Primordial non-Gaussian correlations of inflationary perturbations with gauge fields

Rajeev Kumar Jain

Department of Physics
Indian Institute of Science
Bangalore

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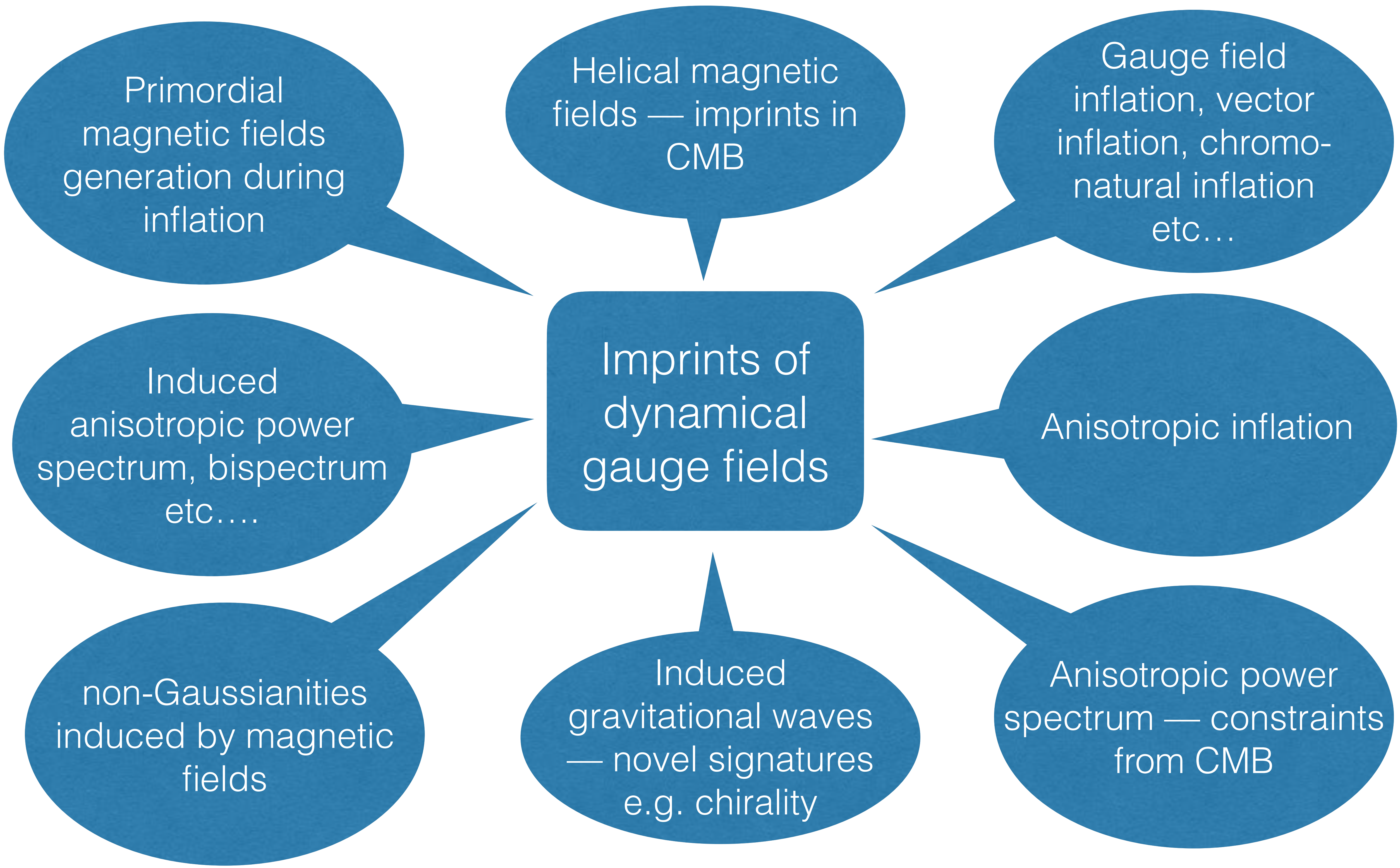
Outline of the talk

- Dynamical gauge fields during inflation — interesting cosmological signatures
- Non-Gaussian correlations — novel consistency relations
  - Cross-correlations with primordial curvature perturbations
  - Cross-correlations with gravitons
- Conclusions

Dynamical gauge fields — cosmological imprints



Dynamical gauge fields — cosmological imprints

Primordial
magnetic fi
generation
inflatio

Helical magnetic
fields — imprints in

Gauge field
inflation, vector
ion, chromo-
ral inflation
etc…

Induc
anisotropic
spectrum, bi
etc…

ropic inflation

Whether or not the gauge field takes a vacuum expectation value (vev) during inflation !!

non-Gaussianities
induced by magnetic
fields

gravitational waves
— novel signatures
e.g. chirality

tropic power
spectrum — constraints
from CMB

Non-Gaussian imprints of primordial gauge fields

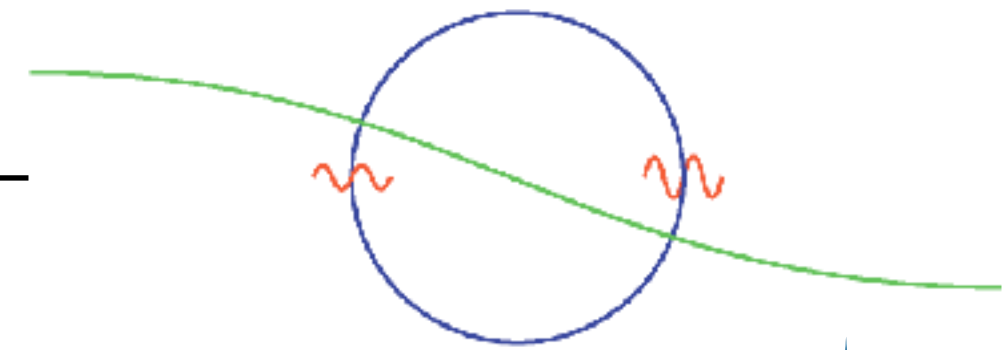
- If gauge field are produced during inflation, they must be correlated with inflationary scalar and tensor perturbations.
- Such cross-correlations will be non-Gaussian in nature — could be large also.
- An interesting scenario for inflationary magnetogenesis is

$$\lambda(\phi, \mathcal{R}) F_{\mu\nu} F^{\mu\nu}$$

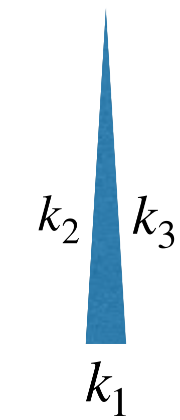
- A model-independent calculation can not be done as these correlations depend on the coupling function.

$$\langle \zeta(k_1) \mathbf{B}(k_2) . \mathbf{B}(k_3) \rangle \qquad \langle \gamma(k_1) \mathbf{B}(k_2) . \mathbf{B}(k_3) \rangle$$

Semi-classical estimate in the squeezed limit



- Squeezed limit: $k_1 \ll k_2 \sim k_3$
- Consider $\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle$ in the squeezed limit i.e. $k_1 \to 0$



  - The long wavelength mode rescales the background for short wavelength modes

$$ds^2 = -dt^2 + a^2(t)\, e^{2\zeta(t,\mathbf{x})} d\mathbf{x}^2$$

  - Taylor expand in the rescaled background

$$\langle \zeta_{k_2} \zeta_{k_3} \rangle_{\zeta_1} = \langle \zeta_{k_2} \zeta_{k_3} \rangle + \zeta_1 \frac{\partial}{\partial \zeta_1} \langle \zeta_{k_2} \zeta_{k_3} \rangle + \ldots$$

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle_{\zeta_1} \approx \left\langle \zeta_{k_1} \langle \zeta_{k_2} \zeta_{k_3} \rangle_{\zeta_1} \right\rangle \sim \langle \zeta_{k_1} \zeta_{k_1} \rangle\, k \frac{d}{dk} \langle \zeta_{k_2} \zeta_{k_3} \rangle$$

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle \sim -(n_s - 1) \langle \zeta_{k_1} \zeta_{k_1} \rangle \langle \zeta_{k_2} \zeta_{k_3} \rangle$$

$$f_{NL}^{\rm local} = -(n_s - 1)$$

Maldacena, JHEP 0305, 013 (2002)

Non-Gaussian cross-correlation

- Define the cross-correlation bispectrum of the curvature perturbation with magnetic fields as

$$\langle\zeta(\mathbf{k}_1)\mathbf{B}(\mathbf{k}_2)\cdot\mathbf{B}(\mathbf{k}_3)\rangle \equiv (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_{\zeta BB}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

$$B_{\zeta BB}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \equiv b_{NL}P_\zeta(k_1)P_B(k_2)$$

- *Local* resemblance between f_{NL} and b_{NL}

$$\zeta = \zeta^{(G)} + \frac{3}{5}f_{NL}^{local}\left(\zeta^{(G)}\right)^2$$

$$\mathbf{B} = \mathbf{B}^{(G)} + \frac{1}{2}b_{NL}^{local}\zeta^{(G)}\mathbf{B}^{(G)}$$

RKJ & Sloth, Phys. Rev. D 86, 123528 (2012)

A novel magnetic consistency relation

- Using Maldacena's approach, the cross-correlation becomes

$$\langle \zeta(\tau_I, \mathbf{k}_1) \mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3) \rangle$$
$$= -\frac{1}{H} \frac{\dot{\lambda}}{\lambda} (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_B(k_2)$$

- With the coupling $\lambda(\phi(\tau)) = \lambda_I (\tau/\tau_I)^{-2n}$, we obtain

$$\boxed{b_{NL} = n_B - 4}$$

- For scale-invariant magnetic field spectrum, $n_B = 0$ and therefore,

$$b_{NL} = -4$$

RKJ & Sloth, Phys. Rev. D 86, 123528 (2012)

A novel magnetic consistency relation

- In the squeezed limit $k_1 \ll k_2, k_3 = k$, we obtain a *new* magnetic consistency relation

$$\langle \zeta(k_1) \mathbf{B}(k_2) \cdot \mathbf{B}(\mathbf{k_3}) \rangle = (n_B - 4)(2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_B(k)$$

with $b_{NL}^{\text{local}} = (n_B - 4)$

- Compare with Maldacena's consistency relation

$$\langle \zeta(k_1) \zeta(k_2) \zeta(k_3) \rangle = -(n_s - 1)(2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_\zeta(k)$$

with $f_{NL}^{\text{local}} = -(n_s - 1)$

RKJ & Sloth, Phys. Rev. D 86, 123528 (2012)

Full in-in calculation

- One has to cross-check the consistency relation by doing a complete in-in calculation.

- The final result is

$$\langle\zeta(\tau_I,\mathbf{k}_1)\mathbf{B}(\tau_I,\mathbf{k}_2)\cdot\mathbf{B}(\tau_I,\mathbf{k}_3)\rangle = -\frac{1}{H}\frac{\dot{\lambda}_I}{\lambda_I}(2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)|\zeta^{(0)}_{k_1}(\tau_I)|^2|A^{(0)}_{k_2}(\tau_I)||A^{(0)}_{k_3}(\tau_I)| \times\left[\left(\mathbf{k}_2\cdot\mathbf{k}_3+\frac{(\mathbf{k}_2\cdot\mathbf{k}_3)^3}{k_2^2k_3^2}\right)k_2k_3\tilde{\mathcal{I}}^{(1)}_n+2(\mathbf{k}_2\cdot\mathbf{k}_3)^2\tilde{\mathcal{I}}^{(2)}_n\right]\ .$$

- The two integrals can be solved exactly for different values of n.

RKJ & Sloth, JCAP 1302, 003 (2013)

Full in-in calculation

- **The flattened shape:** In this limit, $k_1 = 2k_2 = 2k_3$, the cross-correlation becomes

$$\langle \zeta(\tau_I, \mathbf{k}_1)\mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3)\rangle \simeq 96 \ln(-k_t\tau_I) P_\zeta(k_1) P_B(k_2)$$

  - For the largest observable scale today, $\ln(-k_t\tau_I) \sim -60,$

$$\left| b_{NL}^{flat} \right| \sim 5760$$

- **The squeezed limit:** In this limit, $k_1 \to 0$ and

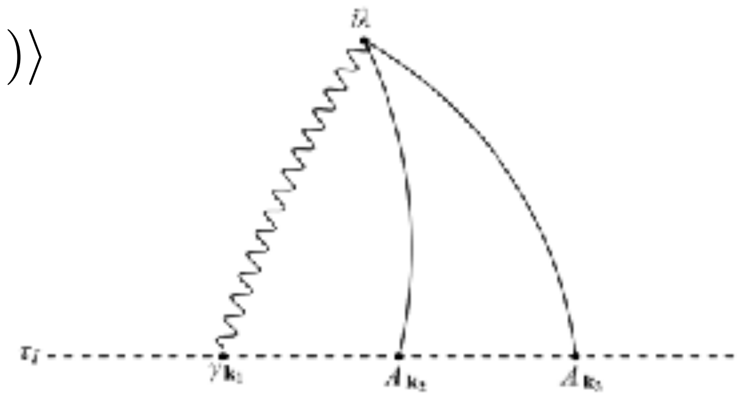
$$\langle \zeta(\tau_I, \mathbf{k}_1)\mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3)\rangle = -\frac{1}{H}\frac{\dot{\lambda}_I}{\lambda_I}(2\pi)^3\delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_B(k_2)$$

with $b_{NL} = -\frac{1}{H}\frac{\dot{\lambda}_I}{\lambda_I} = n_B - 4$ in agreement with the consistency relation.

RKJ & Sloth, JCAP 1302, 003 (2013)

Cross-correlations with gravitons

$$\langle\gamma(\mathbf{k_1})\mathbf{A}(\mathbf{k_2})\cdot\mathbf{A}(\mathbf{k_3})\rangle\ , \qquad \langle\gamma(\mathbf{k_1})\mathbf{B}(\mathbf{k_2})\cdot\mathbf{B}(\mathbf{k_3})\rangle\ , \qquad \langle\gamma(\mathbf{k_1})\mathbf{E}(\mathbf{k_2})\cdot\mathbf{E}(\mathbf{k_3})\rangle$$



$$ds^2 = -dt^2 + a^2(t)\left[e^{\gamma}\right]_{ij}dx^i dx^j \approx -dt^2 + a^2(t)\left[\delta_{ij}+\gamma_{ij}\right]dx^i dx^j$$

In the squeezed limit

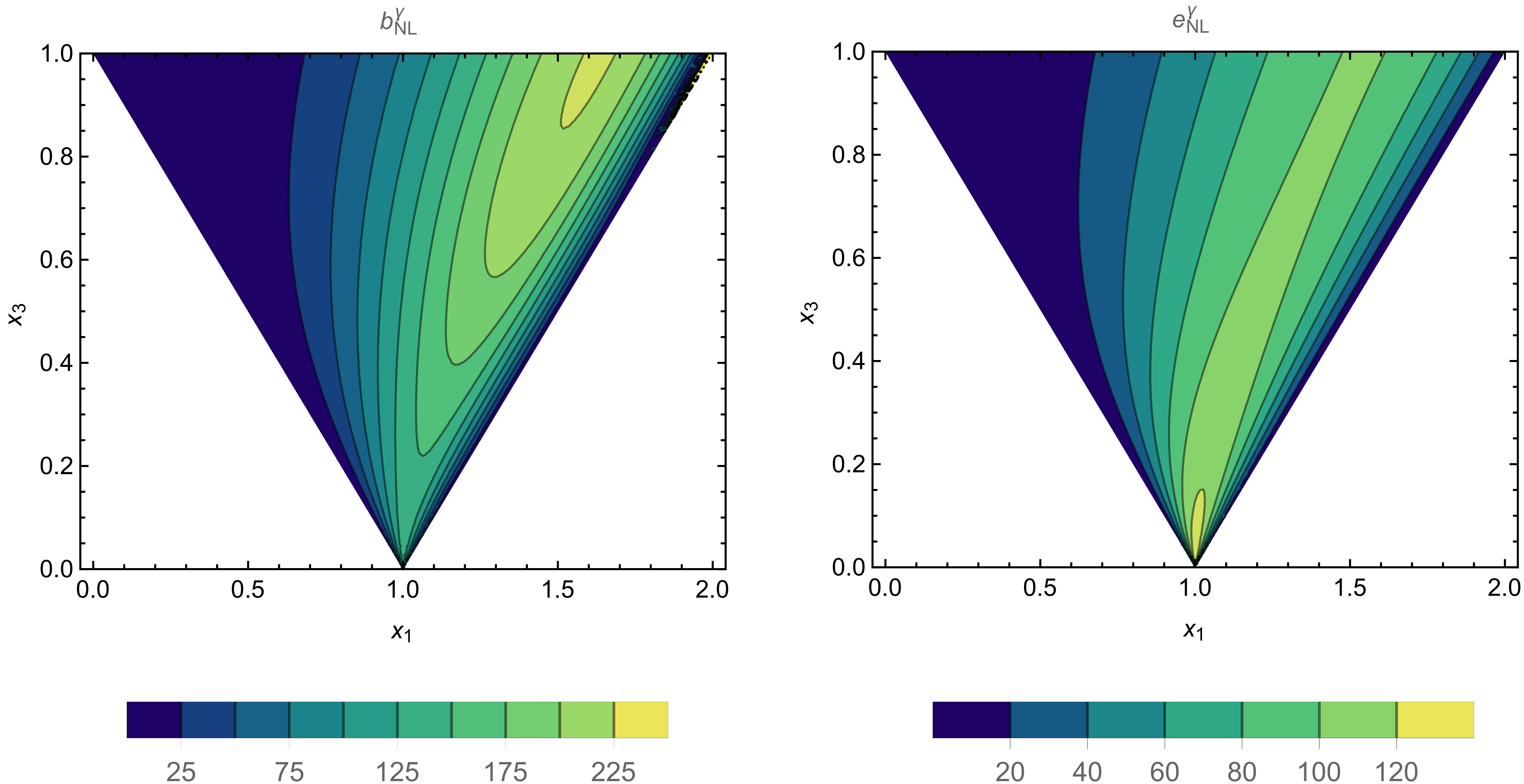
$$\lim_{k_1\to 0}\langle\gamma(\tau_I,\mathbf{k}_1)B_\mu(\tau_I,\mathbf{k}_2)B^\mu(\tau_I,\mathbf{k}_3)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\left(n-\frac{1}{2}\right)\epsilon_{ij}\frac{k_{2i}k_{2j}}{k_2^2}P_\gamma(k_1)P_B(k_2)$$

$$\lim_{k_1\to 0}\langle\gamma(\tau_I,\mathbf{k}_1)E_\mu(\tau_I,\mathbf{k}_2)E^\mu(\tau_I,\mathbf{k}_3)\rangle = -(2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\left(n+\frac{1}{2}\right)\epsilon_{ij}\frac{k_{2i}k_{2j}}{k_2^2}P_\gamma(k_1)P_E(k_2)$$

$$\begin{aligned} b_{NL}^{\gamma} &= \left(n-\frac{1}{2}\right)\epsilon_{ij}\frac{k_{2i}k_{2j}}{k_2^2}, \quad n > -1/2 \\ e_{NL}^{\gamma} &= -\left(n+\frac{1}{2}\right)\epsilon_{ij}\frac{k_{2i}k_{2j}}{k_2^2}, \quad n < 1/2 \end{aligned}$$

RKJ, Sai & Sloth, JCAP 03, 054 (2022)

Cross-correlations with gravitons



RKJ, Sai & Sloth, JCAP 03, 054 (2022)

Conclusions

- Dynamical gauge fields during inflation lead to a very rich phenomenology with interesting cosmological imprints.
- Interesting cross-correlations with curvature and tensor perturbations — new consistency relations — large contributions in specific shapes.
- Such correlations will also induce observable imprints in the CMB — need to explore further.

Thank you.