A geometric look at neutrino oscillation with nonstandard interactions

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(in preparation)

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Plan of the talk

- Motivation & past literature
- Geometric interpretation: Leptonic Unitarity triangle (LUT)
- LUT and oscillation in vacuum & matter
- LUT and oscillation with non standard interactions (NSI)
- Role of NSI phases
- Conclusion

Status of oscillation parameters

Table: de Salas, Forero, Ternes, Tortola, Valle: 1708.01186

Oscillation parameter	Best fit value	3σ range
$\theta_{12}/^\circ$	34.5	$31.5 \rightarrow 38.0$
$\theta_{23}/^\circ$	47.7	$41.8 \rightarrow 50.4$
$\theta_{13}/^\circ$	8.45	$8.0 \rightarrow 8.9$
$\delta_{\rm CP}/\pi$	-0.68	$[-\pi, \pi]$
$\Delta m^2_{21}/10^{-5} eV^2$	7.55	$7.05 \rightarrow 8.14$
$\Delta m^2_{31}/10^{-3} eV^2$	2.5	$2.41 \rightarrow 2.6$

3 mixing angles

1 CP phase

2 mass squared differences

Leptonic CP violation?

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Is the CP phase non-zero?
could help explain baryon asymmetry

$$
\begin{aligned}
P_{\mu e} &= \sin^2 2\theta_{13} \sin^2\theta_{23} \sin^2 \frac{(1-A)\Delta}{1-A} \\
&+ \alpha^2 \sin^2 2\theta_{12} \cos^2\theta_{23} \sin^2 \frac{A\Delta}{A} \\
&+ \alpha \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos(\delta + \Delta)
\end{aligned}
$$

where,

$$A = \frac{2\sqrt{2} E G_F n_E}{\Delta m^2_{31}}$$

$$\Delta = \frac{\Delta m^2_{31} L}{4E}$$

$$\alpha = \frac{\Delta m^2_{21}}{\Delta m^2_{31}}$$

Motivation

$$V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} = 0.$$

- In quark sector CKM mixing matrix generates six unitarity triangles.

- CP violation can be probed through their nonzero angles of the triangles

  Cabibbo (1957), Kobayashi & Maskawa (1973)

Motivation

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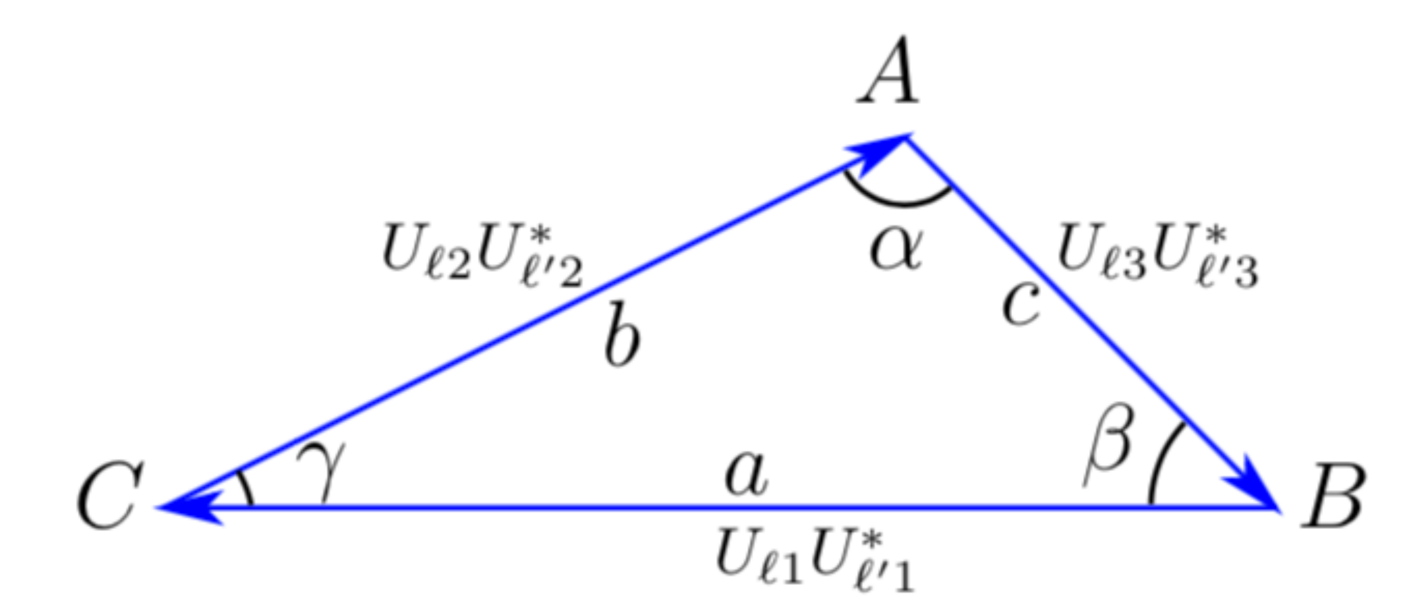
- CP violation can be probed through their nonzero angles of the triangles

Cabibbo (1957), Kobayashi & Maskawa (1973)

- Can the unitarity triangles of PMNS mixing matrix guide us in probing leptonic CPV?

Motivation



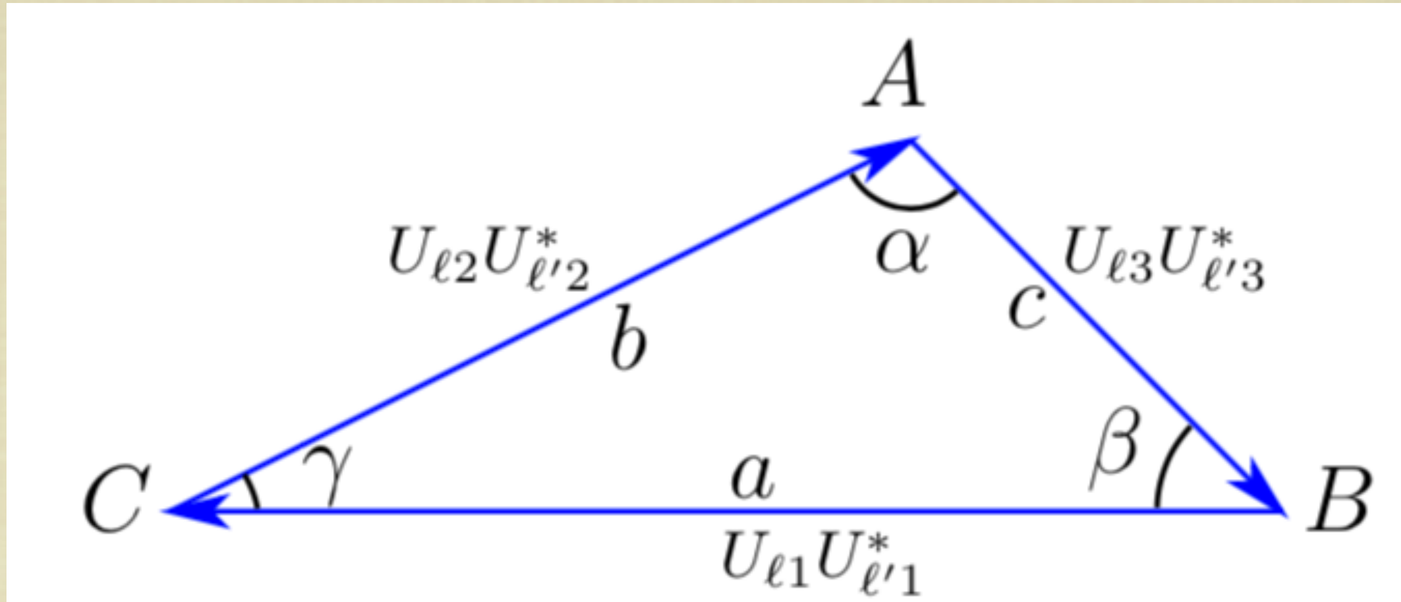
$$(a, b, c) = (|U_{\ell 1}U^*_{\ell' 1}|, |U_{\ell 2}U^*_{\ell' 2}|, |U_{\ell 3}U^*_{\ell' 3}|) \ ,$$

$$(\alpha, \beta, \gamma) = \arg\left(-\frac{U_{\ell 3}U^*_{\ell' 3}}{U_{\ell 2}U^*_{\ell' 2}}, -\frac{U_{\ell 1}U^*_{\ell' 1}}{U_{\ell 3}U^*_{\ell' 3}}, -\frac{U_{\ell 2}U^*_{\ell' 2}}{U_{\ell 1}U^*_{\ell' 1}}\right)$$

Literature survey

- Aguilar-Saavedra & Branco (2000):
  Leptonic CPV with LUT for majorana neutrinos
- Farzan & Smirnov (2000),
  Bjorken et. al (2006),
  Rodejohan et al. (2010)…:
  properties for the sides and angles of LUT
- Zhang & Xing (2005),
  H.J. He & X.J.Xu (2013),
  H.J. He & X.J.Xu (2016):
  study of LUT for the appearance channel for both vacuum and matter

LUT parameters



$$(a, b, c) = (|U_{\ell 1}U^*_{\ell' 1}|, |U_{\ell 2}U^*_{\ell' 2}|, |U_{\ell 3}U^*_{\ell' 3}|) \; ,$$

$$(\alpha, \beta, \gamma) = \arg\left(-\frac{U_{\ell 3}U^*_{\ell' 3}}{U_{\ell 2}U^*_{\ell' 2}}, -\frac{U_{\ell 1}U^*_{\ell' 1}}{U_{\ell 3}U^*_{\ell' 3}}, -\frac{U_{\ell 2}U^*_{\ell' 2}}{U_{\ell 1}U^*_{\ell' 1}}\right)$$

$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$$

$$\Longrightarrow \; a + be^{i(\gamma-\pi)} + ce^{i(\pi-\beta)} = 0$$

Standard neutrino oscillation (vacuum)

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}L}|\nu(L)\rangle = H|\nu(L)\rangle$$

$$H_0 = \frac{1}{2E}U\begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix}U^\dagger \quad \rightarrow \quad |\nu(L)\rangle = e^{-\mathrm{i}H_0 L}|\nu(0)\rangle$$

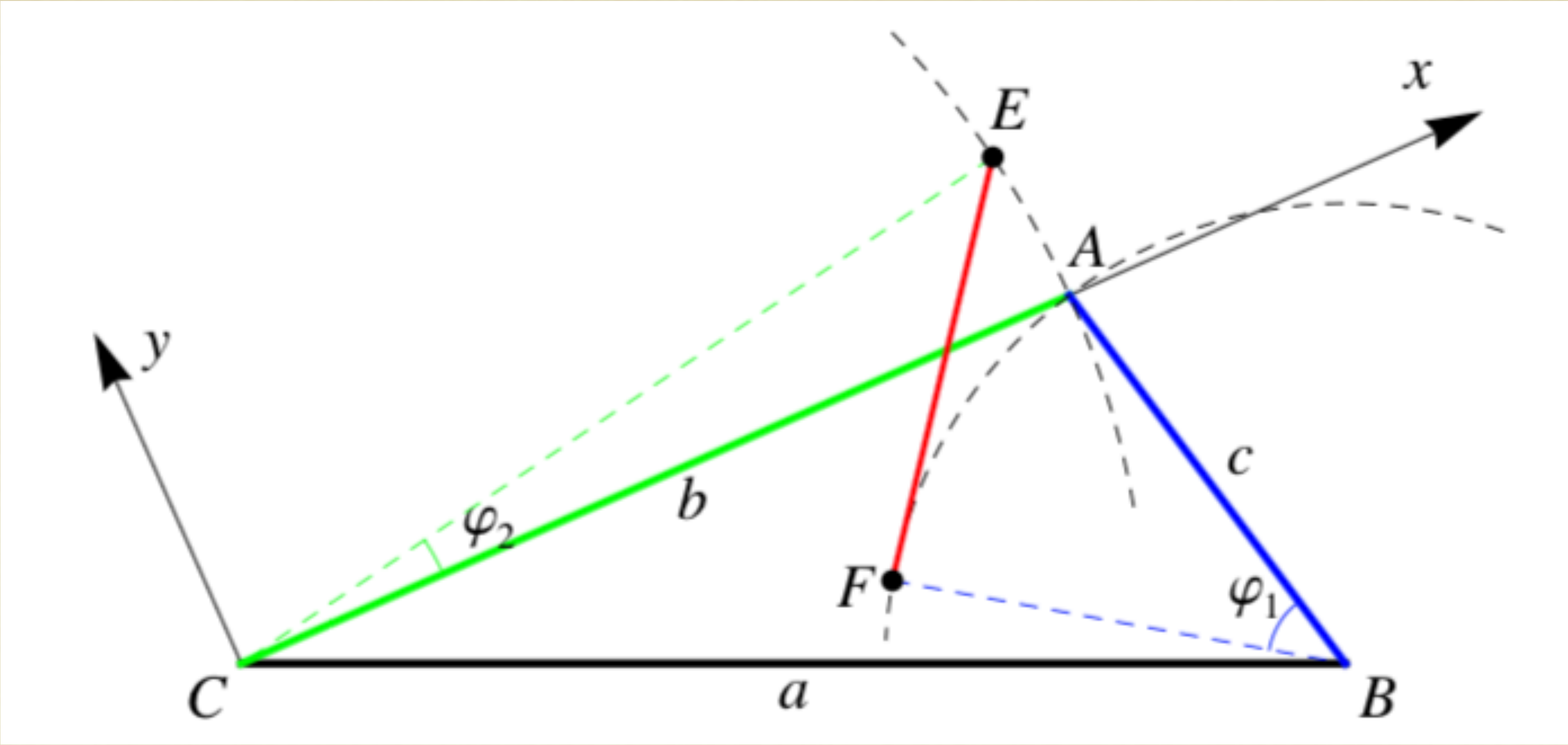
Flavor transtion amplitude: $A_{ll'} = \sum_j U_{lj}U^*_{l'j}e^{2i\Delta_j}$ with $\Delta_j = \frac{\Delta m^2_{jk}L}{4E}$

$$\rightarrow \quad A_{ll'} = a + be^{i(\gamma-\pi)}e^{i\phi_2} + ce^{i(\pi-\beta)}e^{i\phi_1}$$

where $\phi_j = \frac{\Delta m^2_{2j}}{2E}$

LUT → Probability

He & Xu (2016)



$$\overrightarrow{CA} \rightarrow be^{i(\gamma-\pi)}e^{i\phi_2}$$

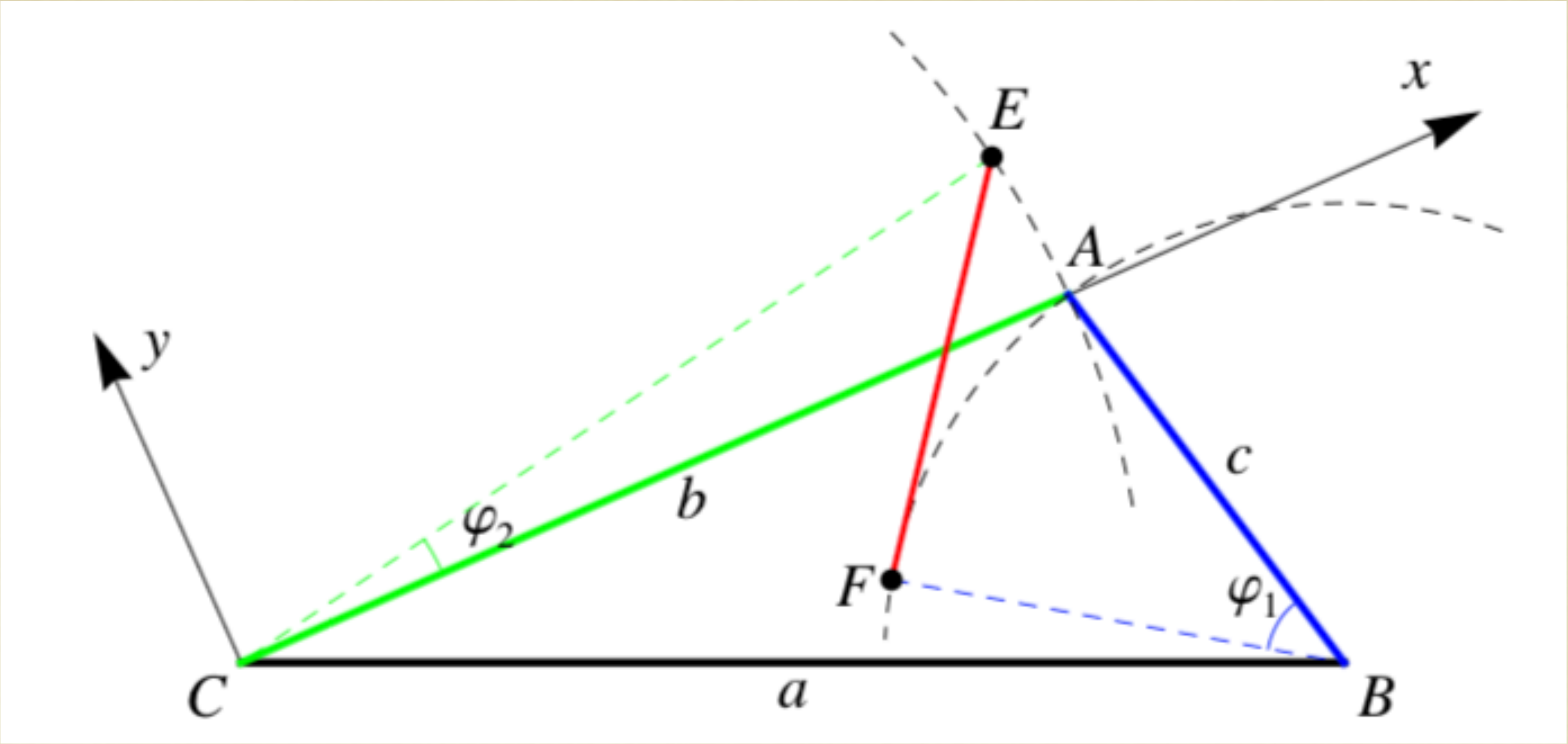
$$\overrightarrow{BA} \rightarrow ce^{i(\pi-\beta)}e^{i\phi_1}$$

$$\overrightarrow{EF} = a + be^{i(\gamma-\pi)}e^{i\phi_2} + ce^{i(\pi-\beta)}e^{i\phi_1}$$

$$P_{ll'} = |\overrightarrow{EF}|^2$$

LUT → Probability

He & Xu (2016)



$$P_{\ell\to\ell'} = 4c^2\sin^2\Delta - 8bc\sin\Delta\sin\epsilon\Delta\cos[(1-\epsilon)\Delta+\alpha] + 4b^2\sin^2\epsilon\Delta\,,$$

$$\Delta \equiv \Delta_{31} = \frac{\Delta m^2_{31}L}{4E}, \qquad \epsilon \equiv \frac{\Delta_{21}}{\Delta_{31}} = \frac{\Delta m^2_{21}}{\Delta m^2_{31}}$$

$$a = \sqrt{b^2+c^2-2bc\cos\alpha}\,,$$

$$\gamma = \arccos\left(\frac{a^2+b^2-c^2}{2ab}\right), \qquad \beta = \pi-(\alpha+\gamma)$$

→ **Only 3 LUT parameters**

Standard neutrino oscillation (std. matter) He & Xu (2016)

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}L}|\nu(L)\rangle = H|\nu(L)\rangle$$

$$H = H_0 + \sqrt{2}G_F N_e \mathrm{Diag}(1,0,0)$$

$$H = \frac{1}{2E}U_m \begin{pmatrix} \tilde{m}_1^2 & & \\ & \tilde{m}_2^2 & \\ & & \tilde{m}_3^2 \end{pmatrix} U_m^\dagger$$

$$\begin{aligned} P_{\ell\to\ell'} = \; & 4c^2\sin^2\Delta \\ & -8bc\sin\Delta\sin\epsilon\Delta\cos[(1-\epsilon)\Delta+\alpha] \\ & +4b^2\sin^2\epsilon\Delta\,, \end{aligned}$$

replaced with b_m, c_m, α_m

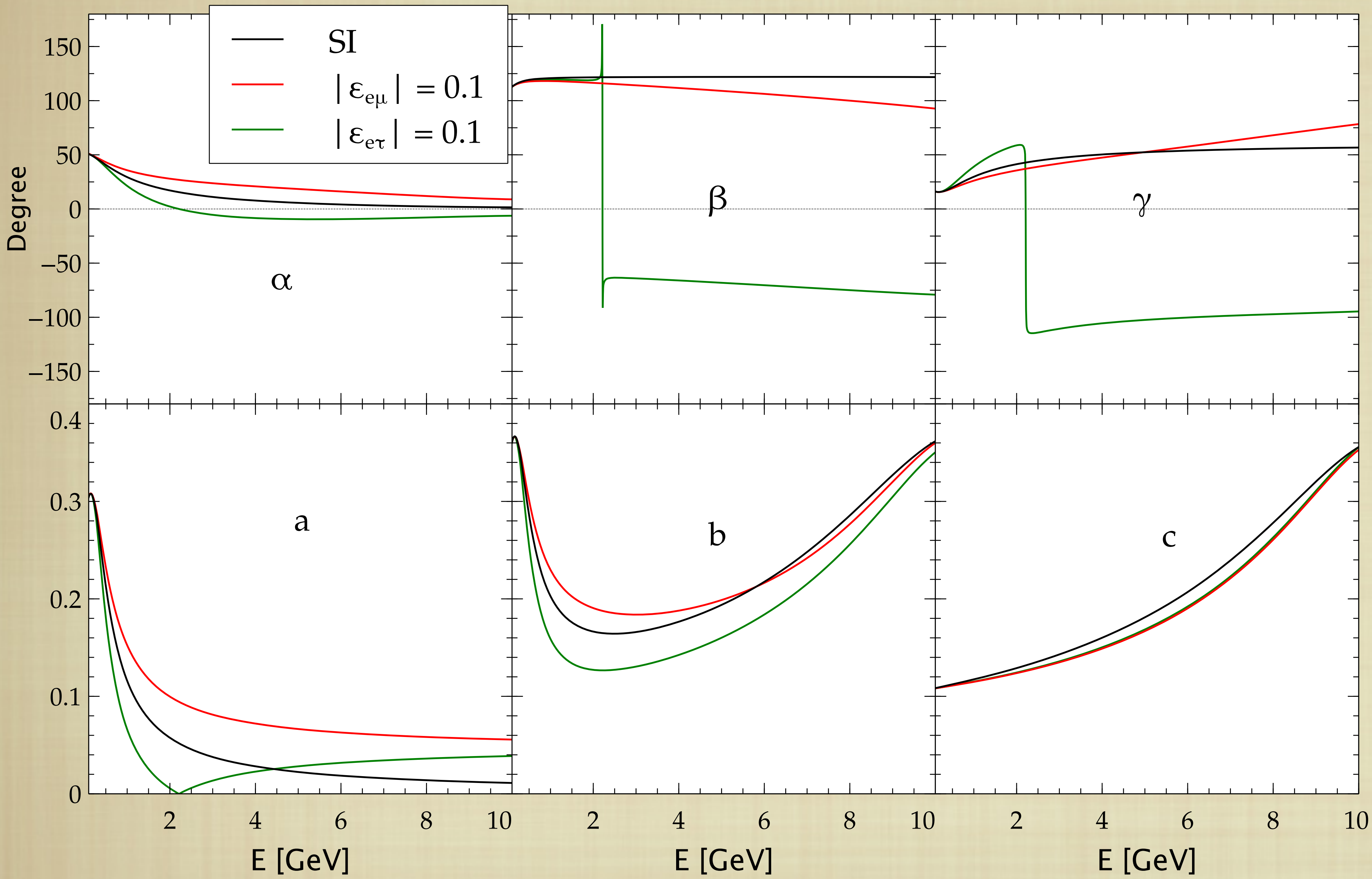
Neutrino oscillation (NSI)

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}L}|\nu(L)\rangle = H|\nu(L)\rangle$$

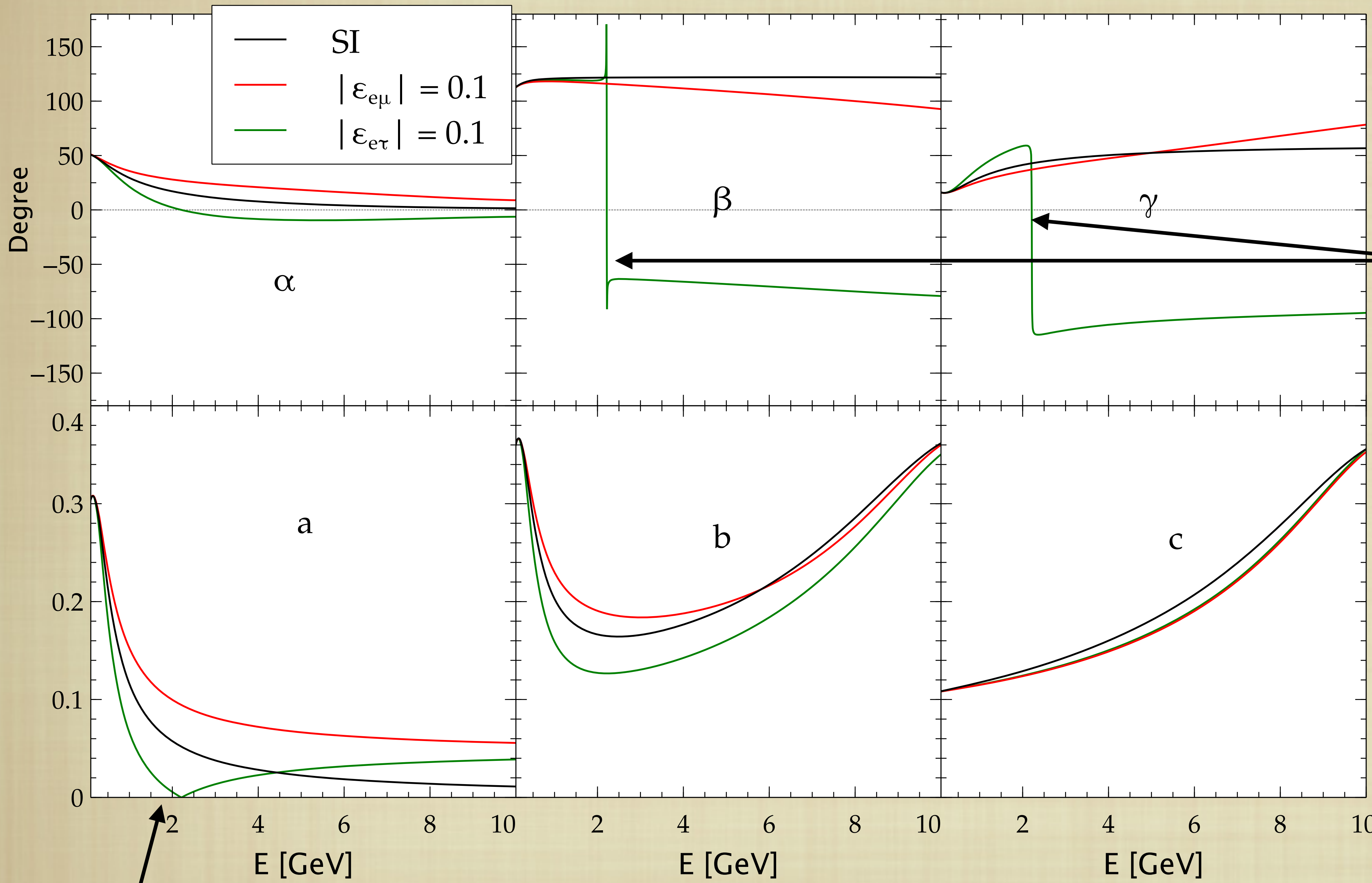
$$\begin{aligned} H &= H_0 + H_{SI} + H_{NSI} \\ &= \frac{1}{2E}U\begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix}U^\dagger + \sqrt{2}G_F N_e \begin{pmatrix} 1 & & \\ & 0 & \\ & & 0 \end{pmatrix} + \sqrt{2}G_F N_e \begin{pmatrix} \varepsilon_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau} \\ \varepsilon_{e\mu}^* & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon_{e\tau}^* & \varepsilon_{\mu\tau}^* & \varepsilon_{\tau\tau} \end{pmatrix} \end{aligned}$$

Proceed similarly and find the LUT angles and sides

Variation of LUT parameters with energy



Variation of LUT parameters with energy

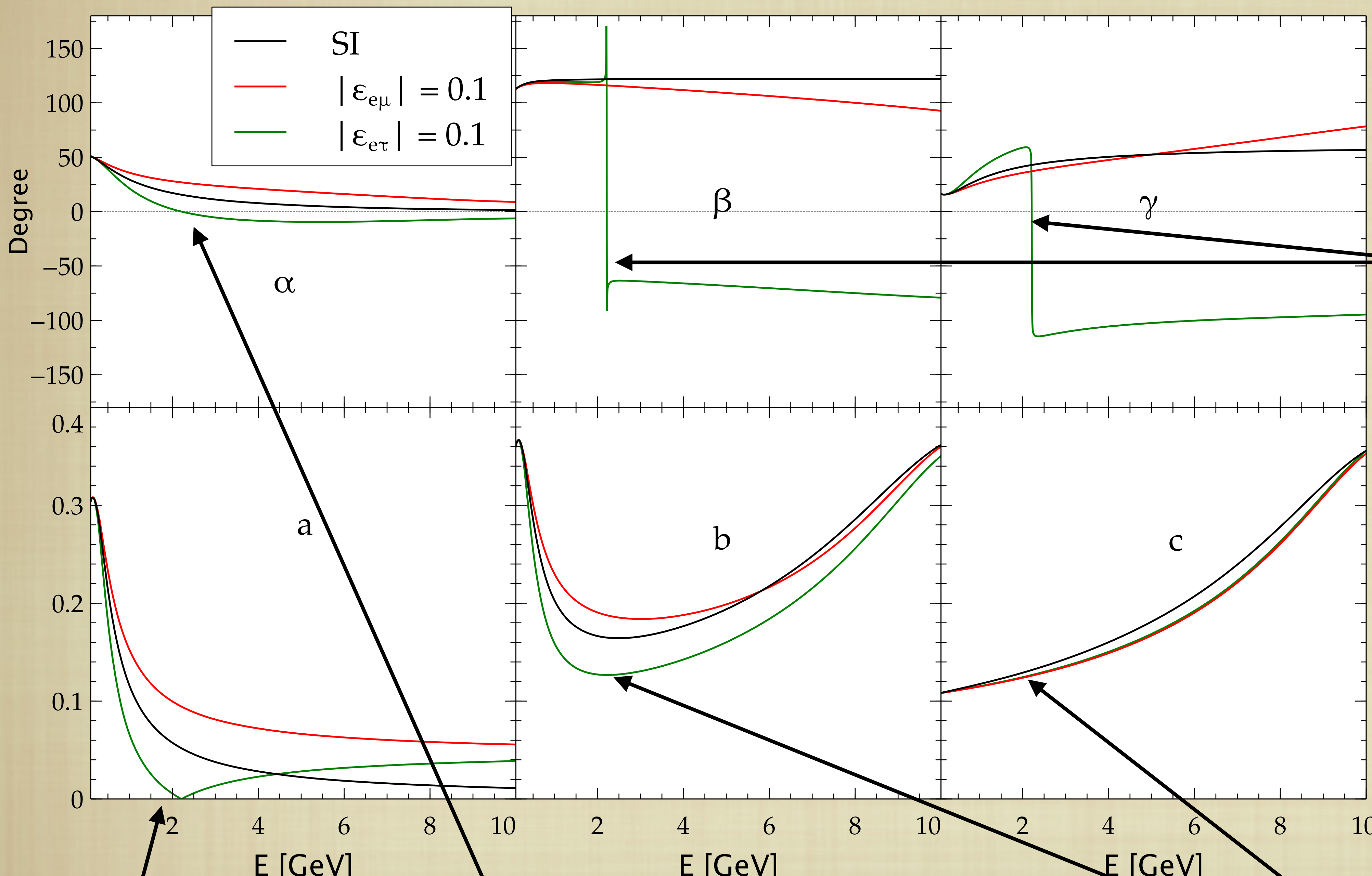


"jumps" in presence of $\epsilon_{e\tau}$

~2.2 GeV

a goes to zero

Variation of LUT parameters with energy



"jumps" in presence of $\epsilon_{e\tau}$

~2.2 GeV

a goes to zero

alpha goes to zero

b=c

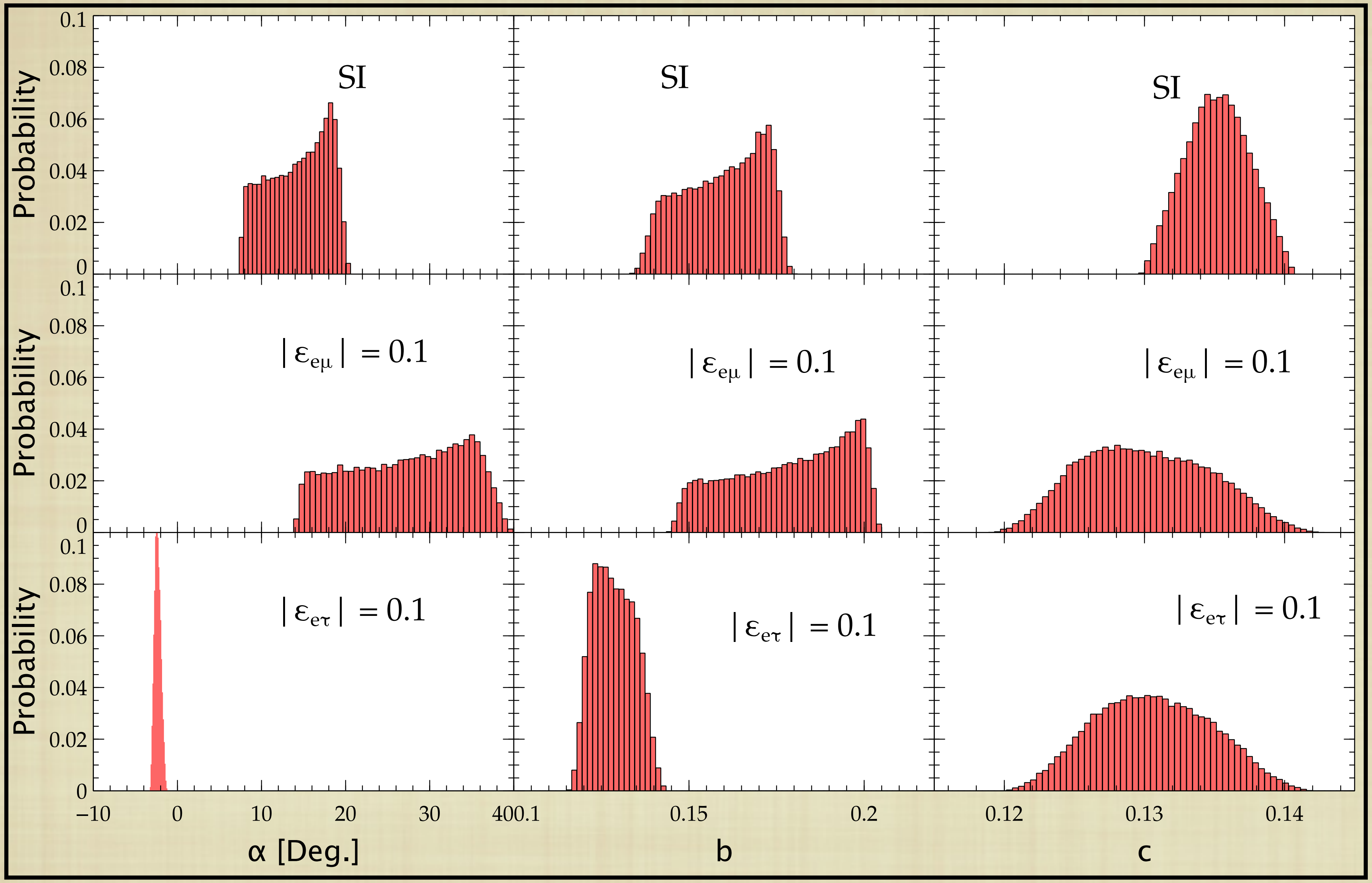
1σ variation of the oscillation parameters

Table: de Salas, Forero, Ternes, Tortola, Valle: 1708.01186

Oscillation parameter	Best fit value	1σ range
$\theta_{12}/^\circ$	34.5	[33.5, 35.7]
$\theta_{23}/^\circ$	47.7	[46.0, 48.9]
$\theta_{13}/^\circ$	8.45	[8.31, 8.61]
$\delta_{\rm CP}/\pi$	−0.68	[−0.83, 0.47]
$\Delta m^2_{21}/10^{-5}eV^2$	7.55	[7.39, 7.75]
$\Delta m^2_{31}/10^{-3}eV^2$	2.55	[2.47, 2.53]

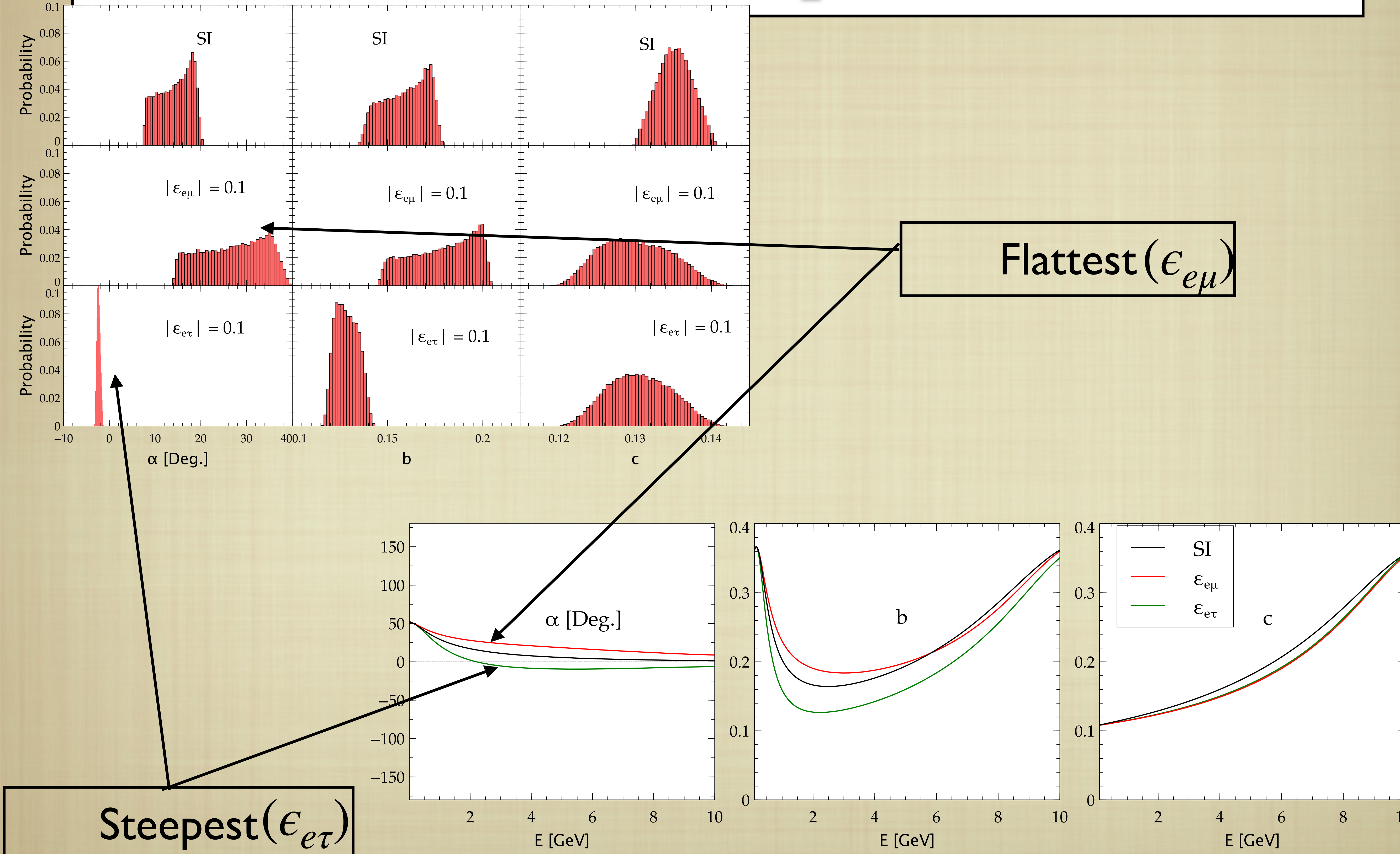
How these variations are connected to LUT parameters?

Distribution of LUT parameters with variation in osc. parameters

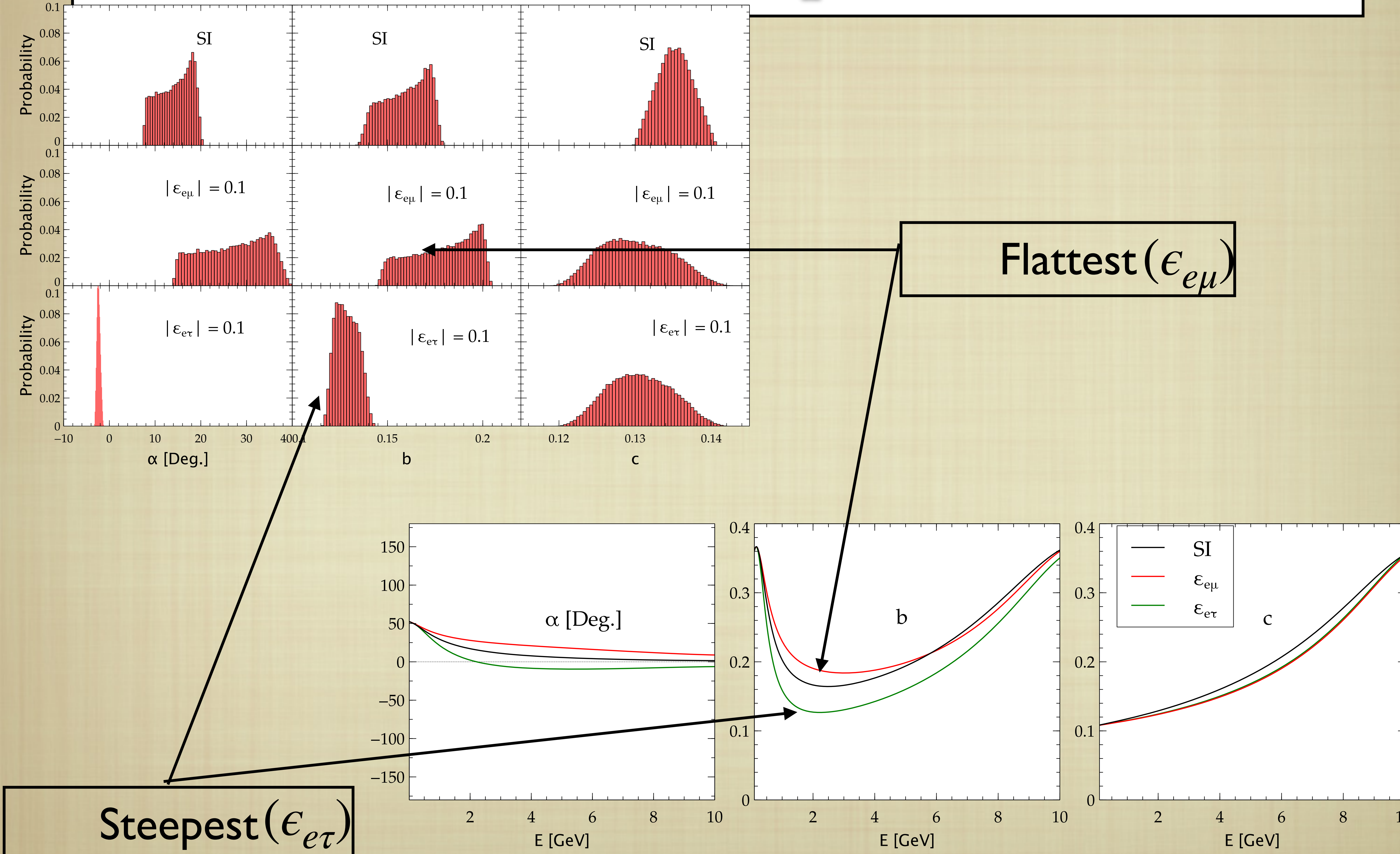


- Random variation (within 1σ) of osc. parameters → a distribution for LUT parameters α, b, c
- E = 2.5 GeV

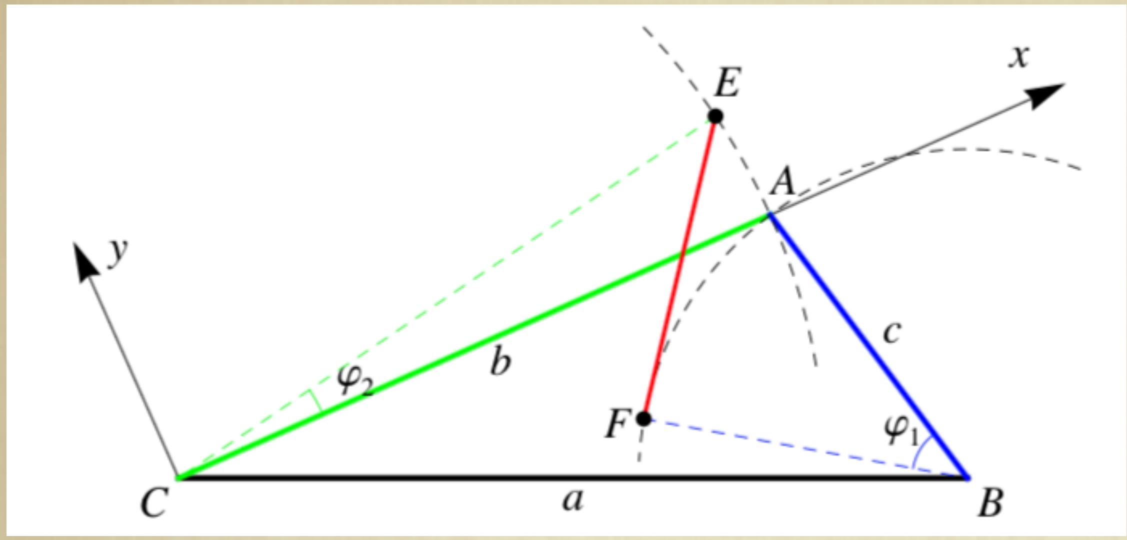
Distribution of LUT parameters with variation in osc. parameters

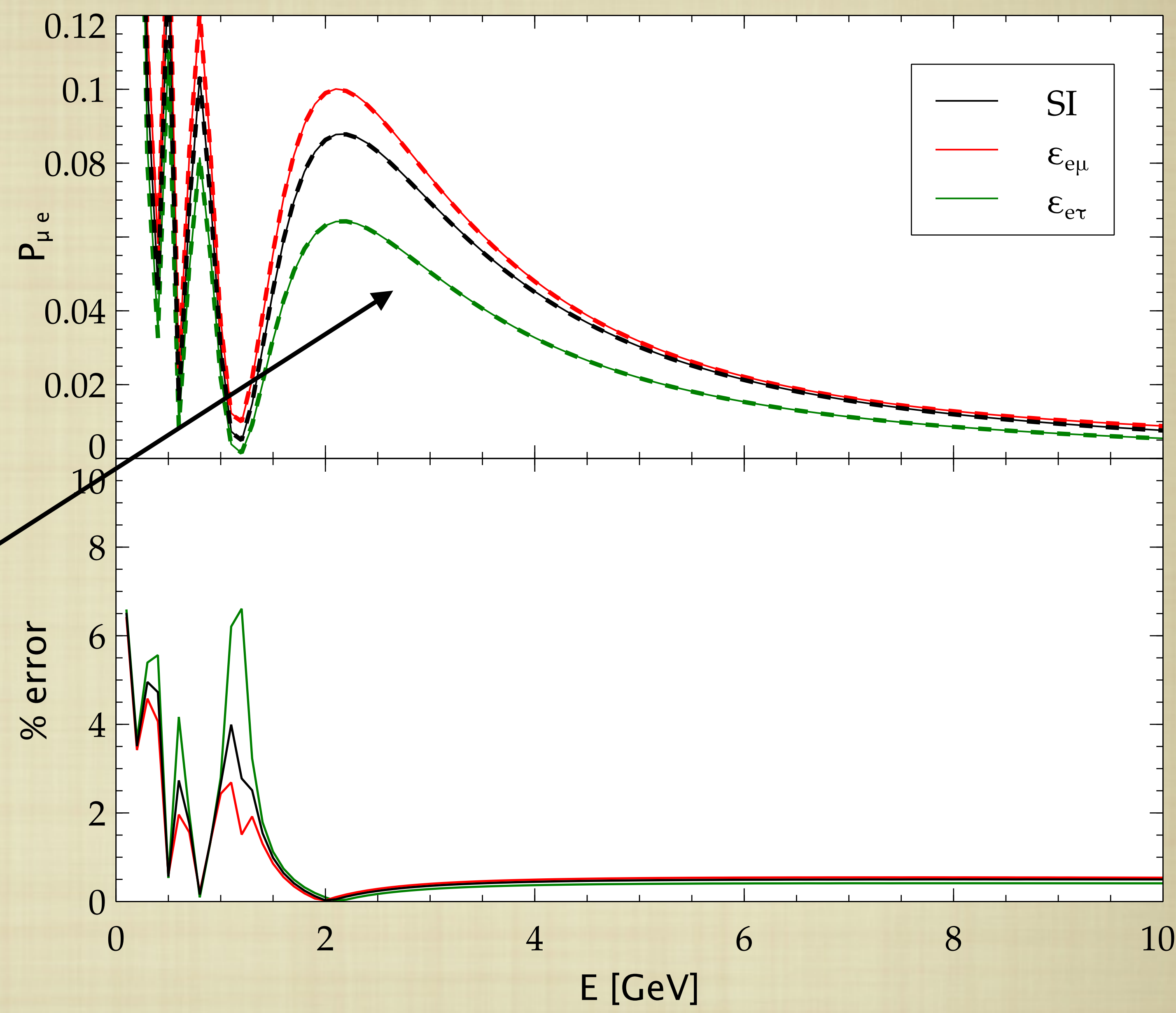


Distribution of LUT parameters with variation in osc. parameters



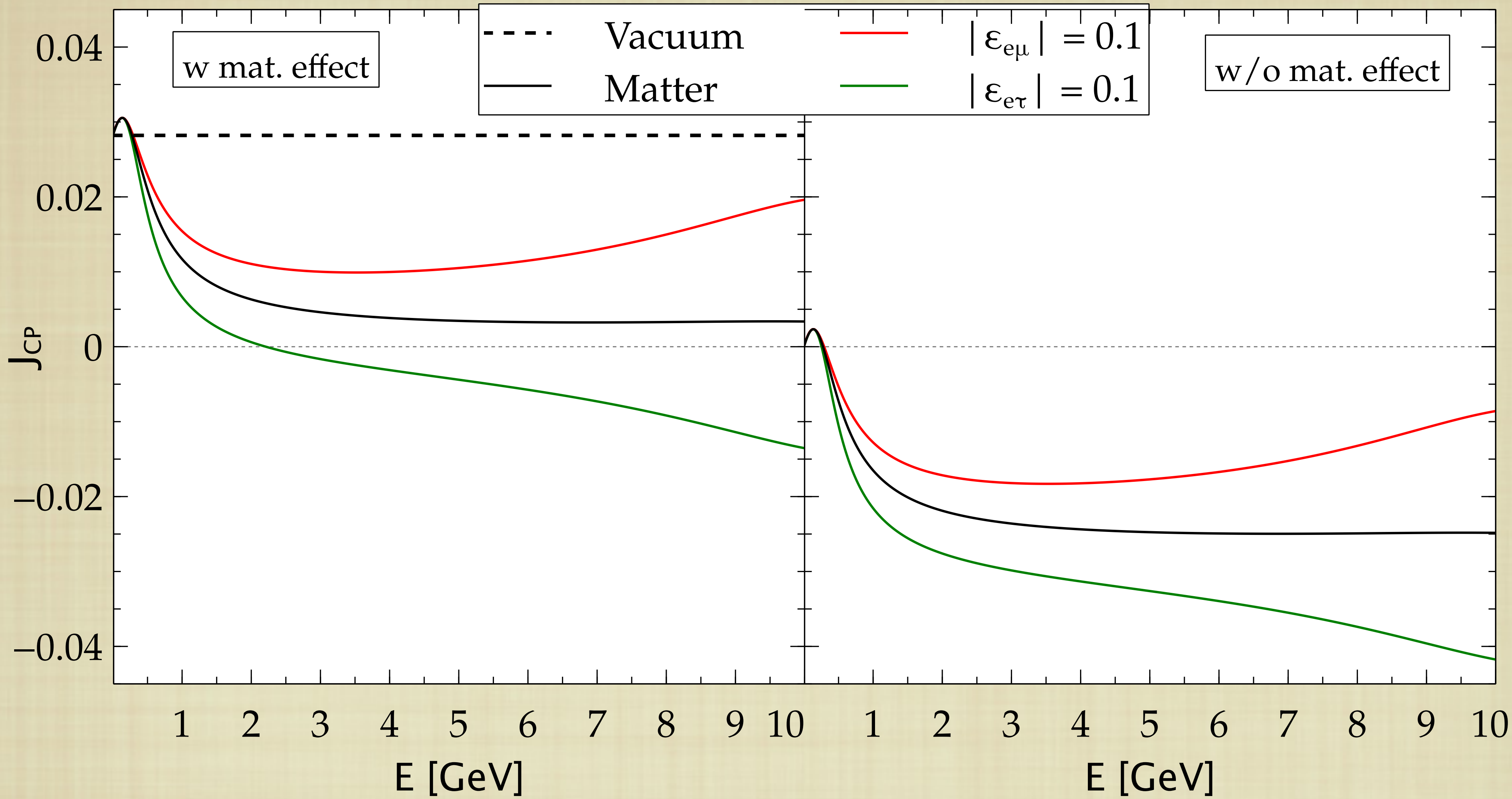
Probability calculation





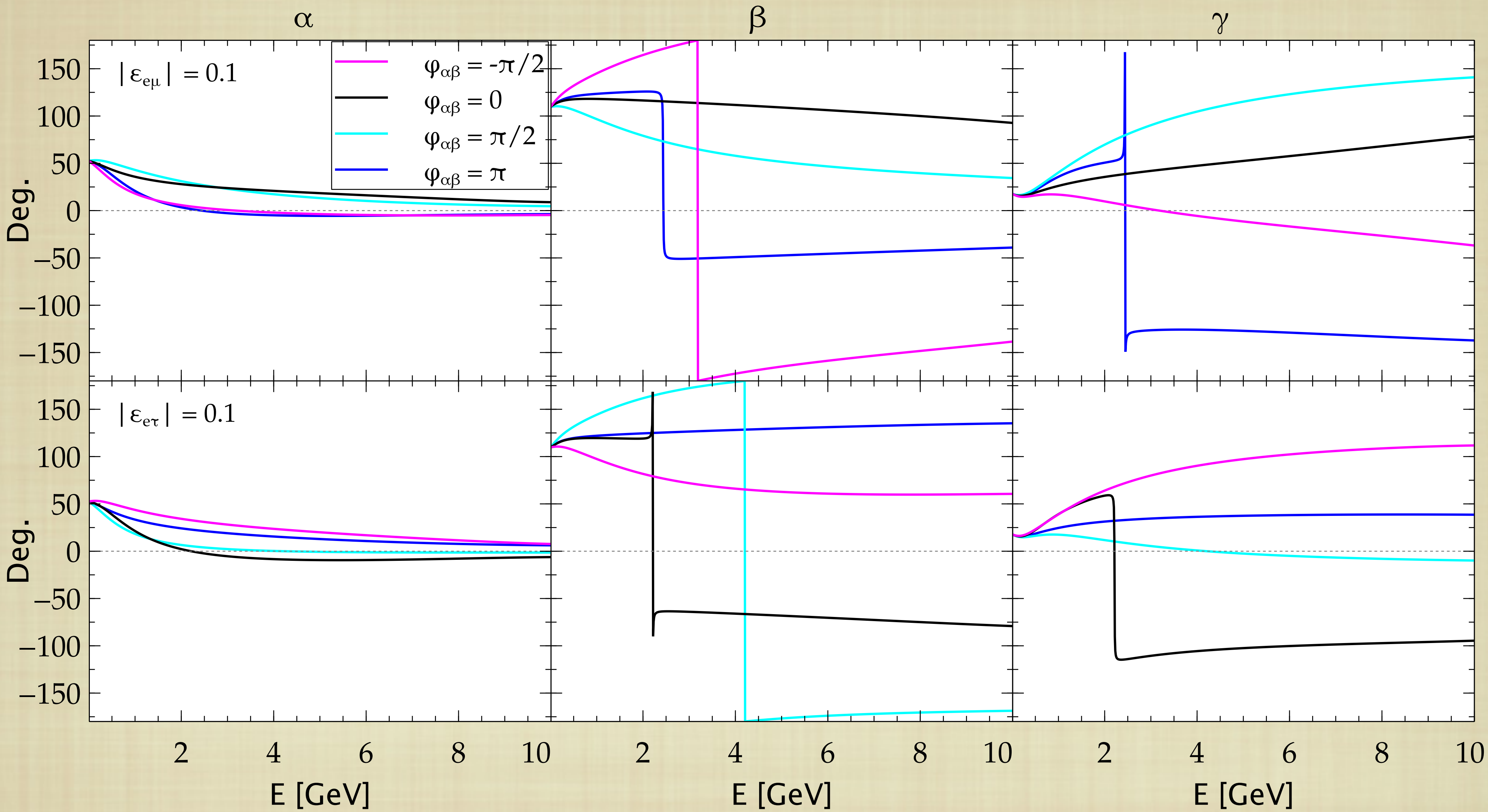
Solid : from GLoBES
Dashed: our code

LUT parameters and CP violation



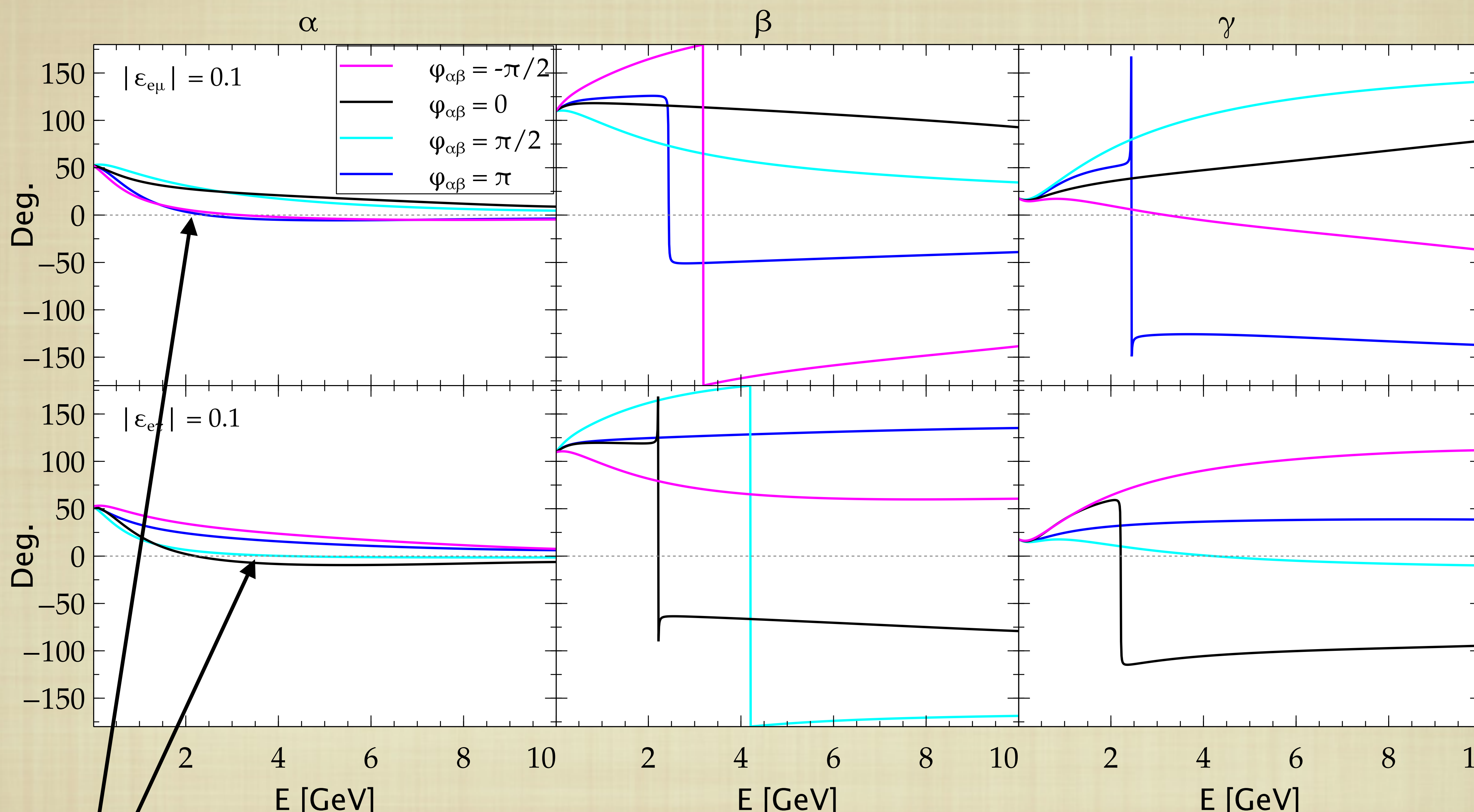
$$J = bc \sin\alpha = 2\Delta$$

Evolution of LUT angles with energy (complex NSI)



$$|\epsilon_{\alpha\beta}|\, e^{i\phi_{\alpha\beta}}$$

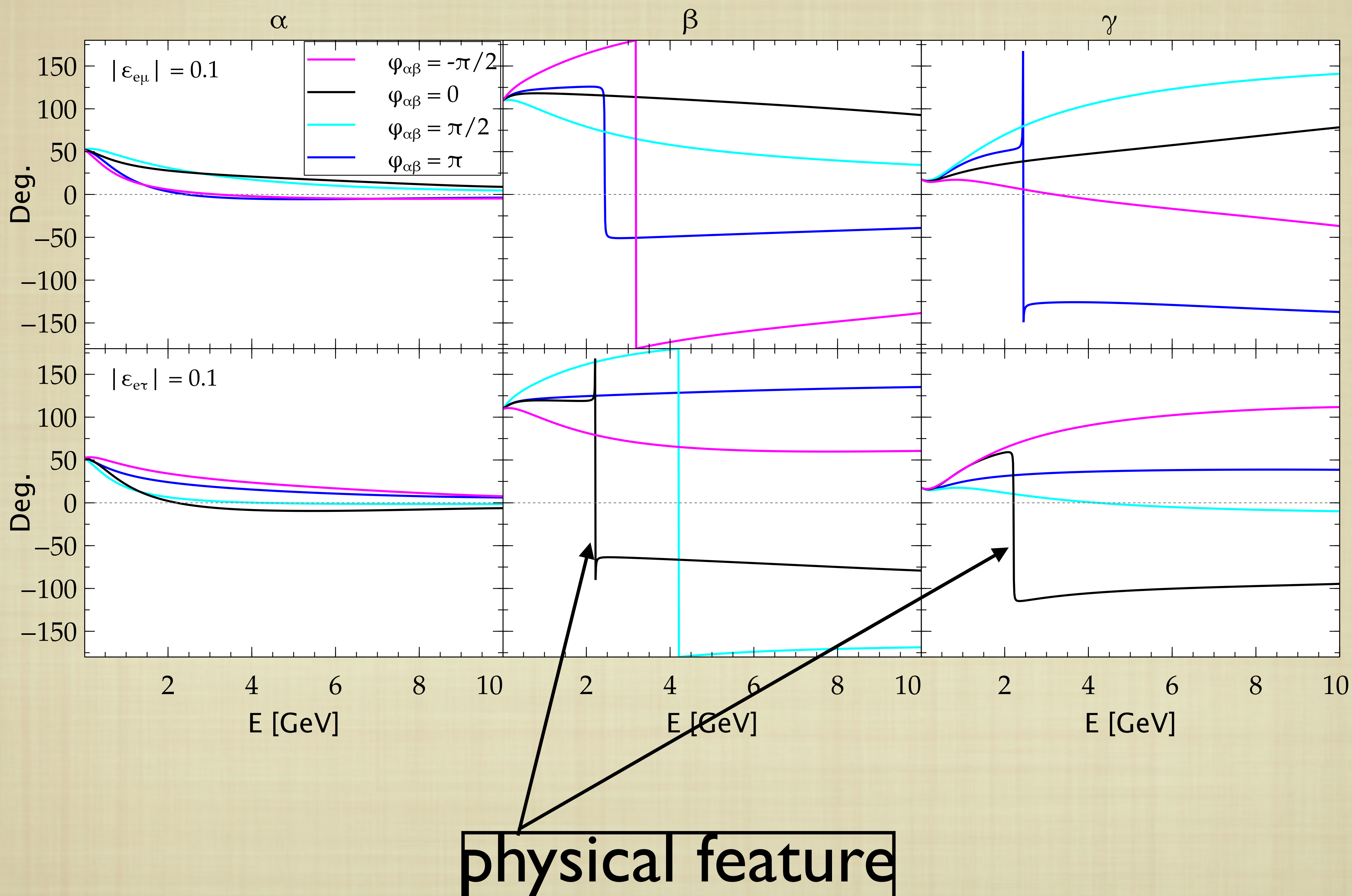
Evolution of LUT angles with energy (complex NSI)



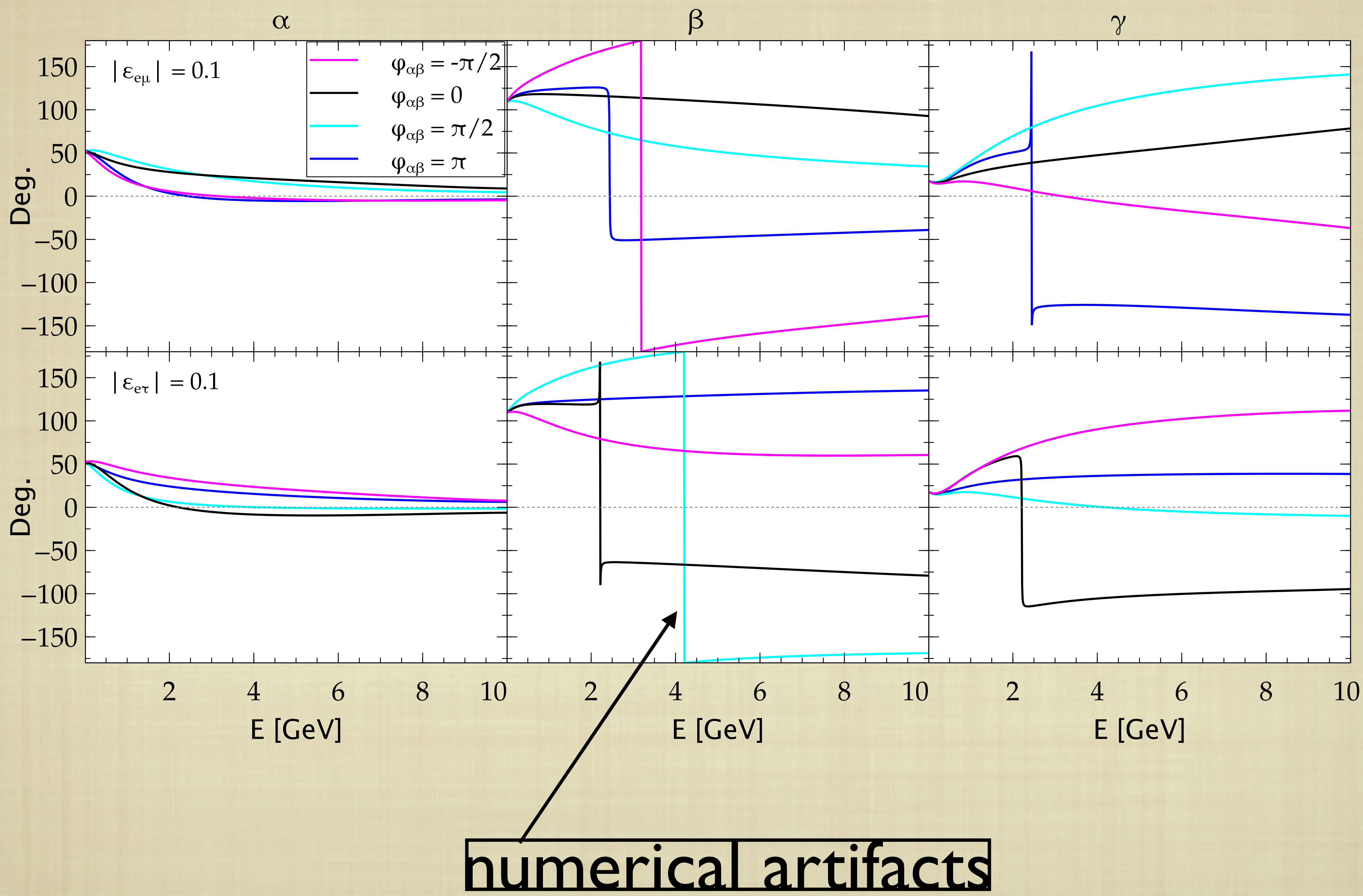
$-\pi/2 \rightarrow \pi/2$ and $0 \rightarrow \pi$

magenta → cyan black → blue

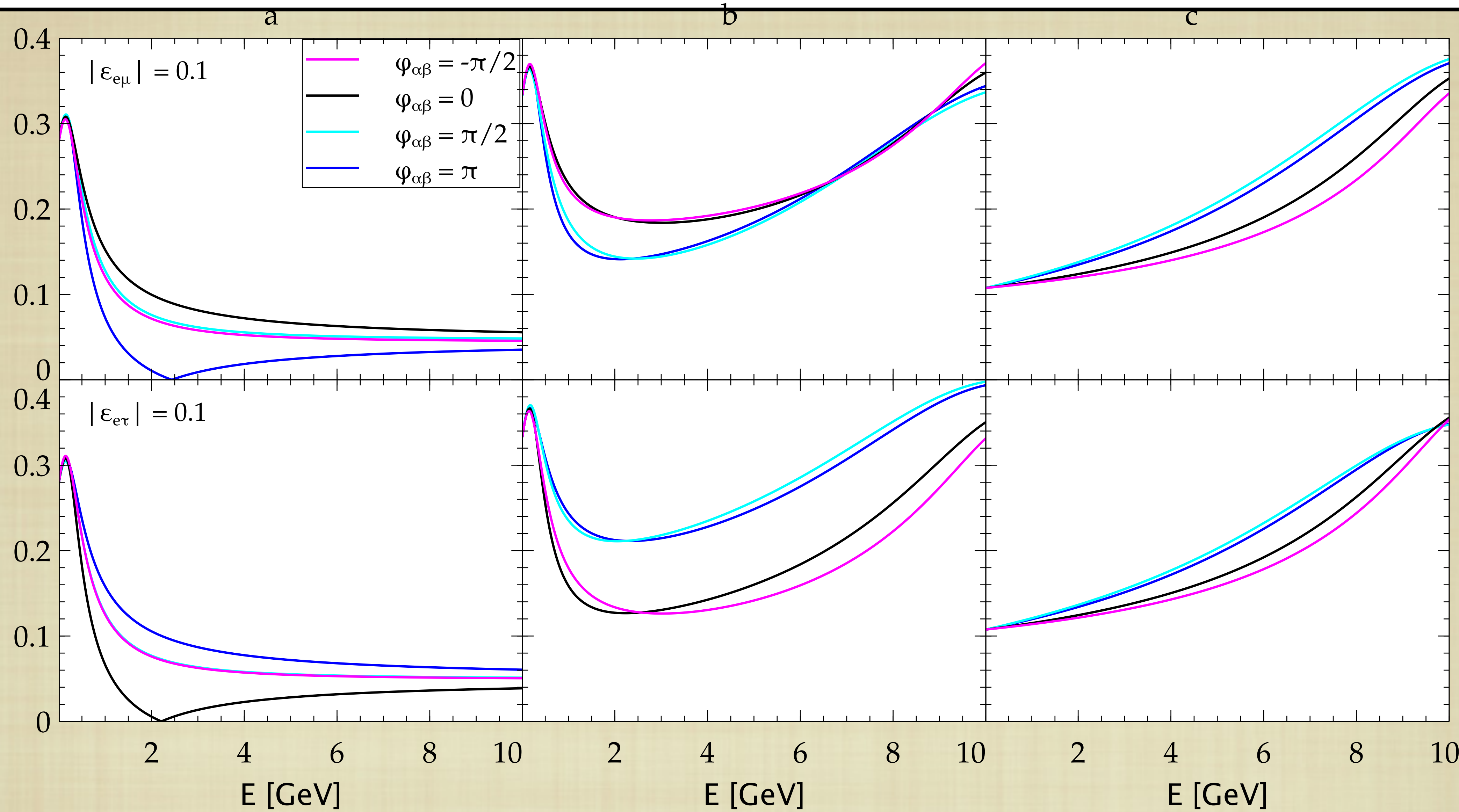
Evolution of LUT angles with energy (complex NSI)



Evolution of LUT angles with energy (complex NSI)



Evolution of LUT sides with energy (complex NSI)



Summary

- An idea of the behaviour of the LUT parameters with variation in energy and std. osc. params. in presence of NSI
- Interesting variation for the LUT parameters in presence of $\epsilon_{e\tau}$
- Quantifying CP violation by calculating the area of LUT
- Numerical determination of oscillation probability in presence of NSI with a compact expression
- Potential application with other new physics scenario

Thank you!

Backup

$$\Delta P_{\mu e}(\varepsilon_{e\mu}) = P_{\mu e}^{NSI}(\varepsilon_{e\mu}) - P_{\mu e}^{SI}$$
$$\approx -4A\Delta \sin\Delta |\varepsilon_{e\mu}| s_{13} s_{2(23)} c_{23} D_1^{e\mu} \sin(\delta + \varphi_{e\mu} - \gamma_1^{e\mu})$$

&

$$\Delta P_{\mu e}(\varepsilon_{e\tau}) \approx 4A\Delta \sin\Delta |\varepsilon_{e\tau}| s_{13} s_{2(23)} s_{23} D_1^{e\tau} \sin(\delta + \varphi_{e\tau} + \gamma_1^{e\tau})$$

where,

$$D_1^{e\mu} = [\sin^2\Delta + (\tan^2\theta_{23}\frac{\sin\Delta}{\Delta} + \cos\Delta)^2]^{1/2} \qquad \gamma_1^{e\mu} = \tan^{-1}(\frac{\tan^2\theta_{23}}{\Delta} + \cot\Delta)$$
$$D_1^{e\tau} = [\sin^2\Delta + (\frac{\sin\Delta}{\Delta} - \cos\Delta)^2]^{1/2}; \qquad \gamma_1^{e\tau} = \tan^{-1}(\frac{1}{\Delta} - \cot\Delta)$$

Backup

$$U_S = \begin{pmatrix} c_s c_x & s_s c_x & e^{-\mathrm{i}\delta} s_x \\ -s_s c_a - e^{\mathrm{i}\delta} c_s s_a s_x & c_s c_a - e^{\mathrm{i}\delta} s_s s_a s_x & s_a c_x \\ s_s s_a - e^{\mathrm{i}\delta} c_s c_a s_x & -c_s s_a - e^{\mathrm{i}\delta} s_s c_a s_x & c_a c_x \end{pmatrix}.$$

Backup:NSI

$$\mathcal{L}_{NSI} = -2\sqrt{2}G_F\varepsilon^{f\,C}_{\alpha\beta}\;[\bar{\nu}_\alpha\gamma^\mu P_L\nu_\beta]\;[\bar{f}\gamma_\mu P_C f]$$

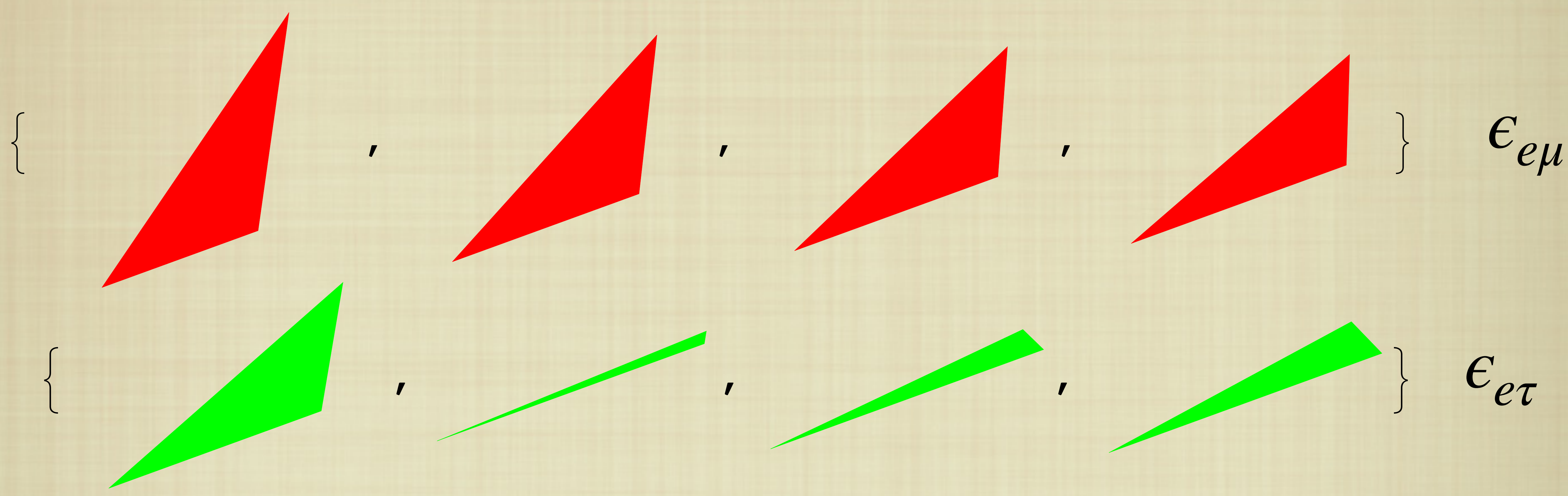
- If NSI arises at $M_{NP} >> M_{EW}$ from some higher dim. operators,
  then $\epsilon_{\alpha\beta} \sim (M_{EW}/M_{NP})^2$

Backup:NSI bound

$$|\varepsilon_{\alpha\beta}| < \begin{pmatrix} 4.2 & 0.3 & 0.5 \\ 0.3 & 0.068 & 0.04 \\ 0.5 & 0.04 & 0.15 \end{pmatrix}$$

- Ohlsson et. al. (2015)

Backup : Evolution of LUT with energy



$E = 1 \qquad 2 \qquad 3 \qquad 4$