Reconstructing the inflaton's speed of sound using cosmological data

arXiv:2012.04640

Guadalupe Cañas-Herrera, Jesús Torrado, Ana Achúcarro

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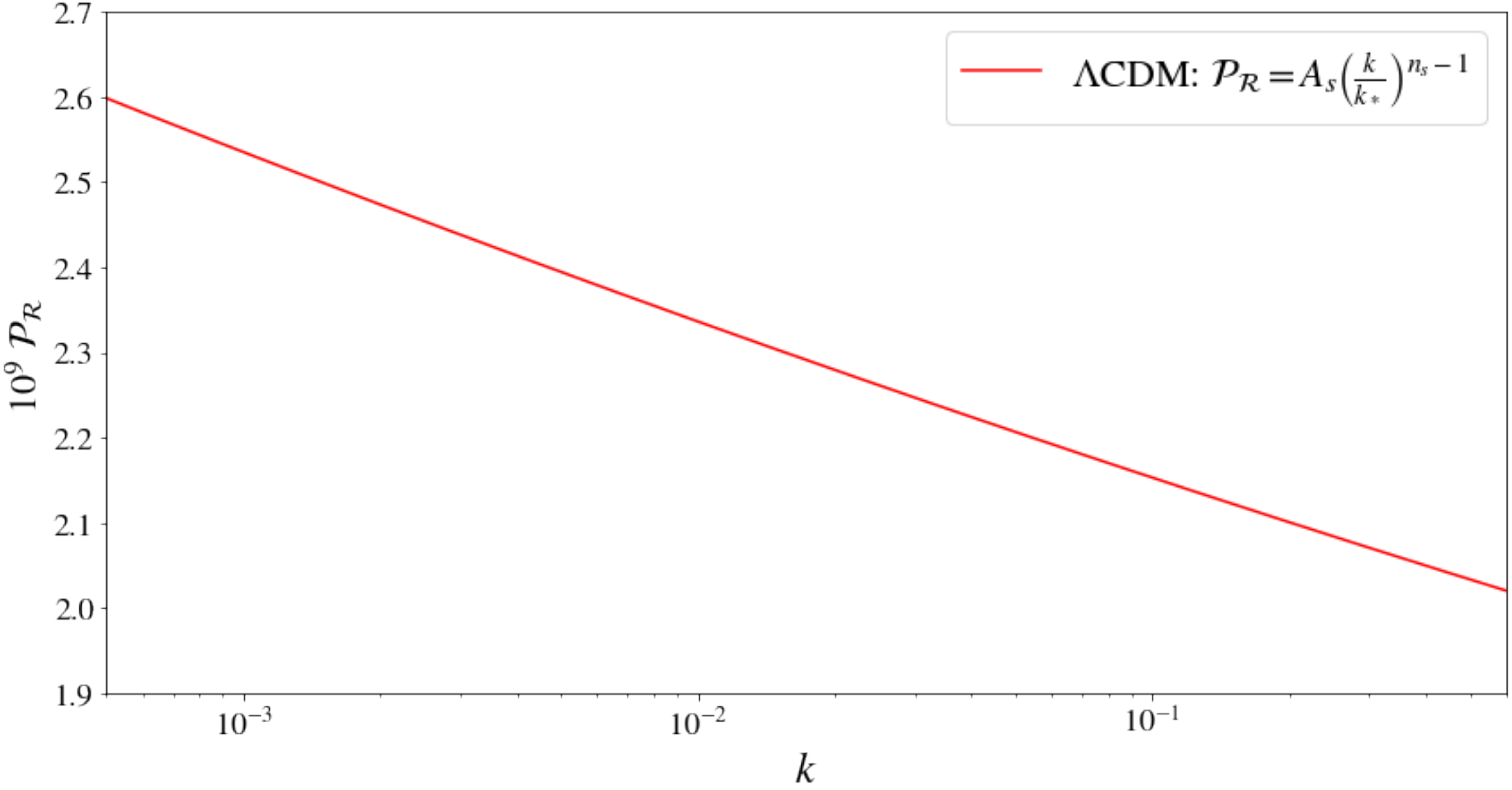
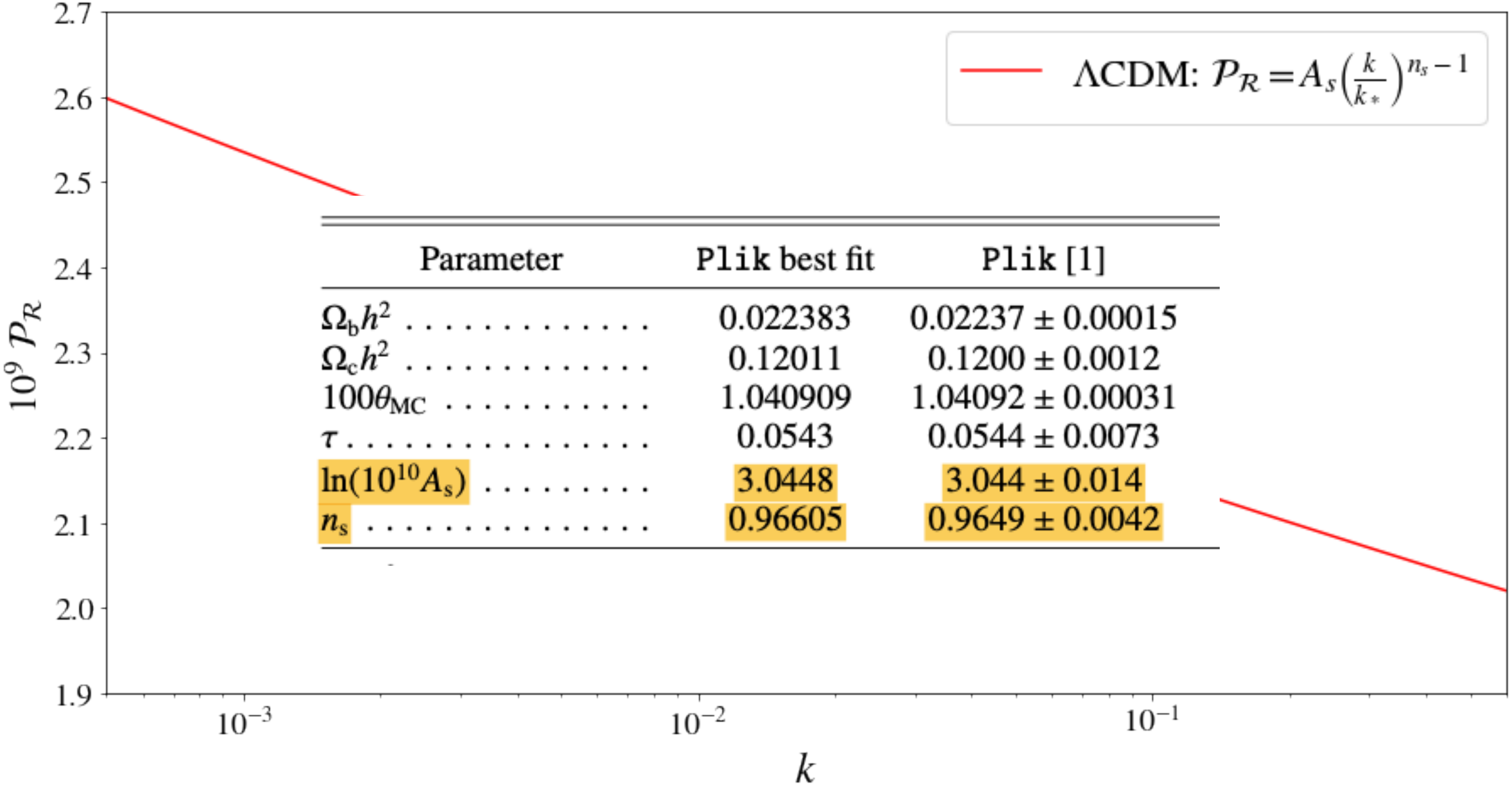
Primordial power spectrum



GCH + Planck 2018

Primordial power spectrum



Parameter	`Plik` best fit	`Plik` [1]
$\Omega_b h^2$	0.022383	0.02237 ± 0.00015
$\Omega_c h^2$	0.12011	0.1200 ± 0.0012
$100\theta_{MC}$	1.040909	1.04092 ± 0.00031
τ	0.0543	0.0544 ± 0.0073
$\ln(10^{10}A_s)$	3.0448	3.044 ± 0.014
n_s	0.96605	0.9649 ± 0.0042

GCH + Planck 2018

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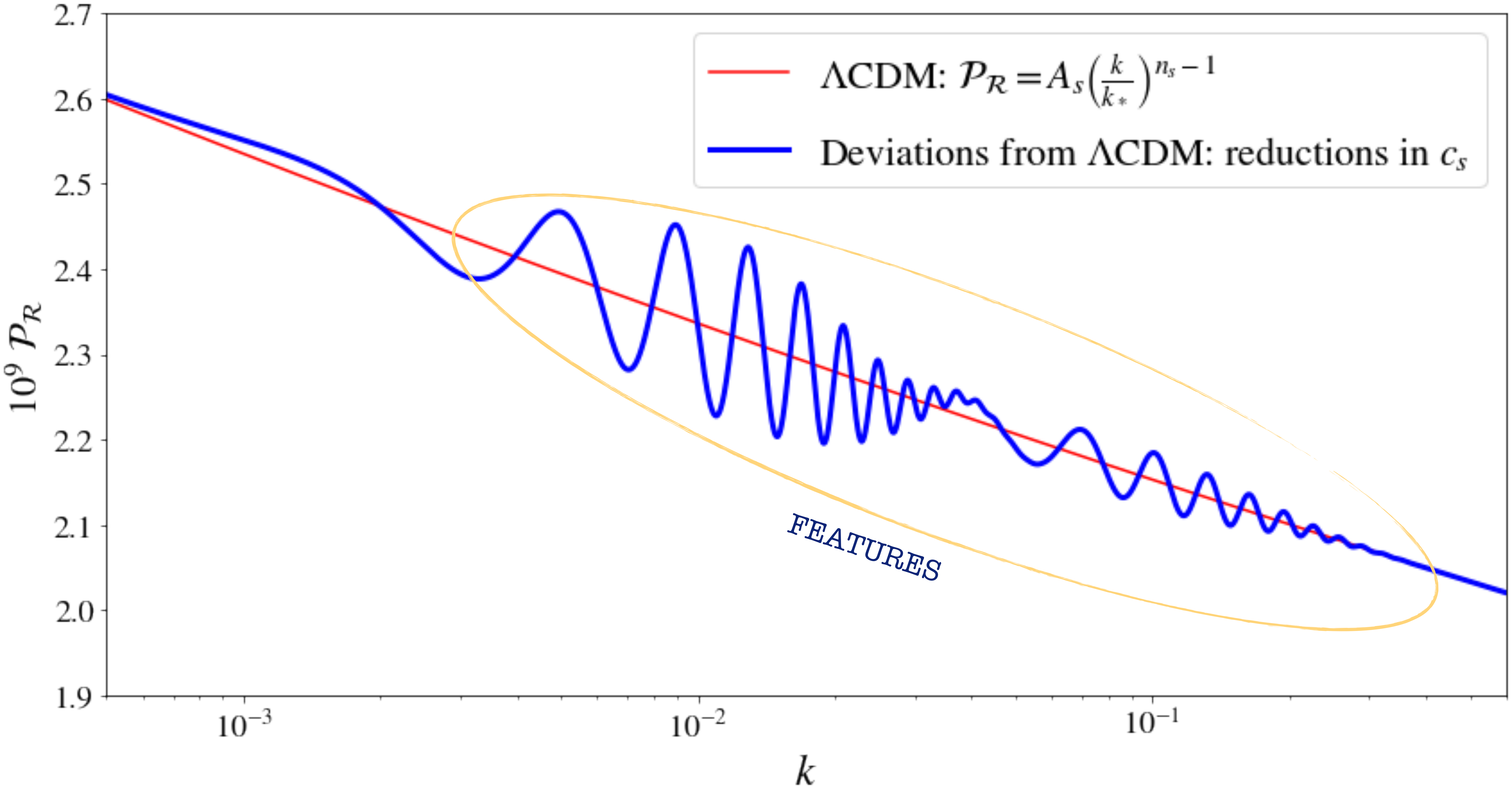


GCH

We know that...

- ΛCDM consistent with Planck 2018 and LSS data

But…

- We know that ΛCDM does not explain everything we observe (i.e: CMB low multipole feature)
- Renewed interest in multi-field inflation recently
- How do we test deviations from ΛCDM and still try to be close to what the data likes?

Rely on an Effective Field Theory

Single field vs. effective single field

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

terms of

perturbations

$$S_2 = M_P^2\int d^4x\, a^3\epsilon_1\left[\dot{\mathcal{R}}^2 - \frac{\left(\partial_i\mathcal{R}\right)^2}{a^2}\right]$$

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\gamma_{ab}(\phi)\partial_\mu\phi^a\partial_\nu\phi^b - V(\phi)\right]$$

terms of

perturbations

Integrating

heavy fields…

$$S_2 = M_P^2\int d^4x\, a^3\epsilon_1(t)\left[\frac{\dot{\mathcal{R}}^2}{c_s^2(t)} - \frac{\left(\partial_i\mathcal{R}\right)^2}{a^2}\right]$$

EFT of inflation

- Framework: effective field theory of inflationary perturbations

$$S_2 = M_P^2 \int d^4x\, a^3 \boxed{\epsilon_1(t)} \left[\frac{\dot{\mathcal{R}}^2}{\boxed{c_s^2(t)}} - \frac{(\partial_i \mathcal{R})^2}{a^2} \right]$$

Physical meaning: turns in field space

$$c_s = 1 + \frac{4\Omega^2}{\frac{k^2}{a^2} + M_{eff}^2}$$

- Features from variable slow-roll parameter

A. Durakovic, P. Hunt, S. Patil, S. Sarkar, D. Hazra, A. Starobinsky

- Features from variable sound speed

A. Achucarro, G. Palma, S. Patil, V. Atal, J. Torrado, B. Hu, J.O. Gong, P. Ortiz…
V. Miranda et al…

Reductions in the speed of sound

- **Action**

$$S_2 = M_{\mathrm{P}}^2 \int \mathrm{d}^4x\, a^3 \cancel{\epsilon_1(t)} \left[\dot{\mathcal{R}}^2 - \frac{(\partial_i \mathcal{R})^2}{a^2}\right] - \boxed{M_{\mathrm{P}}^2 \int \mathrm{d}^4x\, a^3 \cancel{\epsilon_1(t)} \left(1 - \frac{1}{c_s^2(t)}\right) \dot{\mathcal{R}}^2}$$

Small, mild and transient

- **The change in the primordial power spectrum (feature):**

$$\frac{\Delta \mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}}} = k \int_{-\infty}^{0} d\tau \boxed{\left(1 - \frac{1}{c_s^2(\tau)}\right)} \sin(2k\tau)$$

Reductions in the sound speed

- Name:

$$\boxed{u(t) := 1 - \frac{1}{c_s^2(t)}}$$

$$\boxed{\Delta\mathcal{P}_{\mathcal{R}}(k)/\mathcal{P}_{\mathcal{R}}(k)} \propto \mathrm{FourierTranform}(u)$$

Feature

- Constraints in this scenario:

$$u(t) \ll 1$$

small

$$\boxed{s(t) := \frac{\dot{c}_s(t)}{H c_s(t)}} \ll 1$$

mild

Goal

Constrain with data

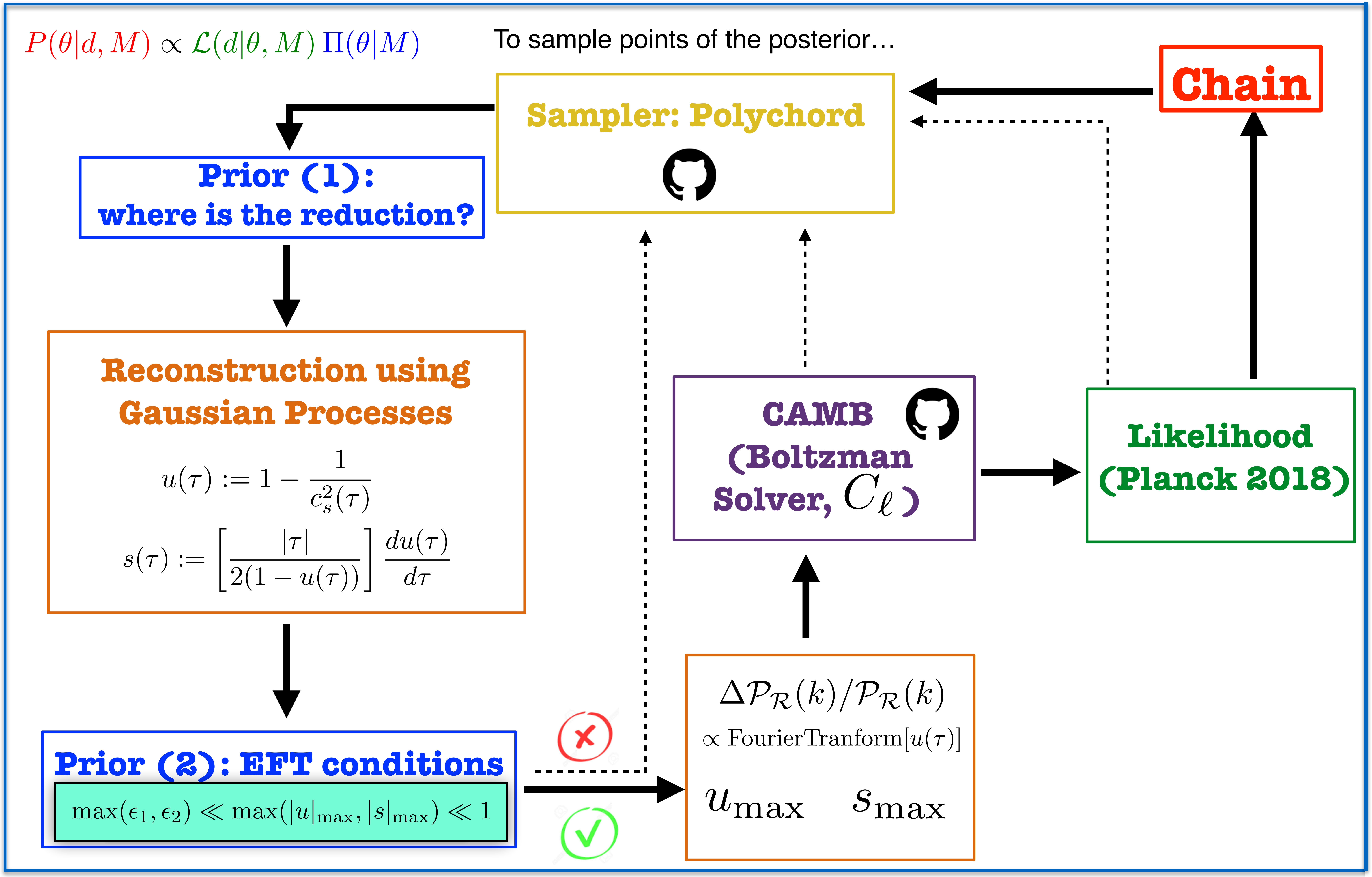
Bayesian statistics comes to rescue

- Summary:

Slow-roll condition still apply

$$\max(\epsilon_1, \epsilon_2) \ll \max(|u|_{\max}, |s|_{\max}) \ll 1$$

Methodology: pipeline



$P(\theta|d,M) \propto \mathcal{L}(d|\theta,M)\,\Pi(\theta|M)$

To sample points of the posterior…

Sampler: Polychord

Chain

Prior (1): where is the reduction?

Reconstruction using Gaussian Processes

$$u(\tau) := 1 - \frac{1}{c_s^2(\tau)}$$

$$s(\tau) := \left[\frac{|\tau|}{2(1-u(\tau))}\right]\frac{du(\tau)}{d\tau}$$

CAMB (Boltzman Solver, C_ℓ)

Likelihood (Planck 2018)

Prior (2): EFT conditions

$\max(\epsilon_1, \epsilon_2) \ll \max(|u|_{\max}, |s|_{\max}) \ll 1$

✗

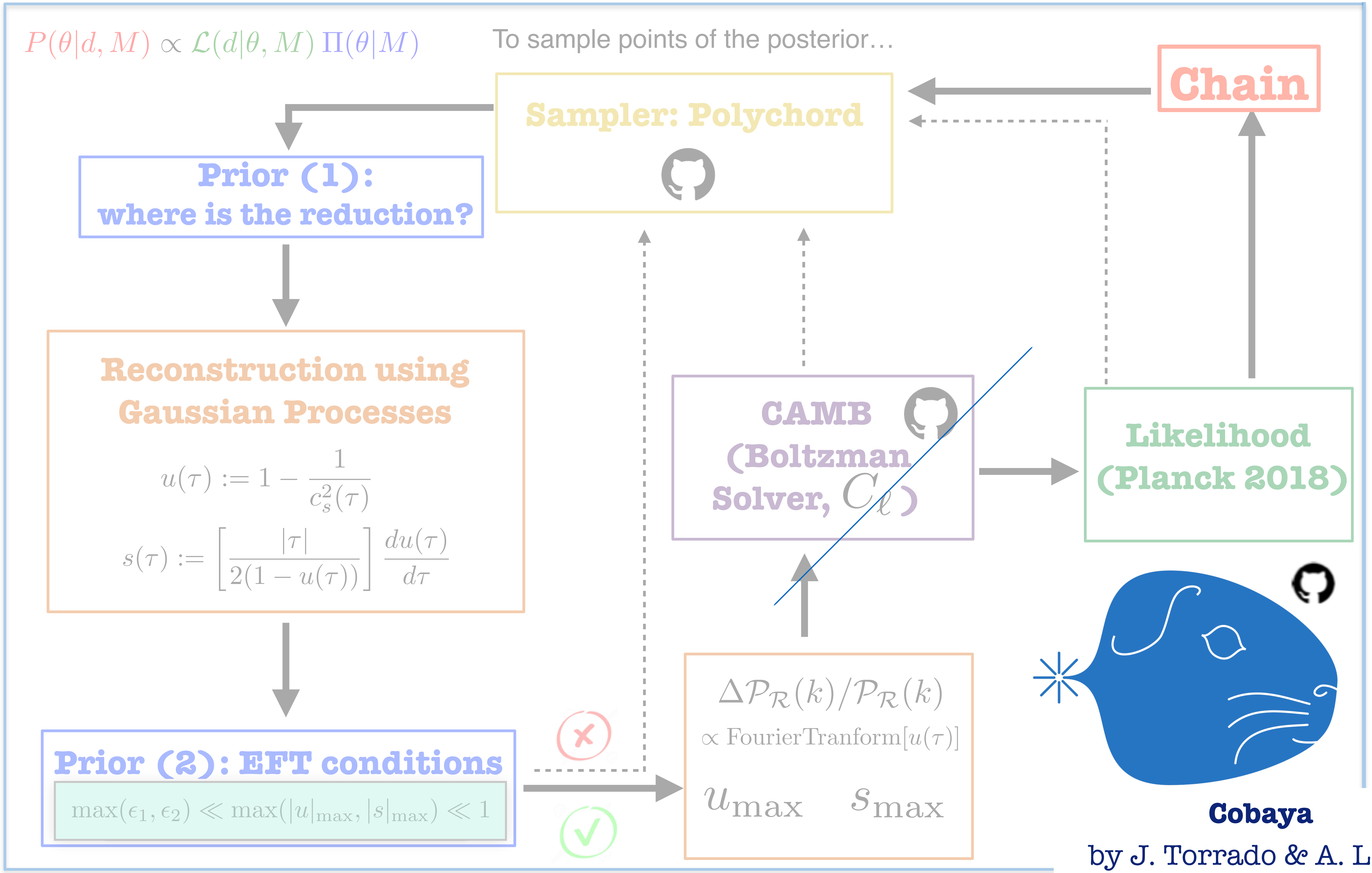
✓

$\Delta\mathcal{P}_\mathcal{R}(k)/\mathcal{P}_\mathcal{R}(k)$

$\propto$ FourierTranform$[u(\tau)]$

$u_{\max}$ $s_{\max}$

Methodology: pipeline



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Likelihood (Planck 2018)

Cobaya

by J. Torrado & A. Lewis

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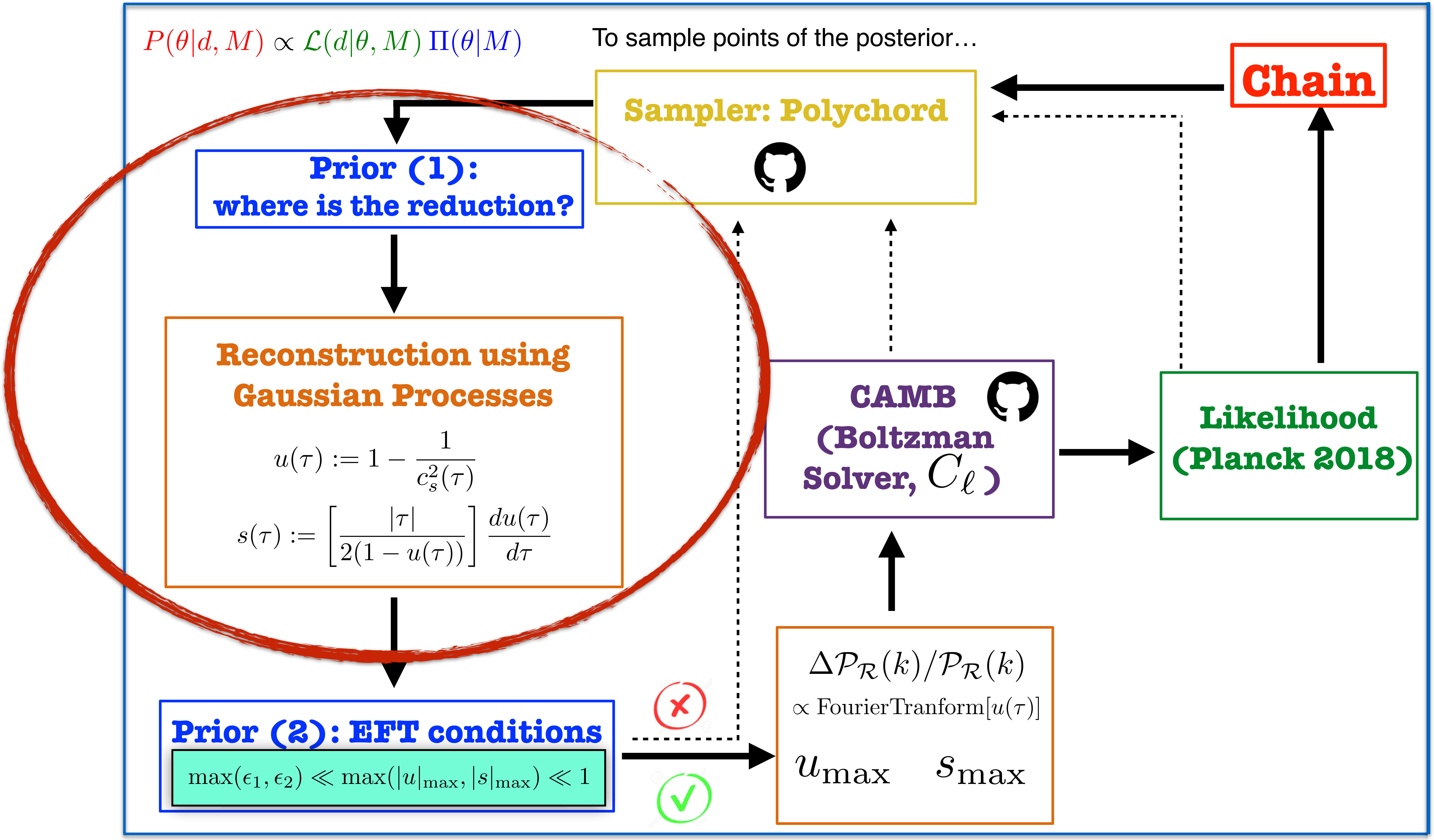

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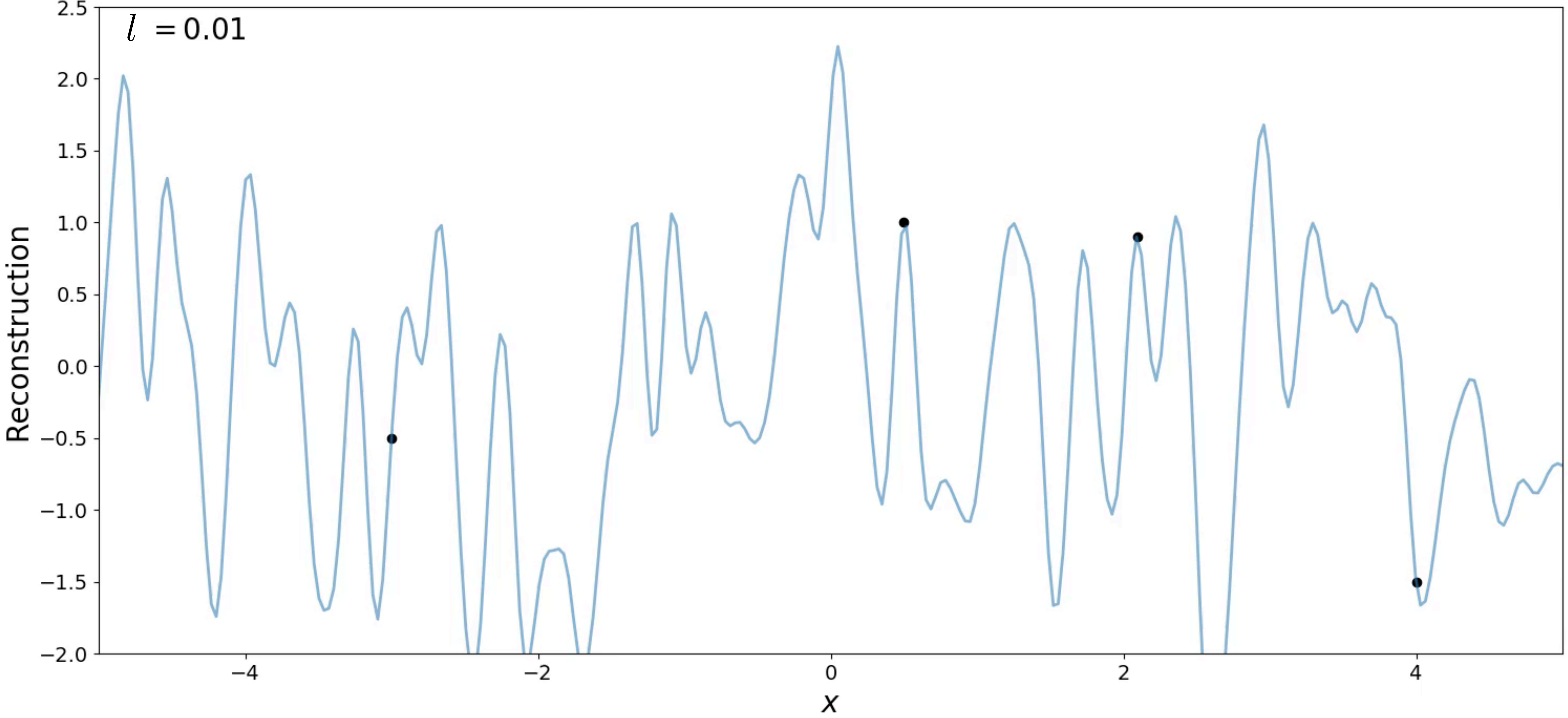
$u_{\max}$ $s_{\max}$

Regression method: Gaussian Processes

Kernel $\boxed{K(x_1 - x_2)} = \exp\left(\frac{-|x_1 - x_2|}{\boxed{2l}}\right)$ Correlation length



Course: Data Basis and Data Mining/**J. Brinchmann + GCH**

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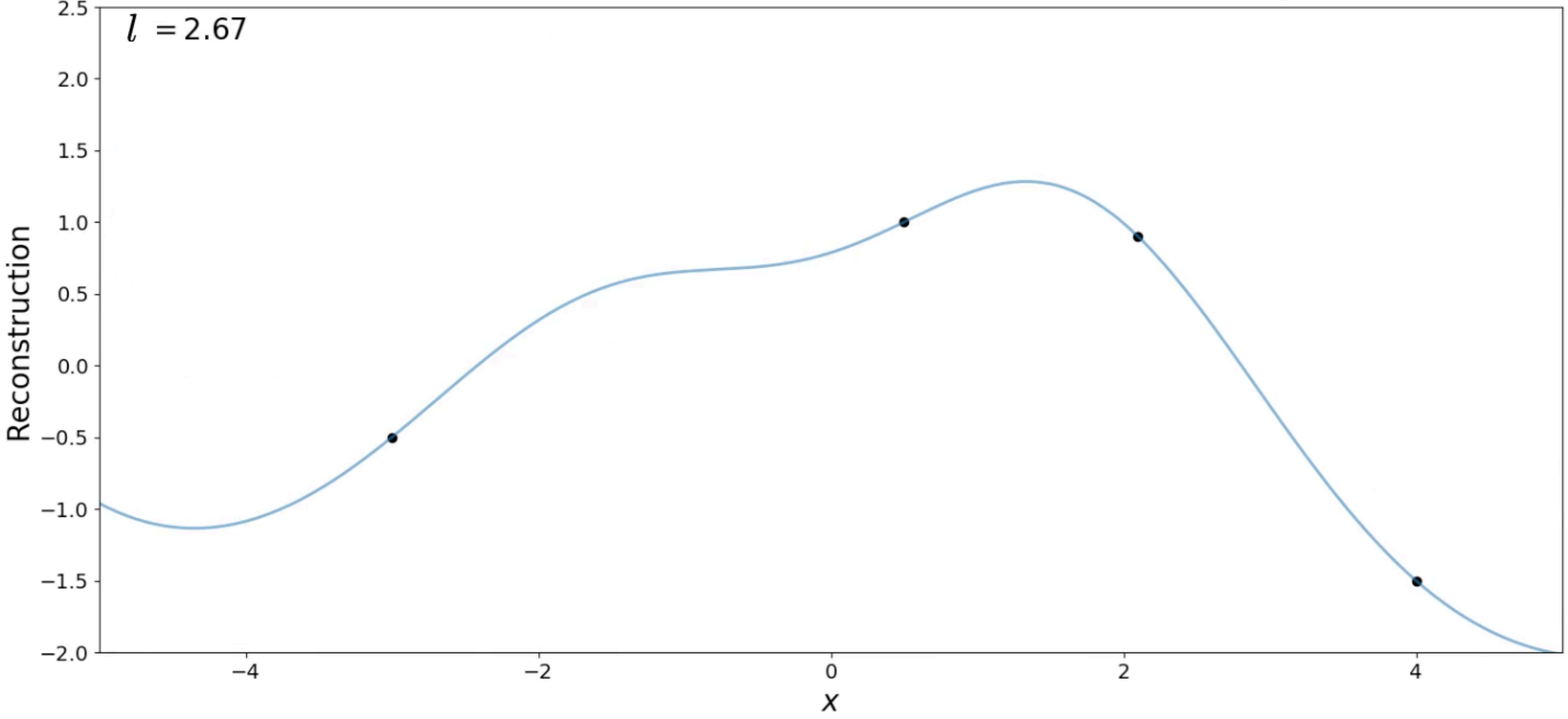

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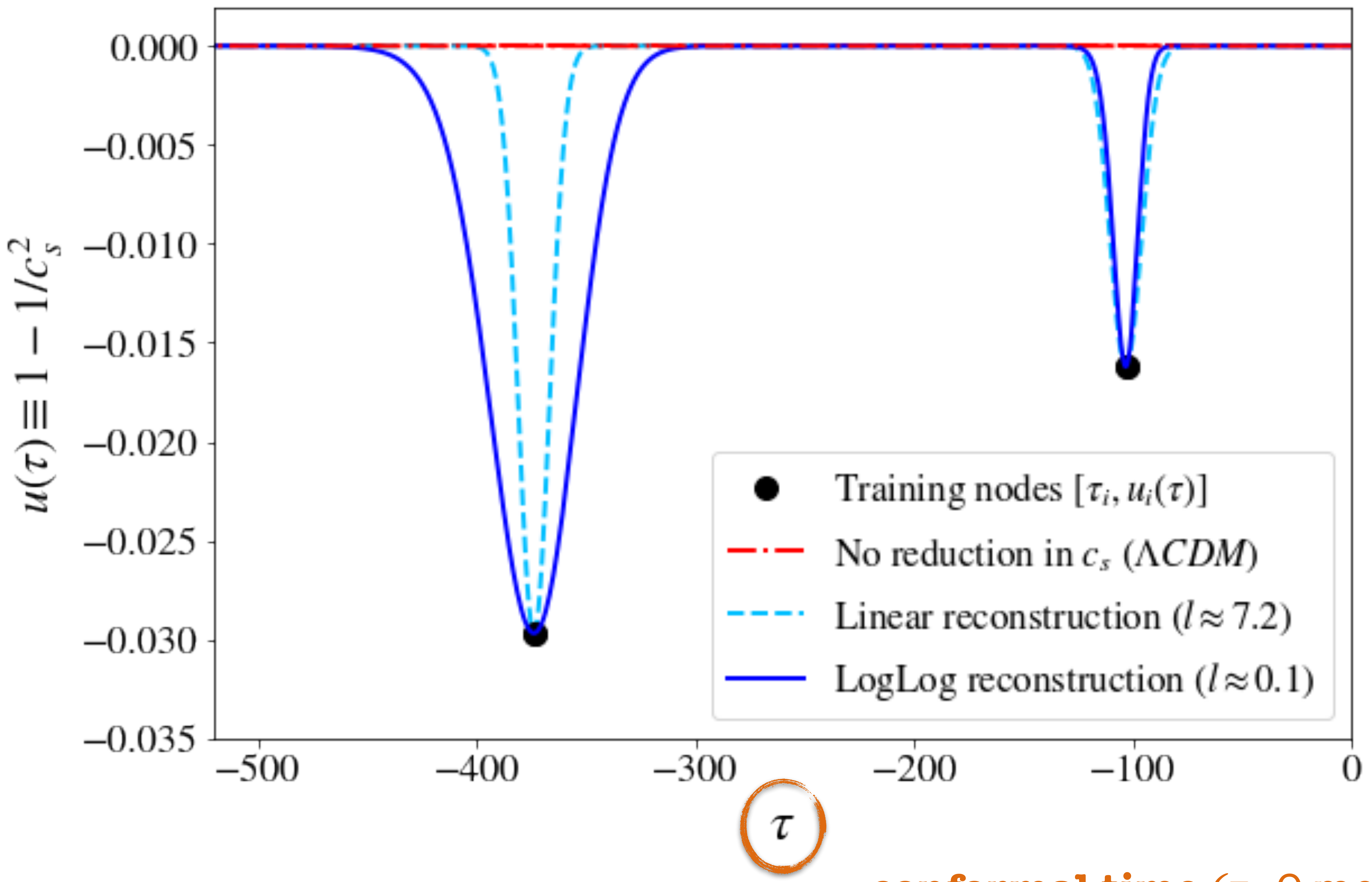
Reconstruction of c_s



conformal time (τ=0 means end of inflation)

- ☐ u(τ) is always negative
- ☐ 2 options: reconstruct u(τ) using GPs in linear space and check numerically or reconstruct in Log-Log

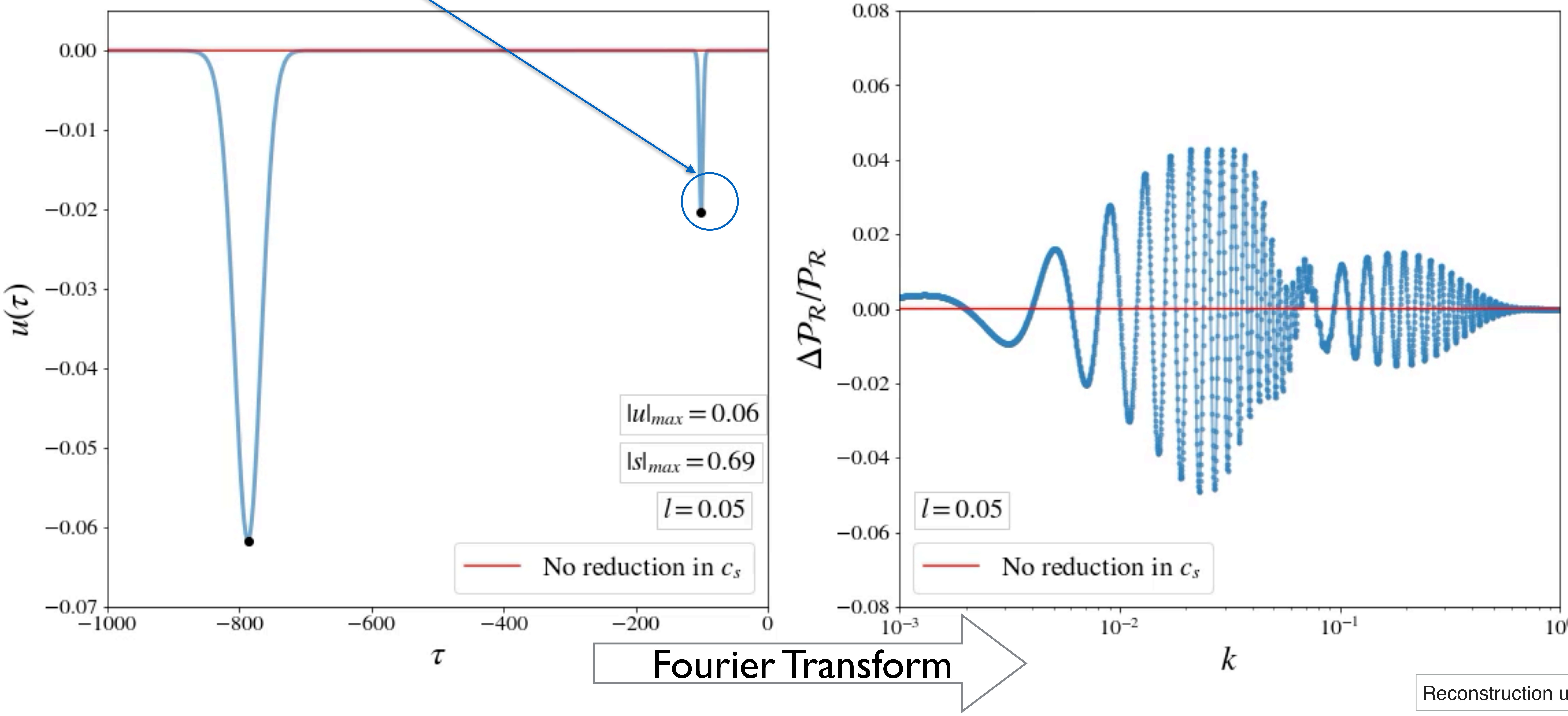
Pipeline visualisation / **GCH**

Reconstruction of c_s

Prior (1):
where is the reduction?

$$\kappa \approx \exp\left\{-\frac{1}{2}\left(\frac{\log|\tau_i| - \log|\tau_{i+1}|}{l}\right)^2\right\}$$

Reconstruction using Gaussian Processes



Reconstruction using GPs/GCH

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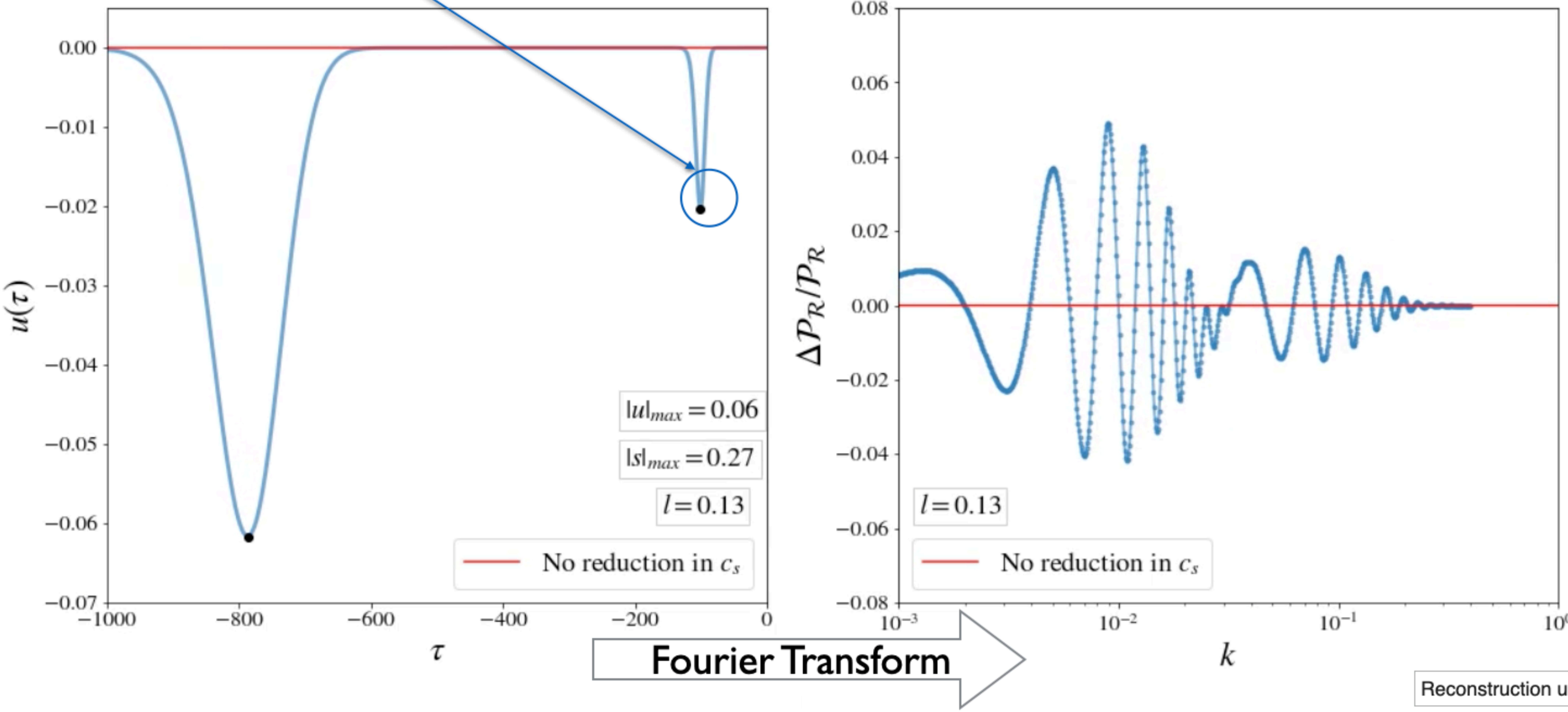
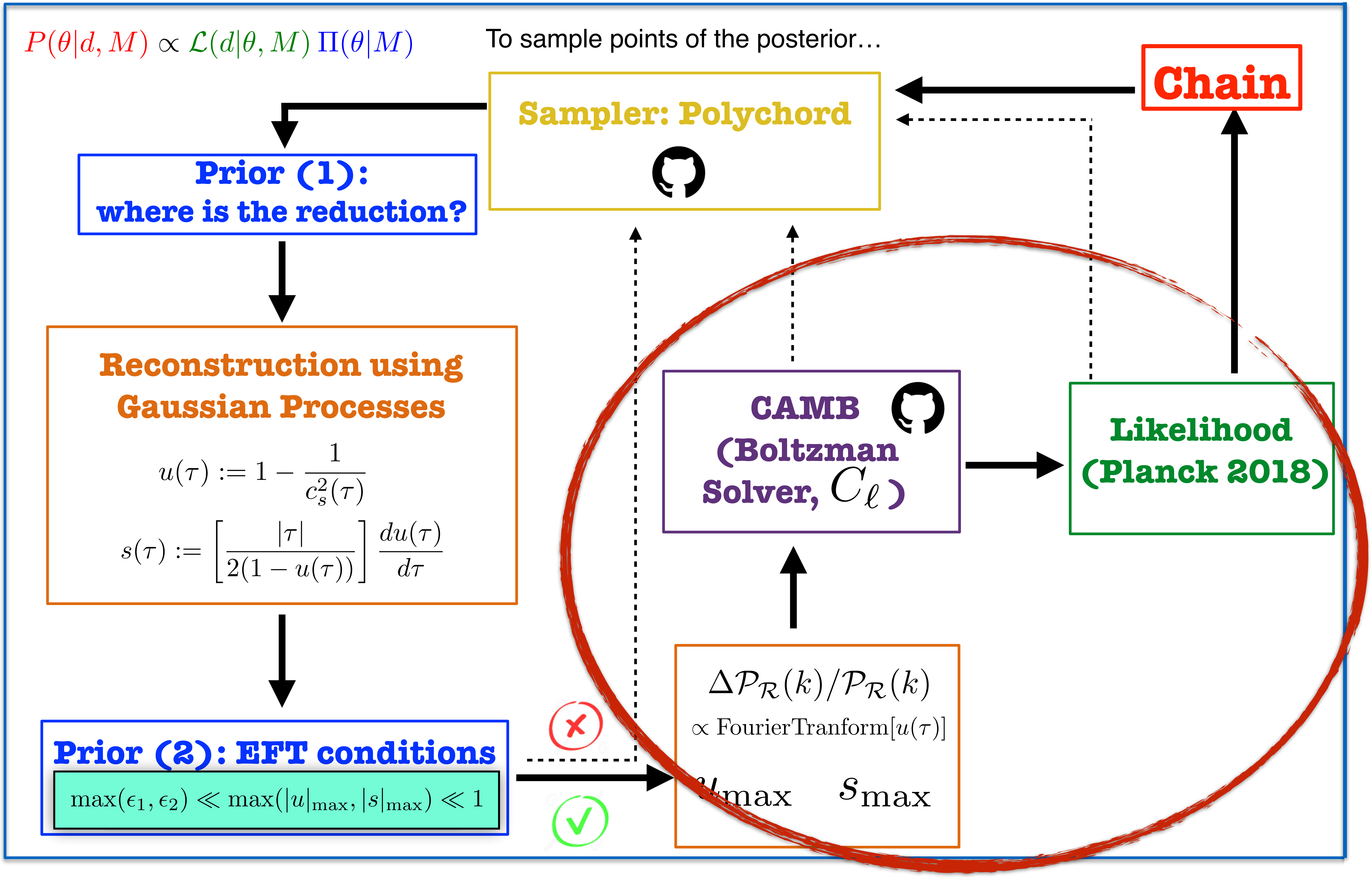


Reconstruction using GPs/GCH

Reconstruction of u(τ)

	Details	
Number of parameters	l (correlation length) + 2 x number of nodes (position of nodes)	
Parameters	$(\log_{10}\lvert\tau_i\rvert, \log_{10}\lvert u_i(\tau)\rvert, l, \epsilon_1, \epsilon_2)$ $(\log_{10}\lvert u\rvert_{\max}, \log_{10}\lvert s\rvert_{\max}, n_s, r)$	Sampled Derived
Theory parameters? $\lvert u\rvert_{\max}$ $\lvert s\rvert_{\max}$	✓ Numerically	

Methodology: pipeline



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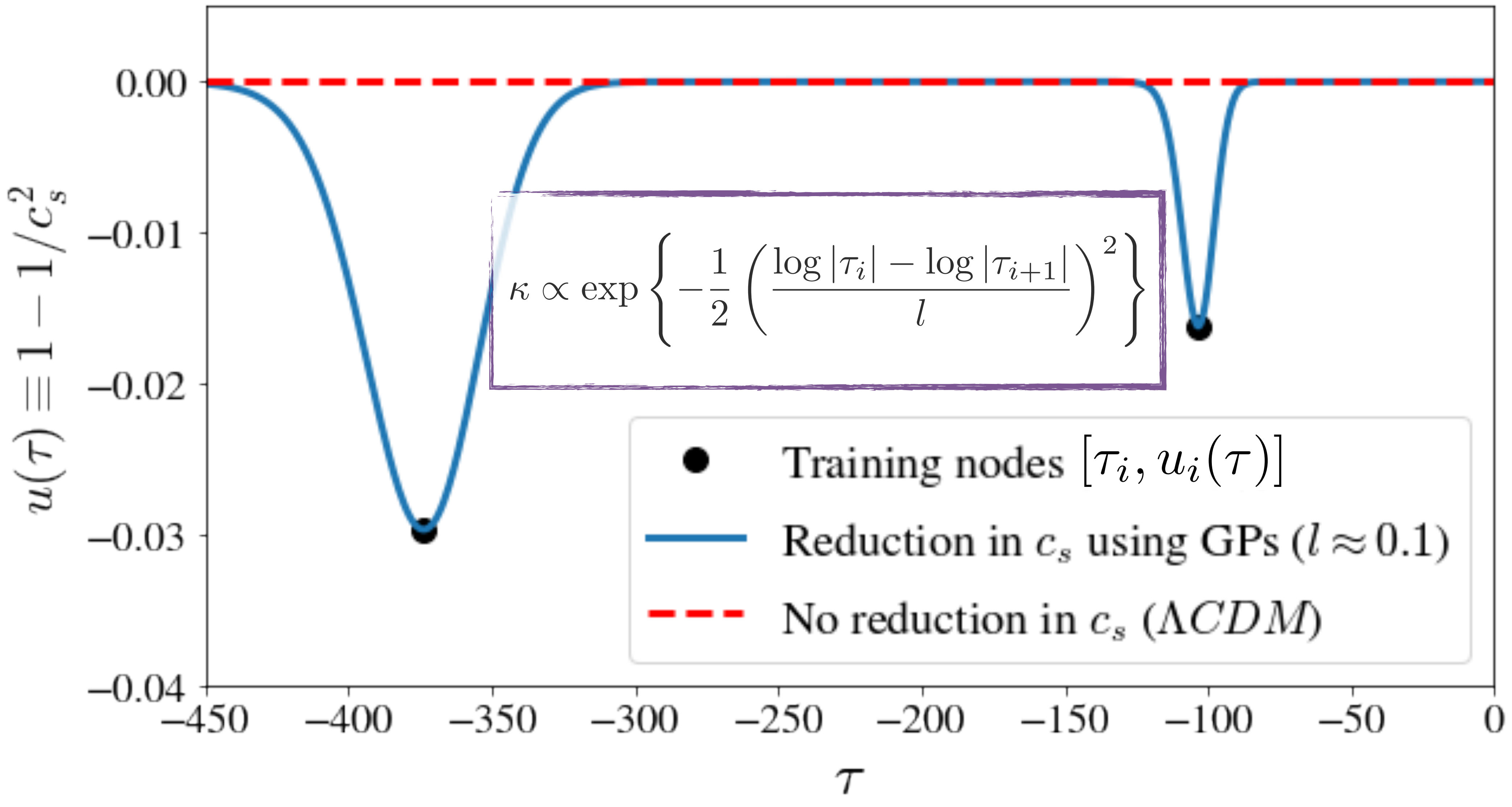
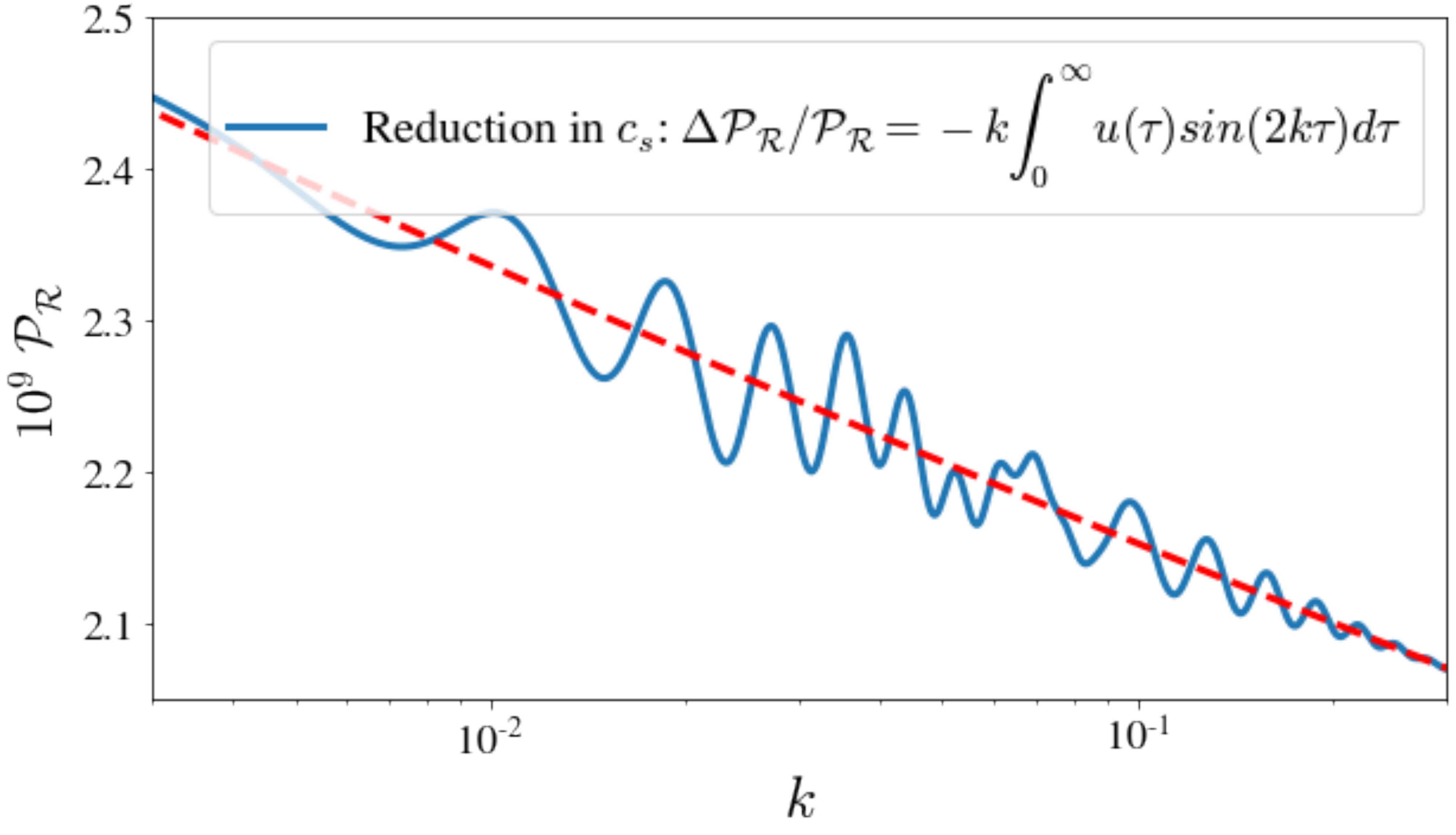
$$\Delta\mathcal{P}_{\mathcal{R}}(k)/\mathcal{P}_{\mathcal{R}}(k)$$

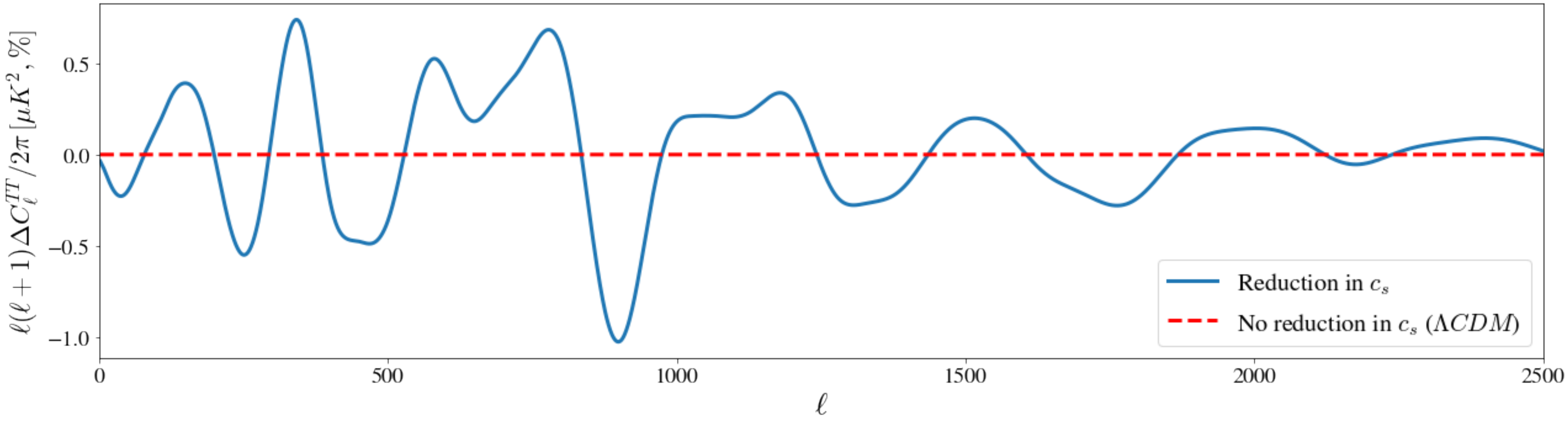
$\propto$ FourierTranform$[u(\tau)]$

$u_{\max}$ $s_{\max}$

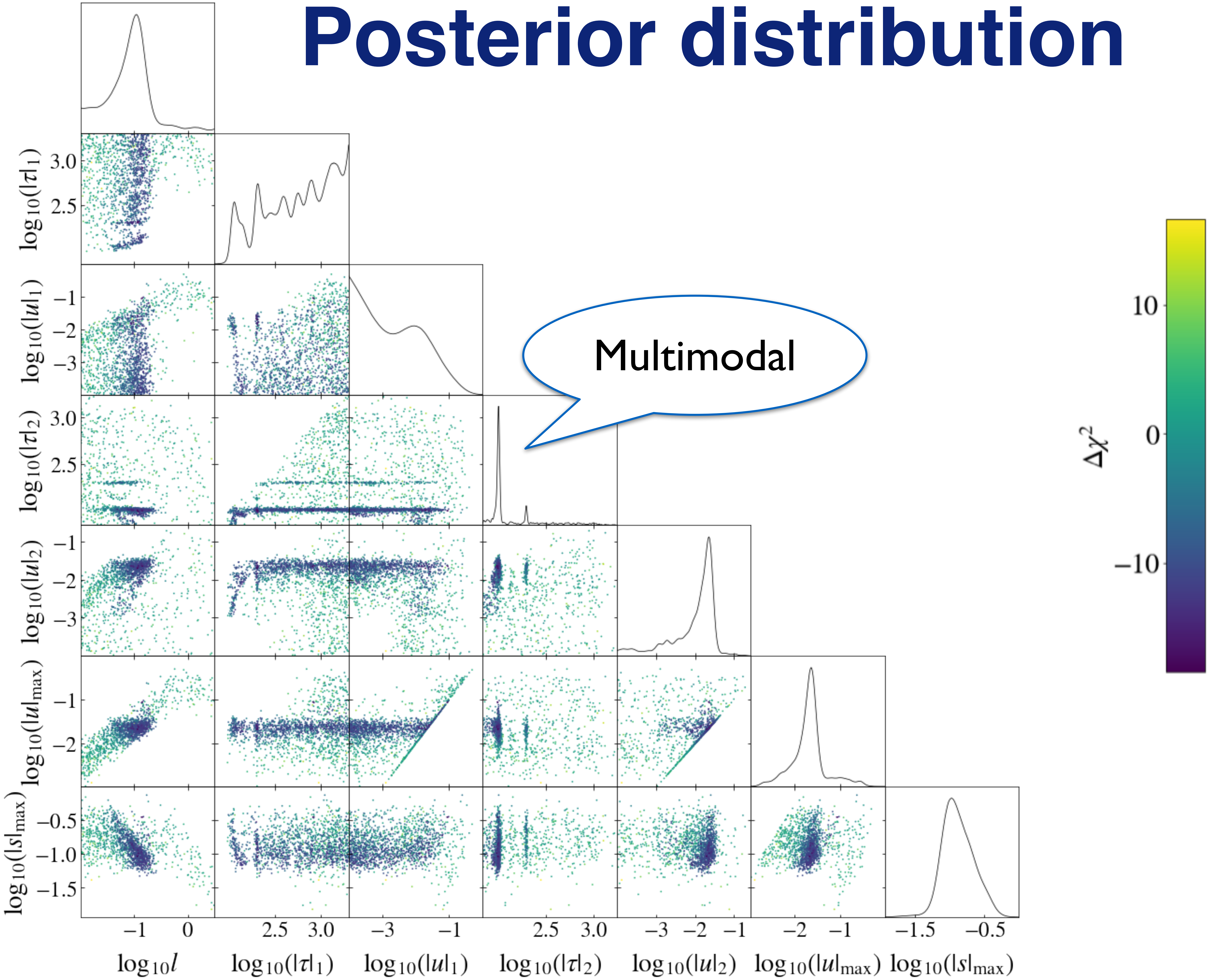
Comparison with the data





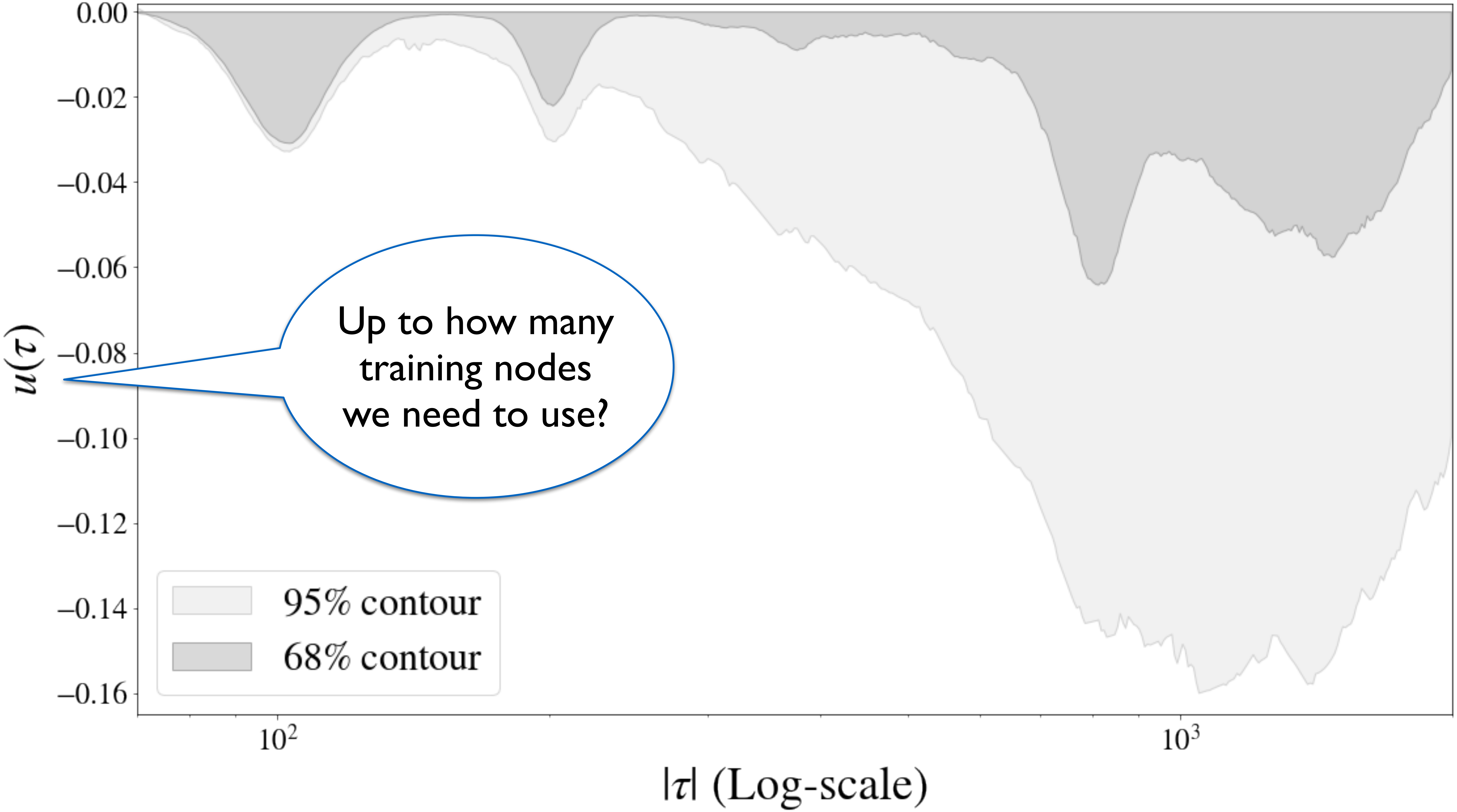


Posterior distribution



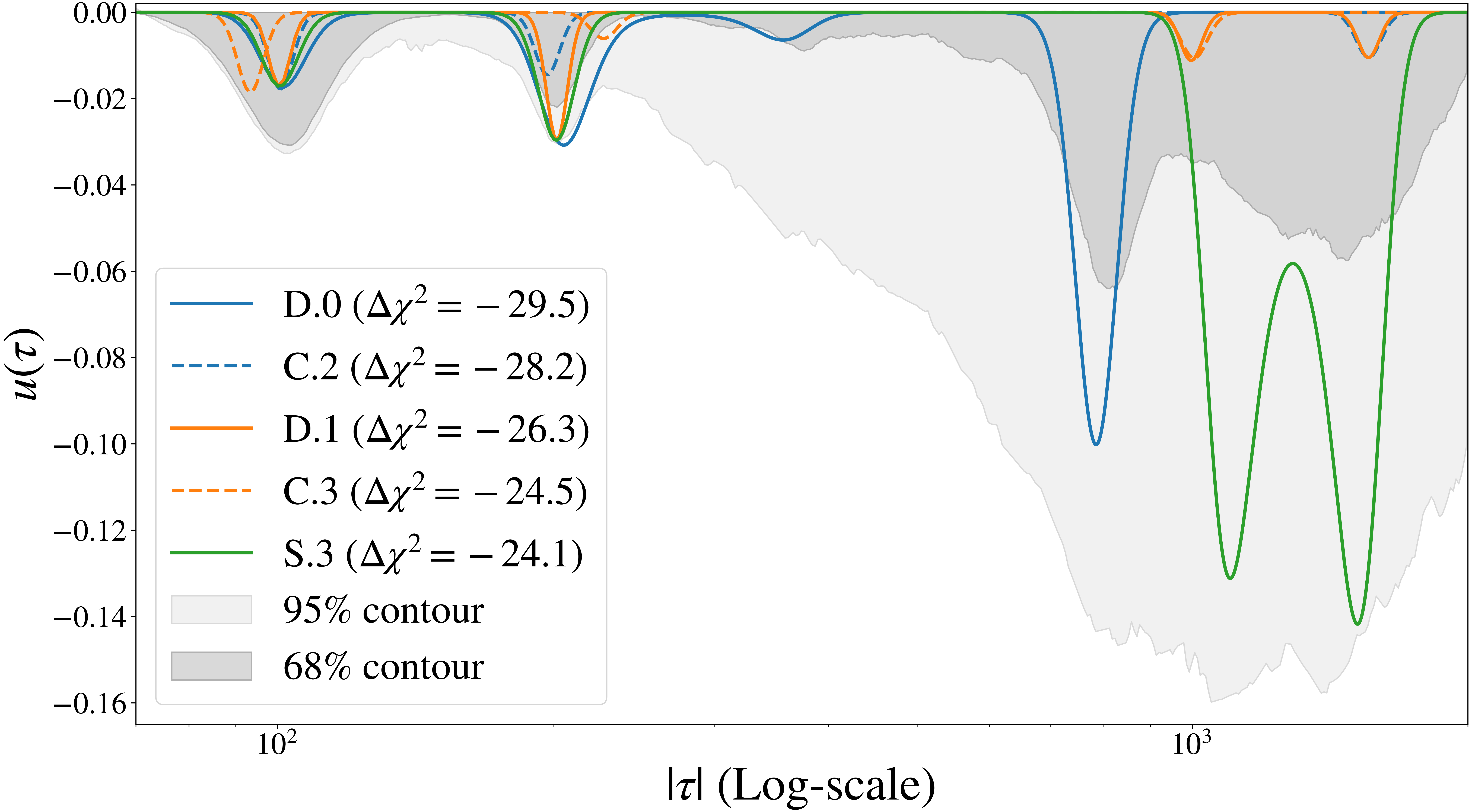
Multimodal

Reconstruction of speed of sound



Up to how many training nodes we need to use?

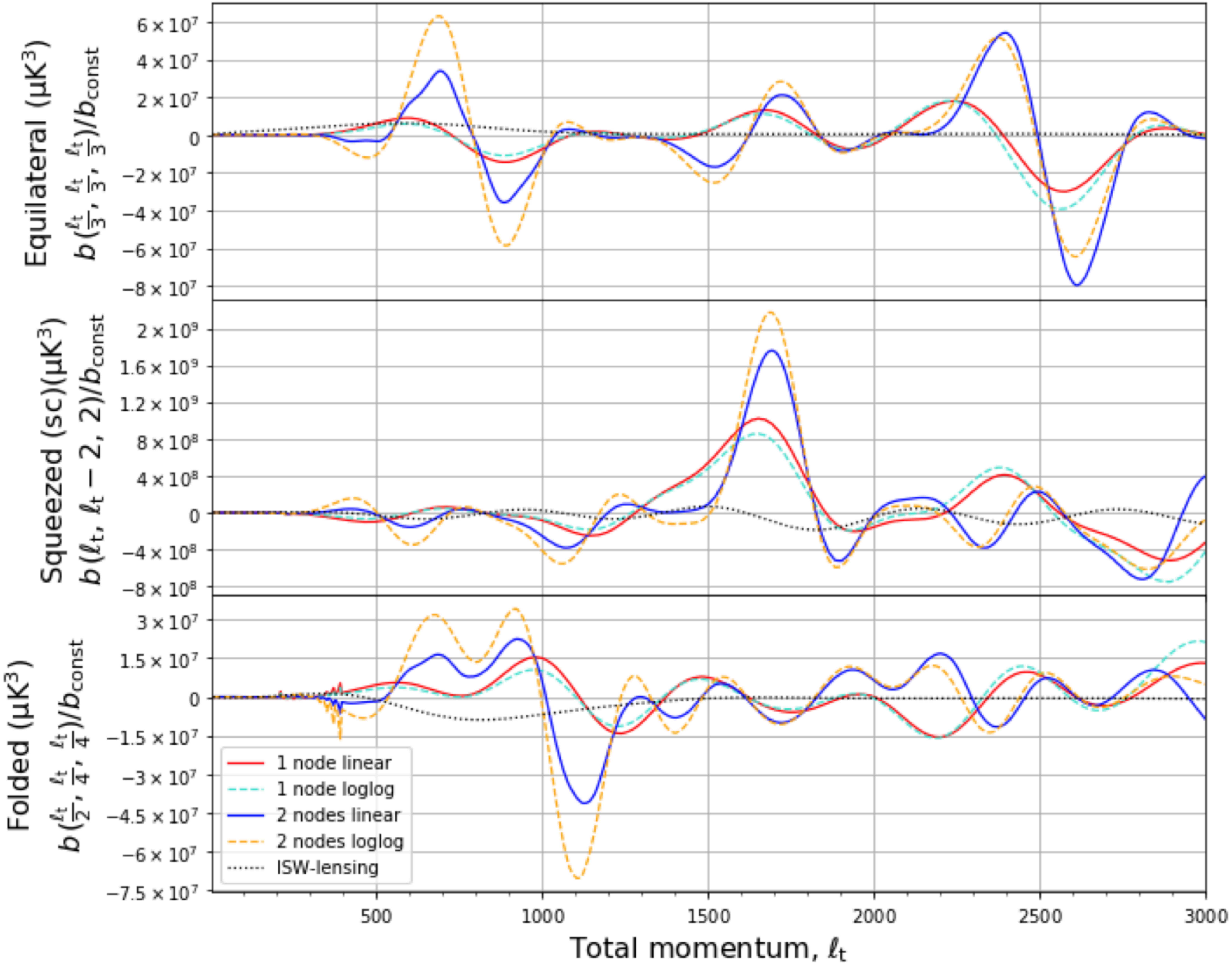
Reconstruction of speed of sound



How do we increase the statistical certainty?

- Use higher order correlation functions (i.e: bispectrum)

$$
\begin{aligned}
\Delta\mathcal{B}_{\mathcal{R}} = \frac{(2\pi)^4 \mathcal{P}_{\mathcal{R}0}^{2}}{(k_1k_2k_3)^2}\Bigg\{ & -\frac{2}{3}\frac{k_1k_2}{k_3}\left[\frac{1}{2k}\left(1+\frac{k_3}{2k}\right)\boxed{\frac{\Delta\mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}0}}} - \frac{k_3}{4k^2}\frac{d}{d\log k}\boxed{\frac{\Delta\mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}0}}}\right] + 2\ \mathrm{perm} + \\
& + \frac{1}{4}\frac{k_1^2+k_2^2+k_3^2}{k_1k_2k_3}\Bigg[\frac{1}{2k}\left(4k^2 - k_1k_2 - k_2k_3 - k_3k_1 - \frac{k_1k_2k_3}{2k}\right)\boxed{\frac{\Delta\mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}0}}} \\
& - \frac{k_1k_2+k_2k_3+k_3k_1}{2k}\frac{d}{d\log k}\left(\boxed{\frac{\Delta\mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}0}}}\right) + \frac{k_1k_2k_3}{4k^2}\frac{d^2}{d\log k^2}\left(\boxed{\frac{\Delta\mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}0}}}\right)\Bigg]\Bigg\}
\end{aligned}
$$


Equilateral (μK³)
$b(\frac{\ell_t}{3}, \frac{\ell_t}{3}, \frac{\ell_t}{3})/b_{\rm const}$
6 × 10⁷
4 × 10⁷
2 × 10⁷
0
−2 × 10⁷
−4 × 10⁷
−6 × 10⁷
−8 × 10⁷
Squeezed (sc)(μK³)
$b(\ell_t, \ell_t - 2, 2)/b_{\rm const}$
2 × 10⁹
1.6 × 10⁹
1.2 × 10⁹
8 × 10⁸
4 × 10⁸
0
−4 × 10⁸
−8 × 10⁸
Folded (μK³)
$b(\frac{\ell_t}{2}, \frac{\ell_t}{4}, \frac{\ell_t}{4})/b_{\rm const}$
3 × 10⁷
1.5 × 10⁷
0
−1.5 × 10⁷
−3 × 10⁷
−4.5 × 10⁷
−6 × 10⁷
−7.5 × 10⁷
1 node linear
1 node loglog
2 nodes linear
2 nodes loglog
ISW-lensing
500
1000
1500
2000
2500
3000
Total momentum, ℓ_t

How do we increase the statistical certainty?

- Use higher order correlation functions (i.e: bispectrum)
- Use LSS data:
  - ☐ Take into account Non-Gaussianities into scale-dependent bias
  - ☐ Model Non-linearities
  - ☐ Advanced computational skills required

Scale dependent bias

- This particular scenario predicts correlated features in the bispectrum (NGs)

$$
\begin{aligned}
\Delta\mathcal{B}_{\mathcal{R}} = \frac{(2\pi)^4 \mathcal{P}_{\mathcal{R}0}^{2}}{(k_1k_2k_3)^2}\Bigg\{ & -\frac{2}{3}\frac{k_1k_2}{k_3}\left[\frac{1}{2k}\left(1+\frac{k_3}{2k}\right)\boxed{\frac{\Delta\mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}0}}} - \frac{k_3}{4k^2}\frac{d}{d\log k}\boxed{\frac{\Delta\mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}0}}}\right] + 2\ \text{perm} + \\
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& \left. - \frac{k_1k_2+k_2k_3+k_3k_1}{2k}\frac{d}{d\log k}\left(\boxed{\frac{\Delta\mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}0}}}\right) + \frac{k_1k_2k_3}{4k^2}\frac{d^2}{d\log k^2}\left(\boxed{\frac{\Delta\mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}0}}}\right)\right]\Bigg\}
\end{aligned}
$$

- They will have an impact in galaxy clustering bias: corrections given by

$$
\Delta b(k) = \left[b_1\delta_c + \frac{d\log\left(\mathcal{F}_*^{(3)}\right)}{d\log(\sigma_*)}\right]\frac{2\mathcal{F}_*^{(3)}(k)}{\mathcal{M}_*(k)}
$$

Giovanni Cabass, Enrico Pajer, Fabian Schmidt (2018)

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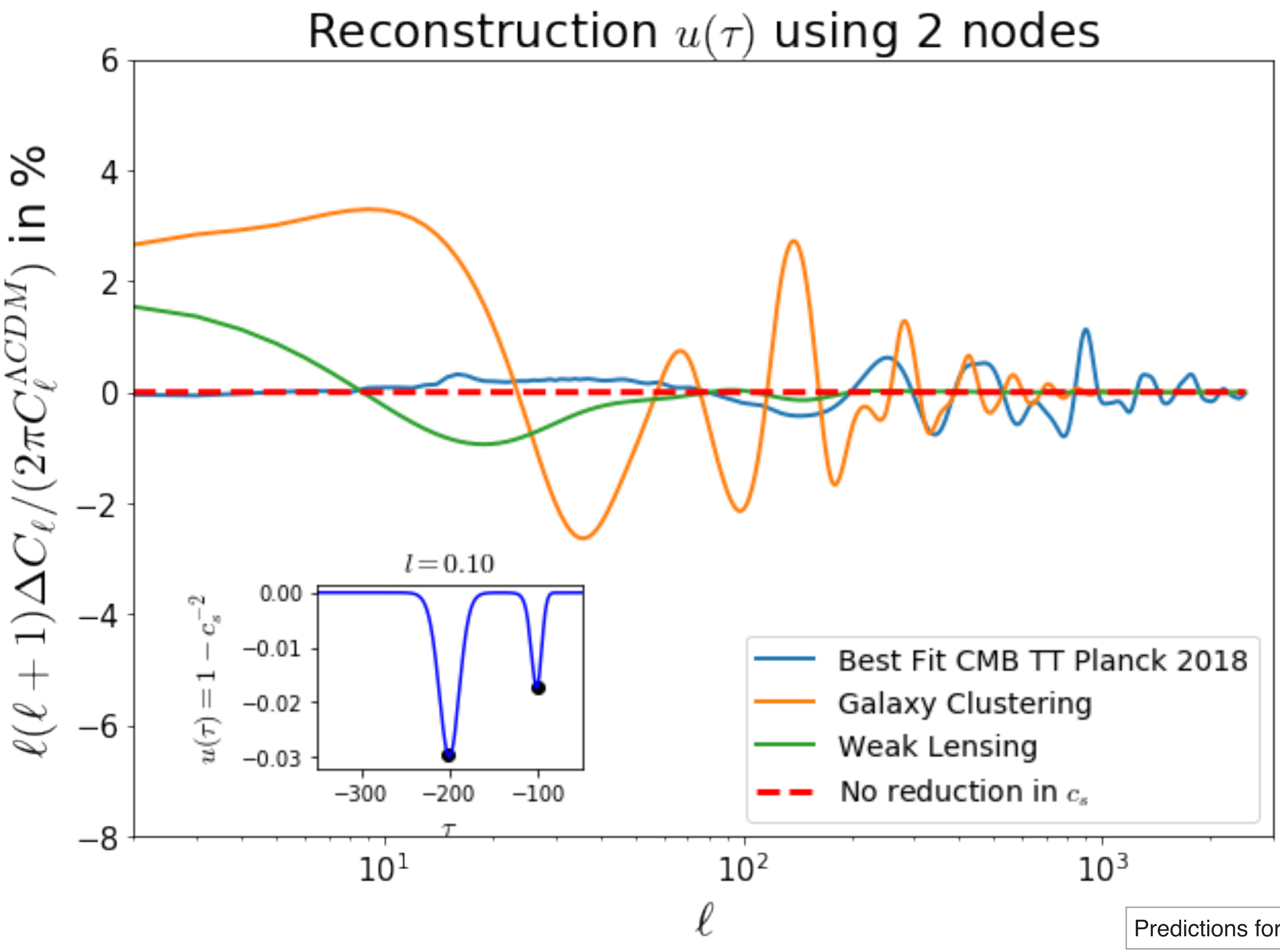
\end{aligned}
$$

- They will have an impact in galaxy clustering bias:
  - Big enough to be detected?
  - Big enough to introduce a significant correction during the statistical analysis?
    - **Forecast needed**

Predictions for LSS



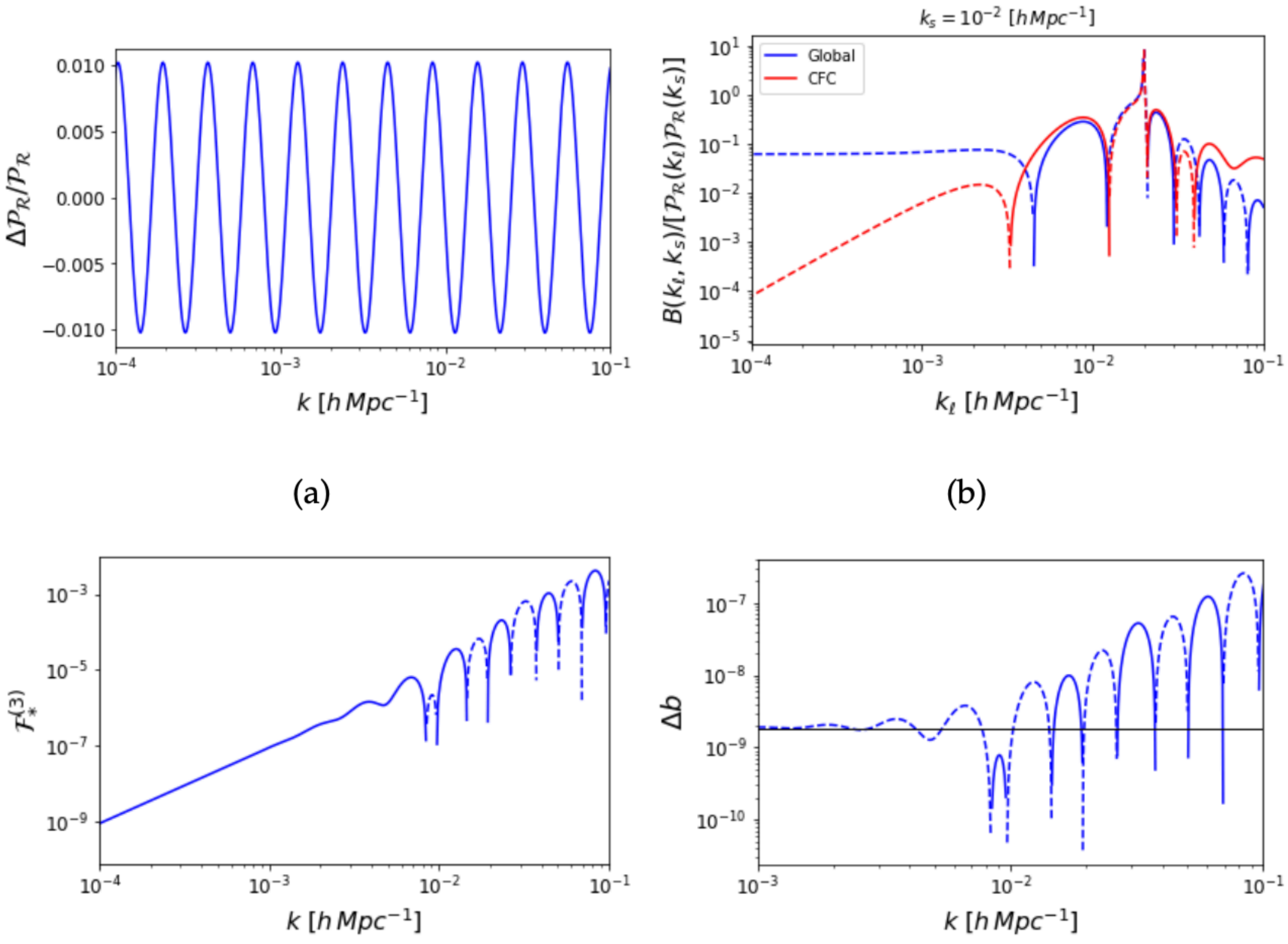
Predictions for Euclid/**GCH**

Summary

- Study of primordial functions (speed of sound or slow-roll parameters as function of time) provide more flexible templates of features:
    - Combination of multiple reductions can take place
    - Interesting substructures (no total *dips*)
- These feature templates have not been checked by Planck
- Ready: efficient and flexible pipeline for the analysis of features from small, mild and transient reductions in sound speed in cosmological data.
- Increase statistical significance using PNGs and LSS

¡Gracias!

Scale dependent bias



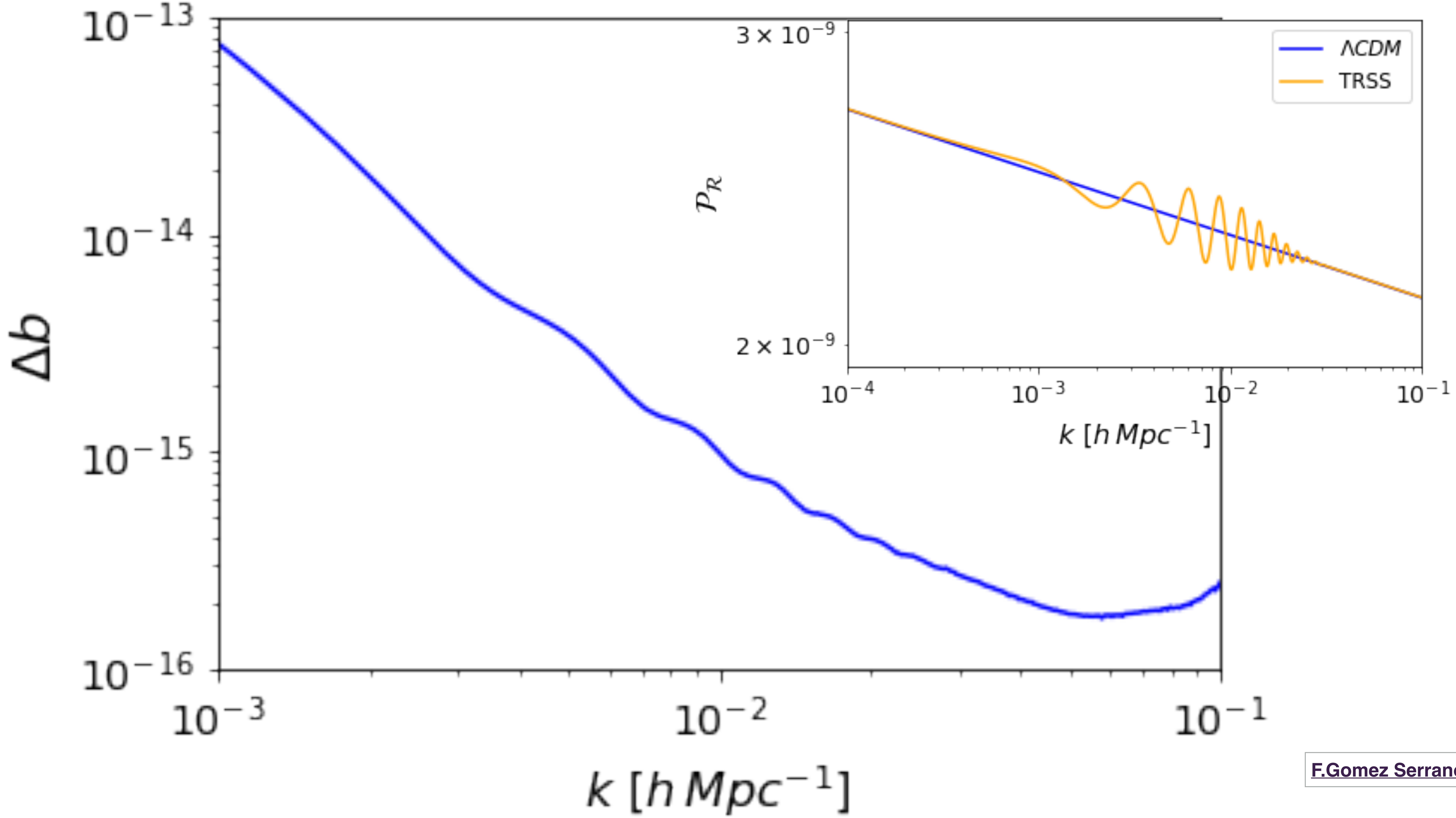
(a)

(b)

(c)

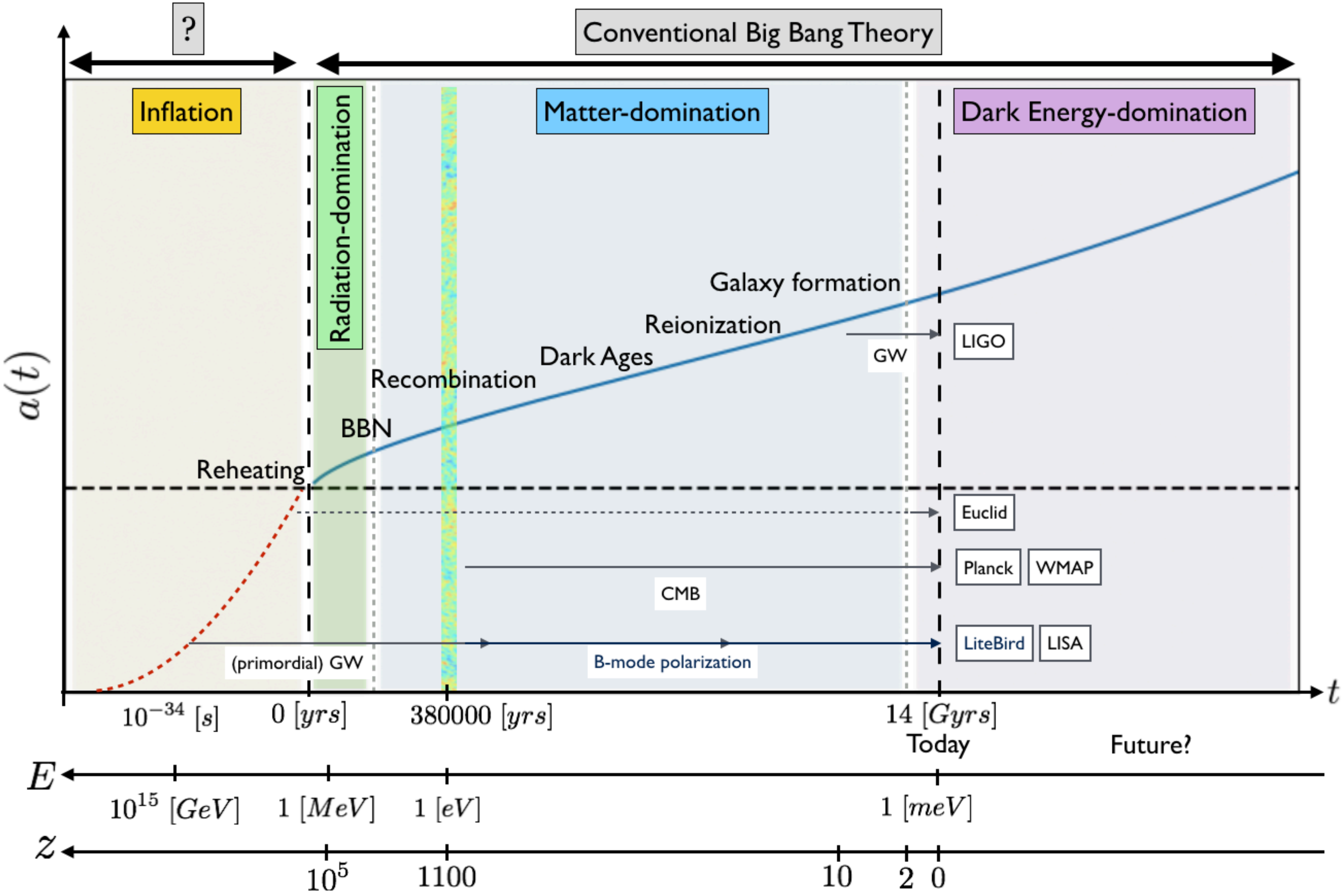
(d)

Scale dependent bias



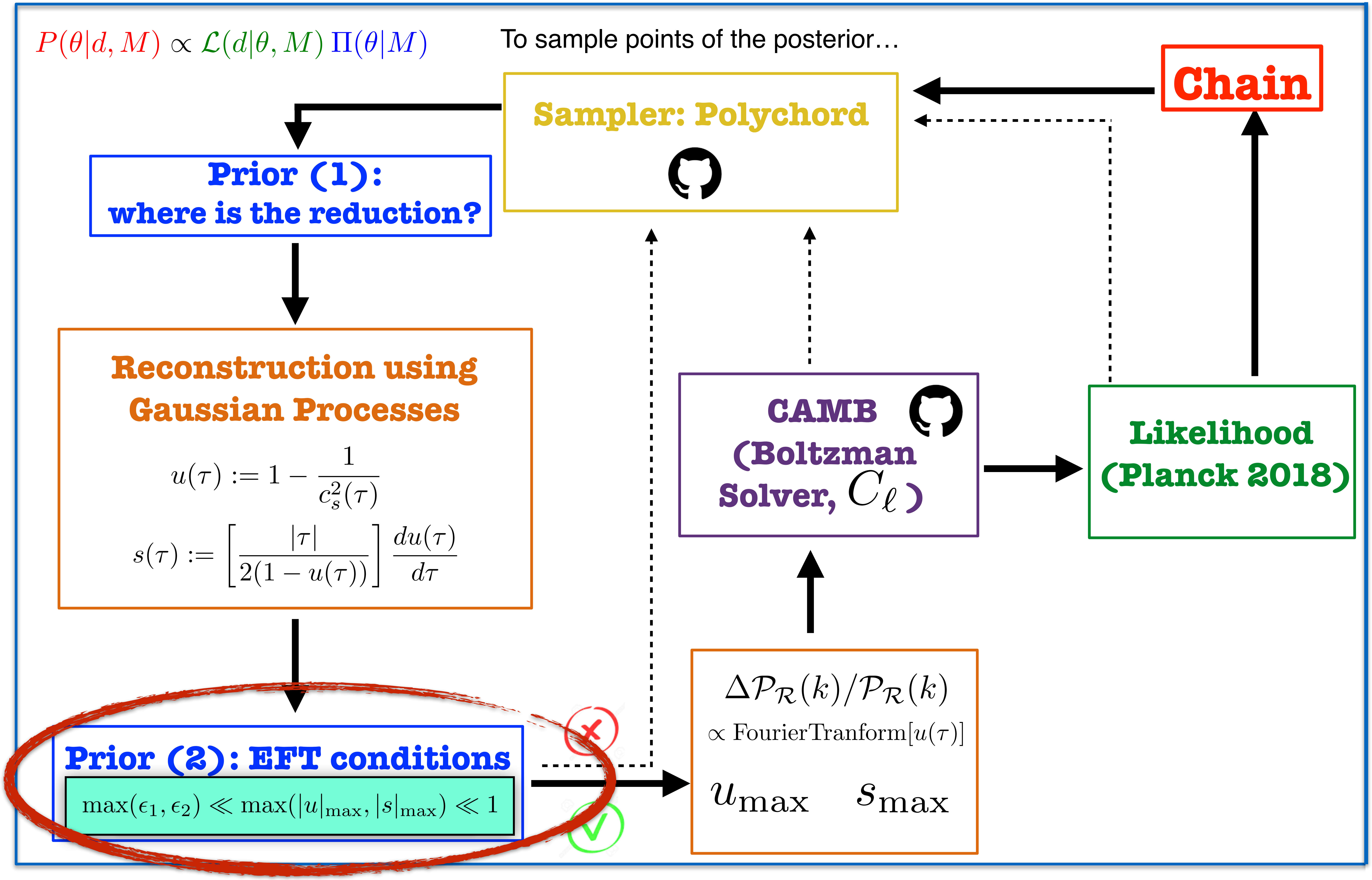
F.Gomez Serrano and GCH

Universe Timeline



GCH

Methodology: pipeline



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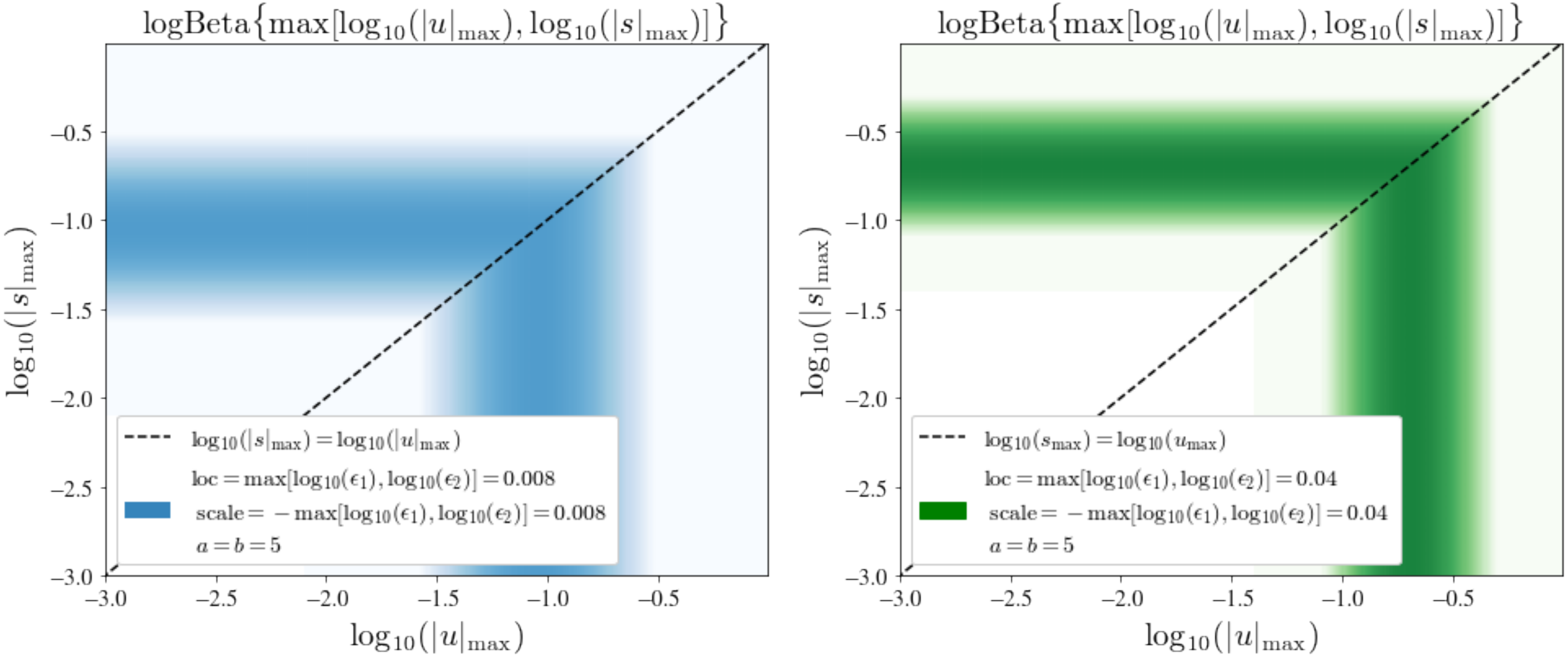
Weakly informative prior

Prior (2): EFT condition

$$\max(\epsilon_1, \epsilon_2) \ll \max(|u|_{\max}, |s|_{\max}) \ll 1$$



- However, we have sorting prior: $|\tau|_{i+1} < |\tau|_i$

1611.10350/Torrado et al.

1804.08143/Handley et al.

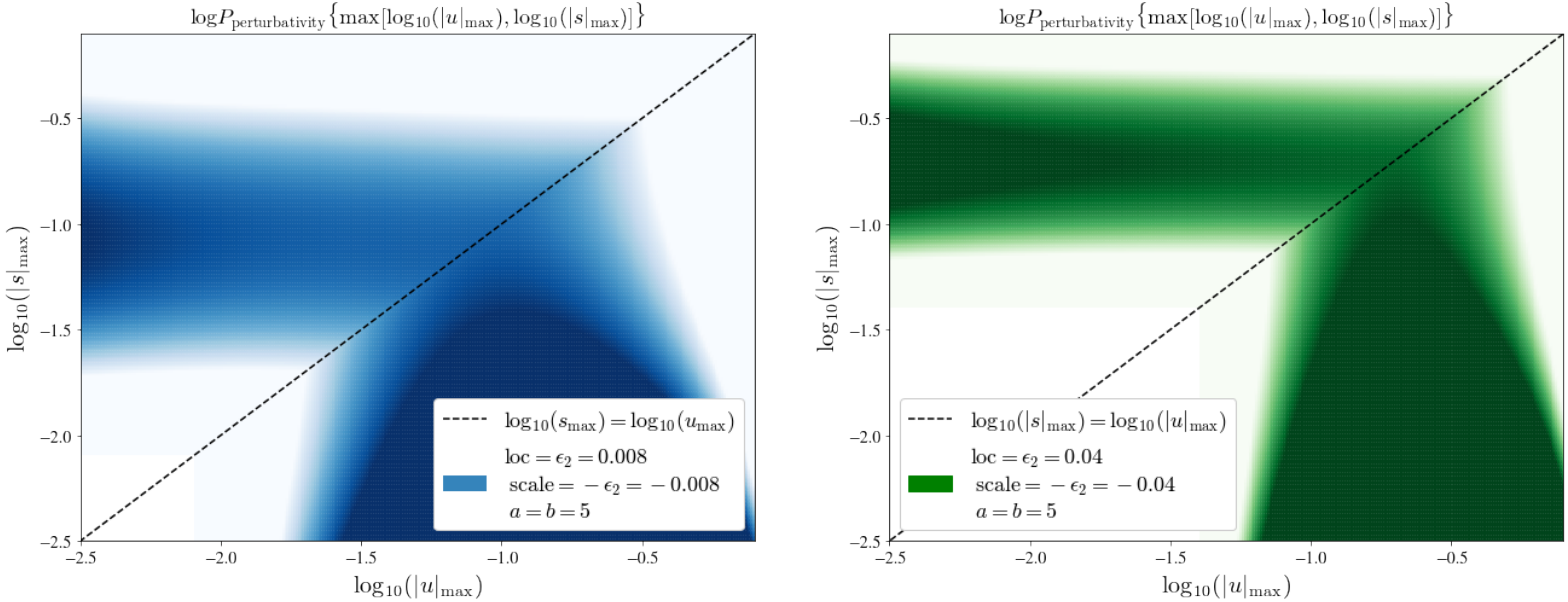
Maximum Entropy

GOAL: actual Beta function

sorting prior

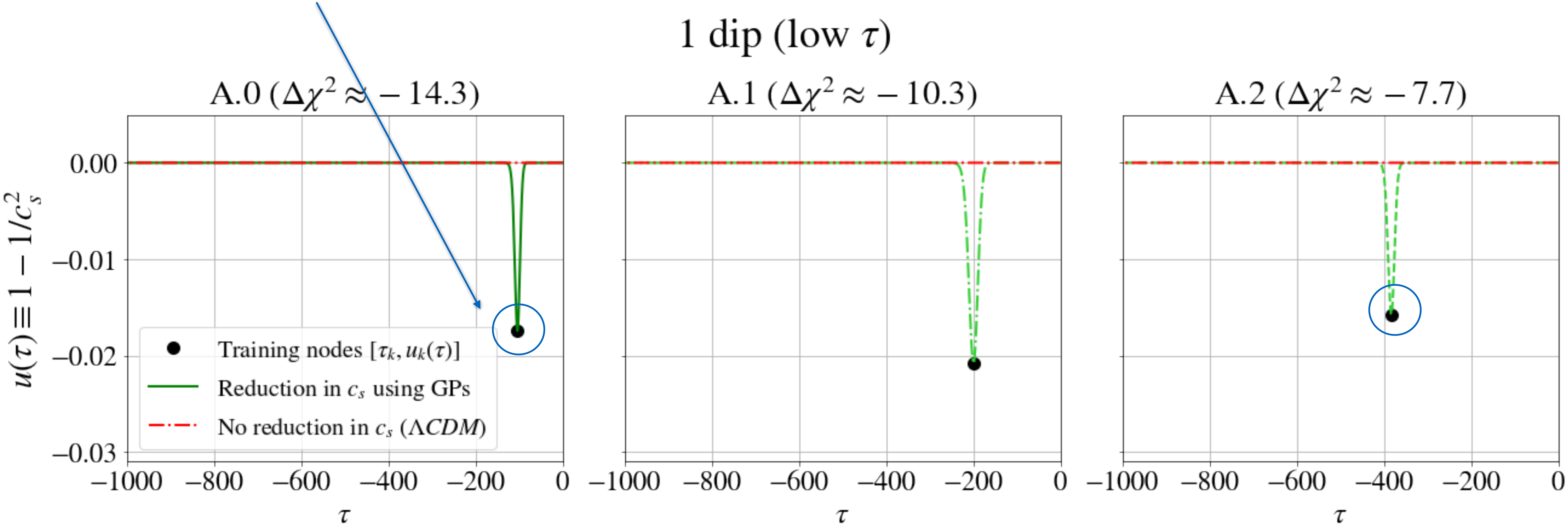
Beta function prior

$$\pi_{\text{pert, maxent}}(\tau_i, u_k, l) = \frac{\pi_0(\tau_i, u_k, l)\,\pi_{\text{pert}}(|u|_{\max}, |s|_{\max})}{P(|u|_{\max}, |s|_{\max} \mid \pi_0)}$$

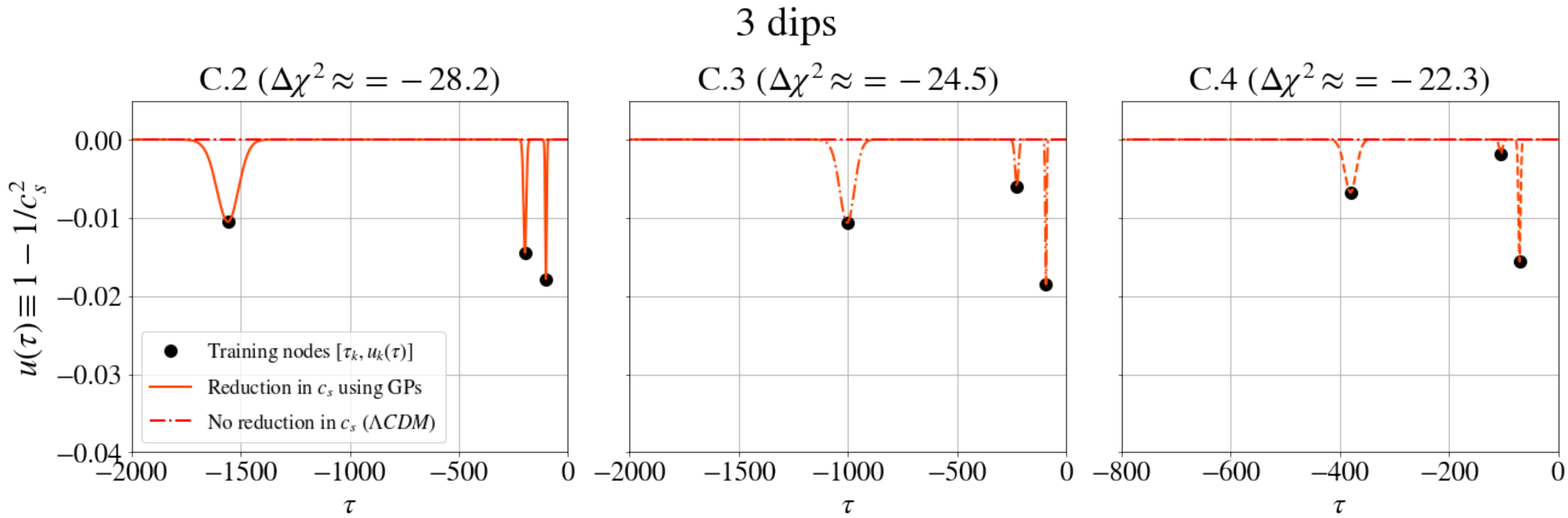


Maxima a posteriori

Training nodes

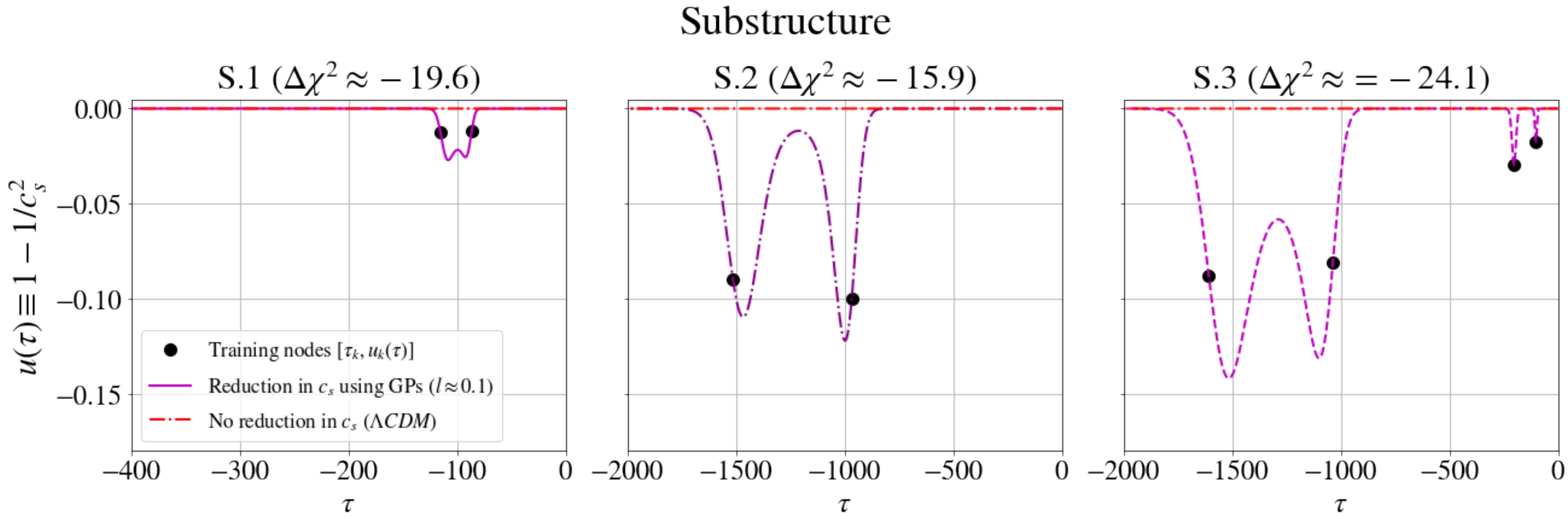


Maxima a posteriori

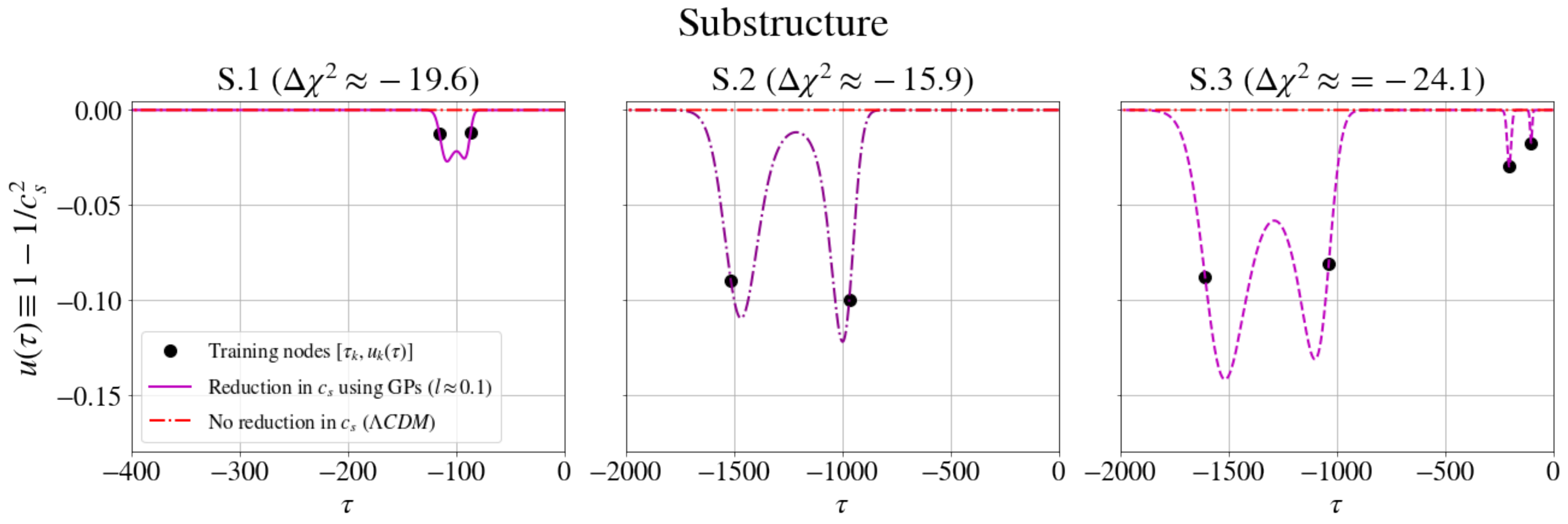


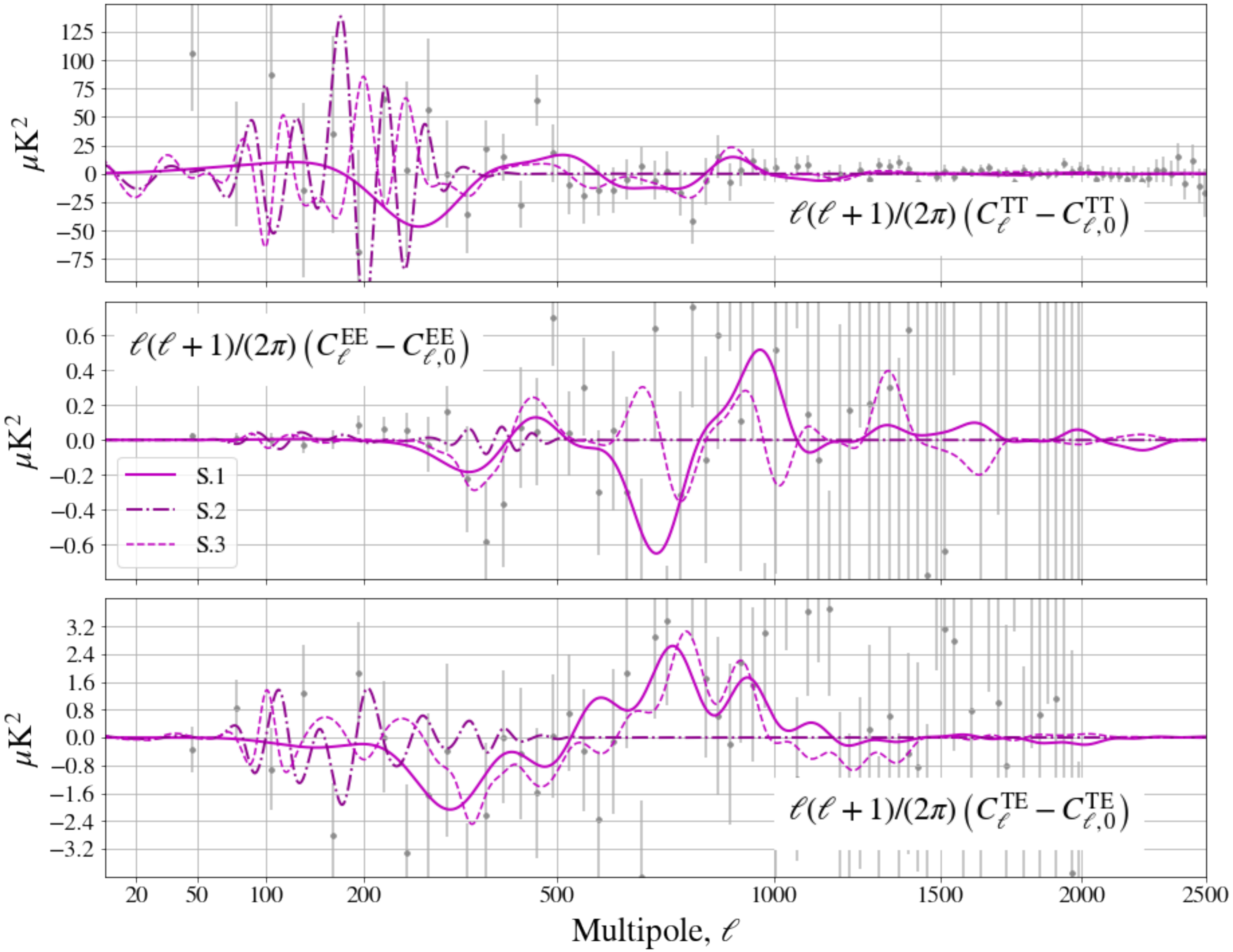
ATTENTION! Number of training nodes is NOT equal to number of dips

Maxima a posteriori



Comparison with Planck 18





Bispectrum

- Multiple Field Inflation — non-gaussianities encoded in:

$$S_3 = \int d^4x a^3 M_P^2 \epsilon H^2 \left[-2H s c_s^{-2} \pi \dot{\pi}^2 - (1 - c_s^{-2}) \dot{\pi} \left(\frac{\dot{\pi}^2}{c_s^2} - \frac{(\partial_i \pi)^2}{a^2} \right) \right]$$

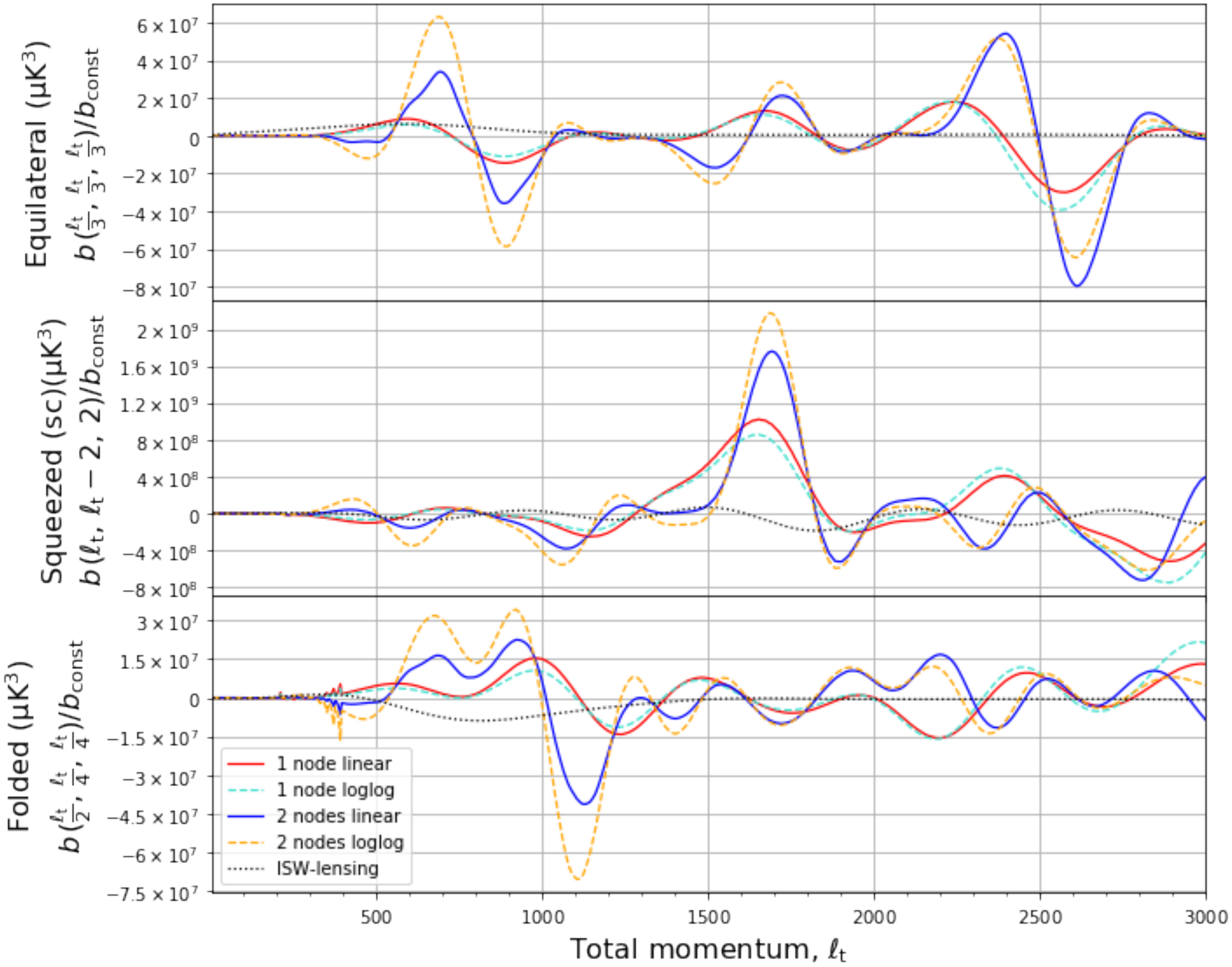
$$\downarrow$$

$$\langle \mathcal{R}(\mathbf{k}_1) \mathcal{R}(\mathbf{k}_2) \mathcal{R}(\mathbf{k}_3) \rangle \equiv (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_{\mathcal{R}}(k)$$

- For a transient and mild reduction of the speed of sound:

$$\begin{aligned} \Delta \mathcal{B}_{\mathcal{R}} = \frac{(2\pi)^4 \mathcal{P}_{\mathcal{R}0}^2}{(k_1 k_2 k_3)^2} \Bigg\{ & -\frac{2}{3} \frac{k_1 k_2}{k_3} \left[\frac{1}{2k} \left(1 + \frac{k_3}{2k} \right) \boxed{\frac{\Delta \mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}0}}} - \frac{k_3}{4k^2} \frac{d}{d \log k} \boxed{\frac{\Delta \mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}0}}} \right] + 2 \text{ perm} + \\ & + \frac{1}{4} \frac{k_1^2 + k_2^2 + k_3^2}{k_1 k_2 k_3} \left[\frac{1}{2k} \left(4k^2 - k_1 k_2 - k_2 k_3 - k_3 k_1 - \frac{k_1 k_2 k_3}{2k} \right) \boxed{\frac{\Delta \mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}0}}} \right. \\ & \left. - \frac{k_1 k_2 + k_2 k_3 + k_3 k_1}{2k} \frac{d}{d \log k} \left(\boxed{\frac{\Delta \mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}0}}} \right) + \frac{k_1 k_2 k_3}{4k^2} \frac{d^2}{d \log k^2} \left(\boxed{\frac{\Delta \mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}0}}} \right) \right] \Bigg\} \end{aligned}$$

Bispectrum



J. Torrado

Scale dependent bias

- The scale dependent bias will have a component based on the bispectrum

$$\Delta b(z,k) = \left[b_1\delta_c + \frac{\partial \log \mathcal{F}_\star^{(3)}}{\partial \log(\sigma_\star)} \right] \frac{\boxed{2\mathcal{F}_\star^{(3)}}(z,k)}{\mathcal{M}_\star(z,k)}$$

Giovanni Cabass, Enrico Pajer, Fabian Schmidt

arXiv:1804.07295

$$\mathcal{F}_\star^{(3)}(z,k_\ell) := \frac{1}{4\sigma_\star^2 P_\phi(k_\ell)} \int d^3k_s \mathcal{M}(z,k_1)\mathcal{M}(z,k_2)\boxed{B_\phi(k_1,k_2,k_\ell)}$$

Fundamental concepts

- Introducing Friedmann equations:

$$H^2 = \left(\frac{1}{a}\frac{da}{dt}\right)^2 = \frac{1}{3}\rho - \frac{k}{a^2}$$

Hubble parameter

scale factor

density

curvature

1st Friedmann equation

$$\frac{1}{a}\frac{d^2a}{dt^2} = -\frac{1}{6}(\rho + 3p)$$

pressure

Acceleration equation

- And the equation of state of a fluid:

$$\frac{d\rho}{dt} + 3H(\rho + p) = 0$$

$$w = \frac{p}{\rho}$$

$$\longrightarrow \quad \frac{d\rho}{dt} + 3H\rho(1 + w) = 0$$

Radiation

$w = 1/3$

Matter

$w = 0$

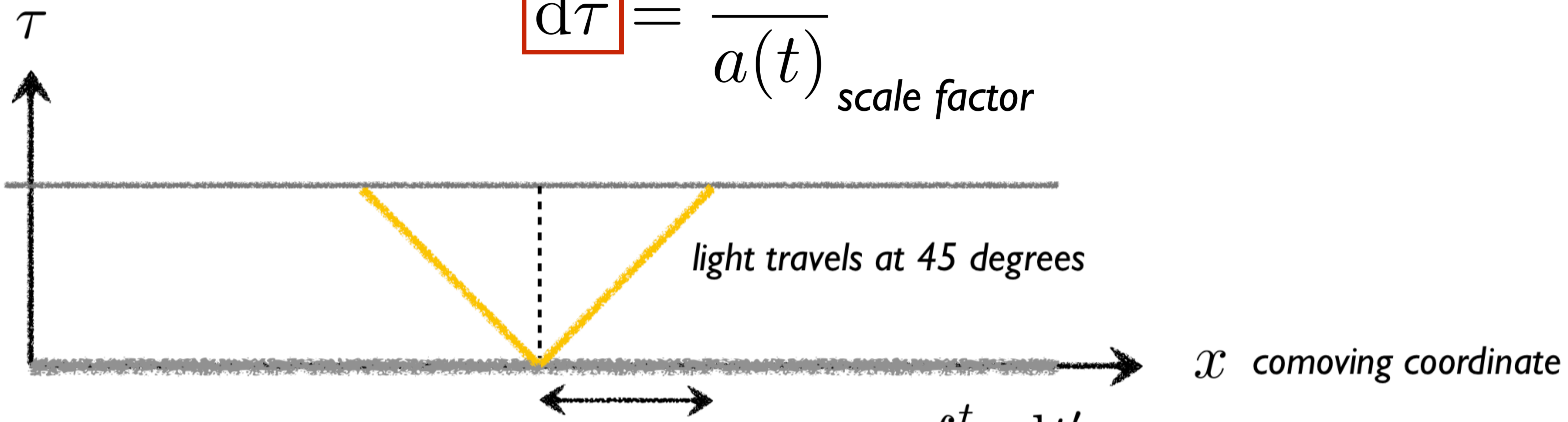
Fundamental concepts

- For convenience, to talk about the history of the universe we use conformal time

$$\boxed{\mathrm{d}\tau} = \frac{\mathrm{d}t}{a(t)}$$

cosmic time

scale factor



$$\Delta x = \boxed{\Delta\tau} = \int_0^t \frac{\mathrm{d}t'}{a(t')} \propto (aH)^{-1}$$

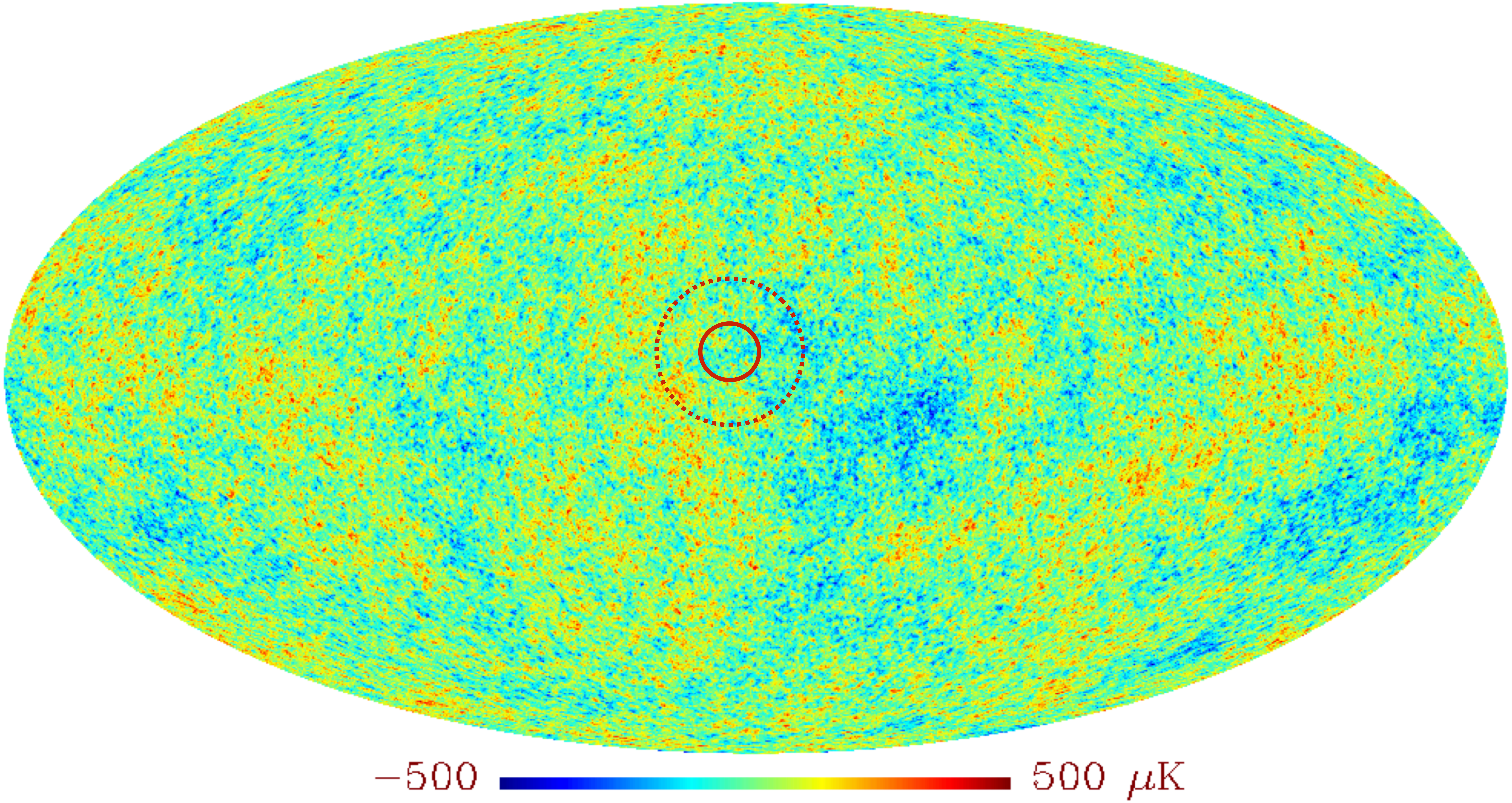
comoving Hubble radius

$$\propto a^{\frac{1}{2}}(1+3\boxed{w})$$

always grows during radiation and matter domination !

Big Bang main problems

Horizon Problem



Big Bang main problems

Flatness Problem

- Manipulating Friedmann Equation + $\Omega(a) = \frac{\rho}{\rho_{crit}}$

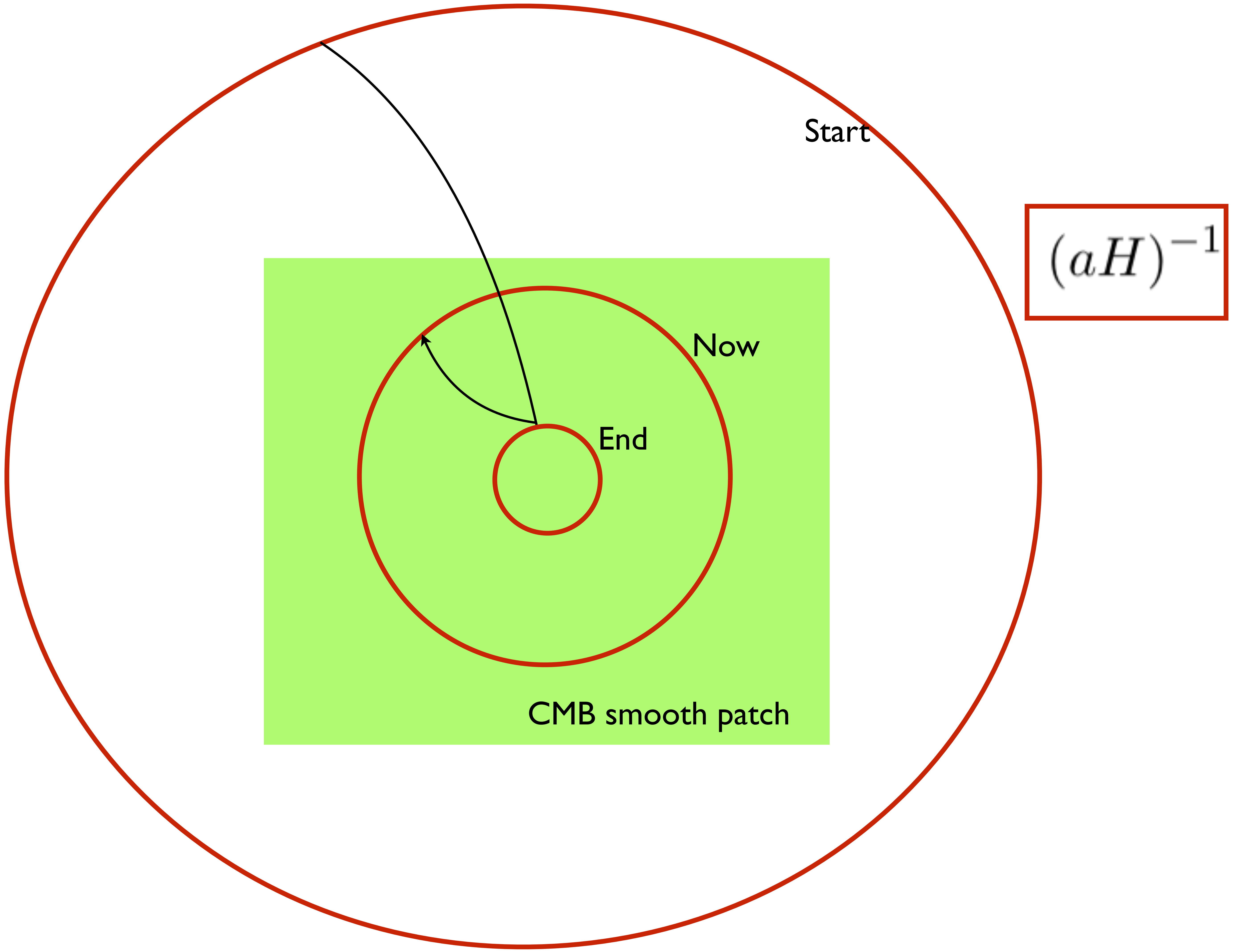
$$1 - \Omega(a) = \frac{-k}{(aH)^2}$$

- This expression diverges in Radiation and Matter domination epochs

- $\Omega = 1$ is an unstable fixed point (need to fine tune)

$$|\Omega(a_{\mathrm{BBN}}) - 1| \quad \leq \quad \mathcal{O}(10^{-16})$$

SOLUTION: Horizon Problem



TASI Lectures on Inflation 2009 + modifications/**Daniel Baumann + GCH**

SOLUTION: Flatness Problem

- If comoving horizon decreases,

$$|\Omega(a) - 1| = \boxed{\frac{1}{(aH)^2}} \rightarrow 0$$

$$\Omega(a) \approx 1$$

- The flatness case becomes a solution!

Physics of Inflation

- First condition: a decreasing comoving Hubble radius
- This is analogous to:
  - Accelerated expansion = Negative pressure = Adiabatic condition

$$\frac{d}{dt}\left(\frac{1}{aH}\right) < 0 \longleftrightarrow \boxed{\frac{d^2a}{dt^2} > 0} \longleftrightarrow \boxed{\rho + 3p < 0} \longleftrightarrow \omega < -\frac{1}{3}$$

$$\Big\downarrow$$

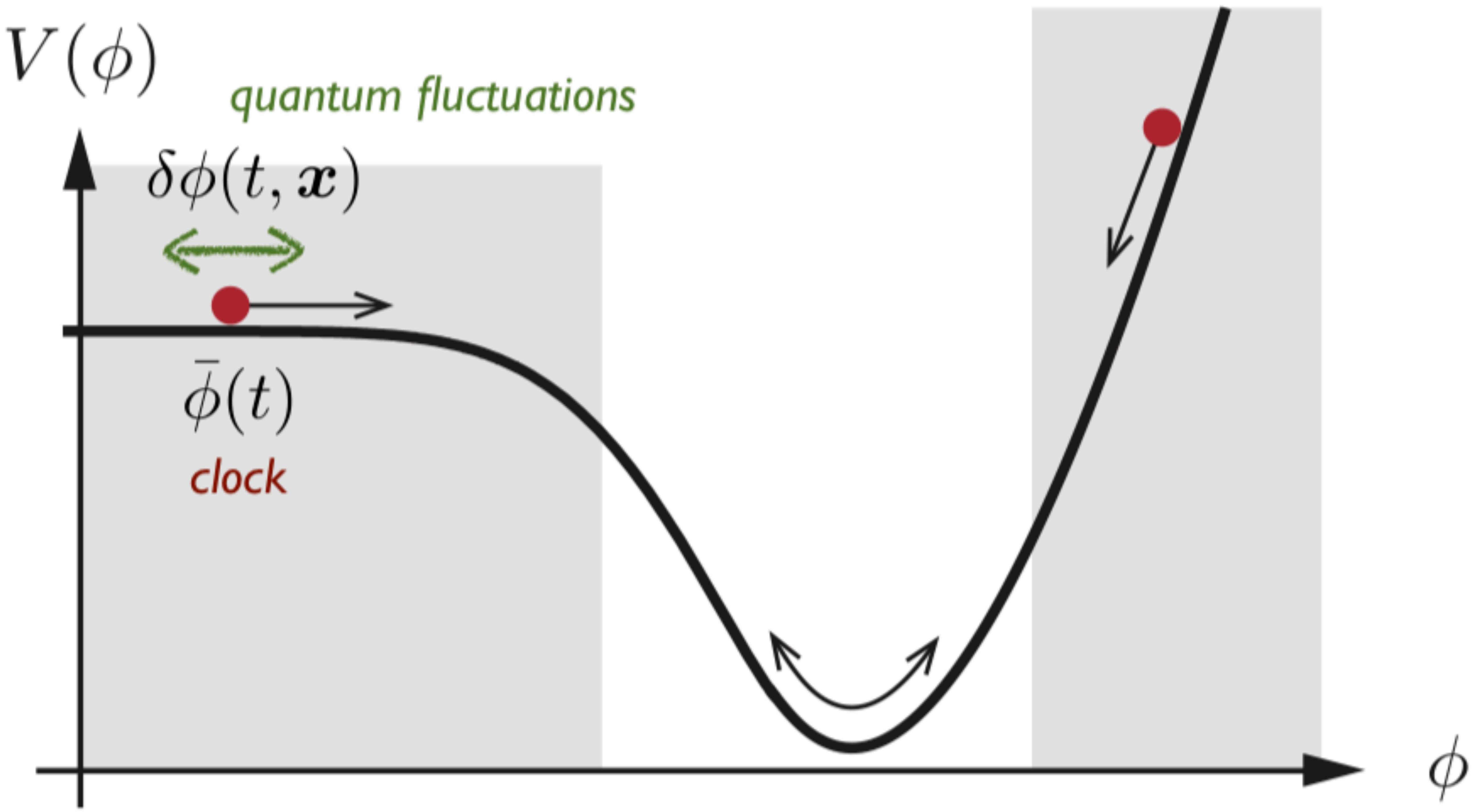
$$\boxed{\epsilon_1 := -\frac{\dot{H}}{H^2} < 1}$$

First slow-roll parameter

$$\boxed{\epsilon_2 := \frac{\dot{\epsilon}}{H\epsilon} < 1}$$

Second slow-roll parameter

From inflation to the CMB



TASI Lectures on Inflation 2009/**Daniel Baumann**

vacuum fluctuations
spread the inflaton vev ...

... which translates into density
fluctuations after inflation

$$\delta\phi(\boldsymbol{x}) \longrightarrow \delta t(\boldsymbol{x}) \longrightarrow \delta\rho(\boldsymbol{x}) \longrightarrow \delta T(\boldsymbol{x})$$

... which induces a local time
delay for the end of inflation

... which become the
CMB anisotropies.

APS/

Slow-roll canonical single-field inflation

$$\boxed{S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu \boxed{\phi}\,\partial_\nu\phi - \boxed{V(\phi)}\right]}$$

$$\boxed{\epsilon_1 := -\frac{\dot{H}}{H^2} < 1}$$

First slow-roll parameter

$$\boxed{\epsilon_2 := \frac{\dot{\epsilon}_1}{H\epsilon_1} < 1}$$

Second slow-roll parameter

- There is only 1 degree of freedom driving inflation
- Dynamics defined in terms of a potential
- Background and perturbations have same origin

$$\boxed{\frac{d^2a}{dt^2} > 0}$$

Accelerated expansion

Standard cosmological model: ΛCDM

- Primordial fluctuations are Gaussian

compatible with a near scale-invariant primordial power spectrum

$$\mathcal{P} = \boxed{A_s} \left(\frac{k}{k_*} \right)^{\boxed{n_s} - 1}$$

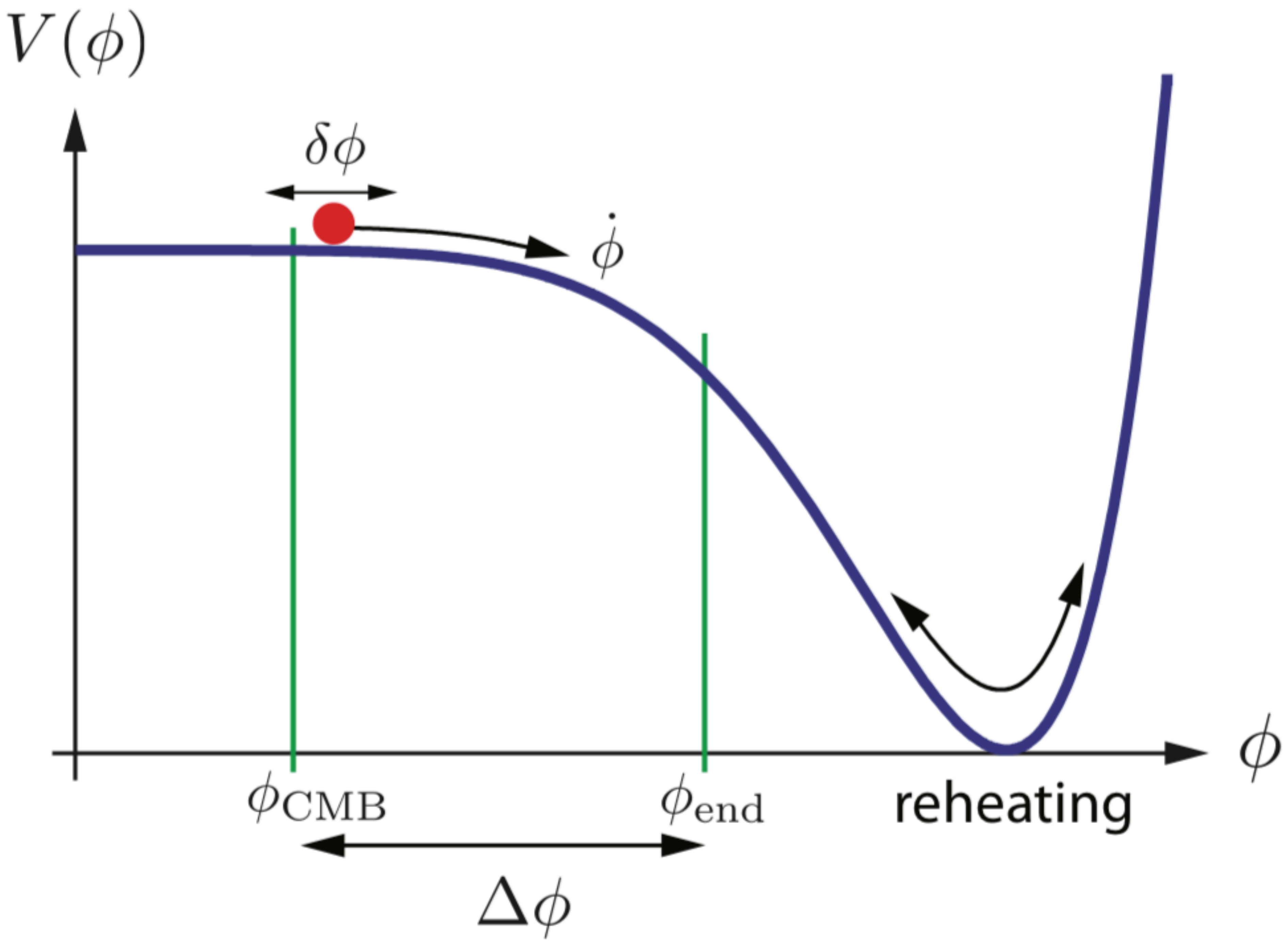
The simplest inflationary model (canonical slow-roll single field inflation)

agrees by construction with this type of spectrum

Single Field Inflation

$$\boxed{\frac{d\phi}{dt} := \dot{\phi} \qquad V_\phi := \frac{dV(\phi)}{d\phi}}$$

$$\boxed{S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - \boxed{V(\phi)}\right]} = S_{\mathrm{EH}} + S_\phi$$



TASI Lectures on Inflation 2009/**Daniel Baumann**

- From the energy-momentum tensor

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,,$$
$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)\,.$$

- Dynamics of the scalar field

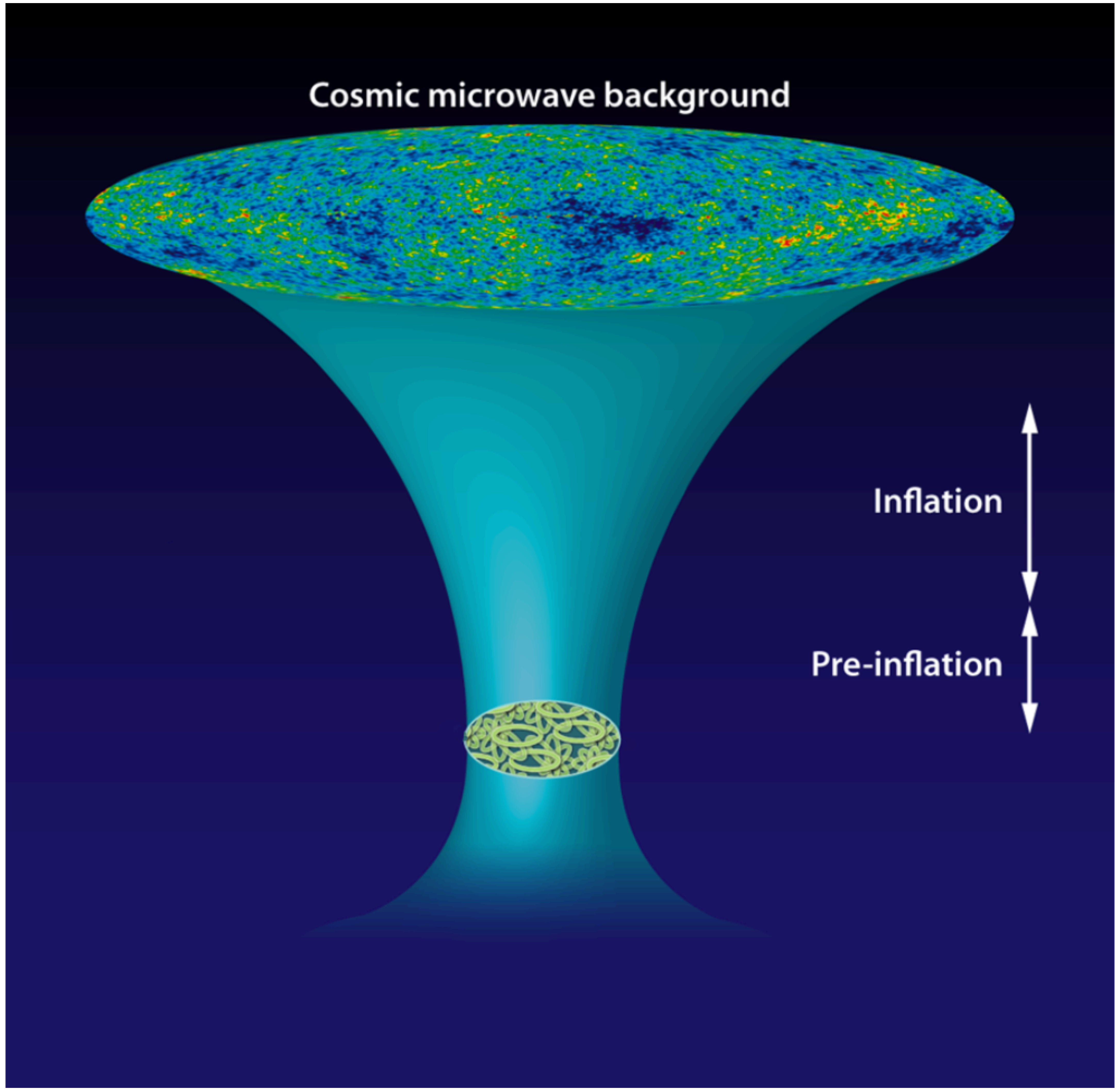
$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

- Slow-roll conditions

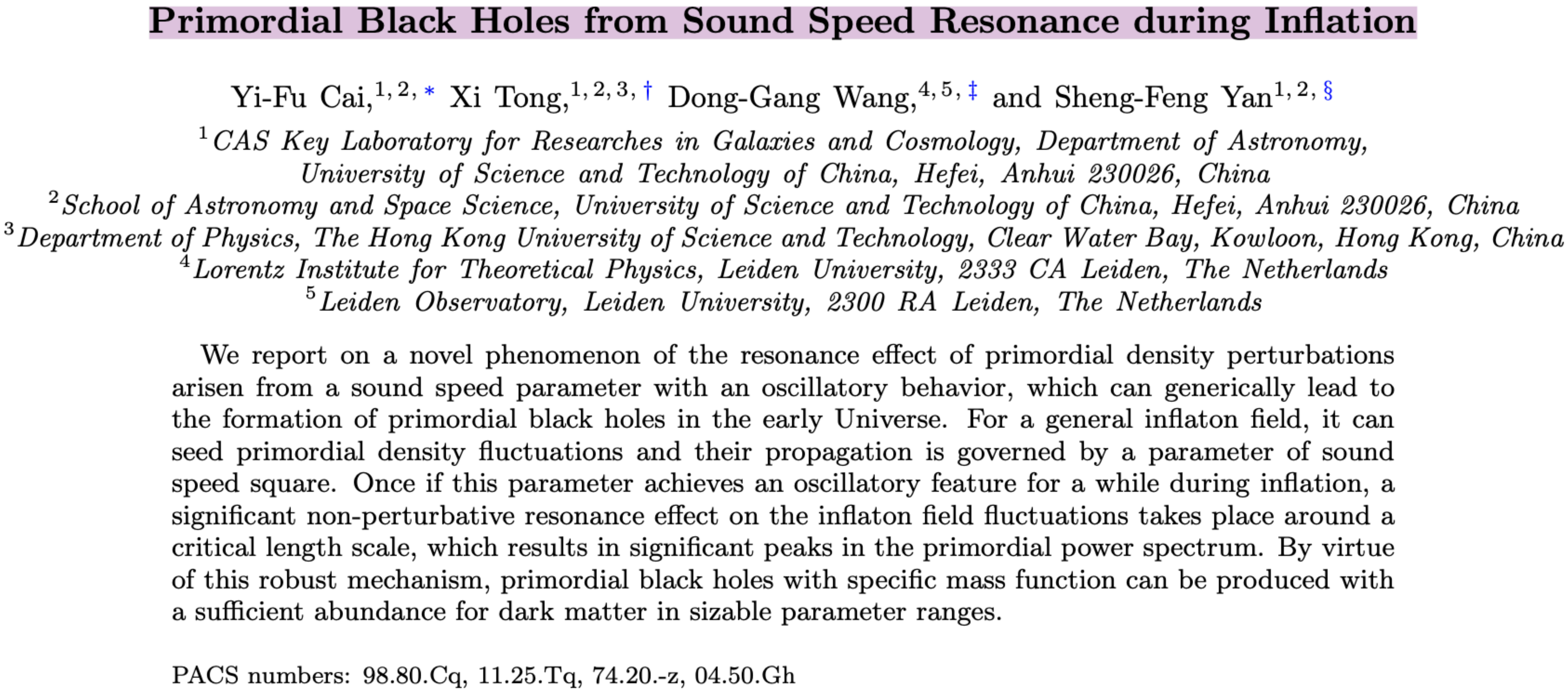
$$\boxed{\epsilon_{\mathrm{v}}(\phi) \equiv \frac{M_{\mathrm{pl}}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2} \qquad \boxed{\eta_{\mathrm{v}}(\phi) \equiv M_{\mathrm{pl}}^2\frac{V_{,\phi\phi}}{V}}$$

Quantum fluctuations



APS/

About the importance of EFT conditions: PBHs



Primordial Black Holes from Sound Speed Resonance during Inflation

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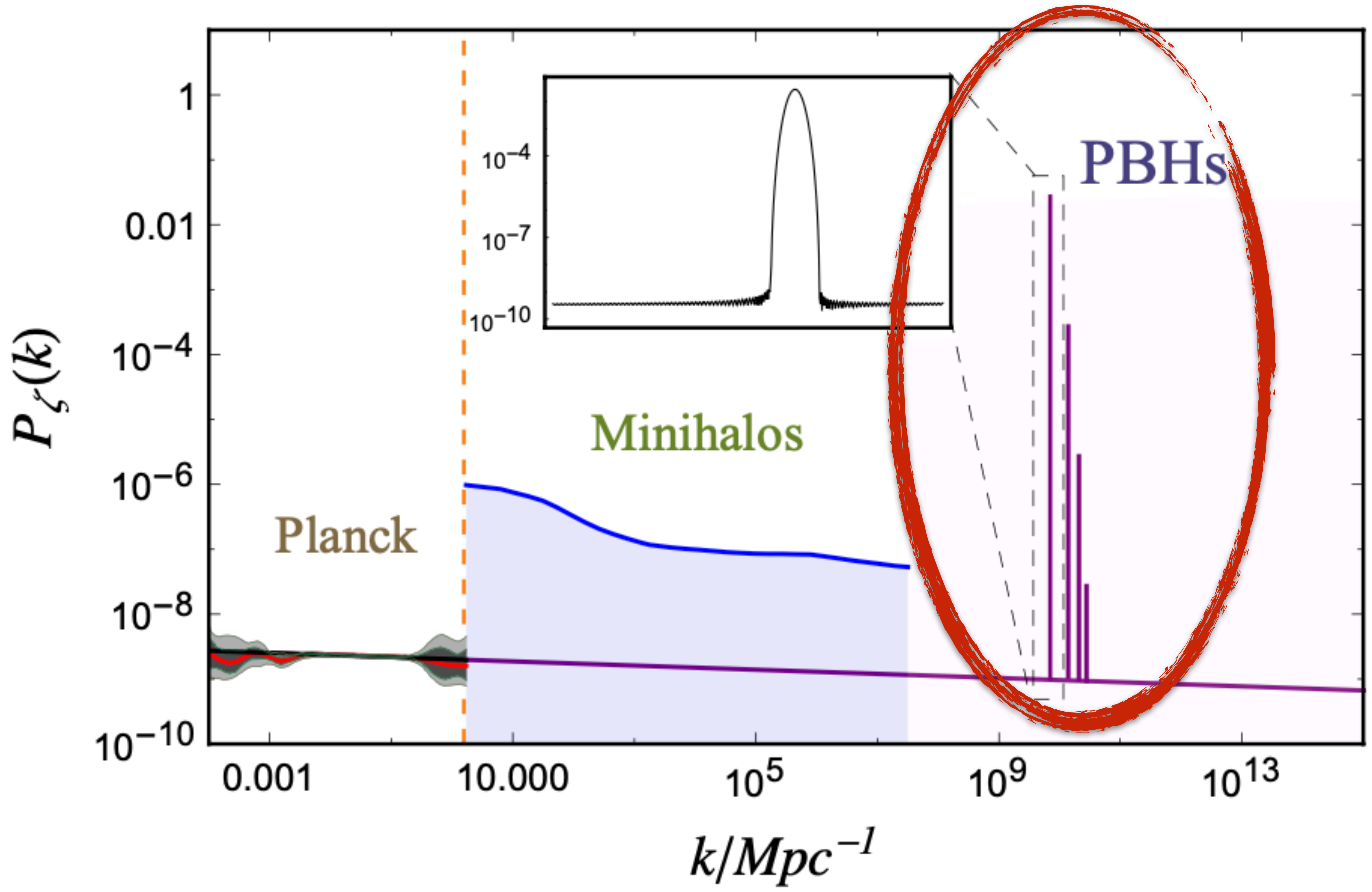
[4] *Lorentz Institute for Theoretical Physics, Leiden University, 2333 CA Leiden, The Netherlands*
[5] *Leiden Observatory, Leiden University, 2300 RA Leiden, The Netherlands*

We report on a novel phenomenon of the resonance effect of primordial density perturbations arisen from a sound speed parameter with an oscillatory behavior, which can generically lead to the formation of primordial black holes in the early Universe. For a general inflaton field, it can seed primordial density fluctuations and their propagation is governed by a parameter of sound speed square. Once if this parameter achieves an oscillatory feature for a while during inflation, a significant non-perturbative resonance effect on the inflaton field fluctuations takes place around a critical length scale, which results in significant peaks in the primordial power spectrum. By virtue of this robust mechanism, primordial black holes with specific mass function can be produced with a sufficient abundance for dark matter in sizable parameter ranges.

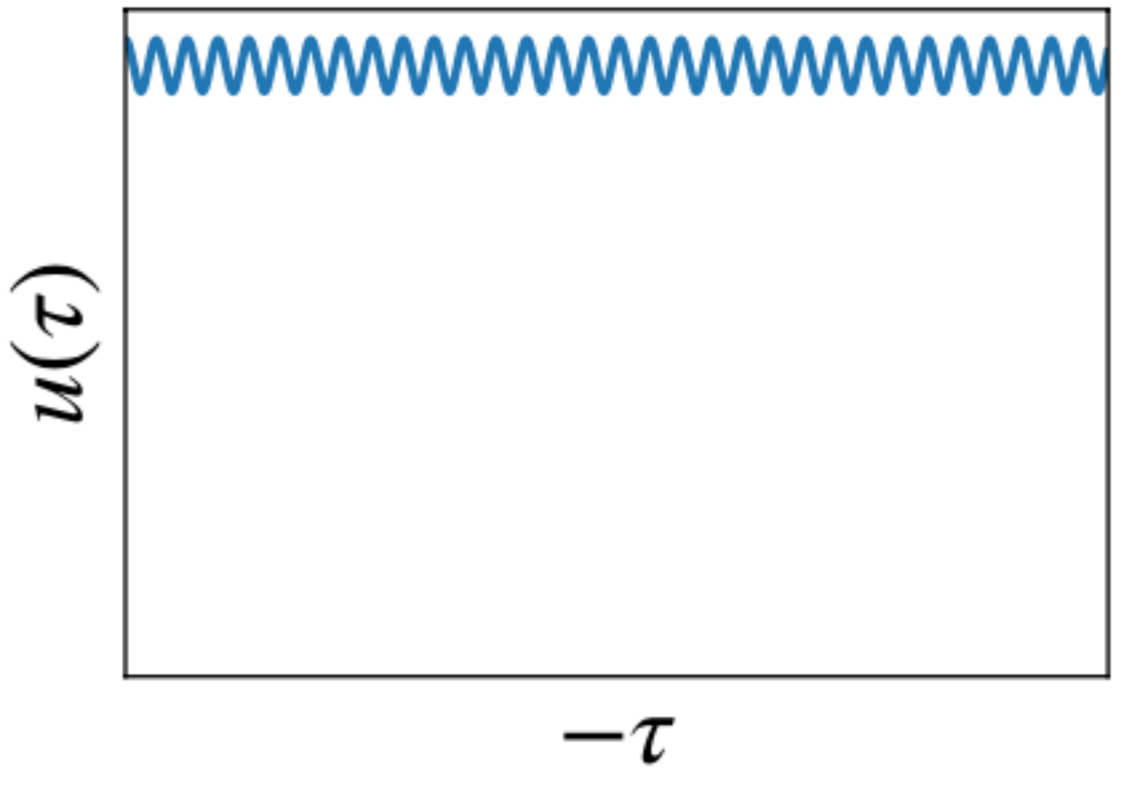
PACS numbers: 98.80.Cq, 11.25.Tq, 74.20.-z, 04.50.Gh

About the importance of EFT conditions: PBHs



$$P_\zeta(k) \simeq A_s\left(\frac{k}{k_p}\right)^{n_s-1}\left[1+\frac{\xi k_*}{2}e^{-\xi k_* \tau_0}\delta(k-k_*)\right]$$

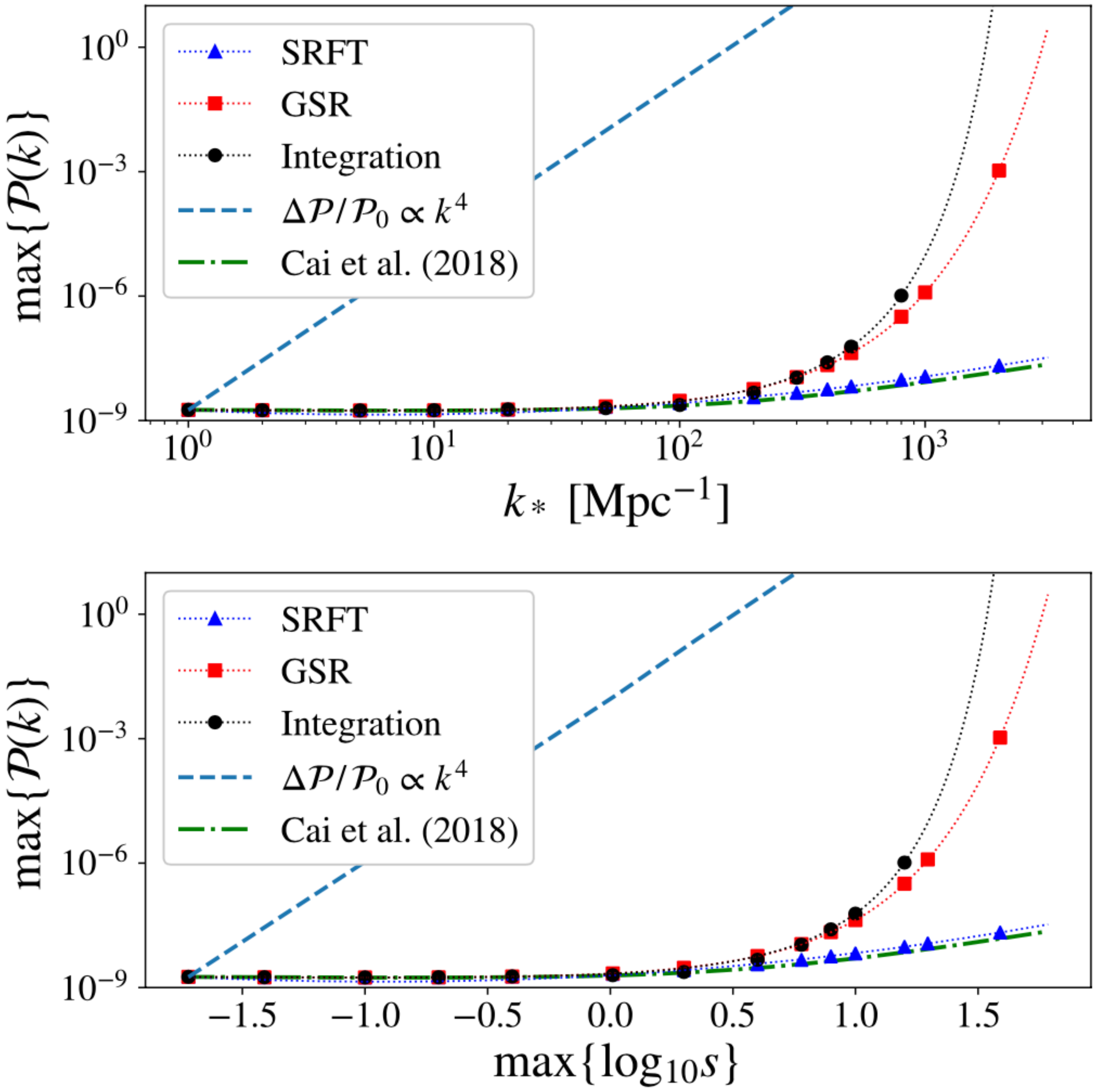
$$\frac{\Delta\mathcal{P}_\mathcal{R}}{\mathcal{P}_\mathcal{R}} = k\int_{-\infty}^{0} d\tau\left(1-\frac{1}{c_s^2(\tau)}\right)\sin\left(2k\tau\right)$$



About the importance of EFT conditions: PBHs

$$S = \int d^4x a^3 \epsilon H^2 \left[-\frac{\dot\pi^2}{c_s^2} + \frac{(\partial_i \pi)^2}{a^2} \boxed{- 2Hs c_s^{-2} \pi \dot\pi^2} - (1 - c_s^{-2}) \dot\pi \left(\frac{\dot\pi^2}{c_s^2} - \frac{(\partial_i \pi)^2}{a^2} \right) \right]$$

$\boxed{\pi = -H\mathcal{R}}$



(In preparation) Hidde Jense + **GCH**

Quantum fluctuations

- Single field inflation action:

$$\boxed{S=\int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}R+\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi-V(\phi)\right]}=S_{\mathrm{EH}}+S_\phi$$

- Perturbations of the homogeneous background

$$\delta\phi=0 \qquad \delta g_{ij}(t,\mathbf{x})=a^2\left[(1-2\mathcal{R}(t,\mathbf{x}))\delta_{ij}+h_{ij}(t,\mathbf{x})\right] \qquad \partial_i h_{ij}=0$$

- Action for the comoving curvature perturbation

$$S_2=\frac{1}{2}\int d^4x a^3\frac{\dot\phi^2}{H^2}\left[-\dot{\mathcal{R}}^2+\frac{(\partial_i\mathcal{R})^2}{a^2}\right]$$

How to characterise perturbations?

- Correlation function:

$$\langle \mathcal{R}(\mathbf{k}_1)\mathcal{R}(\mathbf{k}_2)\rangle \equiv (2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2)P_{\mathcal{R}}(k)$$

- Primordial power spectrum:

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}P_{\mathcal{R}}(k)$$

- Relation with slow-roll parameters

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{1}{8\pi^2}\frac{H_*^2}{\epsilon}$$

- EFT of inflationary perturbations

$$S = \int d^4x a^3 \epsilon H^2 \left[-\frac{\dot{\pi}^2}{c_s^2} + \frac{(\partial_i \pi)^2}{a^2} - 2Hsc_s^{-2}\pi\dot{\pi}^2 - (1 - c_s^{-2})\dot{\pi}\left(\frac{\dot{\pi}^2}{c_s^2} - \frac{(\partial_i \pi)^2}{a^2}\right)\right]$$

- Speed of sound in associated a turn in the field trajectory

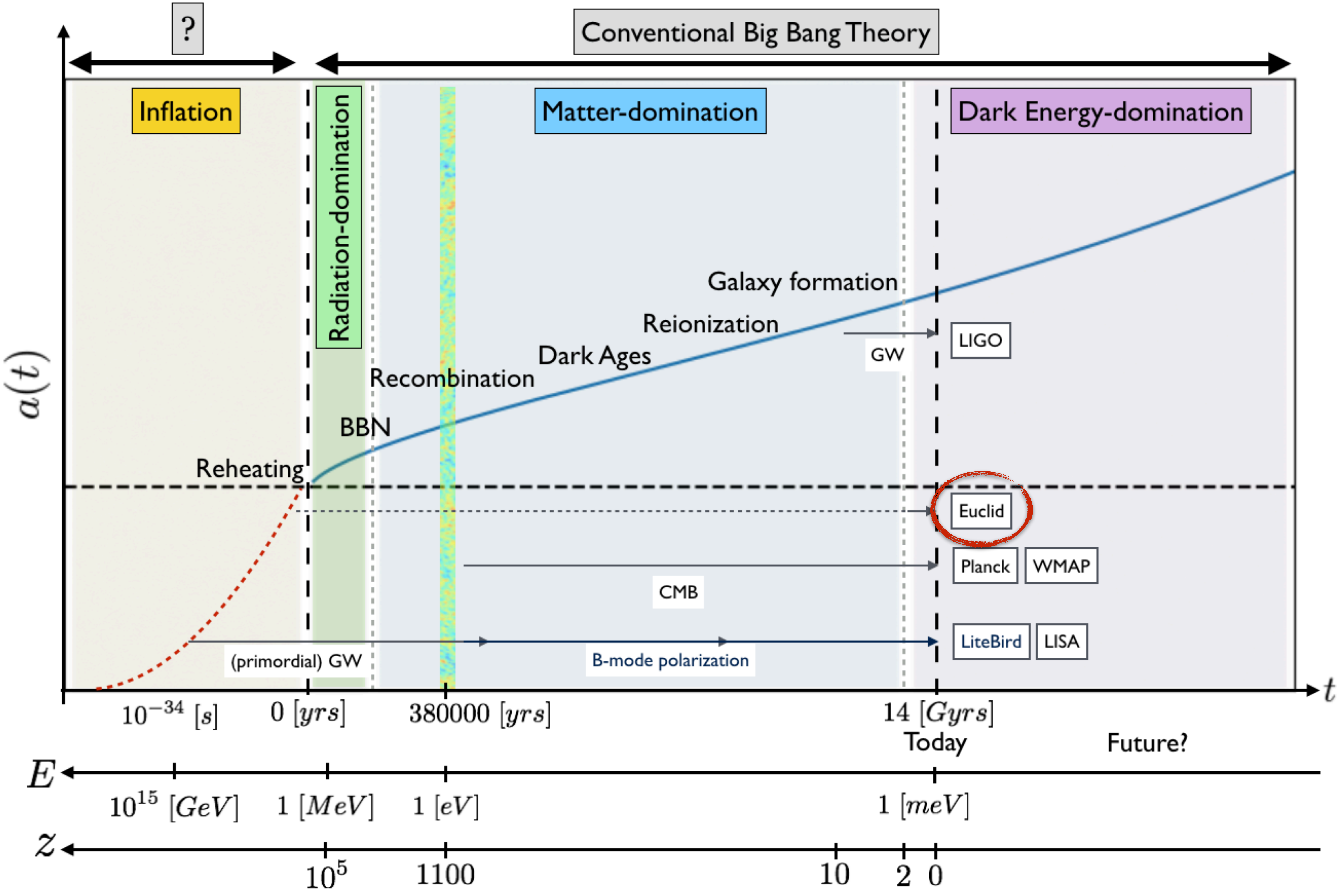
$$c_s^{-2} = 1 + \frac{4\Omega^2}{k^2/a^2 + M_{eff}}$$

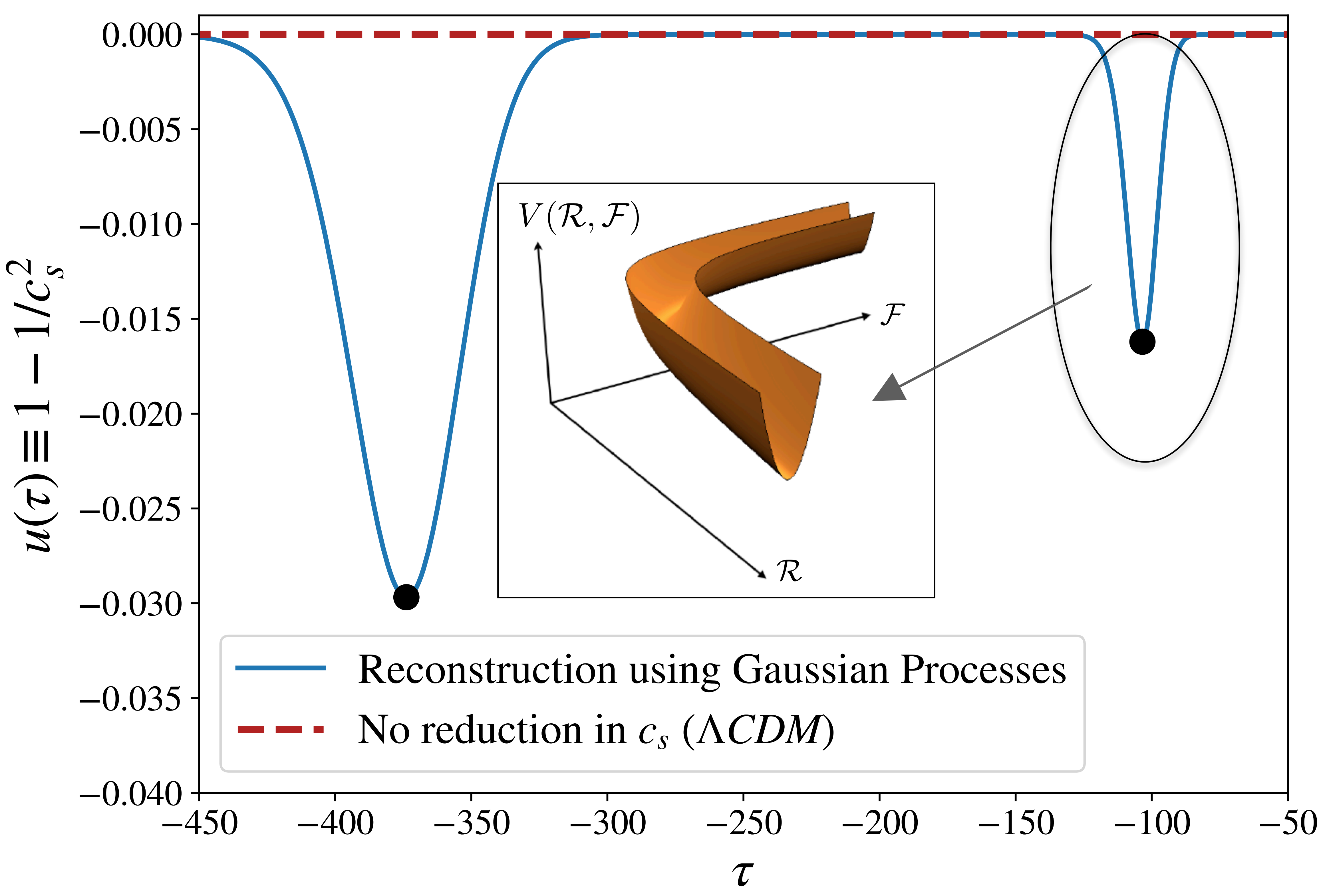
- From where is coming the EFT action?

$$S_2 = \frac{1}{2}\int d^4x a^3 \left[\frac{\dot{\phi}_0^2}{H^2}\dot{\mathcal{R}}^2 - \frac{\dot{\phi}_0^2}{a^2 H^2}(\nabla\mathcal{R})^2 + \dot{\mathcal{F}}^2 - \frac{(\nabla\mathcal{F})^2}{a^2} - M_{eff}^2\mathcal{F}^2 - 4\Omega\frac{\dot{\phi}_0}{H}\mathcal{F}\dot{\mathcal{R}}\right],$$

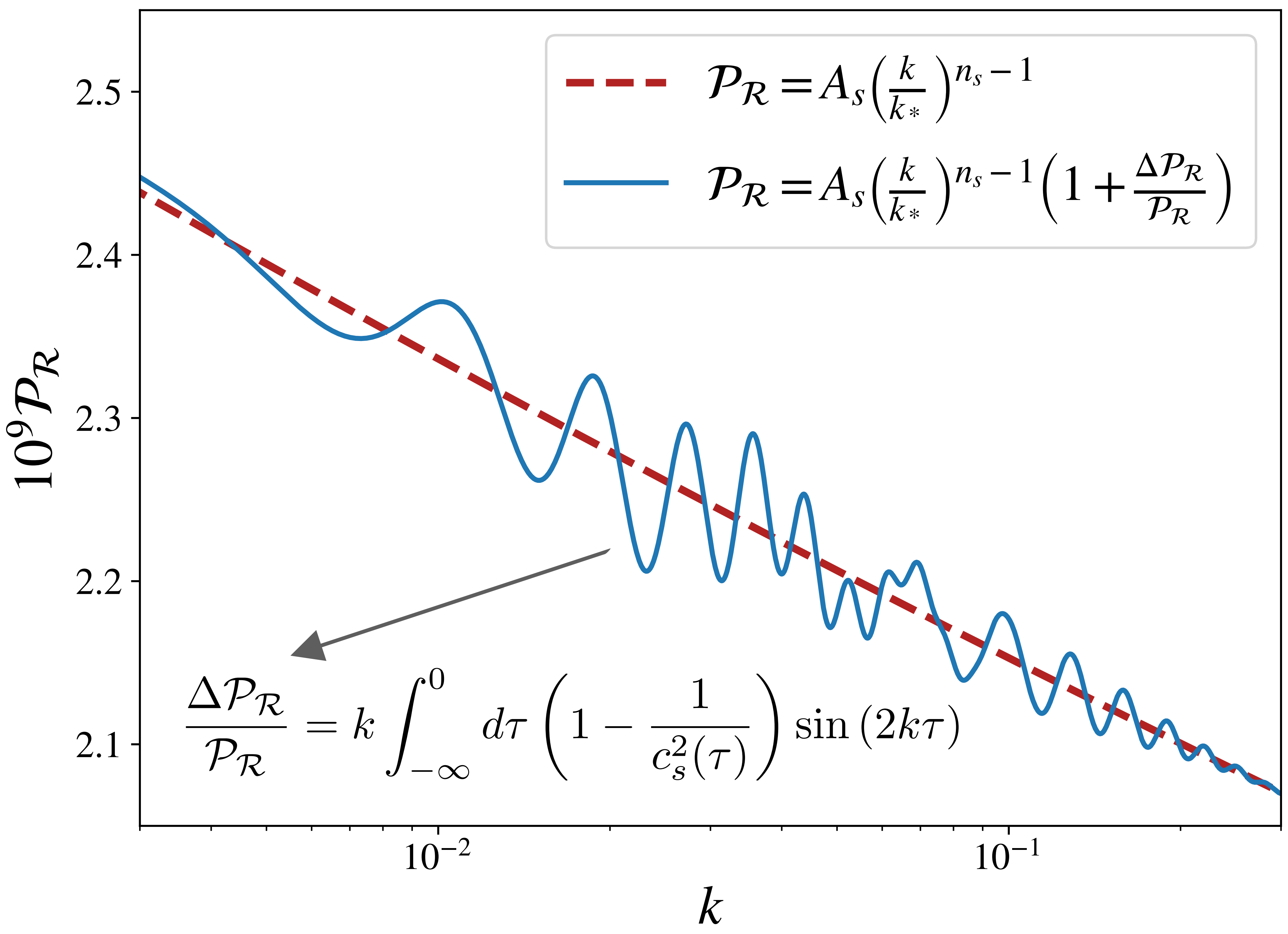
- The Goldstone boson and the comoving curvature perturbation:

$$\pi = -\frac{\mathcal{R}}{H}$$


?
Conventional Big Bang Theory
Inflation
Radiation-domination
Matter-domination
Dark Energy-domination
$a(t)$
Galaxy formation
Reionization
Dark Ages
Recombination
BBN
Reheating
GW
LIGO
Euclid
Planck
WMAP
CMB
LiteBird
LISA
(primordial) GW
B-mode polarization
t
10^{-34} [s]
0 [yrs]
380000 [yrs]
14 [$Gyrs$]
Today
Future?
E
10^{15} [GeV]
1 [MeV]
1 [eV]
1 [meV]
z
10^5
1100
10
2
0



$u(\tau) \equiv 1 - 1/c_s^2$
0.000
−0.005
−0.010
−0.015
−0.020
−0.025
−0.030
−0.035
−0.040
−450
−400
−350
−300
−250
−200
−150
−100
−50
τ
$V(\mathcal{R}, \mathcal{F})$
$\mathcal{F}$
$\mathcal{R}$
Reconstruction using Gaussian Processes
No reduction in c_s (ΛCDM)



$10^9 \mathcal{P}_\mathcal{R}$
2.5
2.4
2.3
2.2
2.1
10^{-2}
10^{-1}
k
$\mathcal{P}_\mathcal{R} = A_s\left(\frac{k}{k_*}\right)^{n_s - 1}$
$\mathcal{P}_\mathcal{R} = A_s\left(\frac{k}{k_*}\right)^{n_s - 1}\left(1 + \frac{\Delta\mathcal{P}_\mathcal{R}}{\mathcal{P}_\mathcal{R}}\right)$
$\frac{\Delta\mathcal{P}_\mathcal{R}}{\mathcal{P}_\mathcal{R}} = k\int_{-\infty}^{0} d\tau \left(1 - \frac{1}{c_s^2(\tau)}\right)\sin(2k\tau)$