Imprints of an early primordial black hole domination in the stochastic gravitational wave background

(Based on JHEP 07, 130 (2022))

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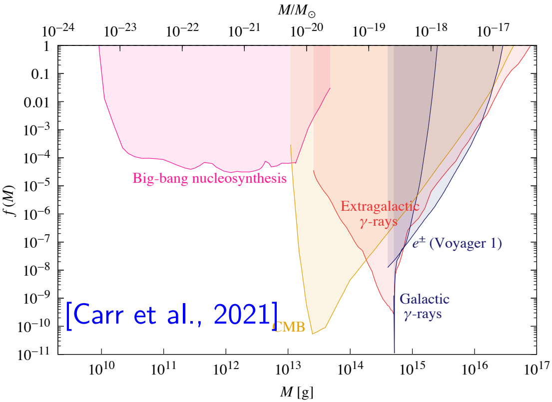
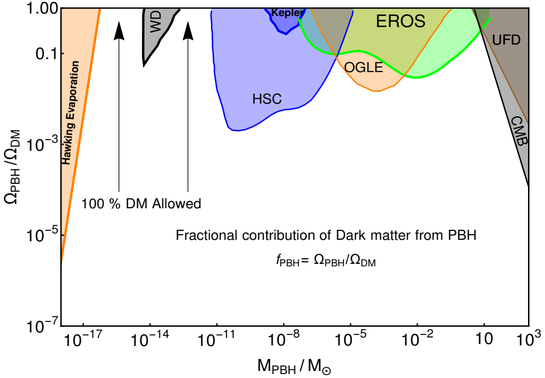
20^{th} September, 2022

Overview

Ultra low mass PBHs





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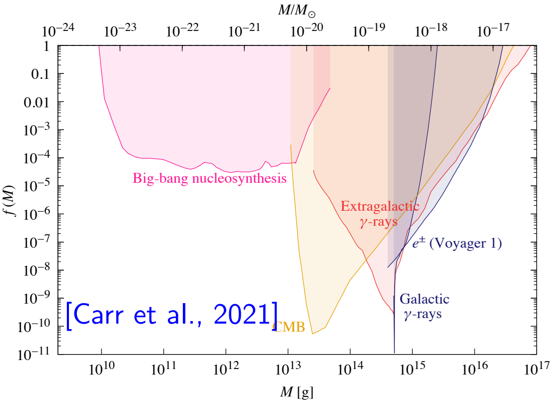
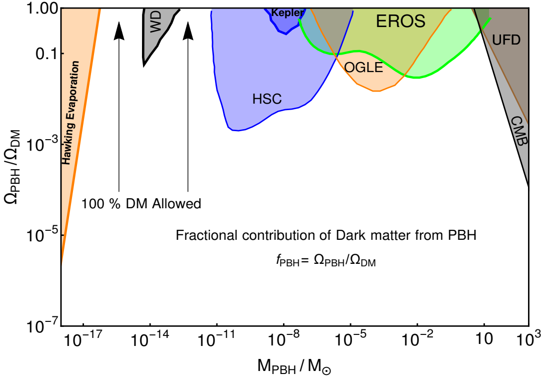




$$\text{Inflation} \xrightarrow{\text{eRD}} \text{PBH formation}(\tau_f) \xrightarrow{\text{eRD}} \text{PBH domination}(\tau_m) \xrightarrow{\text{eMD}} \text{PBH evaporation}(\tau_r) \xrightarrow{\text{RD}}$$

Adiabatic perturbations from two contributions

1. Poisson Distribution of PBHs
2. Cutoff for scales bellow PBH mean distance
3. Finite duration of PBH domination (Non-linearity bound)

$$\mathcal{P}_{\rm PBH}(k,\tau_r) = \frac{2}{3\pi}\left(\frac{k}{k_{\rm UV}}\right)^3\left(5+\frac{4}{9}\frac{k^2}{k_m^2}\right)^{-2}$$

$$k_{\rm UV} = \gamma^{-1/3}\beta^{1/3}k_f$$

[Papanikolaou et al (2021), Domenech et al (2021)]

$$\mathcal{P}_{\rm infl}(k,\tau_r) = A_s\left(\frac{k}{k_p}\right)^{n_s-1}\theta_H(k_m-k)$$

$$\Phi(k,\tau_r) = \Phi_{\rm infl}(k,\tau_r) + \Phi_{\rm PBH}(k,\tau_r)$$

$$\mathcal{P}(k,\tau_r) = \mathcal{P}_{\rm infl}(k,\tau_r) + \mathcal{P}_{\rm PBH}(k,\tau_r)$$

Induced stochastic gravitational wave background (ISGWB)

$$h''_{\mathbf{k}}(\tau) + 2\mathcal{H}h'_{\mathbf{k}}(\tau) + k^2 h_{\mathbf{k}}(\tau) = 4S_{\mathbf{k}}(\tau)$$ [Matarrese et al (1998)]

Dependence on kernel I(u,v,x)

$$\Omega_{\rm GW}(\eta, k) = \frac{1}{6}\left(\frac{k}{\mathcal{H}}\right)^2 \int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u \left(\frac{4v^2-(1+v^2-u^2)^2}{4uv}\right)^2 \overline{I^2(v,u,x)}\mathcal{P}_{\mathcal{R}}(kv)\mathcal{P}_{\mathcal{R}}(ku)$$

[Ananda et al(2006), Baumann et al (2007), Kohri et al (2018) ...]

$$I(u,v,x,x_r) = I_{eRD} + I_{eMD} + I_{RD} \simeq I_{RD}(u,v,x,x_r) = \int_{x_r}^{x} d\bar{x}\frac{a(\bar{x})}{a(x)} f(u,v,\bar{x},x_r) kG(\bar{x},x)$$

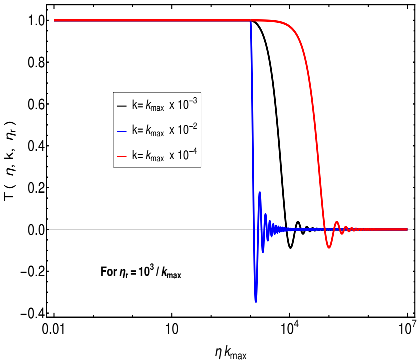
[Inomata et al (2019)][**NB**, R. K. Jain (2021)] [Domènech et al (2021)]

$$f(u,v,\bar{x},x_r) = \frac{4}{9}\Big[(\bar{x}-x_r/2)\partial_{\bar{x}}\mathcal{T}(u\bar{x},ux_r)((\bar{x}-x_r/2)\partial_{\bar{x}}\mathcal{T}(v\bar{x},vx_r)$$

$$+\mathcal{T}(v\bar{x},vx_r))+\mathcal{T}(u\bar{x},ux_r)((\bar{x}-x_r/2)\partial_{\bar{x}}\mathcal{T}(v\bar{x},vx_r)+3\mathcal{T}(v\bar{x},vx_r))\Big]$$

$$x = \eta k \qquad x_r = \eta_r k \qquad \mathcal{H} = aH$$

Two resonant peaks of ISGWB



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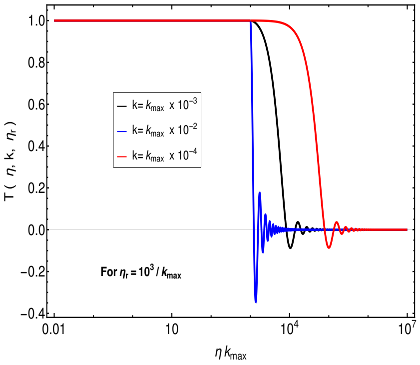
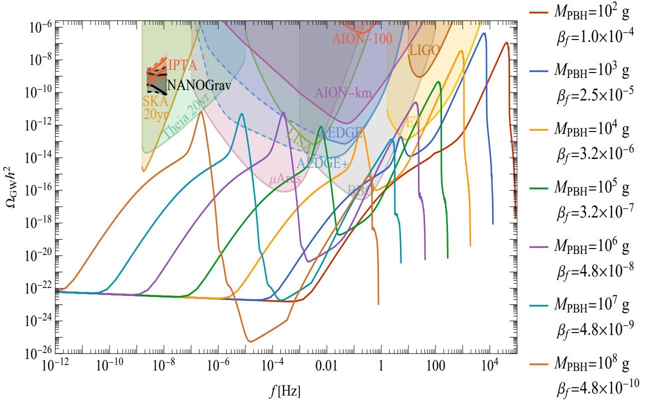
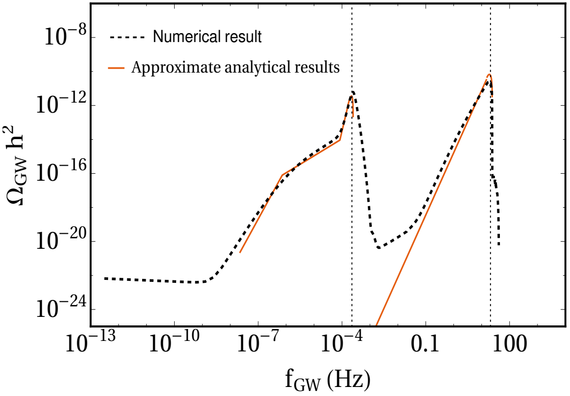
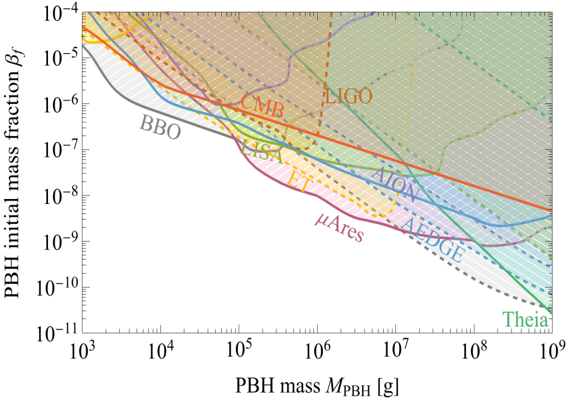




Detection Probability





$$\mathrm{SNR} \geq 10$$

[**NB**, A Ghoshal; M. Lewicki, JHEP 07, 130 (2022)]

Thank You

Summary of Results

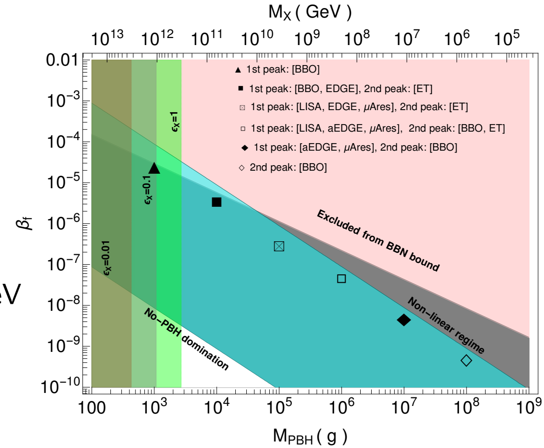
- Ultra-low mass PBHs can dominate the universe before BBN and contribute to isocurvature perturbations.
- Such an early PBH dominated universe leads to an uniquely shaped doubly peaked ISGWB spectrum; one peak from the inflationary adiabatic and another from isocurvature induced adiabatic scalar modes.
- The detection or non-detection of these peaks would act as a probe of such a reheating history.
- The BBN and CMB bounds on the ISGWB constrain the parameter space for PBHs and also for scenarios, like PBH induced Baryogenesis, dark radiation or dark matter relic abundance from PBHs.

Thank You

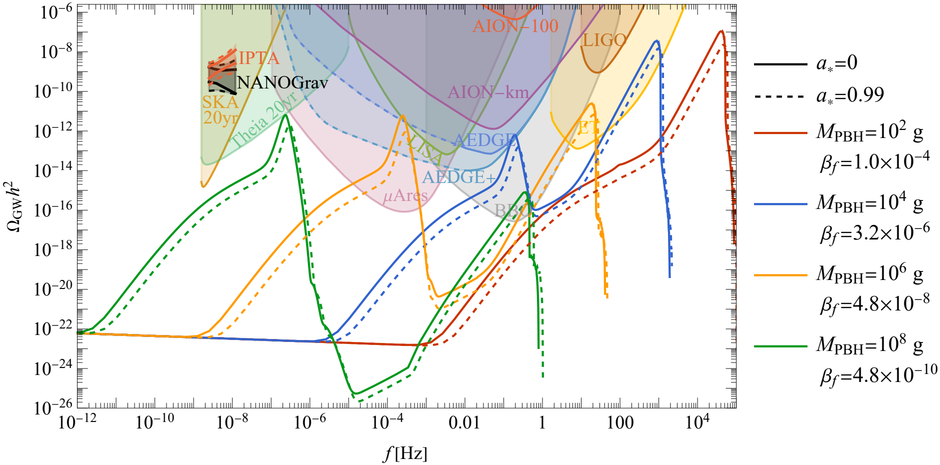
Testing the PBH induced Baryogenesis

$$Y_B = \frac{n_{\rm PBH}}{s}\epsilon_X N_X \approx 8.8\times 10^{-11}$$

$$M_X \approx 6.34\times 10^{15}\sqrt{\epsilon_X\left(\frac{1\text{ g}}{M_{\rm PBH}}\right)^{5/2}}\ \text{GeV}$$



Work in progress: Initial PBH spin

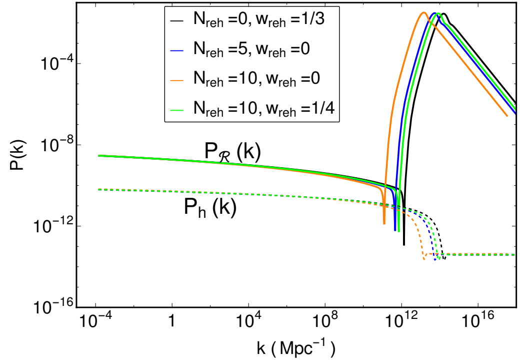


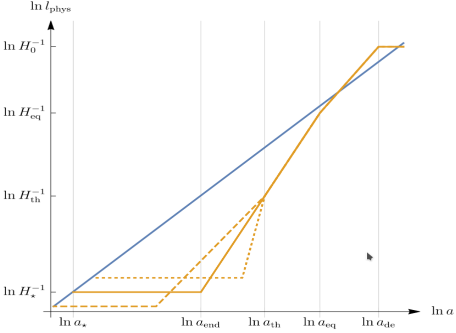
[**NB**, A Ghoshal, R K Jain, M. Lewicki (In preparation)]

Effects of reheating for PBH forming infltionary models

We assume reheating phase with a constant equation of state w_{reh}, and duration N_{reh}.

Remapping of scales : $k_e = k_{no-reheating} \times e^{-\frac{1}{4}N_{reh}(1-3w_{reh})}$





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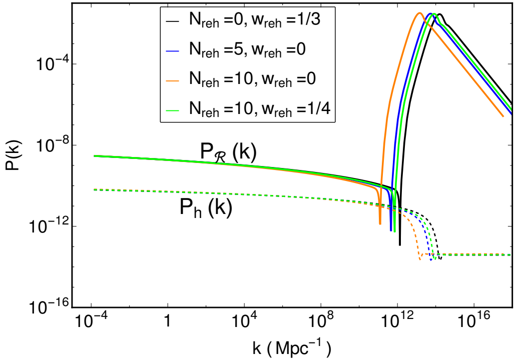
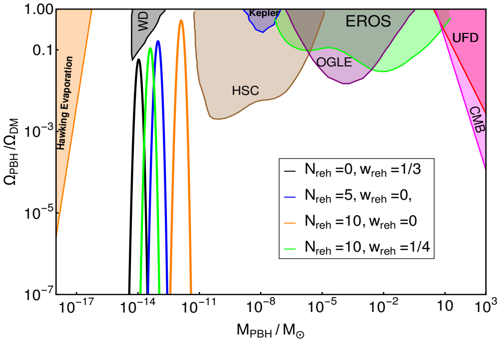
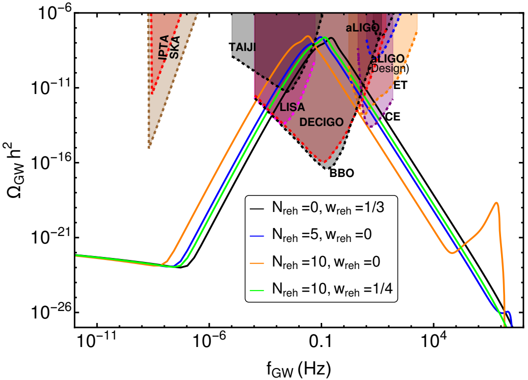
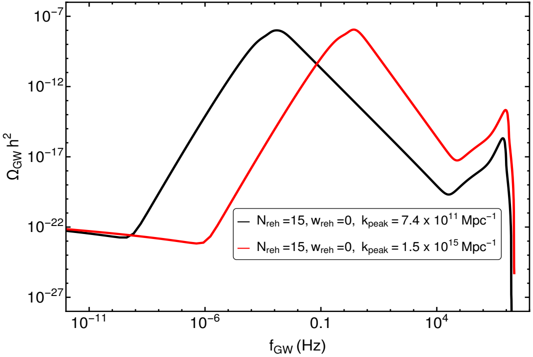




[**NB**, R. K. Jain; JCAP 01(2020), 037]

ISGWB for different reheating histories





[**NB**, R. K. Jain; Phys. Rev. D 104, 023531 (2021)]

Scalar transfer function $\mathcal{T}_k(\eta)$ and Kernel I(u,v,x,x_r)

Oscillating terms

$\mathcal{I} = I(u, v, x, x_r) \times (x - x_r/2)$
$\mathcal{I} = \mathcal{I}_s \sin(x) + \mathcal{I}_c \cos(x) +$ 4 other terms

Oscillation avarage

$$\overline{\mathcal{I}^2} = \frac{1}{2}\left(\mathcal{I}_s^2 + \mathcal{I}_c^2\right)$$

Different k regiemes

$\mathcal{I}_s \simeq \mathcal{I}_{ss} + \boxed{\mathcal{I}_{sl} x_r^4} +$ other terms

$\mathcal{I}_c \simeq \mathcal{I}_{cs} + \boxed{\mathcal{I}_{cl} x_r^4} +$ other terms

$I = I_{eRD} + I_{eMD} + I_{RD}$
$\Phi(k, \tau_r) = \Phi_{\rm infl}(k, \tau_r) + \Phi_{\rm PBH}(k, \tau_r)$

Isocurvature Perturbation

$$d_f \equiv \left(\frac{3M_{\mathrm{PBH},f}}{4\pi\rho_{\mathrm{PBH},f}}\right)^{1/3} = \gamma^{1/3}\beta^{-1/3}H_f{}^{-1}$$

$$\langle\delta\rho_{\mathrm{PBH}}(k)\delta\rho_{\mathrm{PBH}}(k')\rangle = \frac{4\pi}{3}\left(\frac{d}{a}\right)^3\rho_{\mathrm{PBH}}^2\,\delta(k+k')$$

$$S = \frac{\delta\rho_{\mathrm{PBH}}}{\rho_{\mathrm{PBH}}} - \frac{3}{4}\frac{\delta\rho_r}{\rho_r} = \frac{\delta\rho_{\mathrm{PBH}}}{\rho_{\mathrm{PBH}}} + \frac{3}{4}\frac{\delta\rho_{\mathrm{PBH}}}{\rho_r} \approx \frac{\delta\rho_{\mathrm{PBH}}}{\rho_{\mathrm{PBH}}} \quad \text{for } \rho_r \gg \rho_{\mathrm{PBH}}$$

$$\mathcal{P}_S(k) = \frac{2}{3\pi}\left(\frac{k}{k_{UV}}\right)^3$$

$$\Phi_{\mathrm{eMD}}(k; a \gg a_{\mathrm{eq}}) = S\begin{cases}\dfrac{1}{5} & k \ll k_{\mathrm{eq}}\\[2ex] \dfrac{3}{4}\left(\dfrac{k_{\mathrm{eq}}}{k}\right)^2 & k \gg k_{\mathrm{eq}}\end{cases}$$

Analytical form of ISGWB for inflationary adiabatic perturbation

$$\frac{\Omega_{GW}(\tau_0,k)}{A_s^2 c_g\,\Omega_{r,0}} \simeq \begin{cases} 3\times 10^{-7} x_r^3 x_{\max}^5 & 150 x_{\max}^{-5/3} \lesssim x_r \ll 1 \\ 6.6\times 10^{-7} x_r x_{\max}^5 & 1 \ll x_r \lesssim x_{\max}^{5/6} \\ 3\times 10^{-7} x_r^7 & x_{\max}^{5/6} \lesssim x_r \lesssim \frac{2}{1+\sqrt{3}} x_{\max} \\ \mathcal{C}(k) & \frac{2}{1+\sqrt{3}} \le \frac{x_r}{x_{\max}} \le \frac{2}{\sqrt{3}} \end{cases}, \tag{1}$$

where

$$\mathcal{C}(k) = 0.00638 \times 2^{-2n_s-13}\, 3^{n_s}\, x_r^7\, s_0 \left(\frac{x_r}{x_{max}}\right)^{2n_s-2} \times$$
$$\left(-s_0^2\, {}_2F_1\!\left(\frac{3}{2}, -n_s; \frac{5}{2}; \frac{s_0^2}{3}\right) + 4\, {}_2F_1\!\left(\frac{1}{2}, 1-n_s; \frac{3}{2}; \frac{s_0^2}{3}\right) - 3\, {}_2F_1\!\left(\frac{1}{2}, -n_s; \frac{3}{2}; \frac{s_0^2}{3}\right)\right). \tag{2}$$

Analytical form for isocurvature induced adiabatic contribution to ISGWB

$$\Omega_{GW}(\tau_0, k) = c_g\, \Omega_{r,0}\ \mathcal{J} \int_{-s_0}^{s_0} \frac{27\sqrt[3]{3}\left(s^2-1\right)^2}{\left(9-3s^2\right)^{5/3}} \mathrm{d}s \tag{3}$$

$$= c_g\, \Omega_{r,0}\ \mathcal{J} \frac{2}{5} s_0 \left(\frac{3\left(14-3s_0^2\right)}{\left(1-\frac{s_0^2}{3}\right)^{2/3}} - 37\, {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{3}{2}; \frac{s_0^2}{3}\right) \right) , \tag{4}$$

where,

$$\mathcal{J} = \frac{k^3 k_m^8 \left(\frac{k}{k_r}\right)^{2/3}}{1327104\sqrt[3]{2}\sqrt{3}\pi k_r^5 k_{\mathrm{UV}}^6} .$$

and

$$s_0 = \begin{cases} 1 & \frac{k}{k_{\mathrm{UV}}} \leq \frac{2}{1+\sqrt{3}} \\ 2\frac{k_{\mathrm{UV}}}{k} - \sqrt{3} & \frac{2}{1+\sqrt{3}} \leq \frac{k}{k_{\mathrm{UV}}} \leq \frac{2}{\sqrt{3}} \end{cases} . \tag{5}$$

Inflationary adiabatic and isocurvature-induced adiabatic perturbation

$$\Phi^{\mathrm{eISO}}_{\mathrm{eMD}}(k; a \gg a_{\mathrm{eq}}) \approx \begin{cases} \frac{1}{5} & k \ll k_{\mathrm{eq}} \\ \frac{3}{4}\left(\frac{k_{\mathrm{eq}}}{k}\right)^2 & k \gg k_{\mathrm{eq}} \end{cases},$$

$$\Phi^{e\mathrm{CVT}}_{e\mathrm{MD}}(k \gg k_{\mathrm{eq}}; a \gg a_{\mathrm{eq}}) \approx \frac{135}{16}\left(\frac{k_{\mathrm{eq}}}{k}\right)^4 \left(\ln 4 - \frac{7}{2} + \gamma_E + \ln\left(\sqrt{\frac{2}{3}}\frac{k}{k_{\mathrm{eq}}}\right)\right)$$

[KODAMA, SASAKI (1987). International Journal of Modern Physics A, 02(02), 491–560.]

Induced Stochastic Gravitational Wave Background (ISGWB)

$$h''_{\mathbf{k}}(\tau) + 2\mathcal{H} h'_{\mathbf{k}}(\tau) + k^2 h_{\mathbf{k}}(\tau) = 4S_{\mathbf{k}}(\tau)$$

$$S^s_k = \int \frac{\mathrm{d}^3 q}{(2\pi)^{3/2}} e^s_{ij}(k) q_i q_j \left[2\Phi_q \Phi_{k-q} + \frac{4}{3(1+w)}(\mathcal{H}^{-1}\Phi'_q + \Phi_q)(\mathcal{H}^{-1}\Phi'_{k-q} + \Phi_{k-q})\right]$$

$$\Omega_{\mathrm{GW}}(\tau, k) = \frac{1}{6}\left(\frac{k}{\mathcal{H}}\right)^2 \int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u \left(\frac{4v^2 - (1+v^2-u^2)^2}{4uv}\right)^2$$

$$\times \overline{I^2_{\mathrm{RD}}(v, u, x)} \mathcal{P}_{\mathcal{R}}(kv) \mathcal{P}_{\mathcal{R}}(ku)$$

$$I_{RD}(u, v, x) = \int_0^x d\bar{x} \frac{a(\bar{x})}{a(x)} f(u, v, \bar{x}) k G(\bar{x}, x)$$

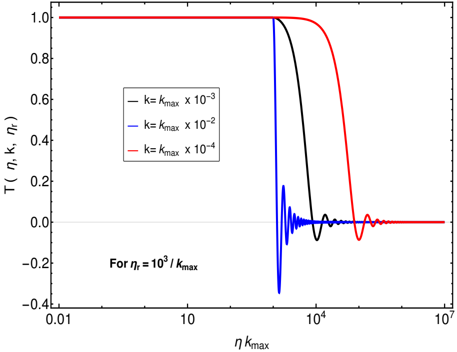
Transfer function in RD in presence of eMD

$$\mathcal{T}_k''(\eta) + 4\mathcal{H}\mathcal{T}_k'(\eta) + \frac{k^2}{3}\mathcal{T}_k(\eta) = 0$$

$$\mathcal{T}(x, x_r) = \frac{3\sqrt{3}\left[A(x_r)\, j_1\left(\frac{x - x_r/2}{\sqrt{3}}\right) + B(x_r)\, y_1\left(\frac{x - x_r/2}{\sqrt{3}}\right)\right]}{x - x_r/2}$$

$$A(x_r) = \frac{x_r}{2\sqrt{3}} \sin\left(\frac{x_r}{2\sqrt{3}}\right) - \frac{1}{36}(x_r^2 - 36)\cos\left(\frac{x_r}{2\sqrt{3}}\right)$$

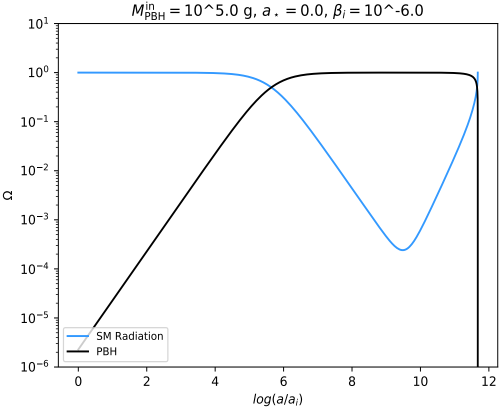
$$B(x_r) = -\frac{1}{36}(x_r^2 - 36)\sin\left(\frac{x_r}{2\sqrt{3}}\right) - \frac{x_r}{2\sqrt{3}}\cos\left(\frac{x_r}{2\sqrt{3}}\right)$$



$x = \eta k \qquad x_r = \eta_r k \qquad \frac{a(\eta)}{a(\eta_r)} = 2\frac{\eta}{\eta_r} - 1 \qquad \mathcal{H} = aH = \frac{1}{\eta - \eta_r/2} \qquad \mathcal{T}_k(\eta_r) = 1 \qquad \mathcal{T}_k'(\eta_r) = 0$

Background expansion

$$a_{eMD}(\tau) = \frac{a_f(\tau + \tau_m)^2}{4\tau_f \tau_m}, \qquad a_{RD}(\tau) = \frac{a_f(\tau_r + \tau_m)(2\tau - \tau_r + \tau_m)}{4\tau_f \tau_m} \simeq a(\tau_r)\left(2\frac{\tau}{\tau_r} - 1\right)$$

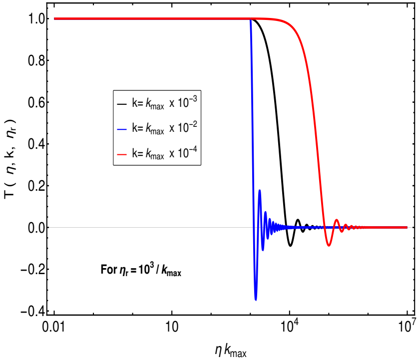


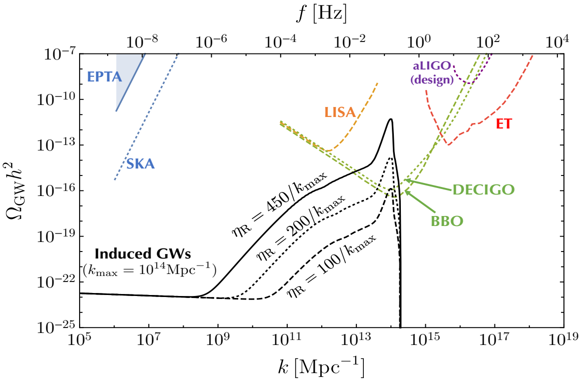
$$k_r \approx 2.1 \times 10^{11} \left(\frac{M_{\rm PBH}}{10^4 \rm g}\right)^{-3/2} \ {\rm Mpc}^{-1}$$

$$k_m \approx 3.4 \times 10^{17} \left(\frac{M_{\rm PBH}}{10^4 \rm g}\right)^{-5/6} \beta_f^{2/3} \ {\rm Mpc}^{-1}$$

$$k_f \approx 3.4 \times 10^{17} \left(\frac{M_{\rm PBH}}{10^4 \rm g}\right)^{-5/6} \beta_f^{-1/3} \ {\rm Mpc}^{-1}$$

Two Peaks of ISGWB





[Inomata et al (2019)]