Imperial College
London

Non-Gaussianity from preheating of non-minimally coupled inflaton

Pulkit S. Ghoderao

Supervisor: Arttu Rajantie

Theoretical Physics Group
Blackett Laboratory, Imperial College London

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Outline

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Non-minimal coupling (NMC) to gravity

What? $\implies$ Term of the form: $\xi\phi^2 R$ in the action. Where,

- ϕ = Inflaton field
- R = Ricci scalar
- ξ = Non-minimal coupling parameter

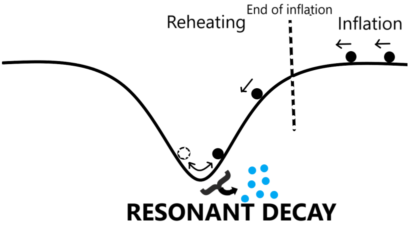
$$S = \int d^4x \sqrt{-g} \left(\frac{M_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi + \frac{1}{2} \xi \phi^2 R \right) \quad (1)$$

Why?

- Necessary for renormalisation at one-loop for scalar QFT in curved spacetime (More natural to include than exclude this term) *Tagirov,1973*
- Makes $\lambda\phi^4$ chaotic inflation observationally compatible ($10^{-4} < \xi < 10^4$) *Planck 2018 results X*

Preheating

What? $\implies$ Reheating through parametric resonance



$$V(\phi, \chi) = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2 \tag{2}$$

Why?

- Integral part of the early universe connecting inflationary and BBN epochs
- Numerical evidence suggests can generate significant non-Gaussianity *Chambers and Rajantie,2008;Bond et al.,2009*

Outline

Action

Our model:

- Massless preheating with only inflaton ϕ non-minimally coupled to gravity
- Inflaton decays to a massless scalar particle χ during reheating

Action :

$$S = \int d^4x \sqrt{-g} \left(f(\phi)R - \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi - V(\phi,\chi) \right)$$

where

$$f(\phi) = \frac{M_P^2}{2} + \frac{1}{2}\xi\phi^2 \;\; \text{and} \;\; V(\phi,\chi) = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2 \qquad (3)$$

Jordan to Einstein frame

Recipe:

1. Conformal rescaling of metric to bring back minimal coupling
2. Field redefinition to bring back canonical kinetic terms

For our action, metric rescaling required is: $\tilde{g}_{\mu\nu} = (1 + \xi \frac{\phi^2}{M_P^2}) g_{\mu\nu}$ and field redefinitions occurring are: $\frac{M_P d\bar{\phi}}{d\tilde{\phi}} = \frac{\xi \bar{\phi}^2 + 1}{\sqrt{\xi(1+6\xi)\bar{\phi}^2 + 1}}$; $\frac{d\tilde{\chi}}{d\chi} = \sqrt{\frac{M_P^2}{2f(\phi)}}$, where $\bar{\phi} = \phi / M_P$.

We choose to work within the case $\xi << 1$.

To get Einstein frame action:

$S = \int d^4x \sqrt{-\tilde{g}} \left(\frac{M_P^2}{2} \tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} \mathcal{G}_{ij} \tilde{\nabla}_\mu \phi^i \tilde{\nabla}_\nu \phi^j - \tilde{V}(\phi^i) \right)$ with potential:

$$\tilde{V} = \frac{\lambda}{4} \left(\frac{M_P}{\sqrt{\xi}} \tanh(\frac{\sqrt{\xi}}{M_P} \tilde{\phi}) \right)^4 + \frac{g^2}{2} \tilde{\chi}^2 \left(\frac{M_P}{\sqrt{\xi}} \tanh(\frac{\sqrt{\xi}}{M_P} \tilde{\phi}) \right)^2 \qquad (4)$$

Simple prescription: $\phi \rightarrow (M_P/\sqrt{\xi}) \tanh(\sqrt{\xi}\phi/M_P)$

Parameter constraints from Planck 2018 observations

Parameters: $\xi, q = g^2/\lambda, \lambda$

- Inflaton power spectrum, $\mathcal{P}_{\text{inf}} = 2.1 \times 10^{-9} \implies \lambda$ is completely fixed by a particular choice of ξ irrespective of g^2/λ
- Tensor to scalar ratio, $r < 0.1 \implies \xi > 0.004$ irrespective of g^2/λ

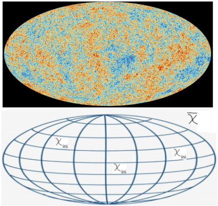
Only g^2/λ remains a truly free parameter

Typical values: $g^2/\lambda = 2$, $\xi = 0.004$, $\lambda = 5 \times 10^{-13}$

Outline

Separate Universe Approximation

During super-horizon evolution, universe is divided into adiabatic Hubble patches. Assumption is each Hubble patch is individually isotropic.



- Quantum fluctuations of spectator field χ amplified during inflation.
- FRW evolution starting from different initial field values in each patch.
- Note presence of cosmic mean $\bar{\chi}$ in $\chi_{\text{ini}} = \bar{\chi} + \delta\chi_{\text{ini}}$. We will have two variances $\langle \delta\chi^2_{\text{ini}} \rangle$ and cosmic variance, $\langle \bar{\chi}^2 \rangle$.

Chaotic spikes in delta N

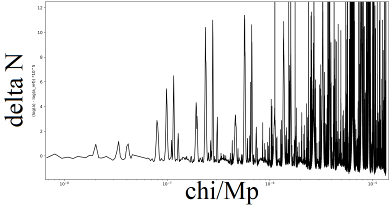
- Curvature using perturbative delta N formalism:

$$\zeta = \delta N(\chi) = N(\chi_0) + N'(\chi_0)(\chi - \chi_0) + \frac{1}{2!}N''(\chi_0)(\chi - \chi_0)^2 + \text{Order}(\chi^3)$$

- Using separate universe approximation,

$$\zeta = \delta N = \ln\left(\frac{a(\rho_*, \chi)}{a_{\text{initial}}}\right) \tag{5}$$

Simulations for the massless preheating potential give spikes:



Bond et al.,2009

- Clearly, perturbative expansion fails!

Non-Gaussianity using Non-perturbative delta N

Non-Gaussianity parameter f_{NL} *Maldacena,2003* :

$$f_{NL} = -\frac{5}{6}\frac{B_\zeta(\vec{k}_1,\vec{k}_2,\vec{k}_3)}{P_\zeta(\vec{k}_1)P_\zeta(\vec{k}_2) + \text{perms}} \tag{6}$$

Upto leading order in field variance around a Gaussian field distribution *Imrith, Mulryne, and Rajantie,2018* :

$$f_{NL} = \frac{5}{6}\frac{\tilde{N}_\chi \tilde{N}_\chi \tilde{N}_{\chi\chi}}{\left(\frac{\mathcal{P}_{\text{inf}}}{\mathcal{P}_{\text{ini}}} + \tilde{N}_\chi \tilde{N}_\chi\right)^2} \tag{7}$$

where, $\tilde{N}_\chi = \Sigma^{-1} \int d\chi\, P_G(\chi)(N(\chi) - \bar{N})(\delta\chi)$
$\tilde{N}_{\chi\chi} = \Sigma^{-1}\Sigma^{-1} \int d\chi P_G(\chi)(N(\chi) - \bar{N})(\delta\chi)^2$
What we require:

- $P_G(\chi)$ i.e. Mean and Variance of χ_{ini}
- $N(\chi)$ from preheating simulations for Gaussian χ_{ini} distribution

Thus, we establish our **Aim**!

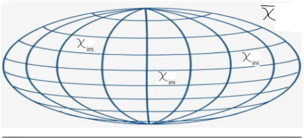
Outline

Importance of Cosmic Variance

$$\langle \delta\chi^2 \rangle = \int \mathcal{P}(k)\frac{dk}{k} = \int \frac{H(N)^2}{4\pi^2} e^{-F(N)} \left(1 - \frac{H'(N)}{H(N)}\right) dN \qquad (8)$$

Integration limits: $N = N_{crit} \rightarrow N_{obs} \implies$ Variance of χ_{ini} while $N = N_{obs} \rightarrow \infty \implies$ Variance of $\bar{\chi}$ = Cosmic Variance

Recall:

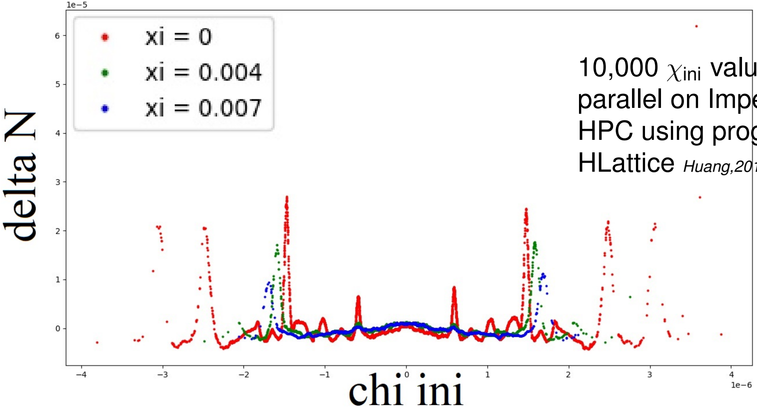


Results:

- A priori, mean $\bar{\chi}$ = 0. Cosmic Variance for minimal coupling $\xi = 0$ is infinite *Chambers and Rajantie,2008*. For even a small non-zero coupling $\xi << 1$, Cosmic Variance becomes finite.
- Cosmic variance $<<$ Variance of χ_{ini} for $g^2/\lambda > 1/2$ $\implies$ only region around $\bar{\chi} = 0$ is important

Non-linear Simulations

Around $\bar{\chi} = 0$



10,000 χ_{ini} values in parallel on Imperial HPC using program HLattice *Huang,2011*

Sub-leading term requirement

Recall: NMC reheating potential symmetric around $\chi = 0$

$$\tilde{V} = \frac{\lambda}{4}\left(\frac{M_P}{\sqrt{\xi}}\tanh(\frac{\sqrt{\xi}}{M_P}\tilde{\phi})\right)^4 + \frac{g^2}{2}\tilde{\chi}^2\left(\frac{M_P}{\sqrt{\xi}}\tanh(\frac{\sqrt{\xi}}{M_P}\tilde{\phi})\right)^2$$

$$\implies \tilde{N}_\chi = \Sigma^{-1}\int d\chi\; P_G(\chi)(N(\chi) - \bar{N})\delta\chi = 0$$

(Useful check: also seen from simulation plot)

$$\implies f_{NL} = \frac{5}{6}\frac{\tilde{N}_\chi \tilde{N}_\chi \tilde{N}_{\chi\chi}}{\left(\frac{\mathcal{P}_{\mathrm{inf}}}{\mathcal{P}_0} + \tilde{N}_\chi \tilde{N}_\chi\right)^2} = 0$$

Need to go sub-leading

Boubekeur-Lyth approximation

At sub-leading order:

$$f_{NL} = -\frac{5}{6}\frac{\tilde{N}^3_{\chi\chi}(\int \Sigma(\vec{q}-\vec{k}_1)\Sigma(\vec{q})\Sigma(\vec{k}_3+\vec{q})d\vec{q})}{\left(\frac{2\pi^2}{k^3}\mathcal{P}_{\rm inf} + \tilde{N}^2_{\chi\chi}\left(\int \Sigma(\vec{q})\Sigma(\vec{k}-\vec{q})d\vec{q}\right)\right)^2} \quad (9)$$

Momentum integrals

Boubekeur and Lyth *Boubekeur and Lyth,2006* take power spectrum to be scale-invariant: $\Sigma(k) = 2\pi^2\mathcal{P}_0/k^3$ along with the approximation $q << k$

$$f^{B-L}_{NL} = -\frac{5}{6}\frac{\tilde{N}^3_{\chi\chi}\frac{8\pi^6\mathcal{P}^3_0}{k^3k^3}\int 4\pi q^2 dq\left(\frac{1}{q^3}\right)}{\frac{4\pi^4\mathcal{P}^2_{\rm inf}}{k^6}} = -\frac{20\pi^3}{3}\frac{\mathcal{P}^3_0}{\mathcal{P}^2_{\rm inf}}\tilde{N}^3_{\chi\chi}\ln(kL) \quad (10)$$

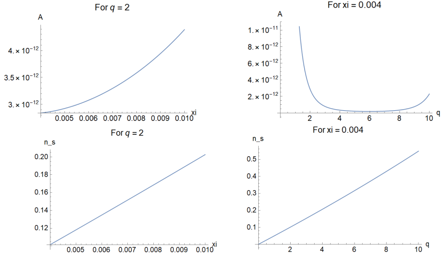
From simulations, $\tilde{N}_{\chi\chi} \sim$ Order(10^6).
Therefore, $f_{NL} \approx 9.1\times 10^{-13}\tilde{N}^3_{\chi\chi} \sim$ Order(10^5)
Detectable Non-Gaussianity?!

Scale dependent power spectrum

Consistency requirement: Use first order theory to calculate momentum integrals ← Power spectrum is not scale-invariant! Assuming fixed H_0, $\Sigma(k) = A/k^{3-n_s}$ where,

$$A = \frac{H_0^{2-n_s}}{2} \exp\left(3\left(N_{\mathrm{crit}} + \frac{\sqrt{32\xi+1}-1}{48\xi}\sqrt{9-48\frac{g^2}{\lambda}\sqrt{\frac{\lambda}{12}}\frac{M_P}{H_0}} - \frac{2\frac{g^2}{\lambda}}{\sqrt{9-48\frac{g^2}{\lambda}\xi}}\tanh^{-1}\left(\sqrt{\frac{3-16\frac{g^2}{\lambda}\sqrt{\frac{\lambda}{12}}\frac{M_P}{H_0}}{3-16\frac{g^2}{\lambda}\xi}}\right.\right.\right. \tag{11}$$

$$n_s = 3 - \sqrt{9-48\frac{g^2}{\lambda}\xi\left(\frac{16\xi N_{\mathrm{obs}}+\sqrt{32\xi+1}+1}{16\xi N_{\mathrm{obs}}+\sqrt{32\xi+1}-1}\right)} \tag{12}$$



Making equilateral assumption $k_1 = k_2 = k_3 = k$ with $q << k$,

$$f_{NL} = -\frac{20\pi^3}{3}\frac{A^3}{\mathcal{P}_{\text{inf}}^2}\tilde{N}_{\chi\chi}^3\frac{k^{3n_s}}{3n_s} \quad (13)$$

Typical values: $A \sim 10^{-12} M_P^{2-n_s}$, $k_{\text{Planck}} = 0.05 MPc^{-1} \sim 10^{-58} M_P$ and $n_s \sim 0.1$ while $\tilde{N}_{\chi\chi}$ remains of Order(10^6) from simulations.
Giving, $f_{NL} \sim$ Order(10^{-16})
Undetectable!

Difference from Boubekeur-Lyth approx. $f_{NL} = -\frac{20\pi^3}{3}\frac{\mathcal{P}_0^3}{\mathcal{P}_{\text{inf}}^2}\tilde{N}_{\chi\chi}^3 \ln(kL)$:

- A is two orders of magnitude less than $\mathcal{P}_0$
- n_s causes power law integration, no log suppression of $k_{\text{Planck}} \sim 10^{-58} M_P$

Outline

Take aways

- We have found that preheating does not produce significant non-Gaussianity in our non-minimal coupling model
- Parameter dependence on $\xi, g^2/\lambda$ remains to be studied
- Using separate universe approximation with the delta N formalism requires us to find out variances. Cosmic variance then becomes important.
- Many inflationary potentials are symmetric and might require sub-leading terms in the non-perturbative delta N formalism
- Tools and methods used can be applied to other reheating scenarios

Thank you for your attention!

Outline

Non-peturbative delta N

Central Object $\rightarrow$ Correlators of the curvature:
$\langle \zeta_1 \zeta_2 ... \rangle = \int d\chi_1 d\chi_2 ... \, P(\chi_1, \chi_2, ...) \, (N_1 - \bar{N})(N_2 - \bar{N})...$
subscript indicates space points $x_1, x_2, ...$
Idea: Expand the joint probability distribution $P(\chi_1, \chi_2, ...)$ around Gaussian distribution $\leftarrow$ early universe fields are near Gaussian

- First, expand around Gaussian joint pdf P_G using Gauss-Hermite expansion
- Second, expand P_G in terms of the variance $\Sigma = \langle \delta\chi^2 \rangle$

Keeping only leading term:

$$\langle \zeta_1 \zeta_2 \rangle = \Sigma_{12} \tilde{N}_\chi^2 \text{ and } \langle \zeta_1 \zeta_2 \zeta_3 \rangle = \Sigma_{12} \Sigma_{23} \tilde{N}_\chi \tilde{N}_{\chi\chi} \tilde{N}_\chi + \text{perms} \tag{14}$$

where,

$$\tilde{N}_\chi = \Sigma^{-1} \int d\chi \, P_G(\chi)(N(\chi) - \bar{N})\delta\chi \tag{15}$$

$$\tilde{N}_{\chi\chi} = \Sigma^{-1}\Sigma^{-1} \int d\chi P_G(\chi)(\delta\chi)^2 (N - \bar{N}) \tag{16}$$

Calculation of variance

$$\phi = \bar{\phi}(t) + \delta\phi(x,t) \;\text{ and }\; \chi = \bar{\chi}(t) + \delta\chi(x,t) \tag{17}$$

Perturbation satisfies damping harmonic oscillator equation:

$$\ddot{\delta\chi} + 3H\dot{\delta\chi} + g^2\hat{\phi}^2\delta\chi = 0 \;\text{ where }\; \hat{\phi} = \frac{M_P}{\sqrt{\xi}}\tanh\left(\frac{\sqrt{\xi}}{M_P}\phi\right) \tag{18}$$

(Hartree approximation in Longitudinal Gauge)
Timeline: Scale-invariant before exiting horizon (=$H^2/4\pi^2$) $\rightarrow$ overdamped oscillator envelope (= $e^{-\int(3H/2-\sqrt{9H^2/4-g^2\hat{\phi}^2})dt}$)
Write everything in terms of N:

$$\langle\delta\chi^2\rangle = \int \mathcal{P}(k)\frac{dk}{k} = \int \frac{H(N)^2}{4\pi^2}e^{-F(N)}\left(1 - \frac{H'(N)}{H(N)}\right)\,dN \tag{19}$$

HLattice

Program written in FORTRAN language. Simulates scalar fields and gravity during inflation and reheating.

- Variable evolved: $\beta_{ij} = \ln(g_{ij})$, where 3×3 metric $g_{ij} = a(t)^2(\delta_{ij} + h_{ij})$ is in synchronous gauge
- Scale factor at each step: $a(t) = \frac{1}{L^3}\left(\int \sqrt{g} d^3x\right)^{1/3}$
- Spatial gradients using a specified discretisation scheme
- Symplectic sixth order integrator with fourth order Runge-Kutta integrator to obtain β_{ij} at each time step