



# The quantum swampland

Ulf Danielsson  
Uppsala University  
Sweden



- The classical swamp
- The non-perturbative swamp
- A way out of the swamp (and back again?)
- The quantum swamp

U.D and T. Van Riet, [arXiv:1804.01120](#)

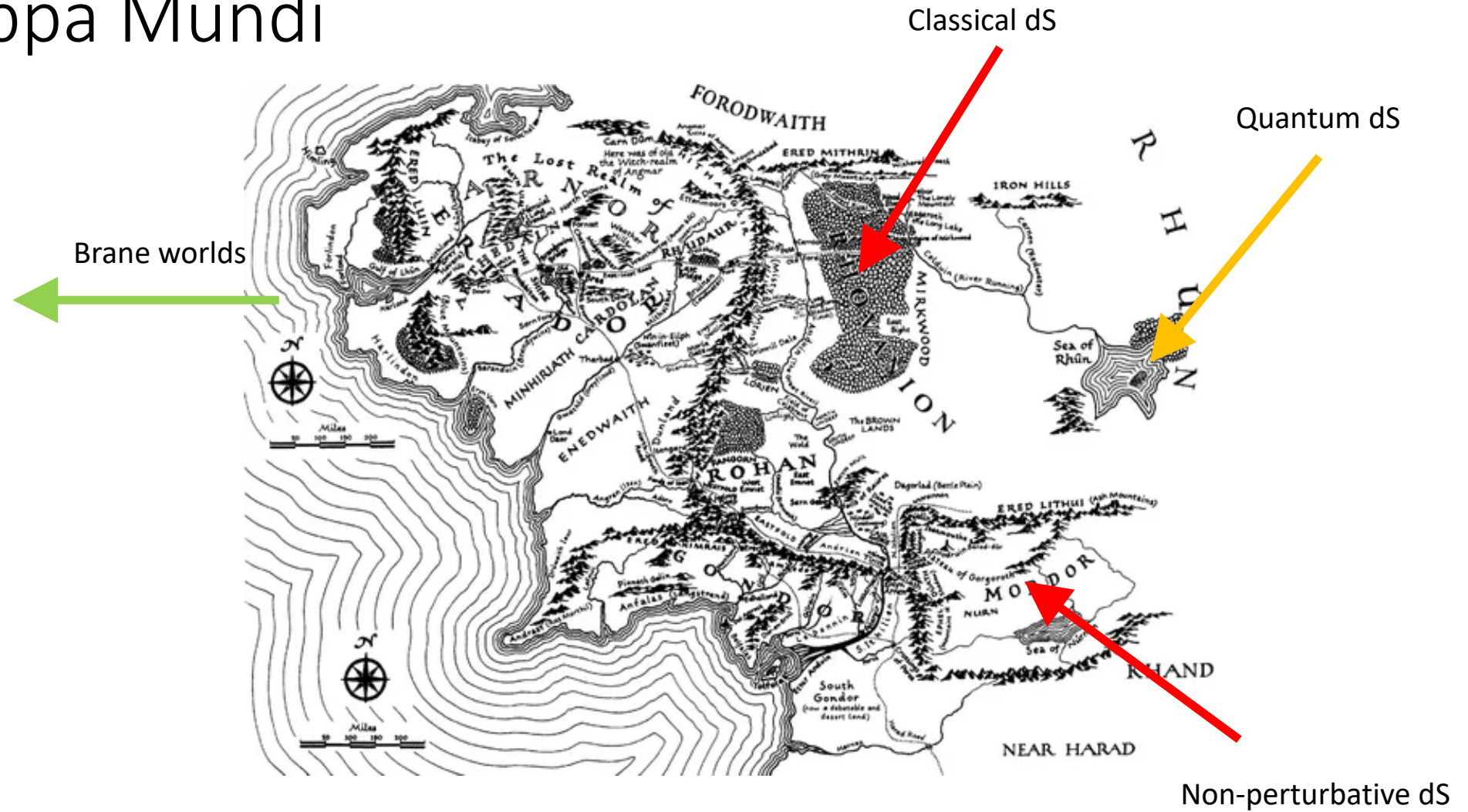
S. Banerjee, U.D, G. Dibitetto, S. Giri and M. Schillo, [arXiv:1807.01570](#)

U.D, [arXiv:1809.04512](#)

$$cV \leq |\nabla V|$$

G. Obied, H. Ooguri, L. Spodyneiko and C.Vafa, [arXiv:1806.08362](#).

# Mappa Mundi



# The classical swamp

The corner that is under best control is possibly type IIA with nothing more exotic than orientifolds.

dS critical points are hard to find and they are **all tachyonic...**

There are no examples of slow roll for more than a couple of e-folds...

[J. Blåbäck, U.D. and G. Dibitetto, arXiv:1310.8300.](#)

... and this is even before such issues as quantization of fluxes are considered...

[U.D., S. Haque, P. Koerber, G. Shiu, T. Van Riet and T. Wrase, arXiv:1103.4858; C. Roupec and T. Wrase, arXiv:1807.09538.](#)

... although non-geometric fluxes (if you can make sense out of them) might help.

[U. D and G. Dibitetto, arxiv:1212.4984; J. Blåbäck, U. D., and G. Dibitetto, arxiv:1301.7073.](#)



# The non-perturbative swamp

KKLT used non-perturbative corrections to stabilize the classical moduli...

Three concerns...

0 Wrong treatment of non-perturbative corrections?

[S. Sethi, arXiv:1709.03554.](#)

1 Back reaction on moduli so that stability is lost?

[J. Moritz, A. Retolaza and A. Westphal, arXiv:1707.08678.](#)

2 The anti-branes are unstable?



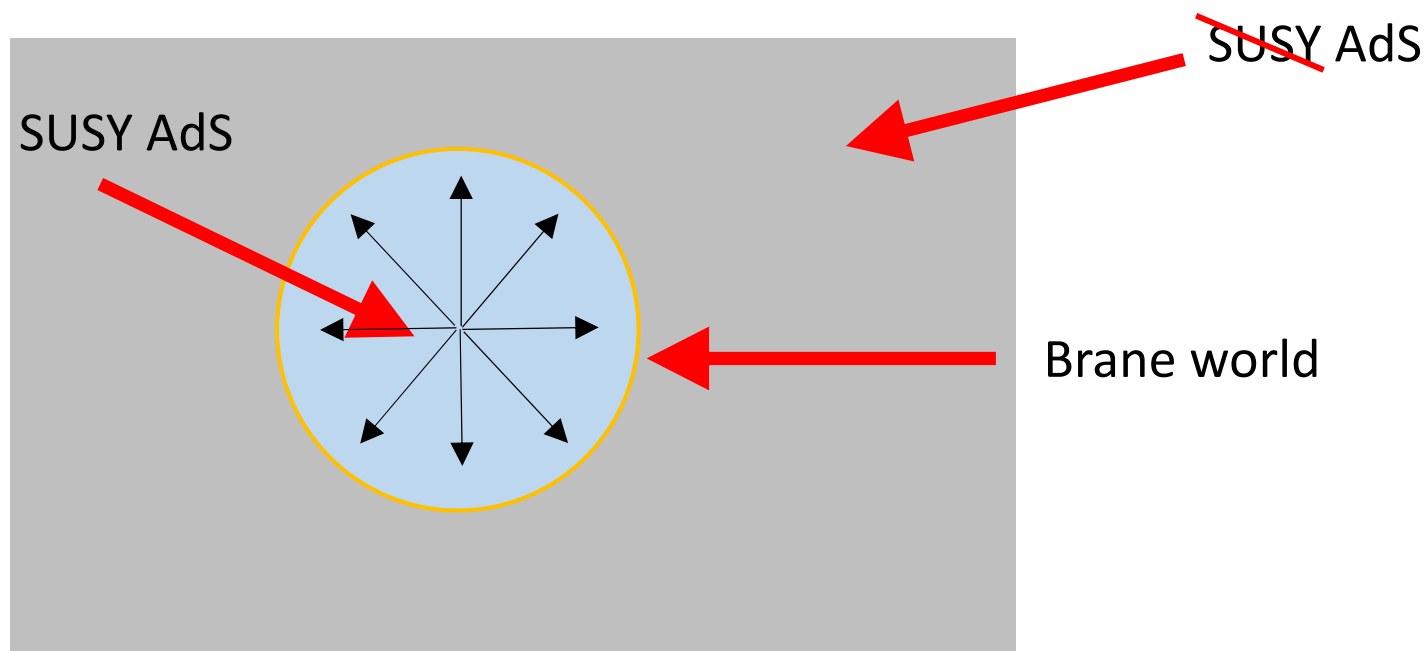
Anti-brane in KS-throat



# Out of the swamp (and back again?)

What if we give up on finding a fundamental vacuum?

[S. Banerjee, U.D, G. Dibitetto, S. Giri and M. Schillo, arXiv:1807.01570.](#)



Junction conditions across brane gives:

$$\sigma = \frac{3}{8\pi G_5} \left( \sqrt{h_-^2 + \frac{1+\dot{a}^2}{a^2}} - \sqrt{h_+^2 + \frac{1+\dot{a}^2}{a^2}} \right)$$

where  $a(t)$  is the radius of the bubble and

$$\Lambda_{\mp} = -6h_{\pm} \quad (h_- > h_+, h_- \text{ inside and } h_+ \text{ outside})$$

... add 5D-matter...

$$1 + h^2 r^2 \rightarrow 1 + h^2 r^2 - \frac{4\pi G_5 m(r)}{\pi r^2}$$

... give rise to Friedmann equations:

$$H^2 = -\frac{1}{a^2} + \frac{8\pi G_4}{3} \left( \Lambda_4 + \frac{3}{4\pi^2} \left( \frac{m_+}{h_+} - \frac{m_-}{h_-} \right) \frac{1}{a^4} \right)$$

$$G_4 = \frac{2h_-h_+}{h_- - h_+} G_5$$

$$\Lambda_4 = \sigma_{ext} - \sigma > 0$$

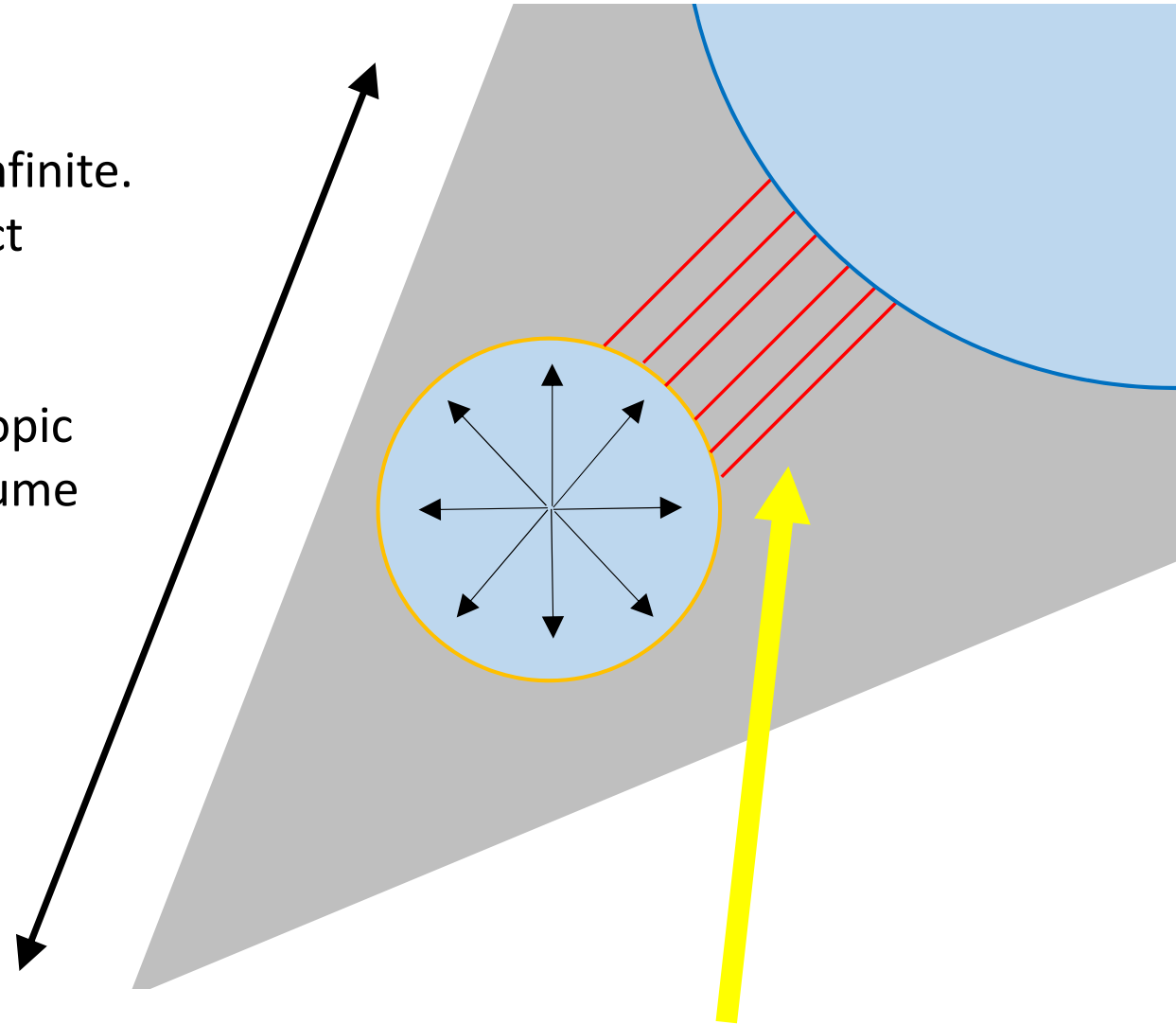
$$\sigma_{ext} = \frac{3}{8\pi G_5} (h_- - h_+)$$

4D matter comes from geometry...



Space exterior to bubble need not be infinite.  
Could be part of finite throat in compact  
extra dimension.

Proper length of throat can be microscopic  
even though coordinate radius and volume  
are astronomical...



Dust realized through stretched strings. Interact as predicted by Friedmann.

# The quantum swamp

What about quantum loops?

Depends on choice of vacuum...



With cutoff in energy...

$$\begin{aligned}
 E &= \frac{1}{2} \int_0^\Lambda \frac{d^4 k}{2\pi^2} \sqrt{k^2 + m^2} = \frac{1}{16\pi^2} (\Lambda^2 + \Lambda^2 m^2) + \frac{1}{64\pi^2} m^4 \ln\left(\frac{m^2}{\Lambda^2}\right) + \dots \\
 P &= \frac{1}{2} \int_0^\Lambda \frac{d^4 k}{2\pi^2} \frac{k^2}{\sqrt{k^2 + m^2}} = \frac{1}{48\pi^2} (\Lambda^2 - \Lambda^2 m^2) - \frac{1}{64\pi^2} m^4 \ln\left(\frac{m^2}{\Lambda^2}\right) + \dots
 \end{aligned}$$

First two terms seem to be artifact of non-invariant cutoff...

... but this one can give a contribution to the cc.

For review and references: J. Martin, arXiv:1205.3365.

We can introduce a cutoff in 4-momentum instead, using...

$$\frac{\partial f}{\partial m^2} = \frac{1}{4} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{\sqrt{k^2 + m^2}} = \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 - m^2 + i\epsilon}$$

... leading to ...

$$\begin{aligned} P &= \frac{1}{2} \int \frac{d^4 k_E}{(2\pi)^4} \ln \frac{k_E^2 + m^2}{\mu^2} = \\ &= \frac{1}{64\pi^2} \left( \Lambda^4 \ln \frac{\Lambda^2}{\mu^2} + 2\Lambda^2 m^2 + m^4 \ln \frac{m^2}{\Lambda^2} \right) \end{aligned}$$

Now the cutoff is invariant.



With all possible particles, we find:

$$V_{loop} \simeq \frac{1}{64\pi^2} \left( \Lambda^4 \text{STr} m^0 \ln \frac{\Lambda^2}{\mu^2} + 2\Lambda^2 \text{STr} m^2 + \text{STr} m^4 \ln \frac{m^2}{\Lambda^2} \right)$$

Compatible with string loop calculations, where terms two and three are generated by corrections to the Kähler potential. [M. Cicoli, J. Conlon and F. Quevedo, arXiv:0708.1873.](#)

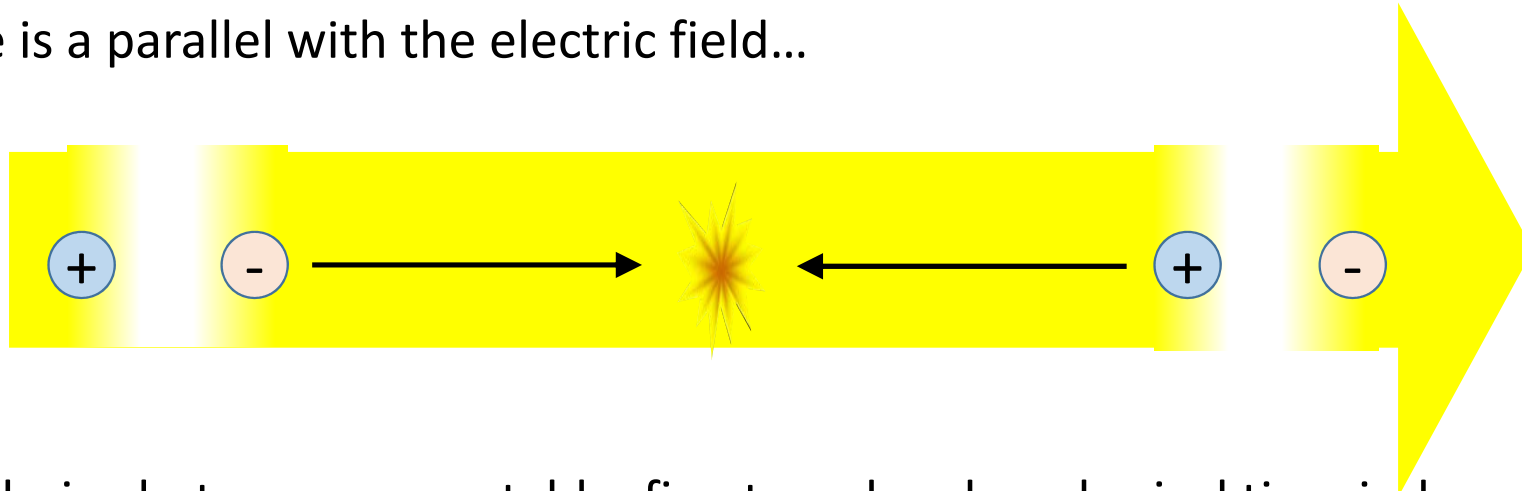
But what about the choice of vacuum?

If naively applied to dS the implicit choice is the **Bunch-Davies vacuum**.

Defined through continuation from Euclidean sphere. Argued by, e.g., Polyakov and Mottola, to be **unphysical**.

[A. Polyakov, arXiv:0709.2899](#); [A. Polyakov, arXiv:0912.5503](#); [D. Krotov and A. Polyakov, arXiv:1012.2107](#); [A. Polyakov, arXiv:1209.4135](#). [E. Mottola, Phys. Rev. D31 \(1985\) 754](#); [P. Anderson and E. Mottola, arXiv:1310.0030](#); [P. Anderson and E. Mottola, arXiv:1310.1963](#); [P. Anderson, E. Mottola and D. Sanders, arXiv:1712.04522](#).

There is a parallel with the electric field...



- 1 Choice between an unstable, fine tuned and unphysical time independent vacuum, and a vacuum with a decaying electric field where time translation invariance is broken.
- 2 If electric field is turned on for a long enough time  $\tau$  , then the created particles may collide with an energy:

$$E_{\tau} = \Lambda$$

... which means that physics will depend on the UV-completion.

There is another way to **define the Bunch-Davies vacuum**. Make a mode expansion....

$$\ddot{\phi} + 3H\dot{\phi} - \nabla^2\phi = 0 \Rightarrow \mu_k'' + \left(k^2 - \frac{a''}{a}\right)\mu_k = 0$$

with  $\mu = a\phi$  and  $\eta = -\frac{1}{aH}$

In Heisenberg approach.....

$$\mu_k(\eta) = \frac{1}{\sqrt{2k}} (a_k(\eta) + a_k^\dagger(\eta)) \quad ; \quad a_k(\eta) = U_k(\eta) a_k(\eta_0) + V_k(\eta) a_k^\dagger(\eta_0)$$

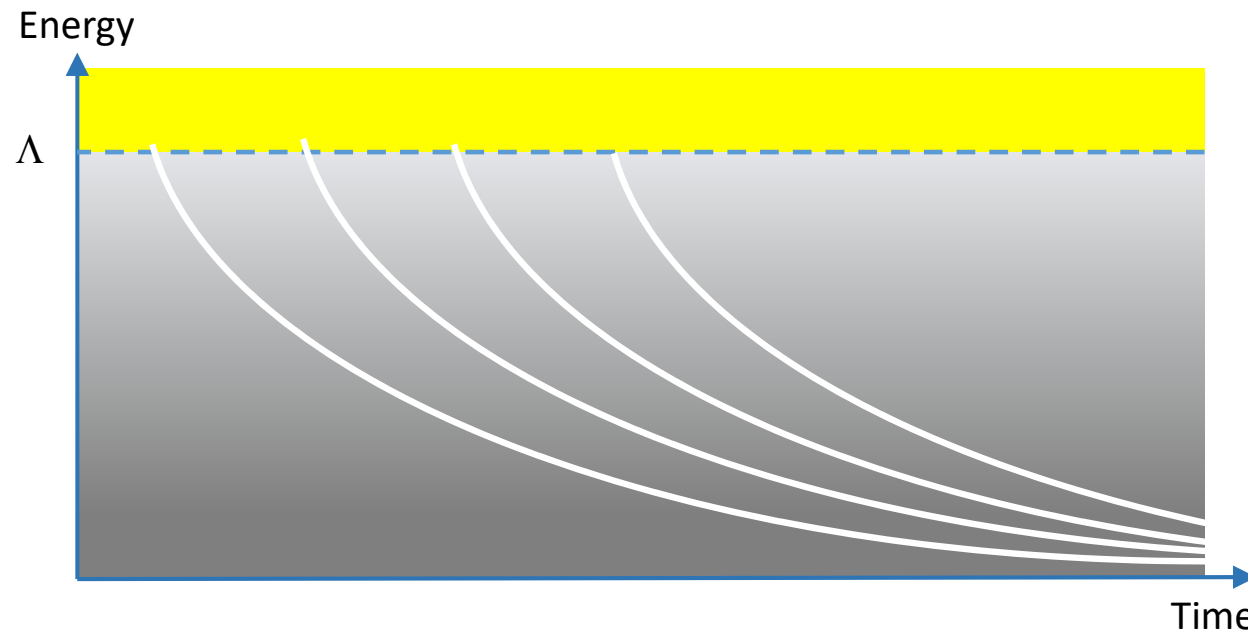
... **trace a given mode back in time** until it is so blue shifted that its wavelength is much shorter than the Hubble scale. Forget that you are in dS space and pick the unique Minkowski vacuum...

$$a_k(\eta_0) |0, \eta_0\rangle = 0 \quad \text{with } \eta_0 \rightarrow -\infty$$

... then you get Bunch-Davies. This is used in inflation. [U. D., hep-th/0411172](#); [U. D., hep-th/0511273](#).

But why should you trust this beyond the string or Planck scale? More natural to impose the instantaneous Minkowski vacuum when mode emerges at cutoff...

$$a_k(\eta_{k,0})|0,\eta_{0,k}\rangle = 0 \text{ at } \eta_{k,0} = -\frac{\Lambda}{Hk}$$



Below cutoff new vacuum will be Bogolubov transformation of Bunch-Davies.



Additional contribution to the vacuum energy...

$$\rho_1(a) = \frac{1}{4\pi^2} \int_{\epsilon}^{\Lambda} d^3p \frac{H^2(\frac{ap}{\Lambda})}{\Lambda^2} \approx \frac{1}{4\pi^2} \frac{\Lambda^3}{a^4} \int_{a_0}^a dx x^3 H^2(x)$$

Need a source term for new the modes fed in at the cutoff...

$$\dot{\rho}_1 \approx 4H \rho_1 = \frac{1}{4\pi^2} \Lambda^2 H^3$$

What about back reaction? Pick this pair of Friedmann equations:

$$\begin{aligned}\dot{H} &= -\frac{4\pi}{M_p^2}(\rho + p) \\ \dot{\rho} + 3H(\rho + p) &= 0\end{aligned}$$

In this way the cosmological constant appears as a constant of integration. Note that the first equation has a thermodynamic interpretation through:

$$\frac{dG}{dt} = T \frac{dS}{dt} = A(\rho + p) \text{ where } S = \frac{M_p^2}{4t} A \text{ and } T = \frac{1}{2H}$$

Should only modify the continuity equation through adding the source.

Integrating we find:

$$H^2 = C_1 a^{-2n_1} + C_2 a^{-2n_2} \text{ where } n_{1,2} = 1 \pm \sqrt{1 - \frac{2\Lambda^2}{3M_p^2}}$$

... that is:

$$H^2 = C_1 a^{-\frac{2\Lambda}{3M_p^2}} + C_2 a^{-4 + \frac{2\Lambda}{3M_p^2}} \quad \text{if } \Lambda \ll M_p$$

... implying that the cosmological constant is drained.

Using

$$-\frac{\dot{H}}{H} = \frac{M_p^2}{2} \left( \frac{V'}{V} \right)^2$$

... we find an estimate for the constant  $c$  in the swampland conjecture:

$$c = \sqrt{\frac{2}{3}} \frac{\Lambda}{M_p} \times \# \text{fields}$$



This is an example of the **UV-sensitivity** of theory. Just as you might worry about contributions to the vacuum energy scaling with the cutoff in (hypothetical) time independent case, there are similar contributions when the vacuum evolves.

Their exact form depend on initial conditions (vacuum choice) as well as details of high energy physics.

Is there a relation through stringy dualities between the quantum decay of dS and the swampland conjecture?

