Forward di-hadron back-to-back correlations in p+A collisions at RHIC and the LHC

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## The context: forward di-hadrons

• large-x projectile (proton) on small-x target (proton or nucleus)



Incoming partons' energy fractions:



so-called "dilute-dense" kinematics

$$\begin{array}{rcl} x_1 & = & \frac{1}{\sqrt{s}} \left( |p_{1t}| e^{y_1} + |p_{2t}| e^{y_2} \right) & \xrightarrow{y_1, y_2 \gg 0} & x_1 & \sim & 1 \\ x_2 & = & \frac{1}{\sqrt{s}} \left( |p_{1t}| e^{-y_1} + |p_{2t}| e^{-y_2} \right) & x_2 & \ll & 1 \end{array}$$

CM (2007)

Gluon's transverse momentum ( $p_{1t}$ ,  $p_{2t}$  imbalance):

 $|k_t|^2 = |p_{1t} + p_{2t}|^2 = |p_{1t}|^2 + |p_{2t}|^2 + 2|p_{1t}||p_{2t}| \cos \Delta \phi \qquad |p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$ 

prediction: modification of the  $k_t$  distribution in p+Pb vs p+p collisions

## **Di-hadron angular correlations**

comparisons between d+Au  $\rightarrow$  h<sub>1</sub> h<sub>2</sub> X (or p+Au  $\rightarrow$  h<sub>1</sub> h<sub>2</sub> X ) and p+p  $\rightarrow$  h<sub>1</sub> h<sub>2</sub> X



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however, when  $y_1 \sim y_2 \sim 0$  (and therefore  $x_A \sim 0.03$ ), the p+p and d+Au curves are almost identical

# Color Glass Condensate (CGC) calculation of forward di-jets

#### Saturation calculation

CM (2007)





b: quark in the amplitudex: gluon in the amplitudeb': quark in the conj. amplitudex': gluon in the conj. amplitude

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collinear factorization of quark density in deuteron  $\frac{d\sigma^{dAu \to qgX}}{d^{2}k_{\perp}dy_{k}d^{2}q_{\perp}dy_{q}} = \alpha_{S}C_{F}N_{c}x_{d}q(x_{d},\mu^{2})\int \frac{d^{2}x}{(2\pi)^{2}}\frac{d^{2}x'}{(2\pi)^{2}}\frac{d^{2}b}{(2\pi)^{2}}\frac{d^{2}b'}{(2\pi)^{2}}\underbrace{d^{2}b'}_{(2\pi)$ 

$$z=\frac{|k_\perp|e^{y_k}}{|k_\perp|e^{y_k}+|q_\perp|e^{y_q}}$$

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interaction with target nucleus

 $z = \frac{|k_\perp|e^{y_k}}{|k_\perp|e^{y_k} + |q_\perp|e^{y_q}}$ 

n-point functions that resums the powers of  $g_s A$  and the powers of  $\alpha_s \ln(1/x_A)$ 

### Scattering on the dense target

 this is described by Wilson lines scattering of a quark:

$$W_F[\alpha](\mathbf{x}) = \mathcal{P} \exp\left\{ig_s \int dx^+ T^c \alpha_c(x^+, \mathbf{x})\right\}$$

lpha dependence kept implicit in the following



in the CGC framework, any cross-section is determined by colorless combinations of Wilson lines  $S[\alpha]$ , averaged over the CGC wave function  $\langle S \rangle_x = \int D\alpha |\Phi_x[\alpha]|^2 S[\alpha]$ 

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• the 2-point function or dipole amplitude

the  $q\overline{q}$  dipole scattering amplitude:

 $\langle T_{q\bar{q}}(\mathbf{x},\mathbf{y}) \rangle_x$  or  $\langle T_{q\bar{q}}(\mathbf{r},\mathbf{b}) \rangle_x$ 

this is the most common Wilson-line average

$$T_{q\bar{q}}(\mathbf{x},\mathbf{y}) = 1 - \frac{1}{N_c} Tr(W_F^+(\mathbf{y})W_F(\mathbf{x}))$$

x : quark transverse coordinatey : antiquark transverse coordinate

### 2- 4- and 6-point functions

coming back to the double-inclusive cross-section

the scattering off the CGC is expressed through the following correlators of Wilson lines:

if the gluon is emitted before the interaction, four partons scatter off the CGC  $S_{qg\bar{q}g}^{(4)}(\mathbf{b},\mathbf{x},\mathbf{b}',\mathbf{x}';x_A) = \frac{1}{C_F N_c} \left\langle \operatorname{Tr}\left(W_F(\mathbf{b})W_F^{\dagger}(\mathbf{b}')T^dT^c\right) [W_A(\mathbf{x})W_A^{\dagger}(\mathbf{x}')]^{cd} \right\rangle_{x_A}$ 

if the gluon is emitted after the interaction, only the quarks interact with the CGC  $S_{q\bar{q}}^{(2)}(\mathbf{b},\mathbf{b}';x_A) = \frac{1}{N_c} \left\langle \operatorname{Tr}\left(W_F(\mathbf{b})W_F^{\dagger}(\mathbf{b}')\right) \right\rangle_{x_A}$ 

interference terms, the gluon interacts in the amplitude only (or c.c. amplitude only)

$$S_{qg\bar{q}}^{(3)}(\mathbf{b},\mathbf{x},\mathbf{b}';x_A) = \frac{1}{C_F N_c} \left\langle \operatorname{Tr}\left(W_F^{\dagger}(\mathbf{b}')T^c W_F(\mathbf{b})T^d\right) W_A^{cd}(\mathbf{x}) \right\rangle_{x_A}$$

Connections with high-energy factorization and TMD factorization

# The linear regime $|p_{1t}|, |p_{2t}|, |k_t| \gg Q_s$

• taking all involved momenta >> Qs, the CGC formula reduces to

 $\frac{d\sigma^{pA \to \text{dijets} + X}}{dy_1 dy_2 d^2 p_{1t} d^2 p_{2t}} = \frac{\alpha_s^2}{\pi (x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1, \mu^2) \ |\overline{\mathcal{M}_{ag^* \to cd}}|^2 \ \mathcal{F}_{g/A}(x_2, k_t^2) \frac{1}{1 + \delta_{cd}} \ .$ 

this is the so-called high-energy factorization (HEF) formula e.g. Kutak and Sapeta (2012)

 $x_1 f_{a/p}(x_1, \mu^2)$  – collinear PDF in *p*, suitable for  $x_1 \sim 1$ 

$$|\overline{\mathcal{M}_{ag^* \rightarrow cd}}|^2$$
 – matrix element with off-shell incoming gluon

 $\mathcal{F}_{g/A}(x_2, k_t)$  – unintegrated gluon PDF in A, suitable for  $x_2 \ll 1$ 

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the unintegrated gluon density involved is also the also involved in deep inelastic scattering, it is related to the dipole scattering amplitude  $\mathcal{N}(x, r)$ 

$$\mathcal{F}_{g/A}(x,k^2) = \frac{N_c}{\alpha_s(2\pi)^3} \int d^2b \int d^2r \ e^{-i\mathbf{k}\cdot\mathbf{r}} \nabla_r^2 \ \mathcal{N}(x,r)$$

# The back-to-back regime $|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$

 a factorization can be established in the small x limit, for nearly back-to-back di-jets
Dominguez, CM, Xiao and Yuan (2011)

 $\frac{d\sigma^{pA \to \text{dijets}+X}}{dy_1 dy_2 d^2 p_{1t} d^2 p_{2t}} = \frac{\alpha_s^2}{(x_1 x_2 S)^2} \left[ \sum_q x_1 f_{q/p}(x_1, \mu^2) \sum_i H_{qg}^{(i)} \mathcal{F}_{qg}^{(i)}(x_2, |\mathbf{p_{1t}} + \mathbf{p_{2t}}|) + \frac{1}{2} x_1 f_{g/p}(x_1, \mu^2) \sum_i H_{gg}^{(i)} \mathcal{F}_{gg}^{(i)}(x_2, |\mathbf{p_{1t}} + \mathbf{p_{2t}}|) \right]$ 

but it involves several unintegrated gluon densities  $\mathcal{F}_{qg}^{(i)}$  and  $\mathcal{F}_{gg}^{(i)}$  and their associated hard matrix elements

this is the so-called Transverse Momentum Dependent (TMD) factorization formula e.g. Bomhof, Mulders and Pijlman (2006)

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e.g. Bomhof, Mulders and Pijlman (2006)

only valid in asymmetric situations

Collins and Qiu (2007), Xiao and Yuan (2010)

does not apply with unintegrated parton densities for both colliding projectiles

### The back-to-back regime

this TMD factorization formula for  $x_2 \ll x_1 \sim 1$  can be derived • in two ways:

from the generic TMD factorization framework (valid up to power corrections): by taking the small-x limit Bomhof, Mulders and Pijlman (2006) Kotko, Kutak, CM, Petreska, Sapeta and van Hameren (2015)

from the CGC framework (valid at small-x): by extracting the leading power

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• at small **x**, the TMD gluon distributions can be written as: (showing here the  $qg^* \rightarrow qg$  channel TMDs only )  $U_{\mathbf{x}} = \mathcal{P} \exp \left[ ig \int_{-\infty}^{\infty} dx^+ A_a^-(x^+, \mathbf{x}) t^a \right]$   $\mathcal{F}_{qg}^{(1)}(x_2, |k_t|) = \frac{4}{g^2} \int \frac{d^2 x d^2 y}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x} - \mathbf{y})} \left\langle \operatorname{Tr} \left[ (\partial_i U_{\mathbf{y}}) (\partial_i U_{\mathbf{x}}^\dagger) \right] \right\rangle_{x_2}$  $\mathcal{F}_{qg}^{(2)}(x_2, |k_t|) = -\frac{4}{g^2} \int \frac{d^2 x d^2 y}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x} - \mathbf{y})} \frac{1}{N_c} \left\langle \operatorname{Tr} \left[ (\partial_i U_{\mathbf{x}}) U_{\mathbf{y}}^{\dagger} (\partial_i U_{\mathbf{y}}) U_{\mathbf{x}}^{\dagger} \right] \operatorname{Tr} \left[ U_{\mathbf{y}} U_{\mathbf{x}}^{\dagger} \right] \right\rangle_{x_2}$ 

these Wilson line correlators also emerge directly in CGC calculations when  $|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$ 

# Evaluating the gluon TMDs at small-x

#### The other TMDs at small-x

- involved in the  $gg^* \to q\bar{q}$  and  $gg^* \to gg$  channels

$$\begin{split} \mathcal{F}_{gg}^{(1)}(x_{2},k_{t}) &= \frac{4}{g^{2}} \int \frac{d^{2}\mathbf{x}d^{2}\mathbf{y}}{(2\pi)^{3}} \ e^{-ik_{t}\cdot(\mathbf{x}-\mathbf{y})} \frac{1}{N_{c}} \left\langle \operatorname{Tr}\left[(\partial_{i}U_{\mathbf{y}})(\partial_{i}U_{\mathbf{x}}^{\dagger})\right] \operatorname{Tr}\left[U_{\mathbf{x}}U_{\mathbf{y}}^{\dagger}\right] \right\rangle_{x_{2}} \ , \\ \mathcal{F}_{gg}^{(2)}(x_{2},k_{t}) &= -\frac{4}{g^{2}} \int \frac{d^{2}\mathbf{x}d^{2}\mathbf{y}}{(2\pi)^{3}} \ e^{-ik_{t}\cdot(\mathbf{x}-\mathbf{y})} \frac{1}{N_{c}} \left\langle \operatorname{Tr}\left[(\partial_{i}U_{\mathbf{x}})U_{\mathbf{y}}^{\dagger}\right] \operatorname{Tr}\left[(\partial_{i}U_{\mathbf{y}})U_{\mathbf{x}}^{\dagger}\right] \right\rangle_{x_{2}} \ , \\ \mathcal{F}_{gg}^{(4)}(x_{2},k_{t}) &= -\frac{4}{g^{2}} \int \frac{d^{2}\mathbf{x}d^{2}\mathbf{y}}{(2\pi)^{3}} \ e^{-ik_{t}\cdot(\mathbf{x}-\mathbf{y})} \left\langle \operatorname{Tr}\left[(\partial_{i}U_{\mathbf{x}})U_{\mathbf{x}}^{\dagger}(\partial_{i}U_{\mathbf{y}})U_{\mathbf{y}}^{\dagger}\right] \right\rangle_{x_{2}} \ , \\ \mathcal{F}_{gg}^{(5)}(x_{2},k_{t}) &= -\frac{4}{g^{2}} \int \frac{d^{2}\mathbf{x}d^{2}\mathbf{y}}{(2\pi)^{3}} \ e^{-ik_{t}\cdot(\mathbf{x}-\mathbf{y})} \left\langle \operatorname{Tr}\left[(\partial_{i}U_{\mathbf{x}})U_{\mathbf{y}}^{\dagger}(\partial_{i}U_{\mathbf{y}})U_{\mathbf{x}}^{\dagger}U_{\mathbf{y}}U_{\mathbf{x}}^{\dagger}\right] \right\rangle_{x_{2}} \ , \\ \mathcal{F}_{gg}^{(6)}(x_{2},k_{t}) &= -\frac{4}{g^{2}} \int \frac{d^{2}\mathbf{x}d^{2}\mathbf{y}}{(2\pi)^{3}} \ e^{-ik_{t}\cdot(\mathbf{x}-\mathbf{y})} \left\langle \operatorname{Tr}\left[(\partial_{i}U_{\mathbf{x}})U_{\mathbf{y}}^{\dagger}(\partial_{i}U_{\mathbf{y}})U_{\mathbf{x}}^{\dagger}U_{\mathbf{y}}U_{\mathbf{x}}^{\dagger}\right] \right\rangle_{x_{2}} \ , \\ \mathcal{F}_{gg}^{(6)}(x_{2},k_{t}) &= -\frac{4}{g^{2}} \int \frac{d^{2}\mathbf{x}d^{2}\mathbf{y}}{(2\pi)^{3}} \ e^{-ik_{t}\cdot(\mathbf{x}-\mathbf{y})} \frac{1}{N_{c}^{2}} \left\langle \operatorname{Tr}\left[(\partial_{i}U_{\mathbf{x}})U_{\mathbf{y}}^{\dagger}(\partial_{i}U_{\mathbf{y}})U_{\mathbf{x}}^{\dagger}\right] \operatorname{Tr}\left[U_{\mathbf{x}}U_{\mathbf{y}}^{\dagger}\right] \operatorname{Tr}\left[U_{\mathbf{y}}U_{\mathbf{x}}^{\dagger}\right] \right\rangle_{x_{2}} \ . \end{split}$$

with a special one singled out: the Weizsäcker-Williams TMD

$$\mathcal{F}_{gg}^{(3)}(x_2,k_t) = -\frac{4}{g^2} \int \frac{d^2 \mathbf{x} d^2 \mathbf{y}}{(2\pi)^3} \ e^{-ik_t \cdot (\mathbf{x}-\mathbf{y})} \left\langle \operatorname{Tr}\left[ (\partial_i U_{\mathbf{x}}) U_{\mathbf{y}}^{\dagger} (\partial_i U_{\mathbf{y}}) U_{\mathbf{x}}^{\dagger} \right] \right\rangle_{x_2}$$

## x evolution of the gluon TMDs

the evolution of Wilson line correlators with decreasing x can be computed from the so-called JIMWLK equation

$$\frac{d}{d\ln(1/x_2)} \left\langle O \right\rangle_{x_2} = \left\langle H_{JIMWLK} \right\rangle_{x_2}$$

Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner

a functional RG equation that resums the leading logarithms in  $y=\ln(1/x_2)$ 

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• qualitative solutions for the gluon TMDs:





as a function of x and  $k_T$ 

### JIMWLK numerical results

using a code written by Claude Roiesnel

CM, Petreska, Roiesnel (2016)

initial condition at y=0 : MV model evolution: JIMWLK at leading log



saturation effects impact the various gluon TMDs in very different ways

### Back to experiments

### STAR forward di-hadrons



new description of the away-side peak suppression

cannot be applied to the overall  $\Delta \phi$  range, but improves the previous approximations near  $\Delta \phi = \pi$  (also gluon initiated process are included)

# LHCb forward di-hadrons

• LHCb measured the di-hadron correlation function at forward rapidities

the delta phi distribution shows:

- a ridge contribution (could be flow, Glasma graphs or something else)
- the remainder of the away-side peak can be qualitatively described in the CGC



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suppression of the away-side peak with increasing centrality seen in the data

#### Conclusions I

 for forward di-hadron production, TMD factorization and CGC calculations are consistent with each other in the overlapping domain of validity

small x and leading power of the hard scale  $|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$ 

- saturation physics is relevant if the di-hadron transverse momentum imbalance  $|k_t|$  is of the order of the saturation scale Qs
- the cross-section involves several gluon TMDs, with different operator definitions

#### Conclusions II

- given an initial condition, the gluon TMDs can all be obtained at smaller values of x, from the JIMWLK equation
- as expected, the various gluon TMDs coincide at large transverse momentum, in the linear regime
- however, they differ significantly from one another at low transverse momentum, in the non-linear saturation regime
- we hope to see at the LHC, a confirmation of the saturation signal seen at RHIC