# *Weak Gravity Conjecture from Unitarity*

***Gary Shiu***
***University of Wisconsin-Madison***

# Based on work with:

**Y. Hamada**

**T. Noumi**

**Y. Hamada, T. Noumi and GS,**
"Weak Gravity Conjecture from Unitarity",
to appear.

**S. Andriolo**

**D.Junghans**

**S. Andriolo, D. Junghans, T. Noumi and GS,**
"A Tower Weak Gravity Conjecture from Infrared Consistency",
Fortsch. Phys. 66, no. 5, 1800020 (2018)
arXiv: 1802.04287 [hep-th].

# A Web of Swampland Conjectures

$$
\begin{tikzcd}
\text{no global symmetry} \arrow[d, "g \neq 0"'] & \\
\text{WGC} \arrow[r] \arrow[d, "\geqq \;\to\; >"'] & \begin{array}{c}\text{tower WGC} \\ \text{sLWGC}\end{array} \\
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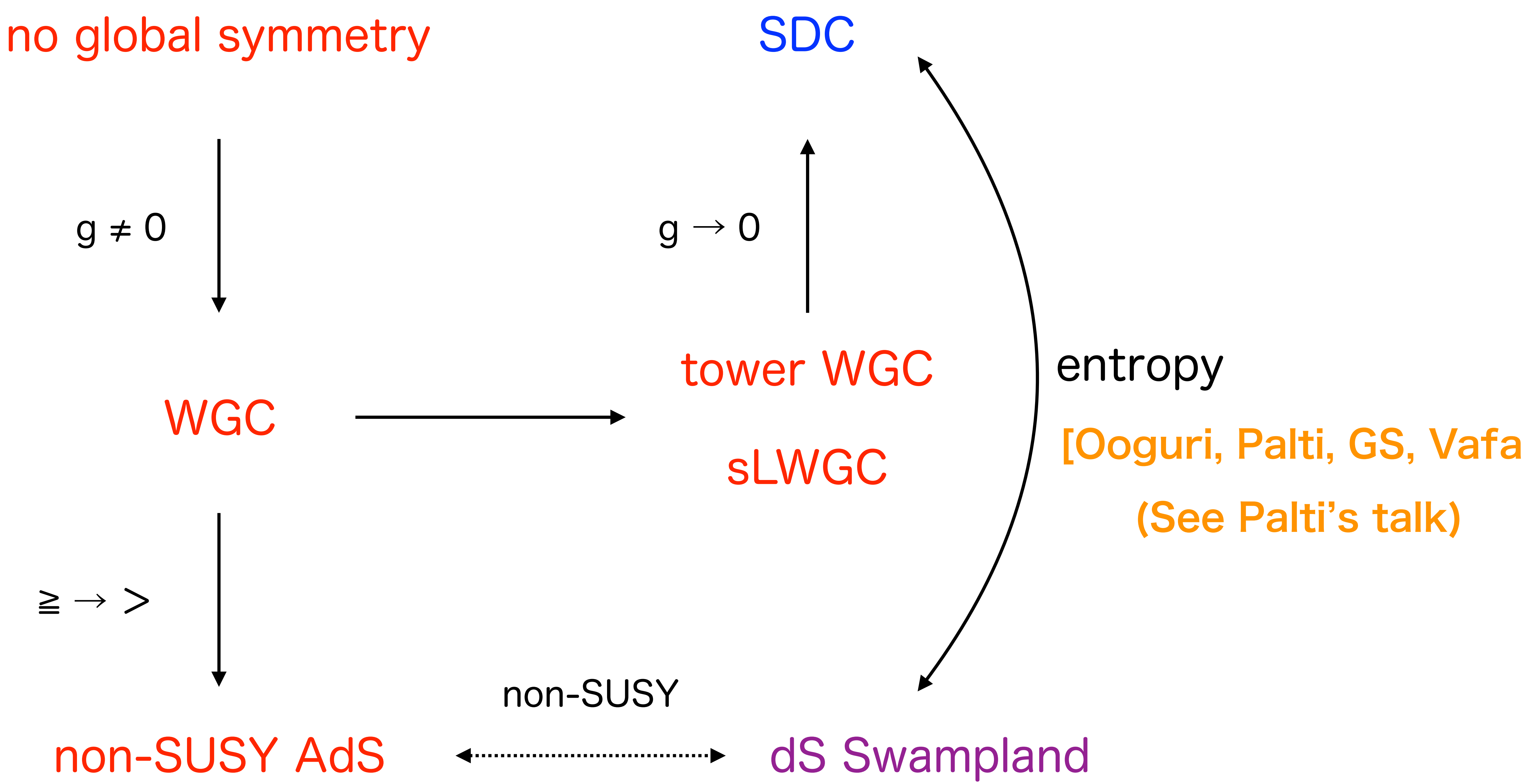

# The Weak Gravity Conjecture

Arkani-Hamed, Motl, Nicolis, Vafa '06

- The conjecture:

**"Gravity is the Weakest Force"**

- For every long range gauge field there exists a particle of charge q and mass m, s.t.

$$\frac{q}{m} M_P \geq \text{``}1\text{''}$$

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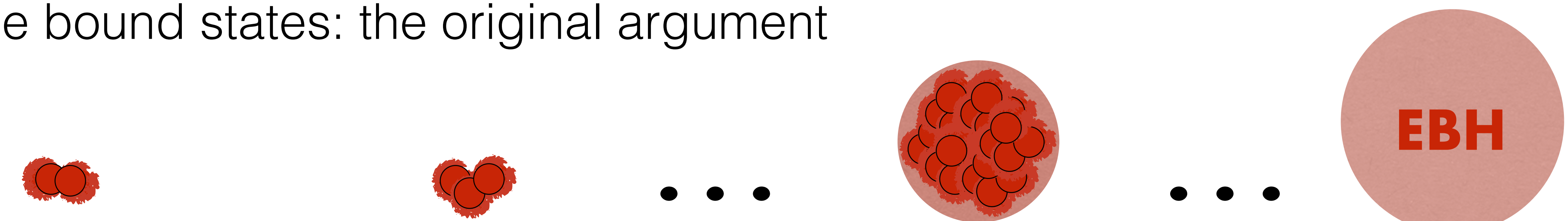
# The Weak Gravity Conjecture

- Take U(1) gauge theory and a scalar with $m > q\, M_p$


- Stable bound states: the original argument




$2m > M_2 > 2q$ $\quad$ $3m > M_3 > 3q$ $\quad \ldots \quad$ $Nm > M_N > Nq$ $\quad \ldots \quad$ $M_\infty \to Q_\infty$

- All these BH states are **exactly stable**. In particular, large bound states (charged black holes) do not Hawking radiate once they reach the extremal limit M=Q, equiv. T=0.

> "...there should not exist a large number of exactly stable objects (extremal black holes) whose stability is not protected by any symmetries."

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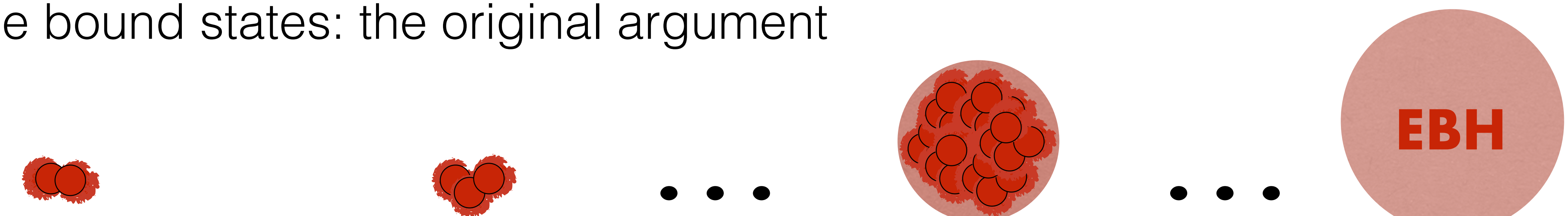


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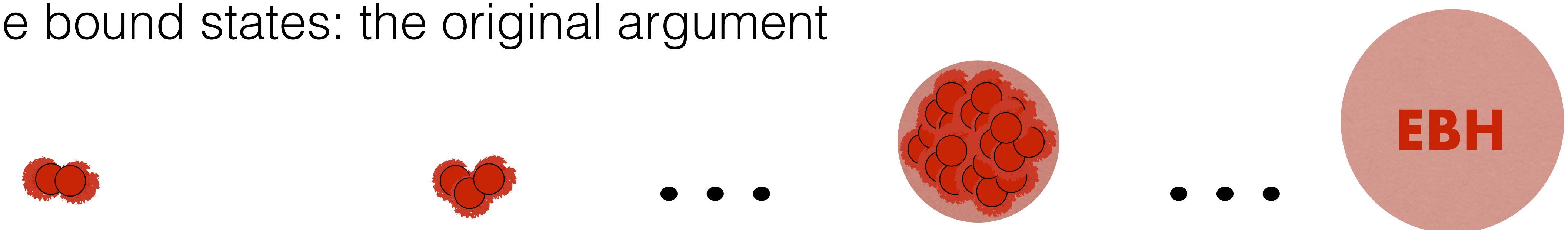


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- All these BH states are **exactly stable**. In particular, large bound states (charged black holes) do not Hawking radiate once they reach the extremal limit M=Q, equiv. T=0.

- In order to avoid a large number of exactly stable states one must demand the existence of some particle with

$$\frac{q}{m} \geq \frac{Q_{ext}}{M_{ext}} = \frac{1}{M_p}$$

# Evidences for the WGC

# Evidences for the Weak Gravity Conjecture

**Several lines of argument have been taken (so far):**

- Holography [Nakayama, Nomura, '15];[Harlow, '15];[Benjamin, Dyer, Fitzpatrick, Kachru, '16];[Montero, GS, Soler, '16] **(see Montero's talk)**
- Cosmic Censorship [Horowitz, Santos, Way, '16];[Cottrell, GS, Soler, '16];[Crisford, Horowitz, Santos, '17] **(see Crisford's talk)**
- Entropy considerations [Cottrell, GS, Soler, '16] [Fisher, Mogni, '17]; [Cheung, Liu, Remmen, '18]). **(see Remmen's and Soler's talks)**
- IR Consistencies (unitarity & causality) [Cheung, Remmen, '14] [Andriolo, Junghans, Noumi, GS,'18]. **(see Arkani-Hamed's talk)**

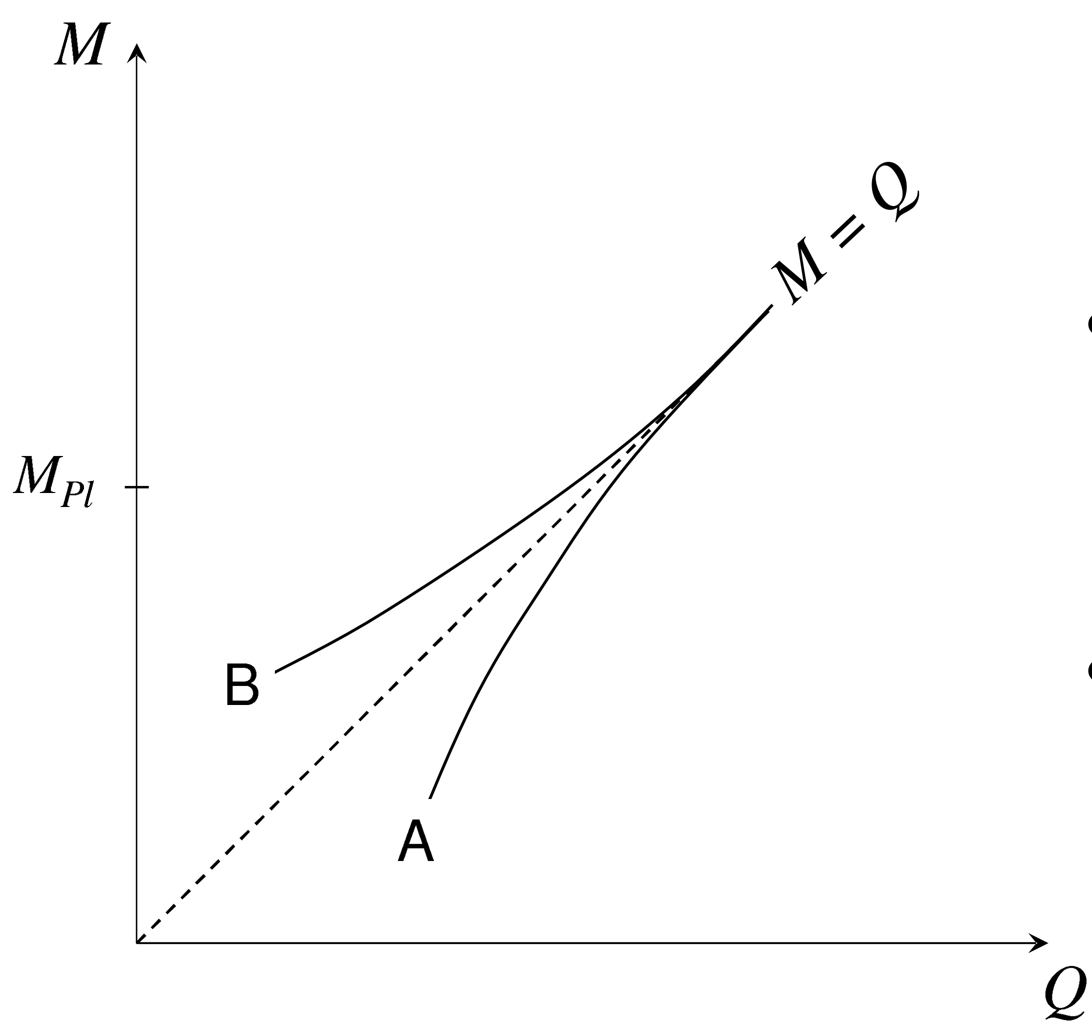
**Evidences for *stronger* versions of the WGC:**

- Consistencies with T-duality [Brown, Cottrell, GS, Soler, '15] and dimensional reduction [Heidenreich, Reece, Rudelius '15].
- Modular invariance + charge quantization suggest a **sub-lattice WGC** [Montero, GS, Soler, '16] (see also [Heidenreich, Reece, Rudelius '16])

# WGC and Blackholes

# Extremality of Blackholes

- The mild form of the WGC requires only ***some*** state for an extremal BH to decay to.
- **Can an extremal BH satisfy the WGC?**




- Higher derivative corrections can make extremal BHs lighter than the **classical bound** Q=M
- Demonstrated to be the case for 4D heterotic extremal BHs. **[Kats, Motl, Padi, '06]**
- We showed that this behavior (A) follows from unitarity (at least for some classes of theories). **[Hamada, Noumi, GS]**

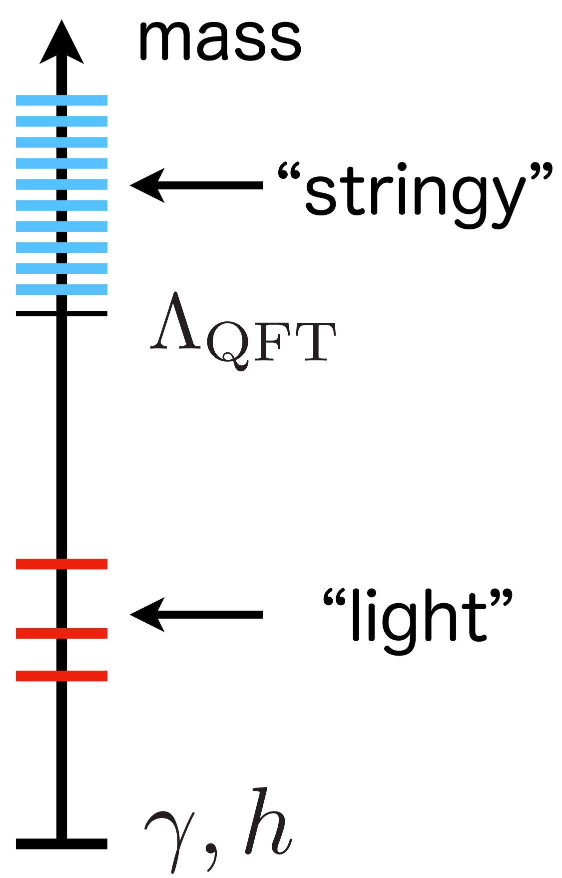
# Theories to which our proof applies

- If ∃ a particle with z=q/m>1, we are done. Even if not, we found that an EBH can satisfy the WGC for:
  - Theories with ***light*** (compared with the UV scale) ***parity-even, scalars*** (e.g., dilaton, moduli), or **spin 2 particles**

***Same limit*** ( $m_\phi \ll \Lambda$ ) considered in the proof of **[Cheung, Liu, Remmen]** using ΔS>0 but we can show more using unitarity:

1) $\phi$ is a parity-even scalar or spin 2 particle

2) ***Strict WGC inequality***




- ***Tree-level SUSY UV completion*** (i.e., higher derivative 4-pt amplitudes generated by tree-level exchange and respect SUSY).

# Higher Derivative Corrections

- In the IR, the BH dynamics is described by an EFT of photon & graviton.
- In D=4, the general effective action up to 4-derivative operators (assume parity invariance for simplicity):

$$S = \int d^4x \sqrt{-g} \left[ \frac{2M_{\rm Pl}^2}{4} R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \Delta\mathcal{L} \right]$$

where $$\begin{aligned}\Delta\mathcal{L} = \quad & c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \\ & + c_4 R F_{\mu\nu} F^{\mu\nu} + c_5 R_{\mu\nu} F^{\mu\rho} F^{\nu}{}_{\rho} + c_6 R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \\ & + c_7 F_{\mu\nu} F^{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} + c_8 F_{\mu\nu} F^{\nu\rho} F_{\rho\sigma} F^{\sigma\mu}.\end{aligned}$$

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$$S=\int d^4x\sqrt{-g}\left[\frac{2M_{\rm Pl}^2}{4}R-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{\alpha_1}{4M_{\rm Pl}^4}(F_{\mu\nu}F^{\mu\nu})^2\right.$$
$$\left.+\frac{\alpha_2}{4M_{\rm Pl}^4}(F_{\mu\nu}\widetilde{F}^{\mu\nu})^2+\frac{\alpha_3}{2M_{\rm Pl}^2}F_{\mu\nu}F_{\rho\sigma}W^{\mu\nu\rho\sigma}\right]$$

by field redefinition. Here, $W^{\mu\nu\rho\sigma}$ is the **Weyl tensor:**

$$R_{\mu\nu\rho\sigma}=W_{\mu\nu\rho\sigma}+\frac{1}{2}\left(g_{\mu[\rho}R_{\sigma]\nu}-g_{\nu[\rho}R_{\sigma]\mu}\right)-\frac{1}{3}Rg_{\mu[\rho}g_{\sigma]\nu}$$

# Extremality Condition

- The higher derivative operators modify the BH solutions, so the charge-to-mass ratio of an extremal BH is corrected:

$$z = \frac{\sqrt{2} M_{\mathrm{Pl}} |Q|}{M} = 1 + \frac{2}{5} \frac{(4\pi)^2}{Q^2} (2\alpha_1 - \alpha_3)$$

**[Kats, Motl, Padi, '06]**

applicable when the BH is sufficiently heavy:

$$M^2 \sim Q^2 M_{\mathrm{Pl}}^2 \gg \alpha_i M_{\mathrm{Pl}}^2$$

because extremal BHs in Einstein-Maxwell theory satisfy:

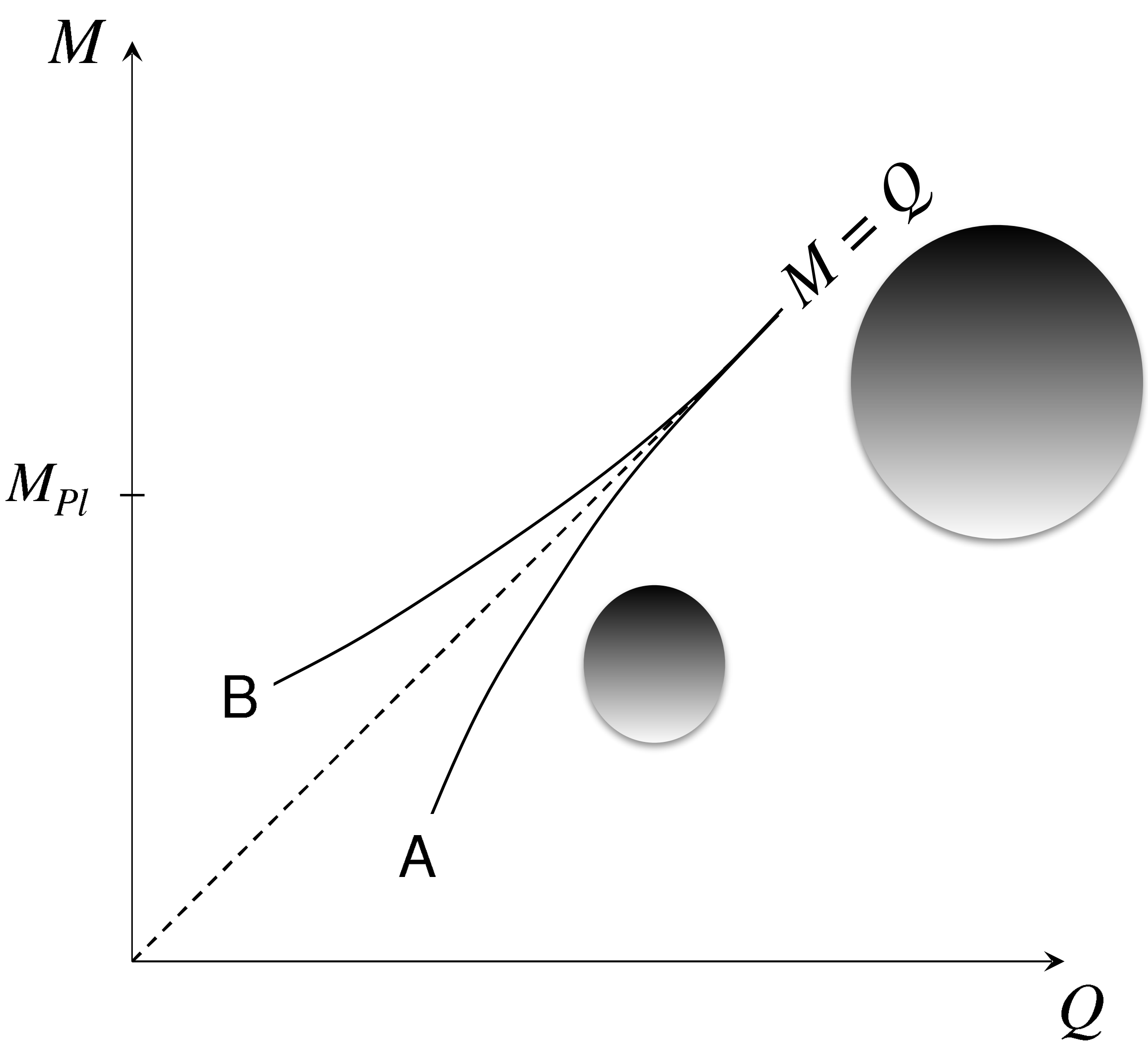
$$R \sim M_{\mathrm{Pl}}^4 / M^2 \text{ and } F^2 \sim M_{\mathrm{Pl}}^6 / M^2$$

# WGC from Unitarity

- Proving the WGC (mild form) amounts to showing:

$$2\alpha_1 - \alpha_3 \geq 0\,.$$

so large extremal BHs can decay into smaller extremal BHs.

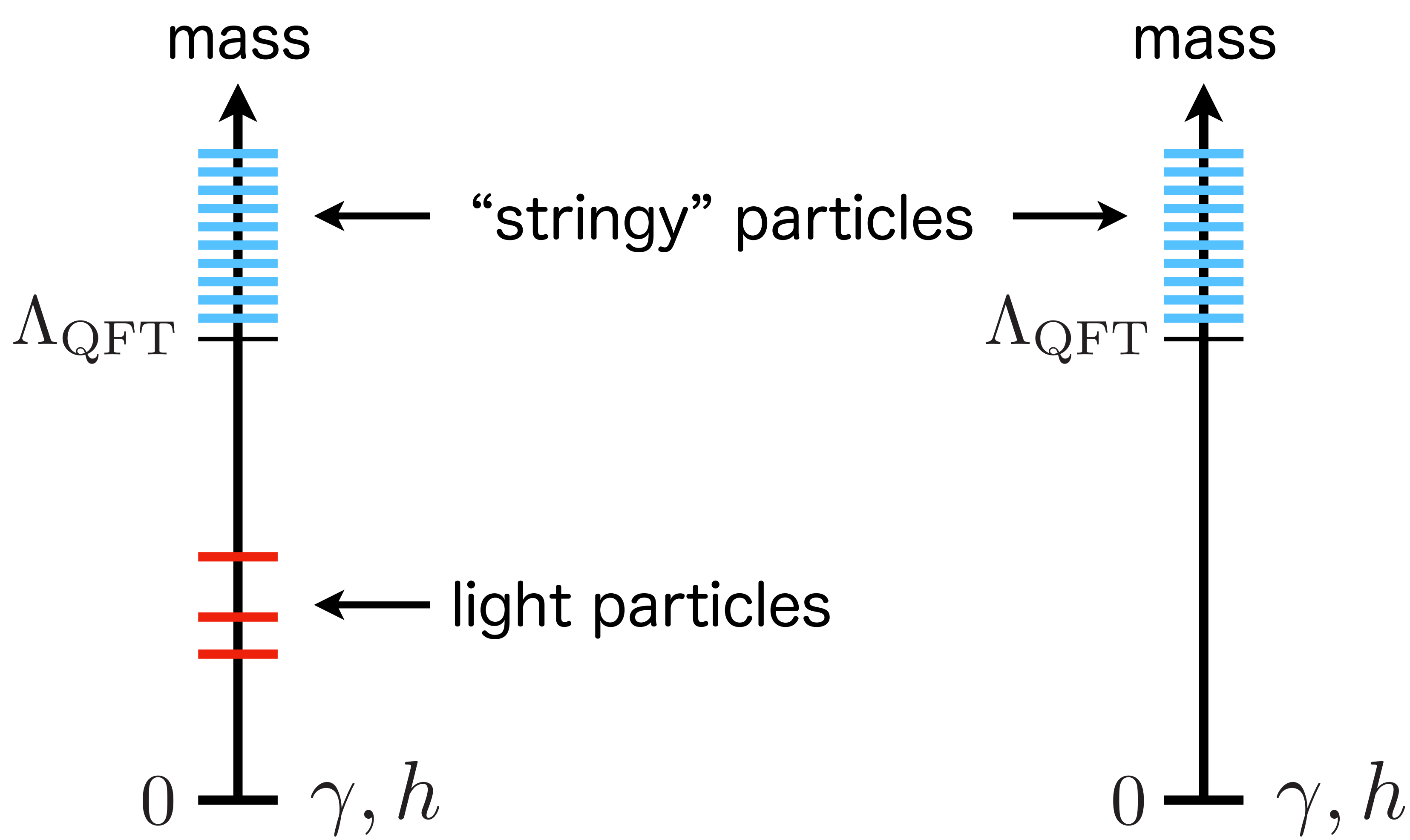



For some classes of theories (defined shortly), this follows from basic principles e.g. unitarity & causality

**[Hamada, Noumi, GS]**

# Sources of Higher Dimensional Operators

- Dominant sources depend on the particle spectrum:



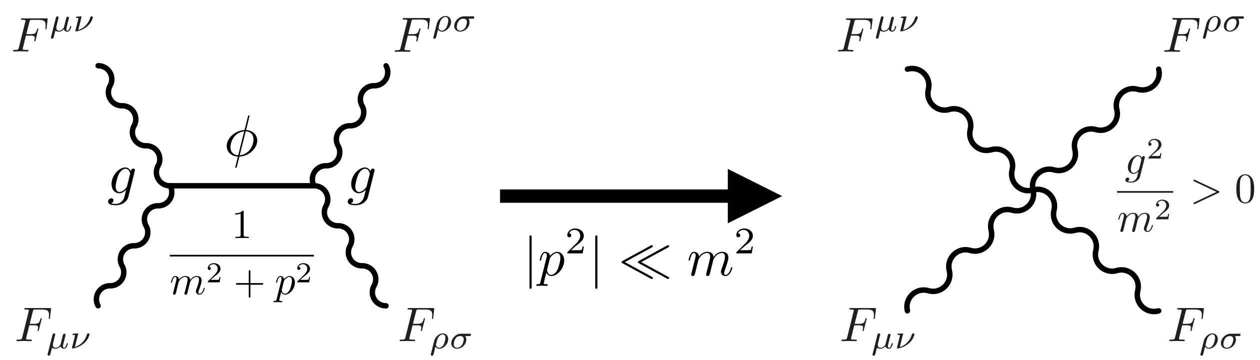

- We assume a **weakly coupled UV completion**. There exists some scale $\Lambda_{QFT} < M_{Pl}$ above which ordinary QFT breaks down. In perturbative string theory, $\Lambda_{QFT} = M_s < M_{Pl}$.

# Sources of Higher Dimensional Operators

- There are 3 sources of higher dimensional operators, which we refer to as (a), (b), (c):

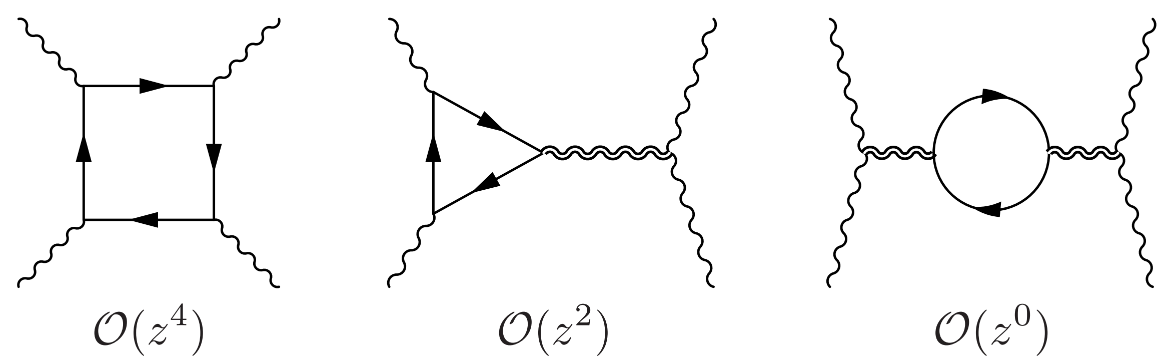
**(a) Neutral Bosons**

(dilaton, axion, moduli)




**(b) Loop Effects**

(charged particles)




**(c) UV Effects**

(string states)

- We now discuss in turn their unitarity constraints.

# (a) Light Neutral Bosons

- Consider a scalar (dilaton) and a pseudoscalar (axion):

$$\mathcal{L}_\phi = -\frac{1}{2}(\partial_\mu \phi)^2 - \frac{m_\phi^2}{2}\phi^2 + \frac{\phi}{f_\phi} F_{\mu\nu}F^{\mu\nu} \,,$$

$$\mathcal{L}_a = -\frac{1}{2}(\partial_\mu a)^2 - \frac{m_a^2}{2}a^2 + \frac{a}{f_a} F_{\mu\nu}\widetilde{F}^{\mu\nu} \,,$$

- Integrating them out leads to tree-level effective couplings:

$$\alpha_1 = \frac{2M_{\rm Pl}^4}{m_\phi^2 f_\phi^2} \,, \quad \alpha_2 = \frac{2M_{\rm Pl}^4}{m_a^2 f_a^2}$$

- We can estimate, for f $\lesssim$ M$_{\rm Pl}$, $|\alpha_i| \gtrsim \mathcal{O}\left(\frac{M_{\rm Pl}^2}{m^2}\right)$

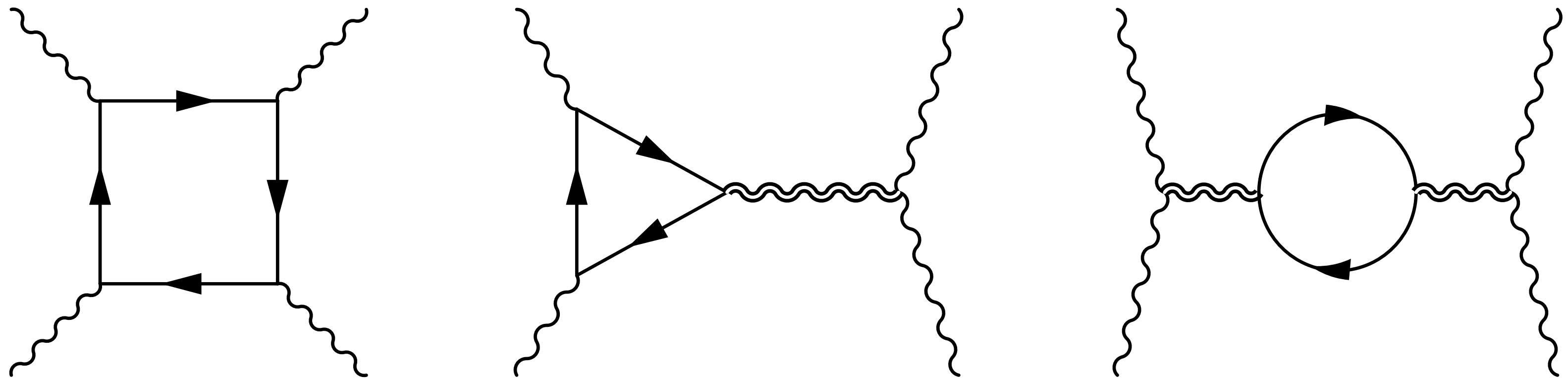
- Positivity of $\alpha_{1,2}$ is consequence of unitarity. More generally, unitarity ⇒

  $\alpha_1$>0 for parity-even neutral scalar or spin 2 neutral particle

  $\alpha_2$>0 for parity-odd neutral scalar or spin 2 neutral particle

# (b) Charged Particles

- Do not contribute at tree-level, leading contribution is 1-loop:



$\mathcal{O}(z^4)$ $\mathcal{O}(z^2)$ $\mathcal{O}(z^0)$

- For example, 1-loop effective couplings generated by minimally coupled charged particles

$$\alpha_{1,2} = \max\left\{\mathcal{O}(z^4), \mathcal{O}(1)\right\} \ , \ \ \alpha_3 = \mathcal{O}(z^2)$$

- If z ≫ 1, $|\alpha_1|$, $|\alpha_2|$ ≫ $|\alpha_3|$ ≫1. In this limit, gravity is negligible and unitarity for QFT (using spectral representations) implies

$$\alpha_1 > 0 \text{ and } \alpha_2 > 0$$

**Adams et al, '06**
**Cheung, Remmen, '14**

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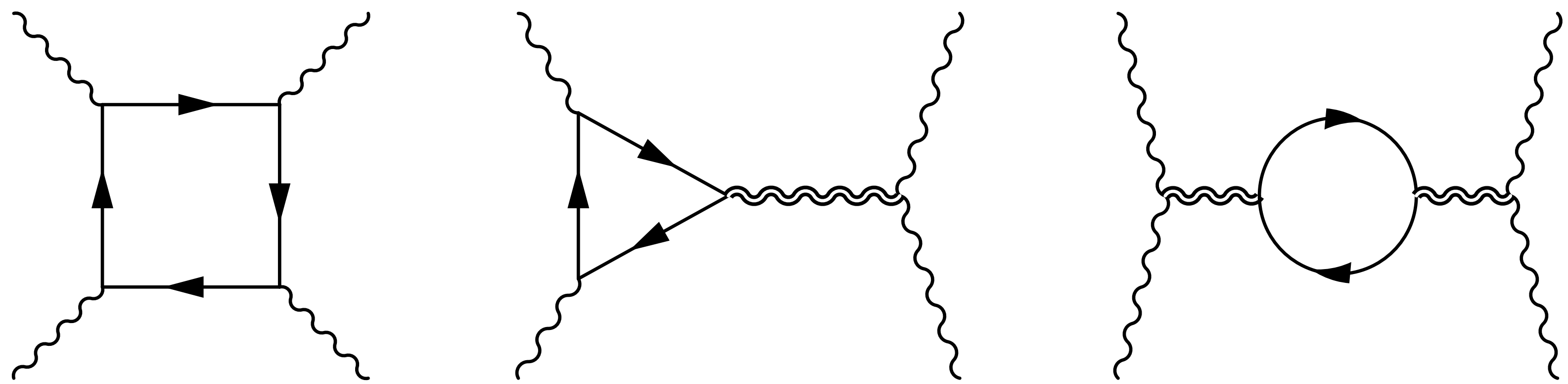

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- If z ≫ 1, $|\alpha_1|$, $|\alpha_2|$ ≫ $|\alpha_3|$ ≫1. Not only do we have superextremal particles, there are extremal BHs with z>1:

$$2\alpha_1 - \alpha_3 \geq 0\,.$$

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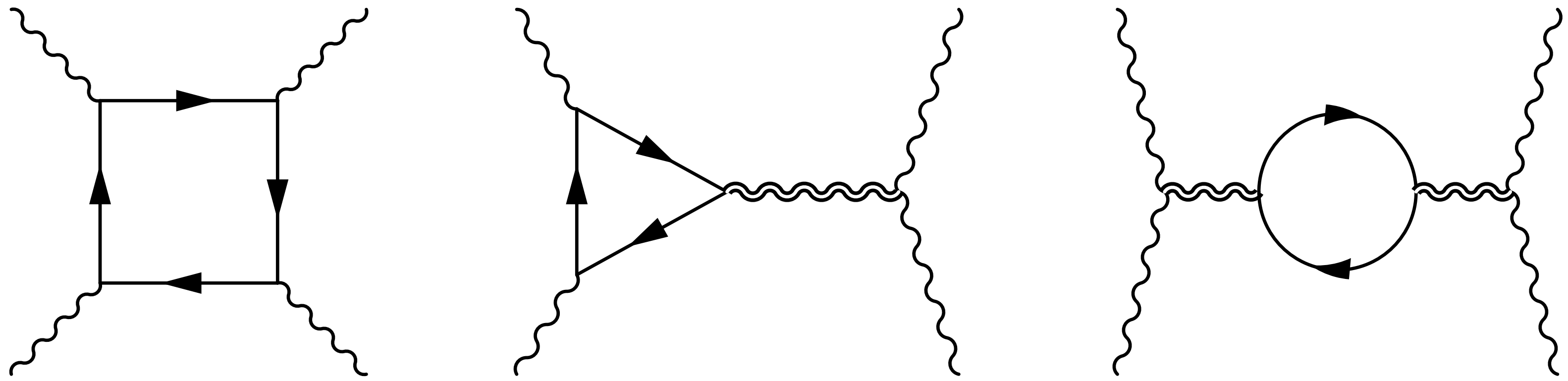

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- If z ≲ 1, $\alpha_\mathrm{i} \sim O$ (1), no rigorous unitarity bound is known, but other effects (A) and (C) dominate.

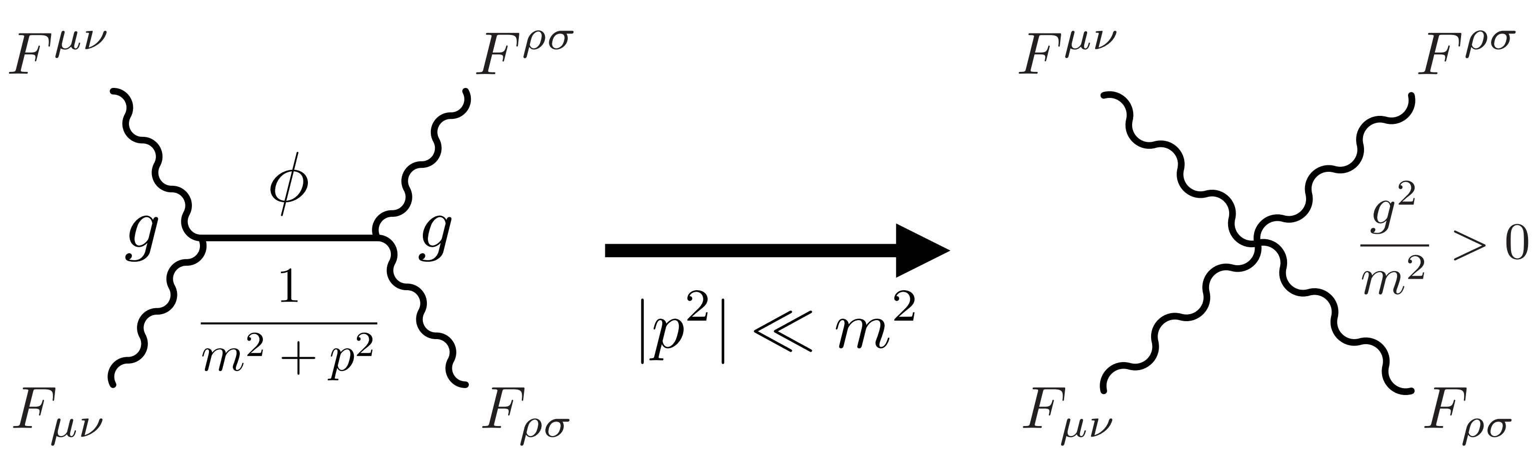
# (c) UV Effects

- Higher derivative operators can also be generated by integrating out UV effects:

$$\alpha_i = \mathcal{O}\left(\frac{M_{\rm Pl}^2}{\Lambda_{\rm QFT}^2}\right),$$

where $\Lambda_{\rm QFT}$ is the scale above which ordinary QFT breaks down. In string theory, these are $\alpha$' effects.

- **Tree-level UV completion:** If 4-pt amplitudes are generated by tree-level exchanges (natural in string theory)




**unitarity ⇒**

$$\alpha_1 > 0 \text{ and } \alpha_2 > 0$$

# WGC from Unitarity

| | magnitude | unitarity |
|---|---|---|
| (a) neutral bosons | $\alpha_i \gtrsim \mathcal{O}\Big(\frac{M_{\rm Pl}^2}{m^2}\Big)$ | $\alpha_1, \alpha_2 > 0$ |
| (b) charged particles | | |
| (b-1) $z \gg 1$ | $\lvert\alpha_1\rvert, \lvert\alpha_2\rvert \gg \lvert\alpha_3\rvert \gg 1$ | $\alpha_1, \alpha_2 > 0$ |
| (b-2) $z = \mathcal{O}(1)$ | $\alpha_i = \mathcal{O}(1)$ | N.A. |
| (c) UV effects | $\alpha_i = \mathcal{O}\Big(\frac{M_{\rm Pl}^2}{\Lambda_{\rm QFT}^2}\Big)$ | $\alpha_1, \alpha_2 > 0\,(\star)$ |

- When (b-1) dominates, $2\alpha_1 - \alpha_3 > 0$

⇒ large extremal BHs can decay but then we already have a superextremal particle satisfying the WGC.

- We are interested in whether extremal BHs may play the role of the WGC state when there are no particles with z ≥ 1

⇒ Effects (a) or (c) (which are tree-effects) dominate.

# Supersymmetry

- The effective operator $\alpha_3$ generates new photon-photon-graviton helicity amplitudes that are not present in Einstein-Maxwell:

$$\mathcal{M}(1^+, 2^-, 3^{\pm 2}) = \mathcal{M}(1^-, 2^+, 3^{\pm 2}) \sim \frac{E^2}{M_{\rm Pl}}\,,$$
$$\mathcal{M}(1^+, 2^+, 3^{+2}) = \mathcal{M}(1^-, 2^-, 3^{-2}) \sim \alpha_3 \frac{E^4}{M_{\rm Pl}^3}\,,$$
$$(\text{other helicity amplitudes}) = 0\,,$$

- $\mathcal{M}$ $(1^+, 2^+, 3^{+2})$ and $\mathcal{M}$ $(1^-, 2^-, 3^{-2})$ are incompatible with the SUSY Wald-Takahashi identity, hence in SUSY theories:

$$\alpha_3 = 0$$

- WGC follows from unitarity $\alpha_1 > 0 \Rightarrow$

$$2\alpha_1 - \alpha_3 > 0$$

# Tree-Level Supersymmetry

- We only used SUSY of ***tree-level*** amplitudes to set $\alpha_3 = 0$.
- Arguments apply to non-SUSY theories as long as tree-level scattering of photons & gravitons is consistent with SUSY.

  e.g., spacetime SUSY is broken in the O(16)xO(16) string **[Alvarez-Gaume, Ginsparg, Moore, Vafa]** but tree-level vertices of the bosonic sector is same as $E_8 \times E_8$ heterotic superstring.

- The bosonic string doesn't enjoy tree-level SUSY, has $\alpha_3 \neq 0$.
- There may be other principles that set $\alpha_3 = 0$, e.g., if the higher derivative cubic interactions are also generated by heavy particle exchange, i.e., no "bare"

$$\frac{\alpha_3}{2M_{\rm Pl}^2} F_{\mu\nu} F_{\rho\sigma} W^{\mu\nu\rho\sigma}$$

# Causality

- The helicity amplitudes $\mathcal{M}$ (1+, 2+, 3+2) & $\mathcal{M}$ (1-, 2-, 3-2) lead to causality violation at the energy scale: $E \sim M_{\rm Pl}/\sqrt{\alpha_3}$

  Moreover, an **infinite tower of massive higher spin particles** with m $\gtrsim$ $M_{\rm Pl}/\sqrt{\alpha_3}$ is required to UV complete the EFT at tree-level **[Camanho, Edelstein, Maldacena, Zhibodev]**

- The scale at which QFT breaks down: $\Lambda_{\rm QFT} \sim M_{\rm Pl}/\sqrt{\alpha_3}$

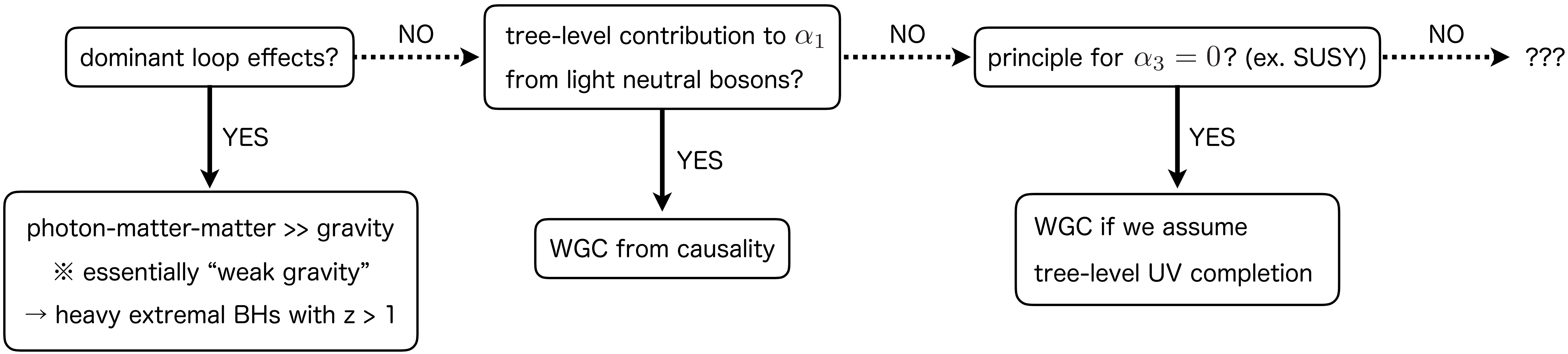
$$\Rightarrow \quad \alpha_3 \sim \frac{M_{\rm Pl}^2}{\Lambda_{\rm QFT}^2} \quad \Rightarrow$$ **effect (c)**

- If tree-level **effect (a)** dominates, casualty implies

$$|\alpha_1|\,,|\alpha_2| \gg |\alpha_3|$$

- The WGC can be satisfied by extremal BHs if ∃ a **parity-even neutral scalar** or a **spin 2 neutral particle** with m « $\Lambda_{\rm QFT}$.

# Summarizing the Unitarity Constraints



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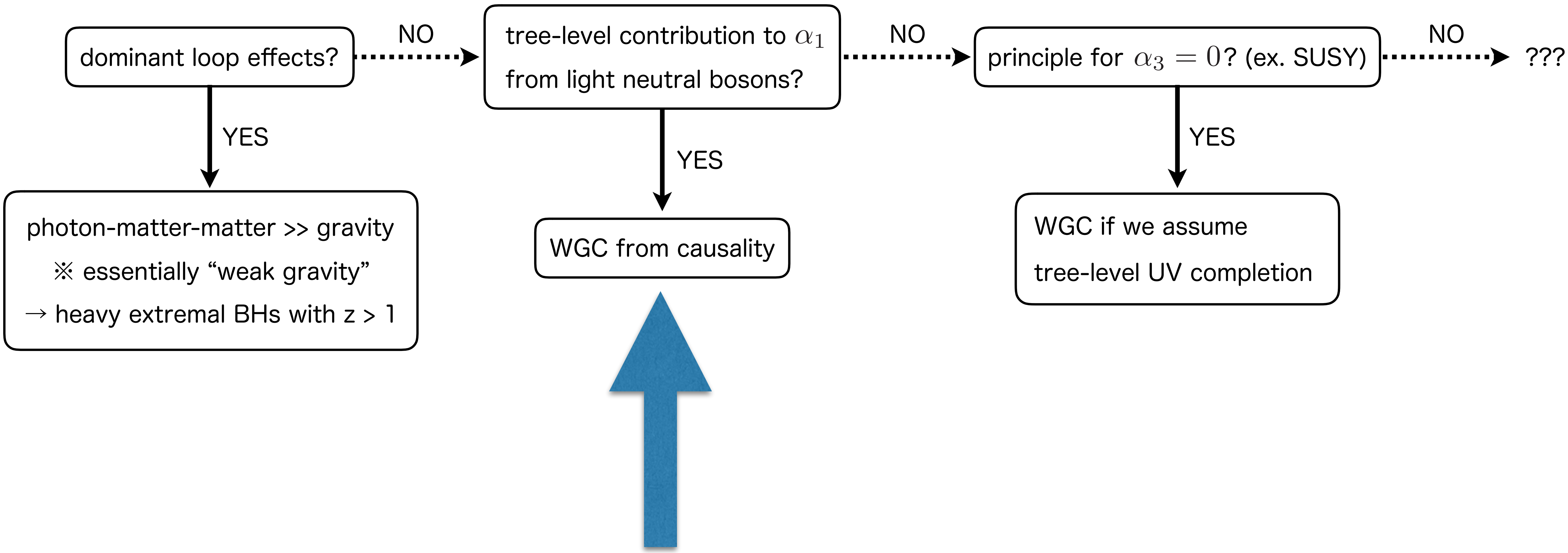


Case covered by entropy consideration

**[Cheung, Liu, Remmen]**

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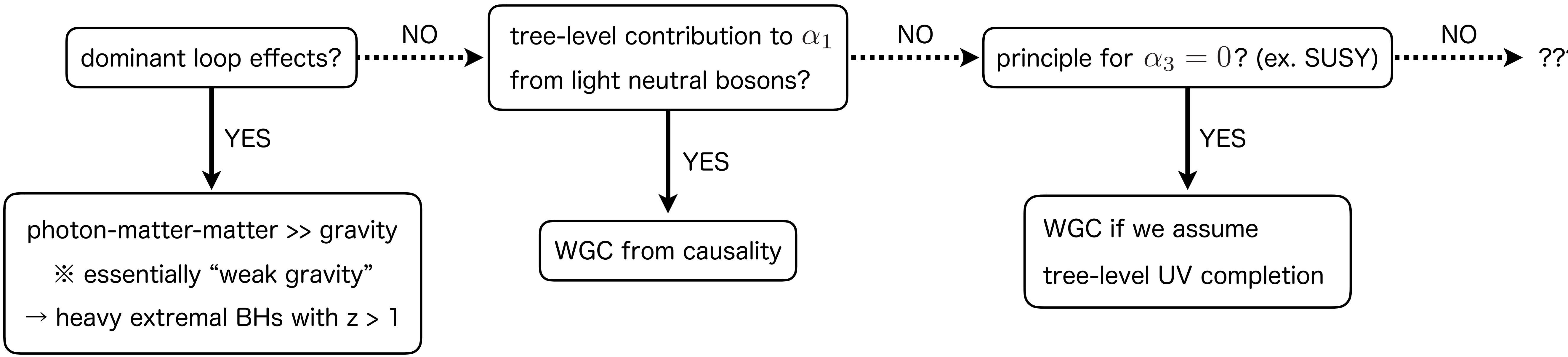


**Open-closed duality interpretation**

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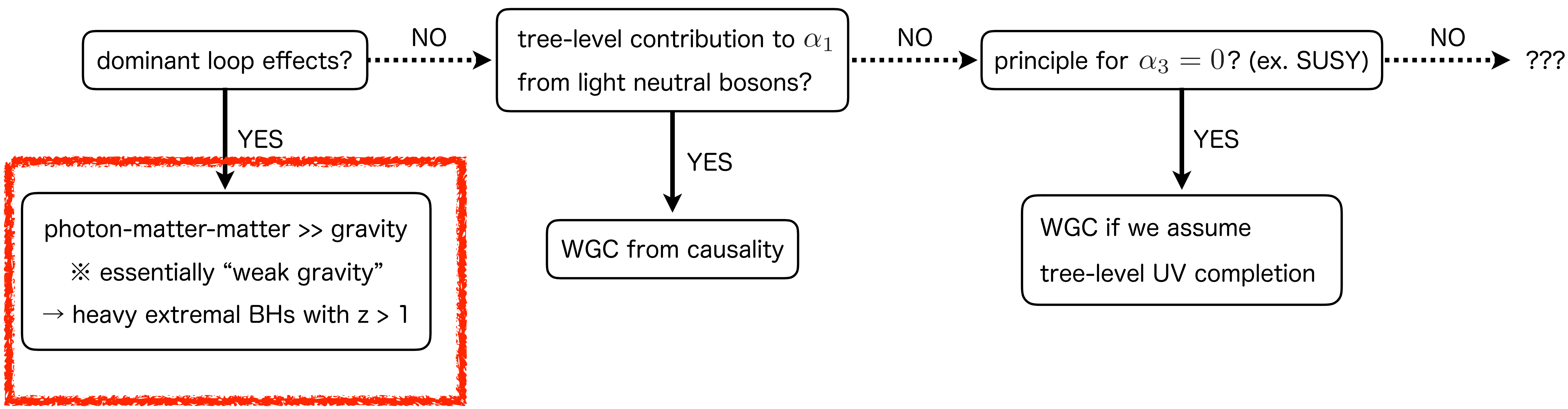


**Open-closed duality interpretation**

**Open**
**z > 1**

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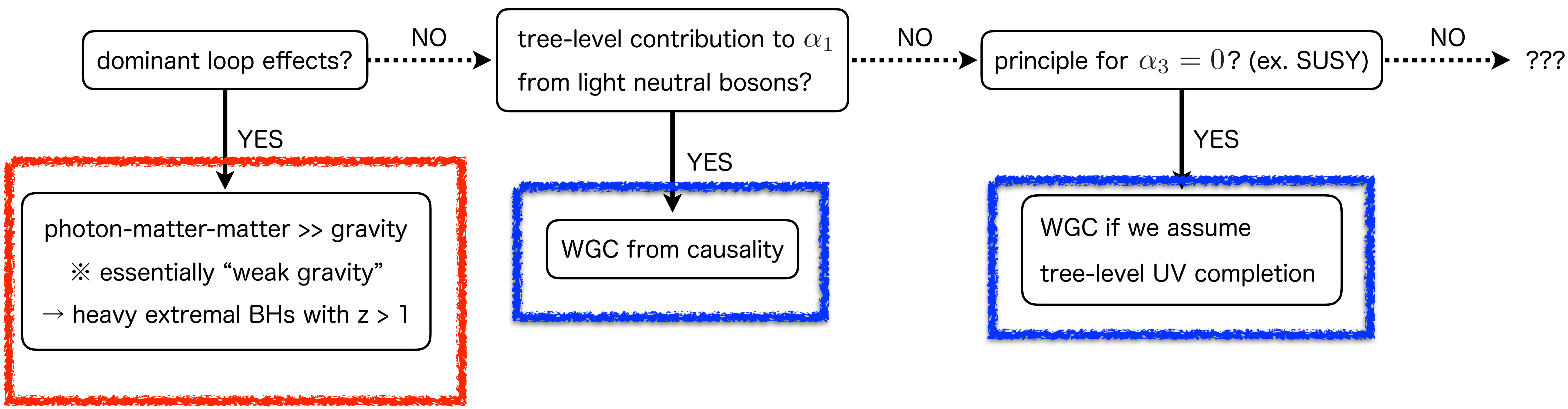



## Open-closed duality interpretation

**Open**
**z > 1**

**Closed**
**z < 1**

# Stronger forms of the WGC

# Stronger forms?

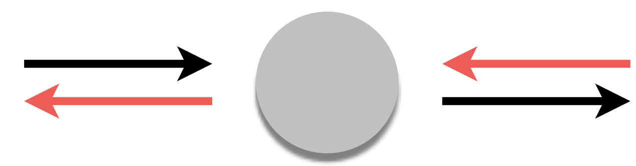
Do stronger forms of the WGC such as:

- WGC satisfied by a ***particle*** with mass $< \Lambda_{QFT} < M_{Pl}$ ?
- Convex hull condition for multiple U(1)'s? **[Cheung, Remmen]**
- Tower WGC **[Andriolo, Junghans, Noumi, GS]** or sLWGC **[Montero, GS, Soler];[Heidenreich, Reece, Rudelius]**?

follow from unitarity?

With some **additional assumptions** on the UV, we can obtain these stronger forms using unitarity constraints.

# Positivity Bounds

Positivity of photon-graviton EFT Im  $\geq 0$

implies $z^4 - z^2 + \gamma \geq 0$ **[Cheung, Remmen]**

→ at lest one of the following two should be satisfied

1) WGC type lower bound on charge-to-mass ratio

in particular when $\gamma = 0$ , WGC $z^2 \geq 1$ is reproduced!

2) not so small value of UV sensitive parameter $\gamma > 0$

In **[Andriolo, Junghans, Noumi, GS]**, we discussed

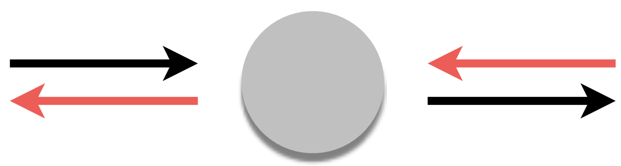
- multiple U(1)'s
- implications for KK reduction

and found **qualitatively new features**.

# Multiple U(1)'s

# for example, let us consider $U(1)_1 \times U(1)_2$

a new ingredient is positivity of $\gamma_1 + \gamma_2 \to \gamma_1 + \gamma_2$

Im  $\geq 0$ implies $z_1^2 z_2^2 - z_1^2 - z_2^2 \geq 0$

- $z_i = q_i/m$ is the charge-to-mass ratio for each U(1)

- we set $\mathcal{O}(z^0) = 0$ for illustration (same as γ = 0 before)

the punchline here:

positivity bound cannot be satisfied unless $z_1^2 z_2^2 \neq 0$

→ **requires existence of a bifundamental particle!**

# Implications for KK Reduction

\# $S^1$ compactify d+1 dim Einstein-Maxwell with single U(1) into d dim Einstein-Maxwell with $U(1) \times U(1)_{\rm KK}$

d+1 dim charged particle (q,m)

→ KK tower with the charged-to-mass ratios

$$(z, z_{\rm KK}) = \left( \frac{q}{\sqrt{m^2 + n^2 m_{\rm KK}^2}}, \ \frac{n}{\sqrt{(m/m_{\rm KK})^2 + n^2}} \right)$$

in the small radius limit $m_{\rm KK} \to \infty$

the lowest mode (n = 0): $(z, z_{\rm KK}) = (q/m, 0)$

KK modes (n ≠ 0): $(z, z_{\rm KK}) \simeq (0, 1)$

※ no bifundamentals → ~~positivity bound~~ generically

# Tower WGC

[Andriolo, Junghans, Noumi, GS]

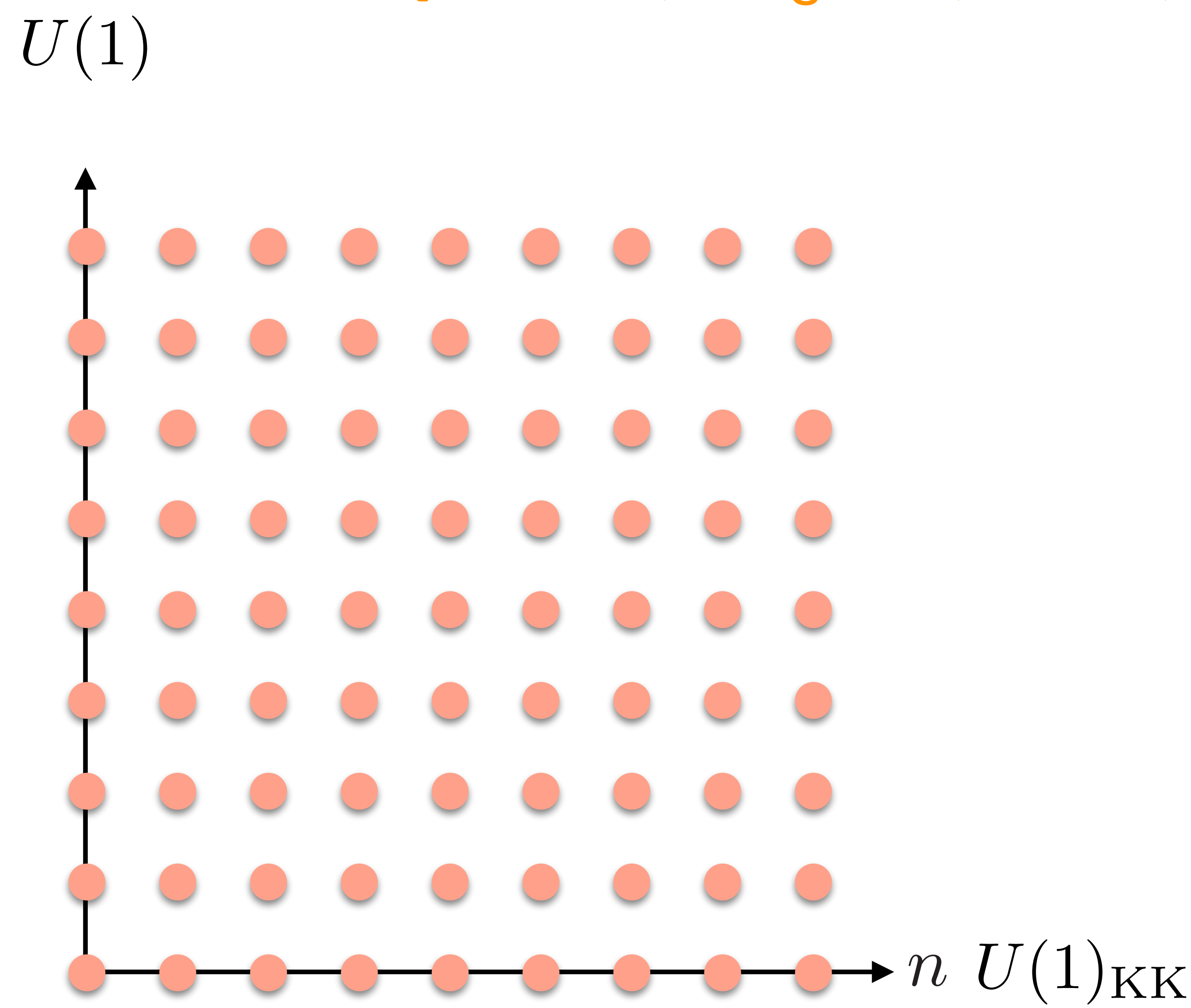
<u>d+1 dim</u>

charged particles

labeled by $\ell = 1, 2, \ldots$

$$(q, m) = (\ell\, q_*, \ell\, m_*)$$

s.t. $z_* = \dfrac{q_*}{m_*} = \mathcal{O}(1)$




d dim charged particles

$$(z, z_{\mathrm{KK}}) = \left( \frac{\ell\, z_*}{\sqrt{\ell^2 (m_*/m_{\mathrm{KK}})^2 + n^2}}, \ \frac{n}{\sqrt{\ell^2 (m_*/m_{\mathrm{KK}})^2 + n^2}} \right)$$

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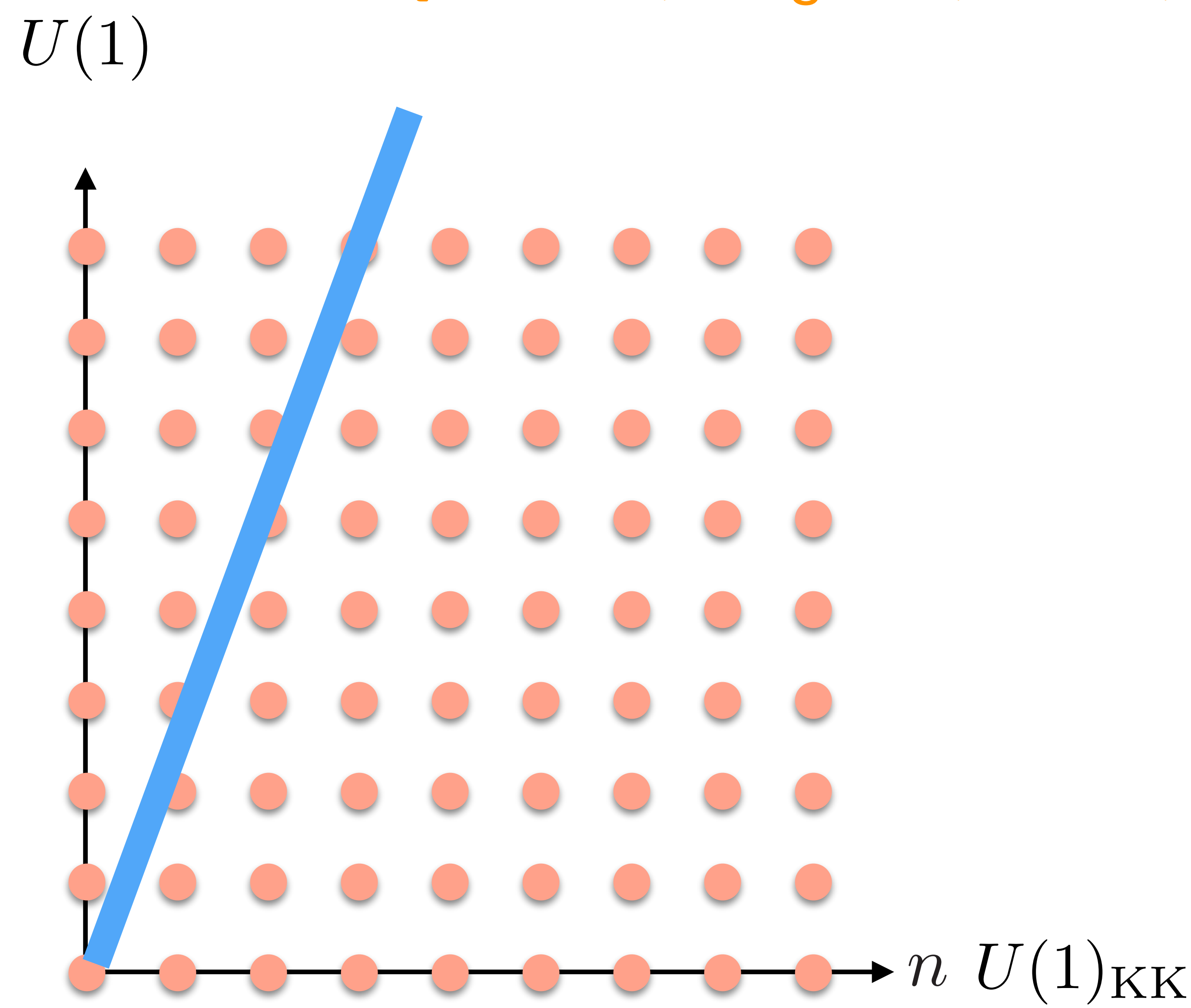


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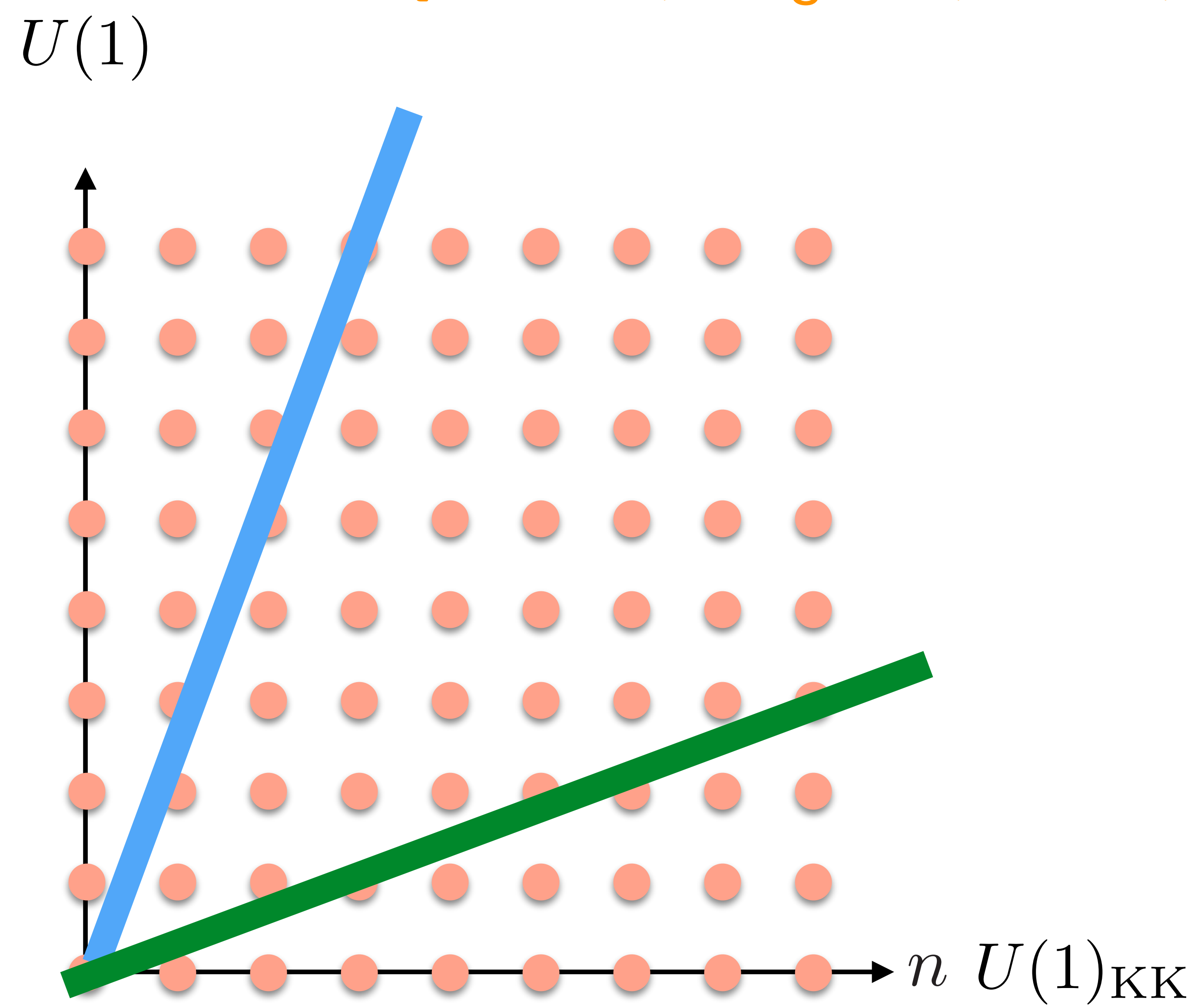


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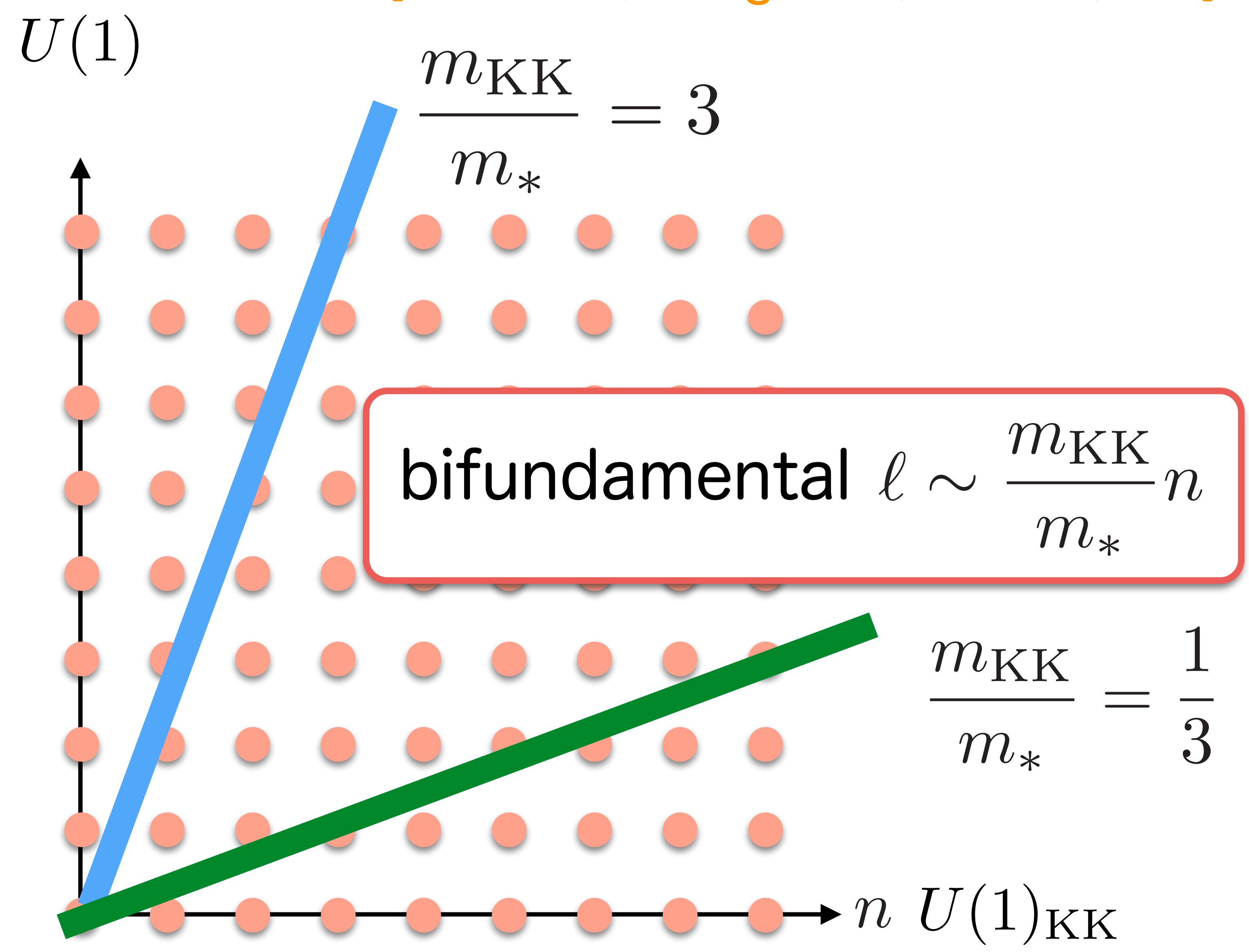


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- We show that the WGC (mild form) can be satisfied by extremal BHs for a wide class of theories, including generic string setups with dilation or moduli with mass below $M_S$.
- Unitarity bounds on the EFT $\Rightarrow$ stronger forms of the WGC (convex hull, tower WGC) under some assumptions on the UV.
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