Swamp in flux

On progress to determine the shape of the flux landscape

Thomas W. Grimm

Utrecht University

Based on: Work in Progress with
Fabian Ruehle, Damian van de Heisteeg - classify CYs
Irene Valenzuela, Chongchuo Li - flux compactifications

and 1811.02571 with Chongchuo Li, Eran Palti

Madrid 2019

Introduction and general comments

Swampland conjectures and potentials

- Recent de Sitter conjectures make strong claims about the shape of potentials arising in string theory

[Obied,Ooguri,Spodyneik,Vafa], [Andriot] [Dvali,Gomez][Andriot,Roupec]
[Garg,Krishnan] [Ooguri,Palti,Shiu,Vafa] 2018

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- Can we systematically study what is possible?

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  flux compactifications

  F-theory
  flux potential

$$V_{\mathrm{F}} = \int_{Y_4} G_4 \wedge *G_4 - \int_{Y_4} G_4 \wedge G_4$$

  Type IIA
  flux potential

$$V_{\mathrm{IIA}} = \int_{\tilde{Y}_3} H_3 \wedge *H_3 + \sum_p \int_{\tilde{Y}_3} F_p \wedge *F_p - \int_{\mathrm{O6/D6}} F_0 H_3$$

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$\Rightarrow$ seems completely impossible!

Emerging perspective

- Focusing on the ‘right’ underlying structure and using a powerful mathematical machinery might make this possible

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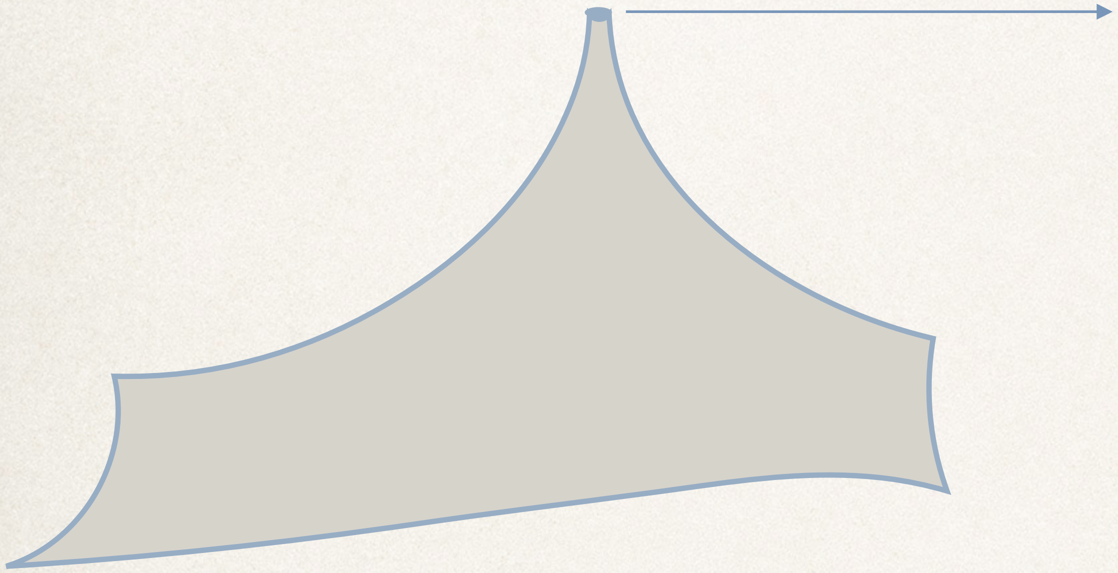

theory should be well-behaved at the boundaries of field space and have a universal singular behavior

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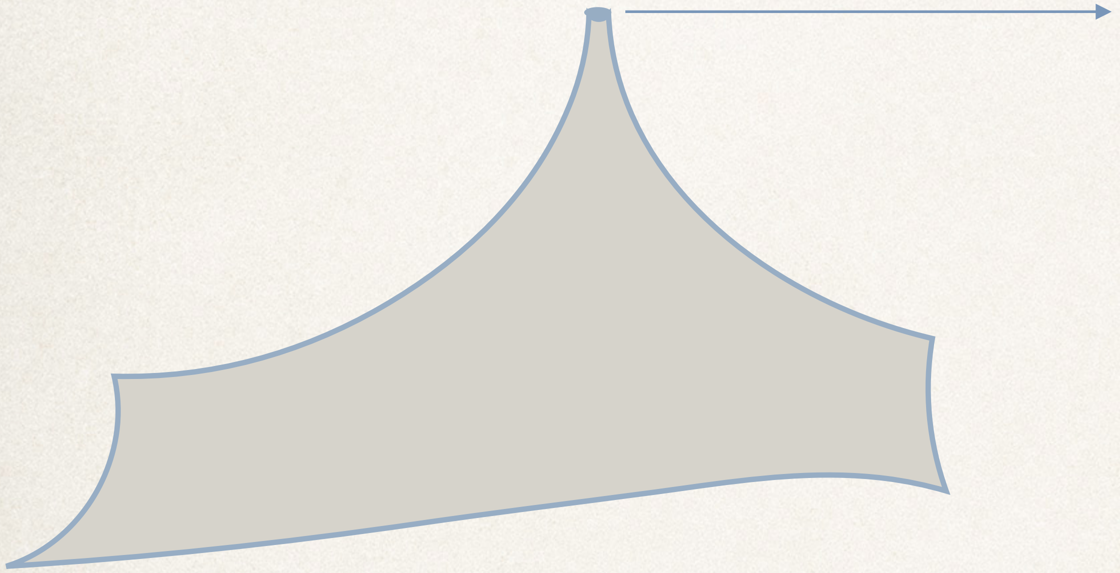


theory should be well-behaved at the boundaries of field space and have a universal singular behavior

Guided by swampland conjectures and their interconnections

- Used machinery: limiting mixed Hodge structure

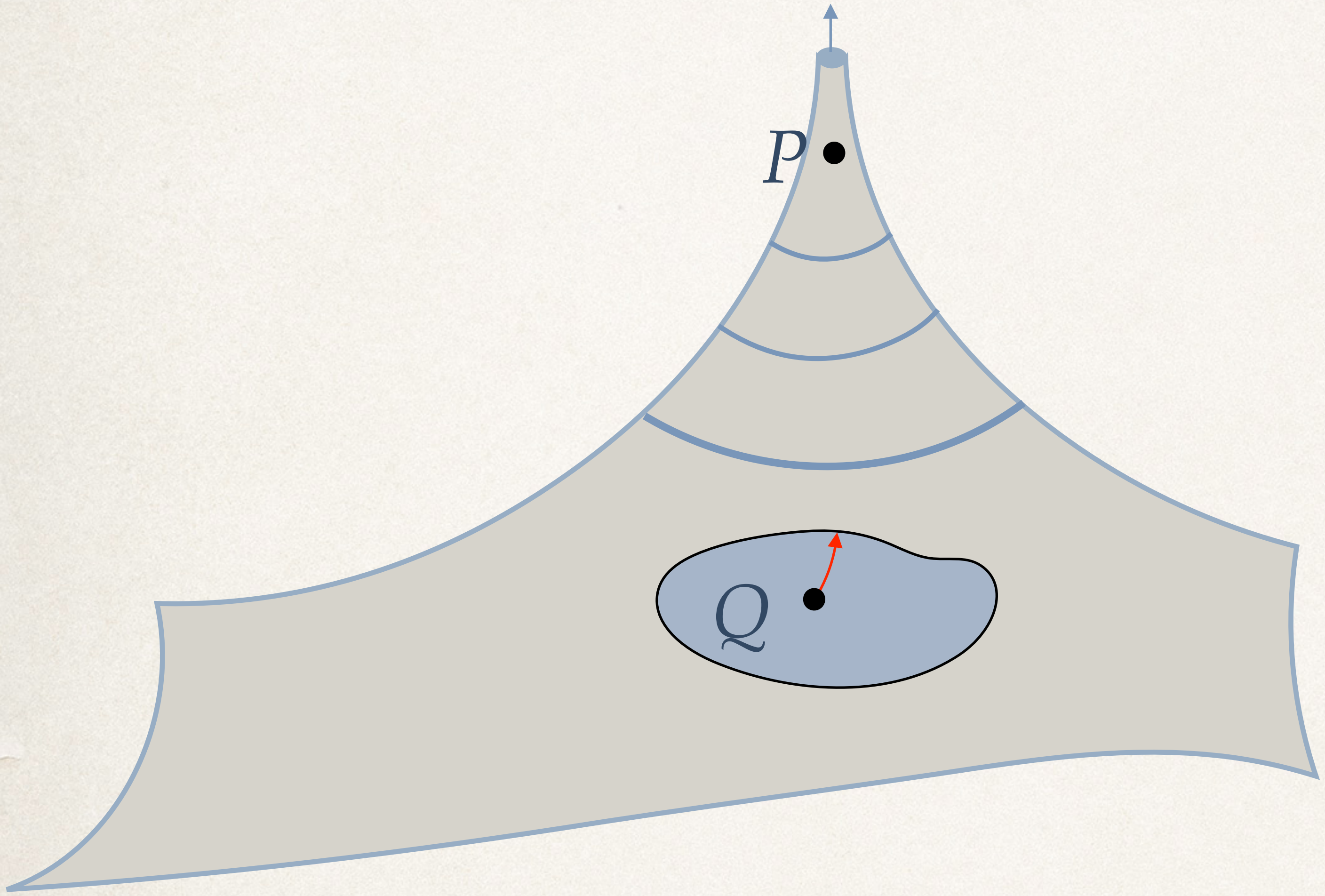
[Deligne,Schmid…]

Aside: arose as an answer to the famous monodromy-weight conjecture (1970) by Deligne… at the time he was working on the Weil conjectures (1949)…

Swampland Distance Conjecture as a Guide

consider a moduli space and two points

[Ooguri,Vafa]



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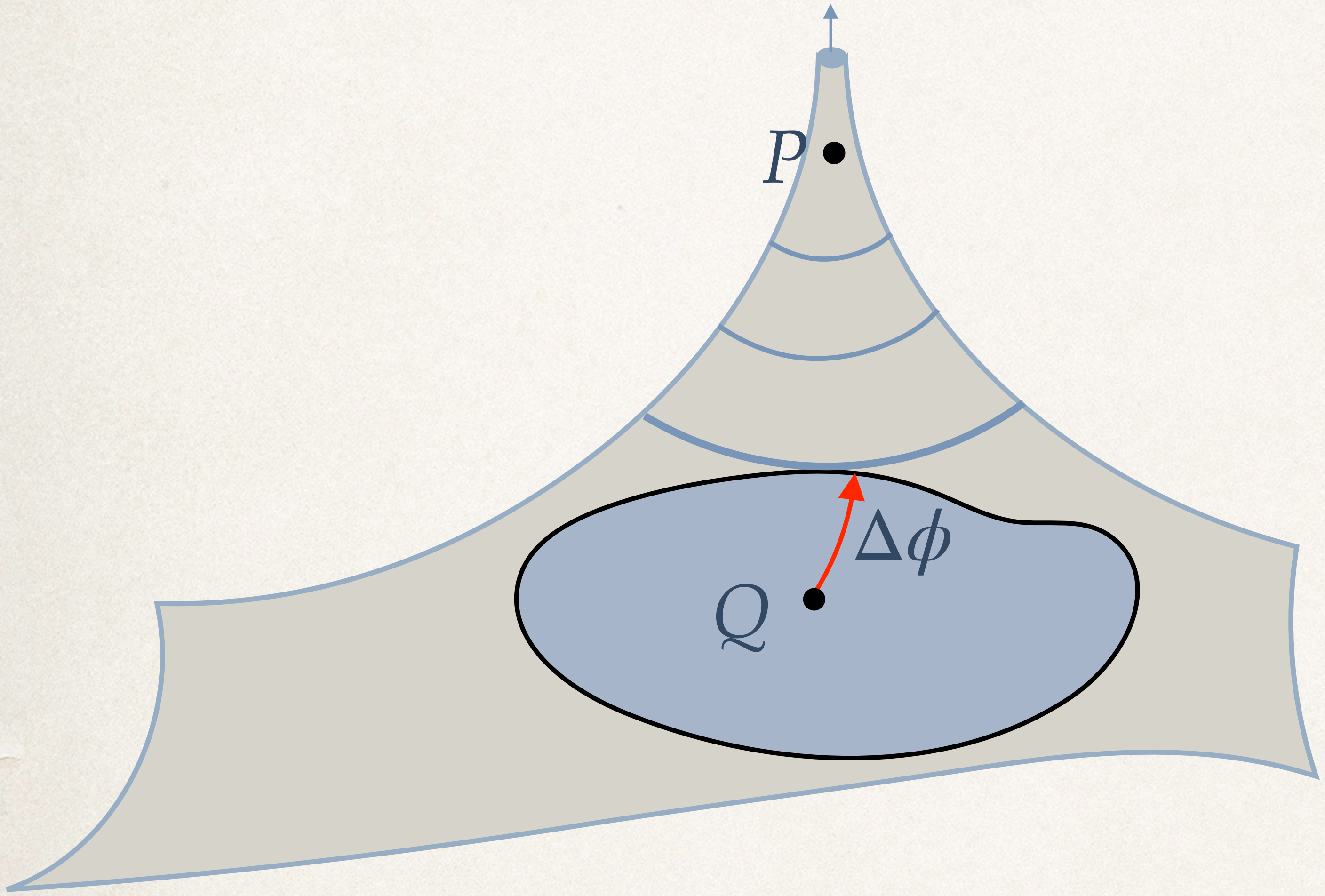

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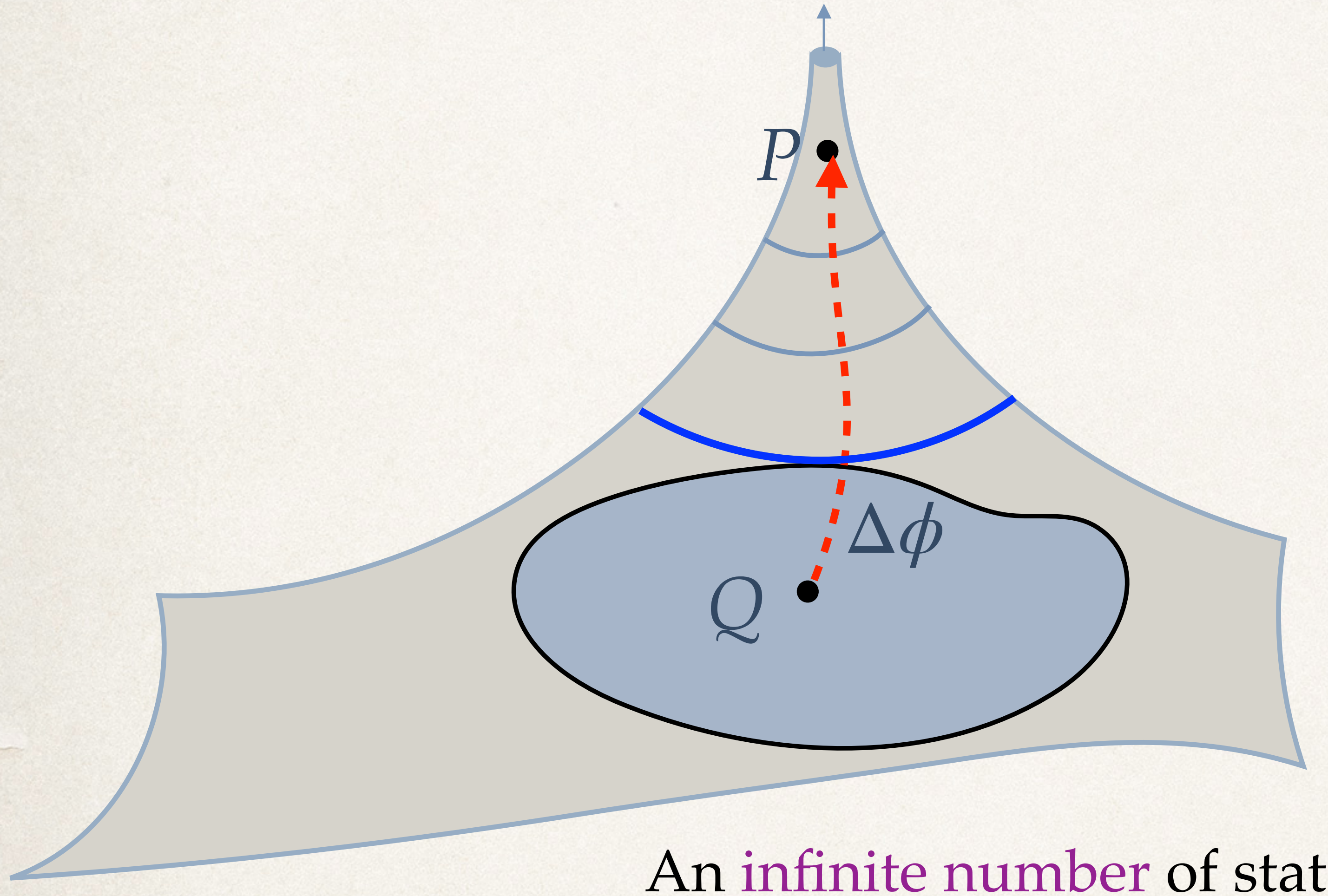


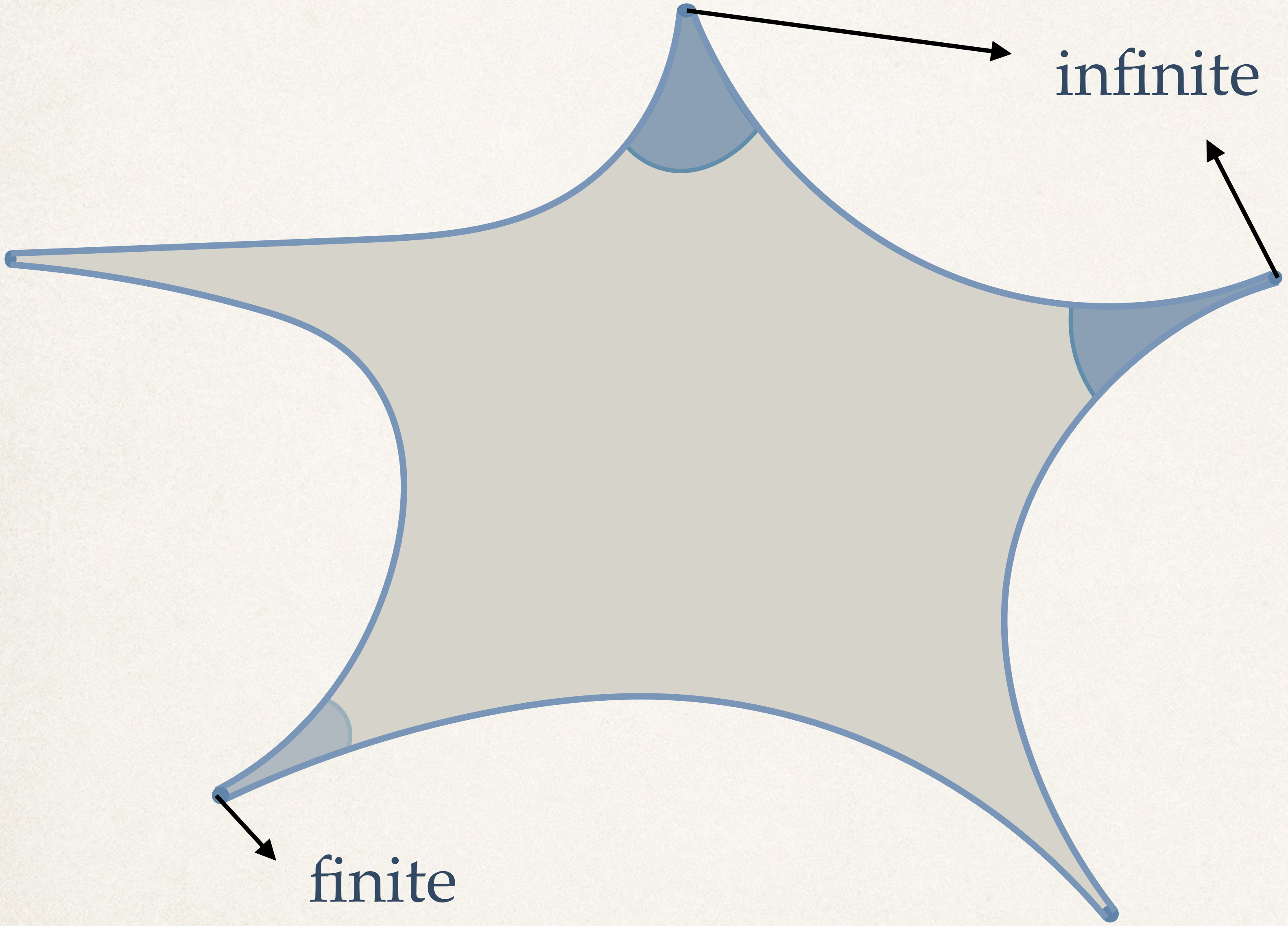
shortest geodesic between P, Q (length $d(P,Q)$)

An infinite number of states become light on paths approaching an infinite distance point: $m(P) \propto M_P\, e^{-\gamma\, d(P,G)}$ as $d(P,Q) \gg 1$

signaling the breakdown of an effective description

$\Rightarrow$ universal structure near infinite distance boundaries

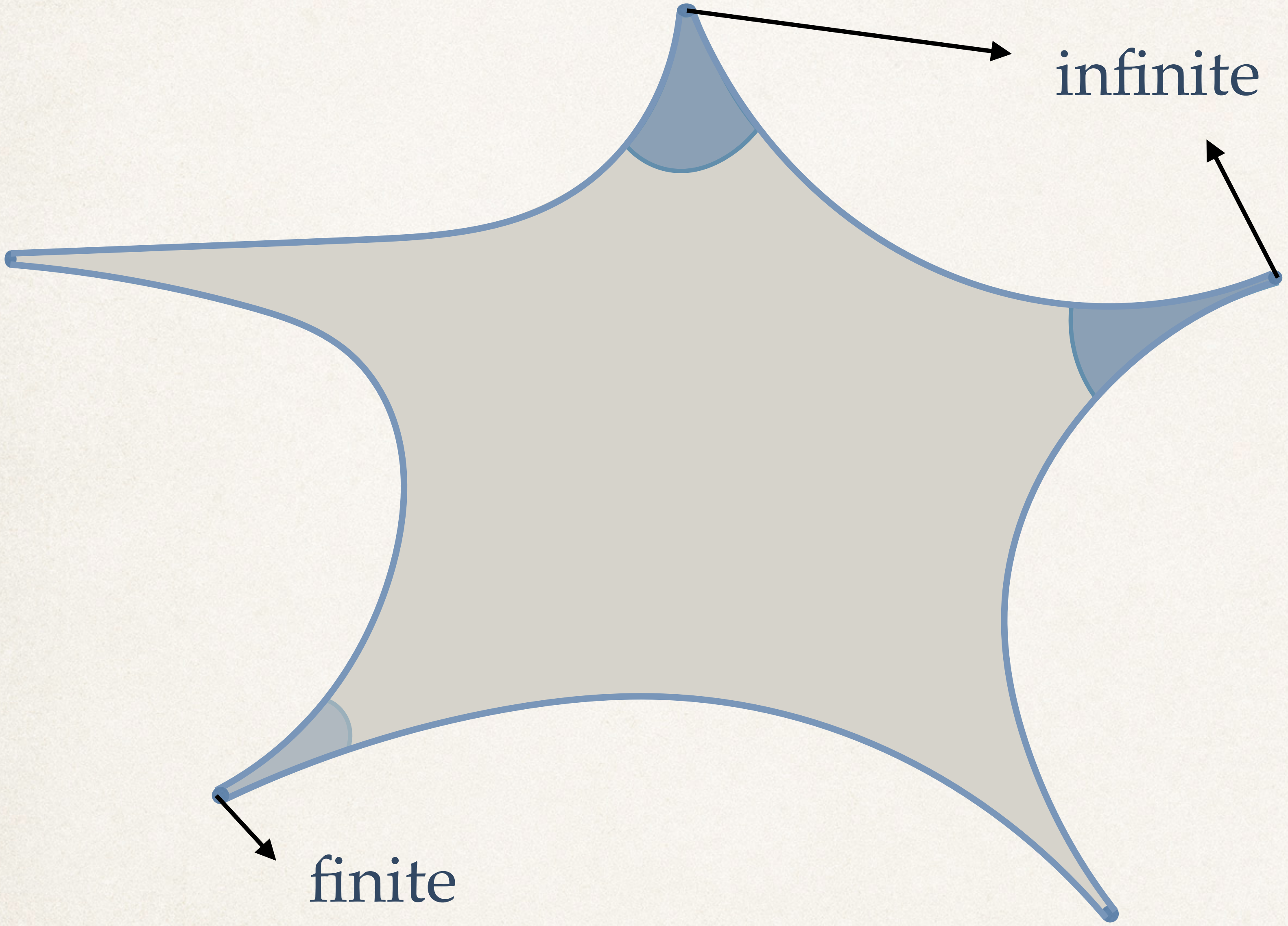
Limits in Moduli Space



infinite

finite

Limits in Moduli Space



- In the following: restrict to geometric moduli spaces arising in Calabi-Yau compactifications: T^2, K3, CY_3 , CY_4

$\Rightarrow$ Universal structure?!

Swampland conjectures and mathematics

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Distance Conjecture + **Weak Gravity Conjecture**

[Ooguri,Vafa] 2006 [Arkani-Hamed,Motl,Nicolis,Vafa] 2006

➡ linked with deep mathematical statements about compactification geometries

[TG,Palti,Valenzuela] [Blumenhagen,Kläwer,Schlechter,Wolf] [Lee,Lerche,Weigand]^3 [Gonzalo,Ibáñez,Uranga] [Corvilain,TG,Valenzuela] [Joshi,Klemm] [Erkinger,Knapp] [Marchesano,Wiesner] [Font,Herráez,Ibáñez] [TG,vd Heisteeg] [TG,Dierigl] to appear

→ see also Timo's talk

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➡ Here: develop tools for scalar potentials

$\Rightarrow$ asymptotic flux compactifications

Universal Structure at the Limits in Moduli Space

Moduli space of Calabi-Yau compactifications

- Consider complex structure moduli space $\mathcal{M}_{\rm cs}$ (Kähler as a mirror)

Kähler metric: $g_{I\bar{J}} = \partial_{z^I}\partial_{\bar{z}^J} K$ $K = -\log\Big[i\int_{\mathrm{CY}_n} \Omega\wedge\bar{\Omega}\Big]$

$\Rightarrow$ knowing the field dependence defines of $\Omega(z)$ the geometry of the moduli space

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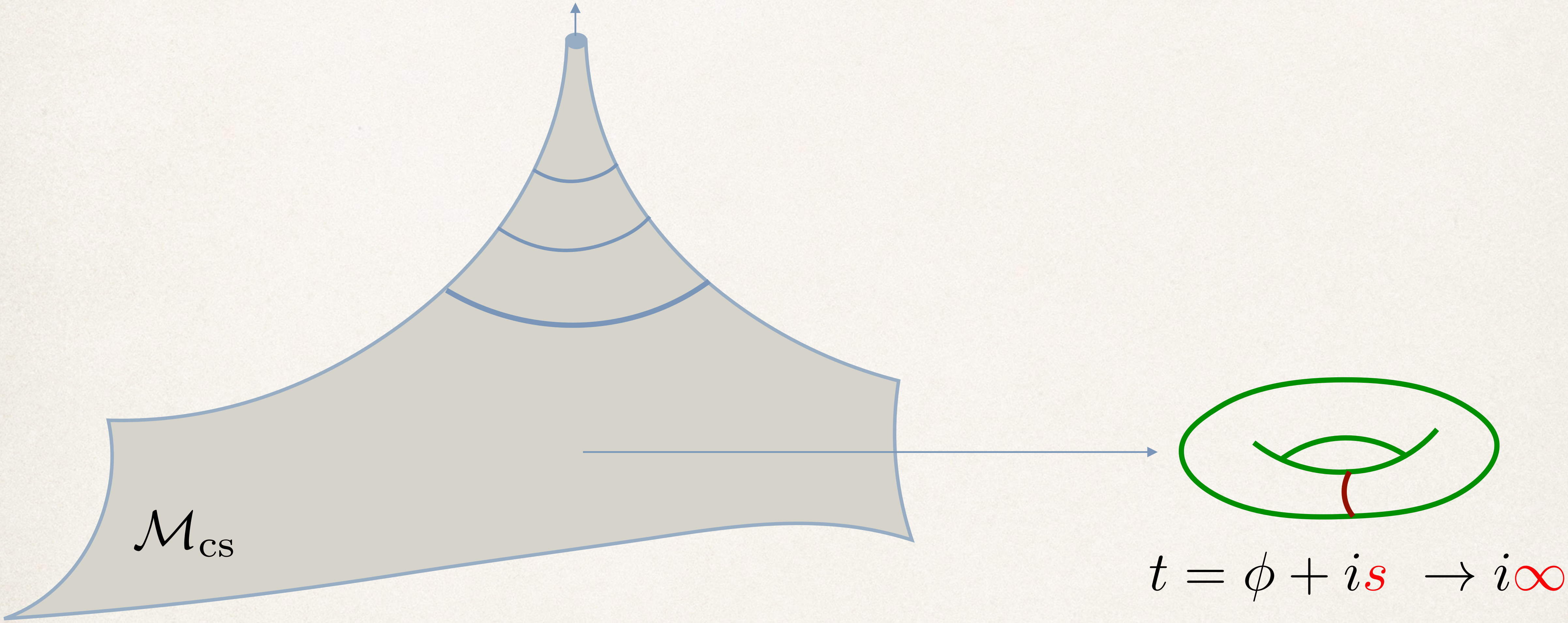
⇒ knowing the field dependence defines of $\Omega(z)$ the geometry of the moduli space

Question: Is there a universal behavior of $\Omega(z)$ at the limits of the moduli space?

Limits in complex structure moduli space

- Limits are the points where Calabi-Yau manifold degenerates/blows up!



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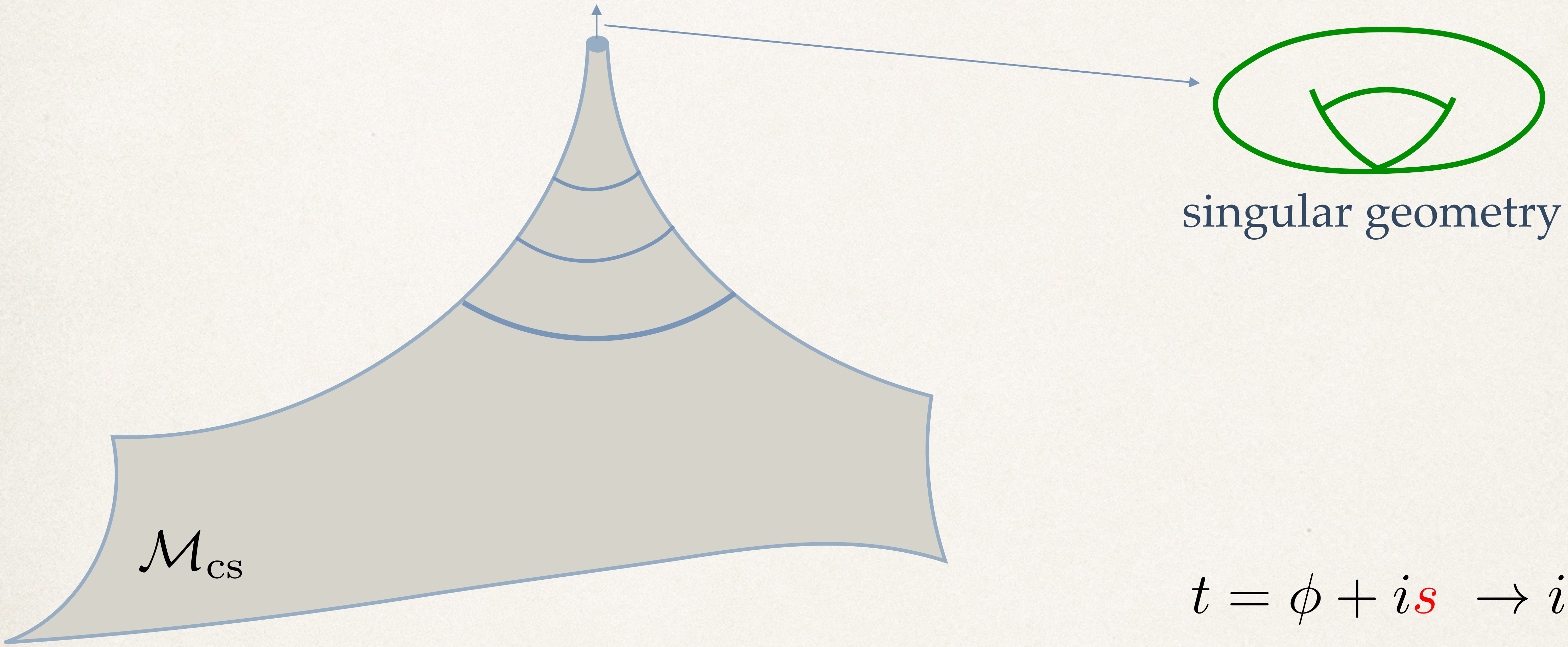

$$t = \phi + i{\color{red}s} \;\; \to i\infty$$

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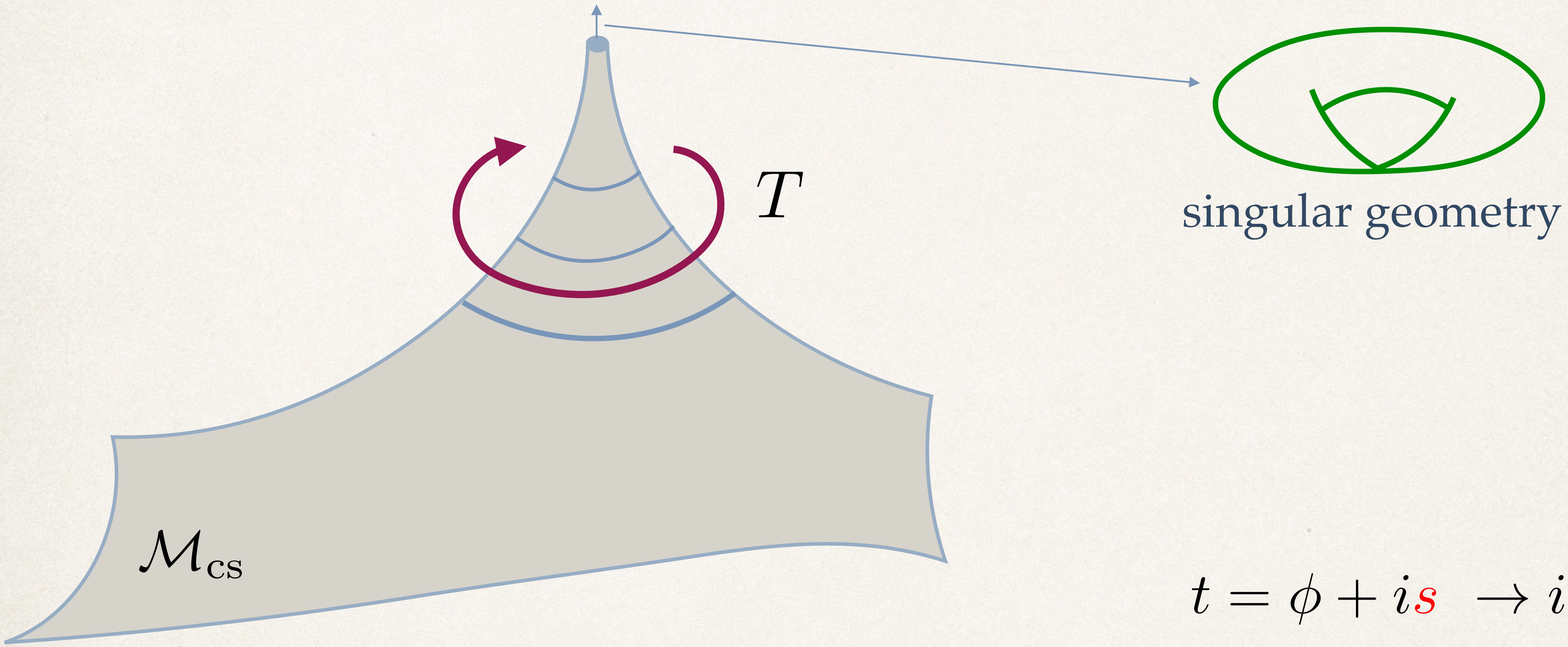

$$t = \phi + i s \ \ \to i\infty$$

⇒ monodromy around singular loci: $\Omega(t{+}1, \ldots) = T \cdot \Omega(t, \ldots)$

Universal behavior of periods

- Limiting behavior of Ω near degeneration points

$$t^1, \ldots, t^n \;\rightarrow\; i\infty \qquad \zeta^\kappa \text{ finite}$$

[Schmid]

$$\boxed{\Omega = e^{t^i N_i}\,\mathbf{a}_0 + \mathcal{O}(e^{2\pi i t})}$$

(up to rescaling)

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- log-monodromy: $N_i = \log T_i^u$ - nilpotent matrix ($N^k = 0$, some k)
- 'limiting' form $\mathbf{a}_0(\zeta)$- can depend on the coords not send to limit

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(up to rescaling)

Polynomial in t^i
nilpotent orbit
("perturbative part")

Strongly suppressed in the limit
⟹ neglect
("non-perturbative part")

Emergence of an $\mathfrak{sl}(2)^n$ - algebra

- Remarkably: can associate an $\mathfrak{sl}(2)^n$ - algebra to N_i, $\mathbf{a}_0$ [Cattani, Kaplan, Schmid]

n commuting $\mathfrak{sl}(2)$-triples: N_i^-, N_i^+, Y_i

$\Rightarrow$ raising, lowering, level-operator

aside: need to fix sector in moduli space, or enhancement chain…later

$$s^1 \gg s^2 \gg ... \gg s^n$$

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$$s^1 \gg s^2 \gg \ldots \gg s^n$$

- Can split form into fine splitting associated to the asymptotic region

$$H^n(Y_n, \mathbb{R}) = \sum_{l_1, \ldots, l_n} V_{l_1, \ldots, l_n}$$

eigenspaces of $Y_{(i)} = Y_1 + \ldots + Y_i$

⇒ full structure: limiting mixed Hodge structure [Deligne,Schmid]

Asymptotic of the Hodge norm

- Hodge norm is omnipresent in string compactifications:

$$\|\alpha\|^2 = \int_{\mathrm{CY}_n} \alpha \wedge \star \alpha \qquad\qquad \alpha \in H^n(Y_n, \mathbb{R})$$

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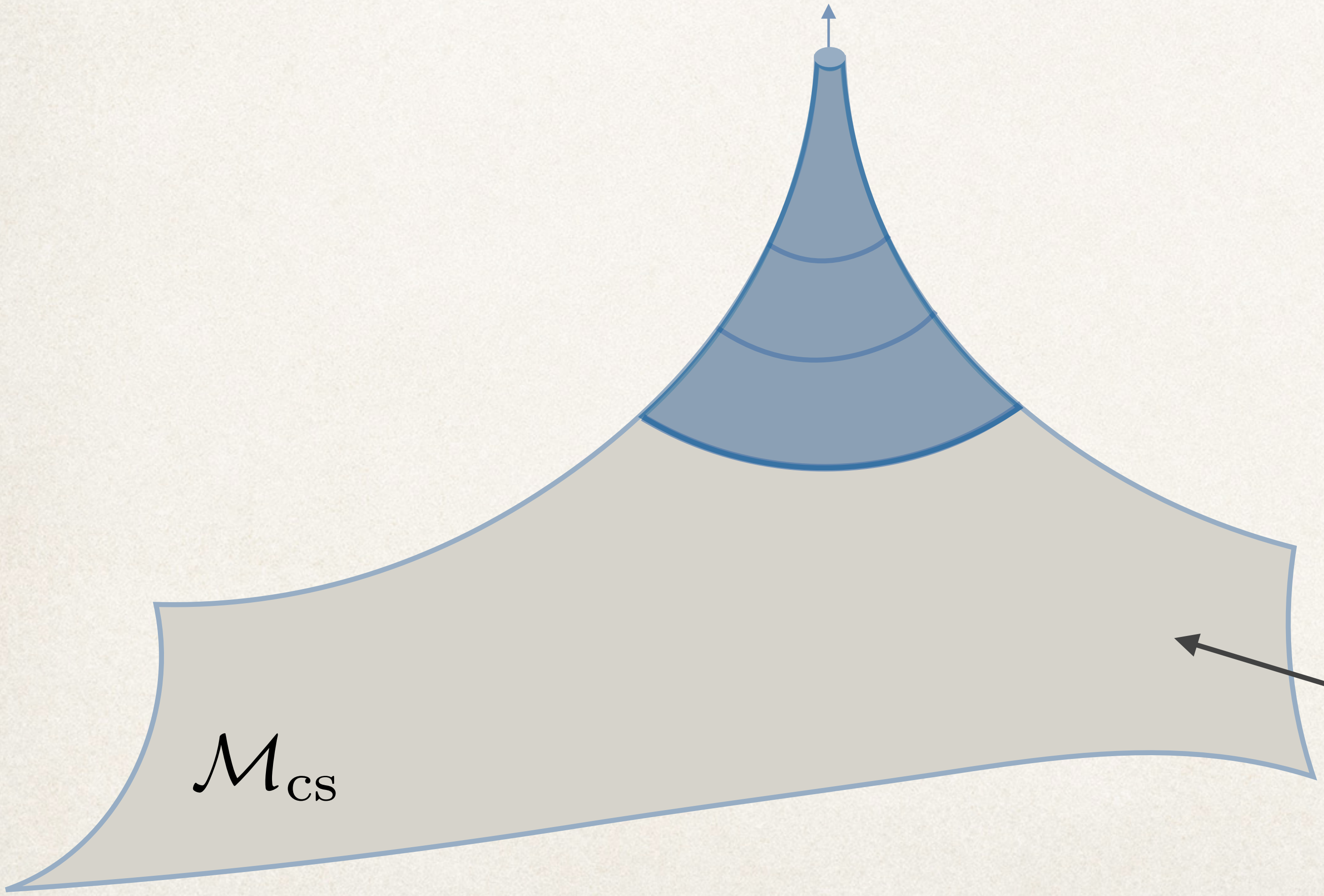
Of importance here: F-theory on Calabi-Yau fourfold Y_4

Flux scalar potential due to background G_4

$$V_{\mathrm{F}} = \frac{1}{\mathcal{V}_{\mathrm{b}}^2}\Big(\underbrace{\int_{Y_4} G_4 \wedge \star G_4}_{\|G_4\|^2} - \int_{Y_4} G_4 \wedge G_4\Big)$$

Asymptotic of the Hodge norm

- Hodge norm in asymptotic region: $t^i = \phi^i + is^i \to i\infty$



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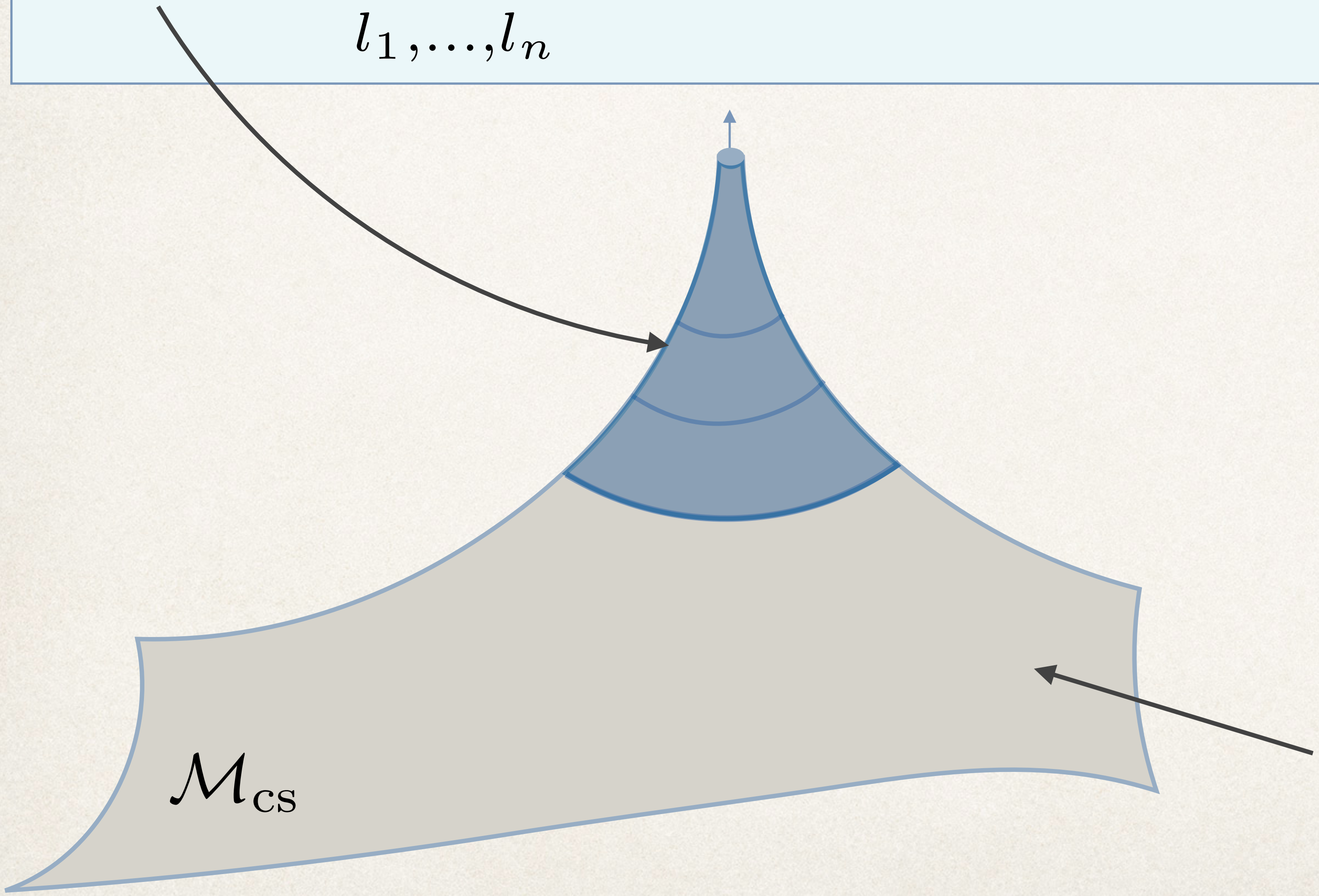
- Hodge norm in asymptotic region: $t^i = \phi^i + i s^i \to i\infty$

$$\|\alpha\|^2 \sim \sum_{l_1,\ldots,l_n} (s^1)^{l_1-n}(s^2)^{l_2-l_1}\ldots(s^n)^{l_{n-1}-l_n}\|\rho_{l_1\ldots l_n}\|_\infty$$



restriction of $e^{\phi^i N_i}\alpha$

to subspaces $V_{l_1\ldots l_n}$

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Upshot:

In asymptotic regime dependence on s^i (saxions) and ϕ^i (axions) is explicit $\Rightarrow$ classification requires to classify all asymptotic limits

Remarkable: there exists a finite $*_\infty$ at each boundary splitting into sl(2)-pieces of the sl(2)^n-algebra

Classification of asymptotic limits (1-parameter)

- K3 surface:

[Kulikov]

$$\text{Types: I, II, III}$$

- Calabi-Yau threefolds: 4 $h^{2,1}$ types of limits

[Kerr,Pearlstein,Robles 2017]
[Green,Griffiths,Robles]…

$$\text{Types: } \mathrm{I}_a,\ \mathrm{II}_b,\ \mathrm{III}_c, \mathrm{IV}_d$$

- Calabi-Yau fourfolds: 8 $h^{3,1}$ types of limits

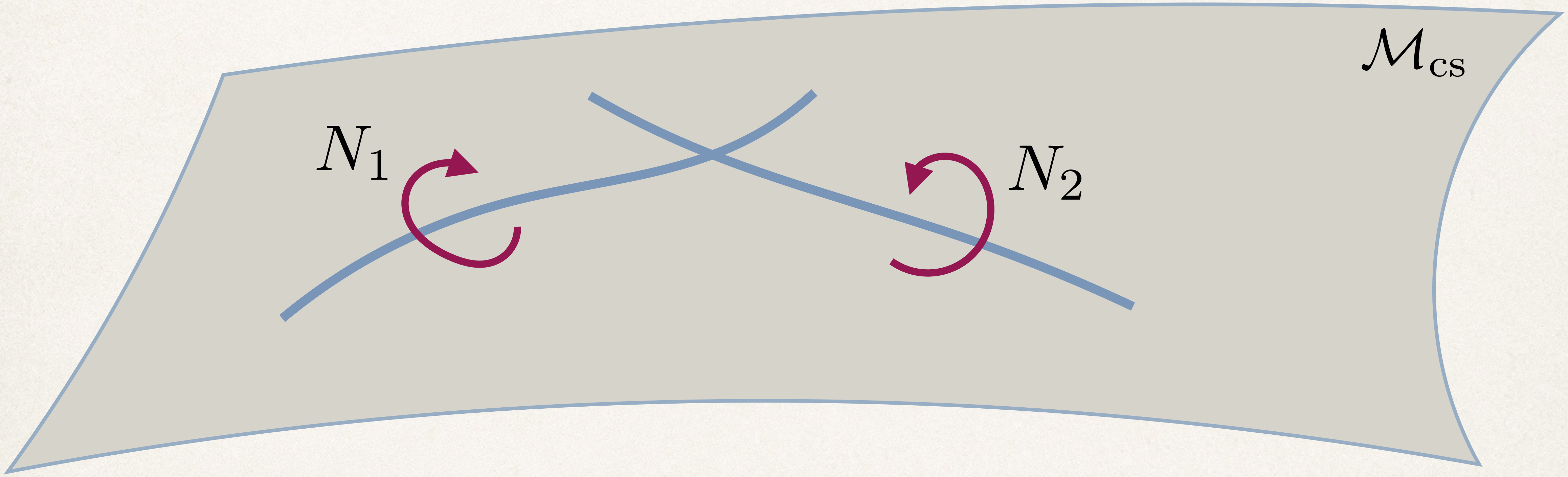
[TG,Li,Zimmermann]

[TG,Li,Valenzuela]

$$\text{Types: } \mathrm{I}_{a,a'},\ \mathrm{II}_{b,b'},\ \mathrm{III}_{c,c'},\ \mathrm{IV}_{d,d'},\ \mathrm{V}_{e,e'}$$

Classification of singularity enhancements

- multi-dimensional moduli spaces:



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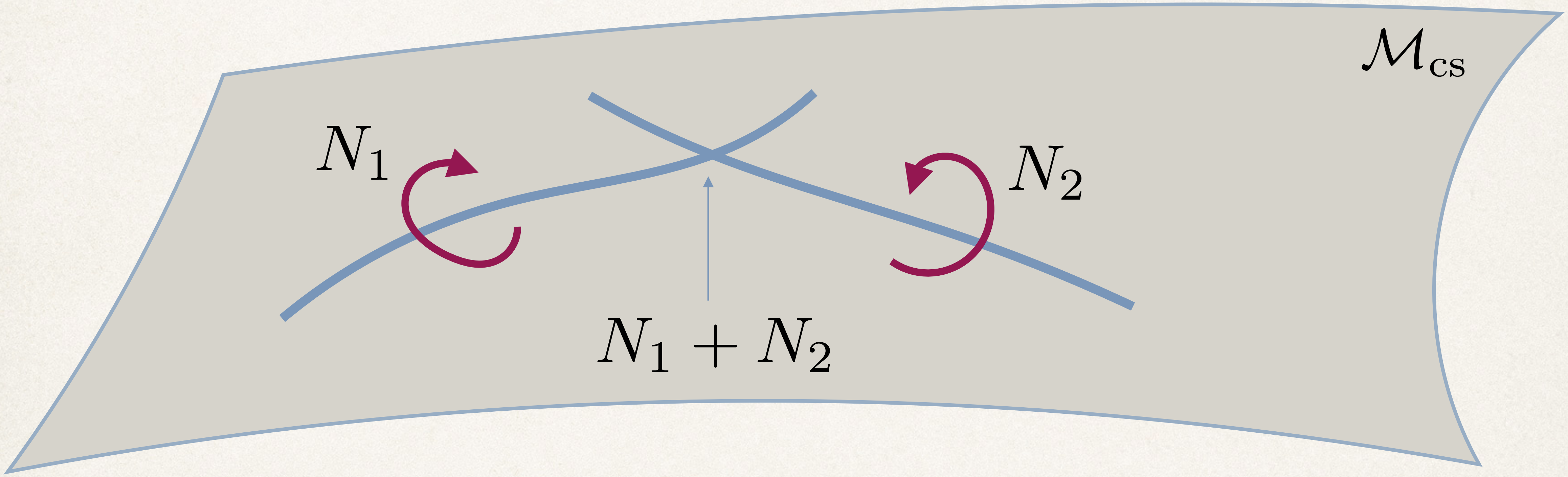

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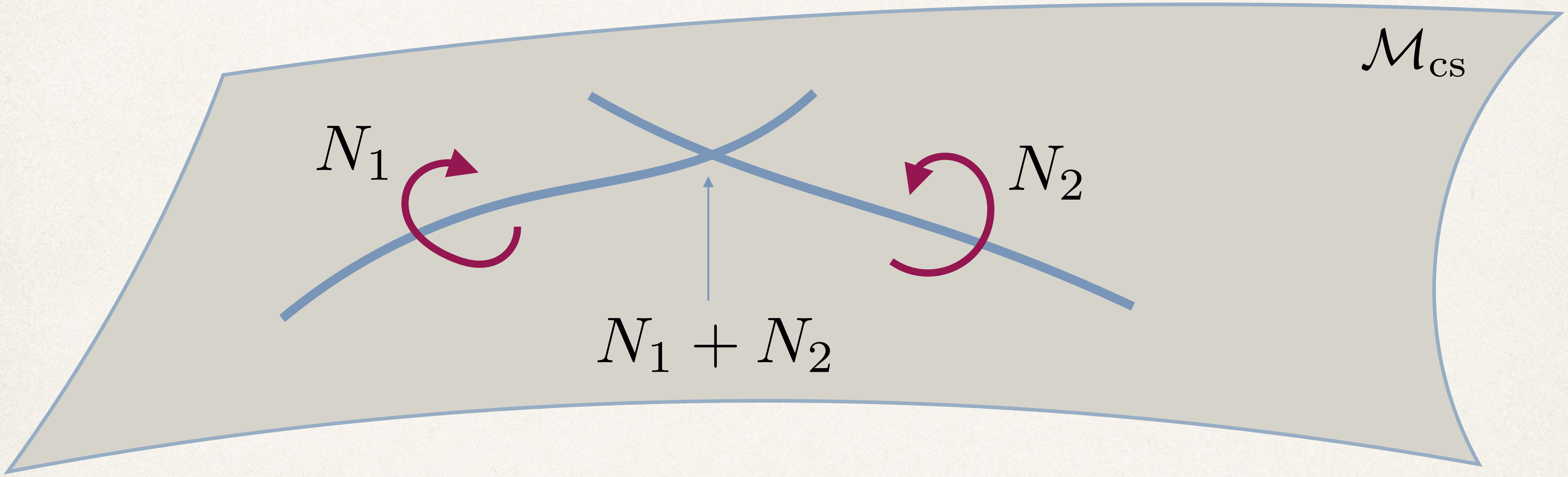
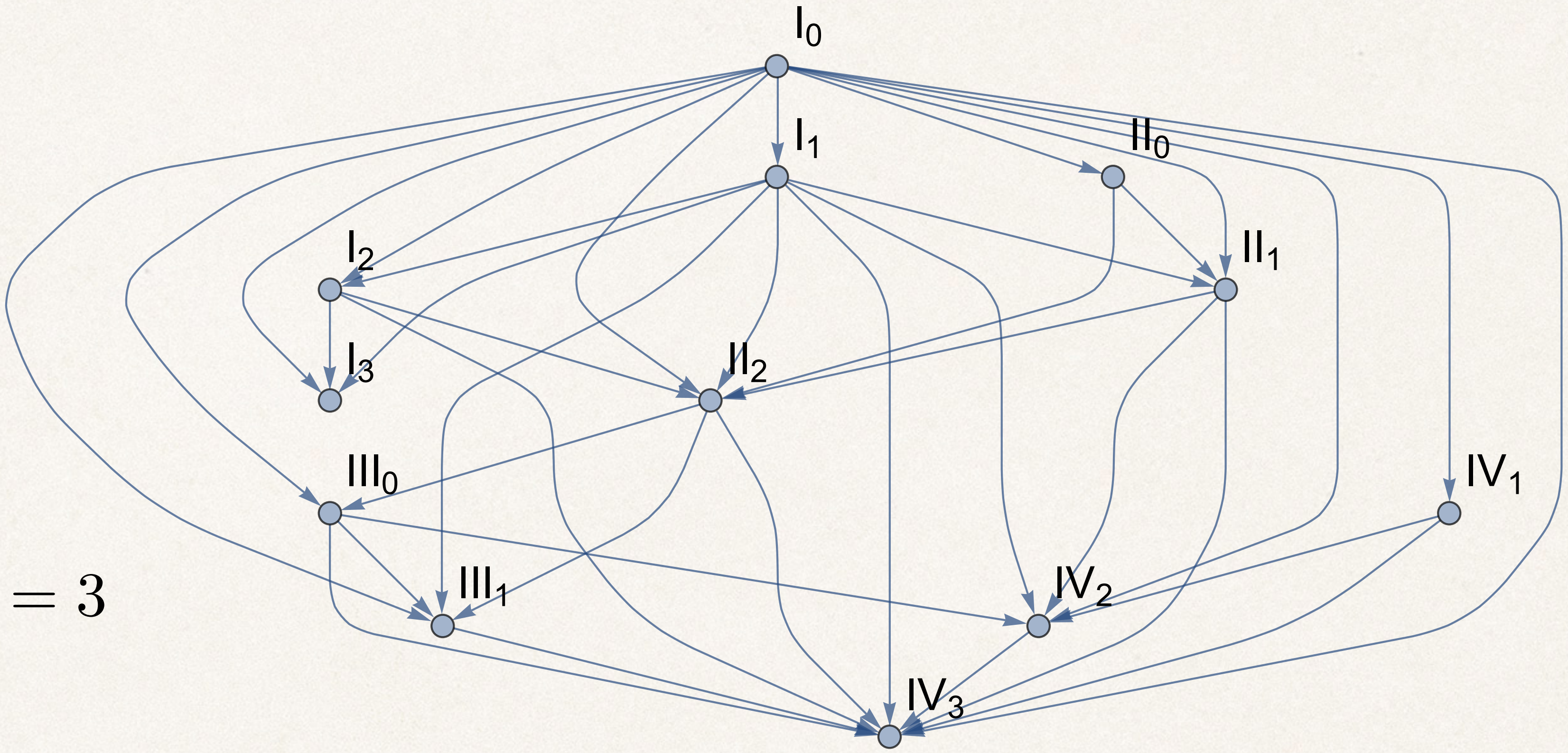


What enhancements are allowed?

- Enhancement rules can be systematically determined:
  - K3, CY$_3$ [Kerr,Pearlstein,Robles]
  - CY$_4$ [TG,Li,Valenzuela], [TG,Li,Zimmermann]

An example: general CY3, with 3 moduli



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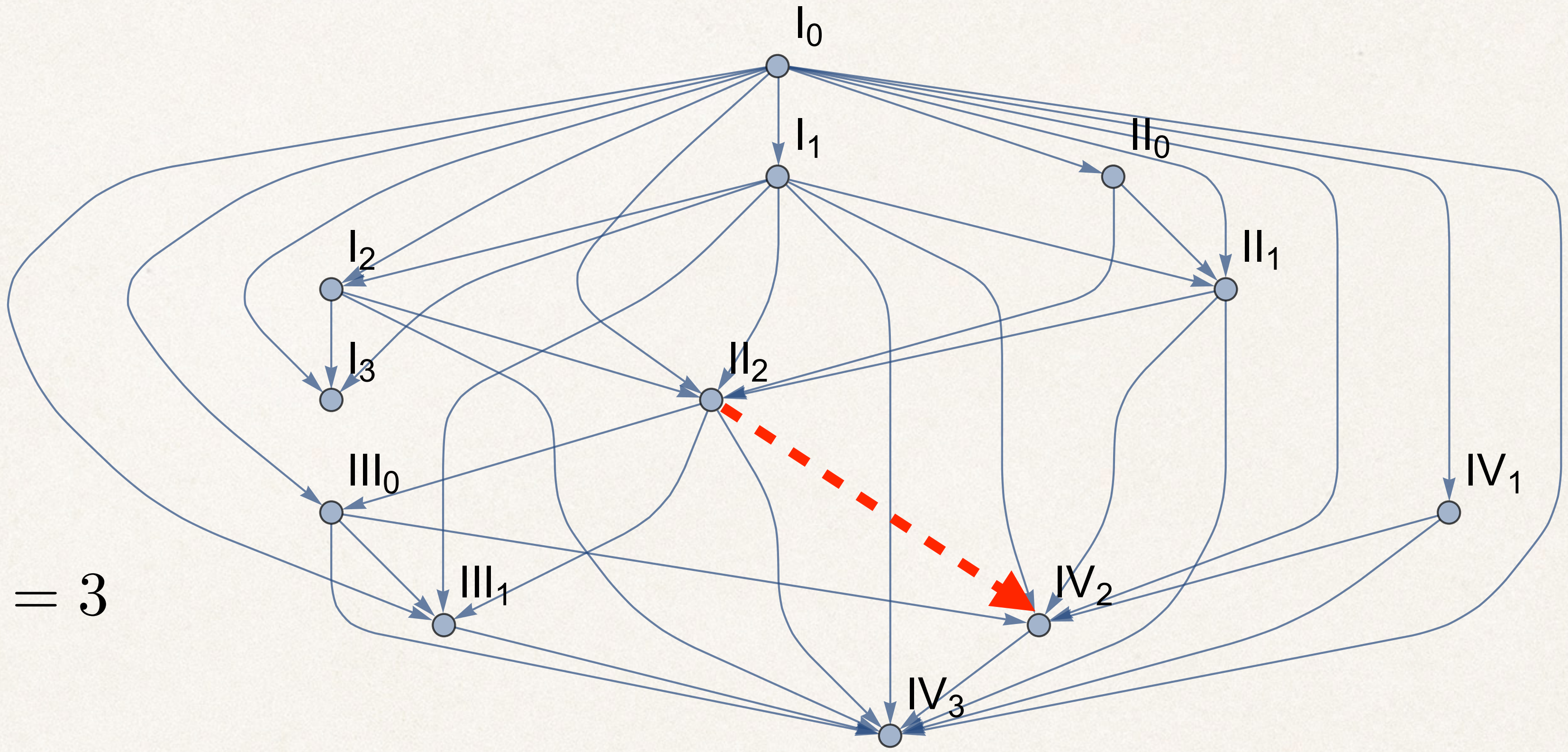

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$\mathrm{II}_2 \rightarrow \mathrm{IV}_2$ not possible

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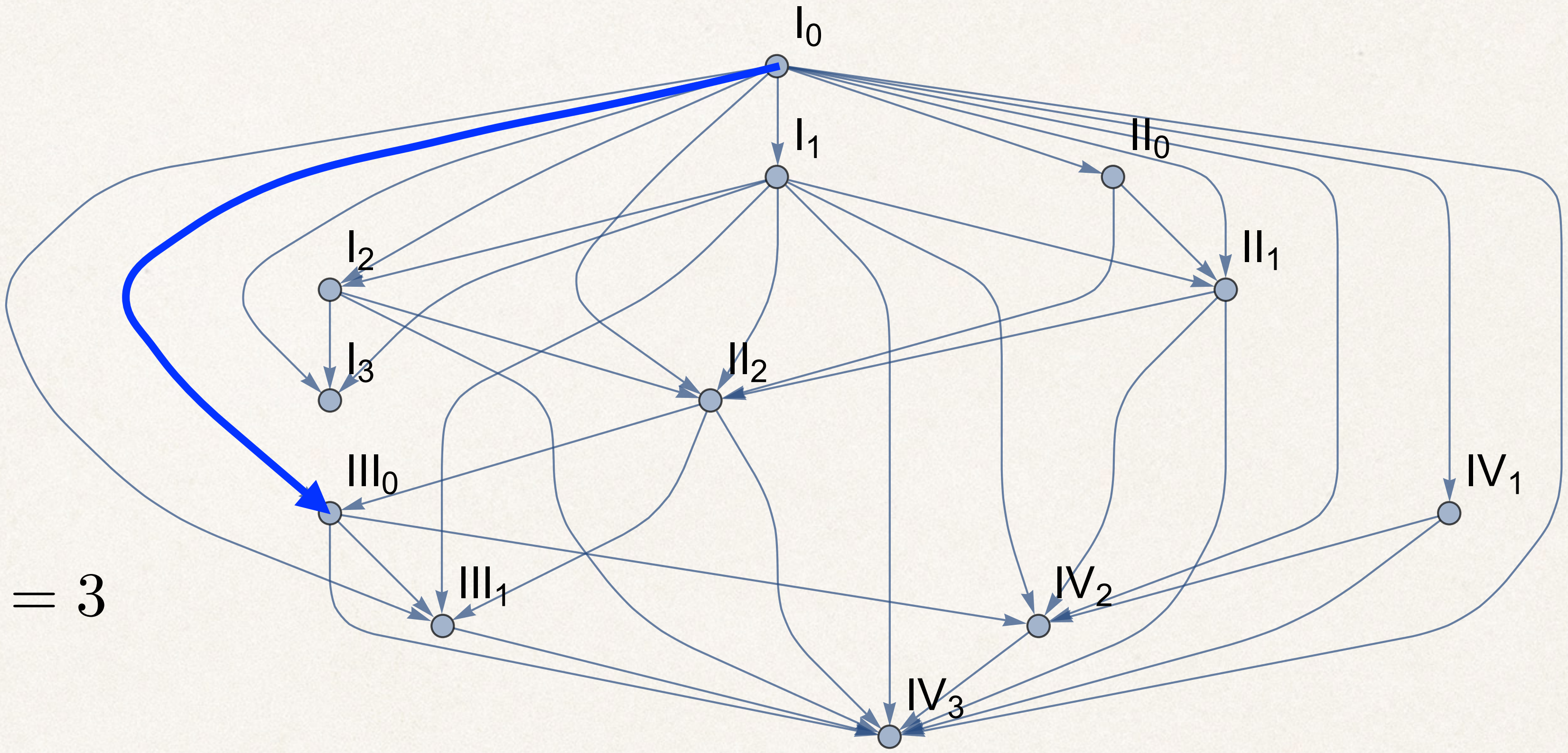

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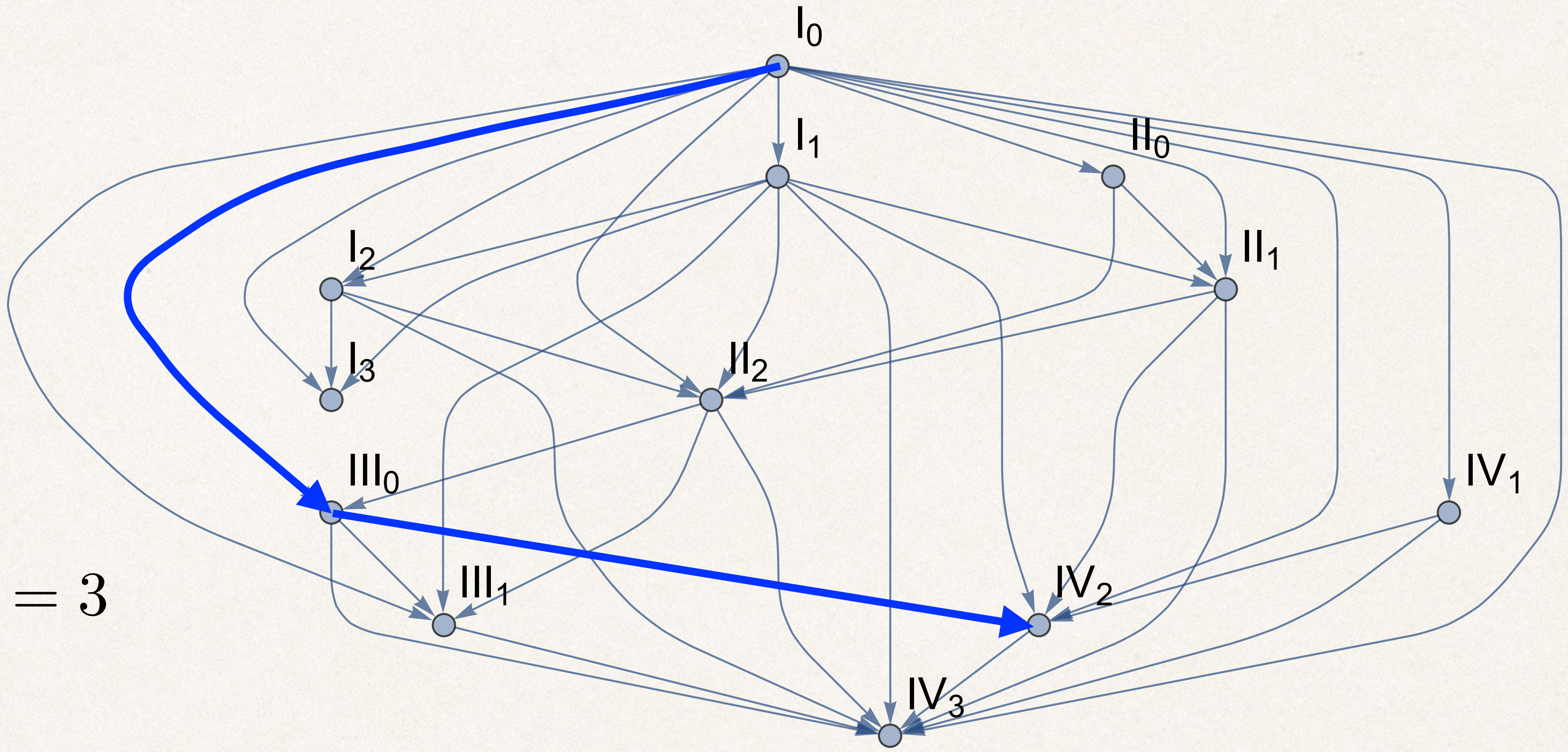

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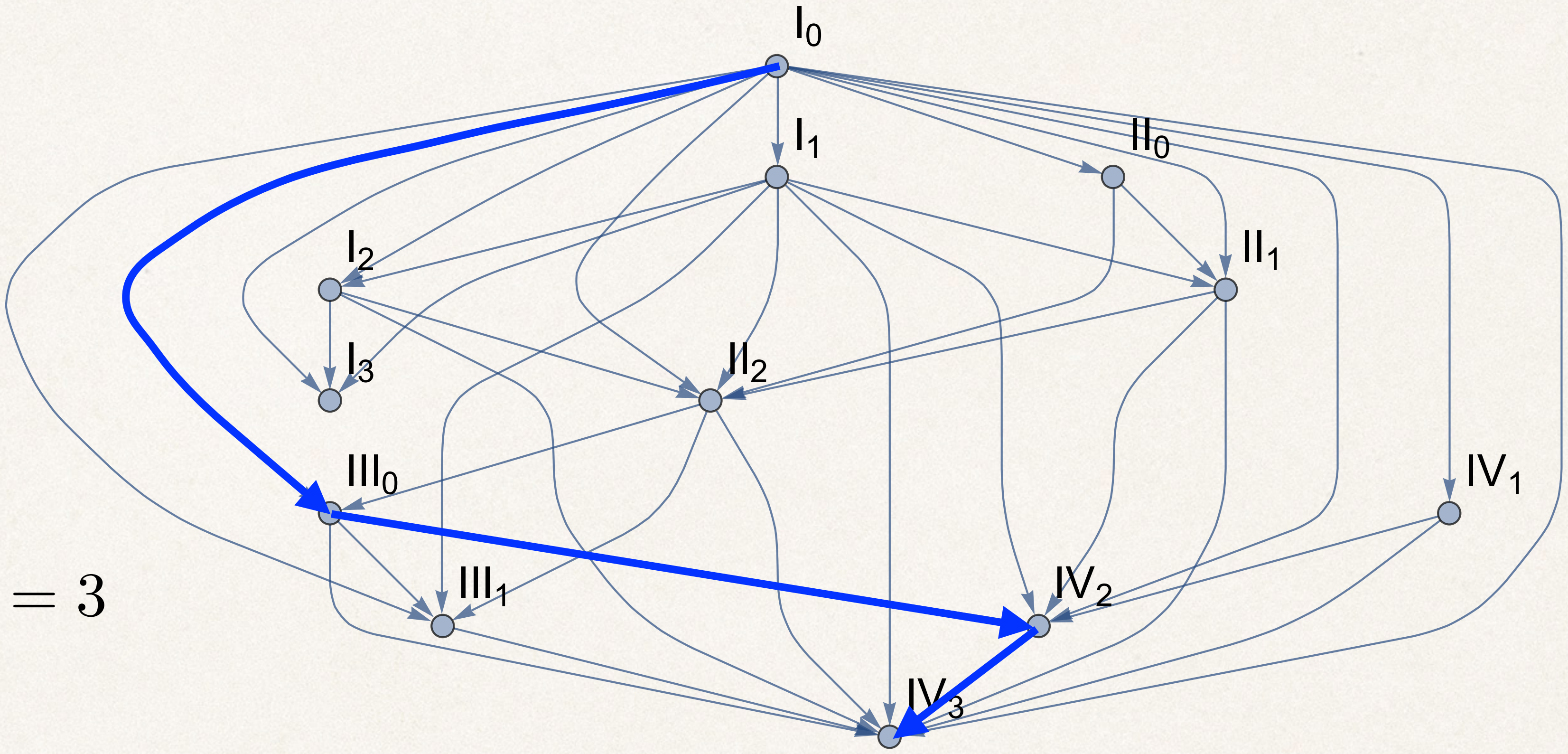

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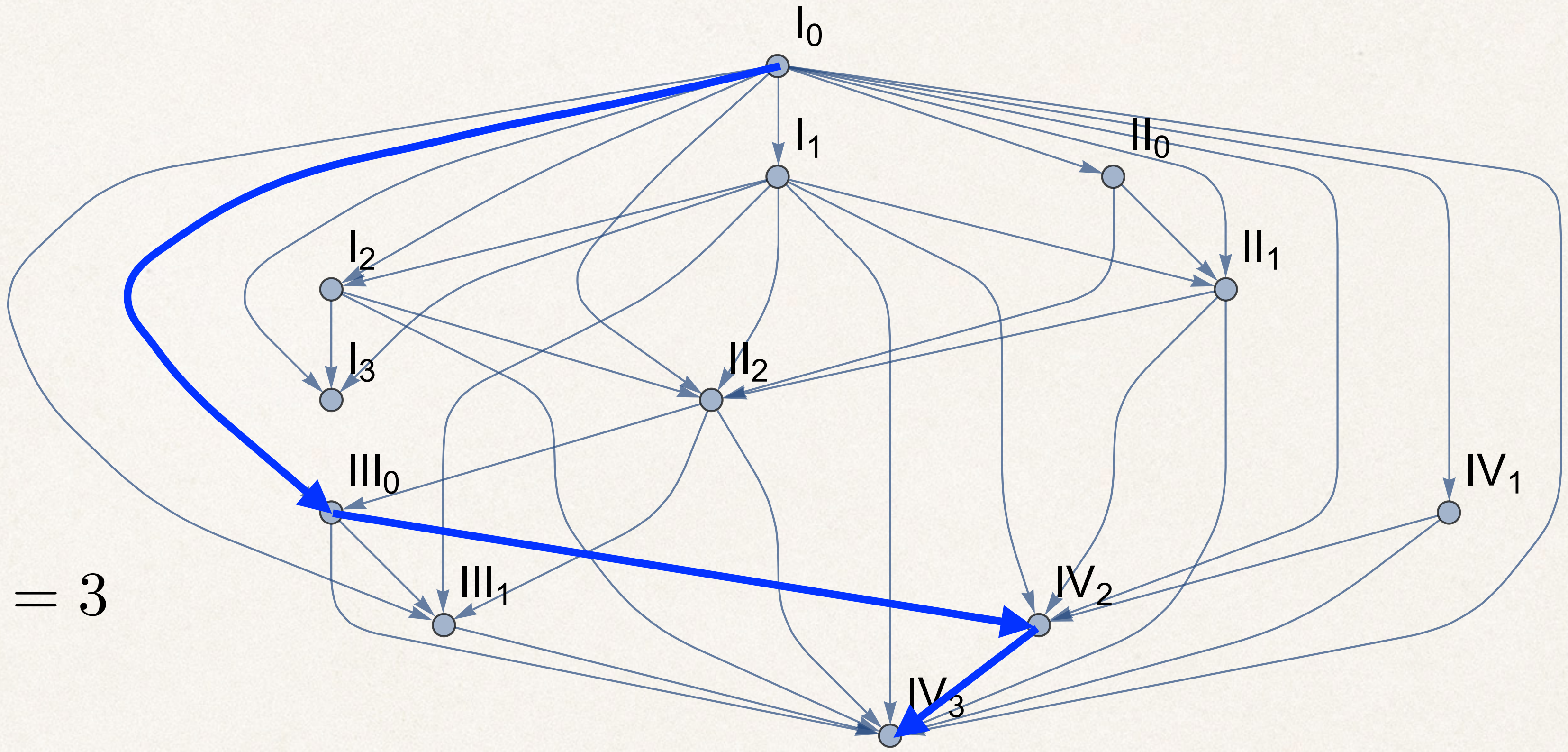

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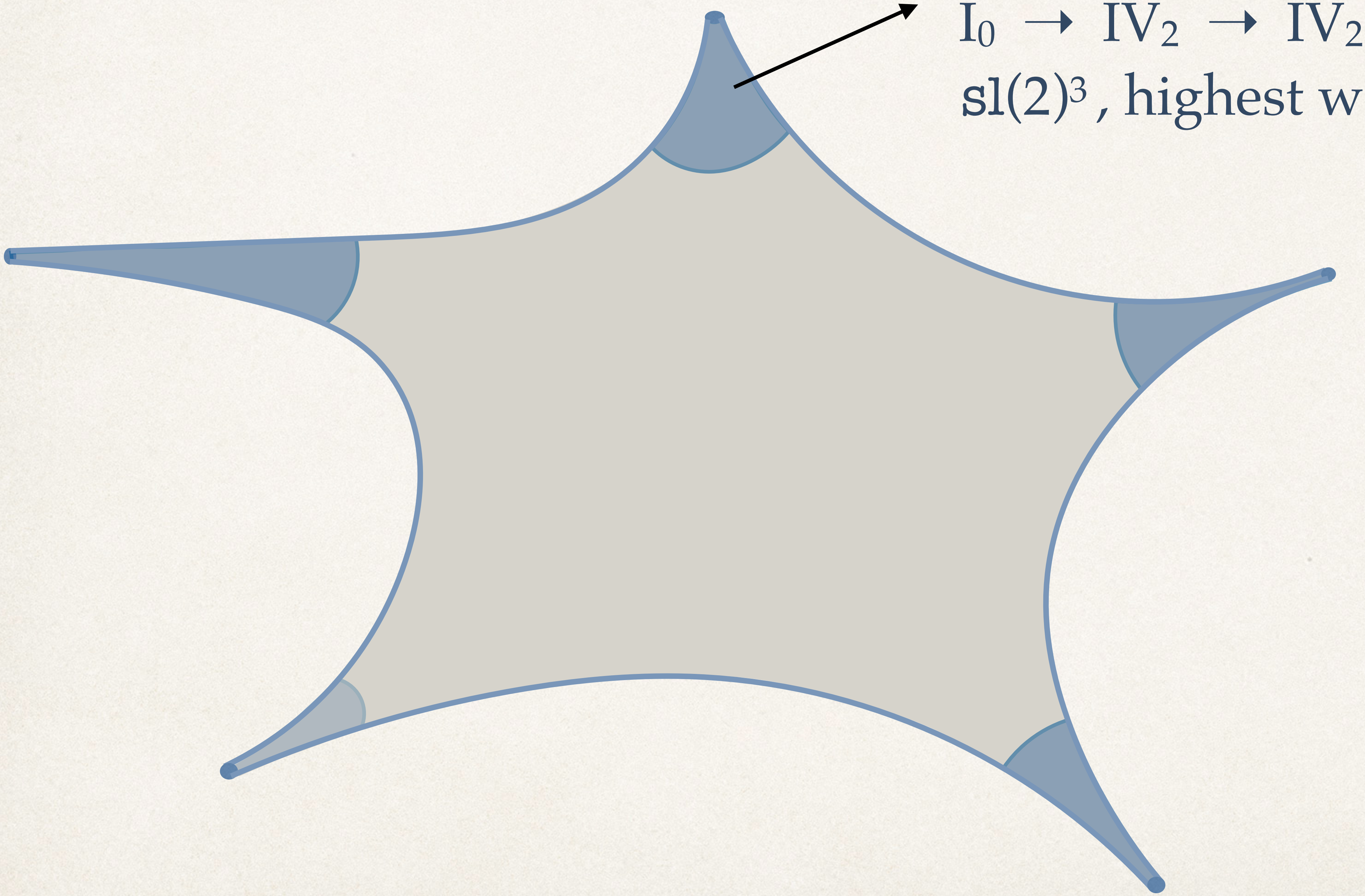
$\mathrm{I}_0 \rightarrow \mathrm{III}_0 \rightarrow \mathrm{IV}_2 \rightarrow \mathrm{IV}_3 \Longrightarrow$ sl(2)^3 - algebra

- each enhancement chain has its associated sl(2)-algebra and highest weight states relevant in the limit → can be computed from $\mathbf{a}_0$, $\mathbf{N}_i$

Limits in Moduli Space



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$\mathfrak{sl}(2)^3$, highest weight states

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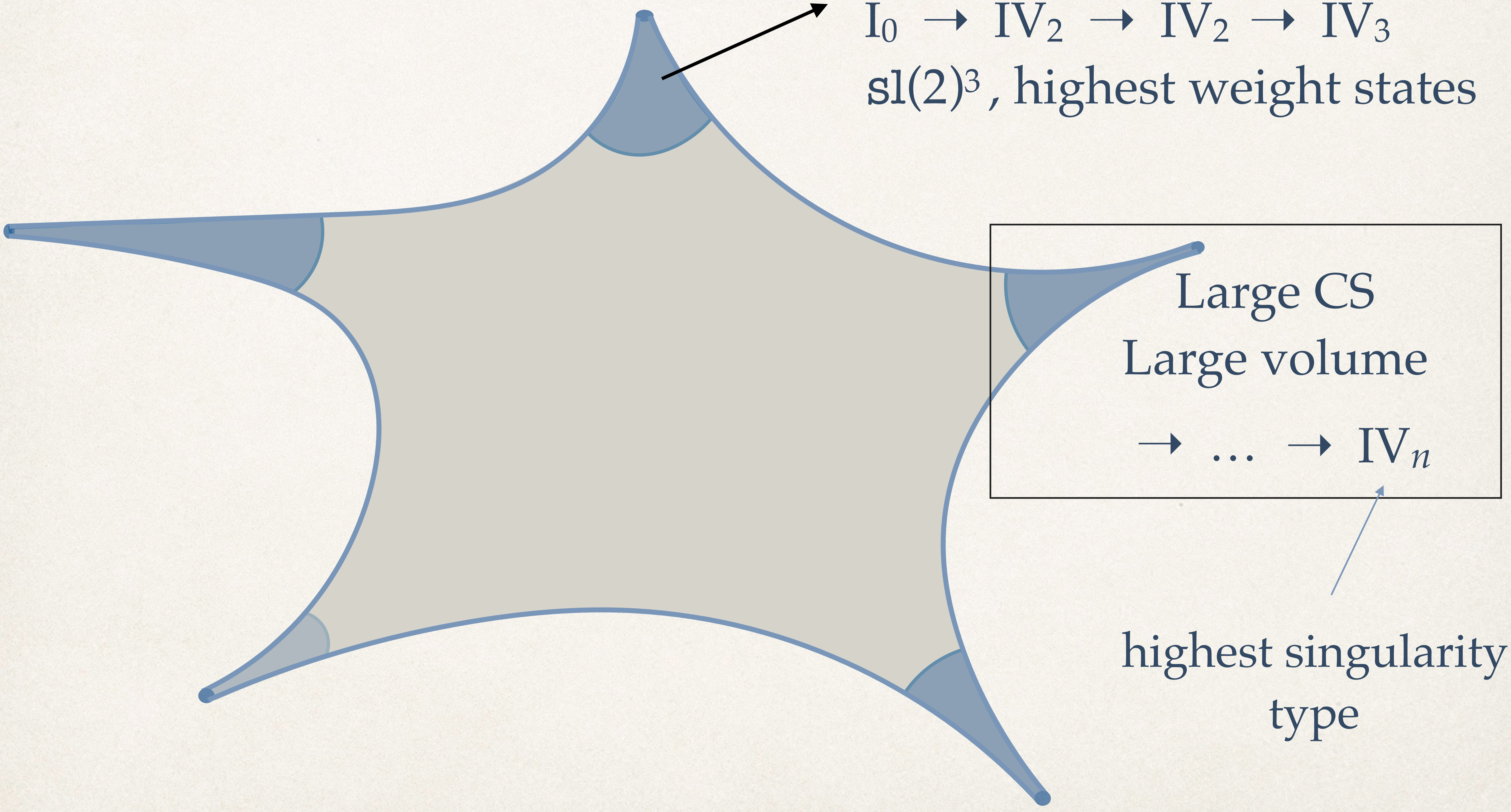

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$\mathfrak{sl}(2)^3$, highest weight states

Large CS

Large volume

$\to \ldots \to \mathrm{IV}_n$

highest singularity type

Classifying Calabi-Yau manifolds

Prospects for classification

- Well-known setting: Large volume compactification
  - Couplings in the effective action are determined by intersection numbers, Chern classes of compact CY space

$$K = -\mathrm{Log}\left(\frac{1}{6}\mathcal{K}_{IJK}v^I v^J v^K + \frac{\zeta(3)\chi}{32\pi^2}\right)$$

⇒ determines also Hodge star in IIA flux potential

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- Can we classify the 'allowed' Calabi-Yau couplings?
  - Wall's theorem: Homotopy types of Calabi-Yau manifolds are classified by the numerical characteristics:

$h^{1,1},\ h^{2,1}$ Hodge numbers

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$\mathcal{K}_{IJK}$	triple intersections
$c_{2\,I}$	second Chern class

→ Hard to classify?!

Classifying Calabi-Yau manifolds

[TG,Li,Palti]
[Corvilain,TG,Valenzuela]

- Monodromies in Kähler moduli spaces (CY$_3$):

$$N_A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ -\delta_{AI} & 0 & 0 & 0 \\ -\frac{1}{2}\mathcal{K}_{AAI} & -\mathcal{K}_{AIJ} & 0 & 0 \\ \frac{1}{6}\mathcal{K}_{AAA} & \frac{1}{2}\mathcal{K}_{AJJ} & -\delta_{AJ} & 0 \end{pmatrix} \qquad \mathbf{a}_0 = \begin{pmatrix} 1 \\ 0 \\ -c_{2\,I} \\ \frac{i\zeta(3)\chi}{8\pi^3} \end{pmatrix}$$

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  $\Rightarrow$ distinguished by: rank(N), rank(N^2), rank(N^3)

see also [Bloch,Kerr,Vanhove]

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- Enhancement rules allow us to rule out non-consistent intersection numbers

Enhancement patters and diagrams

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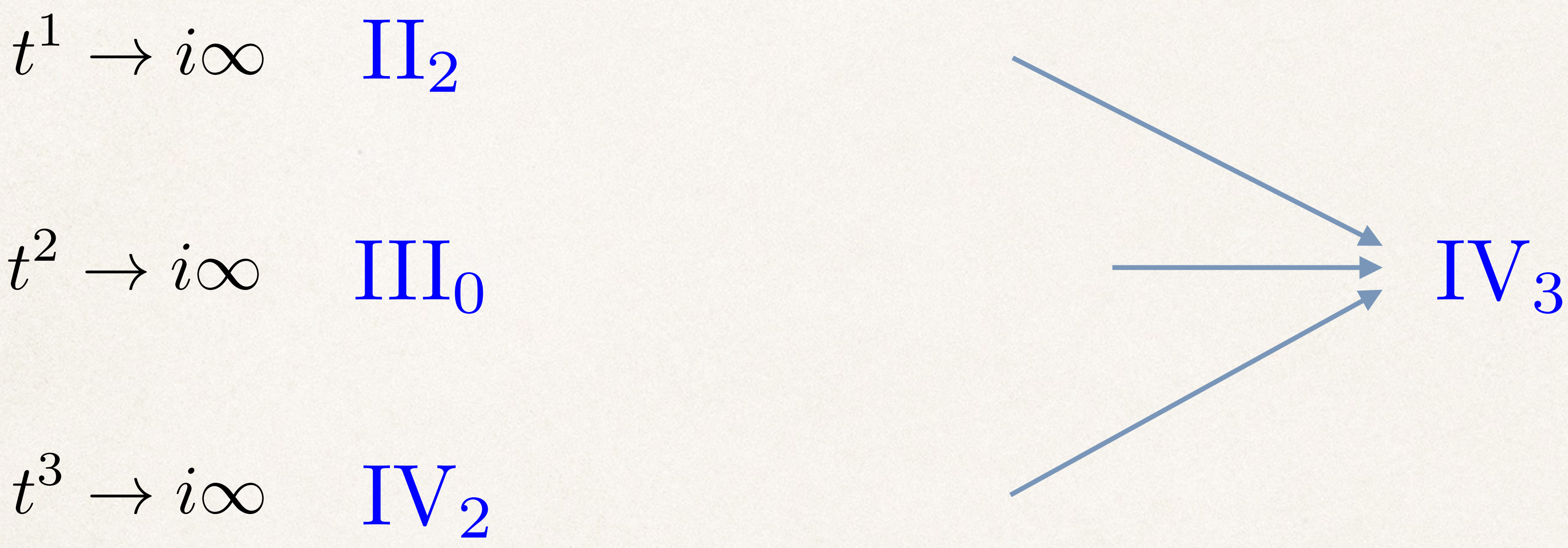

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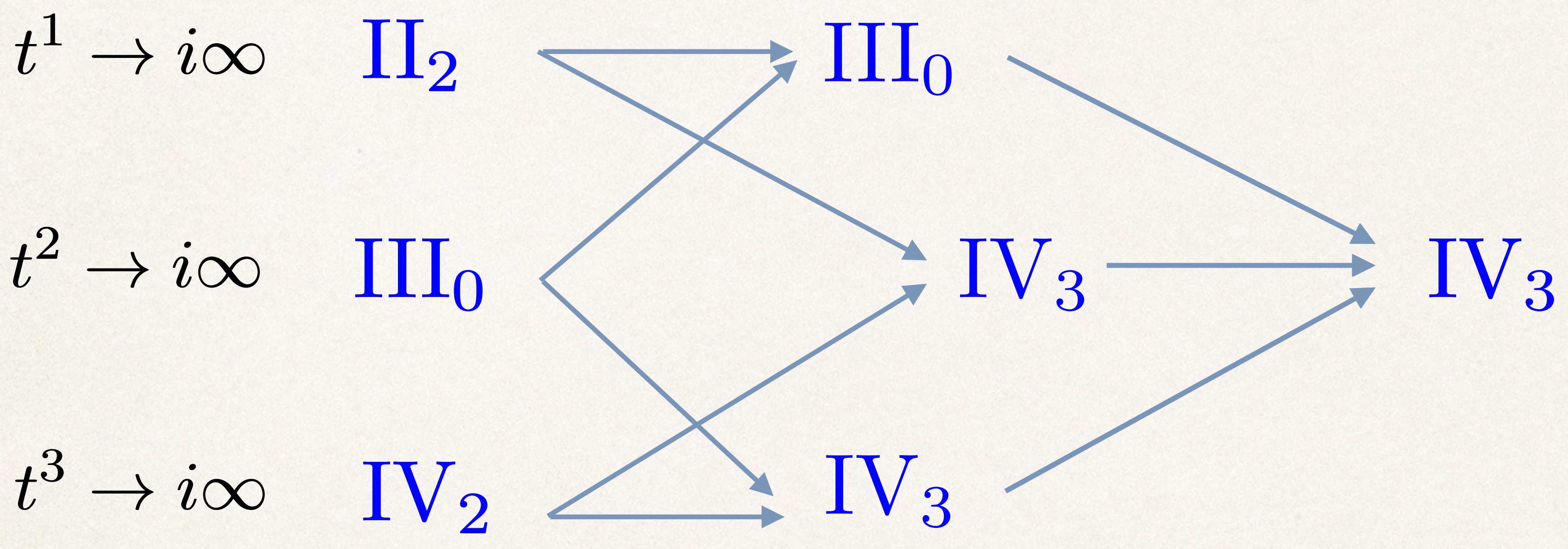

$\Rightarrow$ distinctive enhancement pattern associated to a CY manifold replace by enhancement diagram (Hasse diagrams)

- group examples into equivalence classes
- correlate features of the geometry with the diagram (e.g. count elliptic fibers,…)

[TG,van de Heisteeg,Ruehle]

Asymptotic Flux Compactifications

Reasons for being anti de Sitter

- Consider F-theory with G_4 -flux:

$$V_{\mathrm{M}} = \frac{1}{\mathcal{V}_4^3}\Big(\boxed{\int_{Y_4} G_4 \wedge * G_4} - \int_{Y_4} G_4 \wedge G_4\Big)$$

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$$\boxed{\int_{Y_4} G_4 \wedge * G_4 \sim \sum_{l_1,\dots,l_n} (s^1)^{l_1-n}(s^2)^{l_2-l_1}\dots(s^n)^{l_{n-1}-l_n}\|\rho_{l_1\dots l_n}\|_\infty}$$

$$\rho_{l_1\dots l_n} = e^{\phi^i N_i} G_4|_{V_{l_1\dots l_n}}$$

→ related expression:[Herraez,Ibanez, Marchesano,Zoccarato] [Marchesano,Quirant]

- **Examples:** 2 moduli

$$\mathrm{Im}t^1 = s,\ \mathrm{Im}t^2 = u$$

The complete list of scalar potentials that are geometrically possible for any CY$_4$ at any strict asymptotic limit.

Enhancements	Potential V_{M}
$\begin{matrix}\mathrm{I}_{0,\hat{m}-2}\\ \mathrm{V}_{1,\hat{m}}\end{matrix}\rightarrow\mathrm{V}_{1,\hat{m}-2}$	$\frac{c_1}{s}+\frac{c_2}{u^4}+\frac{c_3}{u^2}+c_4u^2+c_5u^4+c_6s-c_0$
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$\begin{matrix}\mathrm{I}_{1,\hat{m}}\\ \mathrm{V}_{1,\hat{m}}\end{matrix}\rightarrow\mathrm{V}_{2,\hat{m}}$	$\frac{c_1}{s^2}+\frac{c_2}{u^4}+\frac{c_3}{u^2}+c_4u^2+c_5u^4+c_6s^2-c_0$
$\begin{matrix}\mathrm{II}_{0,\hat{m}-2}\\ \mathrm{IV}_{0,\hat{m}-2}\end{matrix}\rightarrow\mathrm{V}_{2,\hat{m}}$	$\frac{c_1}{u^3s}+\frac{c_2}{us}+\frac{c_3u}{s}+\frac{c_4u^3}{s}+\frac{c_5s}{u^3}+\frac{c_6s}{u}+c_7us+c_8u^3s-c_0$
$\begin{matrix}\mathrm{I}_{0,\hat{m}-2}\\ \mathrm{I}_{0,\hat{m}-4}\end{matrix}\xrightarrow{a}\mathrm{I}_{1,\hat{m}-2}$	$\frac{c_1}{us}+\frac{c_2}{u}+\frac{c_3u}{s}+\frac{c_4s}{u}+c_5u+c_6us-c_0$
$\begin{matrix}\mathrm{I}_{0,\hat{m}-2}\\ \mathrm{II}_{0,\hat{m}-2}\end{matrix}\rightarrow\mathrm{II}_{1,\hat{m}}$	
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- Minkowski ‘no-scale’ vacua from self-dual fluxes

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flux ratio

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- Show generally for all Calabi-Yau fourfolds: number of self-dual flux vacua is finite [TG, in progress]

$\Rightarrow$ key point: control the asymptotic regimes $\int G_4 \wedge G_4 = \int G_4 \wedge *G_4 < K$

de Sitter vacua

de Sitter vacua

- No de Sitter vacua at parametric control ⟶ Irene's talk

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Asymptotically massless fluxes

- Keeping coefficients unrelated: list contains the well-known IIA potential [TG,Louis]

$$V_{\mathrm{IIA}} \propto \frac{1}{s^3}\left[\frac{c_1}{u^3 s} + \frac{c_2}{us} + \frac{c_3 u}{s} + \frac{c_4 u^3}{s} + \frac{c_5 s}{u^3} + \frac{c_6 s}{u} + c_7 us + c_8 u^3 s - c_0\right]$$

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- infinite family of AdS$_4$ vacua at parametric control

What is the generalization?

[DeWolfe,Giryavets,Kachru,Taylor]

Asymptotically massless fluxes

- special flux $\hat{G}_4$ that mildly violates the self-duality constraint:

$$G_4 = \hat{G}_4 + G_4^0$$

asymptotically massless: $\|\hat{G}_4\|^2 \to 0$ in the asymptotic limit

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- unbounded, asymptotically massless fluxes are necessary to have infinite family AdS$_4$ vacua at parametric control

$\Rightarrow$ can be classified using limiting mixed Hodge structures

$\Rightarrow$ reminiscent of the constructions for the Distance Conjecture at **infinite distance** singularities [TG,Palti,Valenzuela] [TG,Li,Palti]

Note: models appear to be in conflict with AdS conjecture of [Lüst,Palti,Vafa]

Conclusions

- Motivated by the Swampland Conjectures we uncovered a universal structure emerging in the asymptotic regimes of geometric moduli spaces

⇒ limits characterized by sl(2)^n and its representations

⇒ asymptotic of Hodge norm

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$\Rightarrow$ diagrams representing classes of 'allowed' manifolds

- Asymptotic flux compactification: general analysis of flux vacua at all limits in Calabi-Yau fourfolds

$\Rightarrow$ no de Sitter, finitely many Mink., parametrically controlled AdS (?)
remarkable constraints on axions

Geometry and Strings

28.4. - 1.5. 2020

Organized in Utrecht