The WGC, black holes, and gravitational instantons

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Black holes & 1-form WGC

- The swampland program proposes a web of connected conjectures, most often motivated from string theory

Vafa '05; Ooguri, Vafa '06, '16; Arkani-Hamed et al. '06; Obied et al. '18…

- The underlying expectation is that these represent consistency requirement of theories

  - It is important, for claims of generality and for phenomenologica applications, to understand where these come from.

- Perhaps the most approachable conjecture is the WGC:

$$\frac{Q}{T_p} \geq \text{``1''}$$

  - Expected to derive from entropy bounds

C.f. talks by Shiu, Remmen, Montero, Crisford

Outline

- **1-form WGC:** extremal BH entropy
- **0-form WGC:** Euclidean wormholes
- **2-form WGC:** Axionic BH

Black holes & 1-form WGC

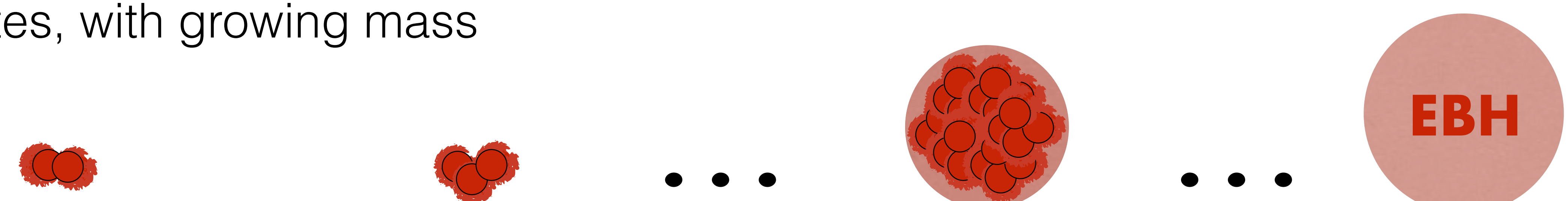
- **Global symmetries** are expected to be problematic in gravity: they lead to an infinite number of (sub-)planckian stable remnants.

Susskind '95

- Intuitively, **gauge symmetries** could become problematic as g -> 0:

$(q/m < 1)$

- If the WGC is violated, one can form an infinite tower of exactly stable bound states, with growing mass



$2m > M_2 > 2q$ $\quad$ $3m > M_3 > 3q$ $\quad \cdots \quad$ $Nm > M_N > Nq$ $\quad \cdots \quad$ $M_\infty \to Q_\infty$

Arkani-Hamed et al. '06

  - Similar towers arise naturally in SUSY situations (e.g. in string theory)
  - Is there an argument to require their decay, i.e. the WGC (m<q)?

Black holes & 1-form WGC

- One typically associates an entropy to the horizon and "hopes" that it will have a microscopic description.
    - Computing microscopic entropy when WGC is violated is out of reach (perhaps impossible!).
- We studied instead corrections to macroscopic entropy of EBH from (sub-)extremal particles
    - Rather involved computation with several pitfalls. Has led to contradictory conclusions.

Cottrell, Shiu, PS '16
Fisher, Mogni '17

Black holes & 1-form WGC

- Take Einstein-Maxwell th. and a scalar/fermion with (m,q). Compute 1-loop correction to EBH entropy using Sen's formalism:

$$\Delta S \approx -a^4\,\Delta\mathcal{L} = -\frac{a^4}{2}\int_{\epsilon^2}^{\infty}\frac{ds}{s}K(s)$$

  - i.e. compute 1-loop correction to the lagrangian in near-horizon AdS_2xS_2, using the heat kernel K(s):

$$K(s) = \frac{e^{-s(\Delta m^2+\frac{1}{4a^2})}}{a^4}\sum_{\ell=0}^{\infty}(2\ell+1)\int_0^{\infty}d\lambda\,\rho(\lambda)\,e^{-\frac{s}{a^2}[\lambda^2+\ell(\ell+1)]}$$

$$\rho_{s,f}(\lambda) = \frac{\lambda\sinh(\lambda)}{\cosh(\frac{qM_p}{a})\pm\cosh(\lambda)}$$

Banerjee, Gupta, Sen
Comtet, Houston
Pioline, Troost

  - a: BH radius, $\Delta m^2 = m^2 - q^2$: extremality bound, λ: AdS momentum, l: angular momentum.

Black holes & 1-form WGC

- Compute 1-loop correction using Sen's entropy function:

. . .

Black holes & 1-form WGC

- Compute 1-loop correction using Sen's entropy function:
  - **Super-extremal $\Delta m<0$:** an IR divergence signals BH decay WGC
  - **Sub-extremal scalars $\Delta m>0$:** no significant (log) corrections
  - **Extremal scalars $\Delta m>0$:**

    Large BH ($a >> 1/\Lambda_{wgc}$) receive no significant (log) corrections

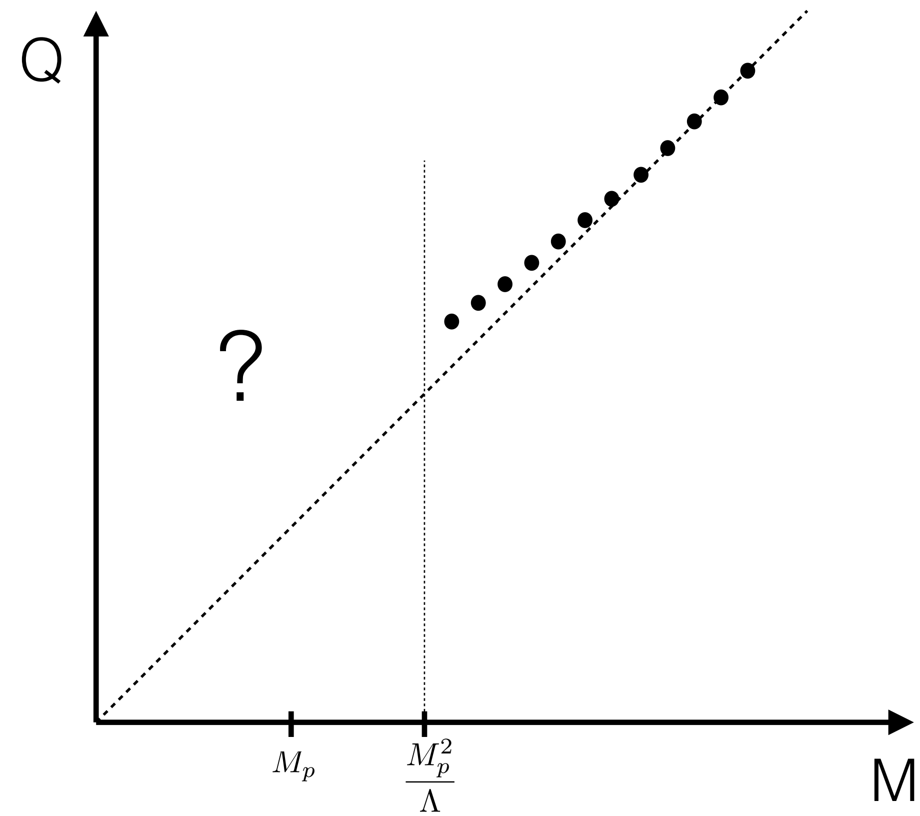
    Intermediate BH ($1/\Lambda_{wgc} >> a >> 1/M_p$): $\Delta S \sim - \log(a\, m)$
  - **(Sub-)extremal fermions $\Delta m \geq 0$:** computations break down because of a pole in the spectral density at the <u>WGC scale</u> qM_p

WGC??

Cottrell, Shiu, PS '16
see Fisher, Mogni '17 for different conclusions.

Black holes & 1-form WGC

- Alternative approach: tree-level corrections to higher dim. ops. make smaller macroscopic black holes become super-extremal



Kats, Motl, Padi ’06
Cheung, Liu, Remmen ’18
Hamada, Noumi, Shiu ‘18

- A consequence of entropy positivity (or unitarity). Matches examples from string theory.

C.f. Remmen and Shiu’s talks

- What about lower masses? Crucial for phenomenological applications

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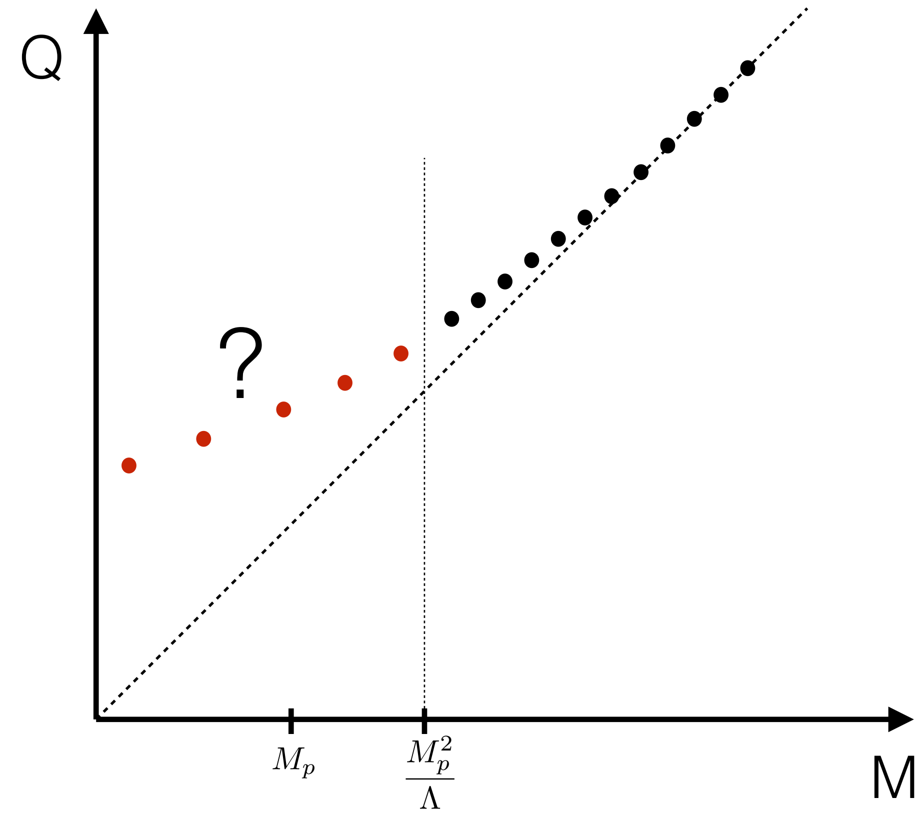

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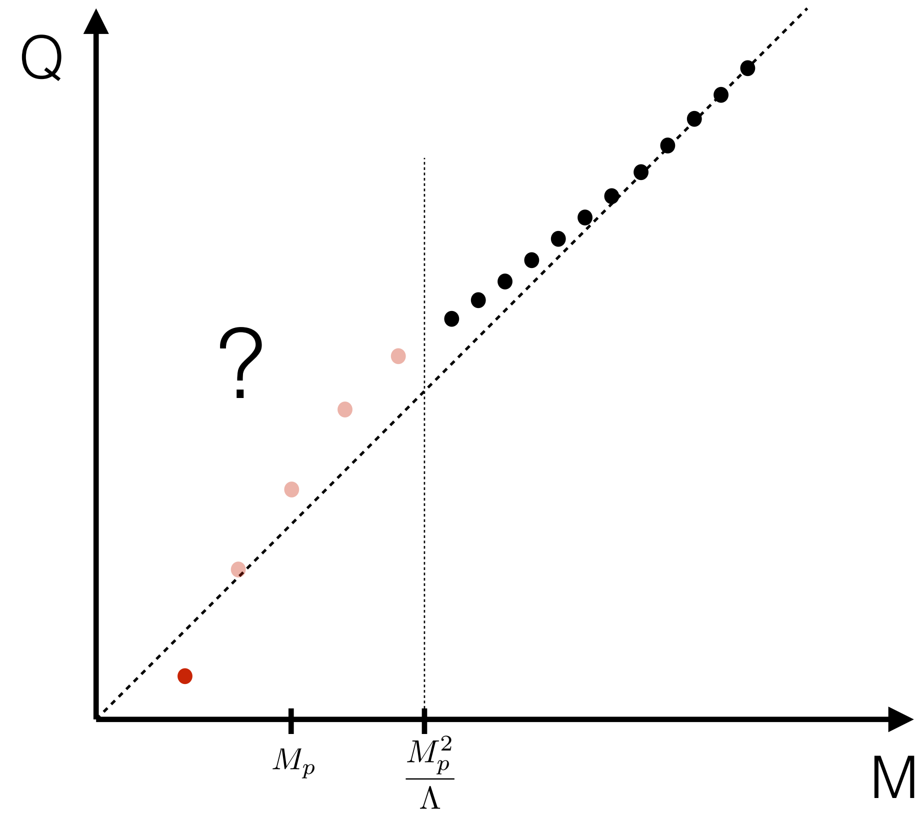

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Euclidean wormholes and 0-form WGC

Wormholes & 0-form WGC

- The standard WGC discusses particles in Einstein-Maxwell theory:
  - Macroscopic: RN (extremal) black holes
  - Microscopic: charged (super-extremal) particles
- Would like to understand analogous objects in Einstein-Axion th.:
  - Macroscopic: gravitational (extremal) instantons
  - Microscopic: elementary instantons
- Particle and instanton perspectives are related by dimensional reduction. With gravity one gets Einstein-Axion-Dilaton theory.
  - Can we understand gravitational instantons and the 0-form WGC in purely Einstein-Axion theory?

Wormholes & 0-form WGC

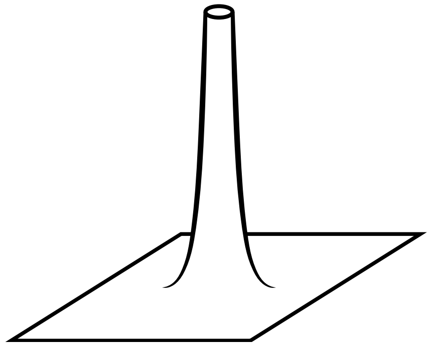
- Let's consider Euclidean Einstein-Axion theory:

$$\mathcal{L} \sim -\mathcal{R} + f^2 |\partial\phi|^2 \qquad \text{with} \qquad \phi \equiv \phi + 2\pi$$

- This can be dualized $(H = dB_2 \equiv f^2 * d\phi)$ to give:

$$\mathcal{L} \sim -\mathcal{R} + \frac{1}{f^2}|dB|^2$$

- This theory admits smooth solutions



- The throat is a three-sphere of minimum radius R, supported by n units of H-flux:

$$M_P^2 R^{-2} \sim \frac{n^2}{f^2} R^{-6} \quad \Rightarrow \quad M_P R^2 \sim \frac{n}{f}$$

Giddings, Strominger '88...

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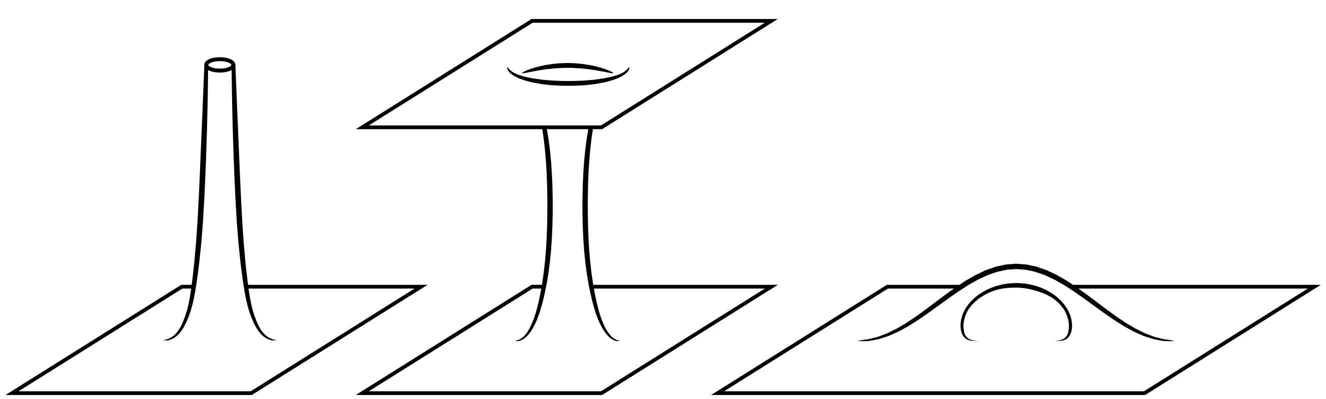

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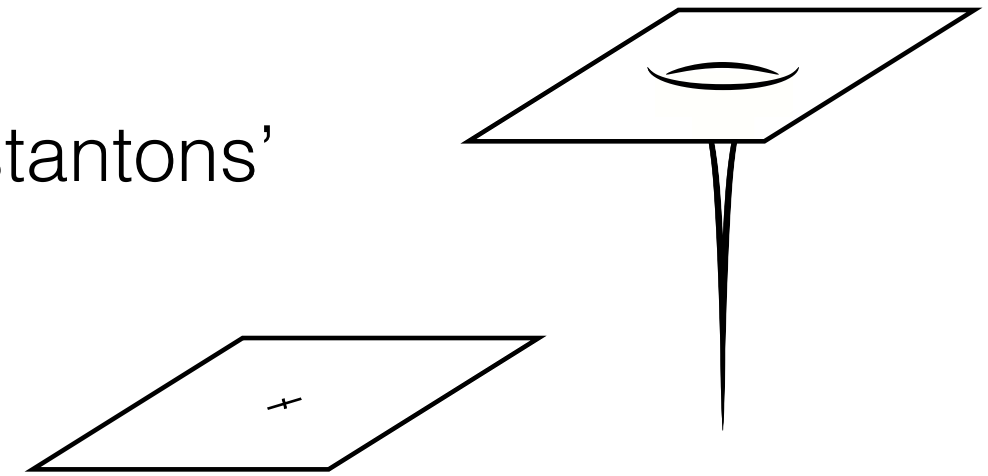
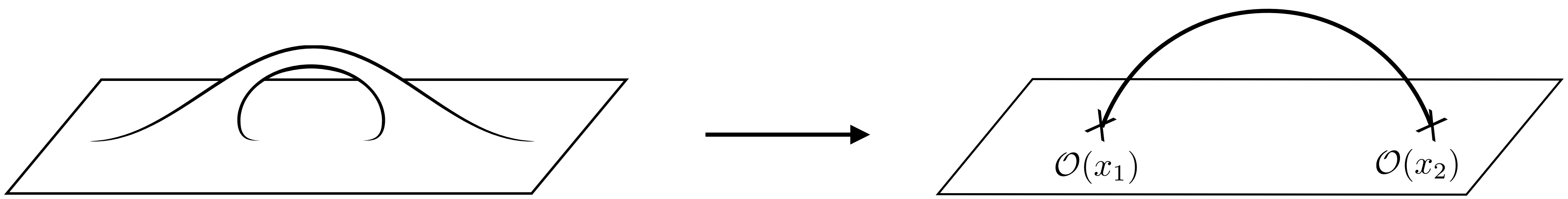
…as well as (singular) 'gravitational instantons'



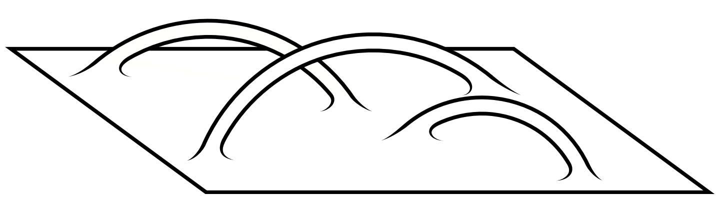
C.f. Bergshoeff et al. '04, '05

Wormholes & 0-form WGC

- In the IR wormholes look like instanton anti-instanton pairs which couple to operators O(x):



- In the dilute gas approximation, their contribution can be exponentiated, leading to a **bi-local effective action**:



$$e^{-\Delta S} \sim \exp\left(\int_{x_1}\int_{x_2} \mathcal{O}(x_1)\mathcal{O}(x_2)\right)$$

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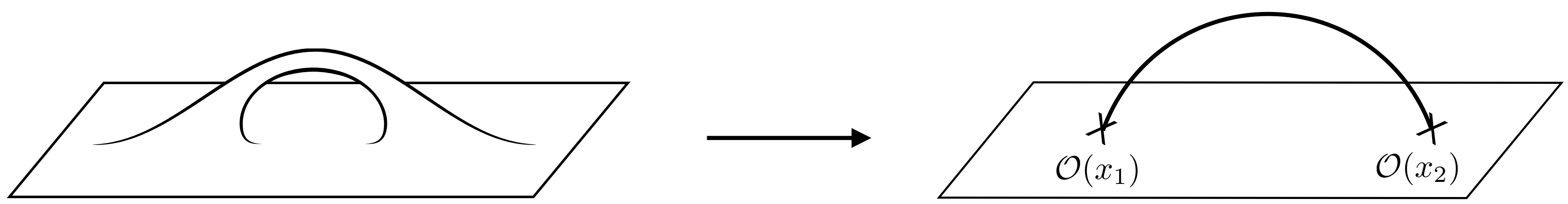

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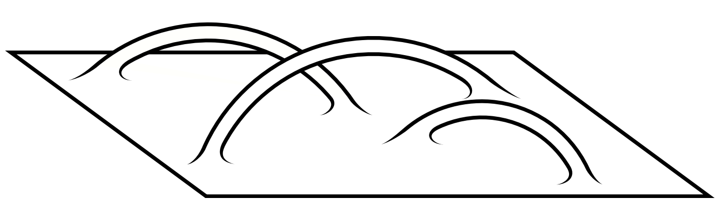


$$e^{-\Delta S} \sim \exp\left(\int_{x_1}\int_{x_2} \mathcal{O}(x_1)\mathcal{O}(x_2)\right)$$

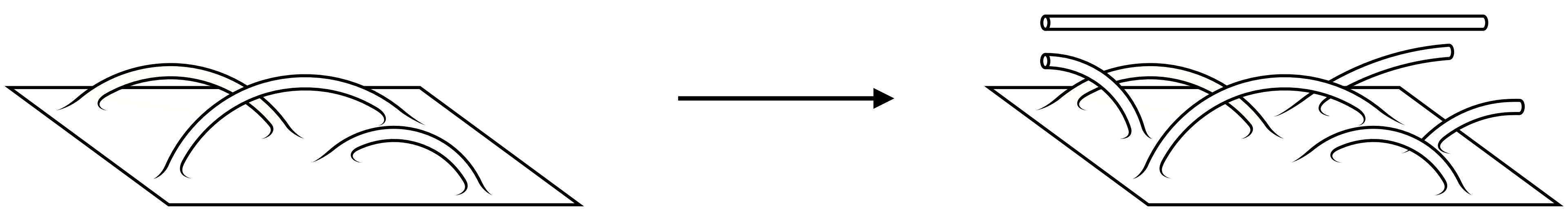
Can be rewritten in a 'local' form with the help of **α-parameters**

$$\sim \int_{\alpha} \exp\left(-\alpha^2 + \alpha \int_x \mathcal{O}(x)\right)$$

Coleman '88, Preskill '89

Wormholes & 0-form WGC

- What is the interpretation of α-parameters? Once wormholes are considered, one needs to introduce "baby-universe states"



- α-parameters determine the state in baby-universe Fock space.
- Superpositions of baby-universe eigenstates have fixed α.
- In 2d (string theory), the sum over topologies (wormholes) is related to the string loop expansion. Couplings in the worldsheet (α-parameters) correspond to target space fields

Polchinski, Banks/Lykken/O'Loughlin, Cooper/Susskind/
Thorlacius, Hawking/Lyons, Strominger '89–'92

Wormholes & 0-form WGC

- Giddings-Strominger wormholes couple like instantons:

$$\mathcal{O}(x) \sim e^{-S_{wh}+i\phi(x)/f} \quad \Longrightarrow \quad e^{-\Delta S} \sim \int_\alpha e^{-\alpha^2} \exp\left(\alpha\, e^{-S_{wh}} \int_x \cos(\phi/f)\right)$$

- GS wormholes hence generate a cos potential for axions, with unfixed coefficient α. Potential pheno applications, but…

- Macroscopic (trustable) if: $R^2 \sim \dfrac{n}{f M_P} > \dfrac{1}{\Lambda_{UV}^2}$

- Non-perturbatively suppressed: $S_{wh} \sim \dfrac{n}{f} M_P \gtrsim \dfrac{M_p^2}{\Lambda_{UV}^2}$

- Generically they give negligible contributions

Montero, Uranga, Valenzuela ’15
Hebecker, Mangat, Theisen, Witkowski ’16
Alonso, Urbano ‘17

Wormholes & 0-form WGC

- GS wormholes break the shift-symmetry of axions. They could play a role (similar to charged black holes) in the axionic WGC
  - What is the notion of extremality for wormholes?
  - What is the role of the α-parameters?
  - Is there a notion of “wormhole/instanton” decay?

- These questions call for a better understanding of wormhole
  - Revisit many long-standing puzzles surrounding wormholes.

Hawking, Coleman, Preskill, Giddings, Strominger, Lee,
Klebanov, Susskind, Rubakov, Fischler, Kaplunovsky…
’88–’92…

  - Perhaps the WGC will help solve some of them.

Wormholes & 0-form WGC

- Wormholes pose deep puzzles in theories of quantum gravity
  - They lead to an intrinsic randomness of coupling constants.
  - Technically, IR divergences often arise.
  - In holography wormholes and α-parameters are problematic from the CFT side (violation of locality/BPS bounds).

Maldacena, Maoz ’04, Arkani-Hamed, Orgera, Polchinski ‘07 Hertog, Trigiante, Van Riet ‘17

- The solution may be to simply forbid topology change, but how?
  - Topology change plays a key role in string theory
  - GS wormholes may posses negative modes and be unstable
  - May be complex saddles not contributing to path integral
- It may be time to revisit these puzzles in the WGC-context.

Work in progress…

Axionic Black Holes and 2-form WGC

ABH & 2-form WGC

- Consider a 2-form gauge theory:

$$S \sim \int \frac{1}{f^2}|dB_2|^2 + \sigma \int_\Sigma \sqrt{g} + \int_\Sigma B_2$$

- Dual to axion case. The WGC postulates

$$\sigma \lesssim f M_p \qquad \text{or} \qquad \Lambda^2 \lesssim f M_p$$

  - WGC predicts **light strings**, and constrains $f \ll M_P$ regime.

c.f. Reese's talk

- Usual WGC arguments would involve (super)-extremal "black strings" or "gravitational instantons" and are rather subtle.
  - Alternative (still speculative) arguments from **axionic BH.**

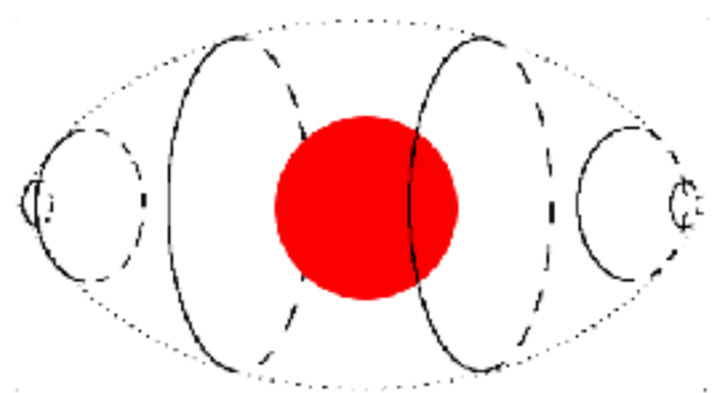
ABH & 2-form WGC

- Schwarzschild BH contains a topologically non-trivial 2-cycle. A "Wilson-line" b~b+1 can be turned on at **no energy cost**.

$$\int_{S^2} B_2 = b\,, \quad dB_2 = 0$$

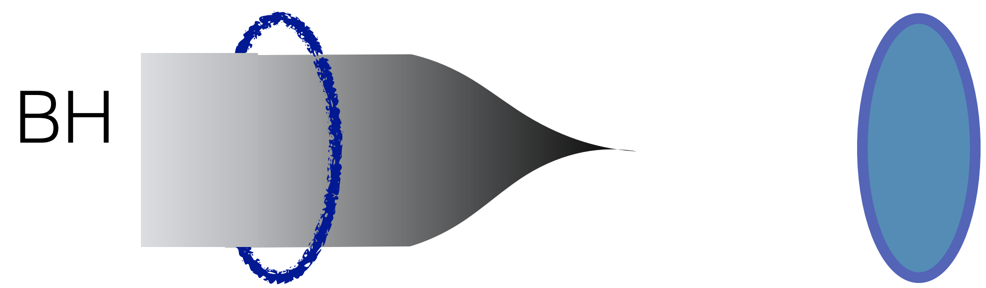
Bowick, Giddings, Harvey, Horowitz, Strominger '88

- Locally 'b' is unobservable, but lassoing strings can measure 'b' a la Ahoronov-Bohm:



ABH & 2-form WGC

- Consider BH evaporation:



$$\int B_2 = b \qquad\qquad b = \int H_3$$

- If the BH 2-cycle disappears completely, the Wilson-line ‘b’ must be supported by an energetically costly flux.
  - Energetic balance requires: $M_{BH}(T_c) \geq \frac{1}{f^2} \int_{B^3} |H_3|^2$
  - By studying evaporation from the time $T_c \sim \sqrt{\sigma}$ at which Wilson line is transferred into flux, we can put a lower bound on f
  - Under certain assumptions: $\sigma \lesssim f M_p$ WGC

Hebecker, P.S. ‘17

Conclusions

Conclusions

- It is crucial to understand how swampland conjectures arise purely from quantum gravity considerations
  - The WGC has phenomenological applications that depend on its precise formulation (strong/mild/sLattice…)
  - BH decay and entropy, holography, cosmic censorship,… give suggestive evidence, but often allow for 'loopholes'

    'Swamp-rangers': take loopholes seriously and try to close them.

    EFT model builders: don't take them too seriously, i.e. for granted.

  - Euclidean wormholes pose long-standing problems that have remained elusive for many years. Perhaps the WGC holds the key to their resolution.