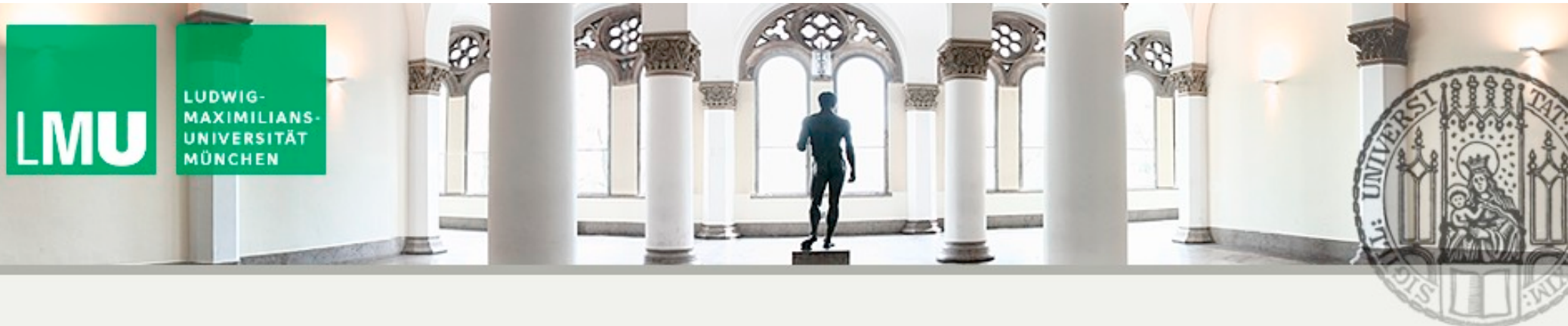


# W-Supergravity

DIETER LÜST (LMU, MPI)



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Joint work with Sergio Ferrara: arXiv:1805.10022  
and with Sergio Ferrara & Alex Kehagias: arXiv:1806.10016

See also earlier work with I. Florakis, I. Garcia-Etxebarria, D. Regalado:  
arXiv:1712.04318

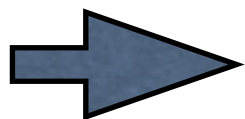
# No-go theorems in Quantum Gravity:

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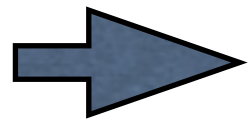


Swampland program

[H. Ooguri, C. Vafa (2006)]

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- Weak gravity conjecture

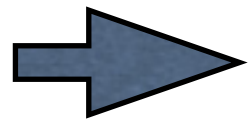
[N.Arkani-Hamed, L. Motl, A. Nicolis, C.Vafa (2007)]

- de Sitter swampland conjecture

[G. Obied, H. Ooguri, L. Spodyneiko, C.Vafa (2018)]

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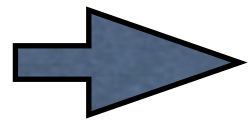
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In general, all (also hidden) assumptions have to be very carefully stated.

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In general, all (also hidden) assumptions have to be very carefully stated.

E.g. de Sitter vacua in **strongly coupled** world-sheet theories: DFT and non-geometric constructions.

[F.Hassler, D.L., S. Massai (2014)]



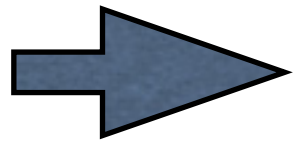
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W - supergravity & W - superstrings

[S. Ferrara, D.L. (2018)]

## Reverse question:

Are there quantum field theories, which according to some no-go theorems, do not exist in the IR, but nevertheless can be realized in the UV?

➡ **W - supergravity & W - superstrings**

[S. Ferrara, D.L. (2018)]

- Exist also for N=7 supersymmetry
- Topological theories: topological twist by S - duality and diffeomorphisms
- **Strongly coupled**, massive spin-four theories

## Classification of rigid, supersymmetric 4D field theories:

exist for  $\mathcal{N} = 1, 2, 4$  supersymmetry

But not for  $\mathcal{N} = 3$  (automatically extends to  $\mathcal{N} = 4$ )

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## Classification of 4D supergravity theories:

exist for  $\mathcal{N} = 1, 2, 3, 4, 5, 6, 8$  supersymmetry

But not for  $\mathcal{N} = 7$  (automatically extends to  $\mathcal{N} = 8$ )

## Main assumptions:

- weak coupling
- CPT invariant lagrangian description
- massless N-extended spin-two gravity multiplet

This agrees with perturbative 4D string constructions:



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4D Heterotic string: # of right-moving supercharges:

$$\mathcal{N}_H = 1, \quad \mathcal{N}_H = 2, \quad \text{and} \quad \mathcal{N}_H = 4$$

$$\mathbb{R}^{1,3} \times \quad CY_3 \quad K3 \times T^2 \quad T^6$$

Again no  $\mathcal{N} = 3$  supersymmetry.

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Again no  $\mathcal{N} = 3$  supersymmetry.

4D Type II string: # of left + right-moving supercharges:

$$\begin{aligned} \mathcal{N}_{II} = 1 &= 1_L + 0_R, & \mathcal{N}_{II} = 2 &= 2_L + 0_R, & \mathcal{N}_{II} = 2' &= 1_L + 1_R, \\ \mathcal{N}_{II} = 3 &= 2_L + 1_R, & \mathcal{N}_{II} = 4 &= 4_L + 0_R, & \mathcal{N}_{II} = 4' &= 2_L + 2_R, \\ \mathcal{N}_{II} = 5 &= 4_L + 1_R, & \mathcal{N}_{II} = 6 &= 4_L + 2_R, & \mathcal{N}_{II} = 8 &= 4_L + 4_R. \end{aligned}$$

Again no  $\mathcal{N} = 7$  supersymmetry.

[H. Kawai, S. Lewellen, H. Tye (1987);  
W. Lerche, D.L., A. Schellekens (1987);  
A. Antoniadis, C. Bachas, C. Kounnas (1987);  
S. Ferrara, D.L., S. Theisen (1989); S. Ferrara, C. Kounnas (1989)]

How to evade „no-go theorems“:

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Field theory:

S-fold construction

[I. Garcia-Etxebarria, D. Regalado (2015/16);  
O. Aharony, M. Evtikhiev (2015);  
O. Aharony, Y. Tachikawa (2016);  
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$$(\mathcal{N} = 3_{SYM}) \equiv (\mathcal{N} = 4_{SYM}) / [(\text{R} - \text{sym.}) \times SL(2)_S] .$$

- Three invariant supercharges
- Strongly coupled theory (  $S = i$  )
- Non-lagrangian, superconformal theory

Follow similar strategy:

$$\mathcal{N} = 7 \text{ W - Sugra: } \quad \text{S-fold of } \mathcal{N} = 8 \text{ Sugra}$$

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Result: massless spin - two multiplet of  $\mathcal{N} = 8$



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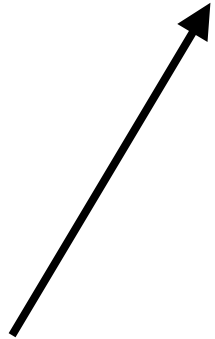
$$\mathcal{N} = 7 \text{ W - Sugra} \equiv (\mathcal{N} = 4 \text{ YM}) \otimes (\mathcal{N} = 3 \text{ YM})$$

Two factors: massive spin-two multiplets of  $\mathcal{N} = 4$  and  $\mathcal{N} = 3$

Result: massive spin - four multiplet of  $\mathcal{N} = 7$

W - supergravity

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Higher spin theory

W - supergravity

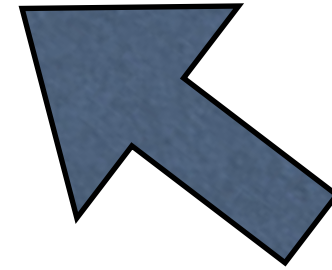
```
graph BT; A[Higher spin theory] --> C[W - supergravity]; B[Weyl] --> C;
```

Higher spin theory

Weyl

# Outline:

II) Double copy construction of  $\mathcal{N}=7$  W-supergravity -  
 $\mathcal{N}=7$  W-supergravity



III) W - Superstring construction

IV) Conclusion and Outlook

## II) Double copy construction of W - supergravity

$$\text{QFT}(\mathcal{N}_L) \otimes \text{QFT}(\mathcal{N}_R) = \text{Sugra}(\mathcal{N}_L + \mathcal{N}_R)$$

[Z. Bern, J. Carrasco, H. Johansson (2008/2010)  
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Open string  $\times$  Open string = Closed string

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Operators/fields:  $\Phi_{L+R} = \Phi_L \otimes \Phi_R$

$\Phi_L, \Phi_R$  : Operators with scaling dimensions  $h_L, h_R$

Require:  $h_L = h_R = h$  ( $m_L = m_R = m$ )

## 4D Standard massless supergravity:

Tensor product of two massless vector multiplets ( $h = 1$ )

Supergravity :    Massless  $\text{Spin}(2) = V_{L, \mathcal{N}_L} \otimes V_{R, \mathcal{N}_R}$



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- (i)  $\mathcal{N}_L = \mathcal{N}_R = 4 \Rightarrow \mathcal{N} = 8$  Sugra
- (ii)  $\mathcal{N}_L = 2, \mathcal{N}_R = 4 \Rightarrow \mathcal{N} = 6$  Sugra
- (iii)  $\mathcal{N}_L = 1, \mathcal{N}_R = 4 \Rightarrow \mathcal{N} = 5$  Sugra
- (iv)  $\mathcal{N}_L = 0, \mathcal{N}_R = 4 \Rightarrow \mathcal{N} = 4$  Sugra
- (v)  $\mathcal{N}_L = \mathcal{N}_R = 2 \Rightarrow \mathcal{N} = 4$  Sugra
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- (viii)  $\mathcal{N}_L = \mathcal{N}_R = 1 \Rightarrow \mathcal{N} = 2$  Sugra
- (ix)  $\mathcal{N}_L = 0, \mathcal{N}_R = 1 \Rightarrow \mathcal{N} = 1$  Sugra

## Non-standard W - supergravities in 4D :

We will need massive multiplets of  $\mathcal{N}$  - extended supersymmetric field theories:

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Massive spin-two Weyl super-multiplets:

$$W_{\mathcal{N}=4} : \text{Spin}(2) + \underline{8} \times \text{Spin}(3/2) + \underline{27} \times \text{Spin}(1) + \underline{48} \times \text{Spin}(1/2) + \underline{42} \times \text{Spin}(0)$$

States build  $USp(8)$  representations.

$$W_{\mathcal{N}=3} : \text{Spin}(2) + \underline{6} \times \text{Spin}(3/2) + (\underline{14} + \underline{1}) \times \text{Spin}(1) + (\underline{14}' + \underline{6}) \times \text{Spin}(1/2) + \underline{14} \times \text{Spin}(0)$$

States build  $USp(6)$  representations.

$$W_{\mathcal{N}=2} : \text{Spin}(2) + \underline{4} \times \text{Spin}(3/2) + (\underline{5} + \underline{1}) \times \text{Spin}(1) + \underline{4} \times \text{Spin}(1/2) + \text{Spin}(0)$$

States build  $USp(4)$  representations.

$$W_{\mathcal{N}=1} : \text{Spin}(2) + \underline{2} \times \text{Spin}(3/2) + \text{Spin}(1)$$

States build  $USp(2)$  representations.

S - fold construction of  $N=3$  field theory:

## S - fold construction of N=3 field theory:

One performs two types of simultaneous twists/projections:

**R-symmetry:**  $Q^{\alpha;1,2,3} \rightarrow e^{-i\pi/k} Q^{\alpha;1,2,3}, \quad Q^{\alpha;4} \rightarrow e^{3\pi/k} Q^{\alpha;4}$

**S-duality:**  $Q^{\alpha;1,2,3,4} \rightarrow e^{i\pi/k} Q^{\alpha;1,2,3,4}$

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Massless spin-one vector field  $V_i(x, \theta)$  is not invariant under the S - fold projection.

Invariant operators:  $\Phi^{2p} = \text{Tr}(V_{i_1} \bar{V}_{i_2} \dots V_{i_{2p-1}} \bar{V}_{i_{2p}})$   
( $h = 2p$ )

Lowest invariant operator:  $(h = 2)$

massive spin - two super-Weyl multiplet:

$$W_{\mathcal{N}=3} : \text{Spin}(2) + \underline{6} \times \text{Spin}(3/2) + (\underline{14} + \underline{1}) \times \text{Spin}(1) + (\underline{14}' + \underline{6}) \times \text{Spin}(1/2) + \underline{14} \times \text{Spin}(0)$$

$W_{\mathcal{N}=3}$  is a true  $\mathcal{N} = 3$  supermultiplet.



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Fields of lowest mass: **massive spin-four super-multiplet:**

Tensor product of two massive, spin-two Weyl multiplets:

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**Closed string = Open string  $\times$  Open string**

$$\Phi_{L+R, \mathcal{N}_L+3}^4 = W_{L, \mathcal{N}_L} \otimes W_{R, \mathcal{N}_R=3}$$

  
spin=4

  
spin=2

  
spin=2

(+matter)

## All possible S-fold cases:

$$(i) \mathcal{N}_L = 4, \mathcal{N}_R = 3 \implies \mathcal{N} = 7 \text{ W - Sugra}$$

$$(ii) \mathcal{N}_L = \mathcal{N}_R = 3 \implies \mathcal{N} = 6 \text{ W - Sugra}$$

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$$(iv) \mathcal{N}_L = 1, \mathcal{N}_R = 3 \implies \mathcal{N} = 4 \text{ W - Sugra}$$

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Most prominent example:  $\mathcal{N} = 7$  W - supergravity:

$$\begin{aligned}
 & \left( \text{Spin}(2) + 8 \times \text{Spin}\left(\frac{3}{2}\right) + 27 \times \text{Spin}(1) + 48 \times \text{Spin}\left(\frac{1}{2}\right) + 42 \times \text{Spin}(0) \right)_{\mathcal{N}=4} \\
 & \otimes \left( \text{Spin}(2) + 6 \times \text{Spin}(3/2) + 15 \times \text{Spin}(1) + 20 \times \text{Spin}(1/2) + 14 \times \text{Spin}(0) \right)_{\mathcal{N}=3} \\
 = & \left( \text{Spin}(4) + \underline{14}_1 \times \text{Spin}\left(\frac{7}{2}\right) + (\underline{90}_2 + \underline{1}) \times \text{Spin}(3) + (\underline{350}_3 + \underline{14}_1) \times \text{Spin}\left(\frac{5}{2}\right) \right. \\
 & + (\underline{90}_2 + \underline{910}_4) \times \text{Spin}(2) + (\underline{350}_3 + \underline{1638}_5) \times \text{Spin}\left(\frac{3}{2}\right) + (\underline{2002}_6 + \underline{910}_4) \times \text{Spin}(1) \\
 & \left. + (\underline{1430}_5 + \underline{1638}_5) \times \text{Spin}\left(\frac{1}{2}\right) + \underline{2002}_6 \times \text{Spin}(0) \right)_{\mathcal{N}=7}
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 \end{aligned}$$

Contains also spin-two and spin-3/2 states.

All massive states form representations of  $USp(14)$

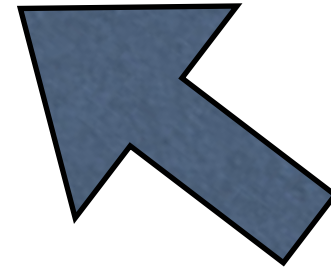
Strongly coupled theory without Lagrangian

Can also be obtained directly by S-fold of  $\mathcal{N} = 8$

Precisely appears at first mass level in  
corresponding string construction.

# Outline:

- II) Double copy construction of  $\mathcal{N}=7$  W-supergravity
- III)  $\mathcal{N}=7$  W - Superstring construction
- IV) Conclusion and Outlook





### III) W - Superstring construction

Basic requirements for  $\mathcal{N} = 7$  (or  $\mathcal{N} = 3$  heterotic) in 4D:

Left-moving sector is untouched:

Type II: Four supercharges

Heterotic: Zero supercharges

Right-moving sector gets projected/twisted as in field theory:

Three invariant supercharges

No massless excitations

4D heterotic (type II) string in light cone gauge on

$$\mathbb{R}^{1,1} \times \mathbb{R}^2 \times T^6$$

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I complex uncompactified, transversal coordinate:

$$X(z, \bar{z}) = X_L(\bar{z}) + X_R(z)$$

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1 complex uncompactified, transversal coordinate:

$$X(z, \bar{z}) = X_L(\bar{z}) + X_R(z)$$

3 complex compactified coordinates:

$$Z^i(z, \bar{z}) = Z_L^i(\bar{z}) + Z_R^i(z) \quad (i = 1, 2, 3)$$

## S-fold projection in string theory:

### I.) R-symmetry projection in the internal sector:

Projection by a discrete element of T-duality group  $SO(6,6)$   
in the internal sector

Can be represented by discrete  $\mathbb{Z}_4$  rotation on the  
internal coordinates:

$$Z_L^i \rightarrow Z_L^i$$

$$Z_R^i \rightarrow e^{-\frac{i\pi}{2}} Z_R^i$$

Moduli have to be frozen at their  
fixed point values:

$$\tau = \rho = i$$

The internal space is a completely left-right asymmetric  
 $\mathbb{Z}_4$  orbifold .

## 2.) S - duality projection in the space-time sector:

It is given by a discrete element of the (string) Geroch group.

[R. Geroch (1972);  
I. Bakas (1994)]

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The Geroch group contains:

(i) Axion-dilaton transformations:

$$S = a + ie^{-2\phi}$$
$$SL(2)_S : \quad S \rightarrow \frac{\tilde{a}'S + \tilde{b}'}{\tilde{c}'S + \tilde{d}'}, \quad \tilde{a}'\tilde{d}' - \tilde{b}'\tilde{c}' = 1$$

[A. Font, L. Ibanez, D.L., F. Quevedo (1990);  
J.H. Schwarz, A. Sen (1993)]

## (ii) Ehlers transformations - G-duality:

Four-dimensional, transversal, complex graviton field:

$$G = \frac{g_{12}}{g_{11}} + i \frac{\sqrt{\det g}}{g_{11}}$$

Large 4D diffeomorphisms (Ehlers transformation):

$$SL(2)_G : \quad G \rightarrow \frac{\tilde{a} G + \tilde{b}}{\tilde{c} G + \tilde{d}}, \quad \tilde{a}\tilde{d} - \tilde{b}\tilde{c} = 1$$



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Combined action of  $SL(2)_S$  and  $SL(2)_G$ :

$$O(2, 2)_{S,G} \simeq SL(2)_S \times SL(2)_G \quad (\text{Geroch group})$$

## (ii) Ehlers transformations - G-duality:

Four-dimensional, transversal, complex graviton field:

$$G = \frac{g_{12}}{g_{11}} + i \frac{\sqrt{\det g}}{g_{11}}$$

Large 4D diffeomorphisms (Ehlers transformation):

$$SL(2)_G : \quad G \rightarrow \frac{\tilde{a} G + \tilde{b}}{\tilde{c} G + \tilde{d}}, \quad \tilde{a}\tilde{d} - \tilde{b}\tilde{c} = 1$$

Combined action of  $SL(2)_S$  and  $SL(2)_G$ :

$$O(2, 2)_{S,G} \simeq SL(2)_S \times SL(2)_G \quad (\text{Geroch group})$$

$SL(2)_S$  and  $SL(2)_G$  also act on the transversal string coordinates  $X_L, X_R$  as certain in general left-right asymmetric rotations.

Particular discrete  $\mathbb{Z}_4$  element:

$$X_L(\bar{z}) \rightarrow X_L(\bar{z}), \quad X_R(z) \rightarrow e^{-\frac{i\pi}{2}} X_R(z)$$

Dilaton and metric must be frozen:  $S = G = i$  .

This is a completely asymmetric  $\mathbb{Z}_4$  rotation of the transversal, uncompactified coordinates.

Geroch transformation becomes T-duality when further compactifying to two dimensions

[I. Florakis, I. Garcia-Etxebarria, D. Lüst, D. Regalado (2018)]

## Invariant right-moving (open string) spectrum:

**64 + 64 invariant states at the first mass level:**

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$$\text{Spin}(2) + (\underline{14} + \underline{1}) \times \text{Spin}(1) + \underline{14} \times \text{Spin}(0)$$

**Fermions:**  $(8)_c + (56)_c : b_{-1}^A |a\rangle$ ,  $(8)_s + (56)_s : \alpha_{-1}^A |\dot{a}\rangle$

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This precisely agrees with the massive  $\mathcal{N} = 3$  Weyl multiplet.

Now we can consider the following type II S - fold:

$$(\mathcal{N} = 7_{II}) \equiv (\mathcal{N} = 8_{II}) / (\text{T - duality} \times \text{Geroch})$$

3 invariant right-moving supercharges .

4 invariant left-moving supercharges .

- 7 invariant supercharges
- $S = i \Rightarrow$  strongly coupled
- $G = i \Rightarrow$  no massless graviton



Spectrum of type II  $\mathcal{N} = 7$  W - superstring theory:

- No massless states, no massless graviton & gravitini.
- Invariant states at first mass level:

Tensor product: (Fermionic left)  $\times$  (fermionic right):

$$\text{B : } [\text{Spin}(2) + 15 \times \text{Spin}(1) + 14 \times \text{Spin}(0)]_R \times [\text{Spin}(2) + 27 \times \text{Spin}(1) + 42 \times \text{Spin}(0)]_L , \\ [6 \times \text{Spin}(3/2) + 20 \times \text{Spin}(1/2)]_R \times [8 \times \text{Spin}(3/2) + 48 \times \text{Spin}(1/2)]_L$$

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$$\text{B : } [\text{Spin}(4) + 91 \times \text{Spin}(3) + 1000 \times \text{Spin}(2) + 2912 \times \text{Spin}(1) + 2002 \times \text{Spin}(0)] ,$$

$$\text{F : } [14 \times \text{Spin}(\frac{7}{2}) + 364 \times \text{Spin}(\frac{5}{2}) + 1988 \times \text{Spin}(\frac{3}{2}) + 3068 \times \text{Spin}(\frac{1}{2})]$$

Massive spin - four multiplet of  $\mathcal{N} = 7$  .

Agrees with double copy construction.

## IV) Conclusion and Outlook

We have provided evidence for the existence of new  
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Strongly coupled theories with frozen graviton !

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W - superstring theories:

- The internal space is non-geometric
  - modding out by T-duality.
- In 4D also modding out by S-duality
  - strongly coupled theory.
- In 4D also modding by large diffeomorphism -  
massive, topological theory.
- 4D space is „non-geometric“ - relation to non-associativity?

No proof (yet) that these theories are fully consistent and full-fledged string theories.

But there exist fully consistent 2D non-geometric string construction with  $N=(20,8)$  supercharges (and also with  $N=(32,0)$ ,  $N=(24,24)$  and  $N=(48,0)$ ).

[I. Garcia-Etxebarria, I. Florakis, D.L., D. Regalado (2017)]

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[I. Garcia-Etxebarria, I. Florakis, D.L., D. Regalado (2017)]

Other exotic, strongly coupled gravity theories:  
(4,0) supergravity in six dimensions.

[C. Hull (2000)]

## Final conjecture about holography:

Following the same kind of arguments as in standard holography, it is tempting to conjecture that  $\mathcal{N} = 4$  spin-two superconformal Weyl supergravity on a 4D boundary (D3-branes) is holographically dual to  $\mathcal{N} = 8$  spin-four W - supergravity in the 5 D bulk theory.

[S. Ferrara, A. Kehagias, D.L., work in progress]

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# Thank you!