Numerical simulations of stochastic inflation using importance sampling

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arXiv:2206.11234, with Hooshyar Assadullahi, Kazuya Koyama, Vincent Vennin and David Wands

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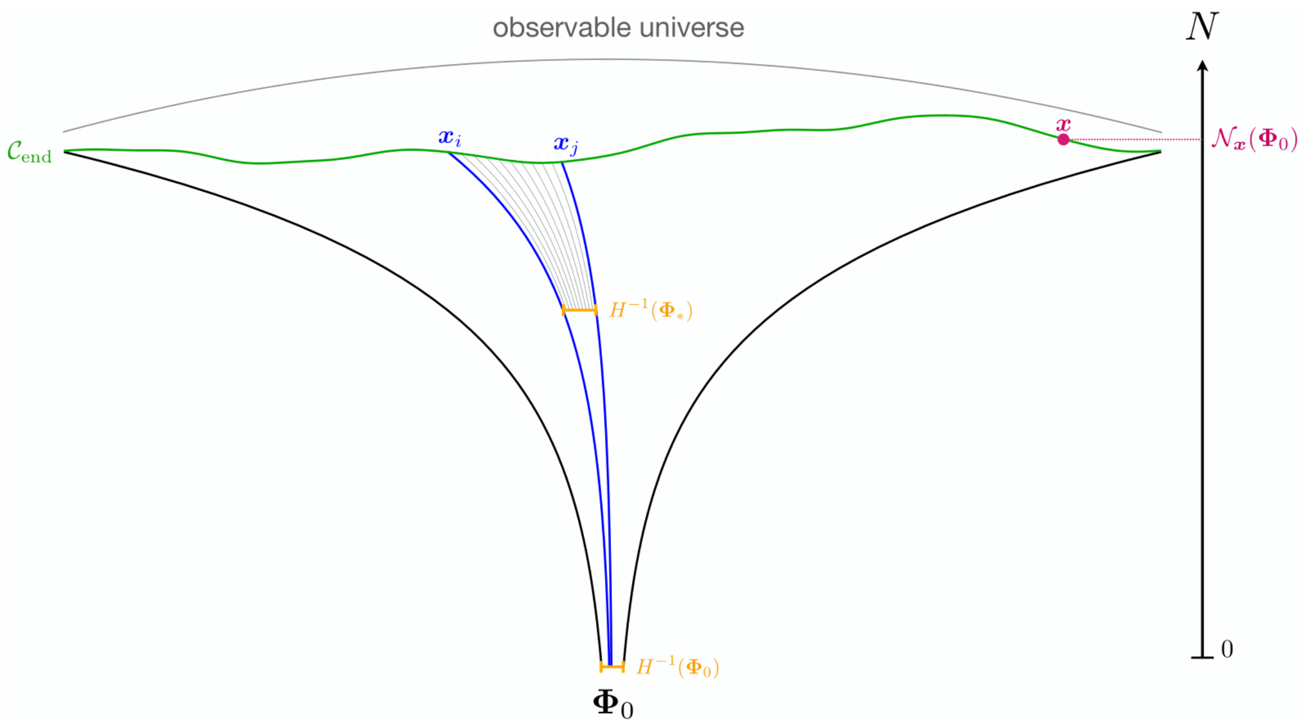
Stochastic Inflation - A Quick Reminder I



Ando+Vennin (2012.0203)

Stochastic Inflation - A Quick Reminder II

The dynamics of stochastic slow-roll inflation are given by

$$\frac{\partial \phi}{\partial N} = \underbrace{-\frac{1}{3H^2(\phi)}\frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi}}_{\text{drift}} + \underbrace{\frac{H(\phi)}{2\pi}\xi}_{\text{diffusion}} .$$

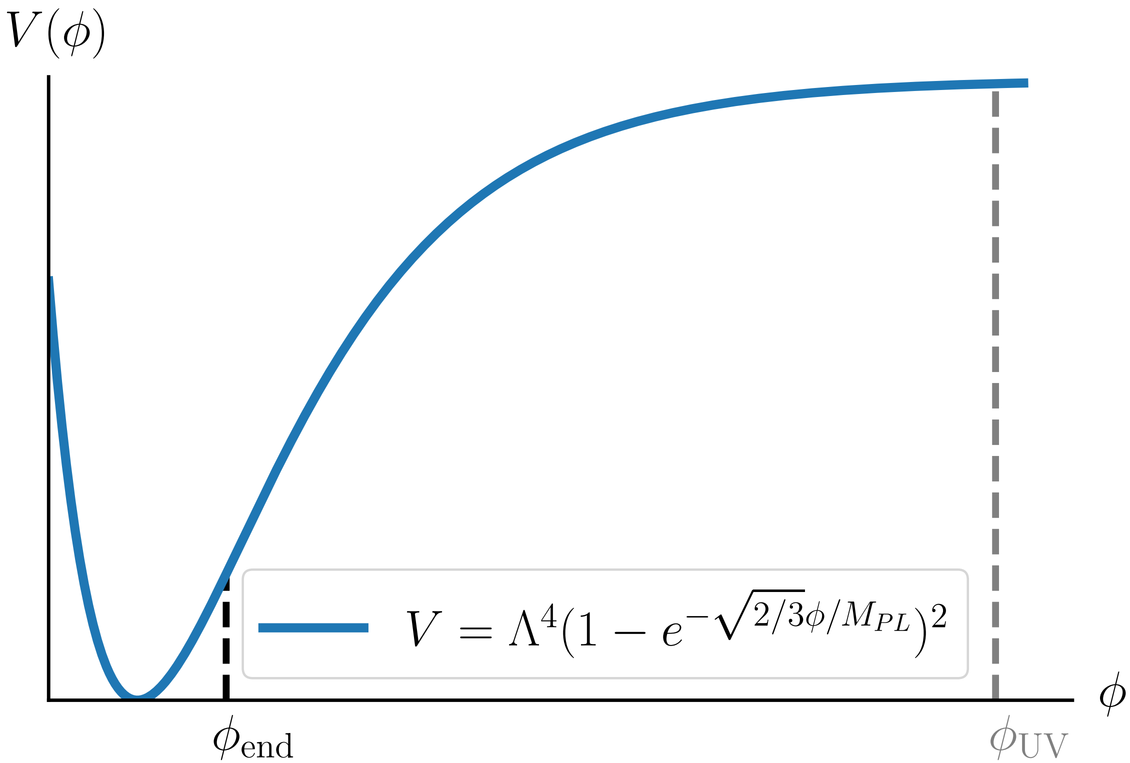
This leads generically to an exponential tail for the probability density of curvature perturbation ζ (through $\zeta = \mathcal{N} - \langle \mathcal{N} \rangle$) for finite UV-cutoff ϕ_{UV}[1]

$$P_\phi(\mathcal{N}) = \sum_n a_n(\phi) e^{-\Lambda_n \mathcal{N}} .$$

The values of the poles Λ_n depend on the potential $V(\phi)$ and ϕ_{UV}.

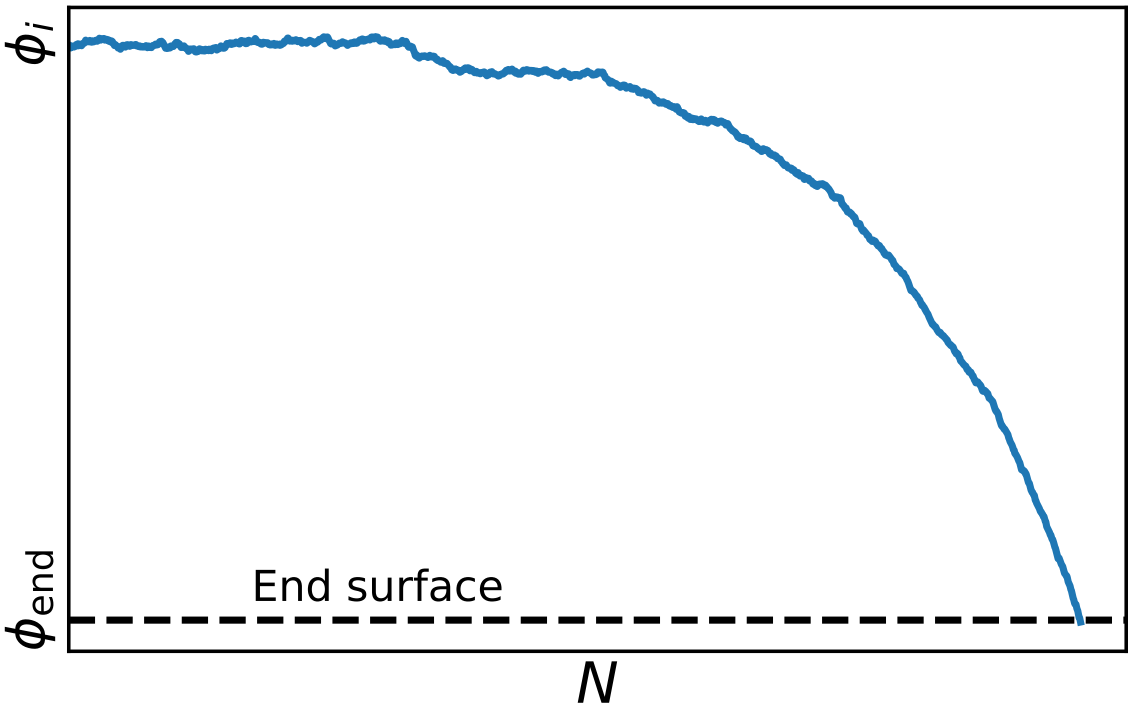
[1] Pattison *et al.* (1707.00537), Ezquiaga *et al.* (1912.05399)

General Potentials, *e.g.* Starobinsky



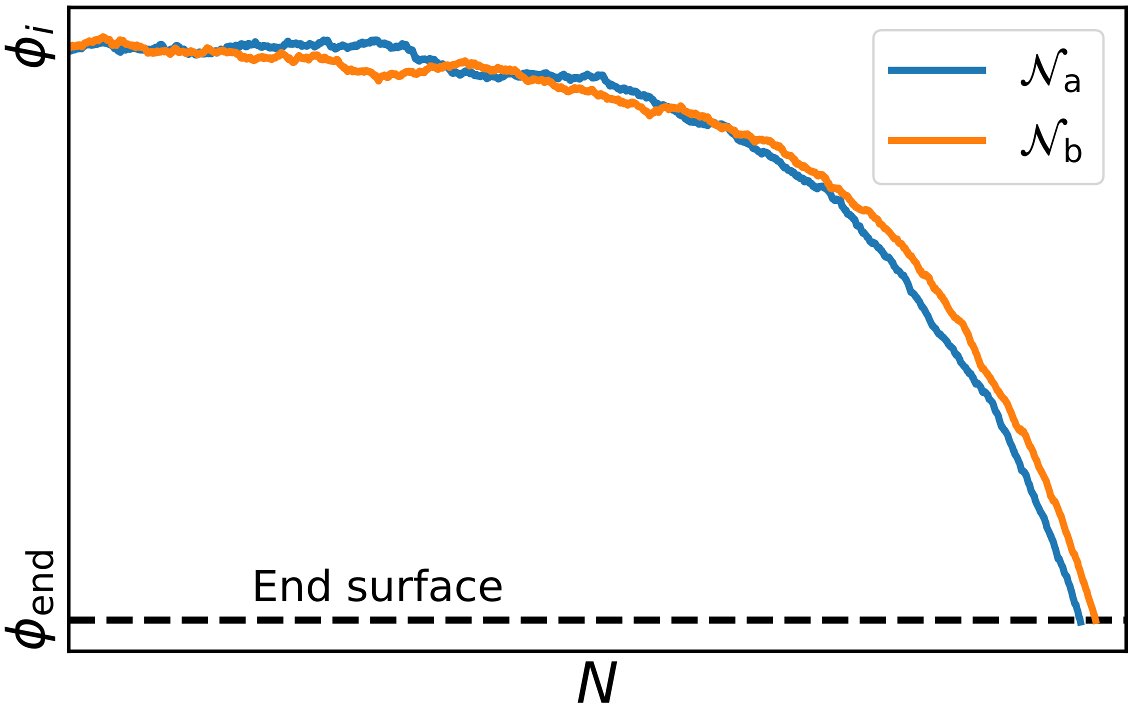
$\Longrightarrow$ Very difficult to solve analytically! Only a few cases are known.

Numerical Approach I



$$\phi_{m+1} = \phi_m + \left[-\frac{1}{3H(\phi_m)} \frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi}\Big|_{\phi=\phi_m} \Delta N + \frac{H(\phi_m)}{2\pi} \xi_m \sqrt{\Delta N} \right]$$

Numerical Approach II



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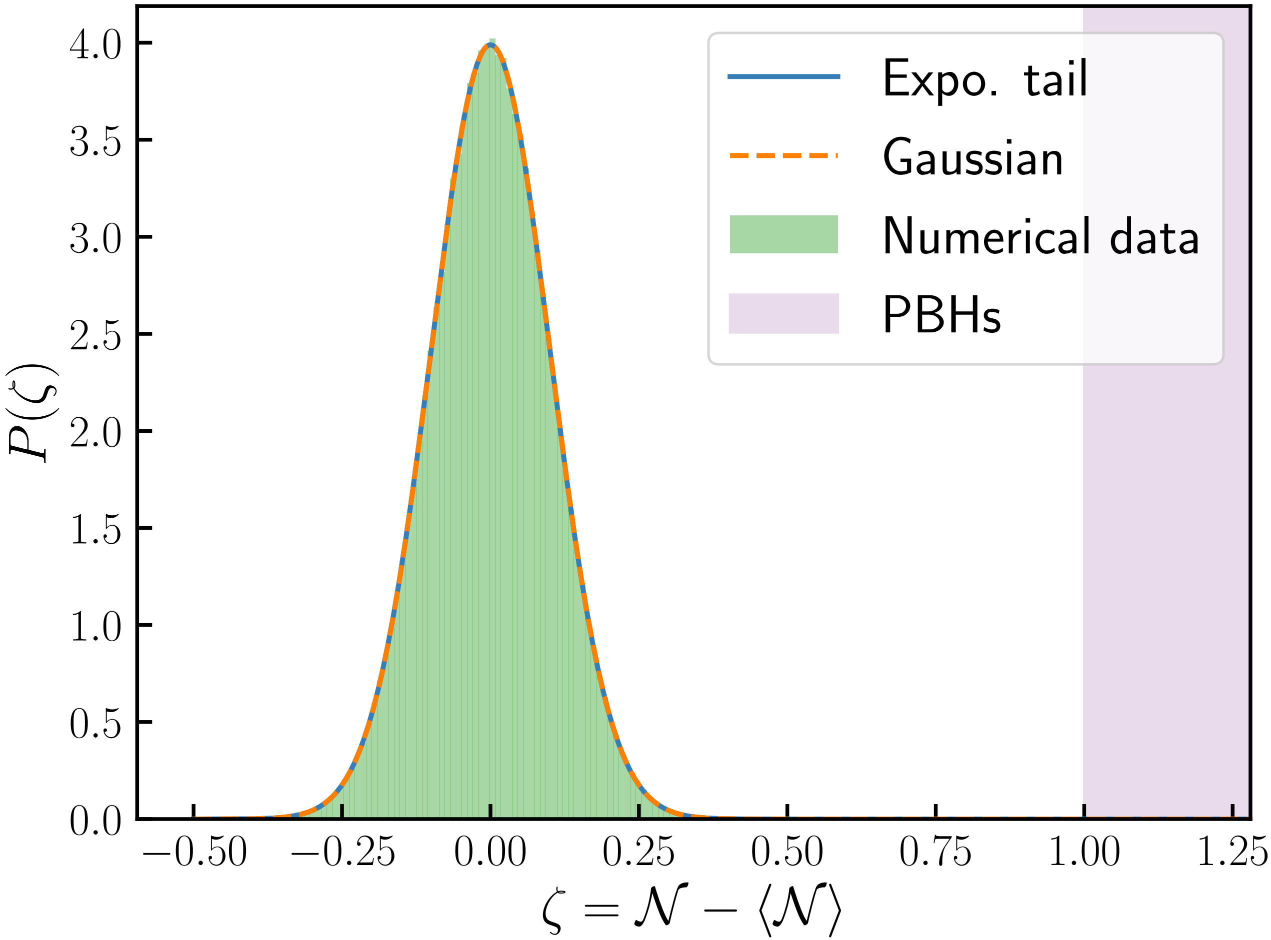
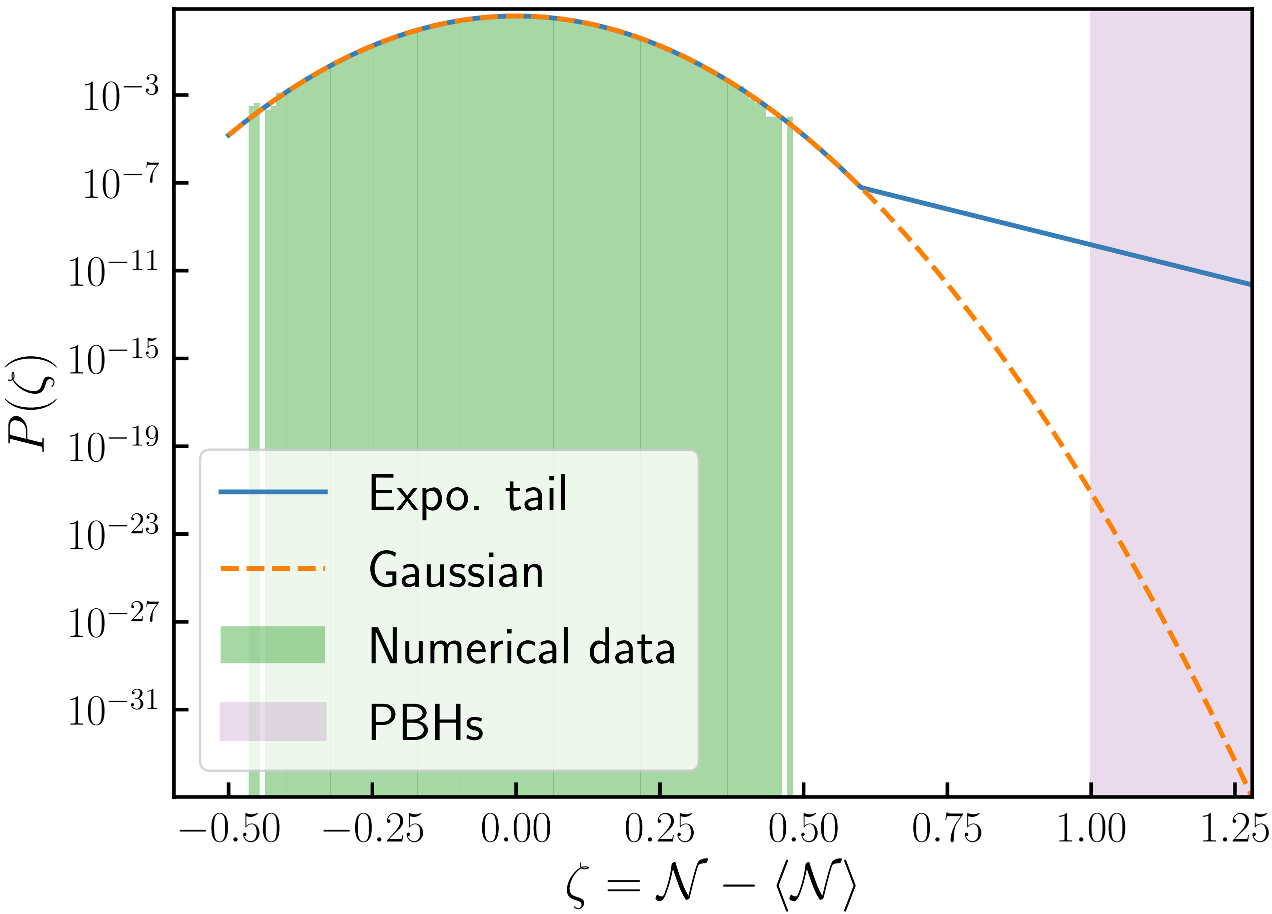
Schematic - Direct Simulation I



Schematic - Direct Simulation II



Introducing Importance Sampling

The numerical step has a bias $\mathcal{B}$ added[2]

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increasing the probability of large ζ events being simulated.

[2] Mazonka *et al.* (nucl-th/9809075)

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increasing the probability of large ζ events being simulated.

The unbiased target distribution (T) is recovered using the weight of the sampled (S) path $\mathbf{X} = (\phi_0, \phi_1, ..., \phi_M)$

$$w = \frac{P_{\mathrm{T}}(\mathbf{X})}{P_{\mathrm{S}}(\mathbf{X})}.$$

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Often we use

$$\mathcal{B}(\phi_m) = \mathcal{A} \frac{H(\phi_m)}{2\pi}$$

.

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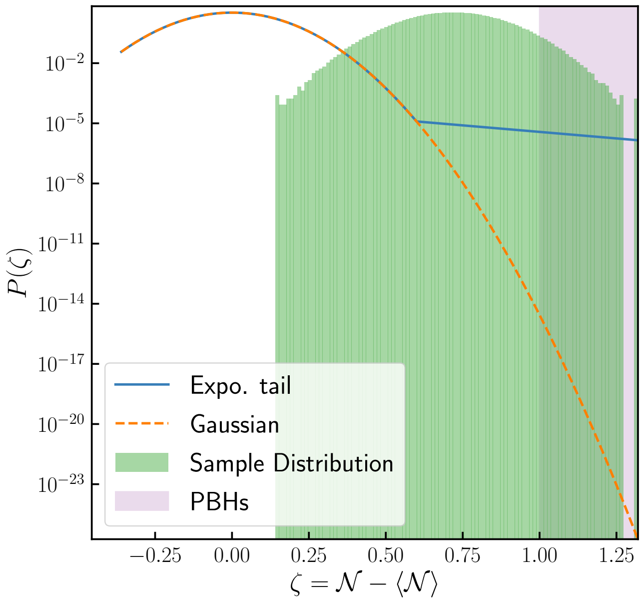
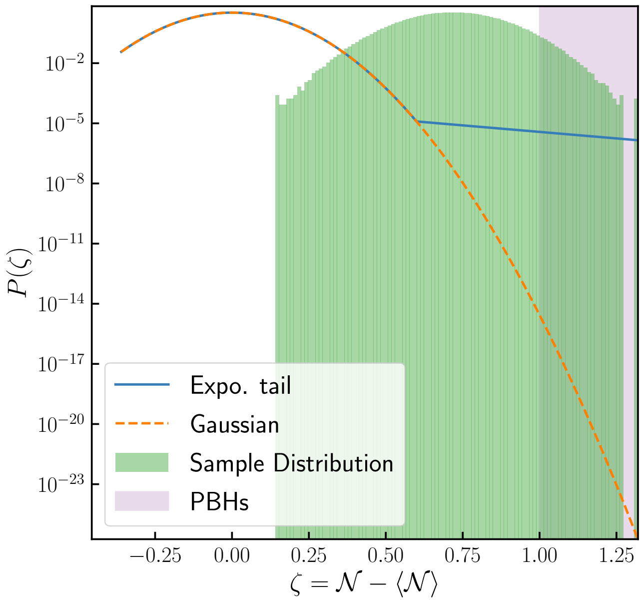
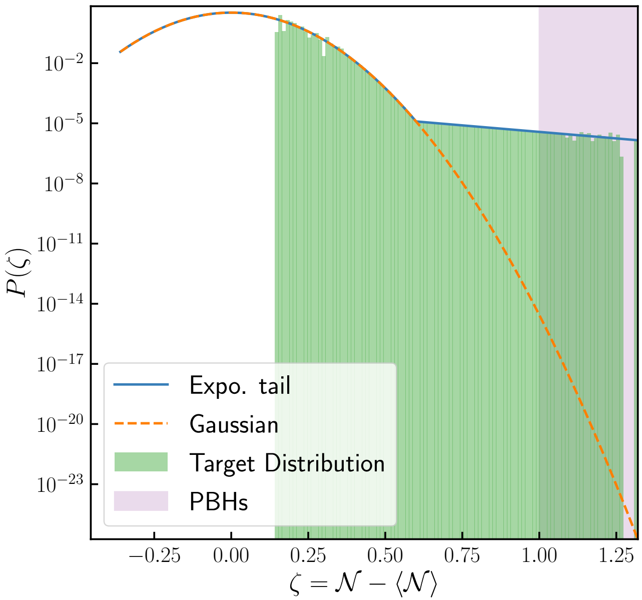
Schematic - Importance Sampling



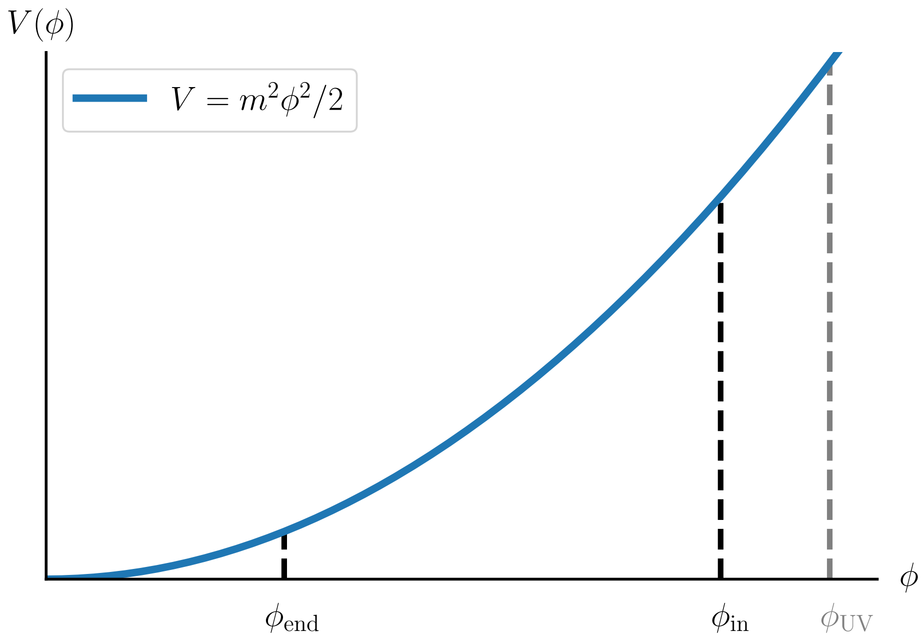
Schematic - Importance Sampling

$$\text{Apply the weights, } w = \frac{P_{\mathrm{T}}(\mathbf{X})}{P_{\mathrm{S}}(\mathbf{X})} \longrightarrow$$



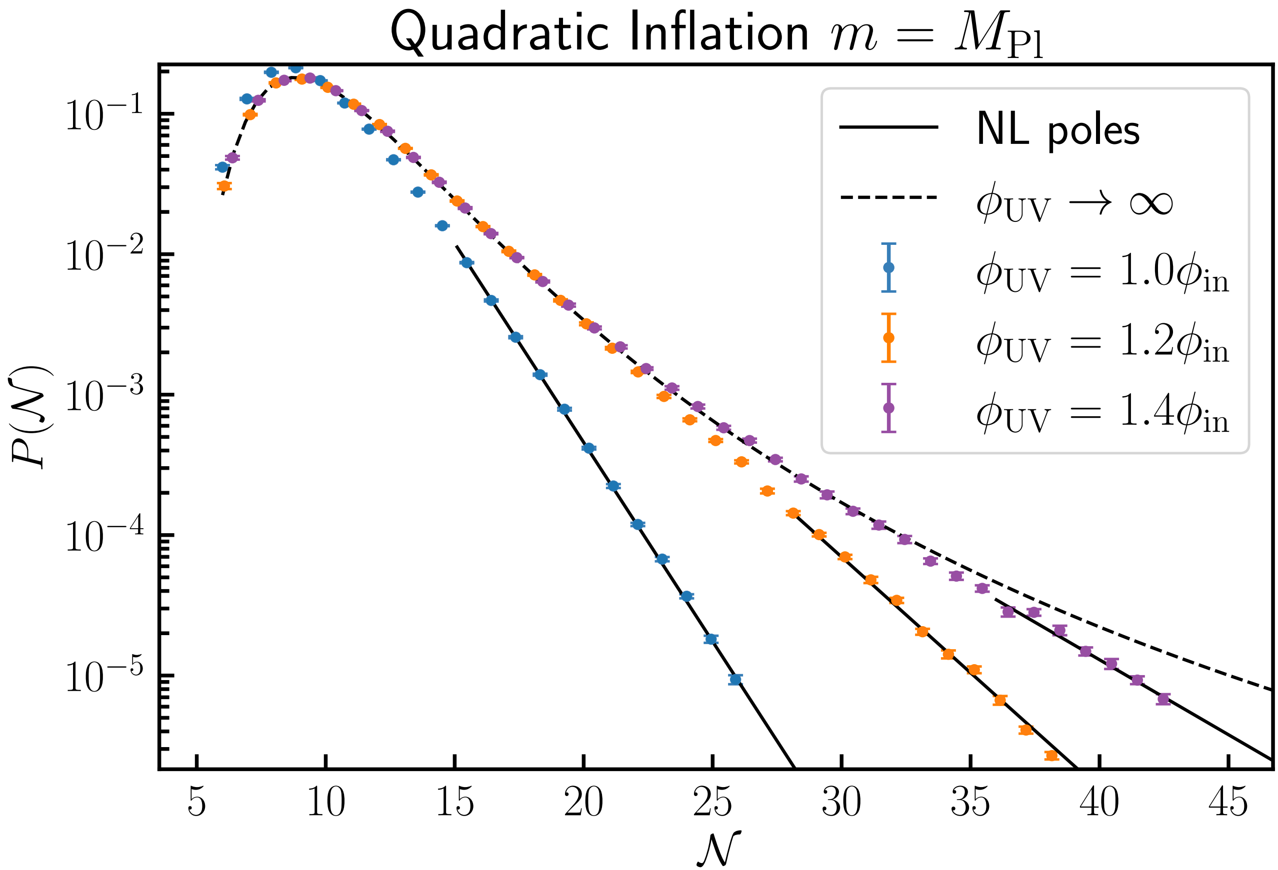


Benchmark Tests - Quadratic Inflation

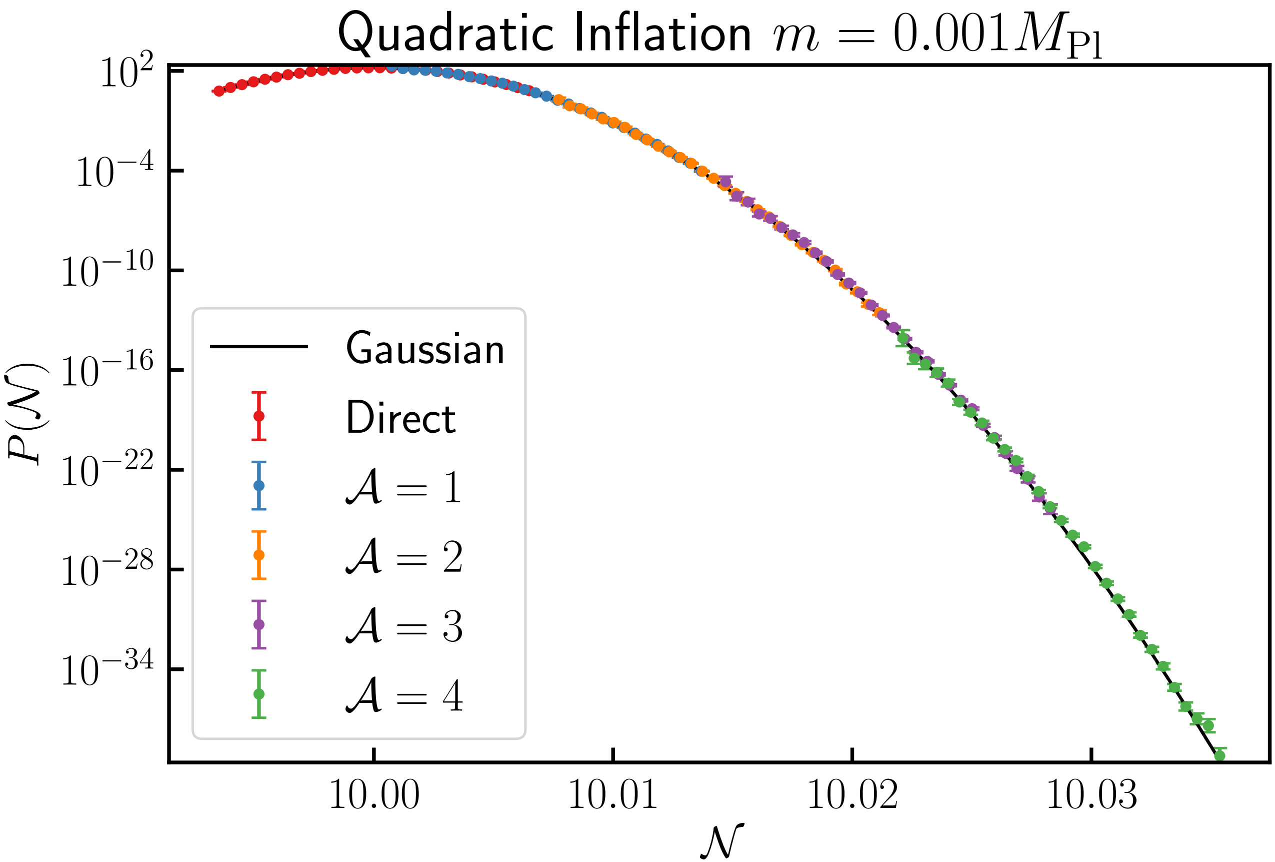


By varying *m* we can investigate the importance sampling method in both drift and diffusion dominated regimes.

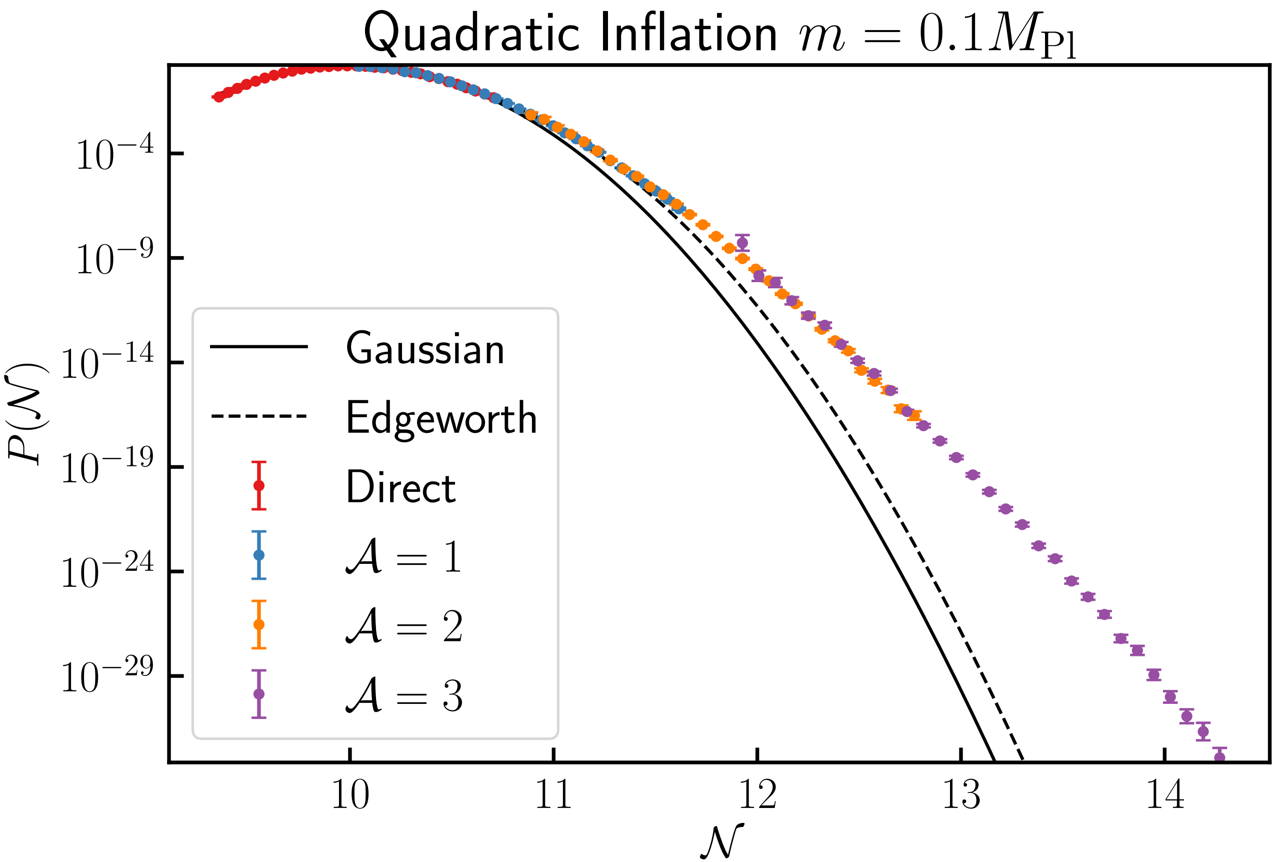
The Exponential Tail



Reconstructing the Full PDF



Non-perturbative Deviations From Gaussianity



Summary

PyFPT

Available at https://github.com/Jacks0nJ/PyFPT

Conclusions

- Numerically expensive to simulate the very large and rare ζ perturbations needed for primordial black holes.
- PyFPT makes these simulations possible with just a laptop!
- We then investigated non-perturbative deviations from Gaussianity.

arXiv: 2206.11234

Future work

- Expand the code to the full non-slow-roll phase space.

Appendix: Weight Calculation

A bias $\mathcal{B}$ is added to the numerical step

$$\phi_{m+1} = \phi_m + \left[-\frac{1}{3H(\phi_m)} \frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi}\bigg|_{\phi=\phi_m} \Delta N + \frac{H(\phi_m)}{2\pi}\xi\sqrt{\Delta N} + \mathcal{B}(\phi_m)\Delta N \right].$$

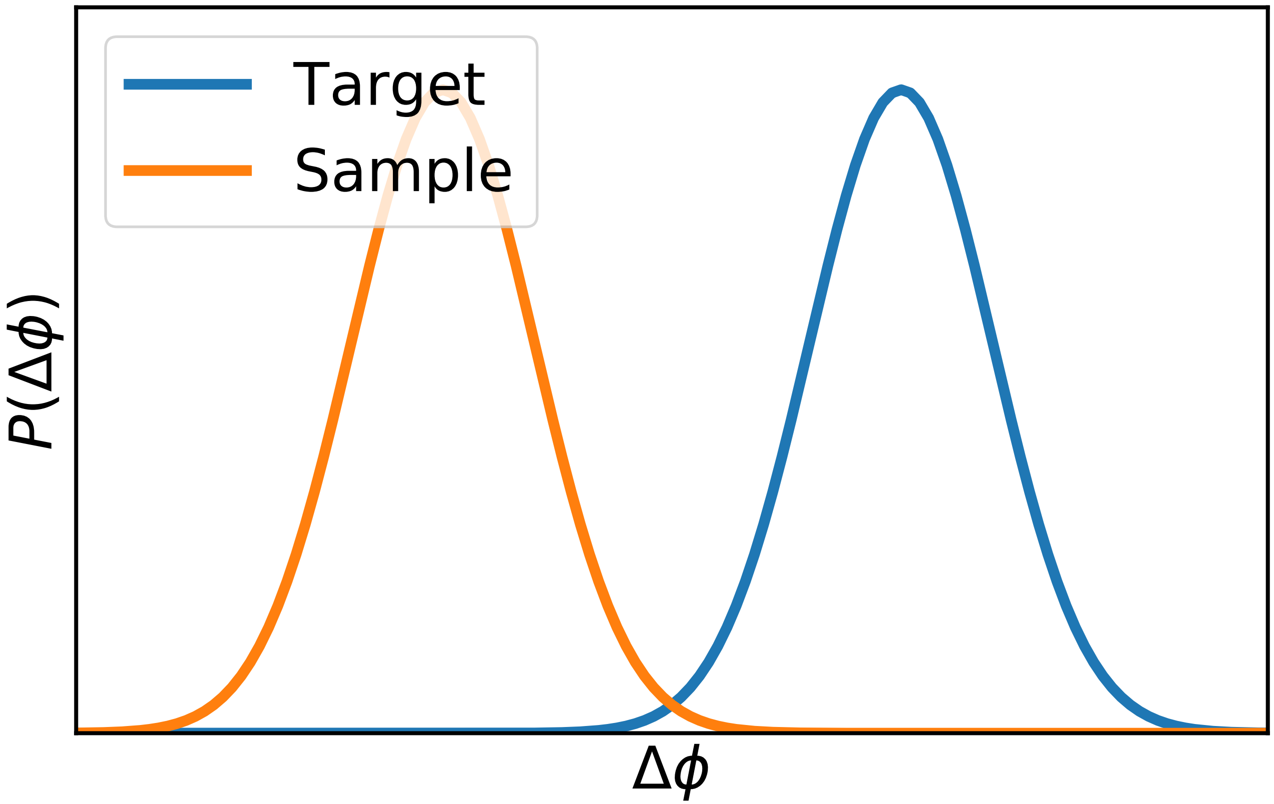
This has a weight

$$w_m = \exp\left\{ \frac{4\pi^2}{H^2(\phi_m)} \left[\phi_{m+1} - \phi_m + \frac{V'(\phi_m)}{3H(\phi_m)}\Delta N - \frac{\mathcal{B}(\phi_m)}{2}\Delta N \right] \mathcal{B}(\phi_m) \right\}.$$

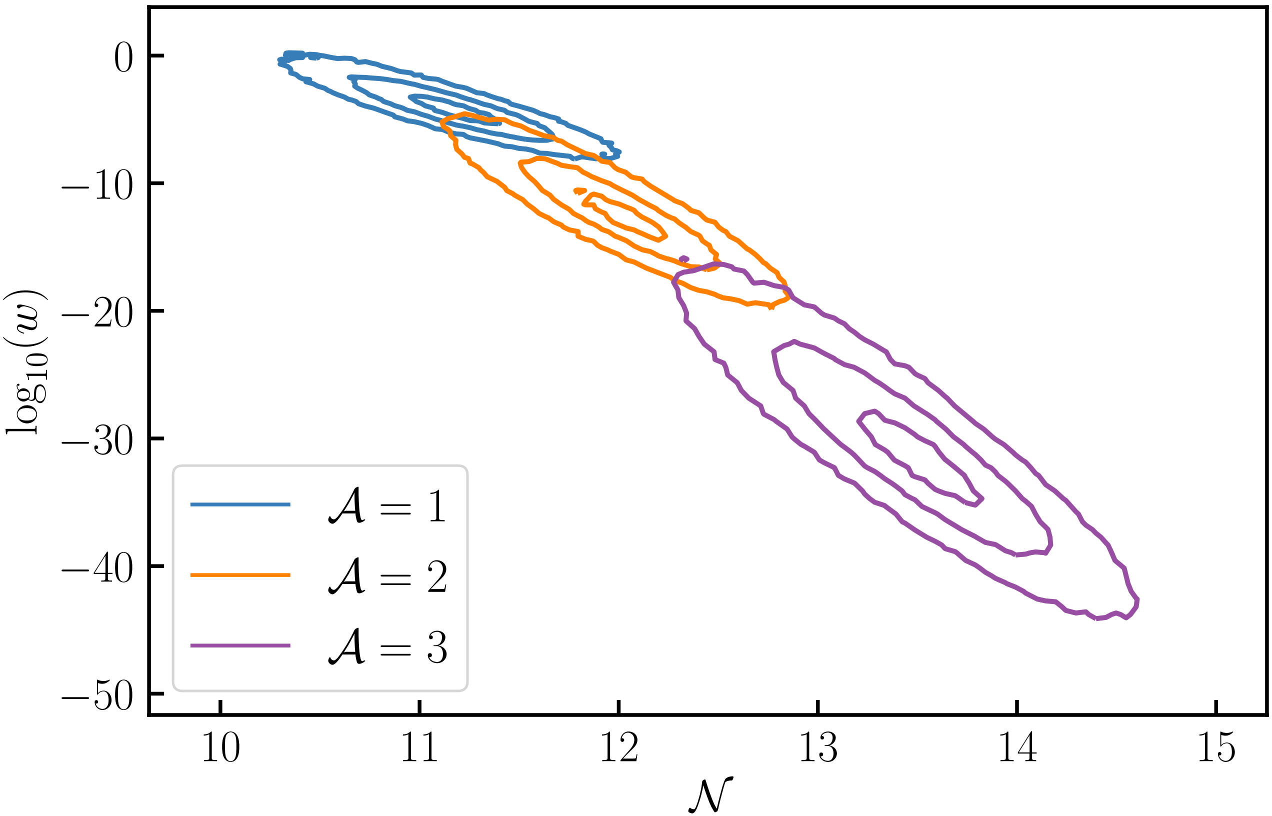
The weight of the whole sampled path $\mathbf{X} = (\phi_0, \phi_1, ..., \phi_M)$ is then

$$w(\mathbf{X}) = \prod_{m=1}^{M} w_m$$

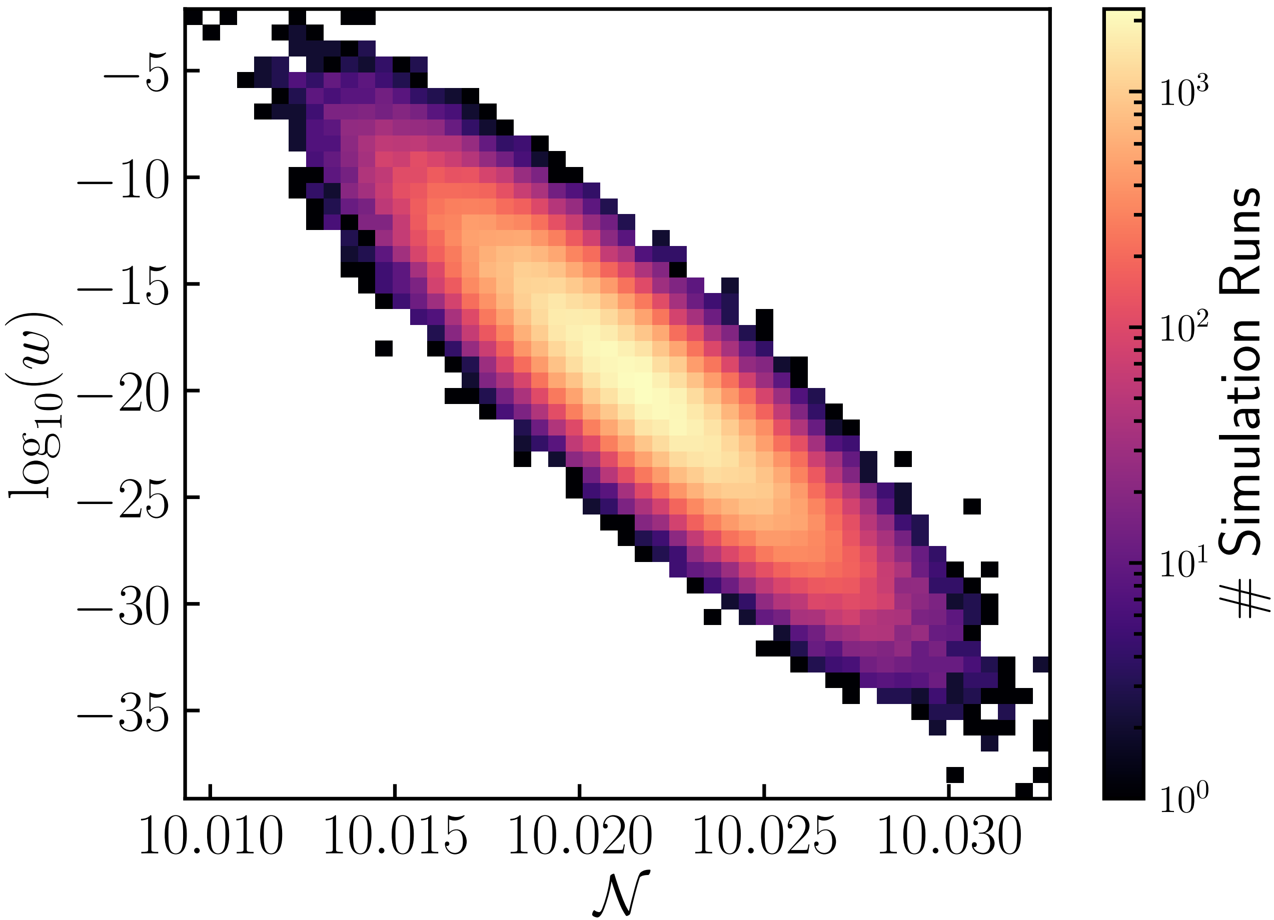
Appendix: Weight Visualized



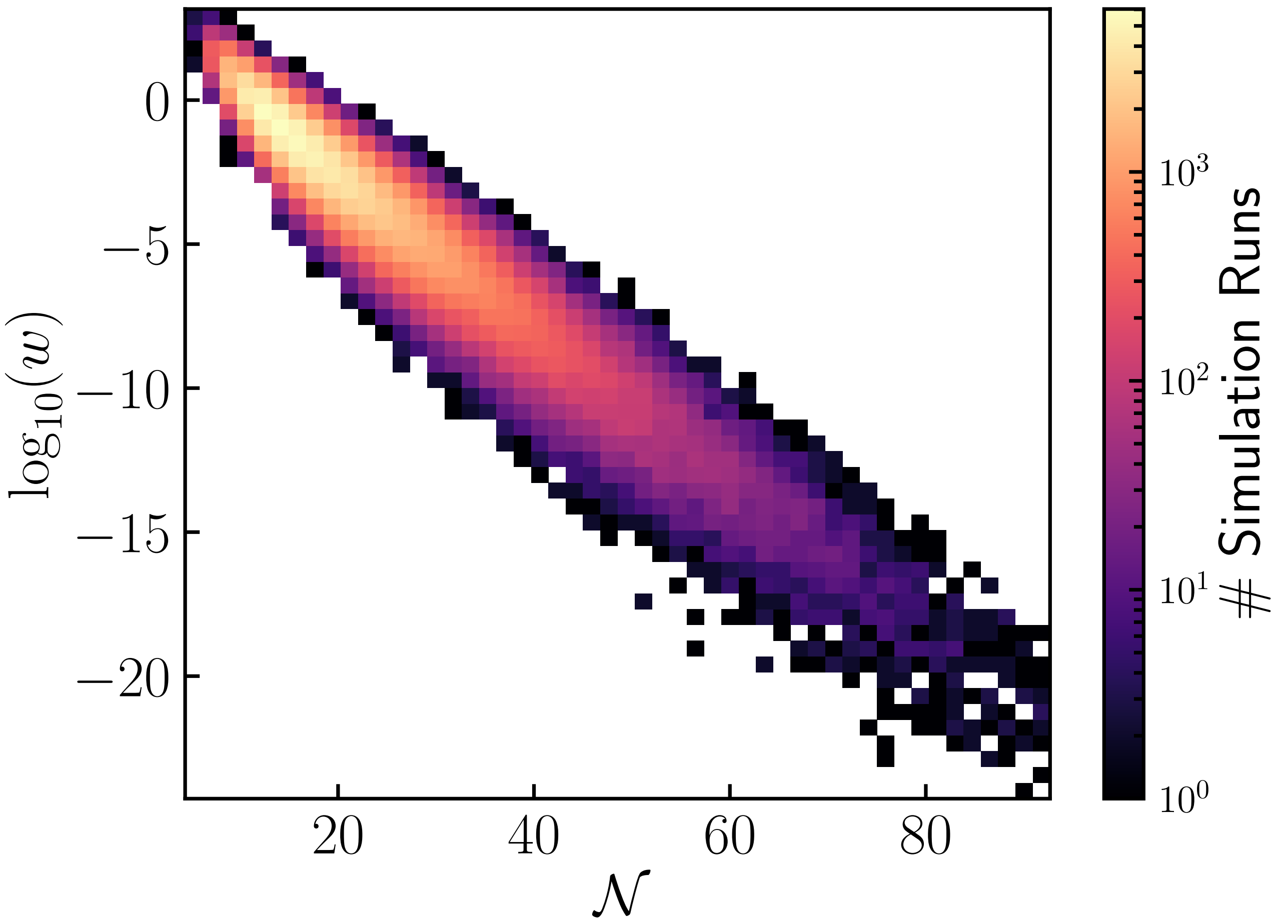
Appendix: $m = 0.1 M_{\mathrm{pl}}$ Weight Contours



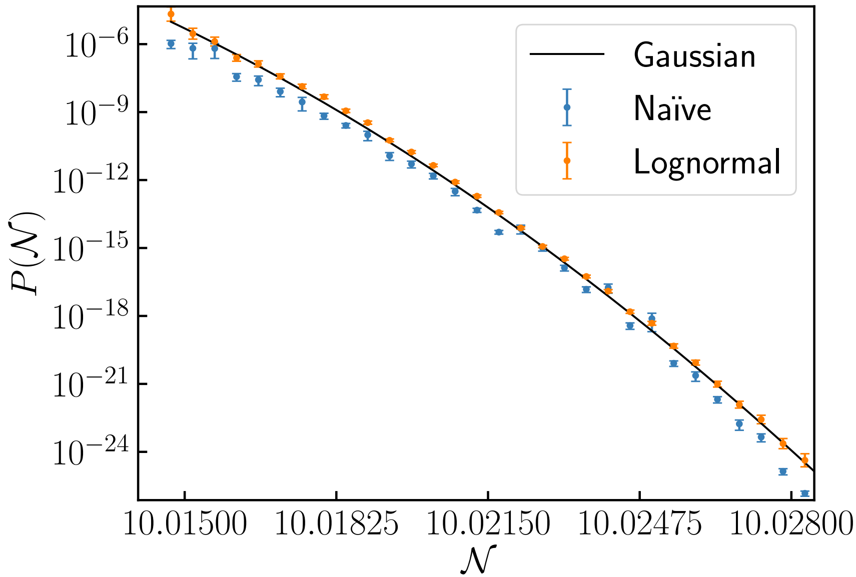
Appendix: $m = 0.001 M_{\rm pl}$ Weight Scatter



Appendix: $m = M_{\rm pl}$ and $\phi_{\rm UV}$ Weight Scatter

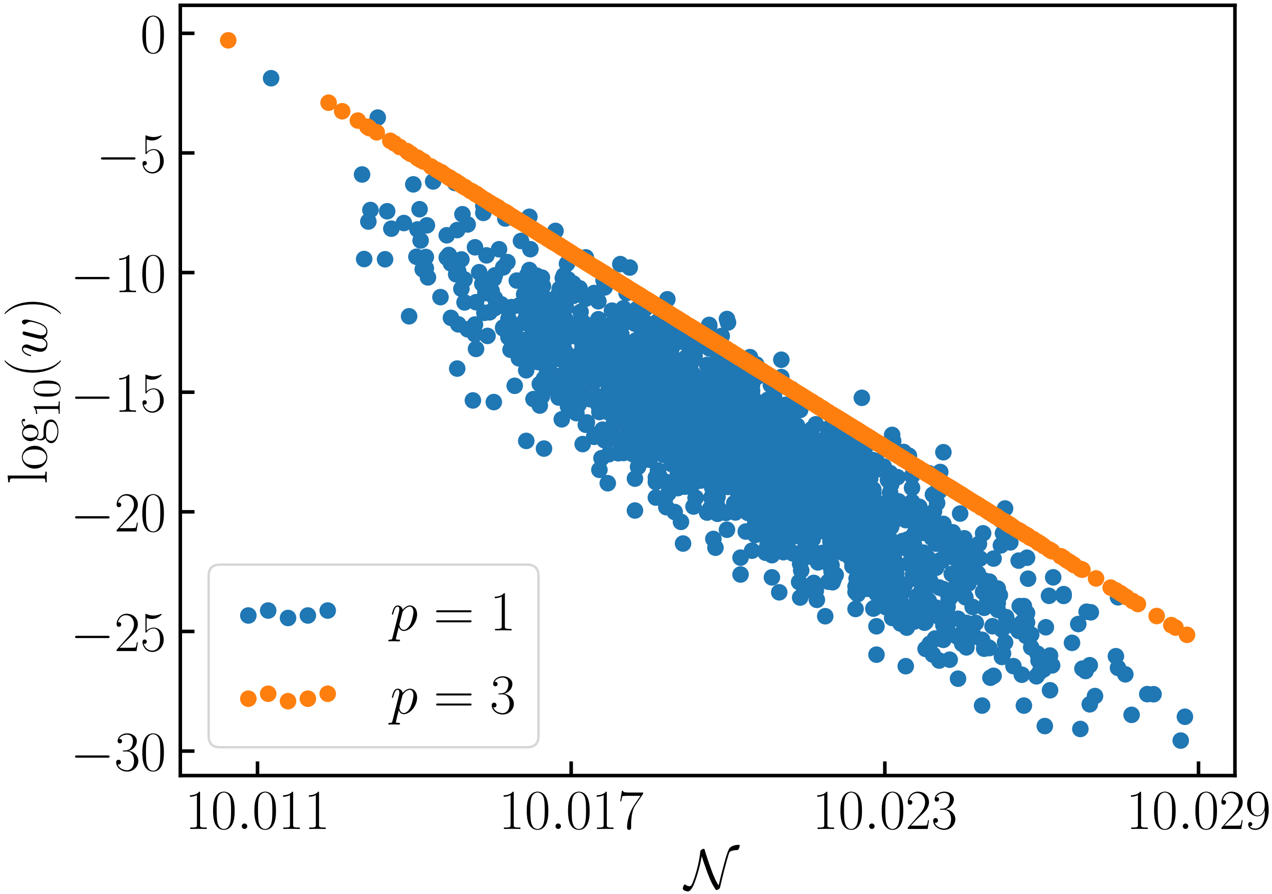


Appendix: $m = 0.001 M_{\mathrm{pl}}$ Lognormal Estimator



$$\langle w \rangle = \exp\left(\langle \ln w \rangle + \frac{\sigma^2_{\ln w}}{2}\right).$$

Appendix: Bias Optimization



Appendix: More Future Work

- Look at ultra-slow-roll inflation.
- Vary ϕ_{in} such that the perturbations on a particular scale can be accurately simulated (stochastic inflation breaks the one-to-one relation between k and ϕ).
- This would allow the compaction $\mathcal{C}$ to be found using the coarse-shelled method[3].

$\implies$ An accurate estimation for primordial black hole abundance!

[3] Tada and Vennin (2111.15280)