Non-perturbative non-Gaussianity and primordial black holes

AG+ (in prep)

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IFT Madrid, 20 September 2022

Motivation

- Stochastic inflation can produce significant non-Gaussianity in the tail of $P(\zeta)$

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- Stochastic inflation can produce significant non-Gaussianity in the tail of $P(\zeta)$
- PBHs form in the tail, so will be affected

Overview

Introduction to Primordial Black Holes

A non-perturbative treatment of general non-Gaussianity

Conclusions

What is a Primordial Black Hole?

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- Density contrast $\delta = \frac{\delta\rho}{\rho} \rightarrow$ "compaction" C
- If $C > C_c$ at horizon entry $\Rightarrow$ PBH

Why do we care?

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- Seeds of supermassive black holes
- LIGO–Virgo–KAGRA merger events
- Constraining early universe physics

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- Typically treated perturbatively (f_{NL}, g_{NL}, ...)
- Not sufficient for non-G in the far tail

Beyond perturbativity

- Recent transformation [Kitajima+ 2021]

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- Not fully non-perturbative
- Want general $P(\zeta_G) \to P(\zeta)$
- Can do in general with CDF transformation:

$$\zeta = F_\zeta^{-1}[F_G(\zeta_G)]$$

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$$C = C_l - \frac{3}{8}C_l^2, \quad C_l = -\frac{4}{3}r\zeta'$$

- Need to get $P(C_l)$ to determine PBH properties

Compaction probability

- Bivariate Gaussian $P(X,Y)$

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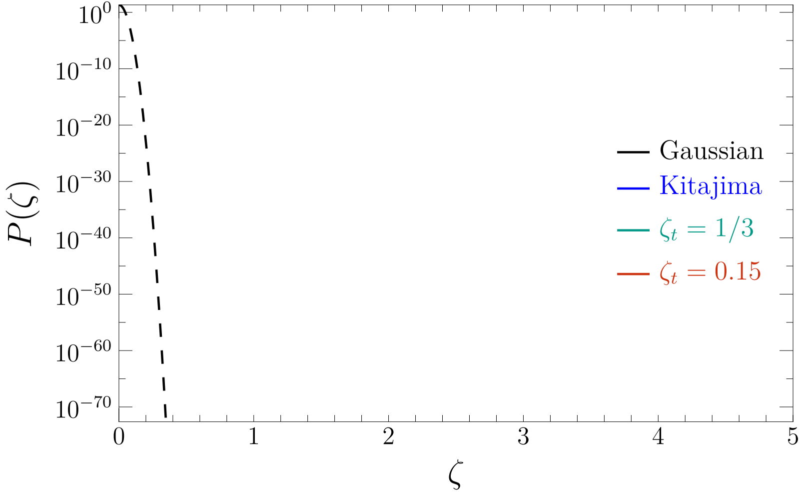
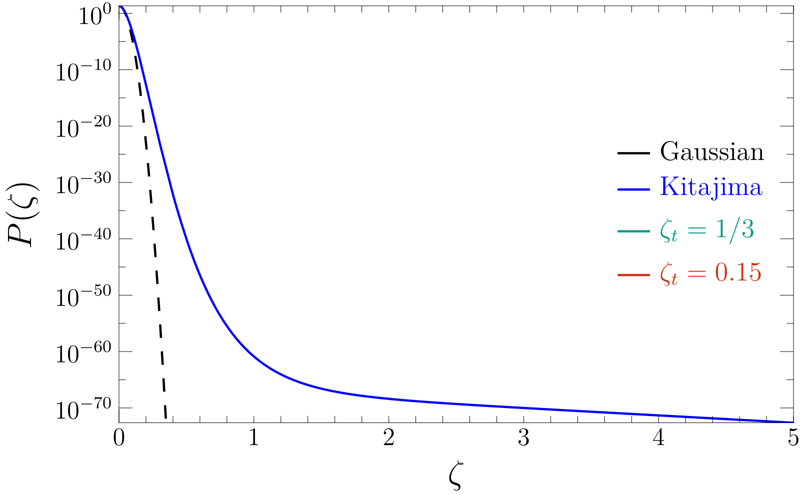
$$P(C_l) = \int \mathrm{d}\zeta_G \, \frac{3}{4|\mathcal{J}_1(\zeta_G)|} P\left[-\frac{1}{\mathcal{J}_1(\zeta_G)}\left(\frac{3}{4}C_l + 2\Sigma_{XY}\mathcal{J}_2(\zeta_G)\right), \zeta_G\right]$$

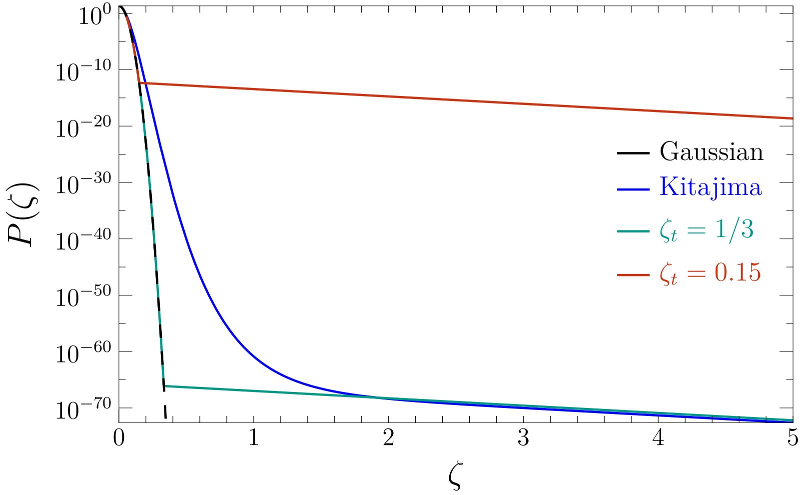
$$\mathcal{J}_1(\zeta_G) = \frac{\mathrm{d}\zeta}{\mathrm{d}\zeta_G}, \quad \mathcal{J}_2(\zeta_G) = \frac{\mathrm{d}\zeta}{\mathrm{d}\Sigma_{YY}}$$


10^{0}
10^{-10}
10^{-20}
10^{-30}
10^{-40}
10^{-50}
10^{-60}
10^{-70}
$P(\zeta)$
0
1
2
3
4
5
ζ
Gaussian
Kitajima
$\zeta_t = 1/3$
$\zeta_t = 0.15$

Tail vs transition



Tail vs transition



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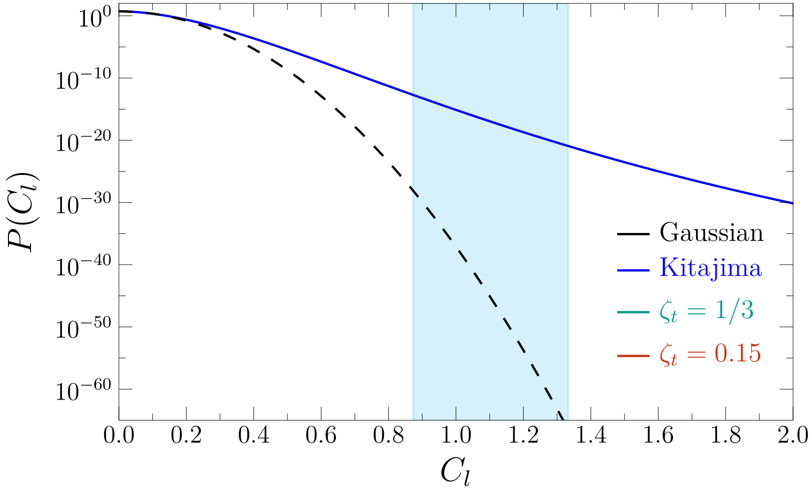

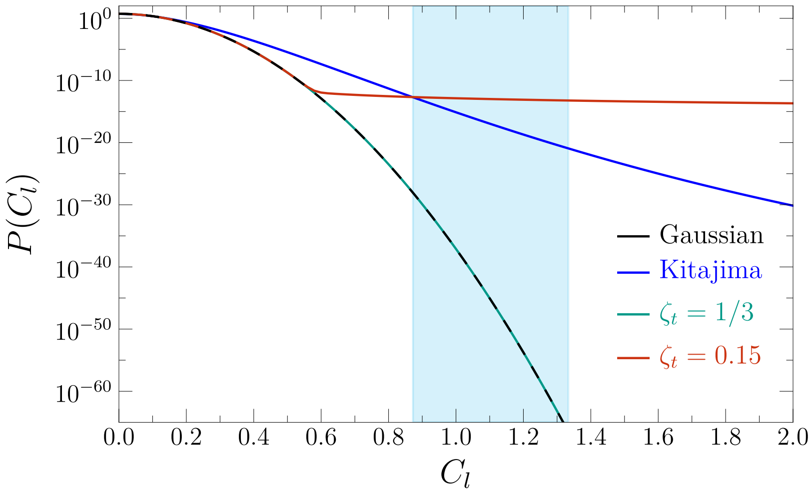

10^{0}
10^{-10}
10^{-20}
10^{-30}
10^{-40}
10^{-50}
10^{-60}
$P(C_l)$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
C_l
Gaussian
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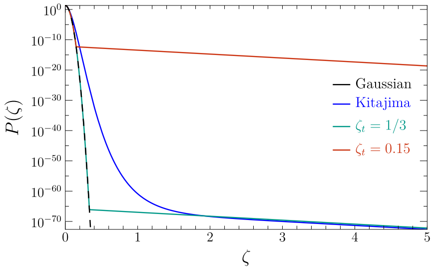
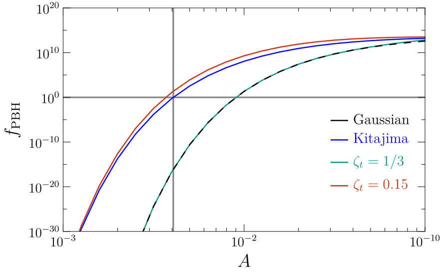
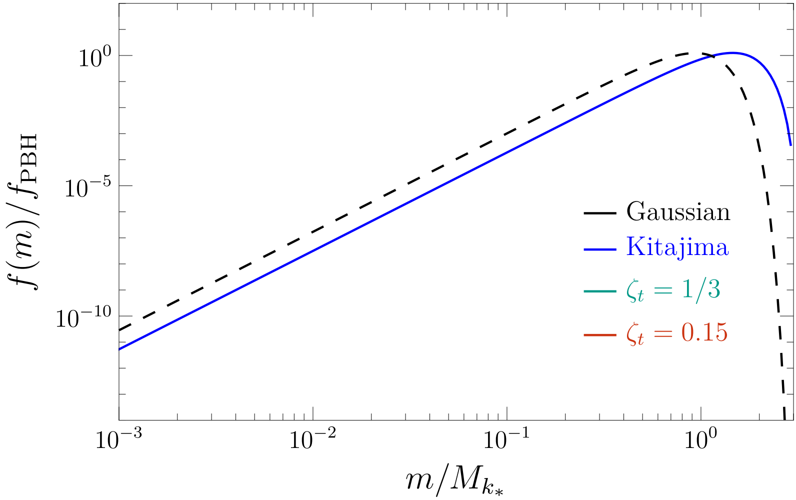
Tail vs transition





PBH mass distribution



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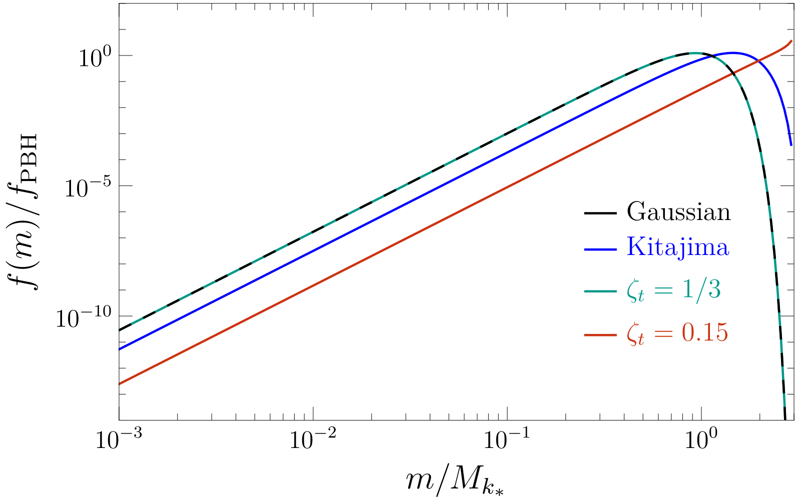

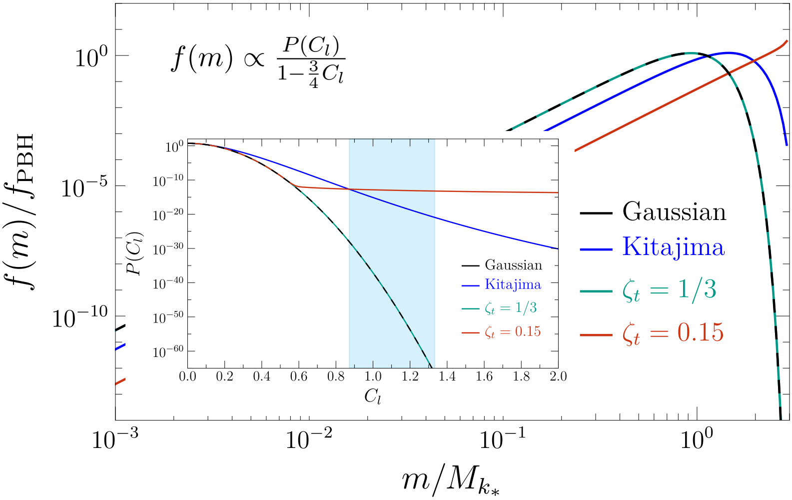

$f(m) \propto \frac{P(C_l)}{1-\frac{3}{4}C_l}$
$f(m)/f_{\rm PBH}$
m/M_{k_*}
10^{0}
10^{-5}
10^{-10}
10^{-3}
10^{-2}
10^{-1}
Gaussian
Kitajima
$\zeta_t = 1/3$
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$P(C_l)$
C_l
0.0
0.2
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1.8
2.0
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10^{-30}
10^{-40}
10^{-50}
10^{-60}

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- Perturbative treatment may miss deviations from Gaussianity in the far tail
- Non-perturbative treatment can be used for any $P(\zeta)$
- Transition between Gaussian and non-Gaussian behaviour is more important than the far tail
- Shallow tail in $P(\zeta)$ highlights divergence in mass distribution

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