Measuring the distortion of time with large-scale structure

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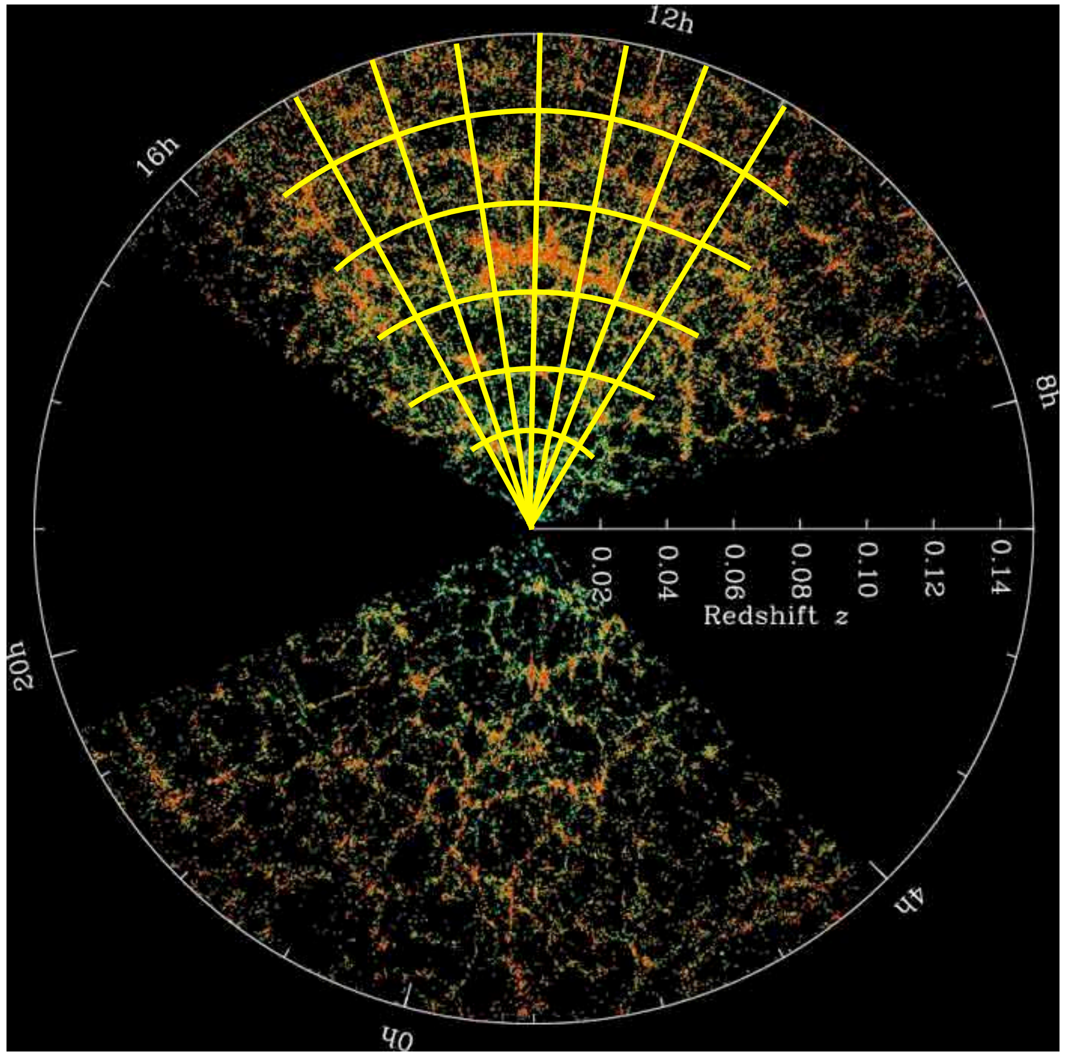
Relativistic effects

- Why are they **useful**?
  - Test of gravity
  - Distinguish between modifications of GR and a dark fifth force

- Measuring the relativistic **dipole** can help **disentangle** relativistic effects from PNG

Galaxy number counts

We count the number of **galaxies** per **pixel**: $\Delta = \dfrac{N - \bar{N}}{\bar{N}}$

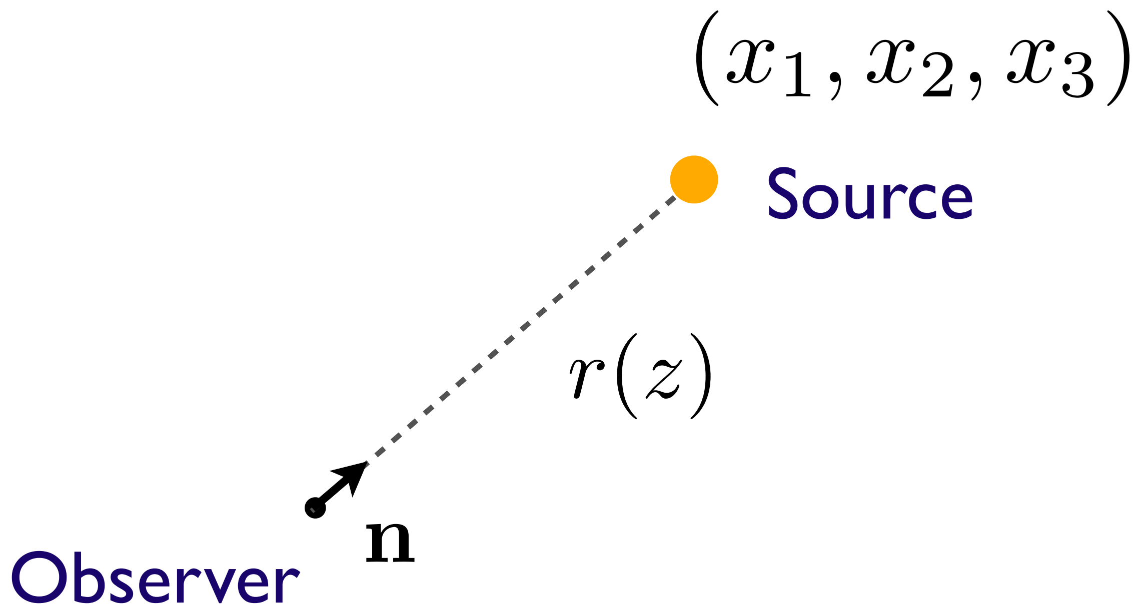


Credit: M. Blanton, SDSS

- Galaxies follow the distribution of matter $\Delta = b \cdot \delta$
- We never observe directly the position of galaxies, we observe the **redshift** z and the **direction** of incoming photons $\mathbf{n}$

In a **homogeneous** universe:

- we calculate the distance $r(z)$
- light propagates on straight lines



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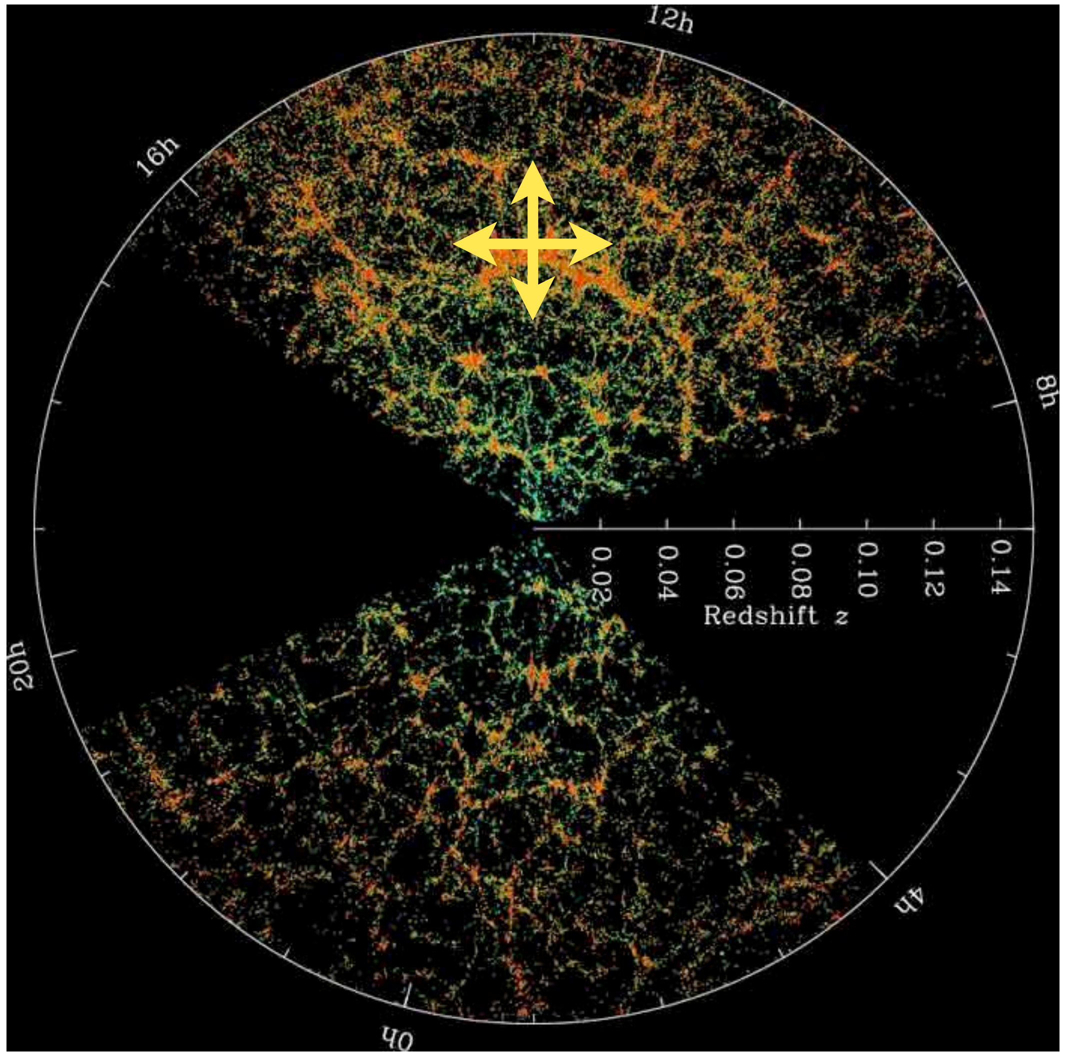

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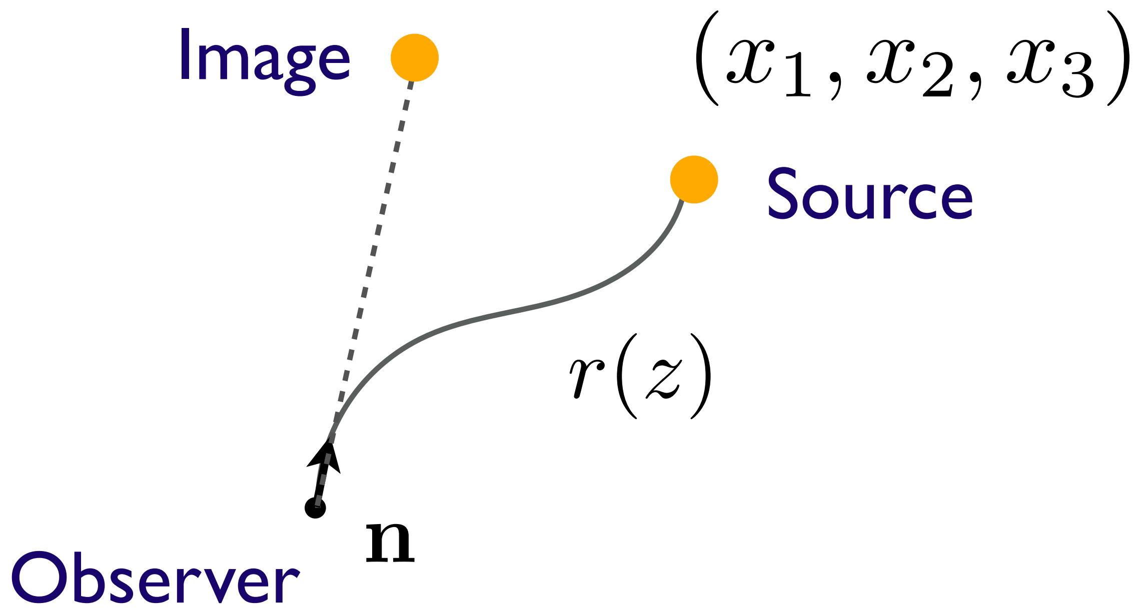

Inhomogeneities modify:

- distance-redshift relation
- angular position of the image

What we really observe

Yoo et al (2010)
CB and Durrer (2011)
Challinor and Lewis (2011)

$$\begin{aligned}
\Delta(z,\mathbf{n}) &= b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n}) \\
&+ (5s-2)\int_0^r dr' \frac{r-r'}{2rr'}\Delta_\Omega(\Phi+\Psi) \\
&+ \left(1-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{5s-2}{r\mathcal{H}}-5s+f^{\mathrm{evol}}\right)\mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\partial_r\Psi \\
&+ \frac{2-5s}{r}\int_0^r dr'(\Phi+\Psi) + 3\mathcal{H}\nabla^{-2}(\nabla\mathbf{V}) + \Psi + (5s-2)\Phi \\
&+ \frac{1}{\mathcal{H}}\dot{\Phi} + \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2-5s}{r\mathcal{H}}+5s-f^{\mathrm{evol}}\right)\left[\Psi+\int_0^r dr'(\dot{\Phi}+\dot{\Psi})\right]
\end{aligned}$$

What we really observe

Redshift-space distortion

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Lensing: measured with quasars and relevant for future surveys

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Relativistic effects

Testing gravity

♦ At late time, our Universe is described by 4 fields

$$\delta \qquad\qquad V$$

$$\Phi \qquad\qquad \Psi$$

♦ Ideally, we want to **measure** the 4 fields and **compare** them

♦ Currently not possible: we have **only 3** measurements

- δ and V from the distribution of galaxies
- $\Phi + \Psi$ from gravitational lensing

Testing gravity

- At late time, our Universe is described by 4 fields

	δ	Continuity	V
Poisson			Euler
	Φ	$=$	Ψ

In GR

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Poisson		**Euler**
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Testing gravity

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δ Continuity V

Poisson Euler

Φ Anisotropic stress Ψ

We cannot test all relations → limit ability to discriminate between models

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Example

Castello, Grimm and CB (2022)
CB and Pogosian (2022)

Distinguish between **modified gravity** and a **dark fifth force**

Modified gravity

$$\Phi = \eta\,\Psi \qquad\qquad -k^2\Psi = 4\pi G a^2 \rho_{\rm m}\mu\delta$$

$$\dot{V}_{\rm dm} + \mathcal{H}V_{\rm dm} + \partial_r\Psi = 0$$

Dark fifth force

$$\Phi = \Psi \qquad\qquad -k^2\Psi = 4\pi G a^2 \rho_{\rm m}\delta$$

$$\dot{V}_{\rm dm} + \mathcal{H}(1+\Theta)V_{\rm dm} + (1+\Gamma)\partial_r\Psi = 0$$

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Example

Enhanced growth
Undistinguishable with RSD

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Example

Measure anisotropic stress

Modified gravity

$$\boxed{\Phi = \eta\,\Psi} \qquad -k^2\Psi = 4\pi G a^2 \rho_{\rm m} \mu \delta$$

$$\ddot\delta + \mathcal{H}\dot\delta = 4\pi a^2 \rho_{\rm m} \underbrace{G\mu}_{G_{\rm eff}} \delta$$

growth of structure

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negligible

growth of structure

Castello, Grimm and CB (2022)
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Example

$\eta \neq 1$ → Smoking gun for modified gravity

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negligible

Measurements

Castello, Grimm and CB (2022)
CB and Pogosian (2022)

- Lensing $\Phi + \Psi$
- Redshift-space distortion $V_{\rm dm} \xrightarrow{\text{Euler}} \Psi$

Modified gravity $\partial_r \Psi = -\dot{V}_{\rm dm} - \mathcal{H} V_{\rm dm}$

$$\frac{\Phi + \Psi}{\Psi} = 1 + \eta \neq 2$$

Dark fifth force $\underbrace{\partial_r (1 + \Gamma) \Psi}_{\Psi^{\rm eff} > \Psi} = -\dot{V}_{\rm dm} - \mathcal{H} V_{\rm dm}$

$$\frac{\Phi + \Psi}{\Psi^{\rm eff}} = 1 + \eta^{\rm eff} \neq 2 = \frac{\Phi + \Psi}{\Psi}$$

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$\boxed{\eta \neq 1 \text{ Not a smoking gun!}}$

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Relativistic effects save the game

$$\Delta(z,\mathbf{n}) = b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$
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Gravitational redshift

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Measure the true Ψ

Relativistic effects save the game

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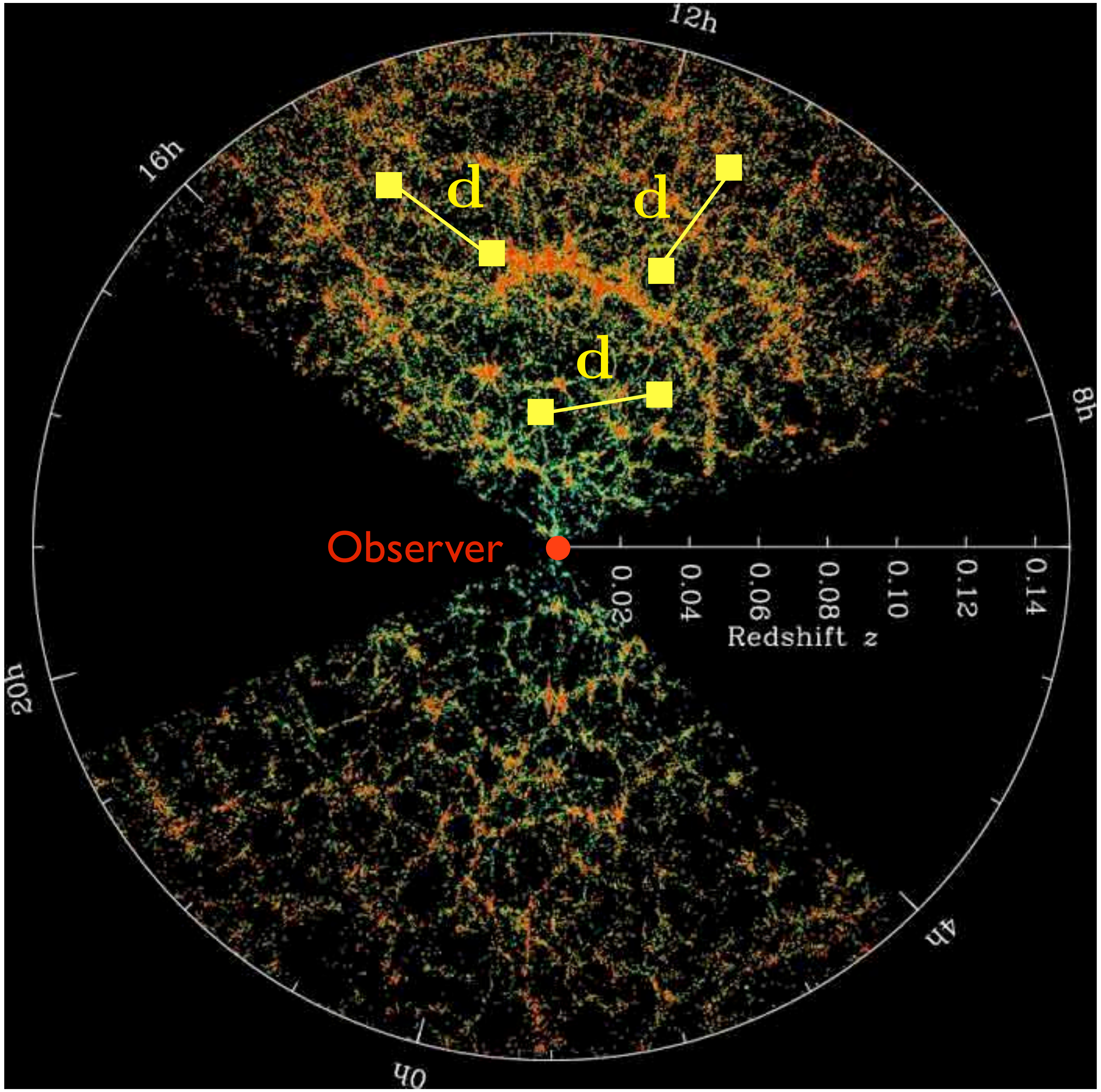
\end{aligned}
$$

How do we isolate this effect?

Measure the true Ψ

Correlation function

Credit: M. Blanton, SDSS



$$\xi = \langle \Delta(\mathbf{x}) \Delta(\mathbf{x}') \rangle$$

The dark matter fluctuations generate **isotropic** correlations

$$\Delta = b \cdot \delta$$

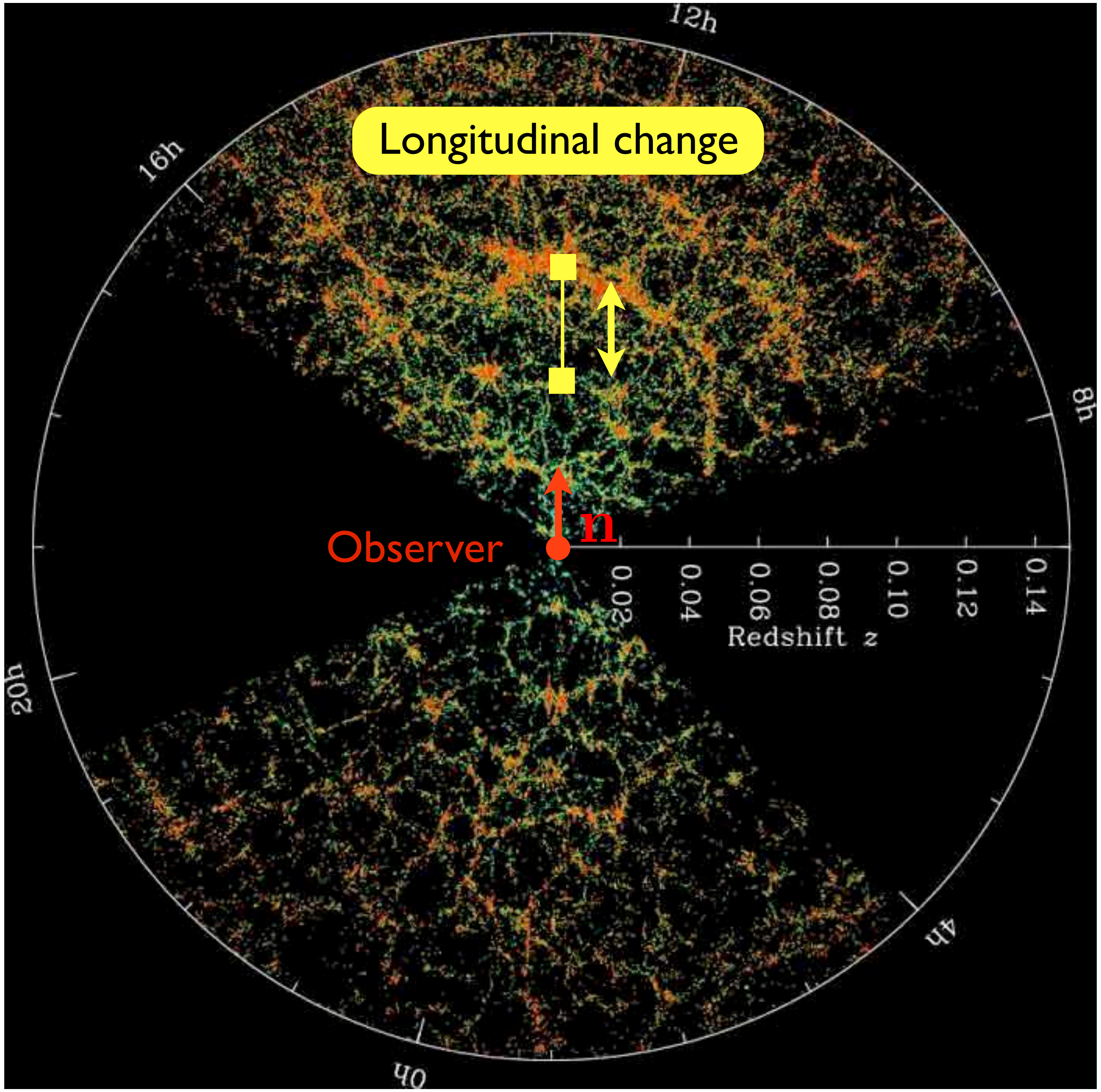
$$\xi(d) = C_0(d)$$

Redshift-space distortions

Kaiser (1987)
Hamilton (1992)

Redshift-space distortions **break** the **isotropy** of ξ.

Credit: M. Blanton, SDSS



$$\Delta = -\frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$

Changes the **redshift** separation but not the **angular** separation.

$$\xi = C_0(d) + C_2(d)P_2(\cos\beta) + C_4(d)P_4(\cos\beta)$$

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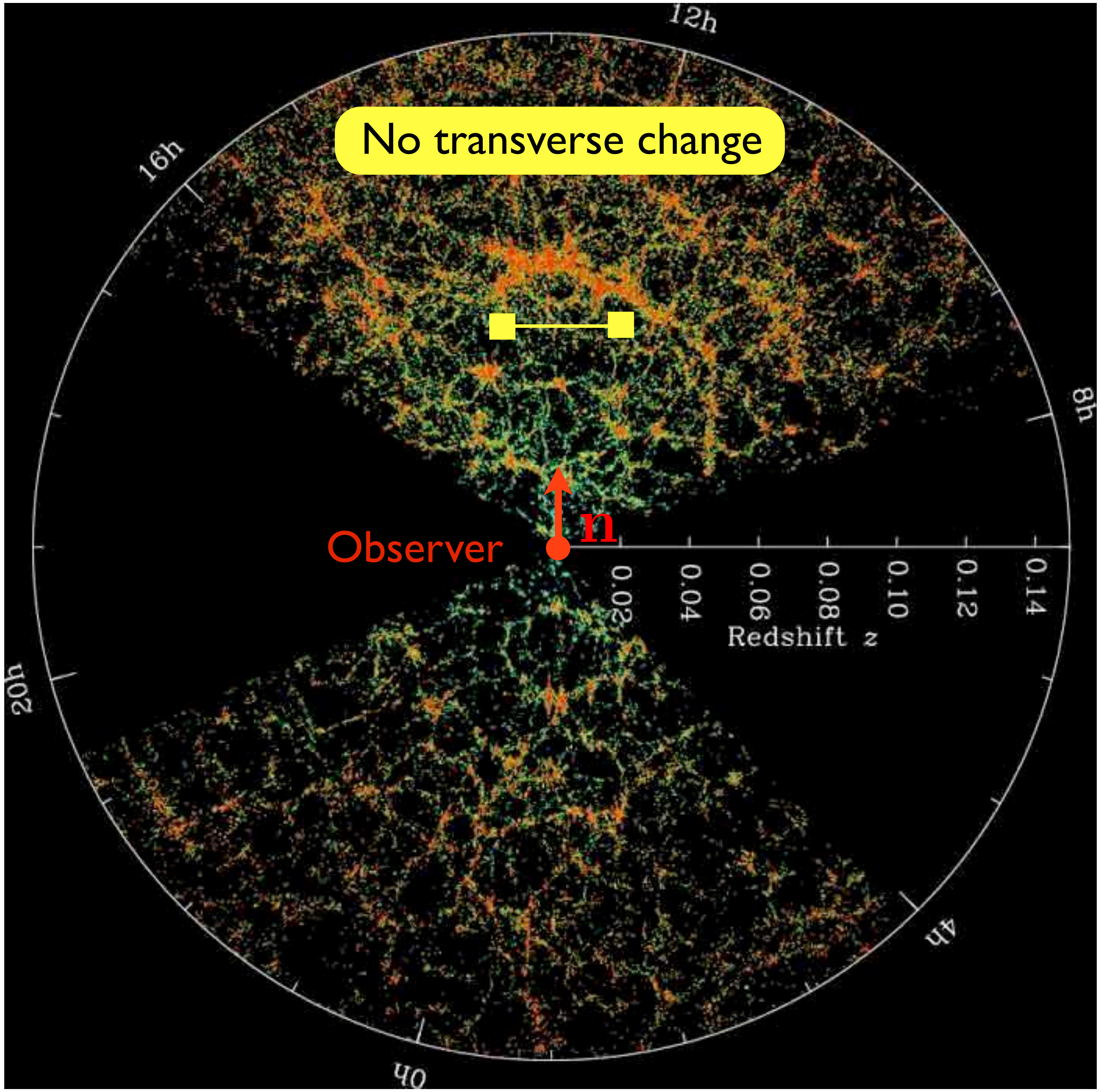

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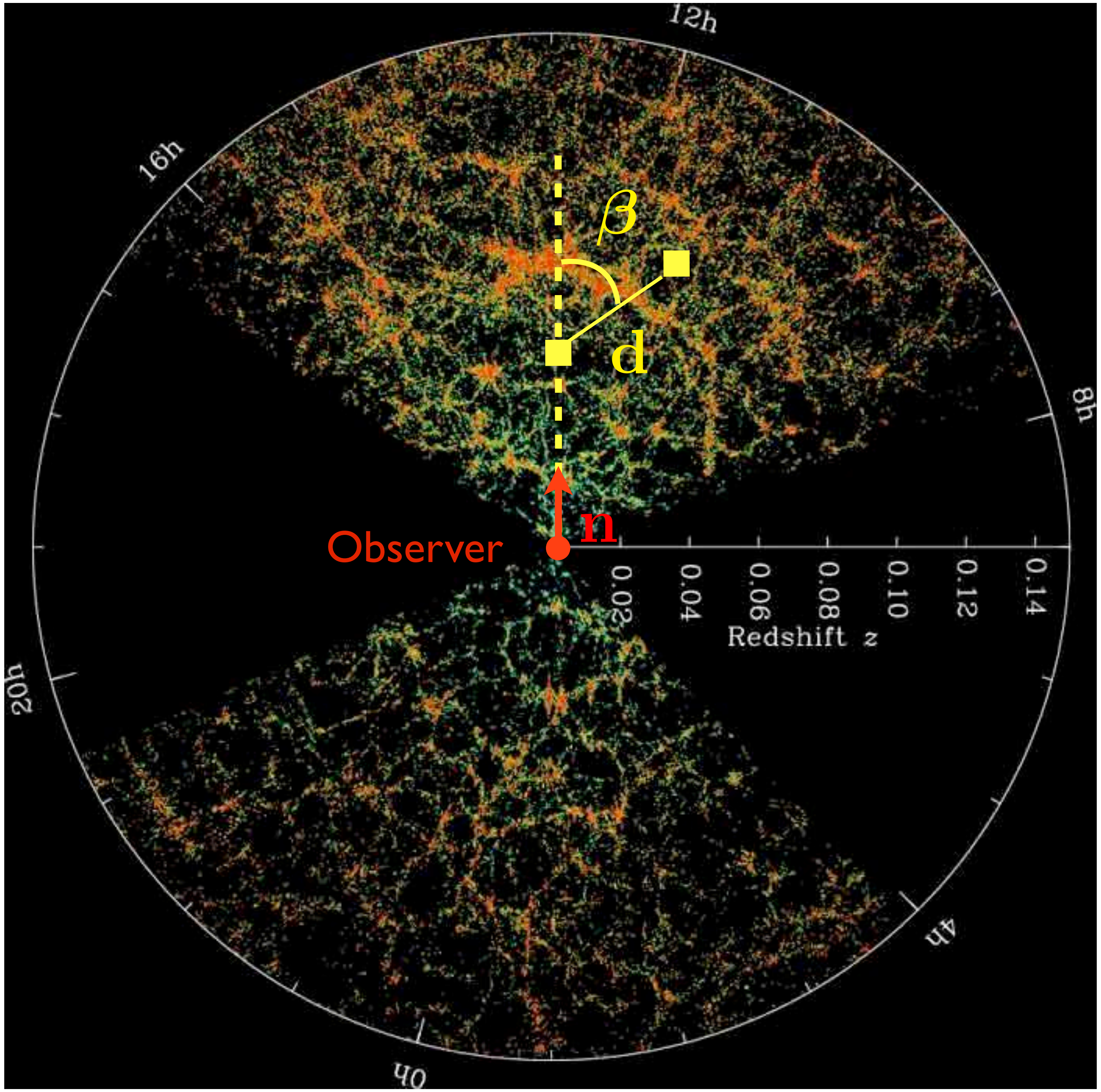

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Legendre polynomials

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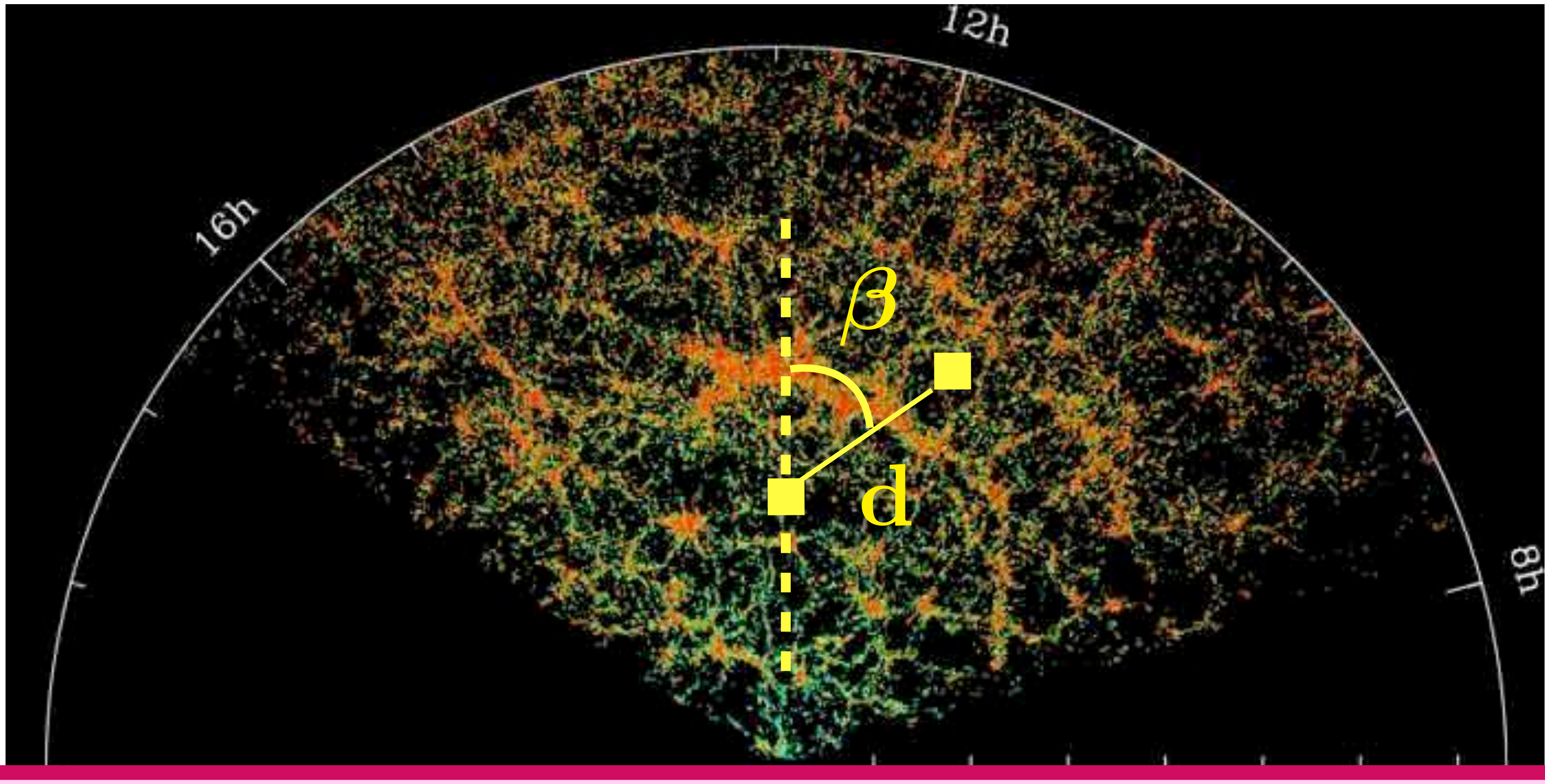

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Changes the **redshift** separation but not the **angular** separation.

$$\sum_{ij}\Delta_i\Delta_j \rightarrow C_0 \text{ monopole}$$

$$\sum_{ij}\Delta_i\Delta_j P_2(\cos\beta_{ij}) \rightarrow C_2 \text{ quadrupole}$$

$$\sum_{ij}\Delta_i\Delta_j P_4(\cos\beta_{ij}) \rightarrow C_4 \text{ hexadecapole}$$

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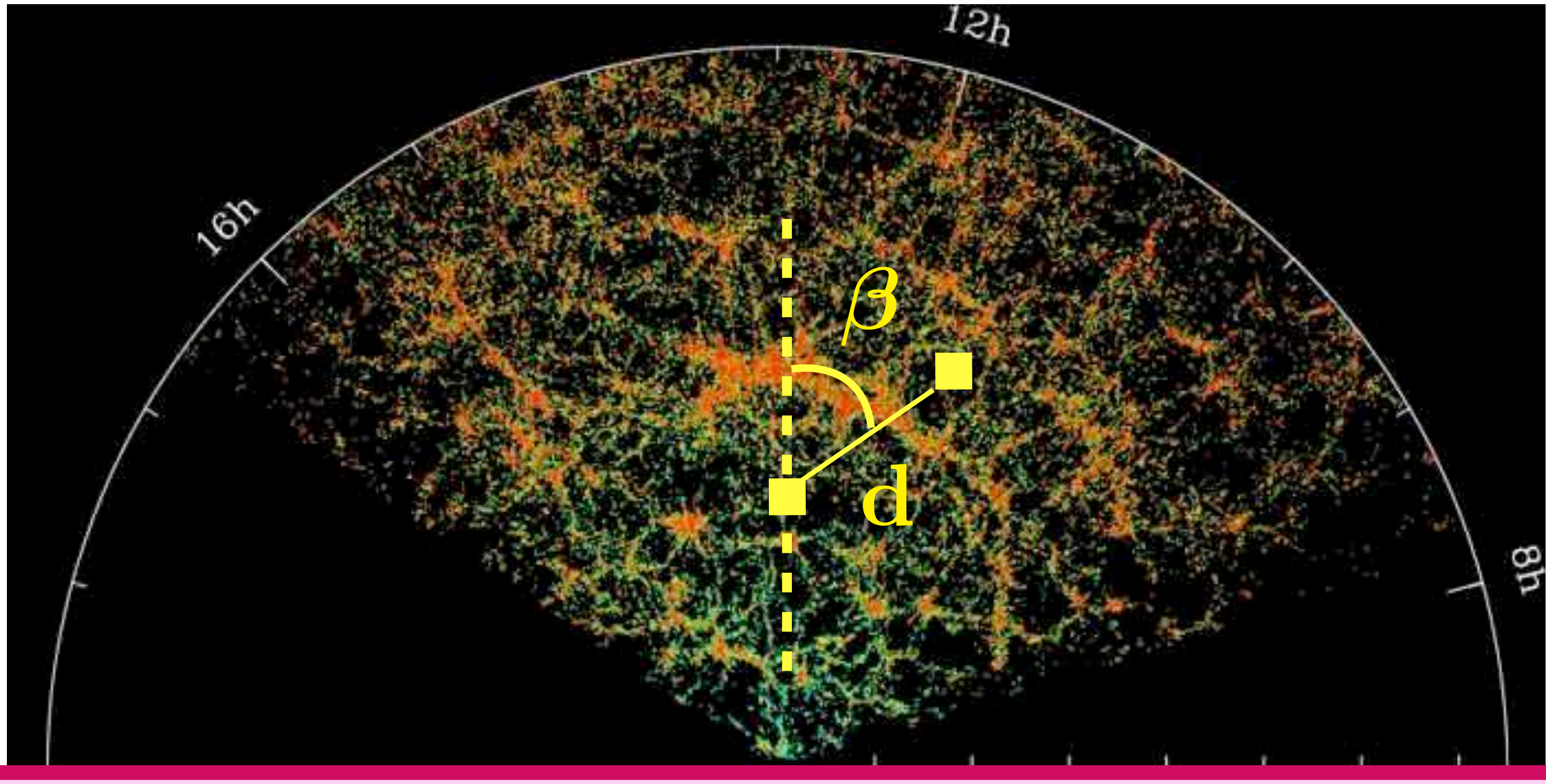

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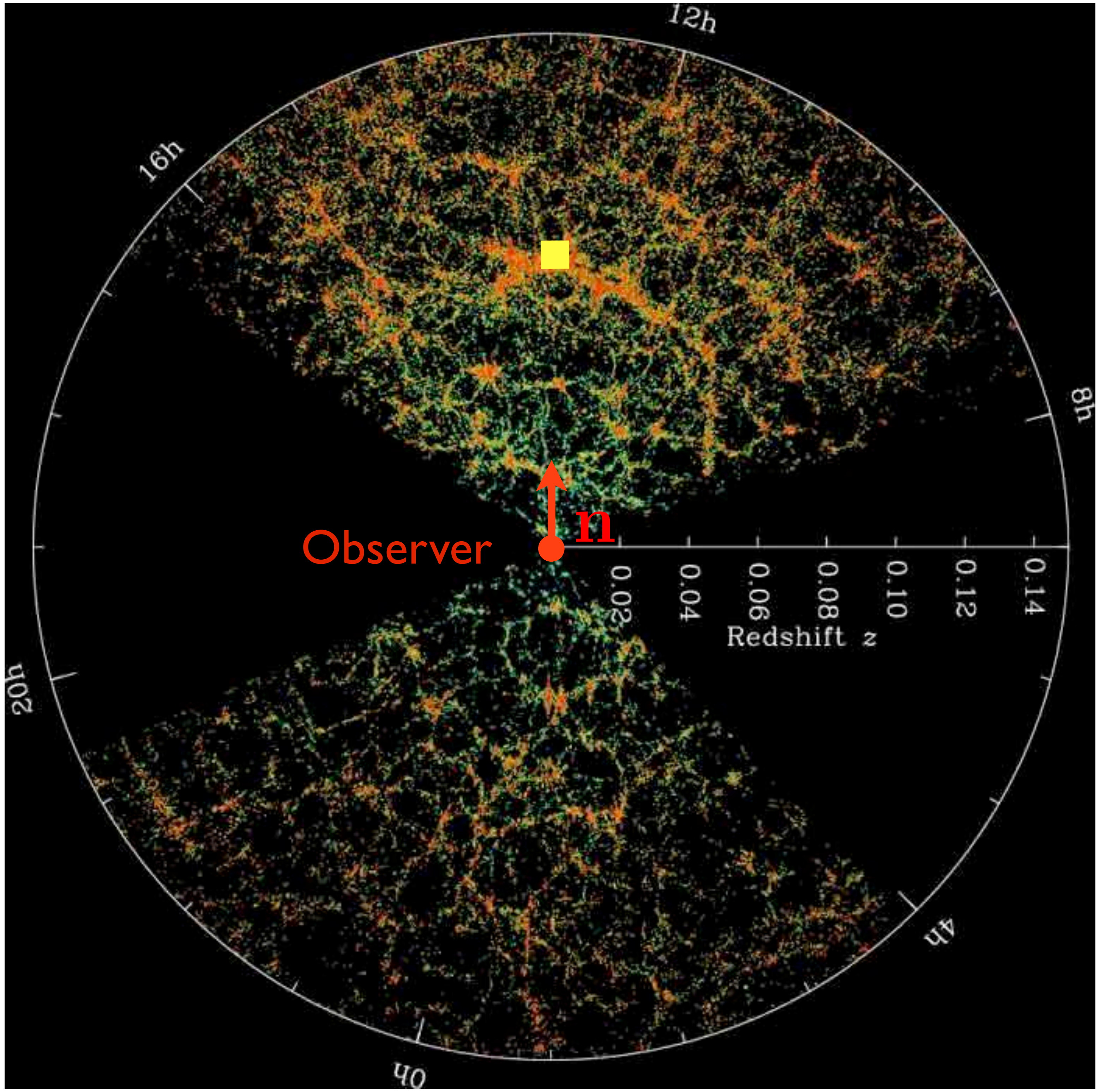
$\rightarrow$ $\boxed{b\,\delta \text{ and } V}$

SDSS, BOSS, Wigglez, eBOSS

Relativistic effects

Relativistic effects **break** the **symmetry** of ξ

Credit: M. Blanton, SDSS



Gravitational redshift

$$\Delta = \frac{1}{\mathcal{H}}\partial_r \Psi$$

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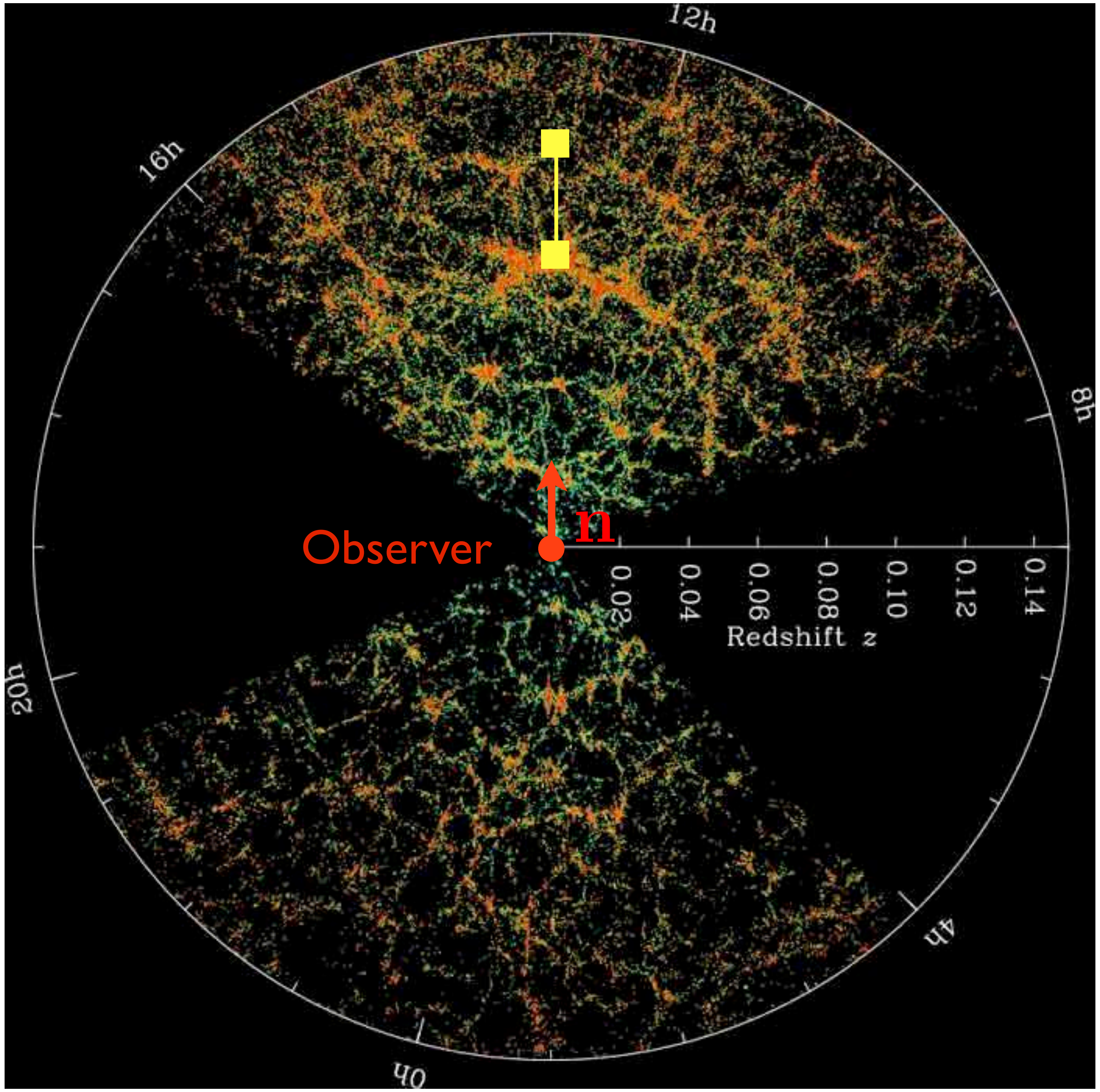

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CB, Hui & Gaztanaga (2014)

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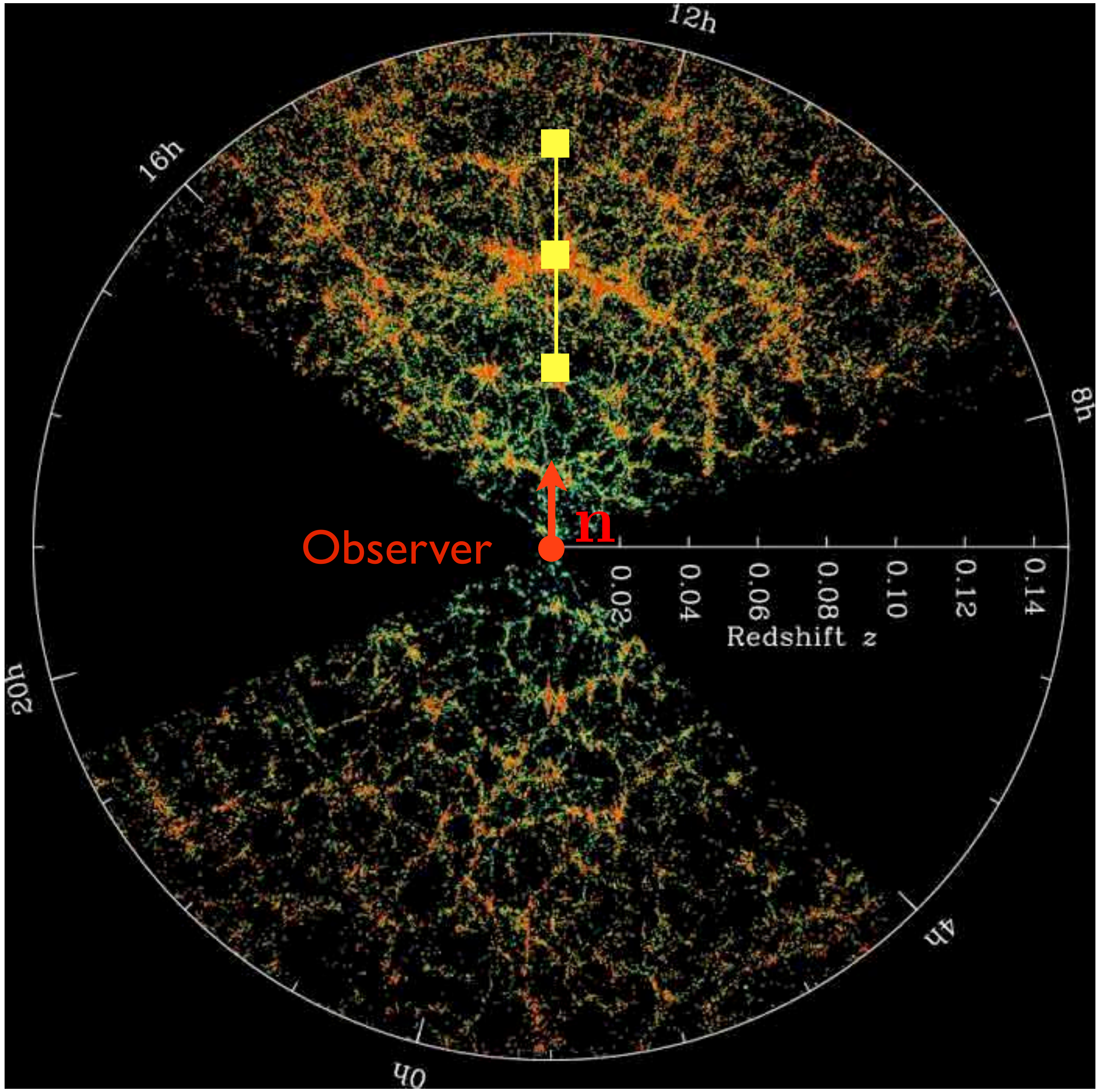

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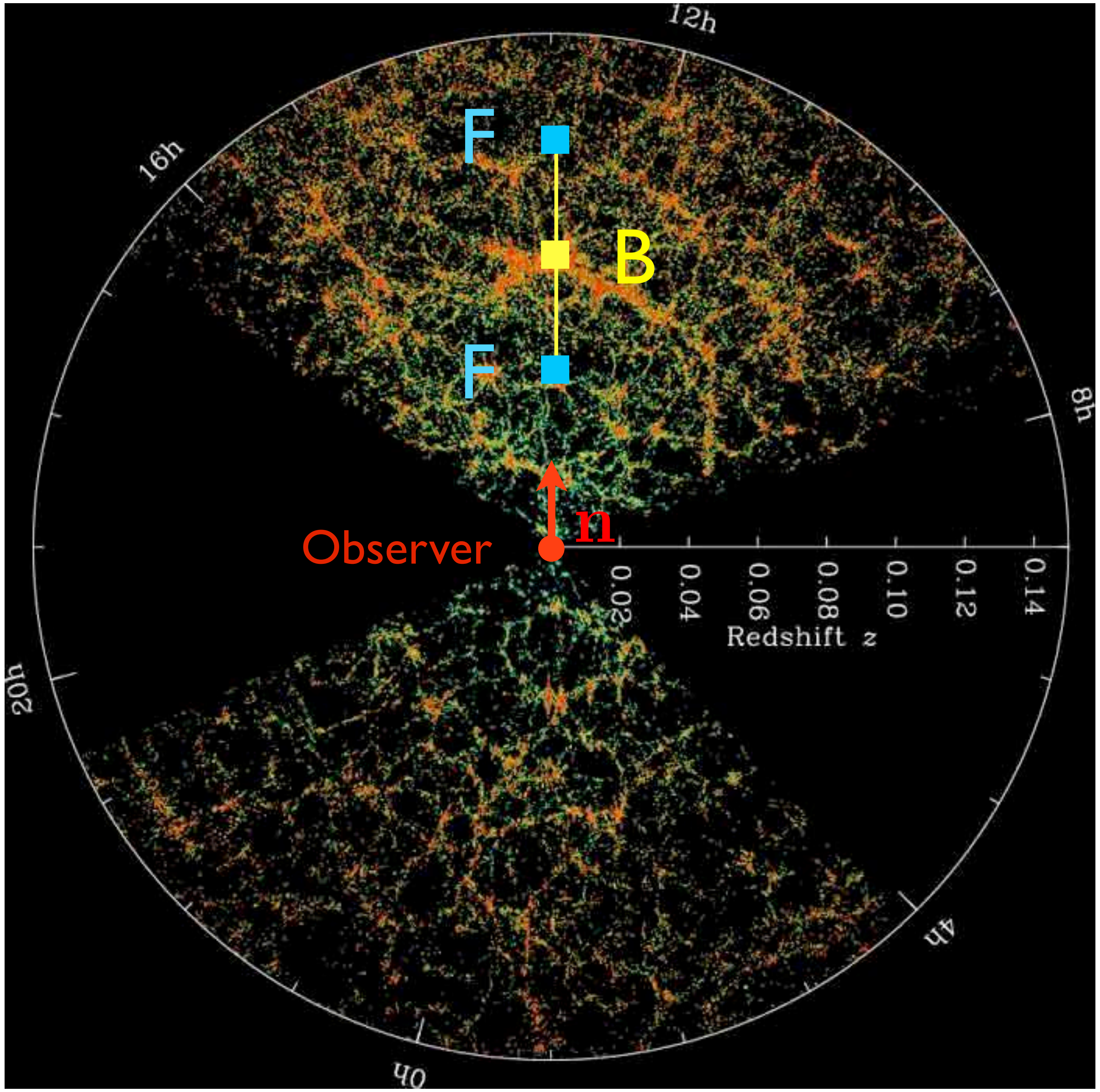

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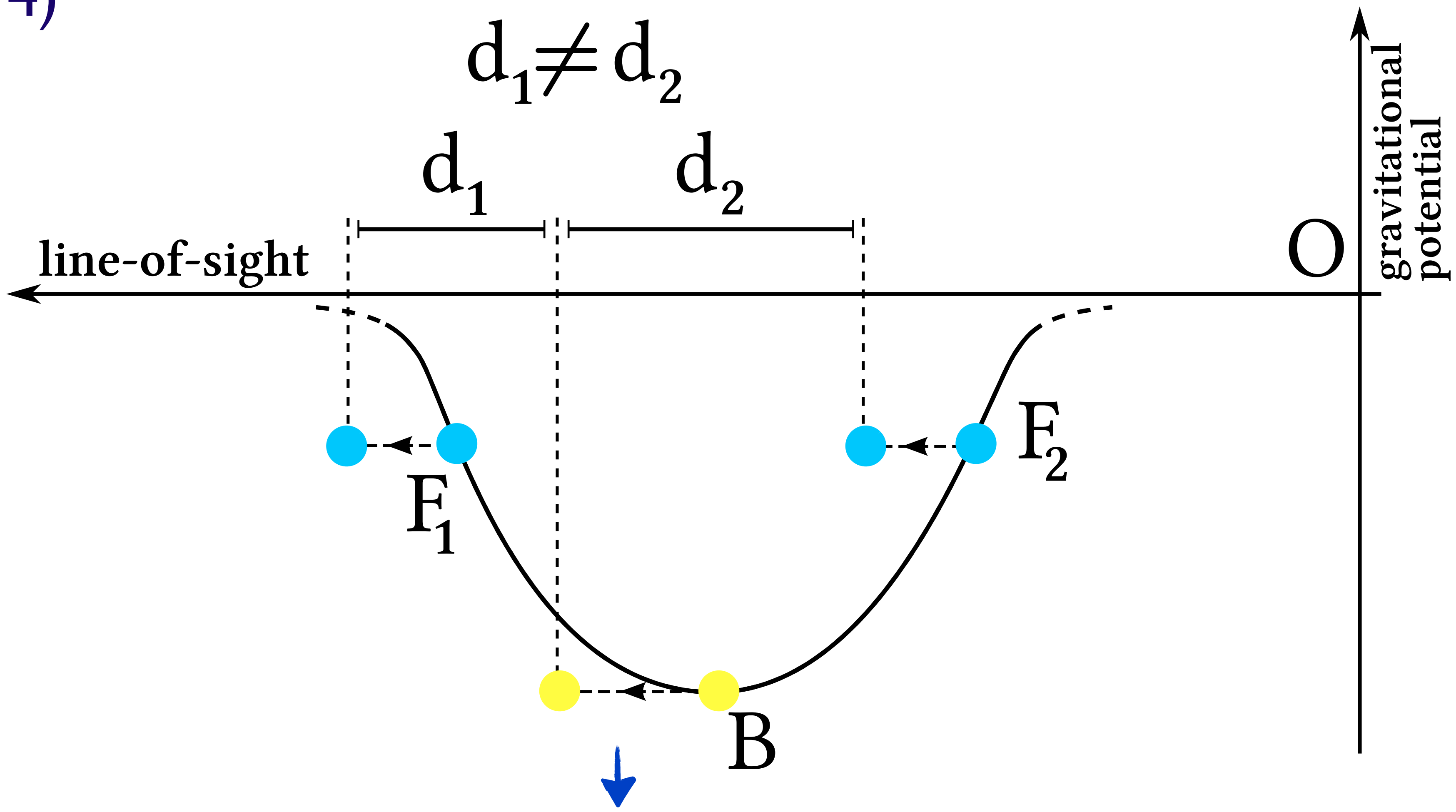
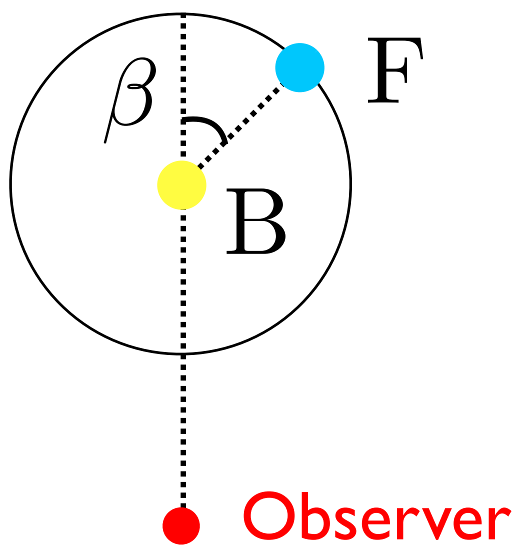
Breaking of symmetry from gravitational redshift

CB, Hui & Gaztanaga (2014)



shift in position due to gravitational redshift

Taking all pairs of galaxies into account: **dipolar** modulation



$$\xi(d,\beta) = C_1(d)\cos\beta$$

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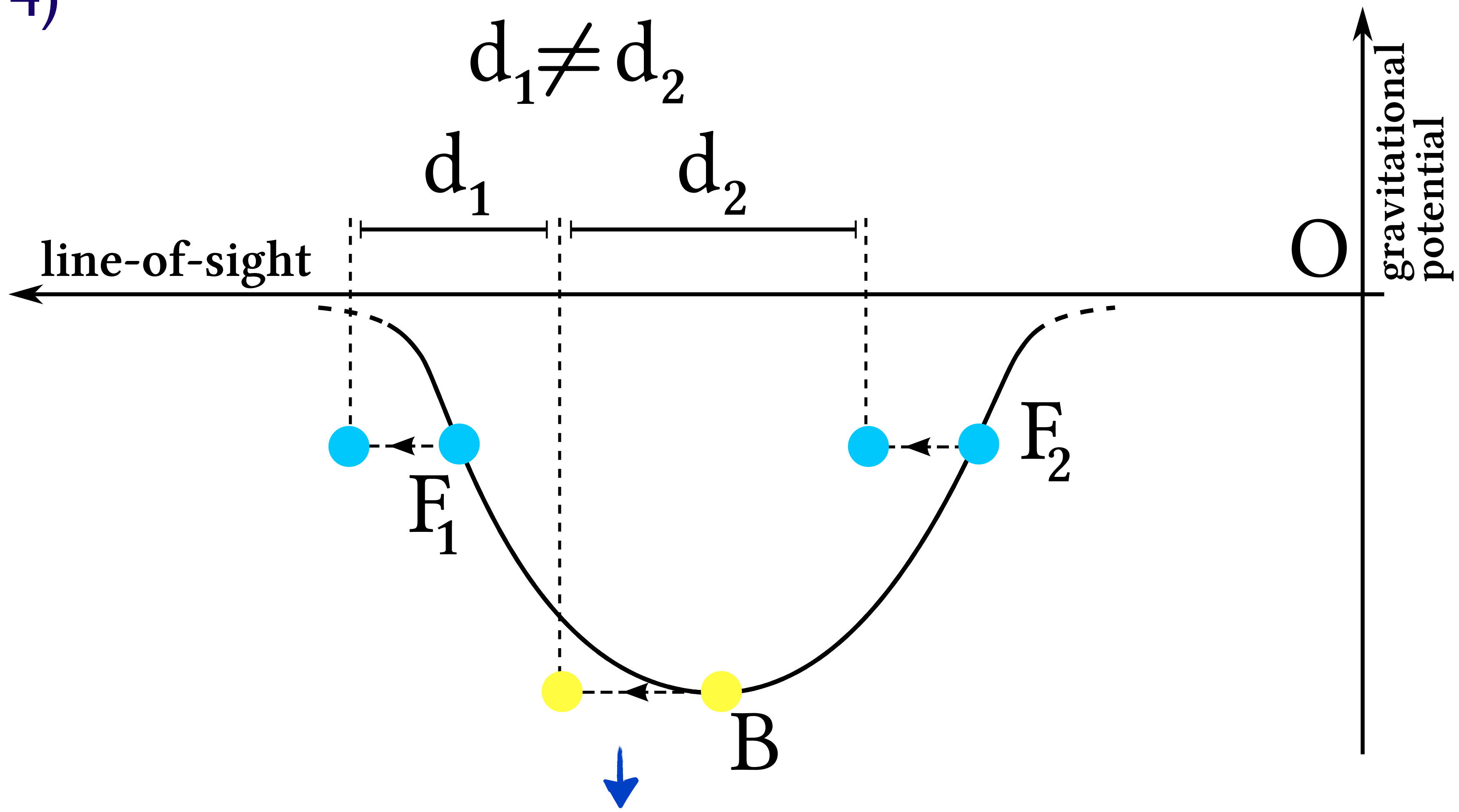
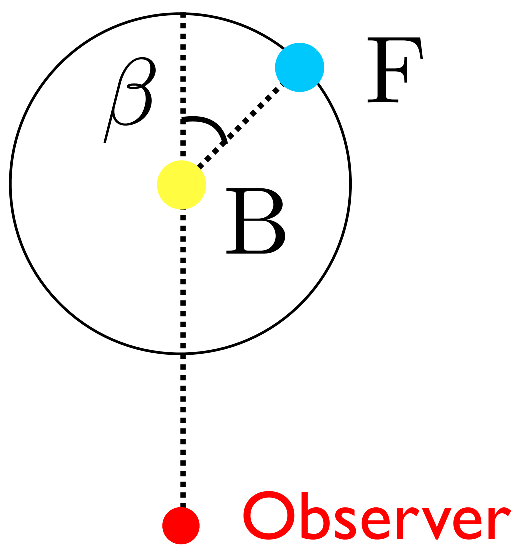


shift in position due to gravitational redshift

Taking all pairs of galaxies into account: **dipolar** modulation



We can **isolate** the effect by fitting for a dipole → $\sum_{ij} \Delta_i^{\mathrm{B}} \Delta_j^{\mathrm{F}} \cos\beta_{ij}$

Number counts

$$
\begin{aligned}
\Delta(z,\mathbf{n}) = {} & \cancel{b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})} \\
& + \cancel{(5s-2)\int_0^r dr'\frac{r-r'}{2rr'}\Delta_\Omega(\Phi+\Psi)} \\
& + \boxed{\left(1-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{5s-2}{r\mathcal{H}}-5s+f^{\mathrm{evol}}\right)\mathbf{V}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\partial_r\Psi} \\
& + \cancel{\frac{2-5s}{r}\int_0^r dr'(\Phi+\Psi) + 3\mathcal{H}\nabla^{-2}(\nabla\mathbf{V}) + \Psi + (5s-2)\Phi} \\
& + \cancel{\frac{1}{\mathcal{H}}\dot{\Phi} + \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2-5s}{r\mathcal{H}}+5s-f^{\mathrm{evol}}\right)\left[\Psi+\int_0^r dr'(\dot{\Phi}+\dot{\Psi})\right]}
\end{aligned}
$$

Dipole

Sobral-Blanco and CB (2022)

Isolating gravitational redshift

We combine the **dipole**, with measurements of redshift-space distortions (monopole, quadrupole and hexadecapole)

- Dipole → Ψ and V
- Redshift-space distortions → V and δ

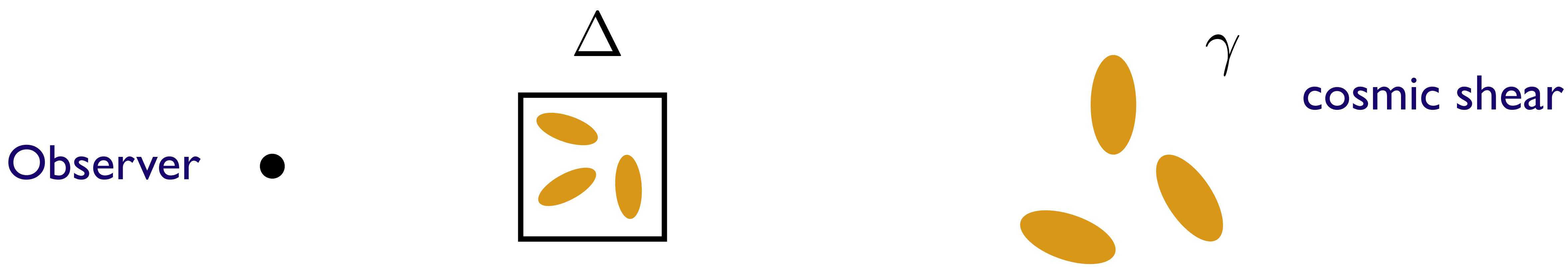
Forecasts for SKA2

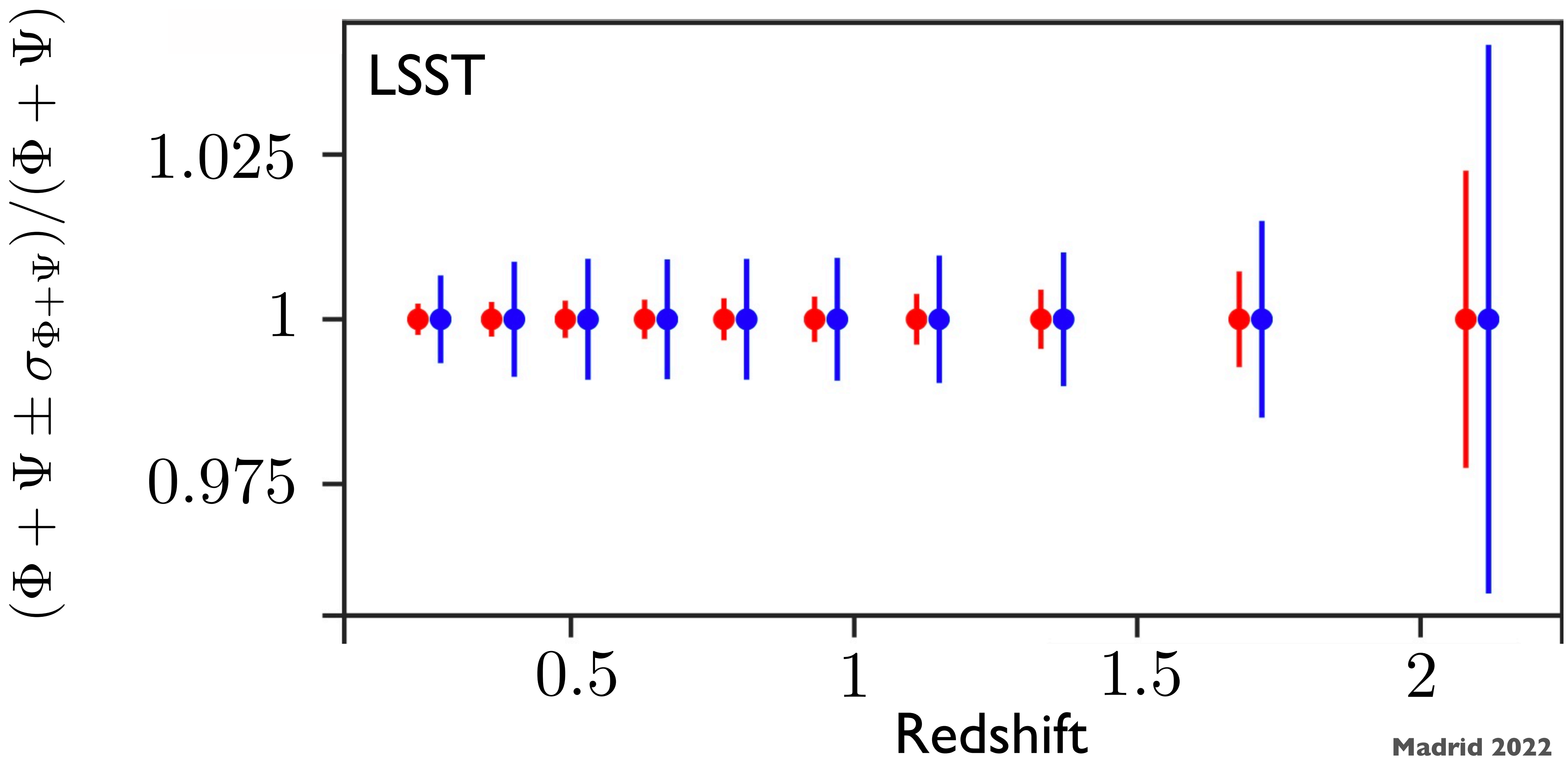
Redshift	0.35	0.45	0.55	0.65	0.75	0.85	0.95
Constraints	23%	24%	28%	33%	40%	48%	60%

Combine with lensing

Tutusaus, Sobral-Blanco and CB (2022)

Galaxy-galaxy lensing allows us to measure directly $\Phi + \Psi$





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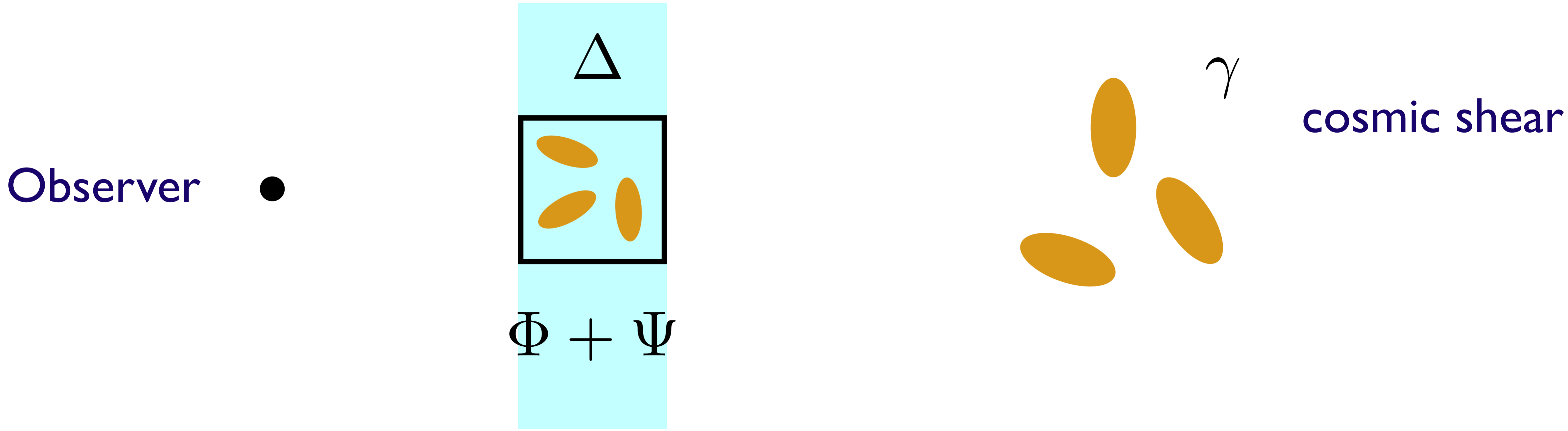
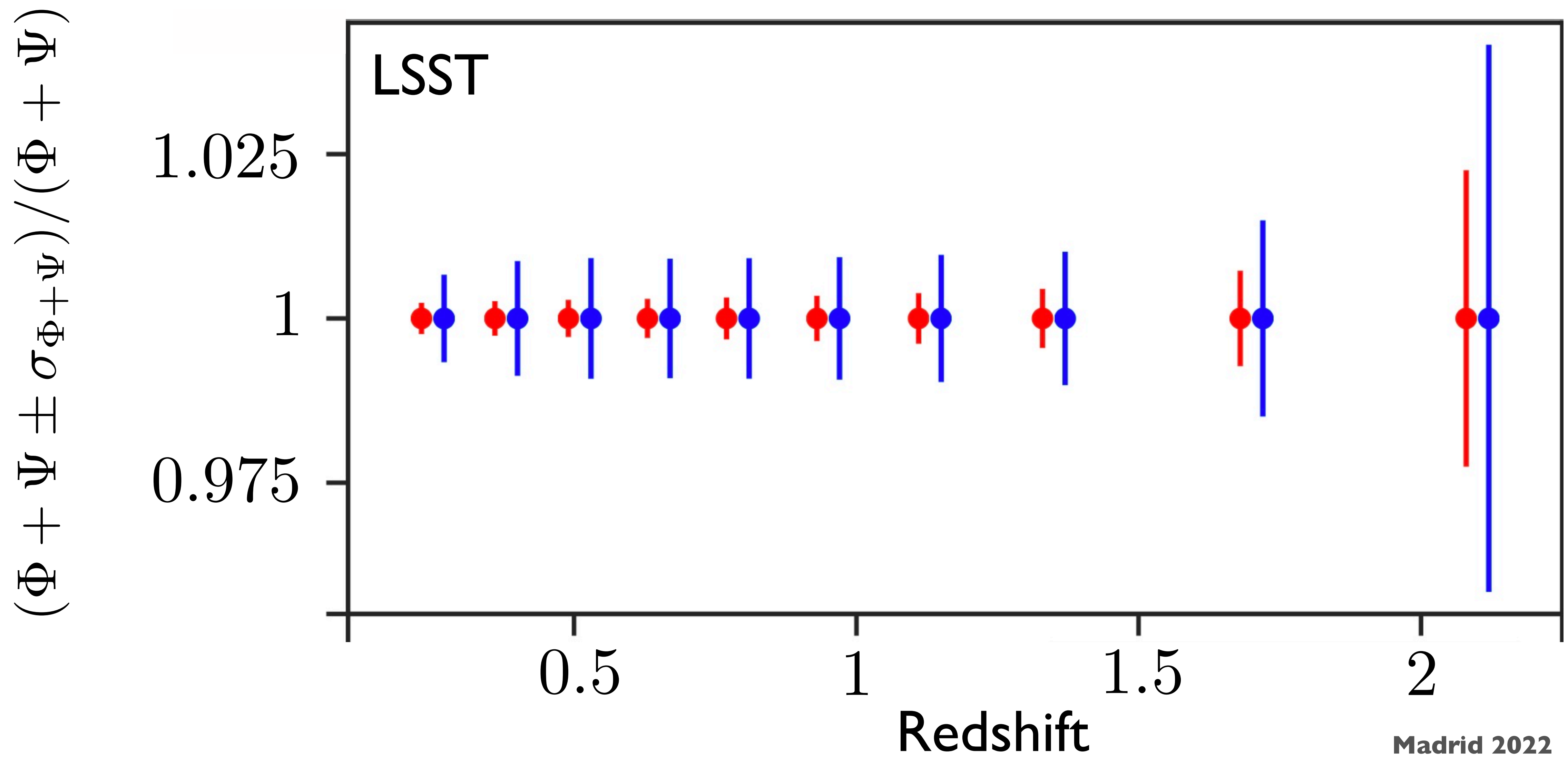
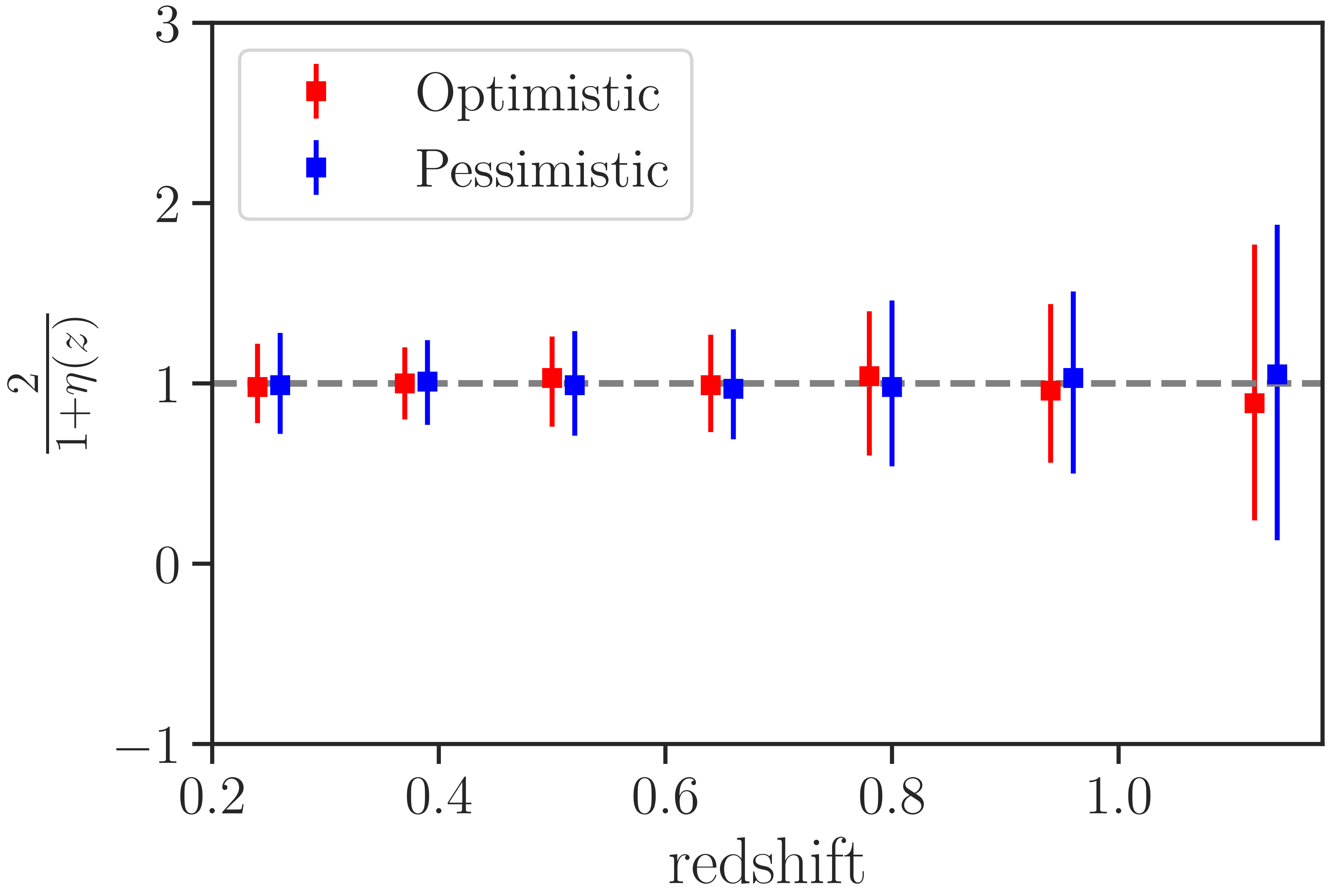




Tutusaus, Sobral-Blanco and CB (2022)

Anisotropic stress



Restore η as **smoking gun** for modified gravity

Dipole for PNG

- PNG generate **scale-dependent** corrections in the monopole and quadrupole scaling as k^{-2} and k^{-4}

- **Relativistic corrections** also generate contributions of this form in $\langle\Delta(\mathbf{k})\Delta(\mathbf{k}')\rangle$

$$\langle\delta(\mathbf{k})\Phi(\mathbf{k})\rangle \sim \left(\frac{\mathcal{H}}{k}\right)^2 P_{\delta\delta} \qquad \langle V(\mathbf{k})V(\mathbf{k})\rangle \sim \left(\frac{\mathcal{H}}{k}\right)^2 P_{\delta\delta}$$

$$\langle\Phi(\mathbf{k})\Phi(\mathbf{k})\rangle \sim \left(\frac{\mathcal{H}}{k}\right)^4 P_{\delta\delta}$$

- **Contamination**! Amplitude depends on s and f^{evol}

Number counts

$$
\begin{aligned}
\Delta(z,\mathbf{n}) = {} & b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n}) \\
& + (5s-2)\int_0^r dr'\frac{r-r'}{2rr'}\Delta_\Omega(\Phi+\Psi) \\
& + \left(1-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{5s-2}{r\mathcal{H}}-5s+f^{\mathrm{evol}}\right)\mathbf{V}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\partial_r\Psi \\
& + \frac{2-5s}{r}\int_0^r dr'(\Phi+\Psi)+3\mathcal{H}\nabla^{-2}(\nabla\mathbf{V})+\Psi+(5s-2)\Phi \\
& + \frac{1}{\mathcal{H}}\dot{\Phi}+\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2-5s}{r\mathcal{H}}+5s-f^{\mathrm{evol}}\right)\left[\Psi+\int_0^r dr'(\dot{\Phi}+\dot{\Psi})\right]
\end{aligned}
$$

Dipole for PNG

Sobral-Blanco, CB, Clarkson and Maartens, in progress

- One needs to know s and $f^{\rm evol}$ → model **luminosity function**

Wang, Beutler and Bacon (2020)

- Dipole can be used as **cross-check**

Known from RSD

Assuming ΛCDM $$\Delta^{\rm rel} = \left(\frac{5s-2}{\mathcal{H}r} - 5s - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + f^{\rm evol}\right)\mathbf{V}\cdot\mathbf{n}$$

- **No** contaminations at this order from **PNG**

$$\langle\delta(\mathbf{k})V(\mathbf{k})\rangle \sim \left(\frac{\mathcal{H}}{k}\right)P_{\delta\delta}$$

Highest contamination $k^{-3}P_{\delta\delta}$ → negligible inside horizon

Conclusion

- Relativistic effects in galaxy clustering are very useful to **test gravity** in a model-independent way

- In particular, gravitational redshift is essential to **distinguish** modified gravity from a dark fifth force

- The **isolate** the effect, we look for a **dipole** in the cross-correlation of bright and faint galaxies

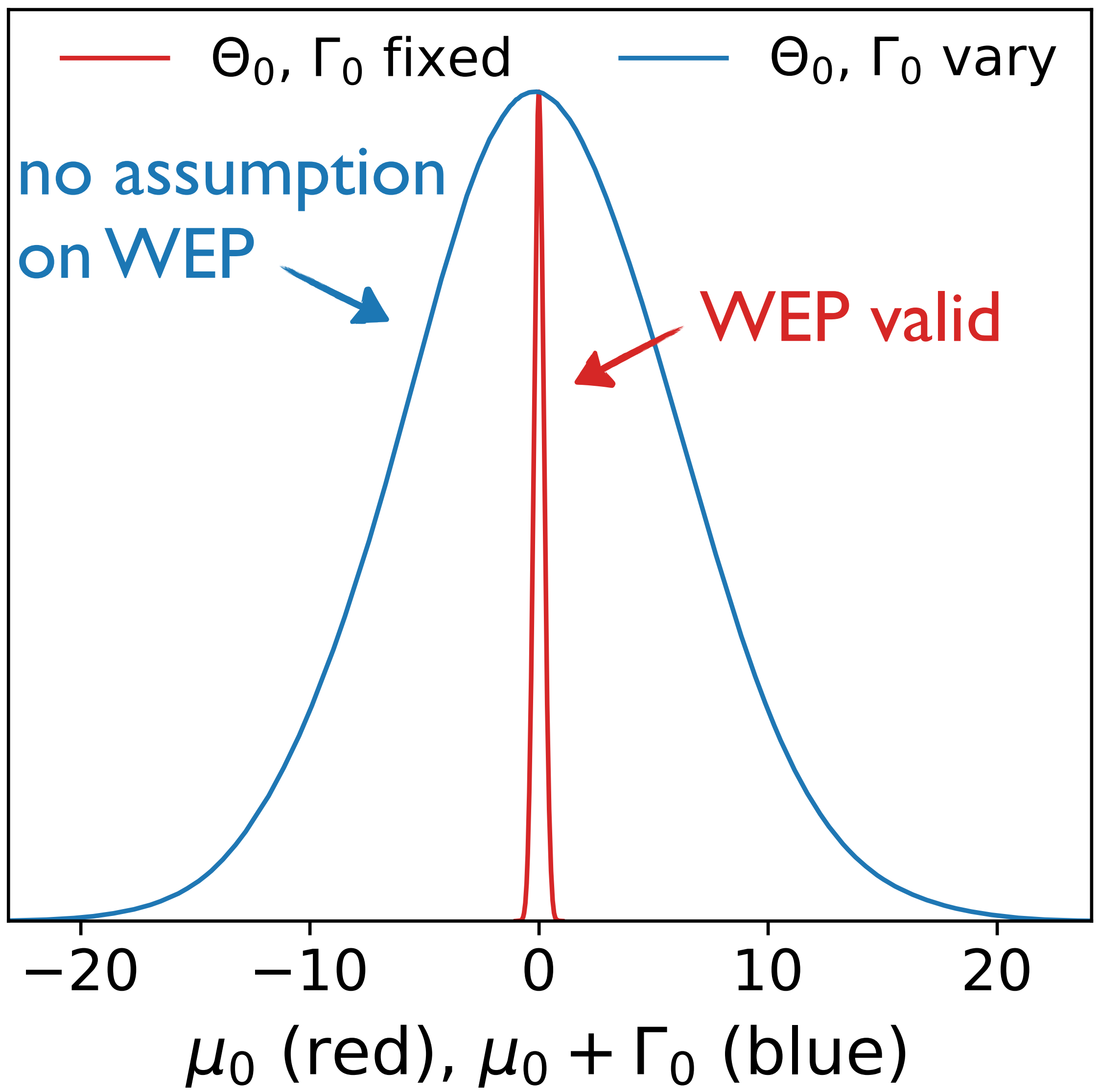
- Measuring the dipole can also help **modelling** the **contaminations** from relativistic effects to PNG

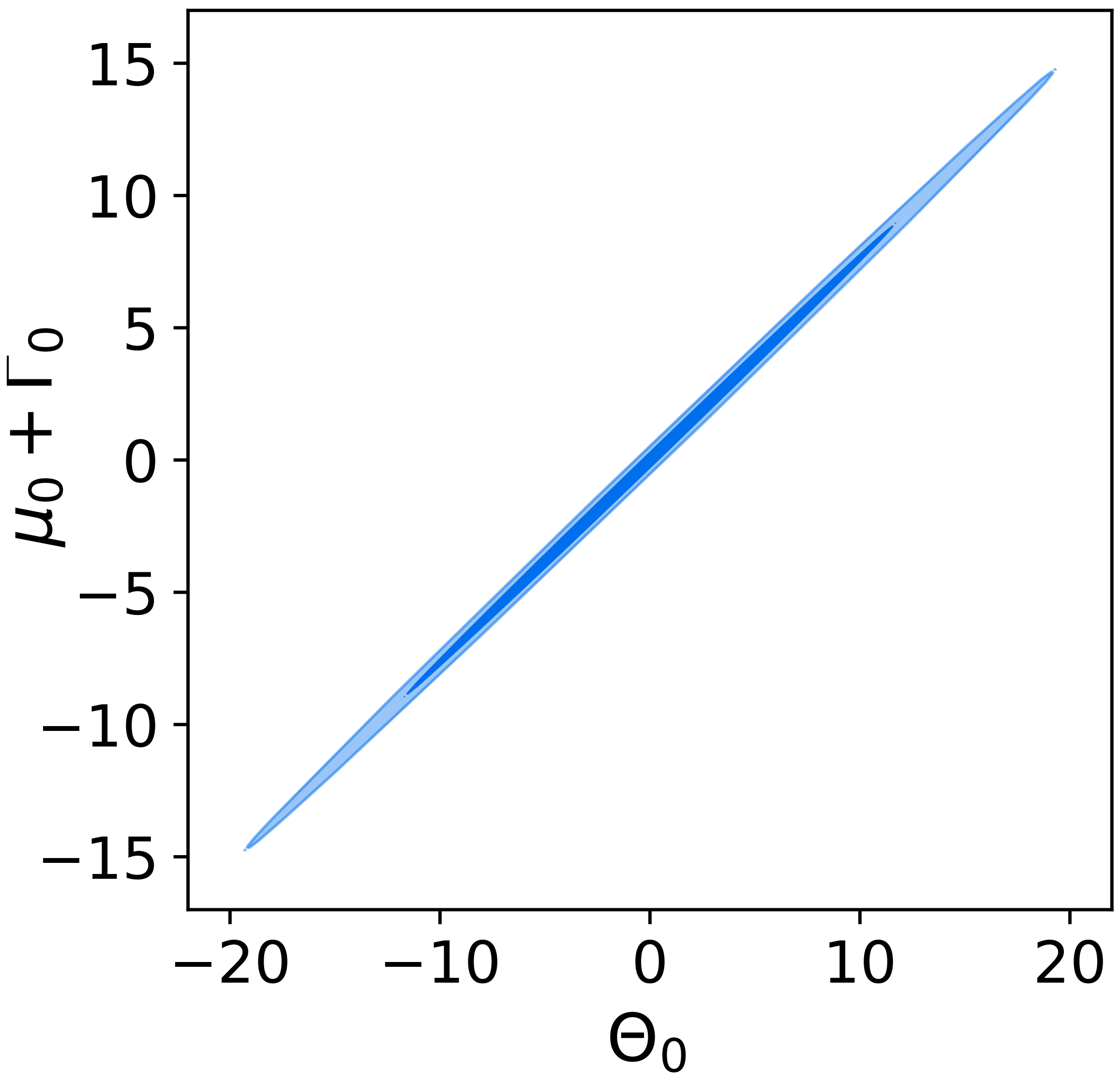
Backup slides

Current constraints from SDSS

Castello, Grimm and CB (2022)

With redshift-space distortion only, we **cannot** test the weak equivalence principle → **degeneracies**

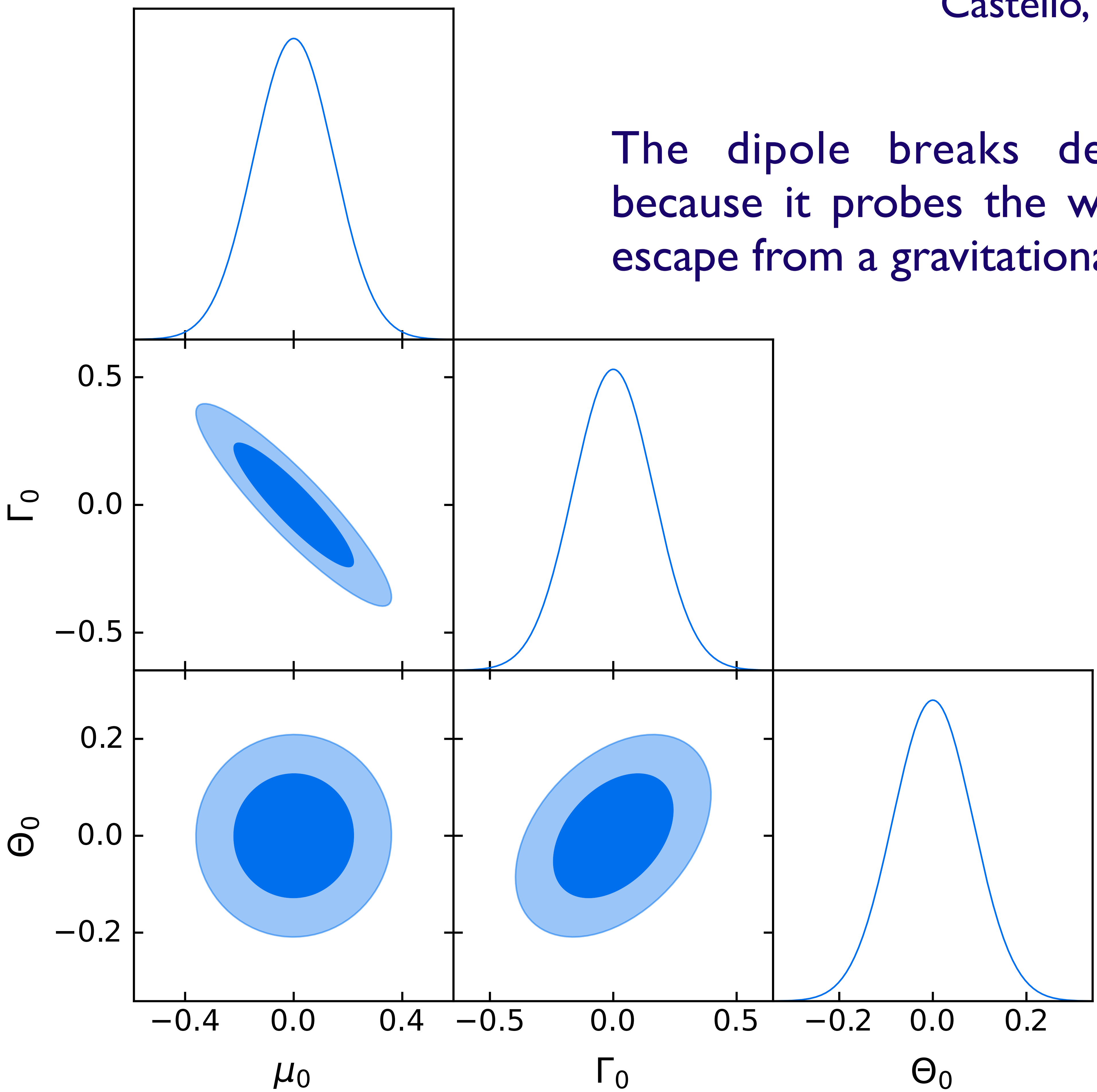




Forecasts with dipole with SKA2

Castello, Grimm and CB (2022)

The dipole breaks degeneracies because it probes the way photons escape from a gravitational potential



$$S^{\mathrm{GBD}} = \int \mathrm{d}^4\sqrt{-g}\left[\frac{A^{-2}(\phi)}{16\pi G}R - \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi) + \mathcal{L}_{\mathrm{m}}(\psi_{\mathrm{DM}}, \psi_{\mathrm{SM}}, g_{\mu\nu})\right]$$

$$S^{\mathrm{CQ}} = \int \mathrm{d}^4\sqrt{-g}\left[\frac{1}{16\pi G}R - \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi) + \mathcal{L}_{\mathrm{SM}}(\psi_{\mathrm{SM}}, g_{\mu\nu}) + \mathcal{L}_{\mathrm{DM}}(\psi_{\mathrm{DM}}, A^2(\phi)g_{\mu\nu})\right]$$

Generalized Brans-Dicke (GBD)

$$k^2\Phi = -4\pi G a^2\left(\rho_b\delta_b + \rho_c\delta_c\right) - \beta k^2\delta\phi \tag{4}$$
$$k^2(\Phi - \Psi) = -2\beta k^2\delta\phi \tag{5}$$
$$\dot{\delta}_b + \theta_b = 0 \tag{6}$$
$$\dot{\theta}_b + \mathcal{H}\theta_b = k^2\Psi \tag{7}$$
$$\dot{\delta}_c + \theta_c = 0 \tag{8}$$
$$\dot{\theta}_c + \mathcal{H}\theta_c = k^2\Psi \tag{9}$$
$$\delta\phi = -\frac{\beta(\rho_c\delta_c + \rho_b\delta_b)}{m^2 + k^2/a^2} \tag{10}$$
$$\Box\phi = V_{,\phi} + \beta(\rho_c + \rho_b) \equiv V^{\mathrm{eff}}{}_{,\phi} \tag{11}$$
$$\ddot{\delta}_m + \mathcal{H}\dot{\delta}_m = 4\pi G a^2\rho_m\delta_m\left[1 + \frac{2\tilde{\beta}^2k^2}{a^2m^2 + k^2}\right] \tag{12}$$

Coupled Quintessence (CQ)

$$k^2\Phi = -4\pi G a^2\left(\rho_b\delta_b + \rho_c\delta_c\right) \tag{13}$$
$$k^2(\Phi - \Psi) = 0 \tag{14}$$
$$\dot{\delta}_b + \theta_b = 0 \tag{15}$$
$$\dot{\theta}_b + \mathcal{H}\theta_b = k^2\Psi \tag{16}$$
$$\dot{\delta}_c + \theta_c = 0 \tag{17}$$
$$\dot{\theta}_c + (\mathcal{H} + \beta\dot{\phi})\theta_c = k^2\Psi + k^2\beta\delta\phi \tag{18}$$
$$\delta\phi = -\frac{\beta\rho_c\delta_c}{m^2 + k^2/a^2} \tag{19}$$
$$\Box\phi = V_{,\phi} + \beta\rho_c \equiv V^{\mathrm{eff}}{}_{,\phi} \tag{20}$$
$$\ddot{\delta}_m + \mathcal{H}\dot{\delta}_m = 4\pi G a^2\rho_m\delta_m\left[1 + \frac{2\tilde{\beta}^2k^2}{a^2m^2 + k^2}\left(\frac{\rho_c}{\rho_m}\right)^2\left(\frac{\delta_c}{\delta_m}\right)\right] \tag{21}$$

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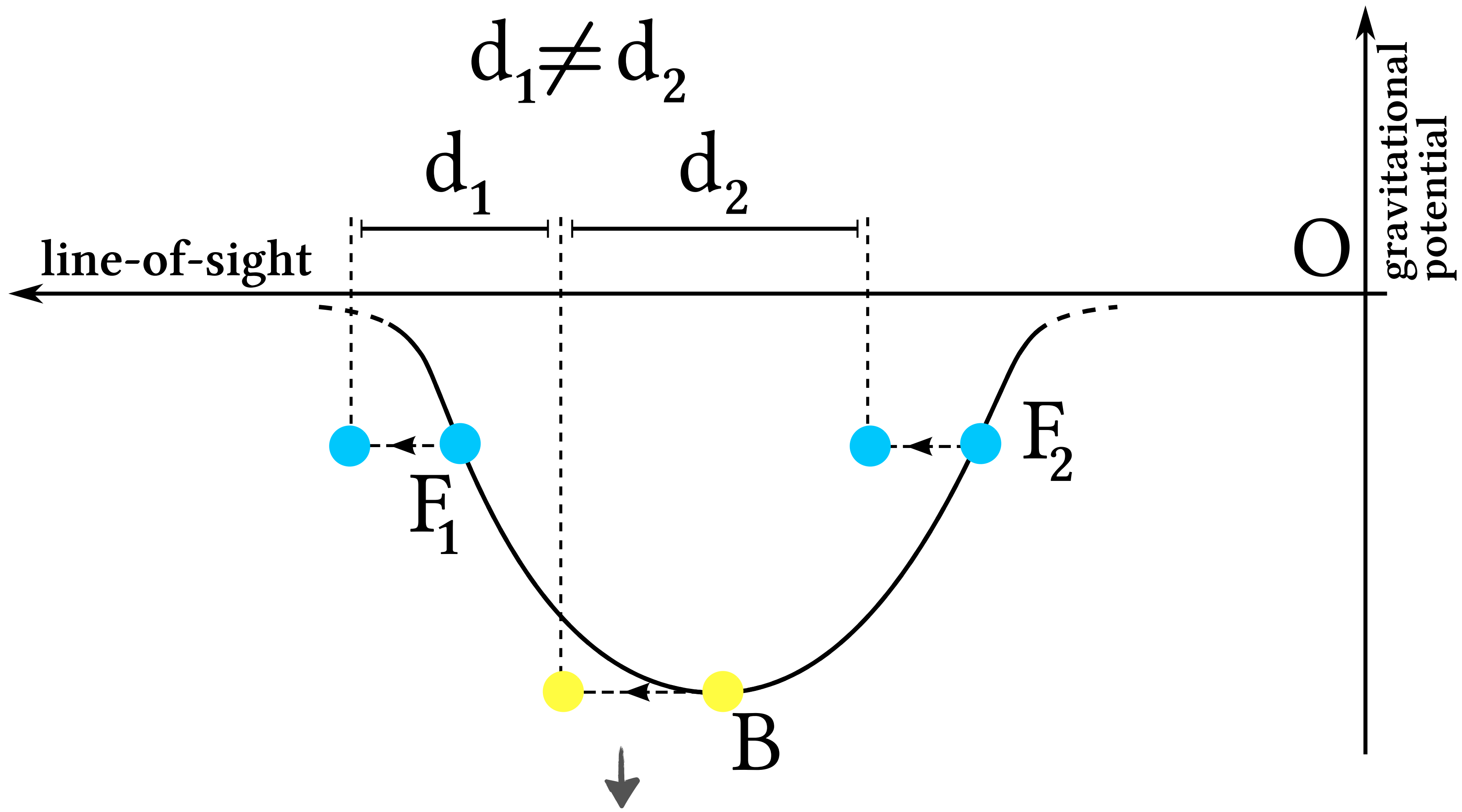

shift in position due to gravitational redshift

$$\xi = \frac{\mathcal{H}}{\mathcal{H}_0}\left(\frac{D_1}{D_{10}}\right)^2 \Big[(b_{\rm B}-b_{\rm F})\left(\frac{2}{r\mathcal{H}}+\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right)+3(s_{\rm F}-s_{\rm B})f^2\left(1-\frac{1}{r\mathcal{H}}\right)$$
$$+5(b_{\rm B}s_{\rm F}-b_{\rm F}s_{\rm B})f\left(1-\frac{1}{r\mathcal{H}}\right)\Big]\nu_1(d)\cos(\beta)$$