On infrared physics and the celestial sphere

Andrea Puhm

X-mas workshop @ IFT Madrid, 11-13 December 2019

The spacetime of the universe

Maximally symmetric spacetimes relevant for our universe:

- $\Lambda > 0$ **de Sitter**
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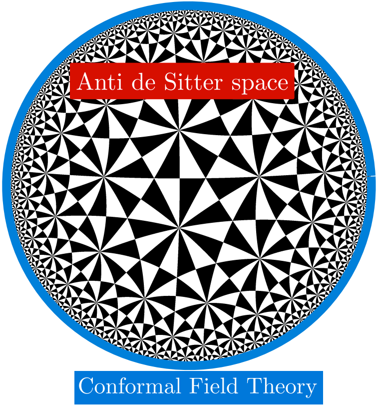
- de Sitter: Identification of symmetries of dS with proposed dual conformal structure at the boundary in the future ⇒ dS/CFT with time emergent. Positive cosmological constant: no box.

- Minkowski: *What are the symmetries? Is there a dual CFT? Where does it live? What are its properties? ...*

Holographic screen of AdS spacetime

- Ground-breaking tool: **AdS/CFT correspondence** (holography)

[Maldacena'97]



Quantum gravity in Anti de Sitter space
=
Conformal Field Theory on boundary

Black hole information in AdS

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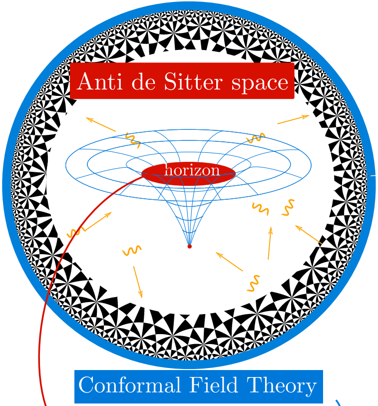
[Strominger,Vafa'96; Strominger'97]



$$S = \frac{A/\ell_P^2}{4} \quad = \quad S = \log N$$

perfect match!

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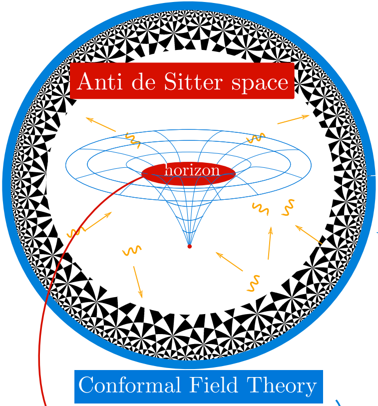

Information encoded at boundary of Anti de Sitter space!

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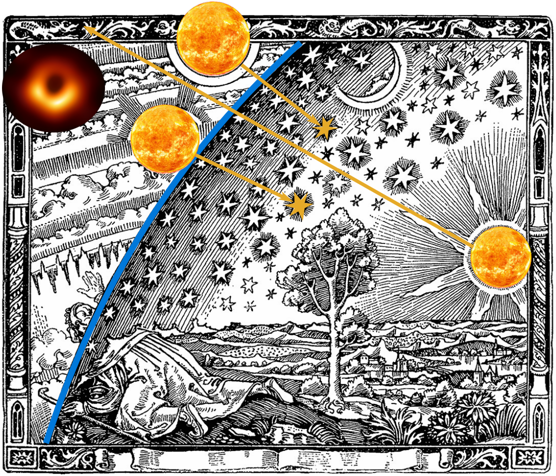
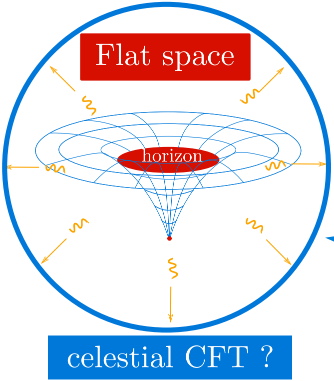
Holographic screen of Minkowski spacetime

- Novel development: **flat space holography?**

4D spacetime encoded on 2D celestial sphere?



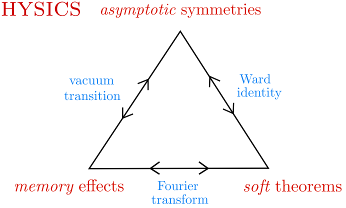
Black hole information in Minkowski



Information encoded at boundary of flat space = celestial sphere ?

Outline

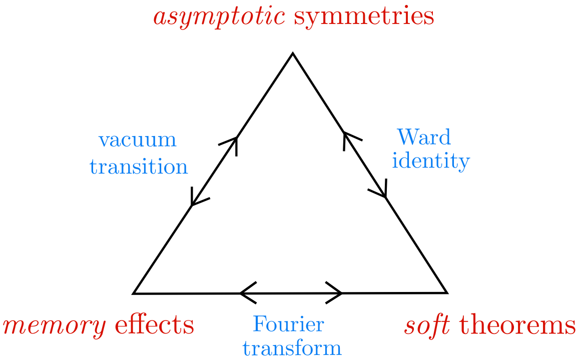
I. Infrared physics



II. Celestial sphere: flat space holography?

- 4D S-matrix $\leftrightarrow$ 2D correlator
- 4D spacetime symmetries $\leftrightarrow$ 2D conformal soft primaries
- 4D memory effects $\leftrightarrow$ 2D conformal memory primaries
- 4D soft theorems $\leftrightarrow$ 2D conformal soft theorems
- 4D collinear singularities $\leftrightarrow$ 2D OPE coefficients

I. Infrared physics



Causal structure of Minkowski spacetime

Natural holographic screen: conformal boundary of Minkowski space

null infinity $\mathcal{I} = \mathcal{C}S^2 \times \mathbb{R}$
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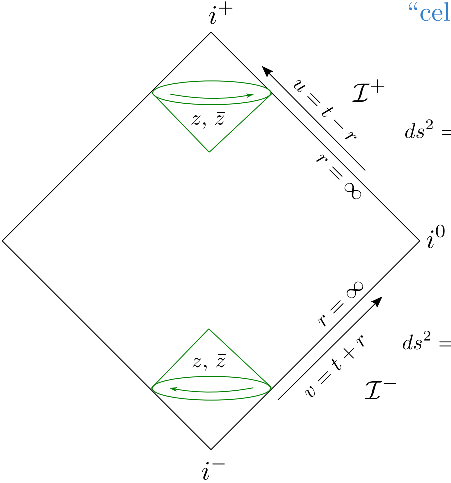

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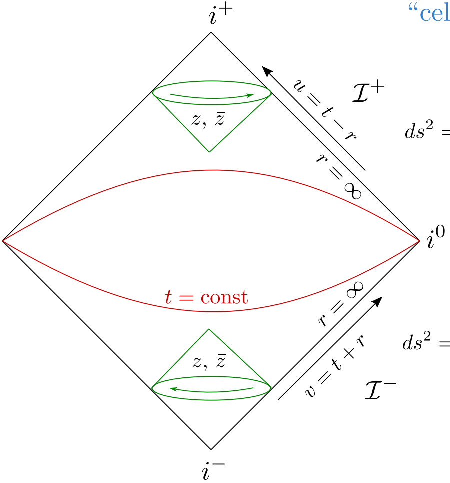

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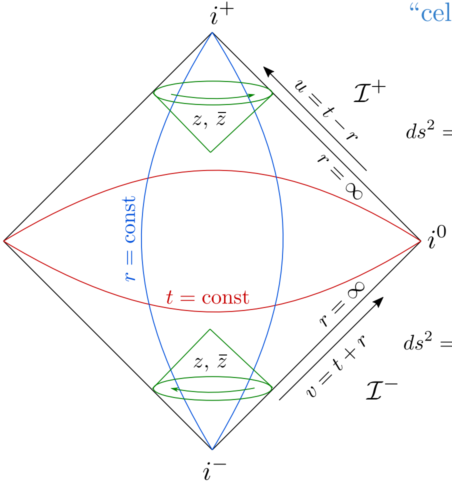

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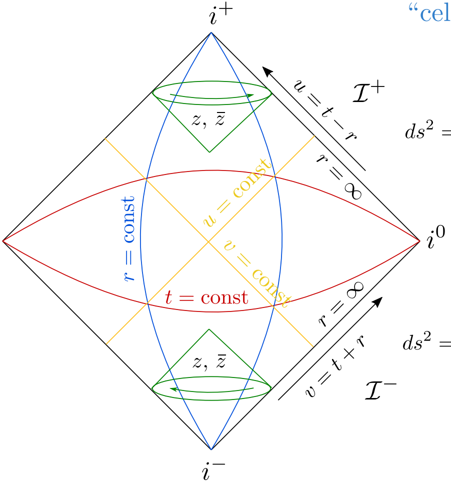

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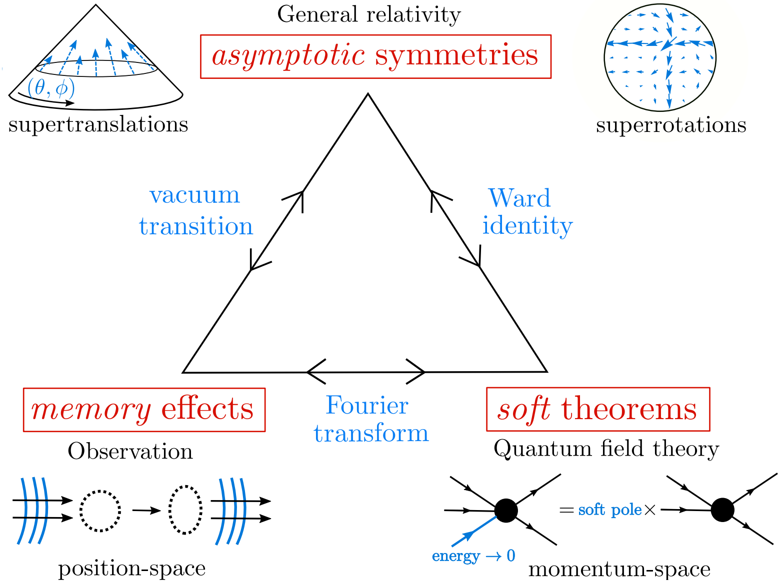
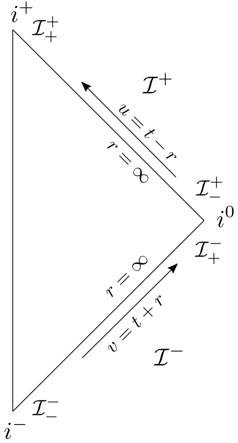
The infrared triangle



Asymptotic symmetries

$$\text{Asymptotic Symmetry Group} = \frac{\text{allowed gauge symmetry}}{\text{trivial gauge symmetry}}$$



- choose boundary conditions
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- find residual gauge/diffeos respecting boundary conditions

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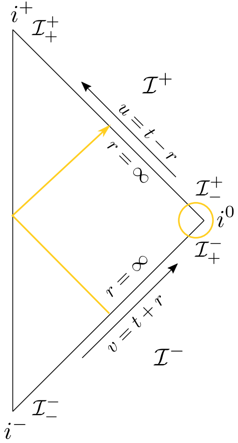

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antipodal matching as $\to i^0$
$\Rightarrow \infty$ of charge conservation laws

[Strominger'13]

Asymptotic symmetries

Large gauge symmetry: $\delta_\varepsilon A_\mu = \partial_\mu \varepsilon$ with $\varepsilon = \varepsilon(z, \bar{z})$

$Q_\varepsilon^+ = \frac{1}{e^2} \int_{\mathcal{I}_-^+} \varepsilon * F = \frac{1}{e^2} \int_{\mathcal{I}_+^-} \varepsilon * F = Q_\varepsilon^-$

$\rightarrow$ Electric charge conservation

Asymptotically flat space

$$ds^2 = -\,du^2 + 2\,du dr + 2r^2\gamma_{z\bar{z}}dz d\bar{z} \qquad \text{... flat space}$$

Asymptotically flat space

[Bondi,van der Burg,Metzner,Sachs'62]

$$
\begin{aligned}
ds^2 =& - du^2 + 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} \qquad\qquad \text{... flat space} \\
&+ \frac{2m_B}{r}du^2 + rC_{zz}dz^2 + D^z C_{zz}dudz + c.c + \\
&+ \frac{1}{r}\left(\frac{4}{3}\left(N_z + u\partial_z m_B\right) - \frac{1}{4}\partial_z(C_{zz}C^{zz})\right)dudz + c.c + \dots
\end{aligned}
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- m_B...Bondi mass aspect
- C_{zz}...free gravitational data
- N_z...angular momentum aspect

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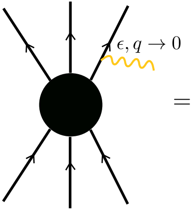
[deBoer,Solodukhin][Banks][Barnich,Troessart]

Superrotations: $\zeta_Y = Y^z \partial_z + \frac{u}{2} D_z Y^z \partial_u + c.c.$ with $Y^z = Y^z(z, \bar{z})$

$Q_Y^+ = \frac{1}{8\pi G} \int_{\mathcal{I}_-^+} d^2 z (Y_z N_{\bar{z}} + Y_{\bar{z}} N_z) = \frac{1}{8\pi G} \int_{\mathcal{I}_+^-} d^2 z (Y_z N_{\bar{z}} + Y_{\bar{z}} N_z) = Q_Y^-$

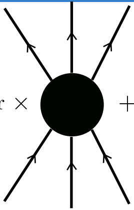
$\rightarrow$ Angular momentum conservation

Soft theorems



$\langle out|a_\pm(\vec{q})\mathcal{S}|in\rangle$

$=$ soft factor $\times$



$+\ \mathcal{O}(q^0)$

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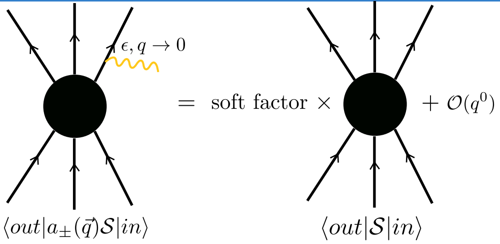
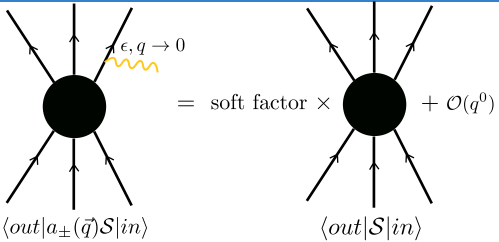


QED:

$$S_0^{\pm} = e \sum_k \frac{\epsilon_\mu^{\pm} p_k^\mu Q_k}{p_k \cdot q}$$

[Bloch,Nordsiek][Low] ...

Soft theorems



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[Weinberg]

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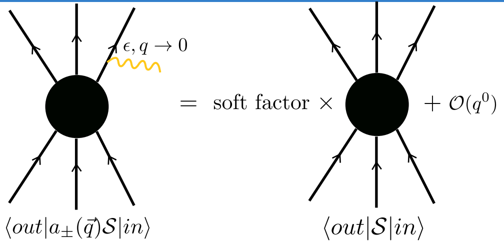

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[Cachazo,Strominger]

Soft theorems ⇔ Asymptotic symmetries

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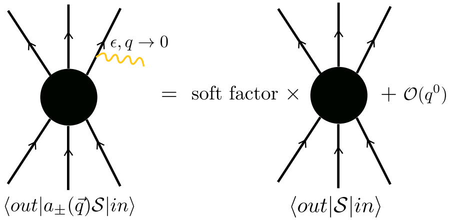

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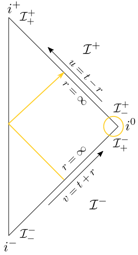

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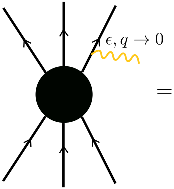
Soft theorems $\Leftrightarrow$ Asymptotic symmetries

Ward ID $\longleftarrow$

soft theorems



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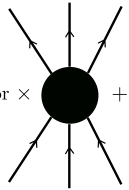

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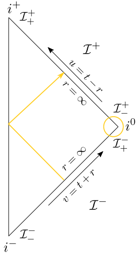

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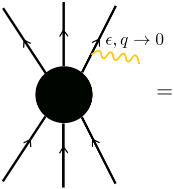

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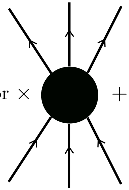

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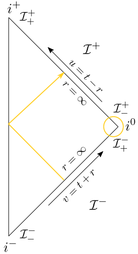

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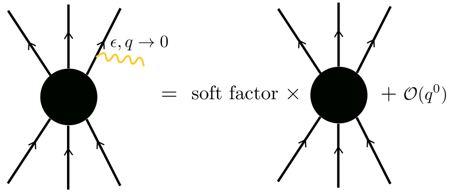

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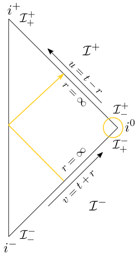

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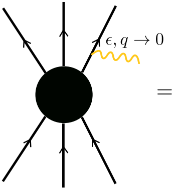

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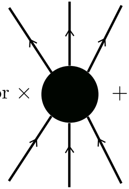

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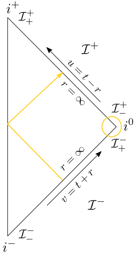


QED:

$$S_0^\pm = e\sum_k \frac{\epsilon^\pm_\mu\, p_k^\mu\, Q_k}{p_k\cdot q}$$

$Q^+_\varepsilon\mathcal{S} - \mathcal{S}\,Q^-_\varepsilon = 0$ $\longleftarrow$

large gauge symmetry:

$$\delta_\varepsilon A_\mu = \partial_\mu \varepsilon$$

Gravity:

$$S_0^\pm = \sqrt{8\pi G}\sum_k \frac{\epsilon^\pm_{\mu\nu}\, p_k^\mu\, p_k^\nu}{p_k\cdot q}$$

$Q^+_f\mathcal{S} - \mathcal{S}\,Q^-_f = 0$ $\longleftarrow$

supertranslations:

$$\xi_f = f\partial_u$$

$$S_1^\pm = -i\sqrt{8\pi G}\sum_k \frac{\epsilon^\pm_{\mu\nu}\, p_k^\mu\, q^\lambda\, J^\nu_{k\,\lambda}}{p_k\cdot q}$$

$Q^+_Y\mathcal{S} - \mathcal{S}\,Q^-_Y = 0$ $\longleftarrow$

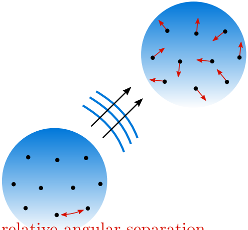
superrotations:

$$\zeta_Y = Y^z\partial_z + \tfrac{u}{2}D_z Y^z \partial_u + c.c.$$

Memory effects

Asymptotically flat space: [Bondi,van der Burg,Metzner,Sachs'62]

$$\begin{aligned} ds^2 = &- du^2 + 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} \\ &+ \frac{2m_B}{r}du^2 + rC_{zz}dz^2 + D^z C_{zz}dudz + c.c + \\ &+ \frac{1}{r}\left(\frac{4}{3}\left(N_z + u\partial_z m_B\right) - \frac{1}{4}\partial_z(C_{zz}C^{zz})\right)dudz + c.c + \dots \end{aligned}$$



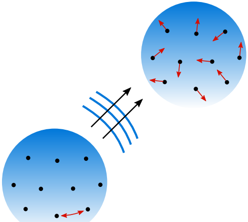
relative angular separation
$s(z,\bar{z})$ of detectors

Memory effects

Asymptotically flat space: [Bondi,van der Burg,Metzner,Sachs'62]

$$\begin{aligned} ds^2 = &- du^2 + 2du dr + 2r^2\gamma_{z\bar z} dz d\bar z \\ &+ \frac{2m_B}{r} du^2 + rC_{zz}dz^2 + D^z C_{zz} du dz + c.c + \\ &+ \frac{1}{r}\left(\frac{4}{3}\left(N_z + u\partial_z m_B\right) - \frac{1}{4}\partial_z(C_{zz}C^{zz})\right) du dz + c.c + \dots \end{aligned}$$

geodesic deviation

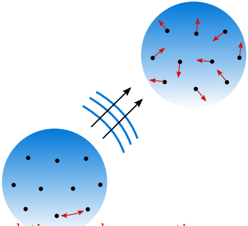


$$\begin{aligned} r^2\gamma_{z\bar z}\partial_u^2 s^{\bar z} &= - R_{uzuz}s^z \\ &= \frac{r}{2}\partial_u^2 C_{zz} s^z \end{aligned}$$

Memory effects

Asymptotically flat space: [Bondi,van der Burg,Metzner,Sachs'62]

$$
\begin{aligned}
ds^2 =& - du^2 + 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} \\
&+ \frac{2m_B}{r}du^2 + rC_{zz}dz^2 + D^z C_{zz}dudz + c.c + \\
&+ \frac{1}{r}\left(\frac{4}{3}\left(N_z + u\partial_z m_B\right) - \frac{1}{4}\partial_z(C_{zz}C^{zz})\right)dudz + c.c + \ldots
\end{aligned}
$$



relative angular separation $s(z,\bar{z})$ of detectors

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=& \frac{r}{2}\partial_u^2 C_{zz}s^z
\end{aligned}
$$

gravitational memory effect

$$
\Delta s^{\bar{z}} = \frac{\gamma^{z\bar{z}}}{2r}\Delta C_{zz}s^z
$$

[Zel'dovich,Polnarev'74][Braginski,Thorne'87][Christodoulou'91] [Blanchet,Damour'92] ...

Soft theorems ⇔ Memory effects

soft theorems

elementary particle collisions

[Weinberg'65]

The dominance of the $1/(p\cdot q)$ pole in (2.5) implies that the effect of attaching one soft-graviton line to an arbitrary diagram is to supply a factor equal to the sum of (2.5) over all external lines in the diagram

$$(8\pi G)^{1/2}\sum_n \eta_n p_n{}^\mu p_n{}^\nu/[p_n\cdot q - i\eta_n\epsilon]. \qquad (2.7)$$

memory effects

black hole/neutron star collisions

[Braginsky, Thorne'87]

permanent change in the gravitational-wave field (the burst's memory) $\delta h_{ij}^{\mathrm{TT}}$ is equal to the 'transverse, traceless (TT) part'[36] of the time-independent, Coulomb-type, $1/r$ field of the final system minus that of the initial system. If $\mathbf{P}^A$ is the 4-momentum of mass A of the system and P_i^A is a spatial component of that 4-momentum in the rest frame of the distant observer, and if $\mathbf{k}$ is the past-directed null 4-vector from observer to source, then $\delta h_{ij}^{\mathrm{TT}}$ has the following form:

$$\delta h_{ij}^{\mathrm{TT}} = \delta\left(\sum_A \frac{4P_i^A P_j^A}{\mathbf{k}\cdot\mathbf{P}^A}\right)^{\mathrm{TT}} \qquad (1)$$

Here we use units with $G=c=1$. In the observer's local Car-

Soft theorems ⇔ Memory effects

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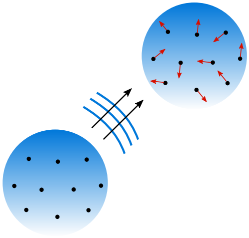
Here we use units with $G=c=1$. In the observer's local Car-

Equivalence:

- $P_i^A \leftrightarrow p_n^\mu$
- different conventions Newton's constant G and normalization
- momentum space soft theorem $\xrightarrow{\int dt e^{i\omega t}}$ position-space memory effect

Gravitational memory in the sky?

- *displacement* memory: Pulsar Timing Array [van Haasteren+'10; Wang et al'15], LIGO [Lasky+'16]
  gravitational wave → relative angular displacement of detectors
- *spin* memory: LISA, Einstein Telescope [Nichols'17]
  gravitational wave → relative time delays of counterorbiting object



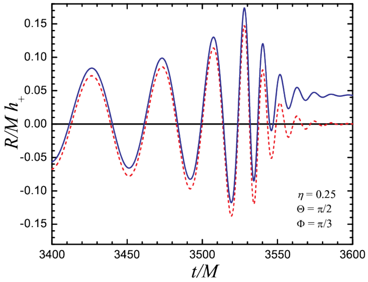
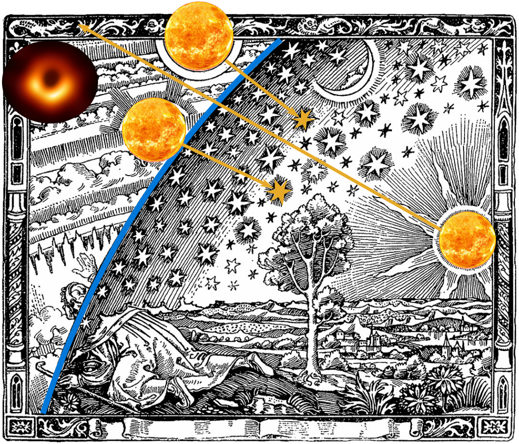


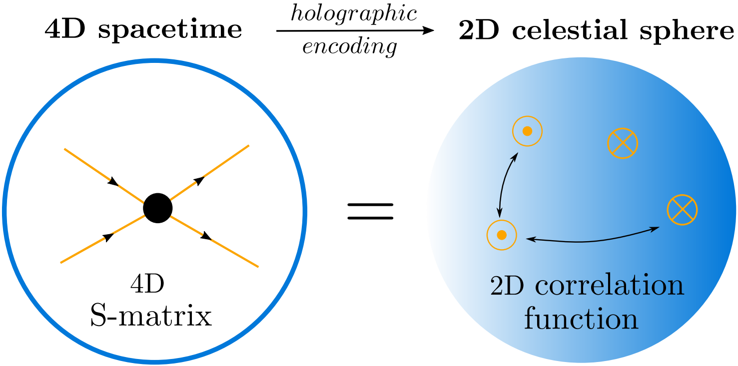
Figure: *Angular displacement of evenly spaced array of detectors caused by gravitational wave (left). h_+ polarization for an equal-mass binary black hole coalescence with and without memory (right) [Favata'10].*

(M total mass, η reduced mass, R distance and (Θ, Φ) angles of source to observer)

II. Celestial sphere: flat space holography?



Evidence for flat space holography



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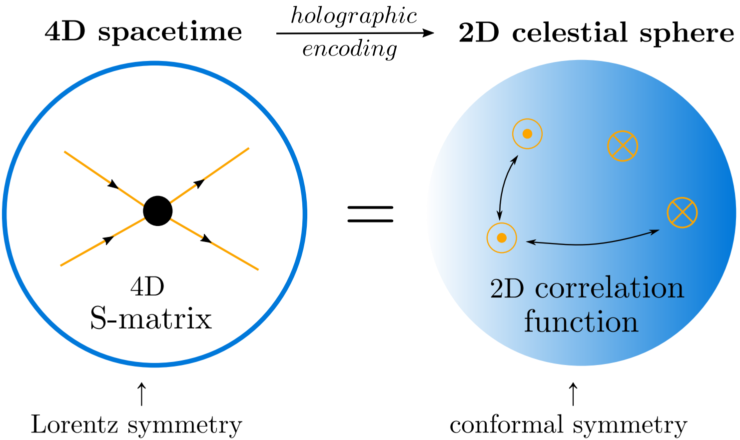

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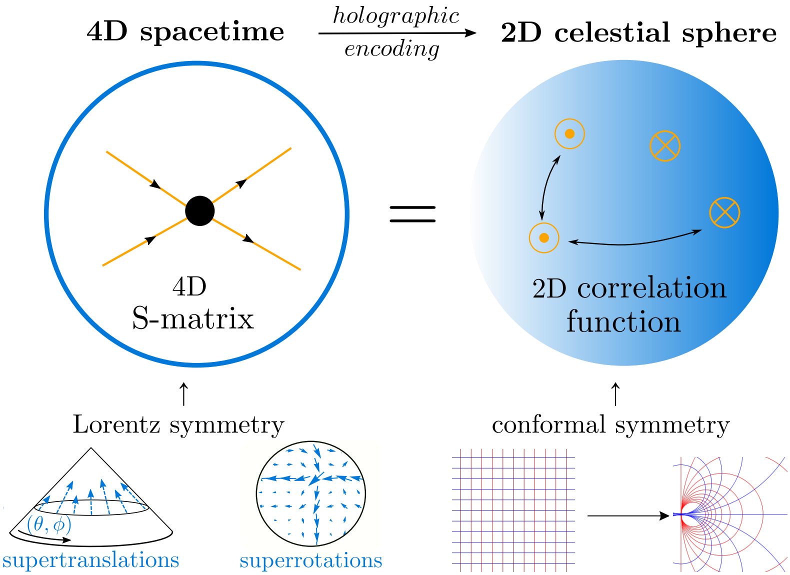
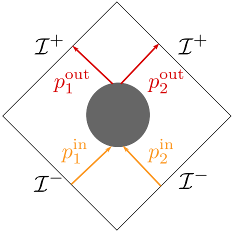
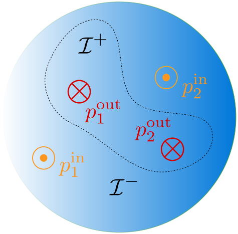


Basic idea of flat space holography

Bulk Mink[1,3]	**Boundary $\mathcal{CS}^2$**
4D $SL(2,\mathbb{C})$ Lorentz	2D global conformal
4D superrotations	2D local conformal
4D supertranslations	2D Kac-Moody



$=$



4D $\mathcal{S}$-matrix	2D conformal correlator
$\langle out \vert \mathcal{S} \vert in \rangle$	$\langle \mathcal{O}_1(z_1, \bar{z}_1) \ldots \mathcal{O}_n(z_n, \bar{z}_n) \rangle$
$\omega, \vec{p}, \ell$	$(z, \bar{z}),\ \Delta,\ J$

Basic elements of flat space holography

standard formulation

energy-momentum basis

$$p_k^\mu = (\omega_k, \vec{p}_k)\,,\ \ell_k$$

holographic formulation

conformal basis

$$z_k = \frac{p_k^1 + i p_k^2}{p_k^3 + p_k^0}\,,\ J_k = \ell_k\,,\ \Delta_k$$

Basic elements of flat space holography

standard formulation

energy-momentum basis

plane waves

$$e^{ip\cdot X} = e^{i\omega\, q\cdot X}$$

$$p_k^\mu = (\omega_k, \vec{p}_k)\,,\ \ell_k$$

holographic formulation

conformal basis

Mellin transform of plane waves*

$$\int_0^\infty d\omega\omega^{\Delta-1} e^{i\omega\, q\cdot X} = \frac{\mathcal{N}(\Delta)}{(-q\cdot X)^\Delta}$$

$$z_k = \frac{p_k^1 + ip_k^2}{p_k^3 + p_k^0}\,,\ J_k = \ell_k\,,\ \Delta_k$$

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Solution to Maxwell:

$$A_\mu = \frac{\Delta-1}{\Delta}\frac{\epsilon_\mu}{(-q\cdot X)^\Delta} + \partial_\mu(...)$$

Solution to linearized Einstein:

$$h_{\mu\nu} = \frac{\Delta-1}{\Delta+1}\frac{\epsilon_{\mu\nu}}{(-q\cdot X)^\Delta} + \partial_{(\mu}(...)_{\nu)}$$

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Transform as (Δ, J) conformal primaries on 2D celestial sphere!

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A holographic formulation of 4D QFT

standard formulation

energy-momentum basis

$\rightarrow$ plane waves

$\mathcal{A}(\omega_1, \vec{p}_1, \ldots, \omega_n, \vec{p}_n)$

$p_k^\mu = (\omega_k, \vec{p}_k)\,,\ \ell_k$

holographic formulation

conformal basis

$\rightarrow$ *Mellin transform* of plane waves*

$\widetilde{\mathcal{A}}(\lambda_1, z_1, \bar{z}_1, \ldots \lambda_n, z_n, \bar{z}_n)$

$z_k = \dfrac{p_k^1 + ip_k^2}{p_k^3 + p_k^0}\,,\ J_k = \ell_k\,,\ \Delta_k = 1 + i\lambda_k$

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$$= \langle \mathcal{O}_1(z_1, \bar{z}_1) \ldots \mathcal{O}_n(z_n, \bar{z}_n) \rangle_{\mathrm{CCFT}}$$

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[de Boer,Solodukhin] [Pasterski,Shao,Strominger]
[Cheung,de la Fuente,Sundrum]...

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translational symmetry manifest
conformal properties obscured

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"soft" particle: $\omega \rightarrow 0$

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→ plane waves

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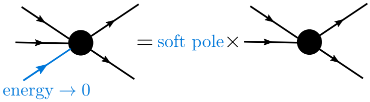
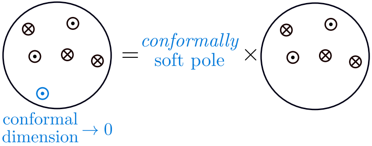
conformal properties manifest
translational symmetry obscured

"conformal soft" particle $\lambda \to 0$**

Conformally soft theorem in gauge theory

Insertions of the $(\Delta, J) = (1, \pm 1)$ soft photon/gluon current:





Soft gluon theorem

[Yennie,Frautschi,Suura'61]

Conformally soft gluon theorem

[Fan,Fotopoulos,Taylor]

[Pate,Raclariu,Strominger]

[Nandan,Schreiber,Volovich,Zlotnikov]

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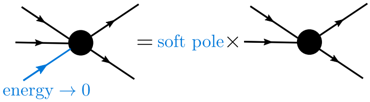

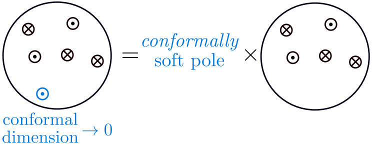

Soft gluon theorem

[Yennie,Frautschi,Suura'61]

$$\lim_{\omega_n \to 0} \omega_n \mathcal{A}_n(\omega_i, z_i, \bar{z}_i) = S^{(0)} \mathcal{A}_{n-1}(\omega_i, z_i, \bar{z}_i)$$

with

$$S^{(0)} = -\frac{1}{2} \frac{z_{n-1\,n+1}}{z_{n-1\,n}\, z_{n\,n+1}}$$

where $z_{ij} = z_i - z_j$

Conformally soft gluon theorem

[Fan,Fotopoulos,Taylor]

[Pate,Raclariu,Strominger]

[Nandan,Schreiber,Volovich,Zlotnikov]

$$\lim_{\lambda_n \to 0} i\lambda_n \widetilde{\mathcal{A}}_n(\lambda_i, z_i, \bar{z}_i) = S^{(0)} \widetilde{\mathcal{A}}_{n-1}(\lambda_i, z_i, \bar{z}_i)$$

with

$$S^{(0)} = -\frac{1}{2} \frac{z_{n-1\,n+1}}{z_{n-1\,n}\, z_{n\,n+1}}$$

where $z_{ij} = z_i - z_j$

Both expected and surprising: amplitude factorization for zero energy in energy basis but Mellin transform = superposition of all energies.

Conformally soft theorem in gravity

Insertions of the $(\Delta, J) = (1, \pm 2)$ BMS supertranslation current:

[Donnay,AP,Strominger]

$$P_w \mathcal{O}_\omega(z) \sim \frac{\omega}{w-z}\mathcal{O}_\omega(z) \quad \rightarrow \quad P_w \mathcal{O}_{(\Delta,J)}(z) \sim \frac{1}{w-z}\mathcal{O}_{(\Delta+1,J)}(z)$$

Conformally soft theorem in gravity

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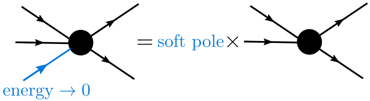
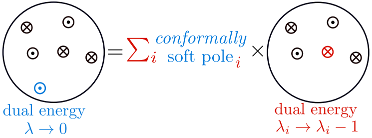




Weinberg's soft graviton theorem

[Weinberg]

$$\lim_{\omega_n \to 0} \omega_n \mathcal{H}_n(\omega_1, \dots, \omega_i, \dots, \omega_n)$$
$$= S^{(0)} \mathcal{H}_{n-1}(\omega_1, \dots, \omega_i, \dots, \omega_{n-1})$$

with

$$S^{(0)} = \sum_{i=1}^{n-1} \omega_i \frac{\epsilon_i}{\epsilon_n} \frac{\bar{z}_{ni}}{z_{ni}} \frac{z_{xi} z_{yi}}{z_{xn} z_{yn}}$$

where x, y is a choice of reference spinors

Conformally soft graviton theorem

[AP] [Adamo,Mason,Sharma] [Guevara]

$$\lim_{\lambda_n \to 0} i\lambda_n \widetilde{\mathcal{H}}_n(\lambda_1, \dots, \lambda_i, \dots, \lambda_n)$$
$$= \sum_{i=1}^{n-1} \widetilde{S}_i^{(0)} \widetilde{\mathcal{H}}_{n-1}(\lambda_1, \dots, \lambda_i - i, \dots, \lambda_{n-1})$$

with

$$\widetilde{S}_i^{(0)} = \frac{\epsilon_i}{\epsilon_n} \frac{\bar{z}_{ni}}{z_{ni}} \frac{z_{xi} z_{yi}}{z_{xn} z_{yn}}$$

where x, y is a choice of reference spinors

State of the art: massless, tree-level, free theory

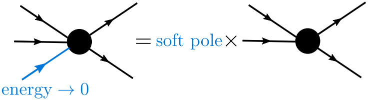
4D scattering amplitudes $\longrightarrow$ 2D celestial correlators

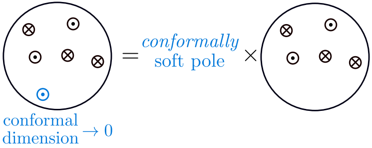
energy, momentum, helicity $\quad$ point on sphere, conformal dimension, spin

4D spacetime symmetries $\longrightarrow$ 2D conserved currents

large gauge symmetry	conformal soft photon/gluon current
supertranslations	supertranslation current
superrotations	stress tensor

4D *"soft"* theorems $\longrightarrow$ 2D *"conformally soft"* theorems





Much remains to be understood!