Inflation with spectator Gauge-flation

Oksana Iarygina

A Cosmic Window to Fundamental Physics: Primordial Non-Gaussianity and Beyond

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NORDITA

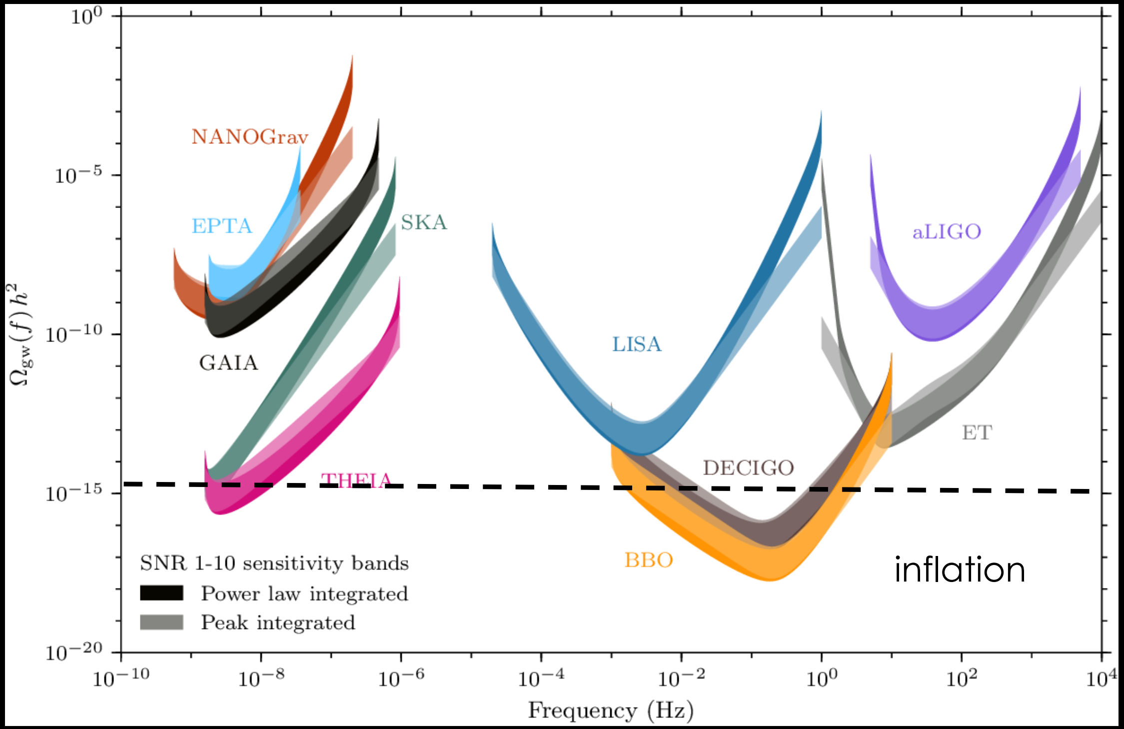
Stockholm University

In the beginning, there was (probably) inflation

From the perturbed Einstein equations:

$$\bar{G}_{\mu\nu} + \delta G_{\mu\nu} = 8\pi G\left(\bar{T}_{\mu\nu} + \delta T_{\mu\nu}\right)$$

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} + k^2 h_{ij} = 0$$



[J. Garcia-Bellido H. Murayama and G. White, 2021]

Amplitude of GWs from inflation is too small to be detected!

Observational predictions: gravitational waves

Presence of a non-zero source term may significantly enhance GW production

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} + k^2 h_{ij} = 16\pi G \Pi_{ij}^{TT}$$

the transverse-traceless part of the anisotropic stress tensor

Several possible sources for Π_{ij}^{TT}:

- gauge fields,
- scalar field gradients,
- gradients of second order scalar perturbations
- tensors

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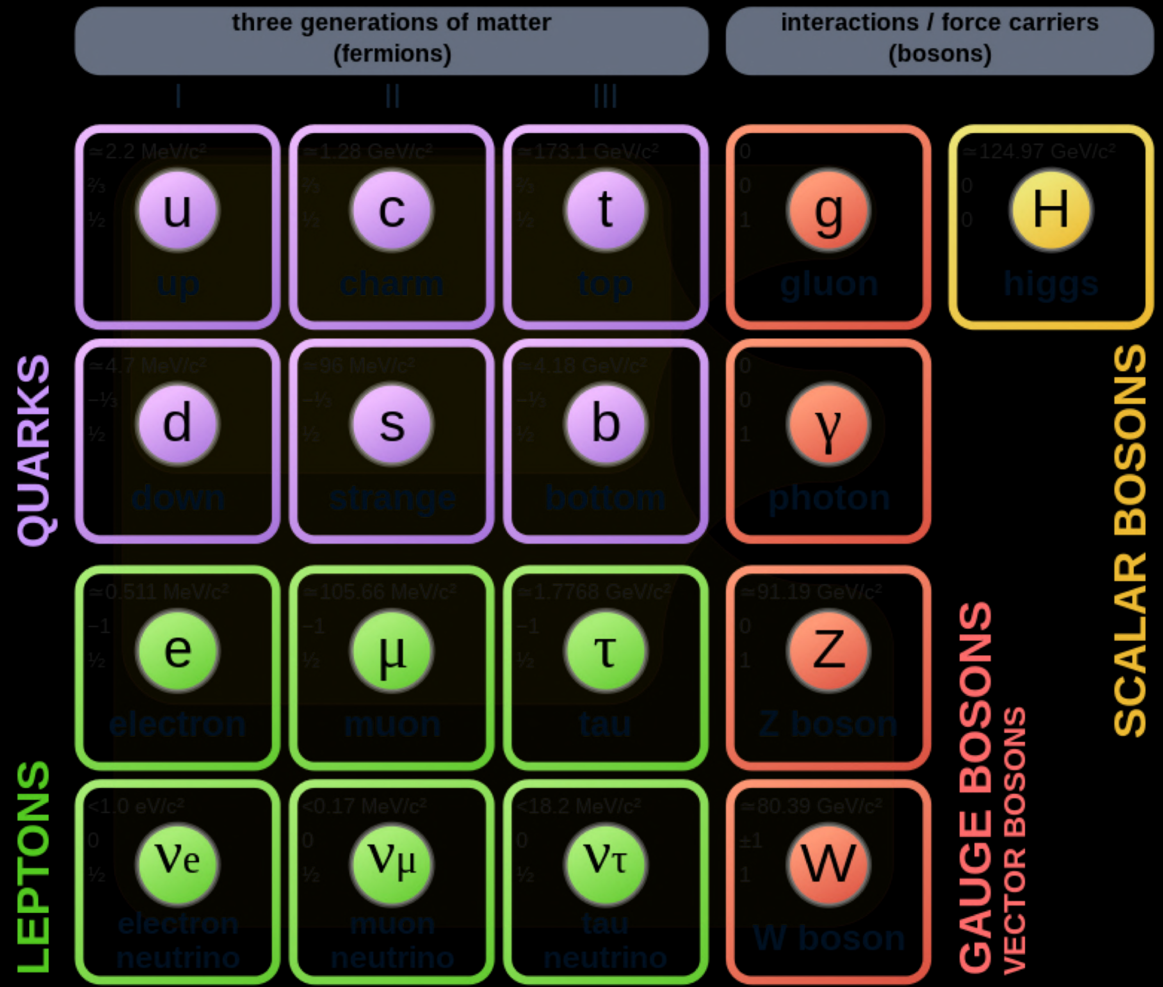
Gauge fields are building blocks of nature

The Standard Model is a non-abelian gauge theory with the symmetry group $U(1) \times SU(2) \times SU(3)$



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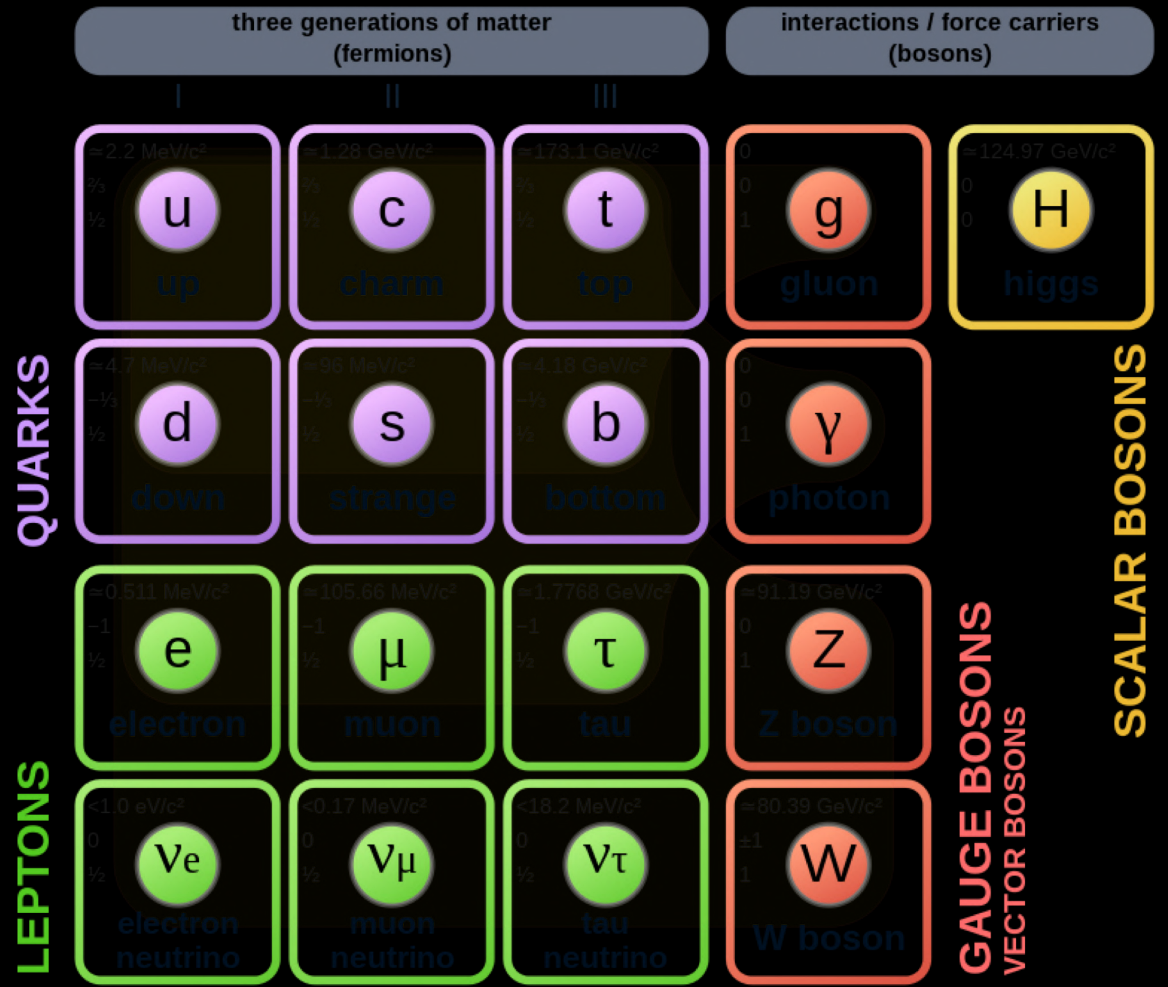

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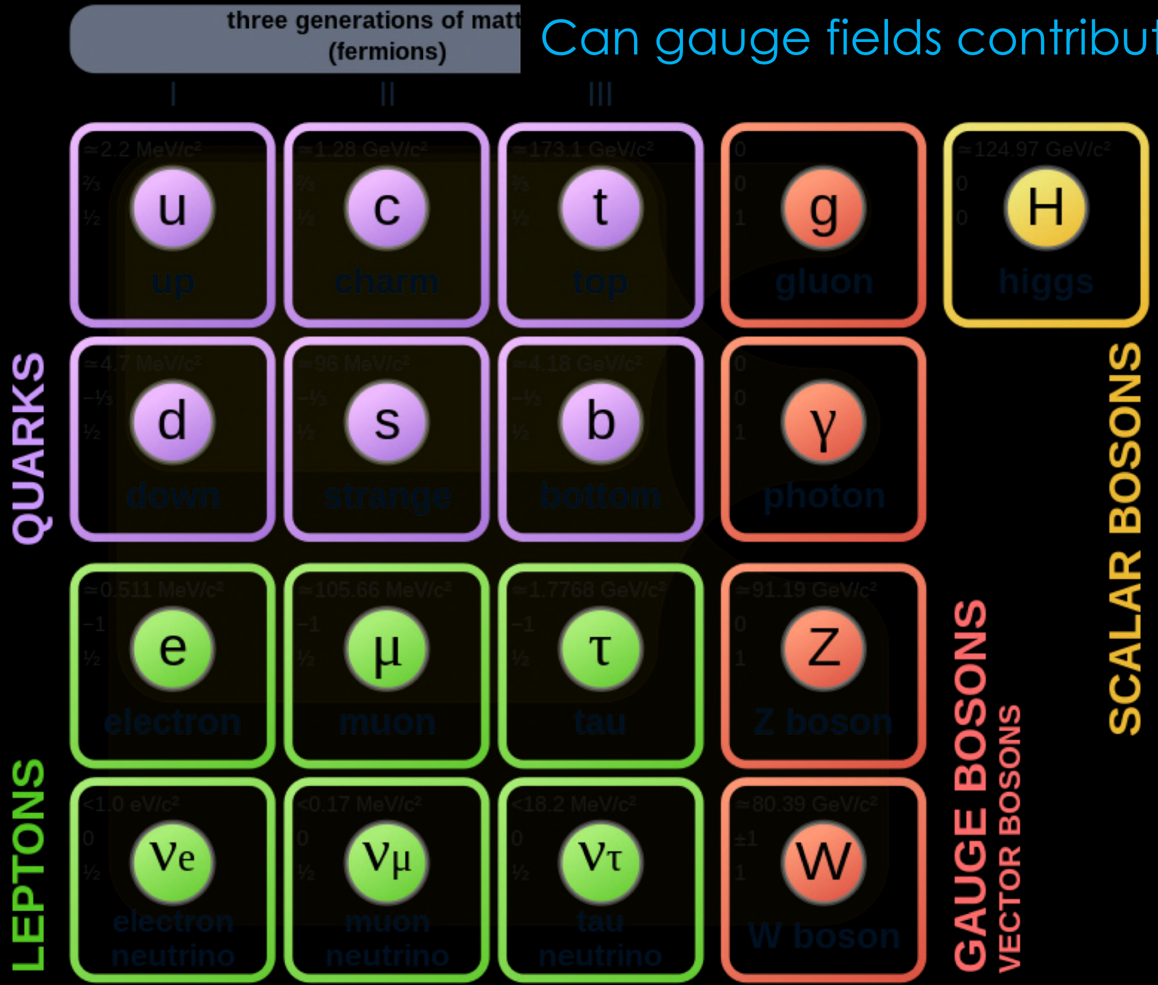
Can gauge fields contribute to the physics of inflation?



Can gauge fields contribute to physics of inflation?

✓ Yes. However, there are some challenges:

1) A homogeneous and isotropic background solution: Can be realised for SU(N) gauge fields.

[A.Maleknejad and M.M.Sheikh-Jabbari, 2011]

2) Dilution: $A_\mu \sim 1/a(t)$

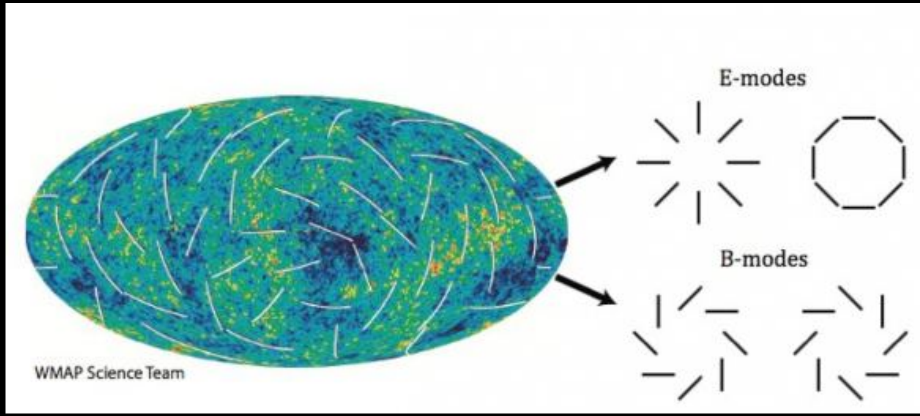
New terms in gauge theory are required or a coupling with inflaton.

3) Respect gauge symmetry:

$$f^2(\phi)F^a_{\mu\nu}F^{a\,\mu\nu}, \quad \chi F\tilde{F}, \quad \left(F\tilde{F}\right)^2$$

Signatures of GWs from gauge fields

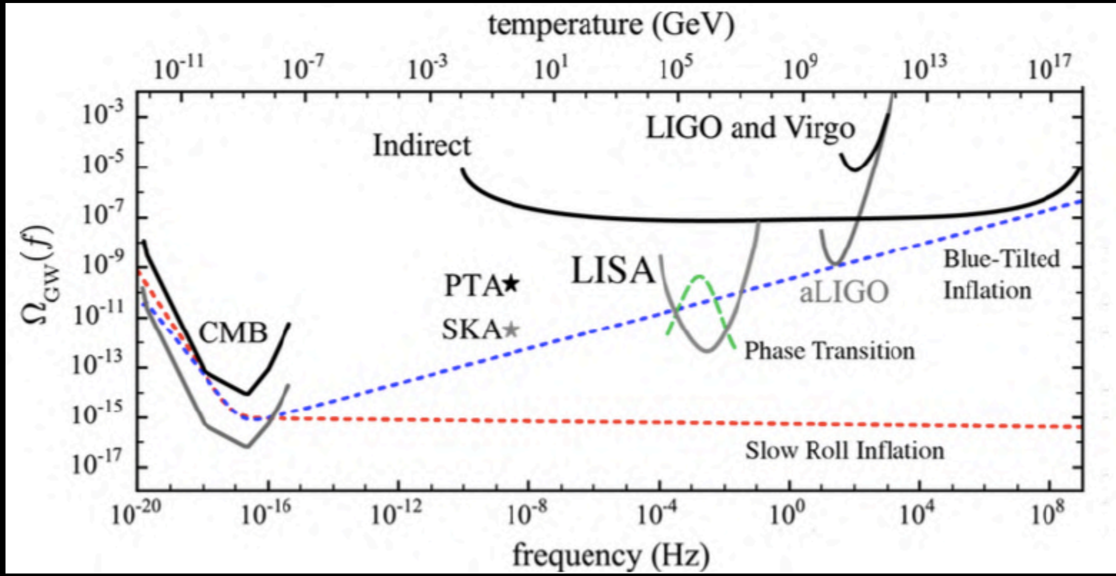
[WMAP]



- Polarization: B-modes
+ parity odd CMB correlations

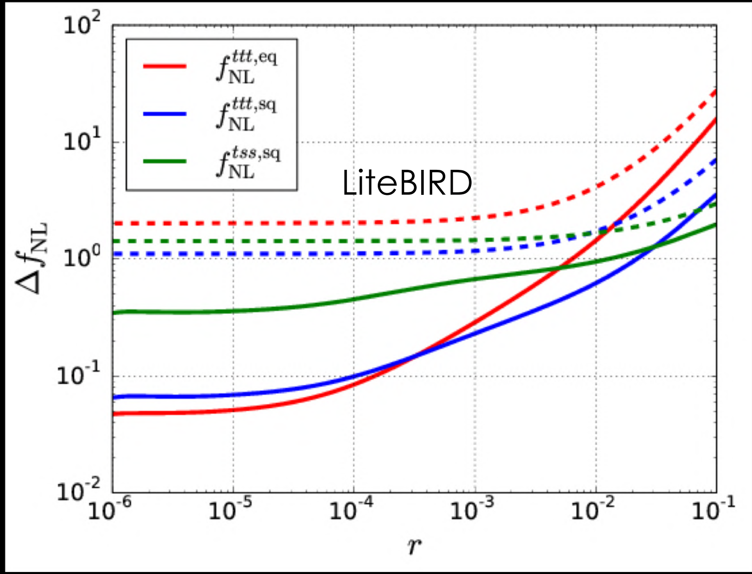
$TB \neq 0, EB \neq 0$

- Enhanced amplitude of GWs



[Caldwell]

- Non-zero tensor non-Gaussianity



[M. Shiraishi, 2019]

Gauge-flation and Chromo-natural inflation models

Gauge-flation: [A.Maleknejad and M.M.Sheikh-Jabbari, 2011]

$$S=\int d^4x\sqrt{-\det(g_{\mu\nu})}\left[\frac{M_{\rm Pl}^2}{2}R-\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu}+\frac{\kappa}{96}\left(F^a_{\mu\nu}\tilde{F}^{a\,\mu\nu}\right)^2\right]$$

Chromo-natural inflation (axion-SU(2)): [P. Adshead , M. Wyman, 2012]

$$S=\int d^4x\sqrt{-\det(g_{\mu\nu})}\left[\frac{M_{\rm Pl}^2}{2}R-\frac{1}{2}\left((\partial\chi)^2-\mu^4\left(1+\cos(\frac{\chi}{f})\right)\right)-\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu}+\frac{\chi}{8f}\lambda F^a_{\mu\nu}\tilde{F}^{a\,\mu\nu}\right]$$

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Natural inflation

[K. Freese, J. A. Frieman and A. V. Olinto, 1990]

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Ruled out by observations!

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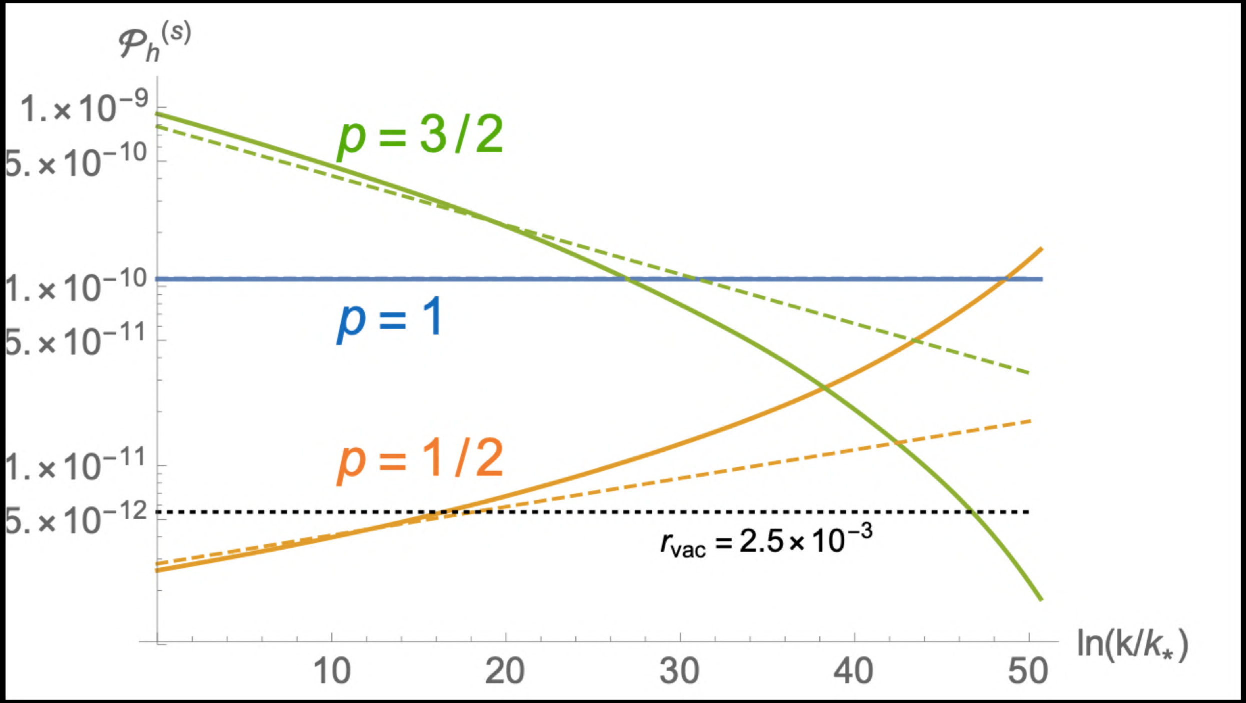
However, Chromo-natural inflation may be realised as a spectator sector.

[E. Dimastrogiovanni, M. Fasiello and T. Fujita, 2017]

The shape of the tensor power spectrum in axion-SU(2) model depends on the form of the axion potential

$$U(\chi) = \mu^4 \left| \frac{\chi}{f} \right|^p \qquad n_T \propto p - 1$$



[T. Fujita, E. I. Sfakianakis, M. Shiraishie(2019)]

Spectator Gauge-flation

[O.I. and E. I. Sfakianakis, *JCAP* 11 (2021) 11, 023]

$$S = \int d^4x \sqrt{-\det(g_{\mu\nu})} \left[\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2}(\partial\varphi)^2 - V(\varphi) - \frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \frac{\kappa}{96} \left(F^a_{\mu\nu} \tilde{F}^{a\,\mu\nu} \right)^2 \right]$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g\epsilon^{abc} A^b_\mu A^c_\nu$$

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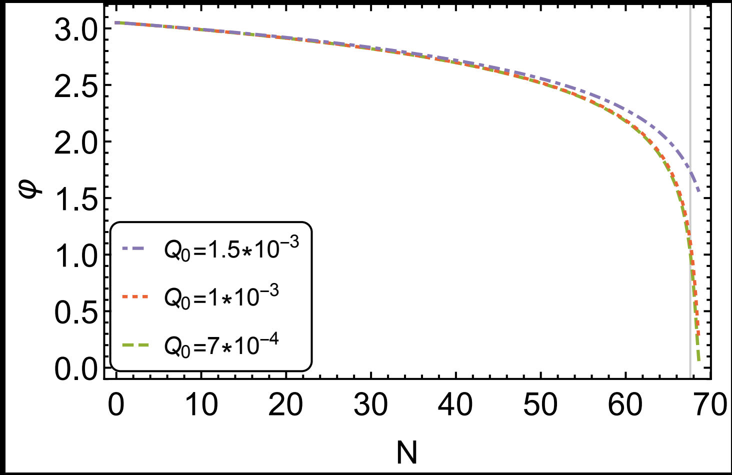
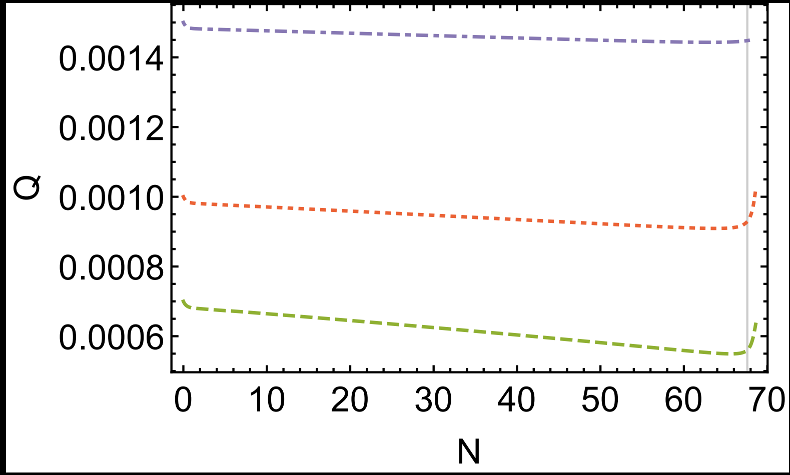
Isotropic solution for the background:

$$A^a_0 = 0,$$

$$A^a_i = \delta^a_i a(t) Q(t)$$





Viability of the model

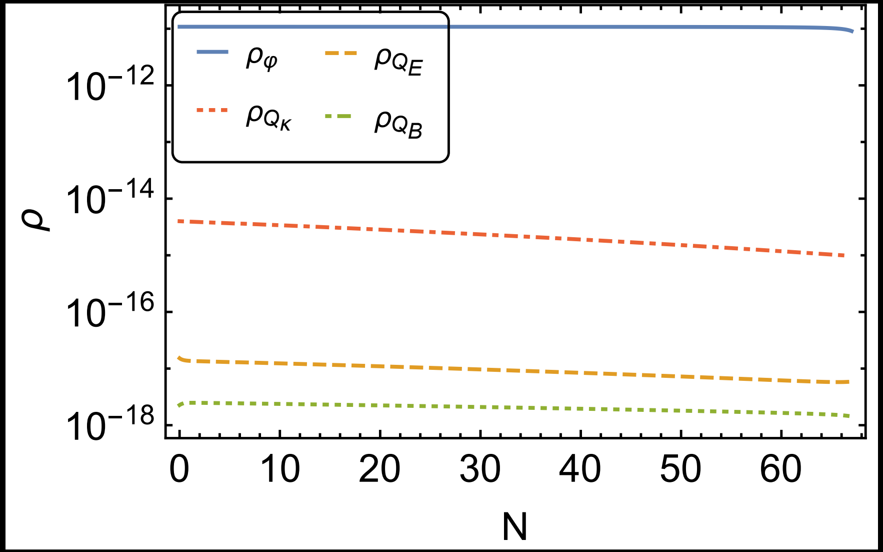
$$\epsilon_\varphi = \frac{\dot\varphi^2}{2M_{\rm Pl}^2 H^2}, \quad \epsilon_{Q_E} = \frac{(\dot Q + HQ)^2}{M_{\rm Pl}^2 H^2}, \quad \epsilon_{Q_B} = \frac{g^2 Q^4}{M_{\rm Pl}^2 H^2}$$

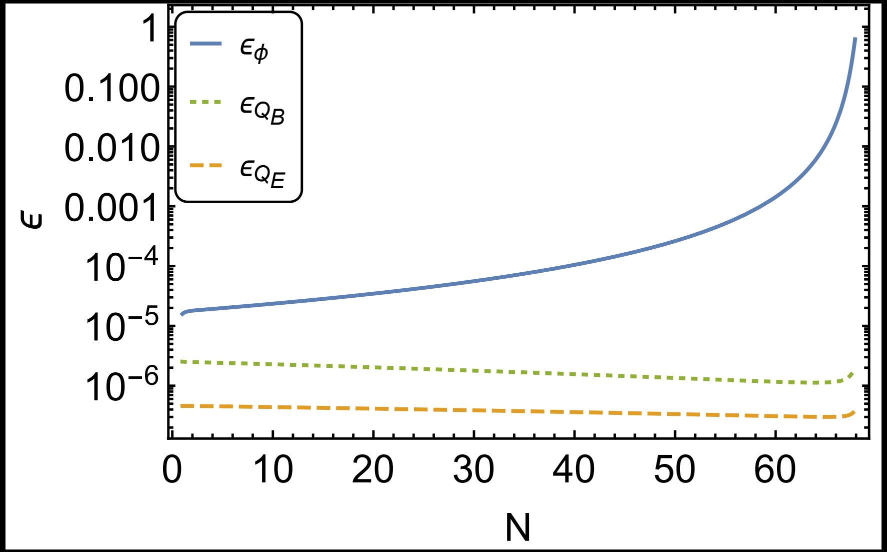
$$\rho_\varphi = \frac{1}{2}\dot\varphi^2 + V(\varphi), \quad \rho_{Q_E} = \frac{3}{2}(\dot Q + HQ)^2, \quad \rho_{Q_B} = \frac{3}{2}g^2Q^4, \quad \rho_{Q_\kappa} = \frac{3}{2}\kappa g^2 Q^4(\dot Q + HQ)^2$$

1. Spectator sector requirement:

$$\epsilon_\varphi \gg \epsilon_{Q_E}, \epsilon_{Q_B},$$

$$\rho_\varphi \gg \rho_{Q_E}, \rho_{Q_B}, \rho_{Q_\kappa}$$



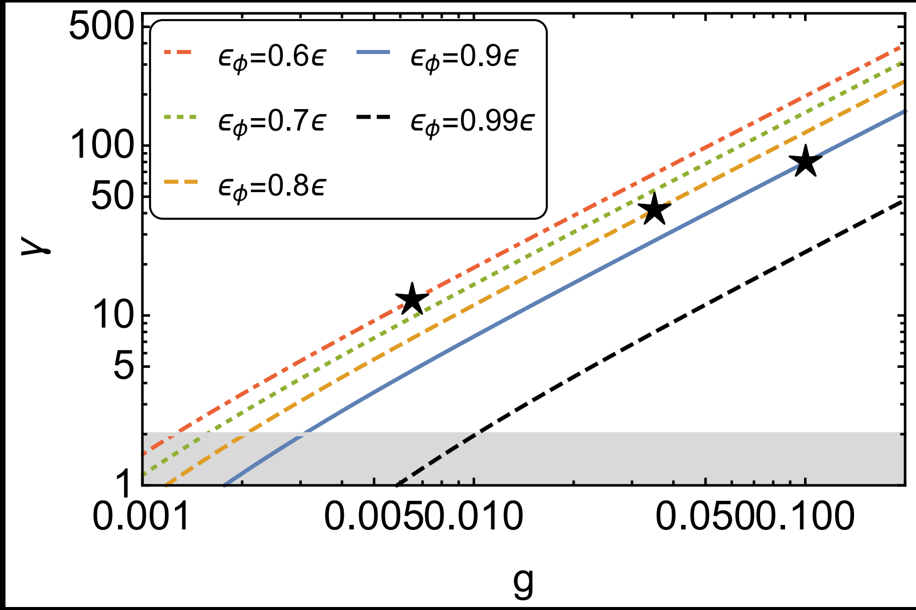


Oksana Iarygina

The requirement for a spectator sector leads to constraints on the amplification and a maximum theoretically allowed value for the parameter γ :

$$\gamma = \frac{g^2 Q^2}{H^2}$$

$$\gamma \simeq -\frac{1}{2} + \frac{1}{2}\sqrt{1 + 4M_{\rm Pl}^2 \frac{g^2\epsilon}{H^2}\left(1 - \frac{\epsilon_\varphi}{\epsilon}\right)}$$



The enhancement of chiral gravitational waves cannot be arbitrarily high!

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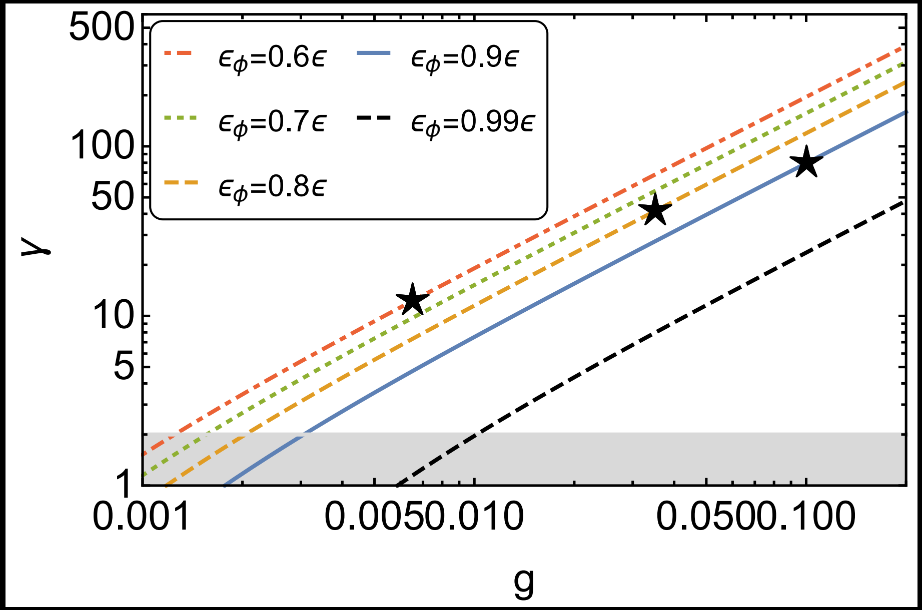
match observations:
big $\epsilon_\varphi/\epsilon$, small γ

amplification of GWs:
small $\epsilon_\varphi/\epsilon$, big γ



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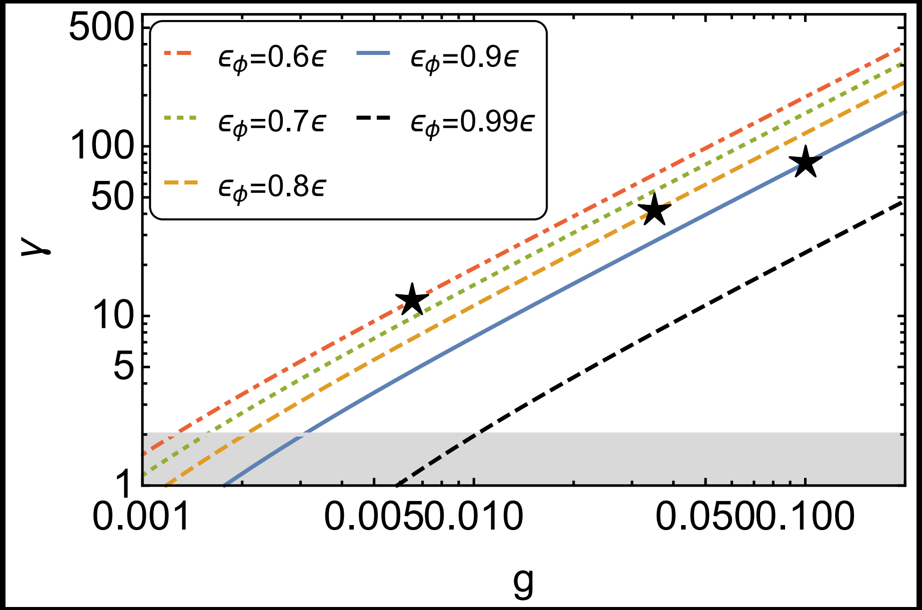
Minimum value
for the gauge coupling:

$$\gamma > 2$$

$$g_{\rm min} \simeq \frac{\sqrt{6}H}{M_{\rm Pl}\sqrt{\epsilon}\sqrt{1 - \frac{\epsilon}{\epsilon_\varphi}}}$$



Maximum value
for the gauge coupling:

loop corrections

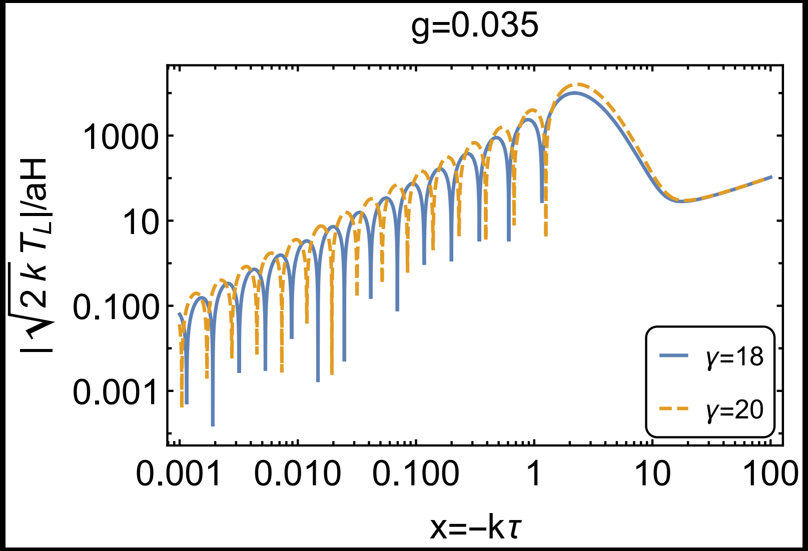
$$\kappa \gg \frac{3g_{\rm max}^4}{H^4}$$

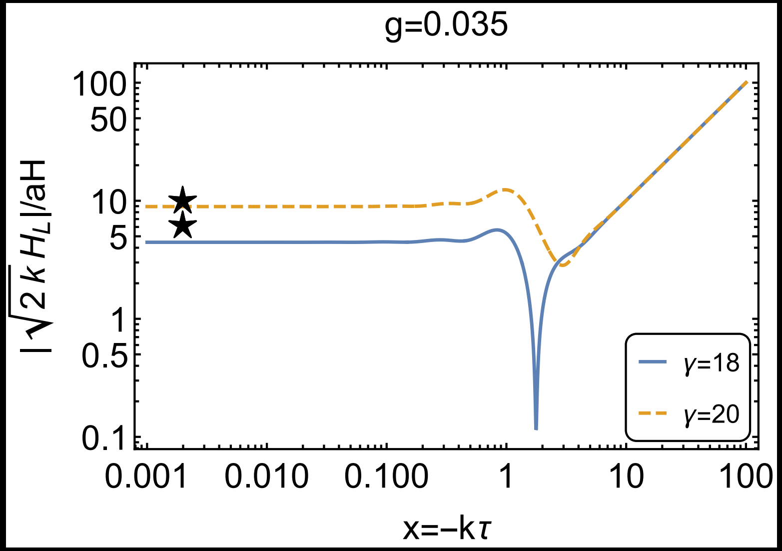
Perturbations: chiral gravitational wave production

$$\delta A^1_\mu = a(0, t_+, t_\times, 0), \quad \delta A^2_\mu = a(0, t_\times, -t_+, 0), \quad \delta g_{11} = -\delta g_{22} = a^2 h_+, \quad \delta g_{12} = a^2 h_\times$$

$$\partial_x^2 T_{L/R} + \left[1 \mp \Omega^2(x, \epsilon_\varphi/\epsilon, \gamma)\right] T_{L/R} = 0,$$

$$\partial_x^2 H_{L/R} + \left(1 - \frac{2}{x^2}\right) H_{L/R} = \hat{f}_{L/R}(x, \epsilon_\varphi/\epsilon, \gamma) T_{L/R}$$





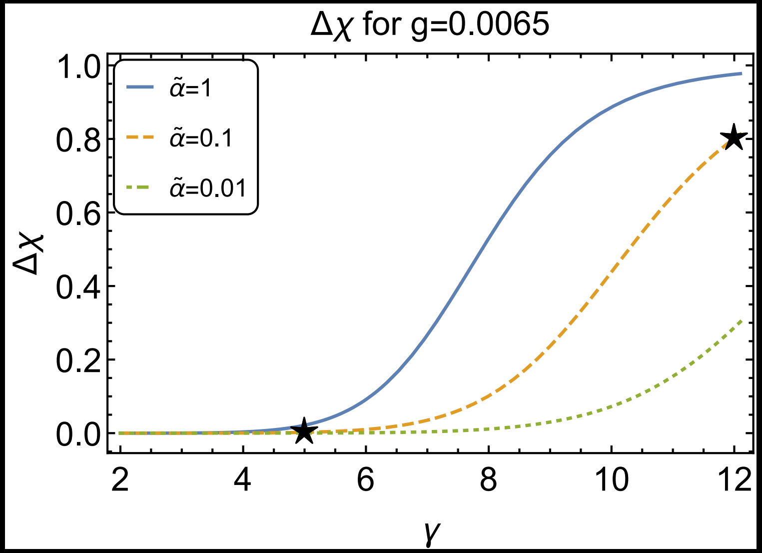
Extra contribution to tensor power spectra and chirality

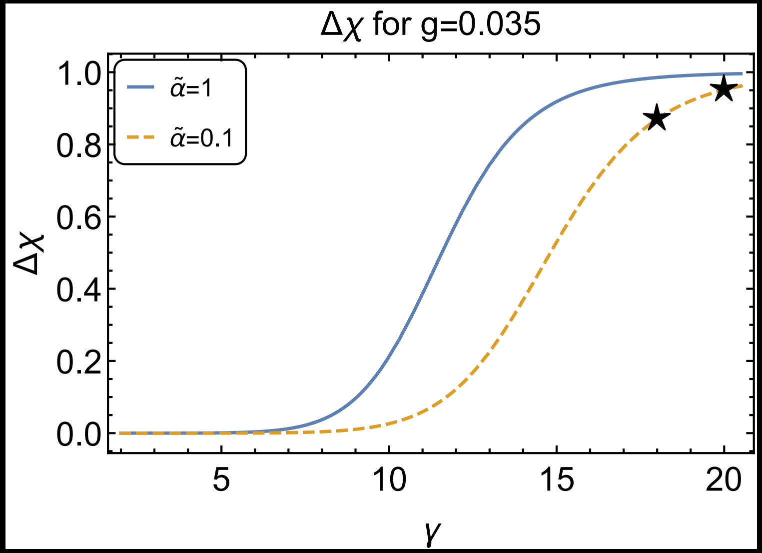
$$H_L = \underbrace{H_L^{(0)}}_{\text{vacuum}} + \underbrace{H_L^{(s)}}_{\text{sourced by } T_L}$$

$$P_L^2(k) = \frac{H^2}{2\pi^2 M_{\rm Pl}^2} + \frac{H^2}{\pi^2 M_{\rm Pl}^2} C^2(\epsilon_\varphi/\epsilon, \gamma),$$

$$P_R^2(k) = \frac{H^2}{2\pi^2 M_{\rm Pl}^2}$$

$$\Delta\chi = \frac{P_L^2 - P_R^2}{P_L^2 + P_R^2}$$





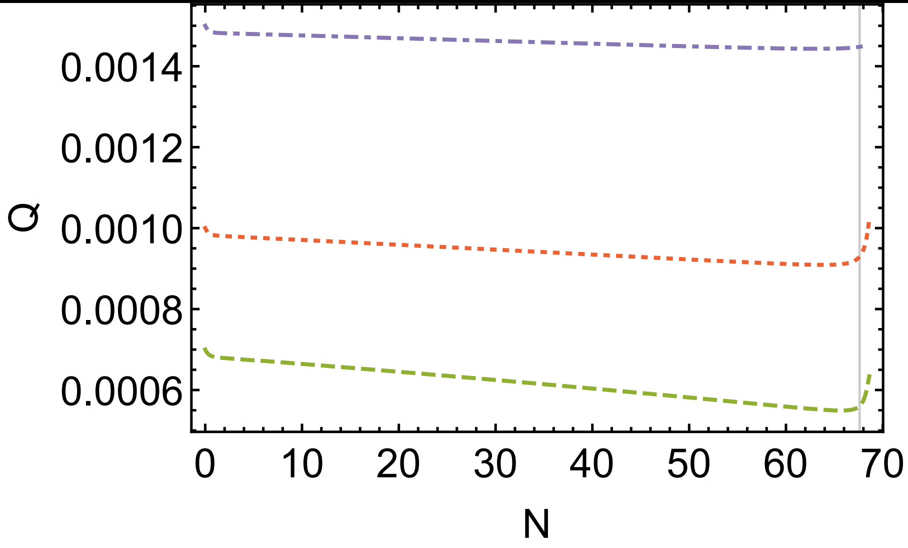
Tensor tilt and tensor-to-scalar ratio

$$P_T^{(s)}(k) = A_T(\gamma_*) \left(\frac{k}{k_*}\right)^{n_T^{(s)}}$$

$$n_T^{(s)} \simeq -\delta_* \left(2.85 + 3.68\sqrt{\gamma_*}\right) \qquad r = \frac{P_T^{(0)} + P_T^{(s)}}{P_\zeta}$$

$$Q(t) \searrow \quad \Rightarrow \quad \dot{Q}(t) < 0 \quad \Rightarrow \quad \delta > 0 \quad \Rightarrow \quad n_T < 0 \quad \text{red tilt,}$$
$$Q(t) \nearrow \quad \Rightarrow \quad \dot{Q}(t) > 0 \quad \Rightarrow \quad \delta < 0 \quad \Rightarrow \quad n_T > 0 \quad \text{blue tilt.}$$

$$\delta = -\frac{\dot{Q}}{HQ}, \quad \delta \ll 1$$



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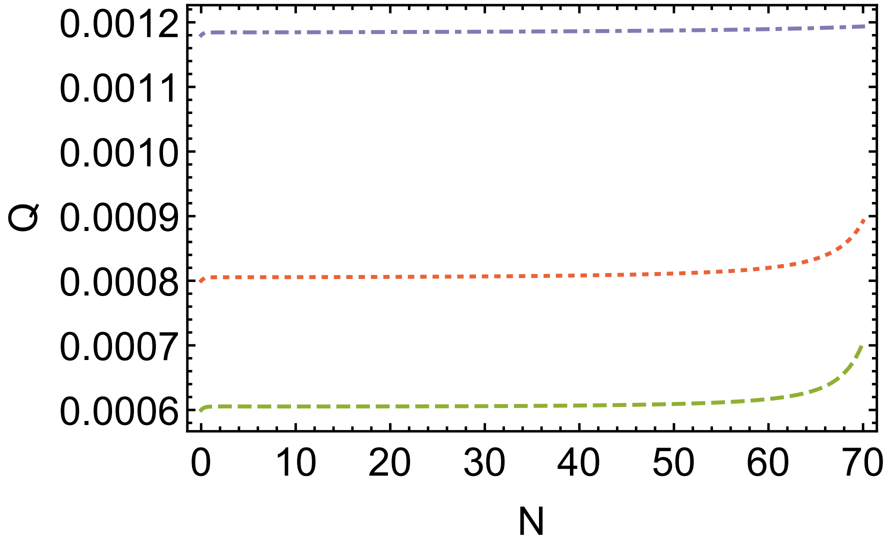

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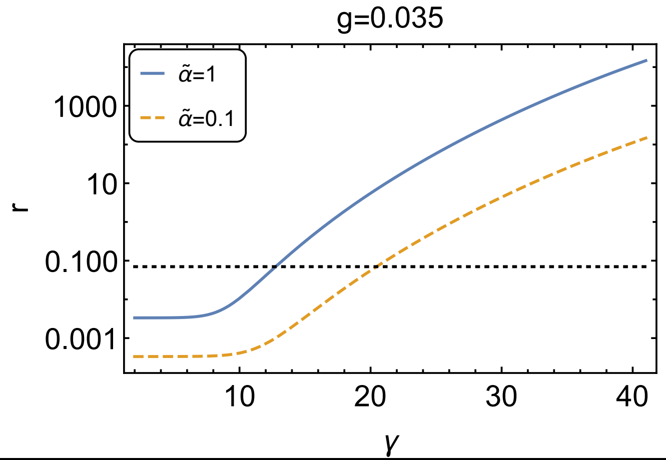
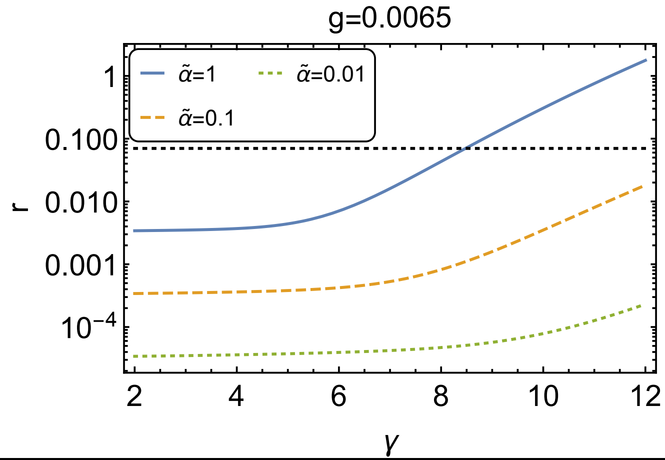
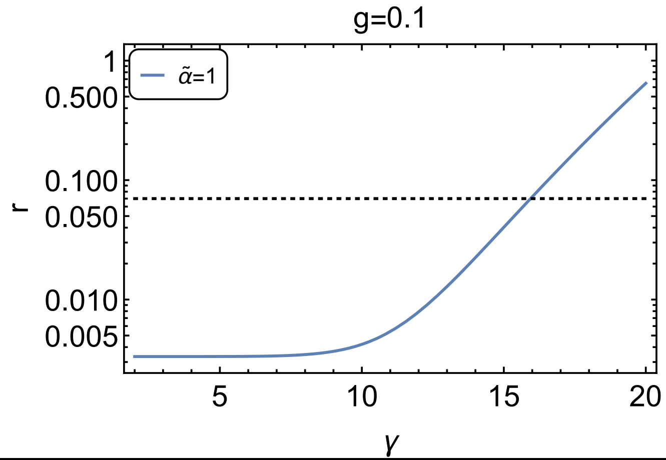
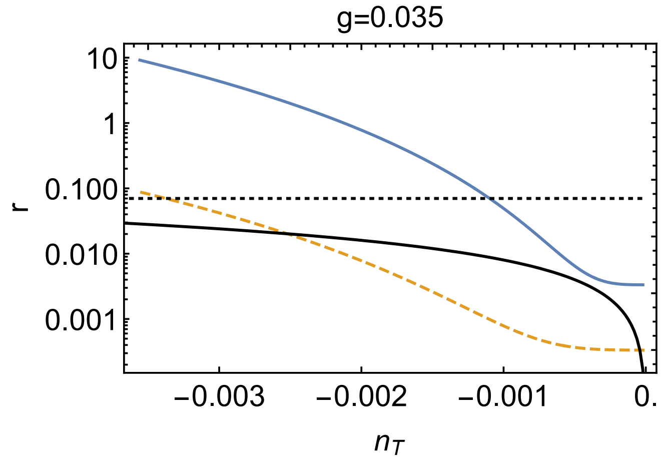
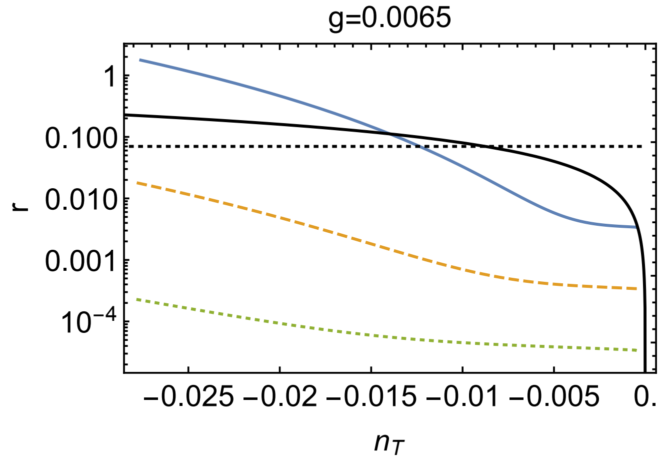
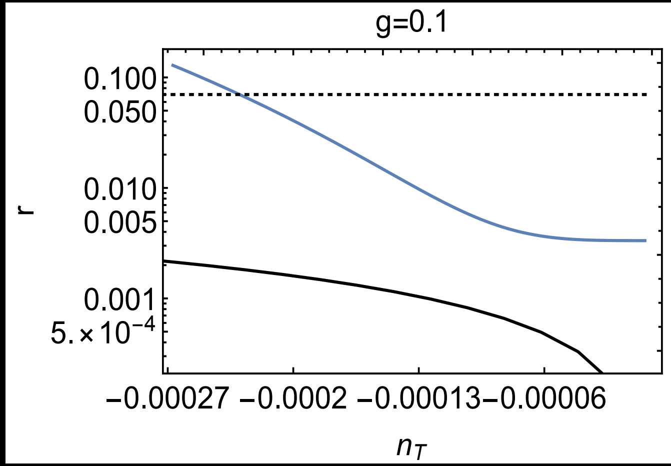






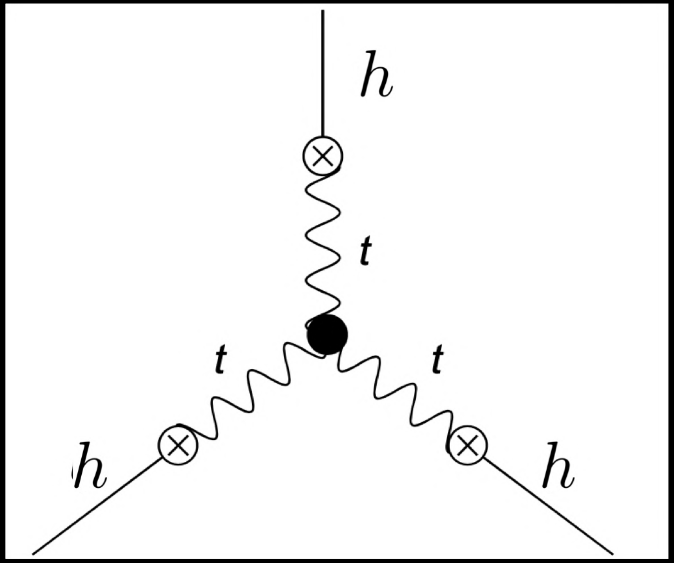


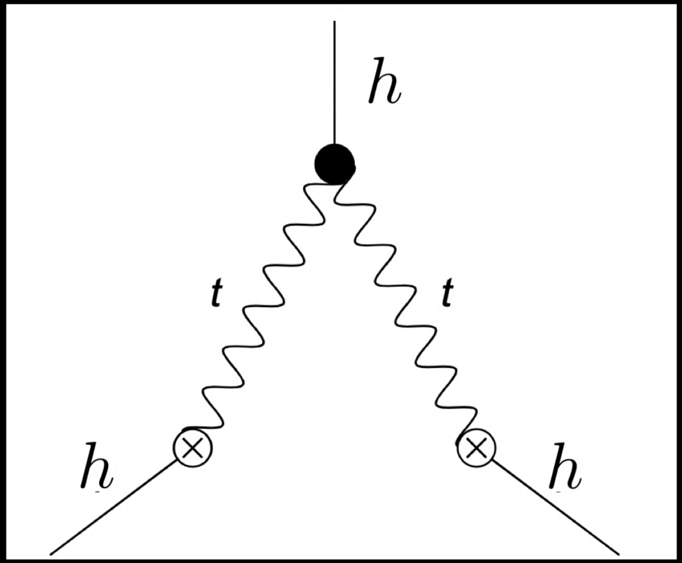




What about tensor non-Gaussianity?

In axion-SU(2) model $$f_{NL} \approx 5\frac{r^2}{\epsilon_B}$$





[A. Agrawal, T. Fujita, E. Komatsu, 2018]

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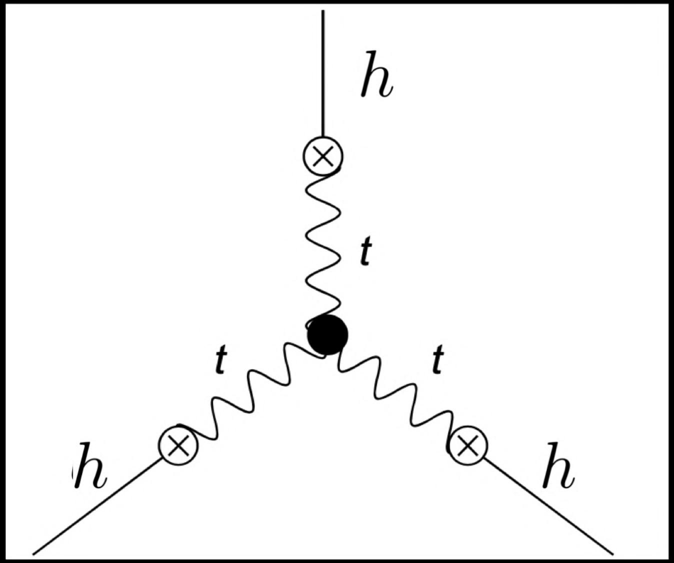

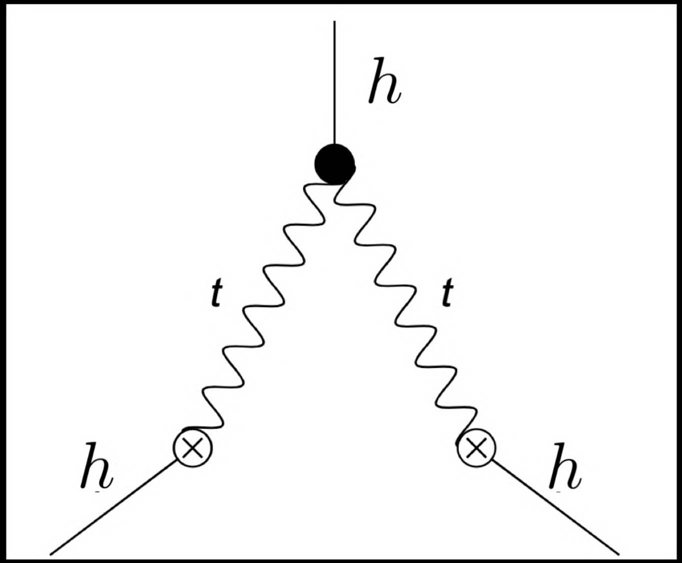

[A. Agrawal, T. Fujita, E. Komatsu, 2018]

A separate check is needed for spectator Gauge-flation.

Summary:

- The enhancement of chiral gravitational waves cannot be arbitrarily high. The amplification reaches its maximal value due to the restrictions for the gauge sector to be a spectator.
- A slightly red-tilted tensor power spectrum is preferred with $n_T = -\mathcal{O}(0.01)$
- Potential observation of chiral gravitational waves with significantly tilted tensor spectra would indicate the presence of additional couplings of the gauge fields to axions or additional gauge field operators.


Thank you!