Lattice simulations of axion-U(1) inflation

Angelo Caravano LMU & MPA @ Munich, Germany

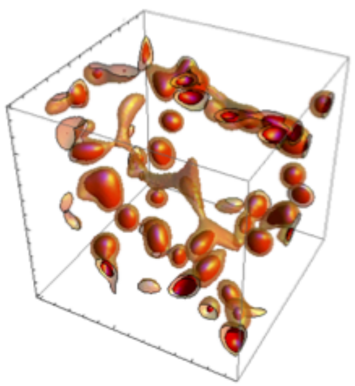
Based on:
A. Caravano, E. Komatsu, K. D. Lozanov, J. Weller
arXiv:2204.12874

LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

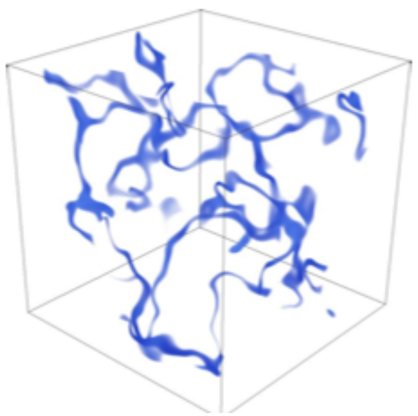
Max-Planck-Institut für Astrophysik

Lattice simulations

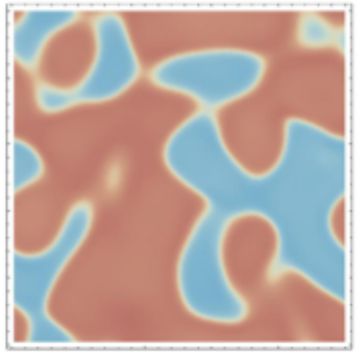
- Numerical tool to study non-linear cosmological phenomena.
- Typically associated with the reheating phase after inflation.



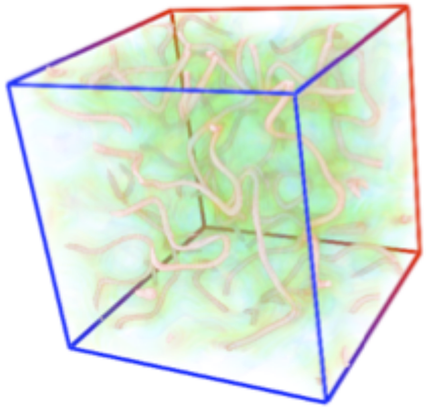
[**M. A. Amin, R. Easther, H. Finkel**, arXiv:1009.2505]



[**A. V. Frolov**, arXiv:1004.3559]



[**M. A. Amin, J. Fan, K. D. Lozanov, M. Reece**, arXiv:1802.00444]



[**J. Dufaux, D.G. Figueroa, J. Garcia-Bellido**, arXiv:1006.0217]

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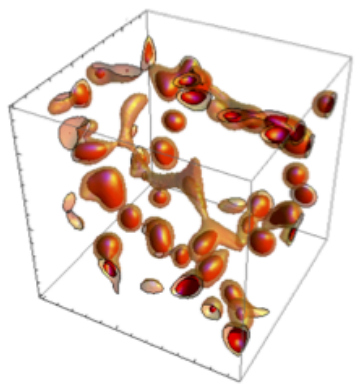

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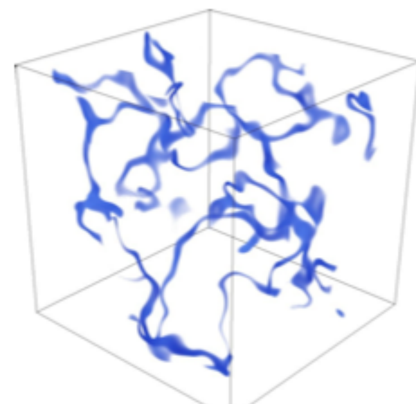

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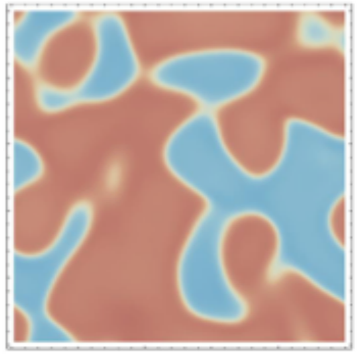

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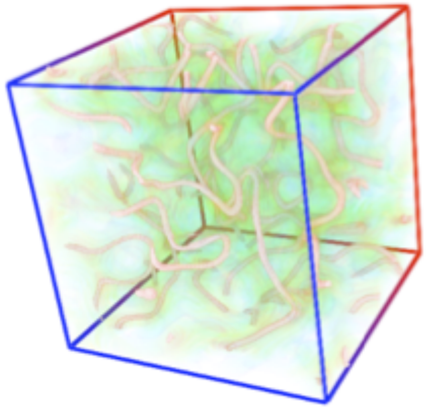

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Our goal:

Generalise lattice techniques to inflationary dynamics

A. Caravano, E. Komatsu, K. D. Lozanov, J. Weller arXiv:2102.06378 arXiv:2110.10695 arXiv:2204.12874

In this talk: **focus on axion-U(1) model.**

Axion-U(1) inflation

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} M_{\rm Pl}^2 \mathscr{R} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\alpha}{4f} \phi F_{\mu\nu} \tilde{F}^{\mu\nu} \right)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Ingredients: (1) Pseudo-scalar (axion) inflaton, (2) U(1) field, (3) Interaction

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M. Anber, L. Sorbo 0908.4089

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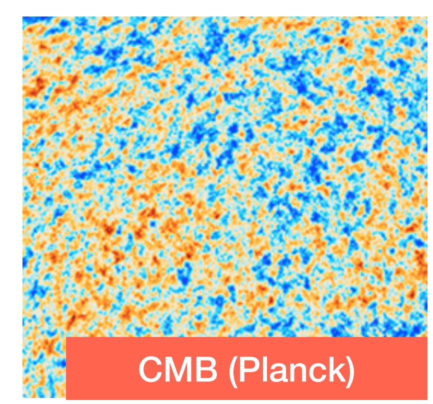
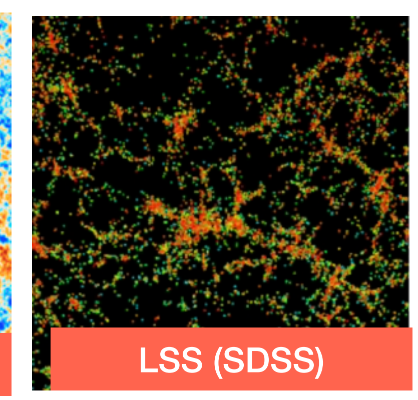
Observational consequences:

Production of gauge field particles → decay into inflaton perturbations





observable!

Axion-U(1) inflation

N. Barnaby, M. Peloso 1011.1500
M. Anber, L. Sorbo 0908.4089

Power spectrum and bispectrum of $\delta\phi$ are known.

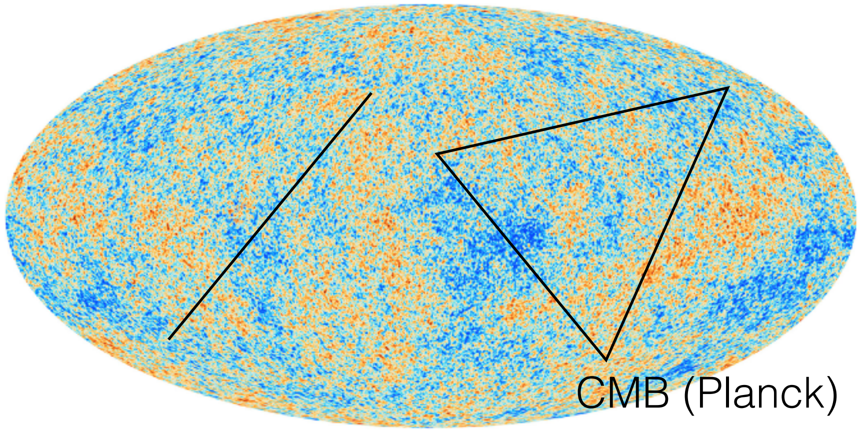
For $k \ll aH$:

$$\xi = \frac{\alpha\dot{\phi}}{2fH}$$

- $\langle \delta\phi_{\mathbf{k}} \delta\phi_{\mathbf{k}'} \rangle \simeq \frac{H^2}{2k^3} \left(1 + f_2(\xi) e^{4\pi\xi}\right) \delta(\mathbf{k} + \mathbf{k}')$

  vacuum (single-field) — sourced

- $\langle \delta\phi_{\mathbf{k}_1} \delta\phi_{\mathbf{k}_2} \delta\phi_{\mathbf{k}_3} \rangle \neq 0 \quad \simeq \frac{9}{80}(2\pi)^{5/2}\frac{H^6}{k^6} f_3\left(\xi, \frac{k_2}{k_1}, \frac{k_3}{k_1}\right) e^{6\pi\xi}\delta(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3})$



CMB (Planck)

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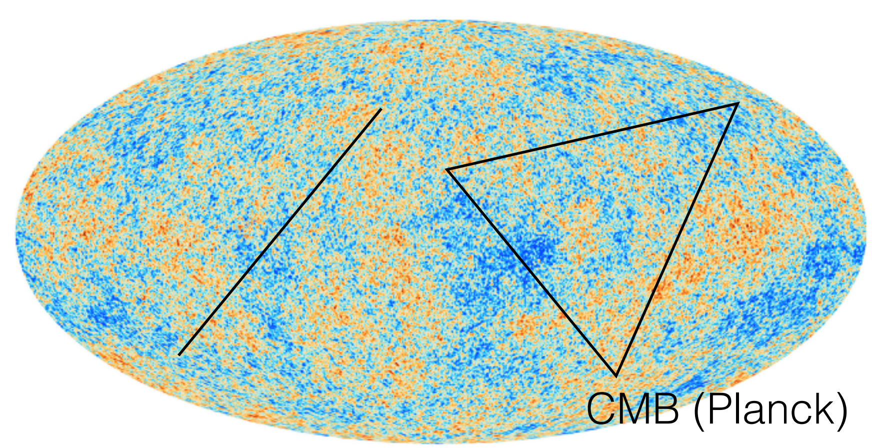
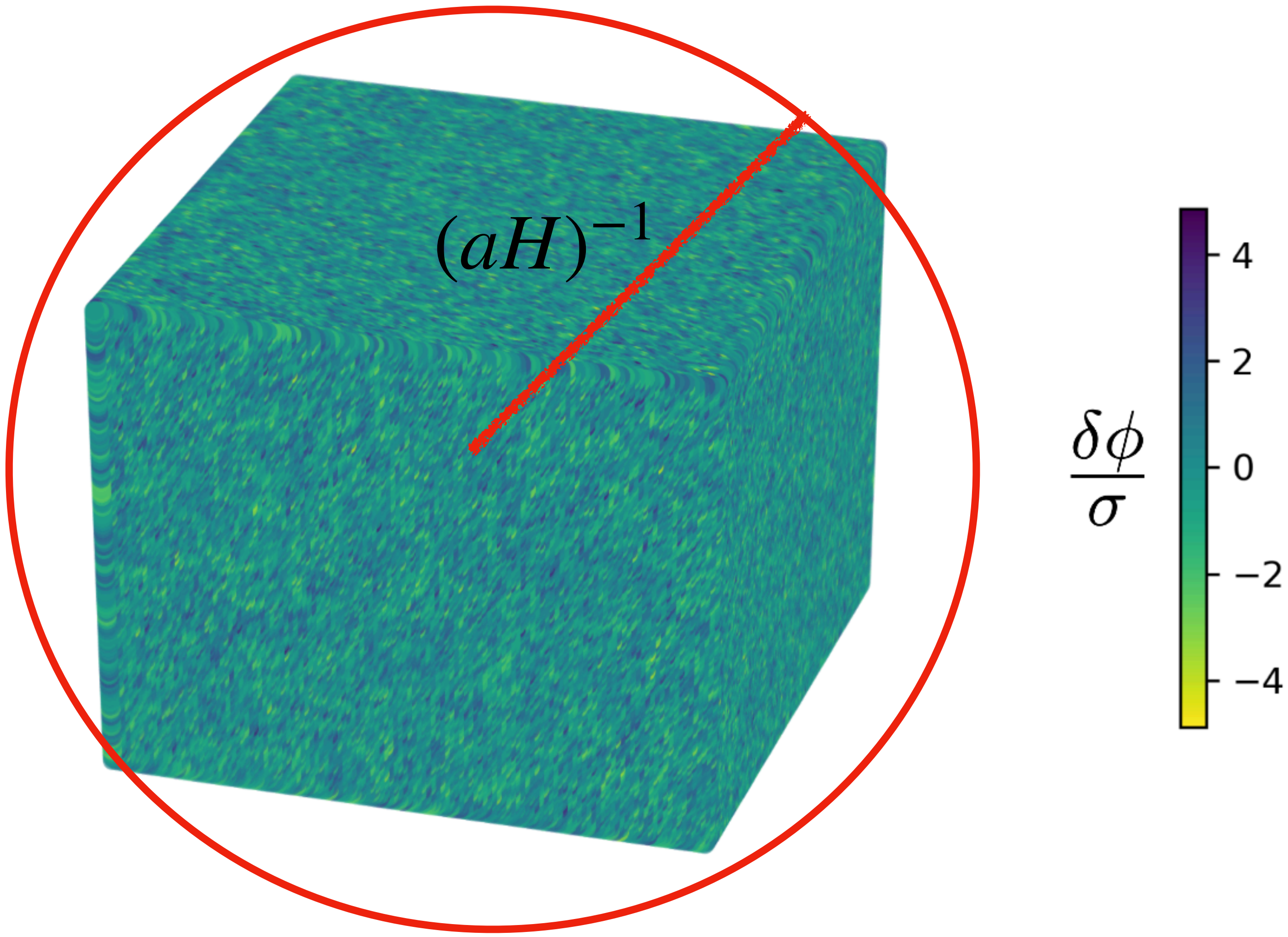
(+ Gravitational waves)



Lattice approach

Start with a **sub-horizon** box



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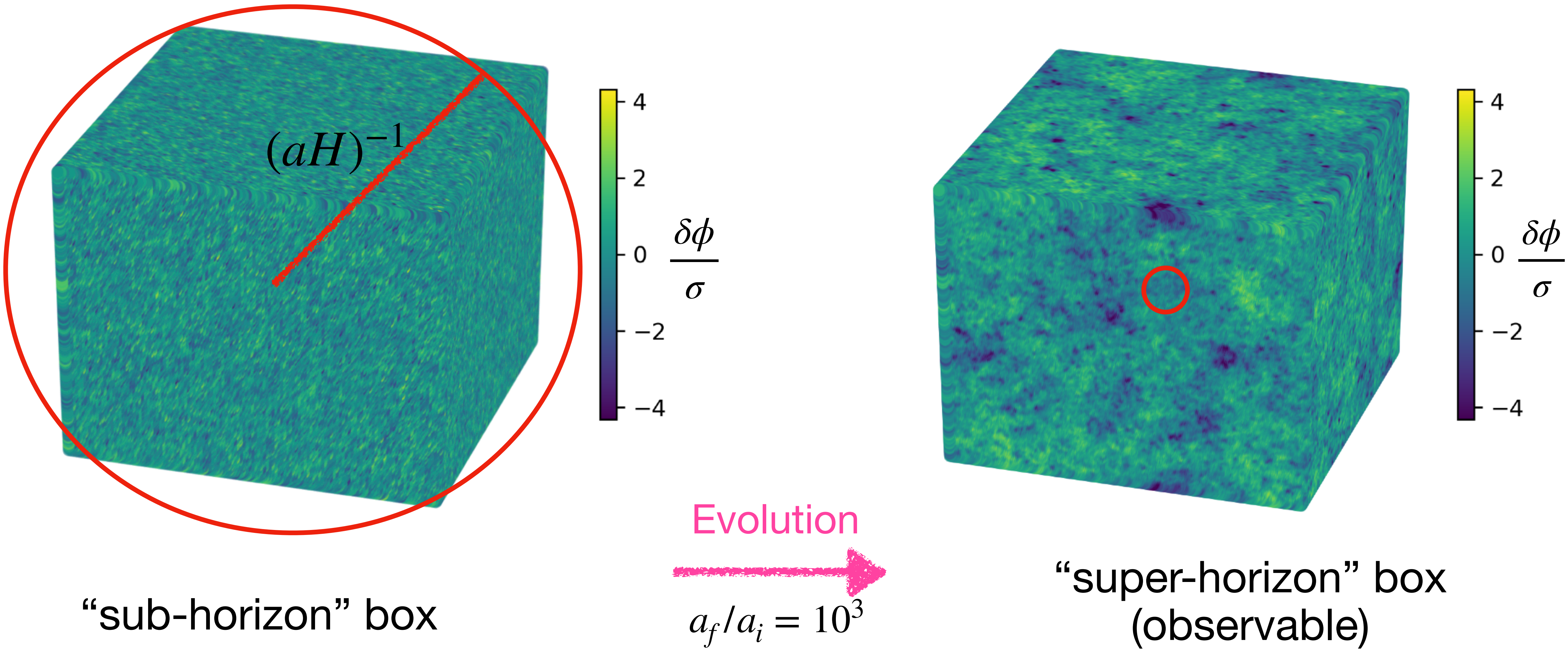

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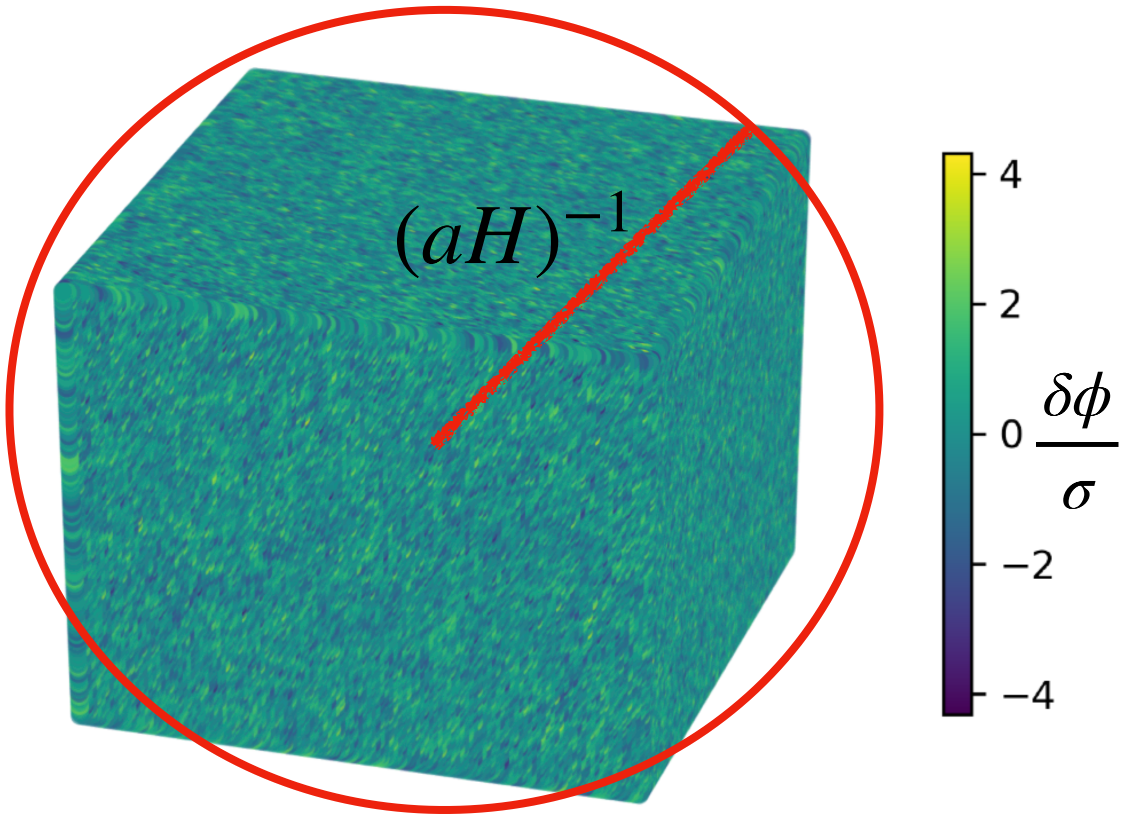
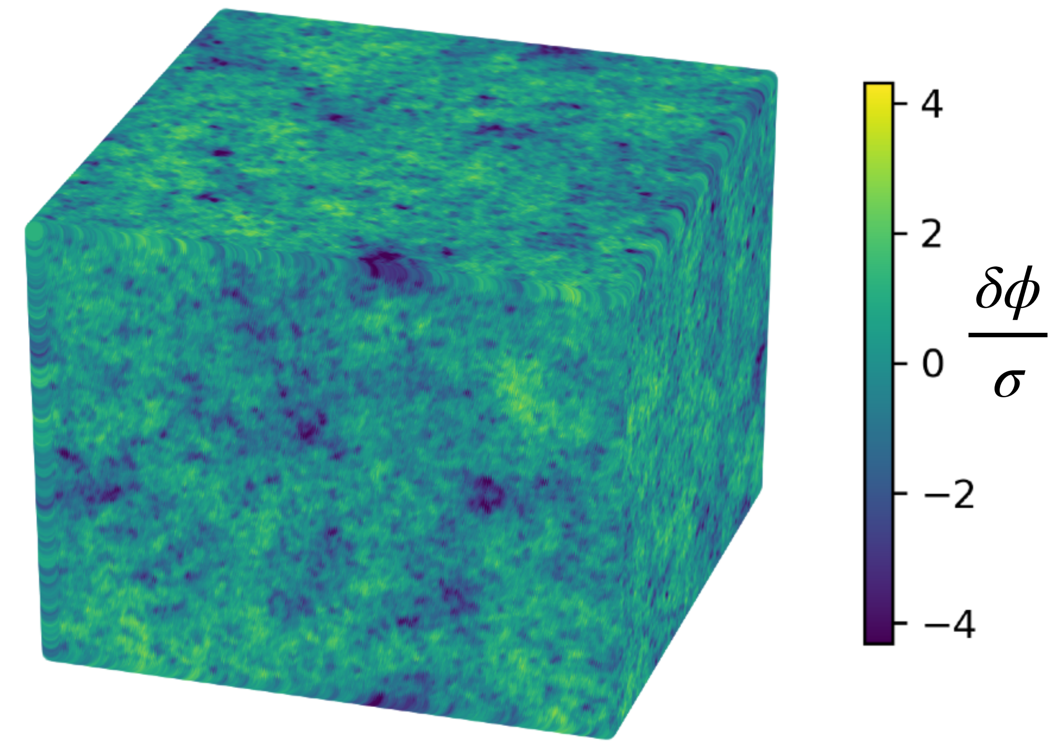


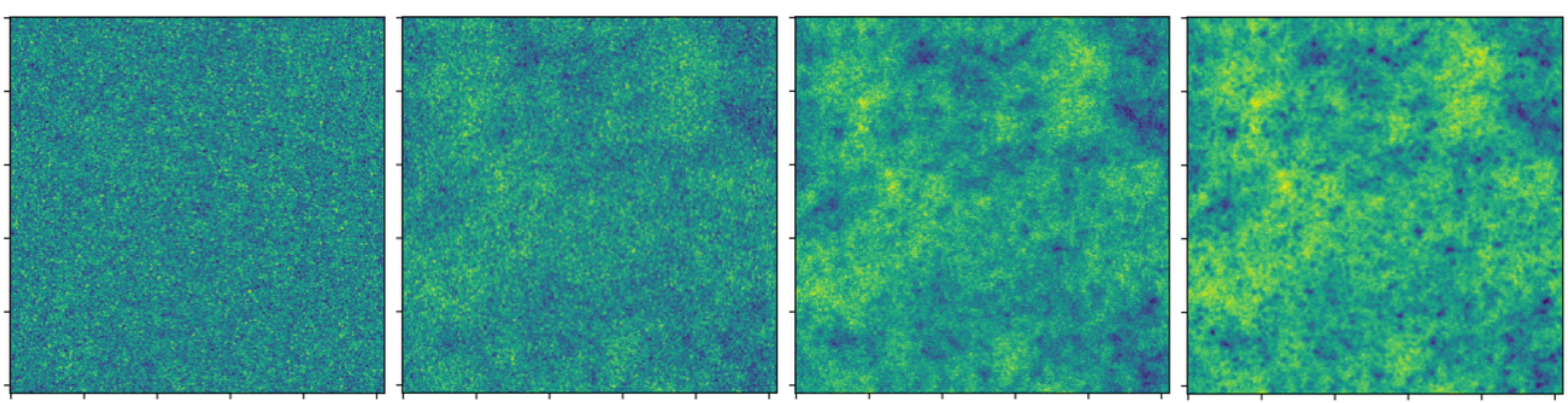


"sub-horizon" box

Evolution

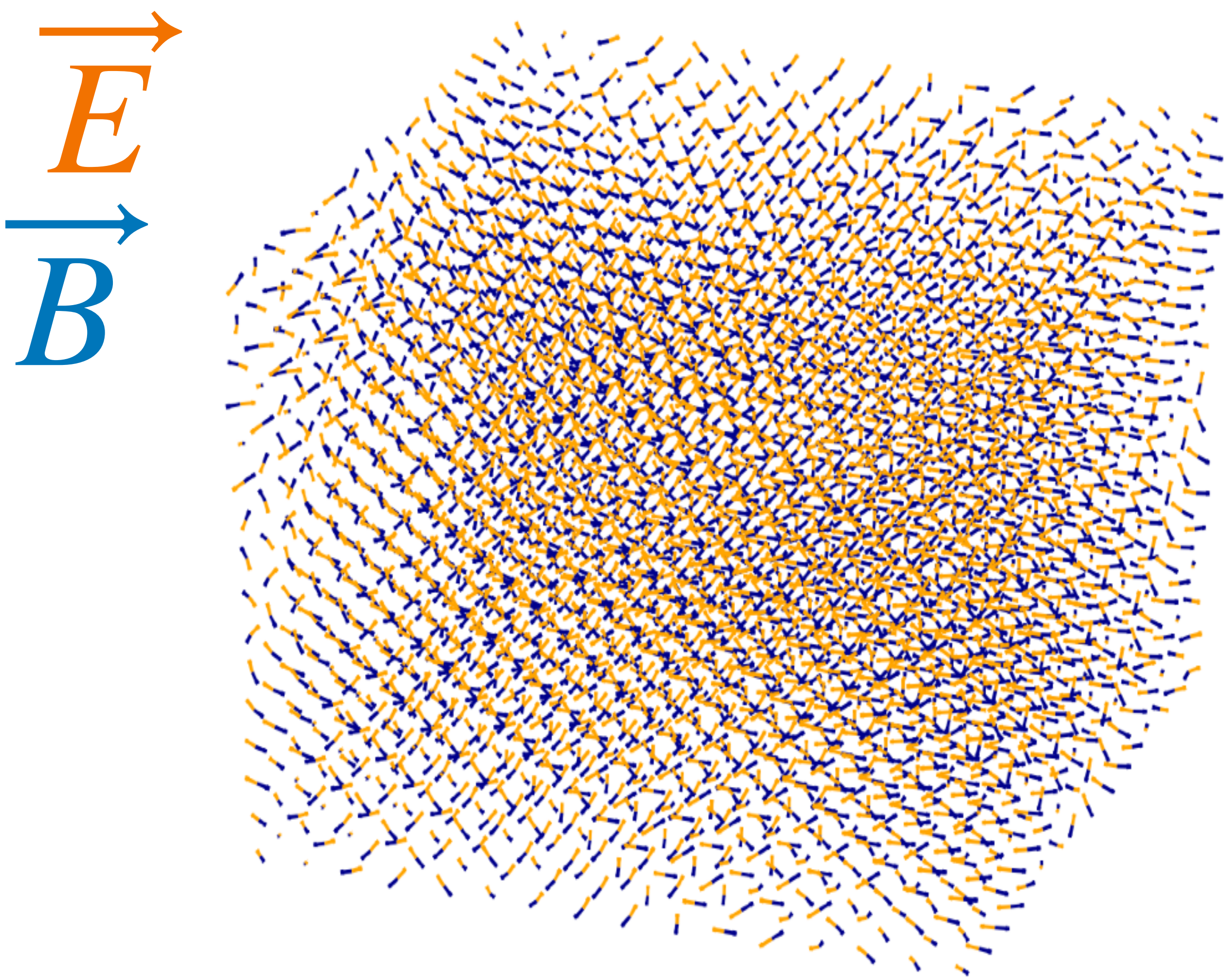
$a_f/a_i = 10^3$

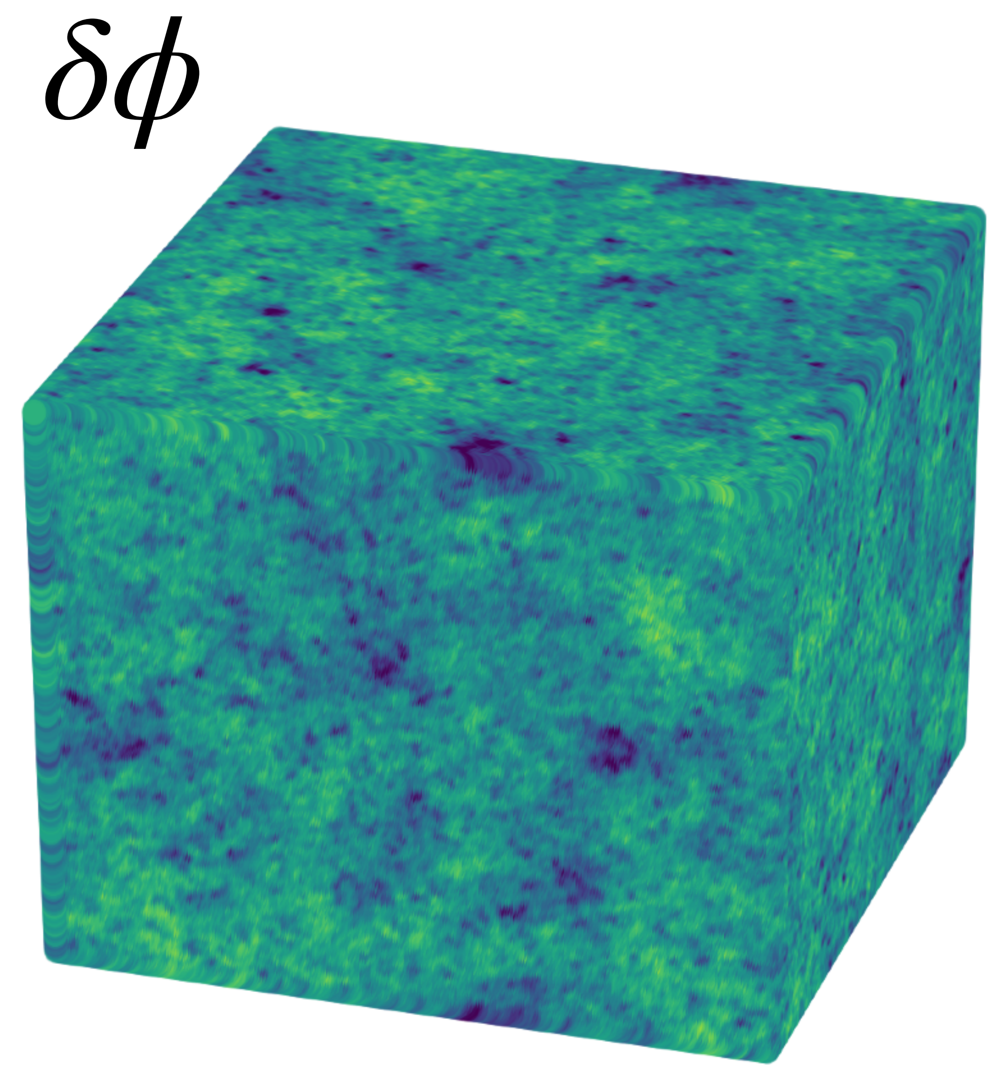
"super-horizon" box
(observable)



Lattice approach

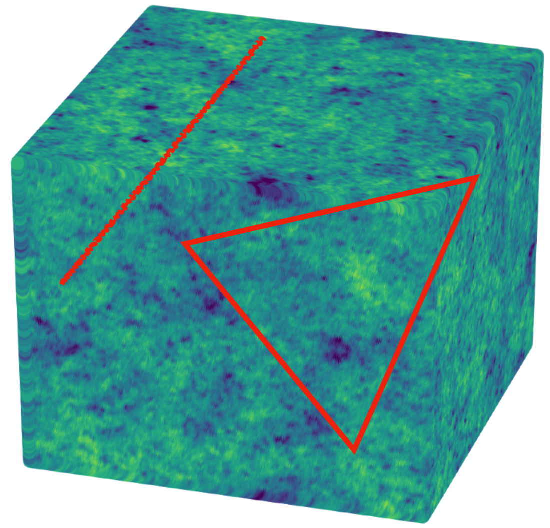
We are interested in the statistical properties of the super-horizon box:

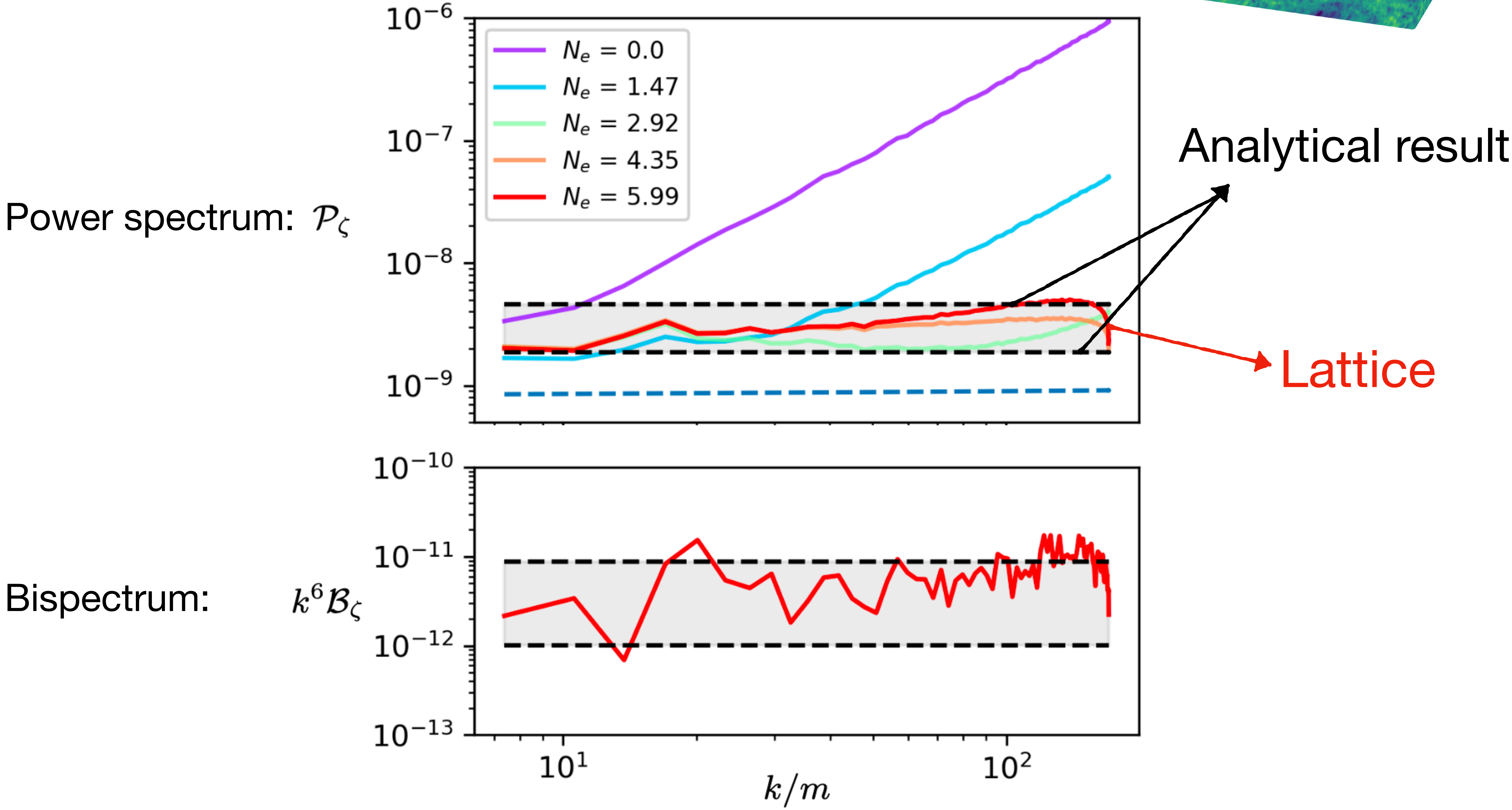




Power spectrum and bispectrum

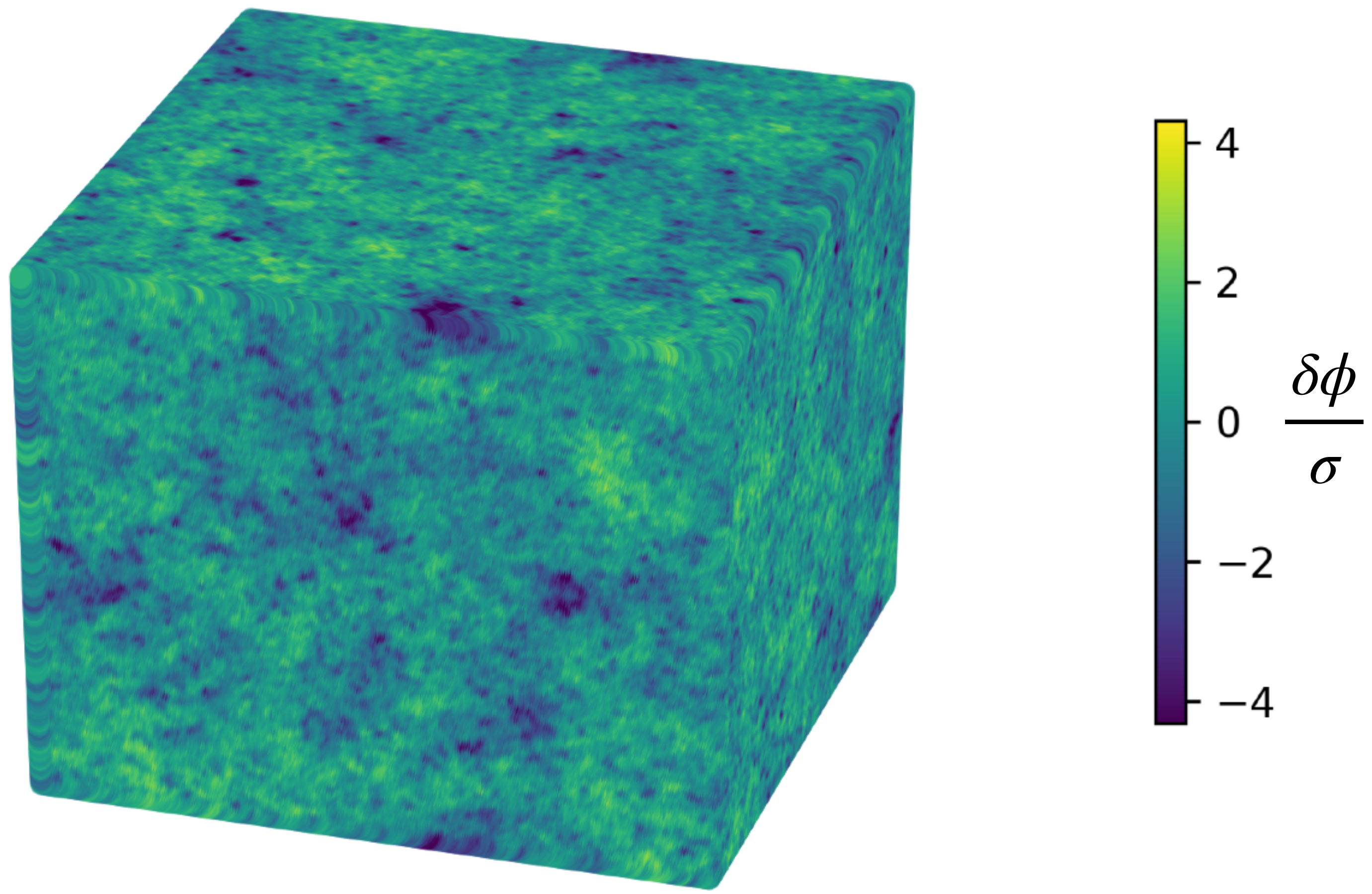
Simulation confirms known results:





And much more!

Thanks to the lattice,
we know the full distribution of $\delta\phi(\mathbf{x})$ in real space!



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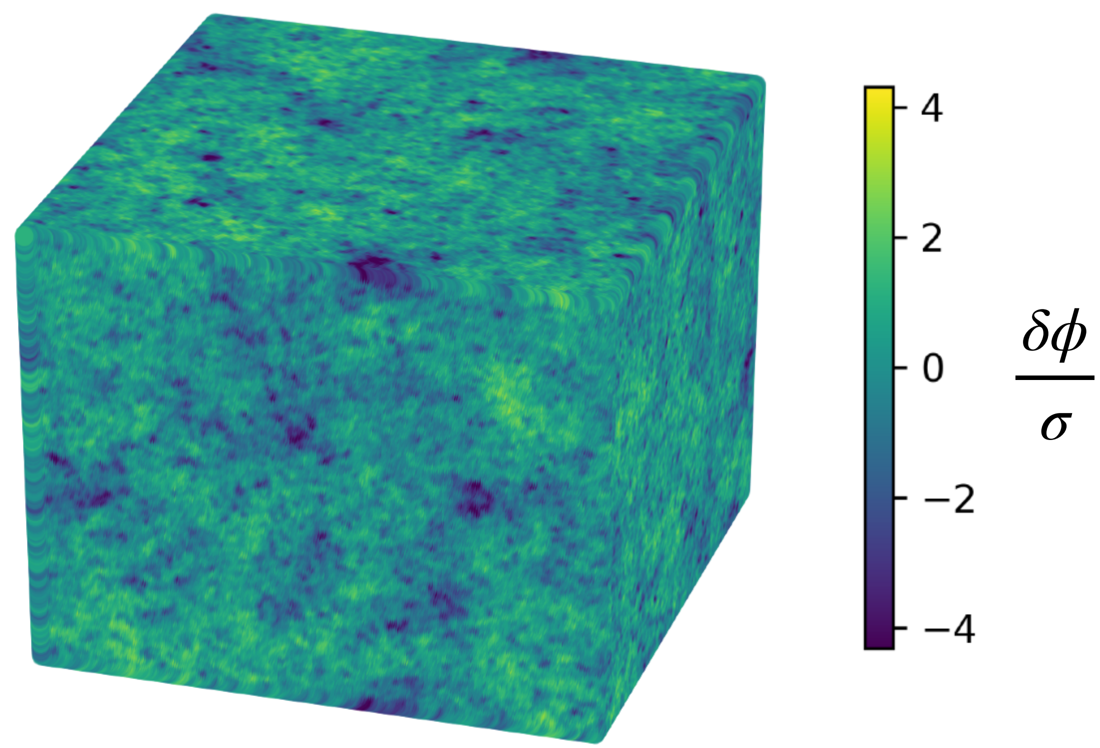
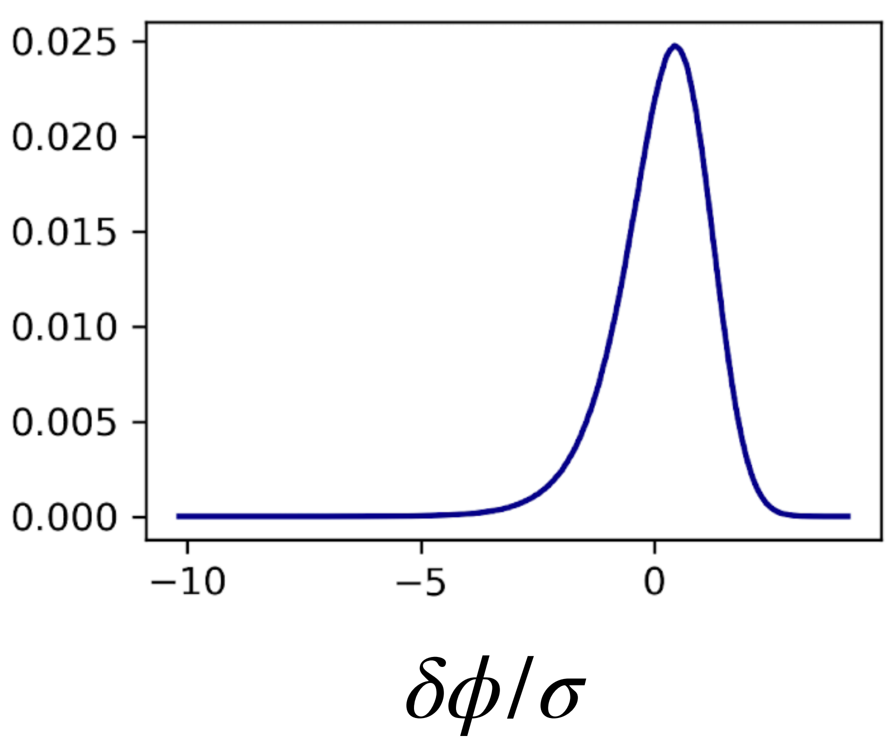
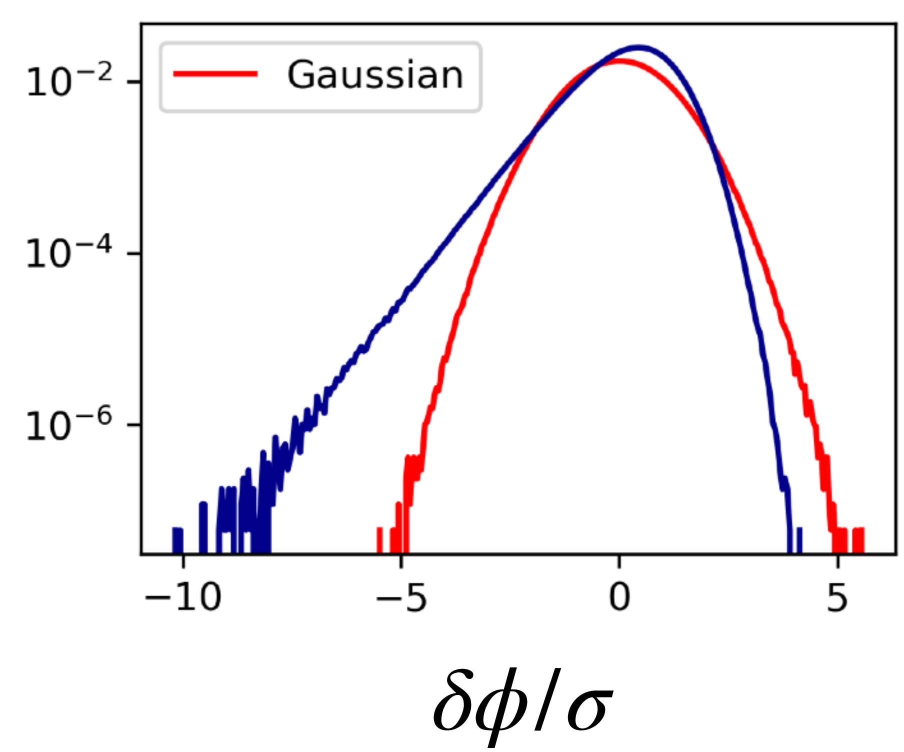




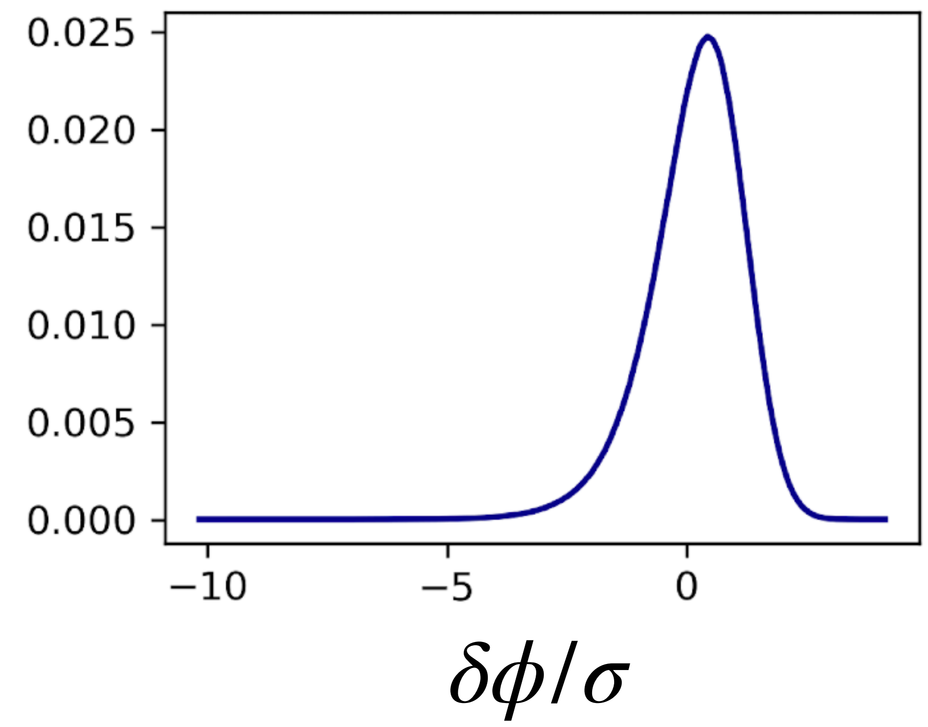


And much more!

Define cumulants:

$$\kappa_n = \frac{\langle \delta\phi^n \rangle_c}{\sigma^n}$$

κ_3 “skewness”, κ_4 “kurtosis”, etc.



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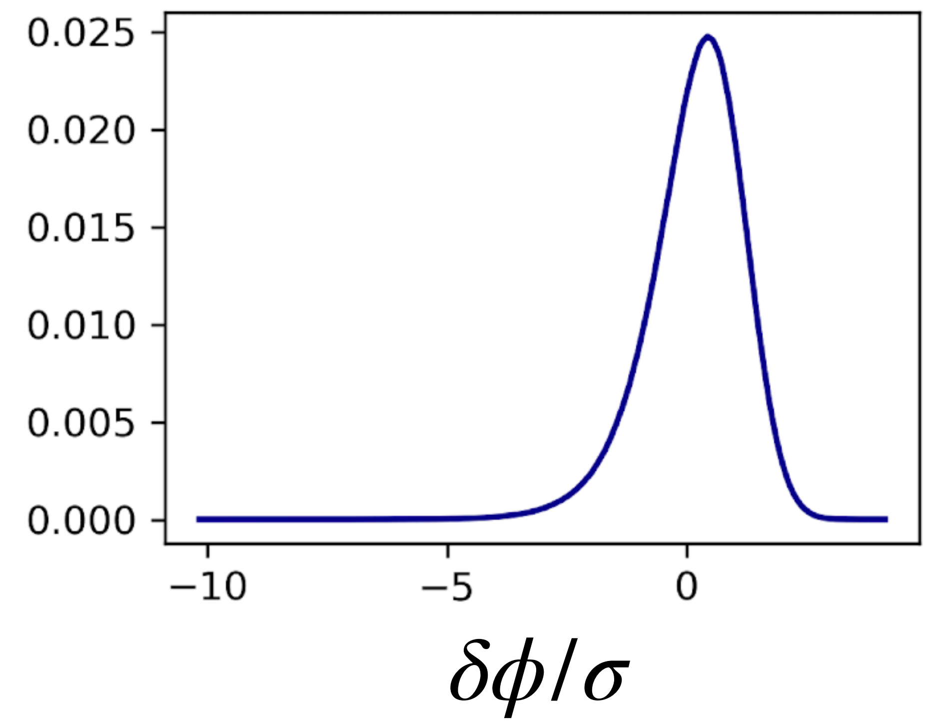

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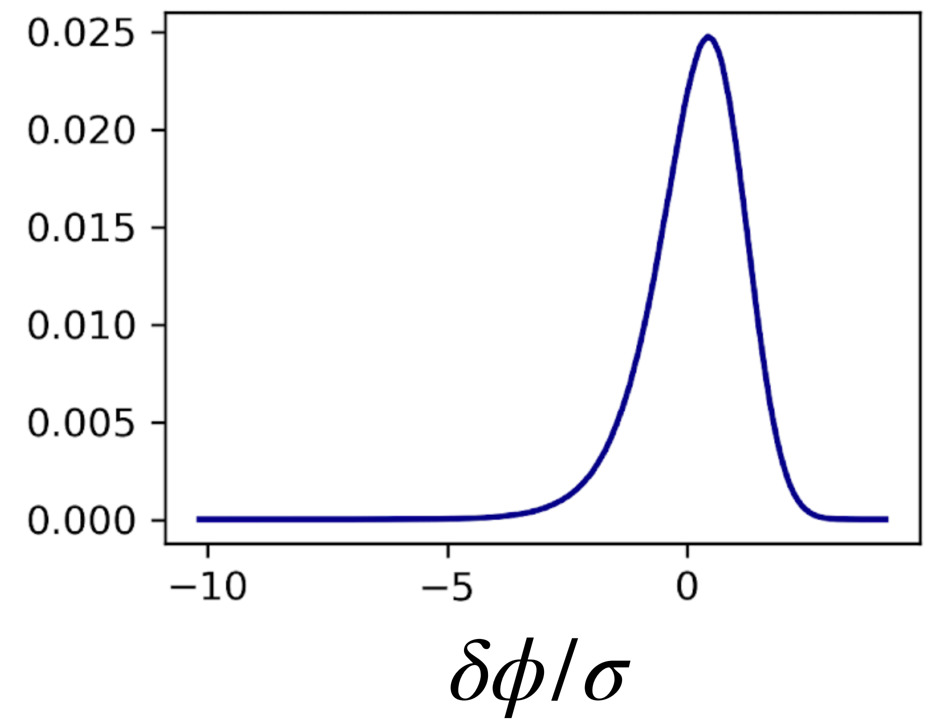

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Higher order polyspectra

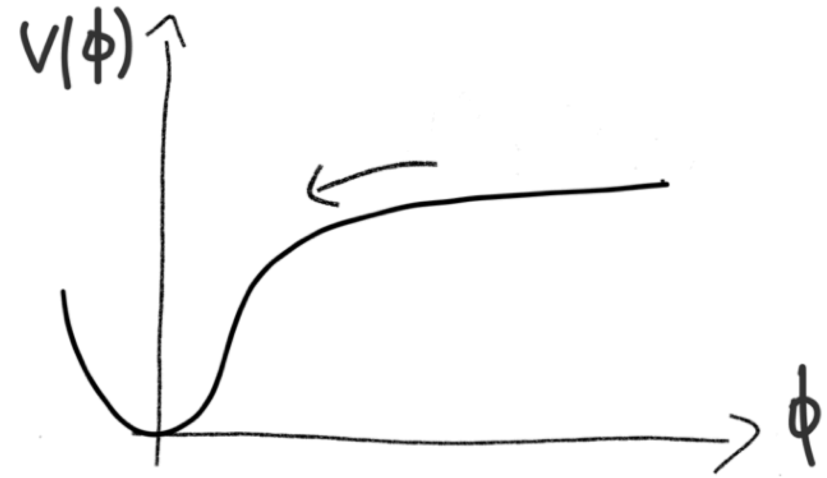
Trispectrum

Bispectrum

Nonlinear dynamics of axion-U(1) inflation

$$\xi = \frac{\alpha\dot{\phi}}{2fH}$$

grows during inflation



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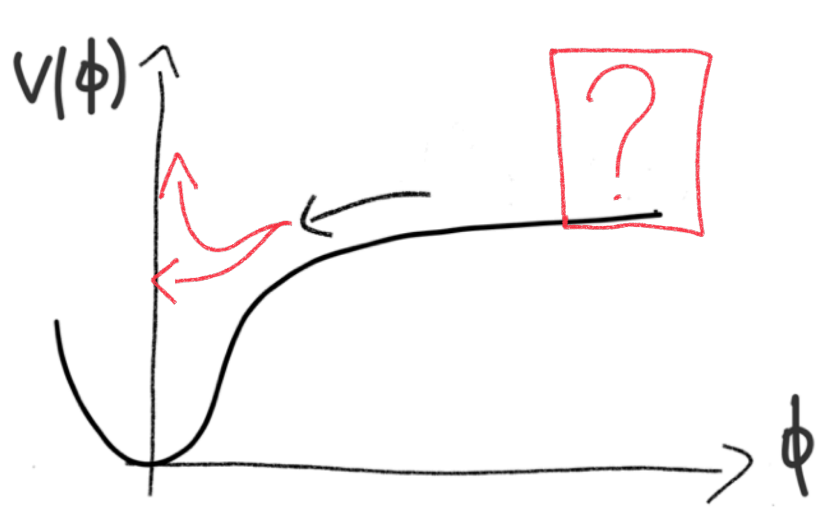

When ξ is large:
nonlinear dynamics
(backreaction)

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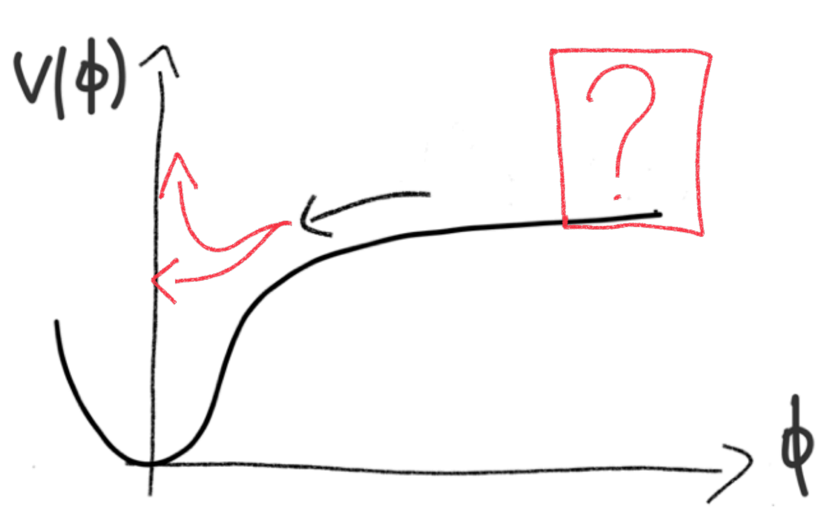
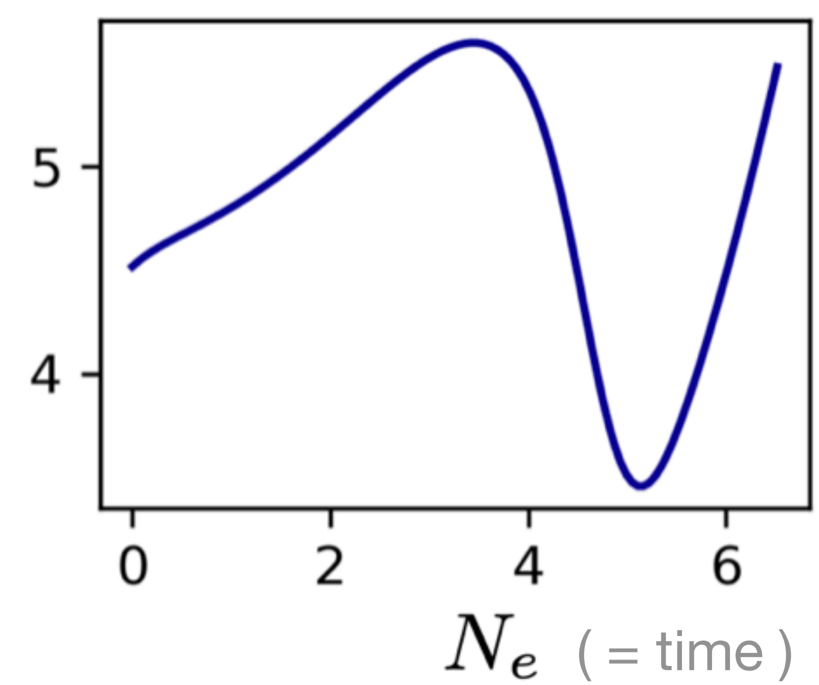


When ξ is large: nonlinear dynamics (backreaction)

Let's study transition linear ⟶ nonlinear



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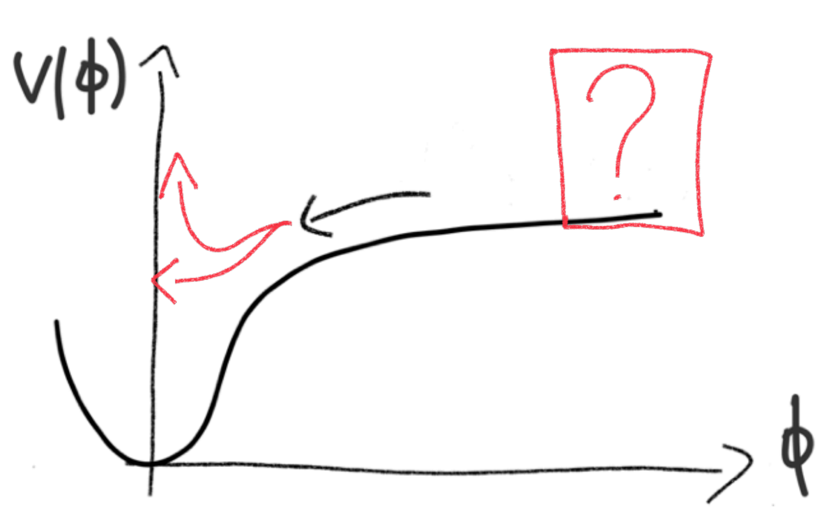

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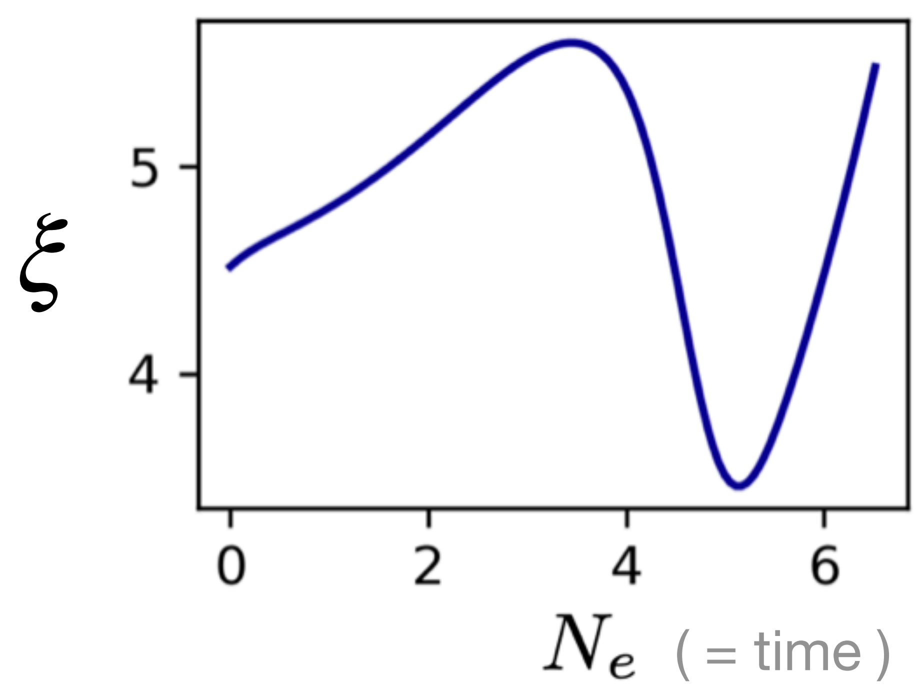

agrees with results of recent semi-analytical studies:

[**V. Domcke, V. Guidetti, Y. Welling, A. Westphal** arXiv:2002.02952]

[**M. Peloso, L. Sorbo** arXiv:2209.08131] ⟵ today!

Nonlinear dynamics of axion-U(1) inflation

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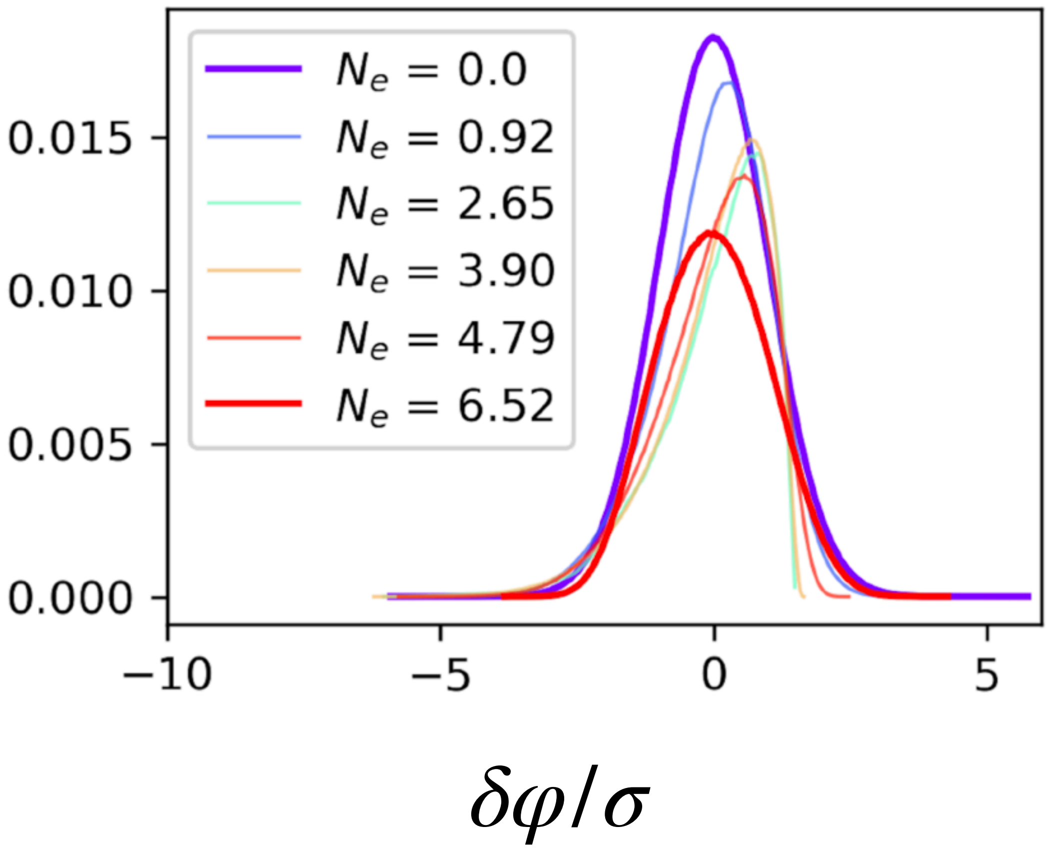

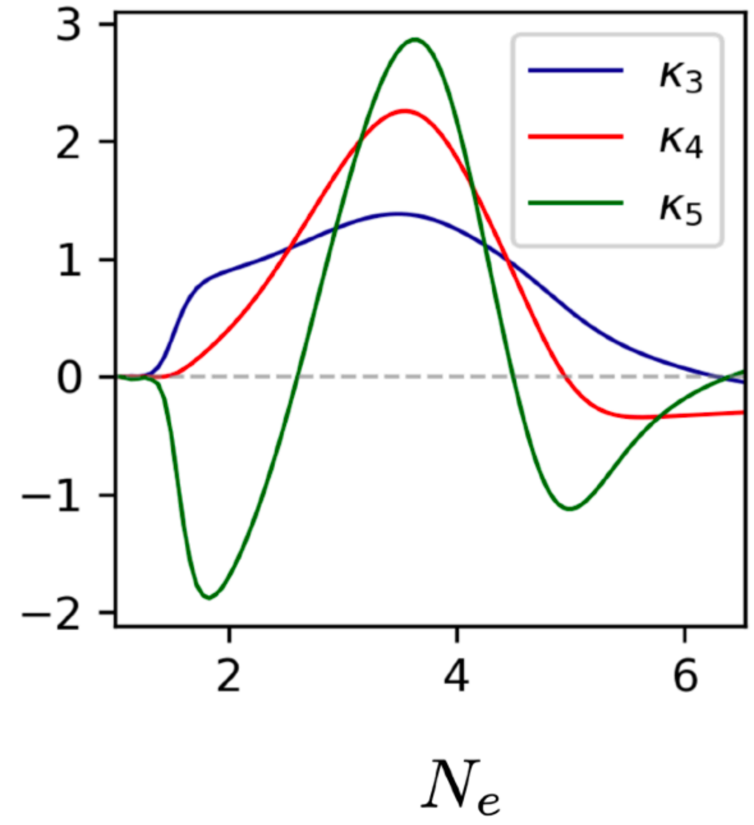

Suppression of non-Gaussianity

A. Linde, S. Mooij, E. Pajer, arXiv:1212.1693

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etc…

Before our study, it was believed that:

Large ξ $\longrightarrow$ large non-Gaussianity

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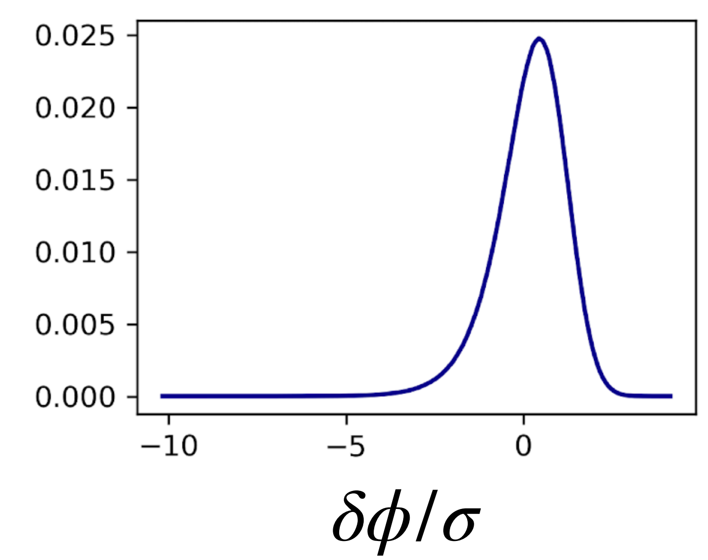

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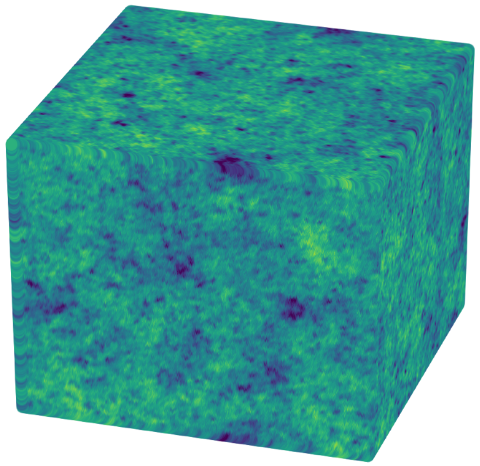
Needs to be revised

Summary:



- First simulation of an axion-gauge model during inflation

We studied both:

- Linear regime:

Full characterisation of $\delta\phi$ and its non-Gaussianity

→ new important properties

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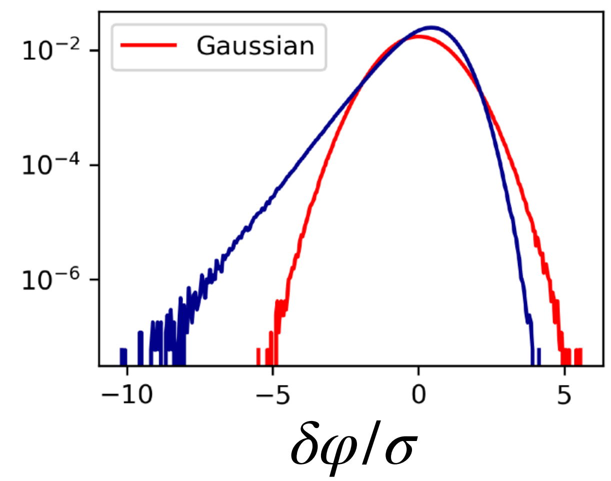

- Nonlinear regime:

Backreaction and its consequences on the phenomenology

→ perturbations become Gaussian, due to nonlinearity!

→ relaxing PBH bound, allowing for more phenomenology (to be explored)