
[MH, Kusina, Kutak, Serino; 1711.04587]

P_{gg} satisfies important constrains

- ✓ from 2→3 scattering amplitude or Lipatov's action in light-cone gauge + generalized CFP projectors
- ✓ current conservation
- ✓ collinear limit: DGLAP splitting
- ✓ low x limit: BFKL kernel
- ✓ soft limit $p_T \to 0$: CCFM kernel byproduct from requesting the first 3 points



just the beginning not the end …

- complete set of 4 ***real*** TMD splitting kernels →satisfies all necessary constraints so far
- virtual corrections = work in progress
- in general: need to properly develop the whole framework → what are we actually doing?
- at the very least: a consistent way to combine DGLAP and BFKL;
- hope: get a handle on kinematic corrections

Conclusions & Summary

- BFKL can be tested in exclusive vector meson production → the most appropriate theoretical framework

- Lipatov's action allows to obtain CGC propagators + Baltisky-JIMWLK evolution

- a definition of (real)TMD splitting kernels which obey correct DGLAP + BFKL + CCFM limits is possible

Appendix

Solve BFKL equation in conjugate (γ) Mellin space

$$G\left(x, \boldsymbol{k}^2, M\right) = \frac{1}{\boldsymbol{k}^2} \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \; \hat{g}\left(x, \frac{M^2}{Q_0^2}, \frac{\overline{M}^2}{M^2}, \gamma\right) \; \left(\frac{\boldsymbol{k}^2}{Q_0^2}\right)^{\gamma}$$

re-introduce two scales: hard scale of process (M) and scale of running coupling ($\overline{M}$)

$\hat{g}$: operator in γ space!

$$\hat{g}\left(x, \frac{M^2}{Q_0^2}, \frac{\overline{M}^2}{M^2}, \gamma\right) = \frac{\mathcal{C} \cdot \Gamma(\delta-\gamma)}{\pi\Gamma(\delta)} \; \cdot \; \left(\frac{1}{x}\right)^{\chi\left(\gamma, \frac{\overline{M}^2}{M^2}\right)} \; \cdot$$

$$\left\{1 + \frac{\bar{\alpha}_s^2 \beta_0 \chi_0\left(\gamma\right)}{8N_c} \log\left(\frac{1}{x}\right)\left[-\psi\left(\delta-\gamma\right) + \log\frac{M^2}{Q_0^2} - \partial_\gamma\right]\right\},$$

resummed NLO BFKL eigenvalue with optimal scale setting ($\rightarrow$ modifies $\chi_1(\gamma)$):

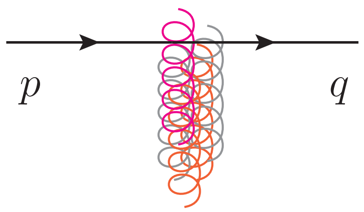
$$\chi\left(\gamma, \frac{\overline{M}^2}{M^2}\right) = \bar{\alpha}_s \chi_0\left(\gamma\right) + \bar{\alpha}_s^2 \tilde{\chi}_1\left(\gamma\right) - \frac{1}{2}\bar{\alpha}_s^2 \chi_0'\left(\gamma\right)\chi_0\left(\gamma\right)$$

$$+ \chi_{RG}(\bar{\alpha}_s, \gamma, \tilde{a}, \tilde{b}) - \frac{\bar{\alpha}_s^2 \beta_0}{8N_c} \chi_0(\gamma) \log\frac{\overline{M}^2}{M^2}.$$

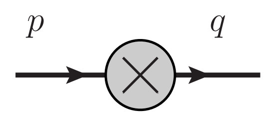
Theory: Propagators in background field

use light-cone gauge, with $k^- = n^+ \cdot k$, $(n^+)^2 = 0$, $n^+ \sim$ target momentum

$$\tilde{S}_F^{(0)}(p) = \frac{i\not{p} + m}{p^2 - m^2 + i0} \qquad \tilde{G}_{\mu\nu}^{(0)}(p) = \frac{i d_{\mu\nu}(p)}{p^2 + i0}$$

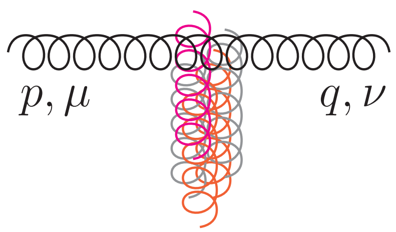


$$= (2\pi)^d \delta^{(d)}(p-q)\tilde{S}_F^{(0)}(p) + \tilde{S}_F^{(0)}(p)$$

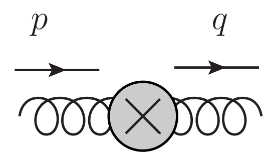


$$\tilde{S}_F^{(0)}(q)$$

$$d_{\mu\nu}(p) = -g_{\mu\nu} + \frac{n_\mu^- p_\nu + p_\mu n_\nu^-}{n^- \cdot p}$$



$$= (2\pi)^d \delta^{(d)}(p-q)\tilde{G}_{\mu\nu}^{(0)}(p) + \tilde{G}_{\mu\alpha}^{(0)}(p)$$



$$\tilde{G}_{\alpha\nu}^{(0)}(q)$$

[Balitsky, Belitsky; NPB 629 (2002) 290], [Ayala, Jalilian-Marian, McLerran, Venugopalan, PRD 52 (1995) 2935-2943], ...

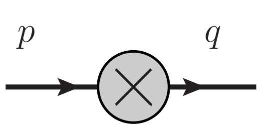
interaction with the background field:

$$V(\boldsymbol{z}) \equiv V_{ij}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig \int_{-\infty}^{\infty} dx^+ A^{-,c}(x^+, \boldsymbol{z}) t^c$$

$$U(\boldsymbol{z}) \equiv U^{ab}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig \int_{-\infty}^{\infty} dx^+ A^{-,c}(x^+, \boldsymbol{z}) T^c$$

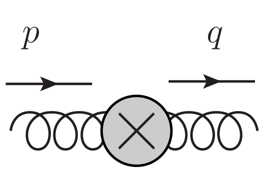
strong background field resummed into path ordered exponentials (Wilson lines)

$$A^-(x^+, x_t) = \delta(x^+)\alpha(x_t)$$



$$= \tau_{F,ij}(p,q) = 2\pi\delta(p^+ - q^+)\, \not{n}$$

$$\times \int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})} \left\{\theta(p^+)\left[V_{ij}(\boldsymbol{z}) - 1_{ij}\right] - \theta(-p^+)\left[V_{ij}^\dagger(\boldsymbol{z}) - 1_{ij}\right]\right\}$$



$$= \tau_G^{ab}(p,q) = 2\pi\delta(p^+ - q^+)\ (-2p^+)$$

$$\times \int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})} \left\{\theta(p^+)\left[U^{ab}(\boldsymbol{z}) - 1\right] - \theta(-p^+)\left[\left(U^{ab}\right)^\dagger(\boldsymbol{z}) - 1\right]\right\}$$

reggeized gluon as log of Wilson line

- proposal made by S. Caron-Huot [1309.6521]:
  2-dim reggeized gluon from Balitsky-JIMWLK evolution

$$R^a(\boldsymbol{z}) \equiv \frac{1}{gN_c} f^{abc} \log U^{bc}(\boldsymbol{z})$$

U satisfies the evolution

- Lipatov's effective action: agrees in this sense with this definition

$$R^a(\boldsymbol{z}) = \frac{1}{gN_c} f^{abc} \left[ig\alpha^d(\boldsymbol{z})T^d_{bc}\right] = \alpha^a(\boldsymbol{z}) = \frac{1}{2}\int dx^+ A^a_+(x^+, \boldsymbol{z})$$

angular averaged TMD splitting functions

$$\bar{P}_{qg}^{(0)} = T_R \left(\frac{\tilde{\boldsymbol{q}}^2}{\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2} \right)^2 \left[z^2 + (1-z)^2 + 4z^2(1-z)^2 \frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2} \right],$$

$$\bar{P}_{gq}^{(0)} = C_F \left[\frac{2\tilde{\boldsymbol{q}}^2}{z|\tilde{\boldsymbol{q}}^2 - (1-z)^2\boldsymbol{k}^2|} - \frac{(2-z)\tilde{\boldsymbol{q}}^4 + z(1-z^2)\boldsymbol{k}^2\tilde{\boldsymbol{q}}^2}{\left(\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2\right)^2} \right],$$

$$\bar{P}_{qq}^{(0)} = C_F \frac{\tilde{\boldsymbol{q}}^2}{\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2} \times \left[\frac{\tilde{\boldsymbol{q}}^2 + (1-z^2)\boldsymbol{k}^2}{(1-z)|\tilde{\boldsymbol{q}}^2 - (1-z)^2\boldsymbol{k}^2|} + \frac{z^2\tilde{\boldsymbol{q}}^2 - z(1-z)(1-3z+z^2)\boldsymbol{k}^2}{(1-z)(\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2)} \right].$$

$$\bar{P}_{gg}^{(0)}\left(z, \frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2}\right) = C_A \frac{\tilde{\boldsymbol{q}}^2}{\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2} \left[\frac{(2-z)\tilde{\boldsymbol{q}}^2 + (z^3 - 4z^2 + 3z)\boldsymbol{k}^2}{z(1-z)\left|\tilde{\boldsymbol{q}}^2 - (1-z)^2\boldsymbol{k}^2\right|} + \frac{(2z^3 - 4z^2 + 6z - 3)\tilde{\boldsymbol{q}}^2 + z(4z^4 - 12z^3 + 9z^2 + z - 2)\boldsymbol{k}^2}{(1-z)(\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2)} \right]$$