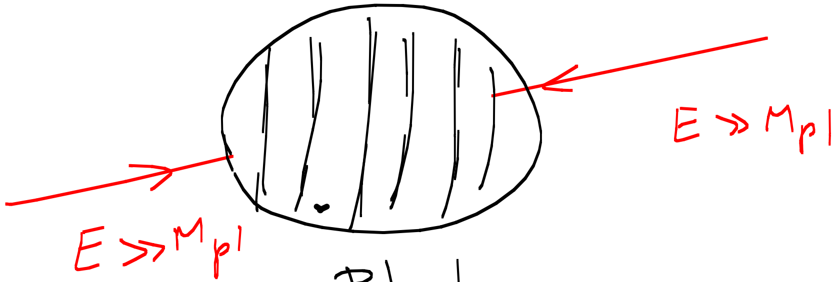
Build the Walls,

Drain the Swamp

Black Holes

End of Reductionism

UV/IR Fusion

* 

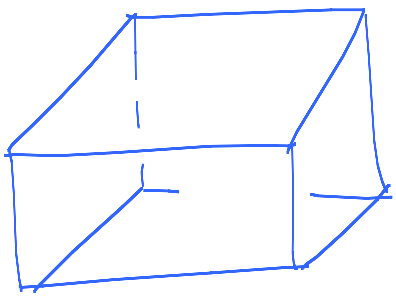

Black Hole $\gg \ell_{pl}$!

High Energies

$\updownarrow$

Long distances!

* Gravitational constraints on low-energy physics



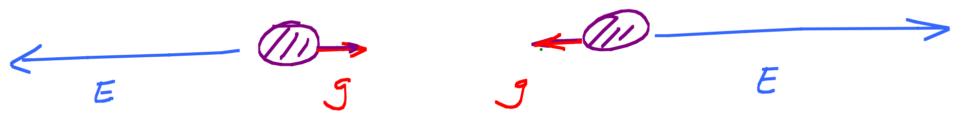
$$N_{max} \sim e^{\text{Volume}} \quad ✗$$

$$\sim e^{\text{Area}/G_N} \quad ✓$$

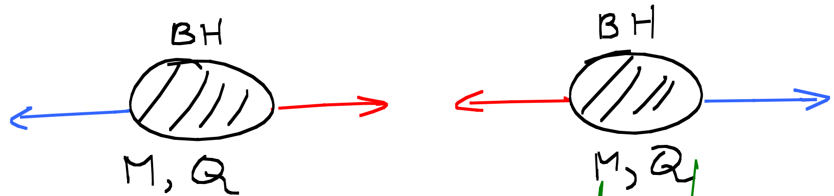
Holographic Bounds

* More surprising constraints appear to be enforced by BH consistency:

e.g.




"gravity is weakest force"

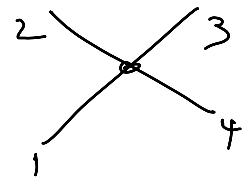



$\Longrightarrow$ very specific signs for deviations from Einstein Gravity, confirmed in all known examples in string theory

POSITIVITY

# Locality [Causality], Unitarity $\Longrightarrow$ Positivity

e.g. $\mathcal{L} = -F_{\mu\nu}^2 + c\,F_{\mu\nu}^4 + \cdots$




$A = c(s^2+t^2+u^2)$

$c > 0$

* Crucial for protecting consistency of Horizon thermodynamics!

* Related to weak gravity: $c<0 \Longleftrightarrow \delta\left(M/Q\right)\Big|_{\substack{\text{extr}\\ \text{BH}}} < 0$

... Meanwhile, in seemingly totally unrelated developments over the past 5–10 years we have been seeing

"Positive Geometry" $\Longrightarrow$ Locality + Unitarity

Concrete examples where physics of space-time and QM are seen to be derivative from ultimately combinatorial ideas

Planar $\mathcal{N}=4$ SYM Amps

$\phi^3$ theory, Pions, Gluons...

$4_{\mho}$ [ $\varphi^N$ theories ]

Amplituhedra

(Generalized) Associahedra

Cosmological Polytopes

Planar $\mathcal{N}=4$ SYM Amps

Amplituhedra

$\phi^3$ theory, Pions, Gluons...

(Generalized) Associahedra

$\Psi_{\mathcal{U}}$ [ $\varphi^N$ theories ]

Cosmological Polytopes

---

Effective Field Theory

"EFThedron"

Hidden Positive Geometry of Causality + Unitarity

Conformal Field Theory

"CFThedra"

Positive Geometry of Conformal Bootstrap.

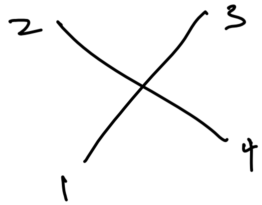
# The EFT-hedron

w/ Huang + Huang

* Universal Geometries controlling EFT

* In particular, infinitely many quantitative predictions about quantum gravity in the real world:

# Higher-Dim Ops in 4-particle Scattering

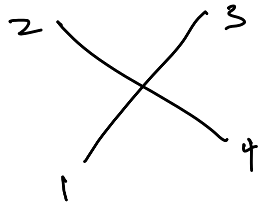



$$A^{LE,\,\text{contact}} = \left(\langle 12\rangle[34]\right)^{2s}\left[\sum{}' a_{D,q}\, s^{D-q} t^{q}\right]$$

carries dim. $\longrightarrow t_D \qquad \hat{a}_{D,q}$ $\longleftarrow$ dimless, projective

$$\left(t_0, t_1, t_2, t_3 \dots\right) \qquad \begin{pmatrix}\hat{a}_{0,0}\end{pmatrix}\begin{pmatrix}\hat{a}_{1,0}\\ \hat{a}_{1,1}\end{pmatrix}\begin{pmatrix}\hat{a}_{2,0}\\ \hat{a}_{2,1}\\ \hat{a}_{2,2}\end{pmatrix}\begin{pmatrix}\hat{a}_{3,0}\\ \hat{a}_{3,1}\\ \hat{a}_{3,2}\\ \hat{a}_{3,3}\end{pmatrix}\dots$$

# Higher-Dim Ops in 4-particle Scattering



$$A^{LE,\,\text{contact}} = \left(\langle 12\rangle[34]\right)^{2s}\left[\sum a_{D,q}\, s^{D-q}\, t^{q}\right]$$

carries dim. $\longrightarrow$ $t_D$ $\qquad$ $\hat{a}_{D,q}$ $\longleftarrow$ dimless, projective

$$\left(t_0, t_1, t_2, t_3 \dots\right)$$

Satisfy $\infty$'ly many non-linear inequalities!

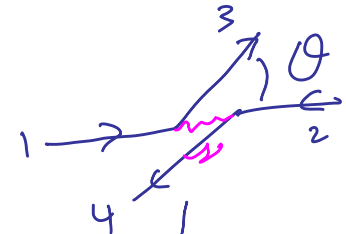
$$\left(\hat{a}_{0,0}\right)\begin{pmatrix}\hat{a}_{1,0}\\ \hat{a}_{1,1}\end{pmatrix}\begin{pmatrix}\hat{a}_{2,0}\\ \hat{a}_{2,1}\\ \hat{a}_{2,2}\end{pmatrix}\begin{pmatrix}\hat{a}_{3,0}\\ \hat{a}_{3,1}\\ \hat{a}_{3,2}\\ \hat{a}_{3,3}\end{pmatrix}\dots$$

Must lie within Specific Polytopes!

# Causality + Unitarity

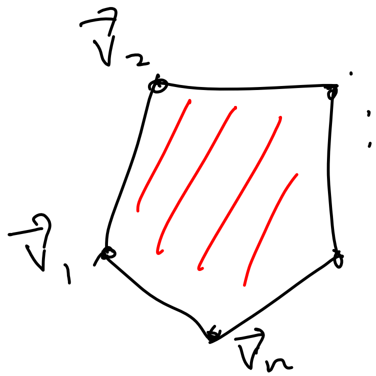
"subtraction terms"

$$-A(s,t) = a(t) + b(t)\,s + \sum_n \frac{R_n\left[\cos\theta = 1+\frac{2t}{m_n^2}\right]}{m_n^2 - s} + \text{u-chann.}$$




* $\boxed{m_n^2 > 0}$
* $R_n[\cos\theta] = \sum_J \boxed{P_{n,J} > 0}\, G_J[\cos\theta]$

# Polytopes 101




$$\vec{A} = \frac{w_1 \vec{v}_1 + \cdots + w_n \vec{v}_n}{w_1 + \cdots + w_n}$$

$$\boxed{w_a > 0}$$

Better Projectively: $A^I = \begin{pmatrix} 1 \\ \vec{A} \end{pmatrix}, \; V_i^I = \begin{pmatrix} 1 \\ \vec{v}_i \end{pmatrix}$

$$A^I = w_1 V_1^I + \cdots + w_n V_n^I$$ "Convex Hull"

Check in $A$ is inside? "Face" description $A^I W_I^{(a)} \geq 0$

Facet structure completely captured by $\langle V_{a_1} \cdots V_{a_D} \rangle \begin{cases} > 0 \\ < 0 \\ = 0 \end{cases}$

Very Special Class of Polytopes

$$\begin{array}{c c} & \begin{matrix} 1 & 2 & \cdots & n \end{matrix} \\ \updownarrow D & \begin{pmatrix} V_1 & V_2 & \cdots & V_n \end{pmatrix} \end{array} \longleftarrow \underline{\text{Ordering}}$$

$\langle V_{a_1} \cdots V_{a_D} \rangle > 0$ for $a_1 < \cdots < a_D$

Matrix: "Total Positivity", "Positive Grassmannian"

Vectors, vertices of "Cyclic Polytope" = "k=1 Amplituhedron"

Know all Facets!

$$\langle A\, V_i V_{i+1}\, V_j V_{j+1} \cdots V_k V_{k+1} \rangle \geq 0$$

Surprise: Hidden Total Positivity In

$$\frac{P_n}{m_n^2 - s}, \quad G_J[\cos\theta]$$

"Locality" "Unitarity"

# Total Positivity of Locality

$$-A(s) = \sum \frac{p_n}{m_n^2 - s} = \sum a_p s^p \Longrightarrow$$

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \end{pmatrix} = \sum \frac{p_n}{m_n^2} \begin{pmatrix} 1 \\ m_n^2 \\ (m_n^2)^2 \\ \vdots \end{pmatrix} = \underset{x \geq 0}{\text{Conv.}} \left\{ \begin{pmatrix} 1 \\ x \\ x^2 \\ \vdots \end{pmatrix} \right\}$$

"Moment Curve"

Now

$$\begin{vmatrix} 1 & \cdots & 1 \\ x_1 & & x_{D+1} \\ \vdots & & \vdots \\ x_1^D & & x_{D+1}^D \end{vmatrix} = \prod_{i<j} (x_j - x_i) > 0$$

non-trivial $\longrightarrow$

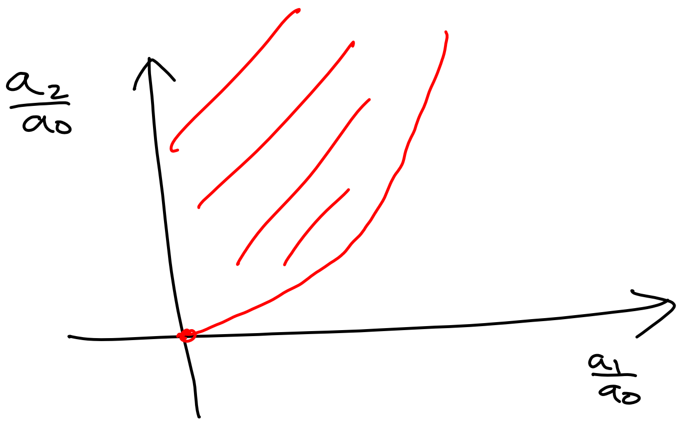
$$\begin{vmatrix} a_0 & a_1 & a_2 & \cdots \\ a_1 & a_2 & a_3 & \cdots \\ a_2 & a_3 & a_4 & \cdots \end{vmatrix}$$

All minors of "Hankel matrix" are Positive!

e.g. $\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \underset{x \geq 0}{\operatorname{conv}} \begin{pmatrix} 1 \\ x \\ x^2 \end{pmatrix}$. Trivially $a_{0,1,2} > 0$

[old known fact]

But

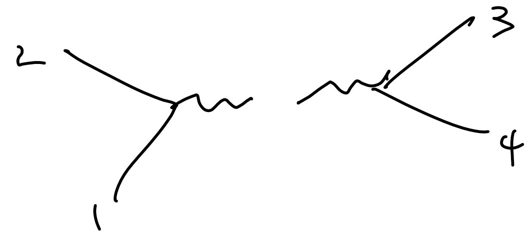


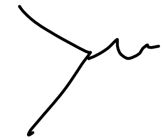

$a_1 > 0$

$a_2 a_0 - a_1^2 > 0$

$$\boxed{\begin{matrix} a_0 & a_1 \\ \boxed{a_1} & a_2 \end{matrix}}$$

# Hidden Positivity of Unitarity

$G_s[\cos\theta] =$ 

$= \Big|$  $\Big|^2$ when $\cos\theta \to 1$

Standard

But expand $\quad G_s[1+y] = G_{s,q}\, y^q \quad \ldots\ldots$

$$
\begin{array}{c|cccccc}
 & s=0 & 1 & 2 & 3 & 4 & \cdots \\
\hline
y^0 & 1 & 1 & 1 & 1 & 1 & \cdot \\
y^1 & 0 & 1 & 3 & 6 & 10 & \cdots \\
y^2 & 0 & 0 & \frac{3}{2} & \frac{15}{2} & \frac{45}{2} & \cdots \\
y^3 & 0 & 0 & 0 & \frac{5}{2} & \frac{35}{2} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots
\end{array}
$$

$$
\begin{array}{c}
\\ y^0 \\ y^1 \\ y^2 \\ y^3 \\ \vdots
\end{array}
\begin{array}{c}
\begin{array}{cccccc} s=0 & 1 & 2 & 3 & 4 & \cdots \end{array} \\
\begin{pmatrix}
\boxed{1} & \boxed{1} & \boxed{1} & \boxed{1} & \boxed{1} & \cdot \\
0 & 1 & 3 & 6 & 10 & \cdots \\
0 & 0 & \frac{3}{2} & \frac{15}{2} & \frac{45}{2} & \cdots \\
0 & 0 & 0 & \frac{5}{2} & \frac{35}{2} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \cdots
\end{pmatrix}
\end{array}
$$

Obvious

$$
\begin{array}{c|cccccc}
 & s=0 & 1 & 2 & 3 & 4 & \cdots \\
\hline
y^0 & 1 & 1 & 1 & 1 & 1 & \cdots \\
y^1 & 0 & \boxed{1} & \boxed{3} & \boxed{6} & \boxed{10} & \cdots \\
y^2 & 0 & 0 & \boxed{\frac{3}{2}} & \boxed{\frac{15}{2}} & \boxed{\frac{45}{2}} & \cdots \\
y^3 & 0 & 0 & 0 & \boxed{\frac{5}{2}} & \boxed{\frac{35}{2}} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots
\end{array}
$$

Trivial, noticed by many (Martin '60s)

$$
\begin{array}{c|ccccccc}
 & s=0 & 1 & 2 & 3 & 4 & \cdots \\
\hline
y^0 & & & & & & \\
y^1 & & & & & & \\
y^2 & & & & & & \\
y^3 & & & & & & \\
\vdots & & & & & &
\end{array}
\quad
\begin{pmatrix}
1 & 1 & 1 & 1 & 1 & \cdots \\
0 & 1 & 3 & 6 & 10 & \cdots \\
0 & 0 & \frac{3}{2} & \frac{15}{2} & \frac{45}{2} & \cdots \\
0 & 0 & 0 & \frac{5}{2} & \frac{35}{2} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots &
\end{pmatrix}
$$

Totally Positive!

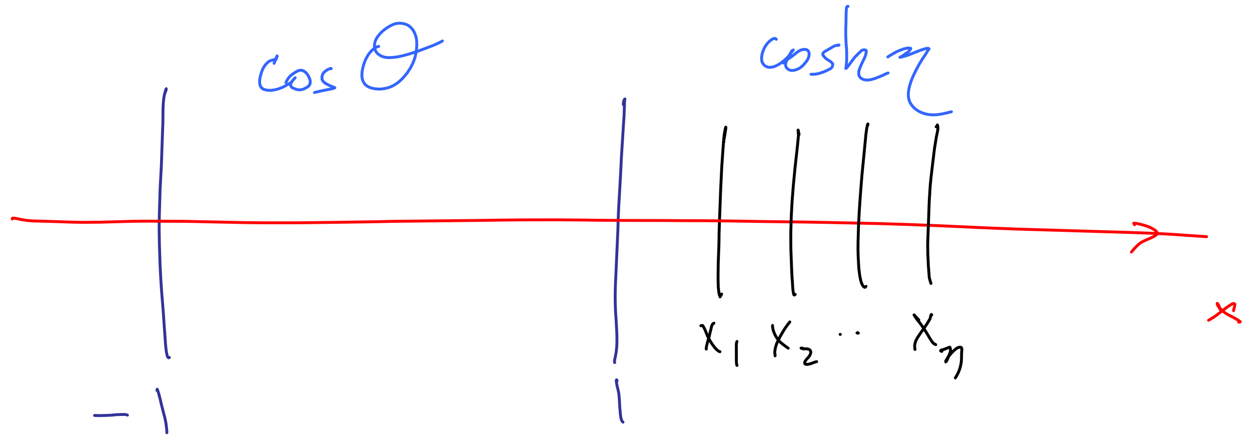
{Proof by direct computation}

$$
\begin{array}{c|cccccc}
 & s=0 & 1 & 2 & 3 & 4 & \cdots \\
\hline
y^0 & 1 & 1 & 1 & 1 & 1 & \cdots \\
y^1 & 0 & 1 & 3 & 6 & 10 & \cdots \\
y^2 & 0 & 0 & \frac{3}{2} & \frac{15}{2} & \frac{45}{2} & \cdots \\
y^3 & 0 & 0 & 0 & \frac{5}{2} & \frac{35}{2} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots
\end{array}
$$

Top $k$ rows: Positive Grassmannian

Cyclic Polytope

# Positivity seen in (2,2) Signature

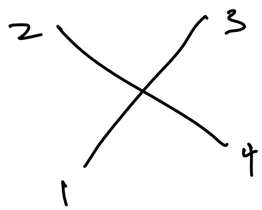



$$\det \begin{pmatrix} G_{\nu_1}(x_1) & G_{\nu_2}(x_1) & \cdots & G_{\nu_n}(x_1) \\ \vdots & \vdots & & \vdots \\ G_{\nu_1}(x_n) & G_{\nu_2}(x_n) & & G_{\nu_n}(x_n) \end{pmatrix} > 0$$

$$x_1 < \cdots < x_n$$

$$\nu_1 < \cdots < \nu_n$$

# Higher-Dim Ops in 4-particle Scattering




$$A^{LE,\,\text{contact}} = \left(\langle 12\rangle[34]\right)^{2S}\left[\sum a_{D,q}\, s^{D-q}\, t^{q}\right]$$

carries dim. $\longrightarrow t_D$ $\qquad \hat{a}_{D,q}$ $\longleftarrow$ dimless, projective

$$\left(t_0, t_1, t_2, t_3 \ldots\right)$$

Satisfy $\infty$'ly many non-linear inequalities!

$$\begin{pmatrix}\hat{a}_{0,0}\end{pmatrix}\begin{pmatrix}\hat{a}_{1,0}\\ \hat{a}_{1,1}\end{pmatrix}\begin{pmatrix}\hat{a}_{2,0}\\ \hat{a}_{2,1}\\ \hat{a}_{2,2}\end{pmatrix}\begin{pmatrix}\hat{a}_{3,0}\\ \hat{a}_{3,1}\\ \hat{a}_{3,2}\\ \hat{a}_{3,3}\end{pmatrix}\cdots$$

Must lie within Specific Polytopes!

# 1 Pure photon

Consider the configuration ( -, +, +, -) where we have

$$\langle 14\rangle^2[23]^2\left(\sum_{i,j} g_{i,j} z^i t^j\right) \tag{1}$$

Note that due to the $\langle 14\rangle^2[23]^2 \sim t^2$ prefactor, we will not have any $t$-pole obstruction.

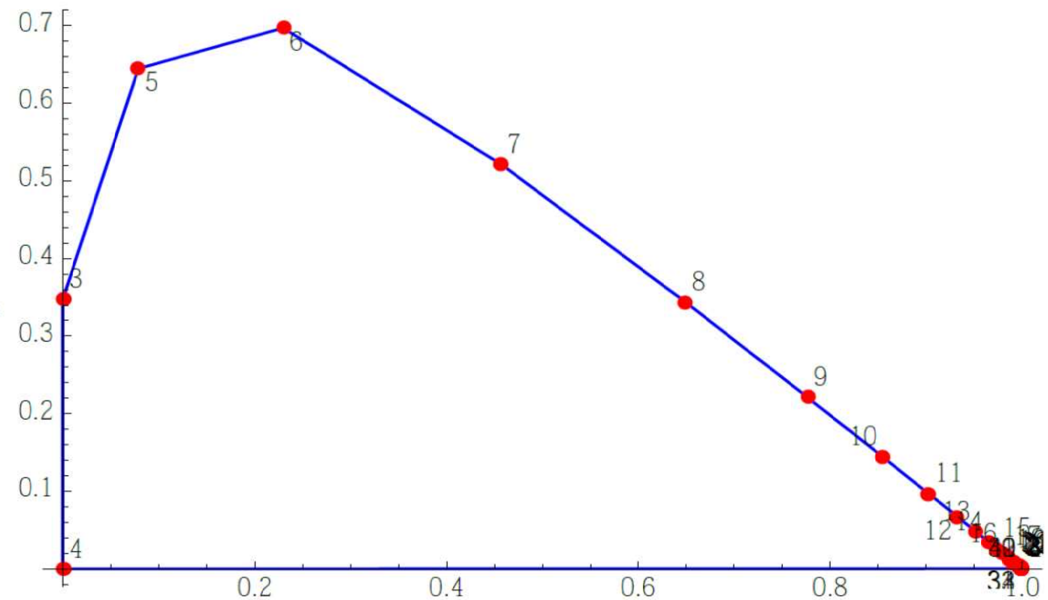
- $(z^2, t^2)$: The space is one-dimensional, and the bound is simply

$$-\frac{7}{20} < \frac{g_{2,0}}{g_{0,2}}$$

- $(z^4, z^2t^2, t^4)$: The critical spin is $s_c = 4$, spin-2 is inside the hull, i.e. not a vertex. The boundaries are:

$$\langle X, i, i+1\rangle > 0 \text{ for}, \quad i \geq 5, \langle X, 4, 3\rangle > 0, \quad \langle X, 3, 5\rangle > 0 \tag{2}$$

Out[120]=




(3)

This plot was made by plotting the vectors

$$\vec{V}_\ell = \left(\frac{\langle V_\ell, 4, 3\rangle}{\langle V_\ell, 4, 3\rangle + \langle V_\ell, \infty, 4\rangle + 1}, \frac{\langle V_\ell, \infty, 4\rangle}{\langle V_\ell, 4, 3\rangle + \langle V_\ell, \infty, 4\rangle + 1}\right)$$

- $(z^6, z^4t^2, z^2t^4, t^6)$ The critical spin is $s_c = 4$, spin-2 is inside the hull. The boundaries are:

$$\langle 4, X, i, i+1\rangle > 0 \text{ for}, \quad i \geq 9, \langle X,3,4,5\rangle > 0, \quad \langle X,3,7,6\rangle > 0, \quad \langle X,5,8,7\rangle > 0, \quad \langle X,5,9,8\rangle > 0, \tag{4}$$

Using the vectors

$$\vec{V}_\ell = \left(\frac{\langle V_\ell,3,4,5\rangle}{\langle V_\ell,3,4,5\rangle + \langle 3,V_\ell,6,7\rangle + \langle 3,V_\ell,\infty_1,\infty_2\rangle + 1}, \frac{\langle 3,V_\ell,6,7\rangle}{\langle V_\ell,3,4,5\rangle + \langle 3,V_\ell,6,7\rangle + \langle 3,V_\ell,\infty_1,\infty_2\rangle + 1}, \right.$$
$$\left. \frac{\langle 3,V_\ell,\infty_1,\infty_2\rangle}{\langle V_\ell,3,4,5\rangle + \langle 3,V_\ell,6,7\rangle + \langle 3,V_\ell,\infty_1,\infty_2\rangle + 1}\right) \tag{5}$$

## 2 Pure Graviton

Consider the configuration ( -2, +2, +2, -2) where we have

$$\langle 14\rangle^4[23]^4\left(\sum_{i,j} g_{i,j}z^it^j\right) \tag{8}$$

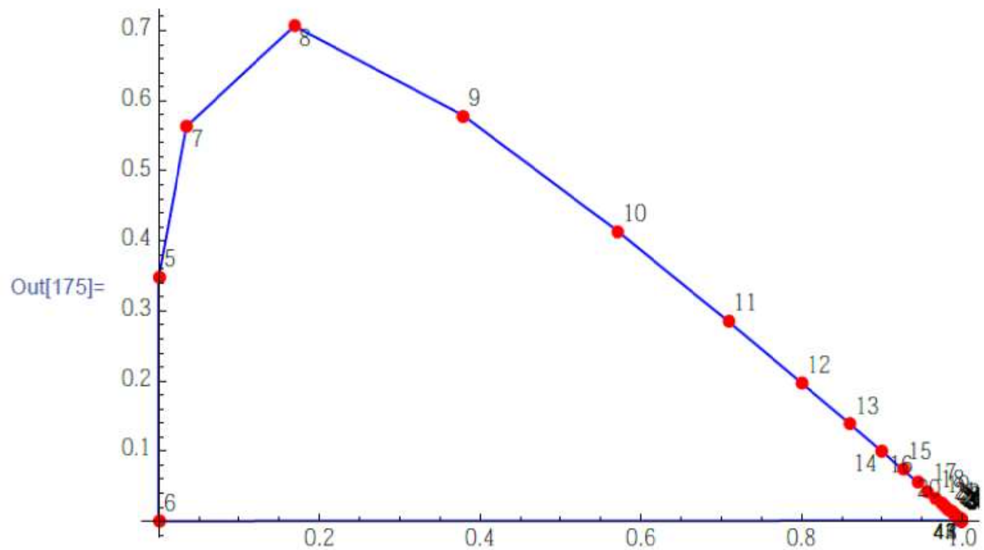
The exchanged spin begins with spin-4

- $(z^2, t^2)$: The space is one-dimensional, and the bound is simply

$$-\frac{11}{36} < \frac{g_{2,0}}{g_{0,2}}$$

- $(z^4, z^2t^2, t^4)$: The critical spin is $s_c = 6$, spin-4 is inside the hull, i.e. not a vertex. The boundaries are:

$$\langle X, i, i+1\rangle > 0 \text{ for}, \quad i \geq 7, \langle X, 6, 5\rangle > 0, \quad \langle X, 5, 7\rangle > 0 \tag{9}$$




(10)

This plot was made by plotting the vectors

$$\vec{V}_\ell = \left(\frac{\langle V_\ell, 6, 5\rangle}{\langle V_\ell, 6, 5\rangle + \langle V_\ell, \infty, 6\rangle + 1}, \frac{\langle V_\ell, \infty, 6\rangle}{\langle V_\ell, 6, 5\rangle + \langle V_\ell, \infty, 6\rangle + 1}\right)$$

# The World is Not a Crappy Metal!

Vastly greater UV Constraints

on IR physiss than

naively expected

* Can we include constraints from actual UV completion? [i.e. fixed angle HE softness]

* Can we prove $\delta\left(\frac{M}{Q}\right)\Big|_{\text{ext BH}} < 0$ ?