Quasi-maximum likelihood estimation of Primordial Non-Gaussianity

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arxiv:
Jung et al 2022 (2206.01624)
Coulton et al 2022 (2206.01619)

Non Gaussianity

Bispectrum: $\langle \delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2)\delta(\boldsymbol{k}_3)\rangle = (2\pi)^3 \delta_D(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) B_\delta(k_1, k_2, k_3)$

Primordial non-Gaussianity:

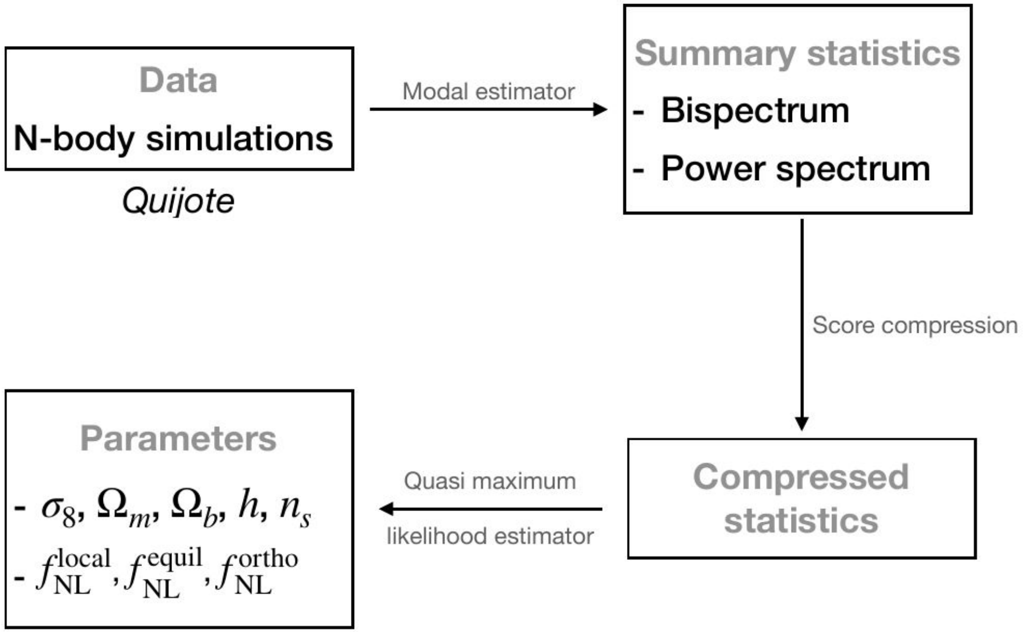
- Different shapes related to different models of inflation
  $\Rightarrow$ **Local, equilateral, orthogonal**
- Parametrized by their **amplitude parameter** $f_{\rm NL}$

Matter bispectrum

- Large non-Gaussian signal due to gravitational evolution
- Analytical predictions are hard to make, perturbation theory only works on large scales ($k_{\rm max} \simeq 0.2h/{\rm Mpc}$ at 1-loop)

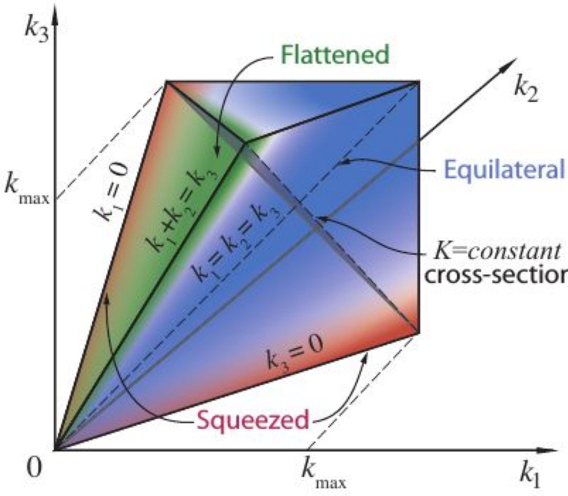
$\Rightarrow$ To probe nonlinear scales, **simulation-based inference** is the solution!

Pipeline


Data
N-body simulations
Quijote
Modal estimator
Summary statistics
- Bispectrum
- Power spectrum
Score compression
Compressed statistics
Quasi maximum
likelihood estimator
Parameters
- σ_8, Ω_m, Ω_b, h, n_s
- $f_{\mathrm{NL}}^{\mathrm{local}}, f_{\mathrm{NL}}^{\mathrm{equil}}, f_{\mathrm{NL}}^{\mathrm{ortho}}$

Data ⟹ Summary statistics

- **Power spectrum** $$\hat{P}(k_i) = \frac{1}{VN_i}\sum_{\mathbf{k}\in\Delta_i}\delta(\mathbf{k})\delta^*(\mathbf{k})$$
- **Bispectrum modes** $$w(k_1,k_2,k_3)B(k_1,k_2,k_3) = \sum_n^N \beta_n Q_n(k_1,k_2,k_3)$$



Separable modal basis: $Q_n(k_1,k_2,k_3) = q_r(k_1)q_s(k_2)q_t(k_3)$ + perms, with $n \equiv \{r,s,t\}$

Only ~100 well chosen modes to describe the bispectrum up to $k_{\max} = 0.5h/\mathrm{Mpc}$

Method originally developed for CMB
- *Fergusson, Liguori & Shellard* (0912.3411)
- Planck NG (1905.05697)

Later implemented in the LSS context
- *Schmittfull, Regan & Shellard* (1207.5678)
- *Hung, Fergusson & Shellard* (1902.01830)
- *Byun, Oddo, Porciani & Sefusatti* (2010.09579)

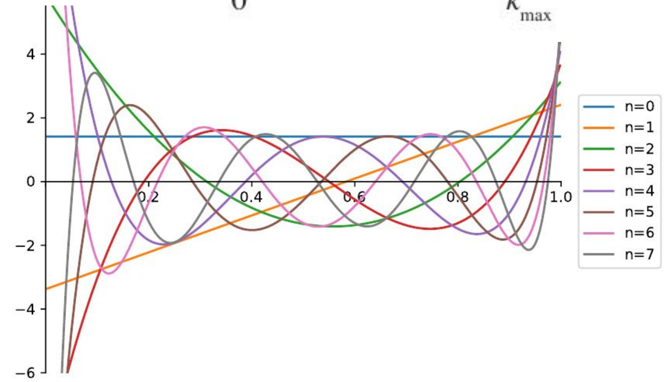
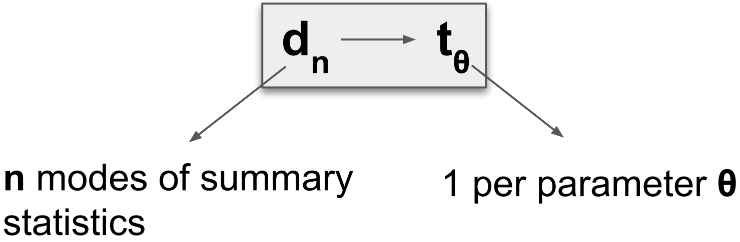


Figure 6.2 – The first eight orthogonal one dimensional polynomials $q_n(x)$ defined on the tetrahedral. They are well defined and well behaved within the boundaries of the domain.

Summary statistics ⟹ Compressed statistics

- **Compression to the score function (J. Alsing, B. Wandelt ; 1712.00012):**



1. Assume an approximate form for the likelihood of the summary statistics [compression equivalent to MOPED astro-ph/9911102 (A. Heavens, R. Jimenez & O. Lahav)]
2. Define the compressed statistics **t** to be the **score-function** – the parameter gradient of the log-likelihood – evaluated at some fiducial parameter set θ_*.

$$\mathbf{t} = \nabla_\theta \boldsymbol{\mu}_*^{\mathrm{T}} \mathbf{C}_*^{-1} (\mathbf{d} - \boldsymbol{\mu}_*)$$

Summary statistics ⟹ Compressed statistics

$$\mathbf{t} = \nabla_{\boldsymbol{\theta}} \boldsymbol{\mu}_*^{\mathrm{T}} \mathbf{C}_*^{-1} (\mathbf{d} - \boldsymbol{\mu}_*)$$

- **μ** and **C** are the mean and the covariance of **d**, evaluated at the fiducial cosmology from simulations.
- The **derivative** is computed from simulations with a step Δθ away from fiducial $\boldsymbol{\theta}_{\mathrm{fid}}$

$$\frac{\partial \mathrm{d}_n}{\partial \theta} \simeq \frac{\mathrm{d}_n(\theta_{\mathrm{fid}} + \Delta\theta) - \mathrm{d}_n(\theta_{\mathrm{fid}} - \Delta\theta)}{2\Delta\theta}$$

1. This compression results in just n numbers, i.e. the number of parameters we aim to infer from data.
2. Compression to the score is **optimal** in the sense that it saturates the information inequality.
3. Covariance does not depend on parameters θ (score-function is equivalent to MOPED).
4. By construction the covariance of **t** is $\mathbf{F} = \nabla_{\boldsymbol{\theta}} \boldsymbol{\mu}^{\mathrm{T}} \mathbf{C}^{-1} \nabla_{\boldsymbol{\theta}}^{\mathrm{T}} \boldsymbol{\mu}.$ computed at fiducial point $\boldsymbol{\theta}_{\mathrm{fid}}$

The quasi-ML estimator of θ is thus:

$$\hat{\boldsymbol{\theta}} = \boldsymbol{\theta}_* + \mathbf{F}_*^{-1} \mathbf{t}$$

Data: N-body simulations

- **Quijote simulations** ($f_{\rm NL} = 0$)

  https://quijote-simulations.readthedocs.io (F. Villaescusa-Navarro)

  Large suite (~44000) of N-body simulations with 512^3 particles and a size of 1 Gpc/h

  ⇒ 8000 simulations at fiducial cosmology to estimate covariances

  ⇒ Sets of 500 simulations for numerical derivatives $(\sigma_8, \Omega_m, \Omega_b, h, n_s)$

- **Non-Gaussian Quijote-like (** $f_{\rm NL} = \pm 100$ **)**

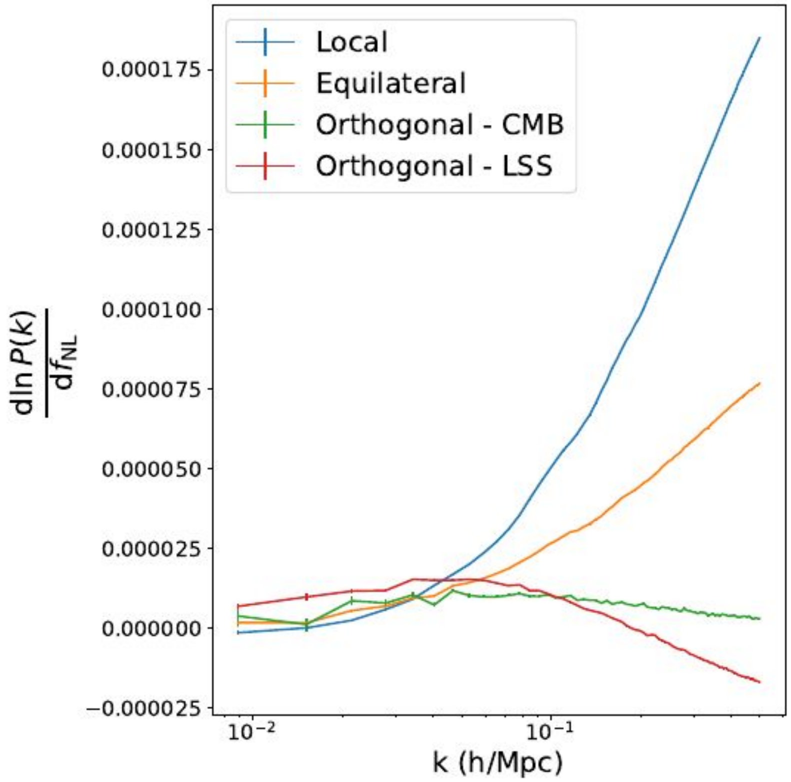
  ⇒ Sets of 500 simulations for three primordial shapes: local, equilateral and orthogonal

  ⇒ Numerical derivatives $(f_{\rm NL}^{\rm local}, f_{\rm NL}^{\rm equil}, f_{\rm NL}^{\rm ortho})$

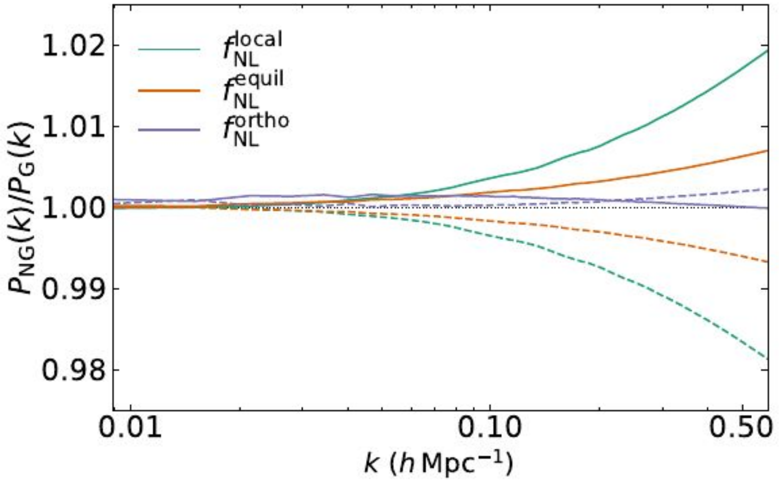
Table 1. The cosmological parameters and PNG amplitudes of the QUIJOTE and QUIJOTE-PNG simulations.

	σ_8	Ω_m	Ω_b	n_s	h	$f_{\rm NL}^{\rm local}$	$f_{\rm NL}^{\rm equil}$	$f_{\rm NL}^{\rm ortho}$
Fiducial	0.834	0.3175	0.049	0.9624	0.6711	0	0	0
Steps	±0.015	±0.01	±0.002	±0.02	±0.02	±100	±100	±100

Quijote-PNG: Power spectrum

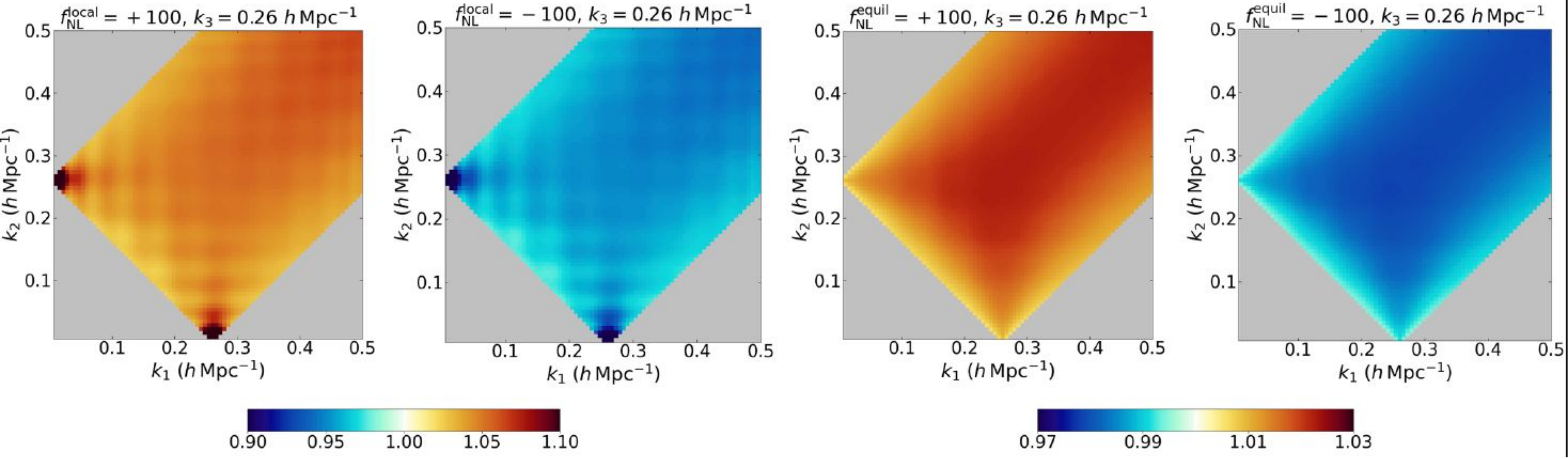


(b) Matter power spectrum at $z = 0.0$



Coulton et al 2022
(2206.01619)

Quijote-PNG: Bispectrum ratio NG/G



Constraints

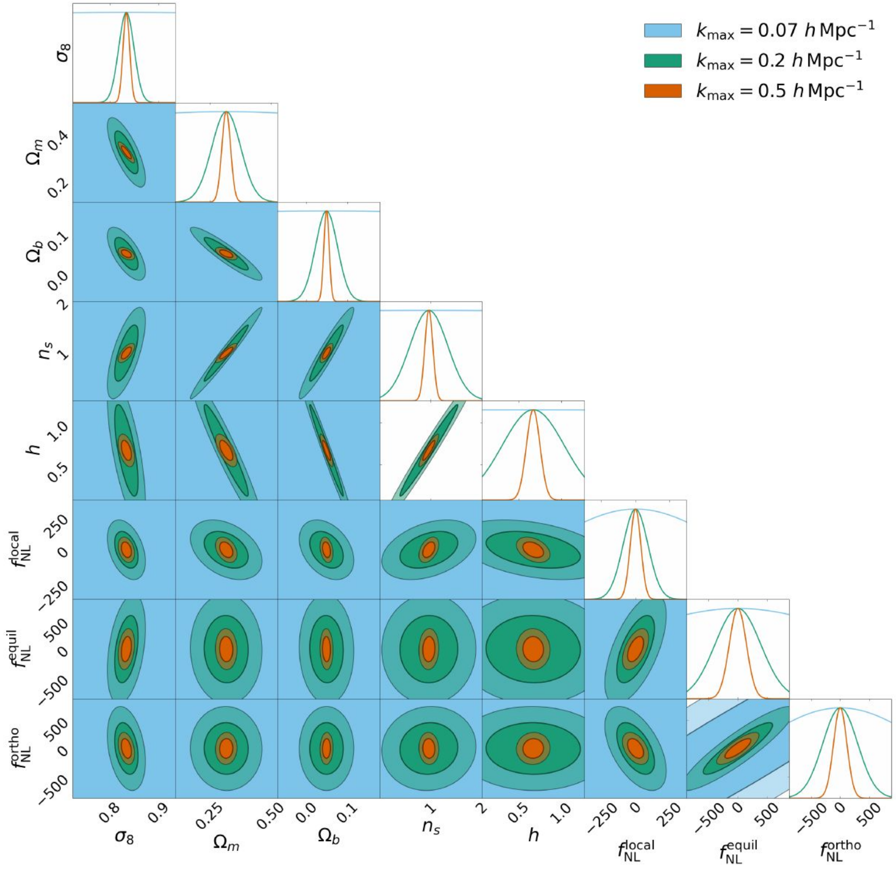
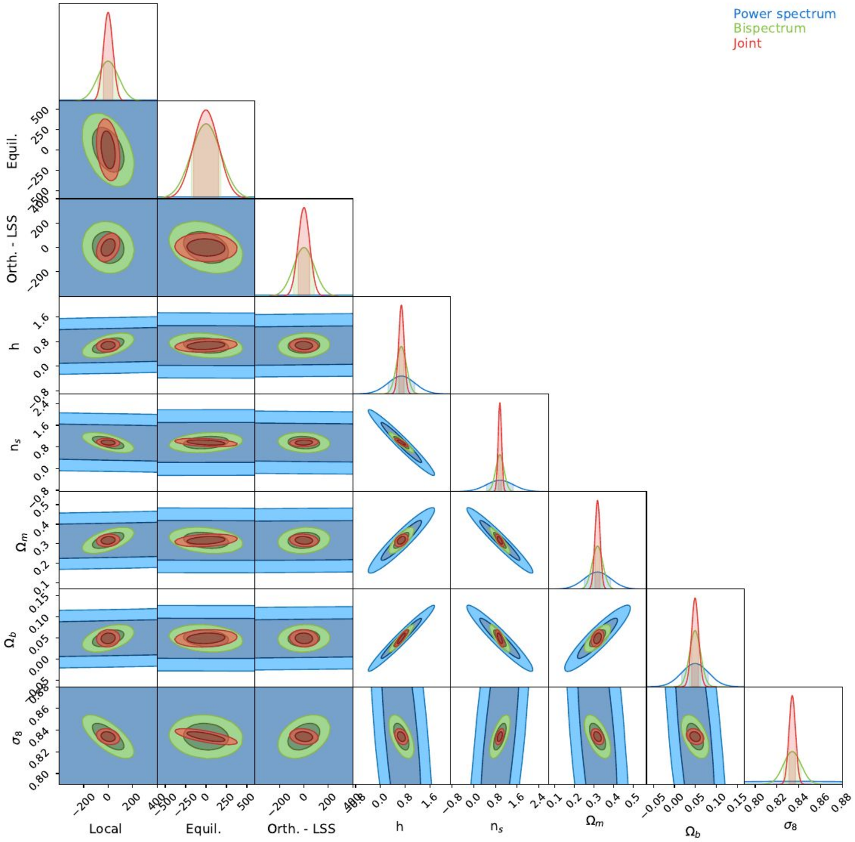
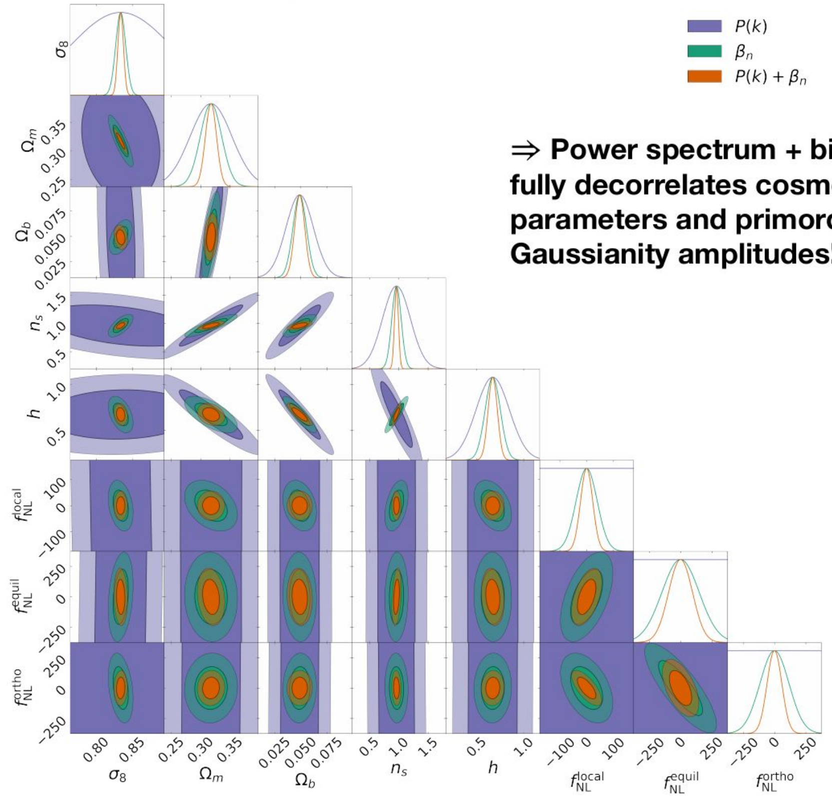


Figure 6. Joint constraints on cosmological parameters and PNG from the power spectrum and the modal bispectrum for different $k_{\max}$ at $z = 1$.

Constraints



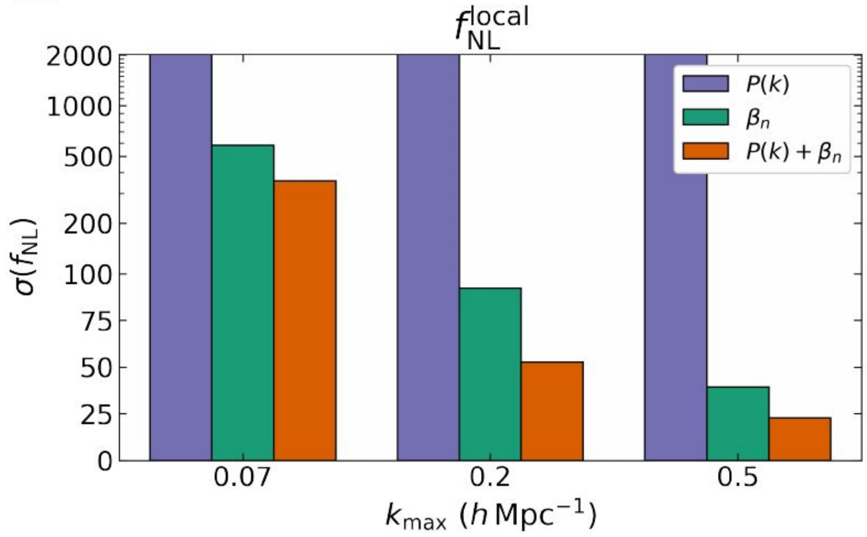


⇒ Power spectrum + bispectrum fully decorrelates cosmological parameters and primordial non-Gaussianity amplitudes!

Coulton et al 2022 (**2206.01619**)

Constraints

- **Constraints on $f_{\rm NL}^{\rm local}$:**



- **Nonlinear regime ($k_{\rm max} = 0.5h/{\rm Mpc}$ here) improves the constraints significantly**
- **Combining power spectrum and bispectrum also improves the constraints by a factor ~2, even if the power spectrum constraining power is very small**

Constraints

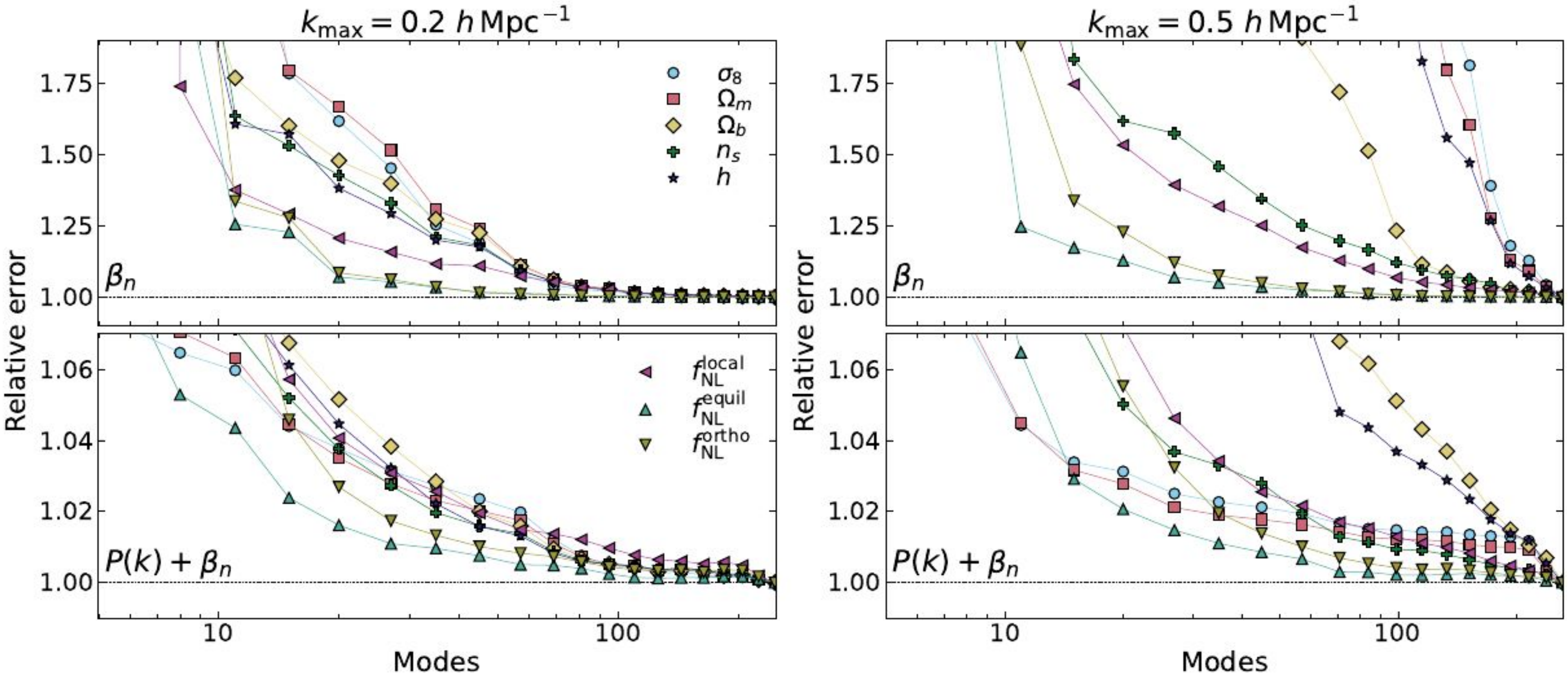
Table 2. Joint constraints on cosmological parameters and PNG from the power spectrum and the modal bispectrum at $z = 1$, for different $k_{\max}$. We analyzed 8000 QUIJOTE N-body simulations of 1 $(\mathrm{Gpc}/h)^3$ volume at fiducial cosmology, and sets of 500 N-body simulations with one adjusted parameter.

	$k_{\max}$ ($h\,\mathrm{Mpc}^{-1}$)	σ_8 0.834	Ω_m 0.3175	Ω_b 0.049	n_s 0.9624	h 0.6711	$f_{\mathrm{NL}}^{\mathrm{local}}$ 0	$f_{\mathrm{NL}}^{\mathrm{equil}}$ 0	$f_{\mathrm{NL}}^{\mathrm{ortho}}$ 0
$P(k)$	**0.07**	±0.17	±0.32	±0.32	±3.9	±4.4			
	0.2	±0.012	±0.039	±0.018	±0.24	±0.24			
	0.5	±0.0045	±0.011	±0.0062	±0.042	±0.062			
β_n	**0.07**	±0.59	±0.95	±1.	±12	±14			
	0.2	±0.014	±0.051	±0.023	±0.32	±0.31			
	0.5	±0.0063	±0.016	±0.006	±0.066	±0.071			
$P(k)+\beta_n$	**0.07**	±0.17	±0.31	±0.31	±3.7	±4.2			
	0.2	±0.011	±0.035	±0.015	±0.21	±0.21			
	0.5	±0.0038	±0.0091	±0.0048	±0.031	±0.048			
$P(k)$	**0.07**	±0.55	±0.41	±0.36	±4.5	±4.9	±200000	±500000	±180000
	0.2	±0.13	±0.07	±0.031	±0.49	±0.39	±31000	±85000	±36000
	0.5	±0.069	±0.035	±0.013	±0.24	±0.18	±9600	±29000	±11000
β_n	**0.07**	±1.1	±1.3	±1.4	±18	±20	±670	±2300	±1500
	0.2	±0.016	±0.059	±0.027	±0.37	±0.36	±91	±390	±300
	0.5	±0.0068	±0.018	±0.0064	±0.078	±0.079	±39	±150	±110
$P(k)+\beta_n$	**0.07**	±0.17	±0.31	±0.31	±3.7	±4.2	±350	±930	±610
	0.2	±0.011	±0.035	±0.015	±0.21	±0.21	±52	±170	±120
	0.5	±0.0038	±0.0093	±0.0048	±0.033	±0.048	±22	±94	±58

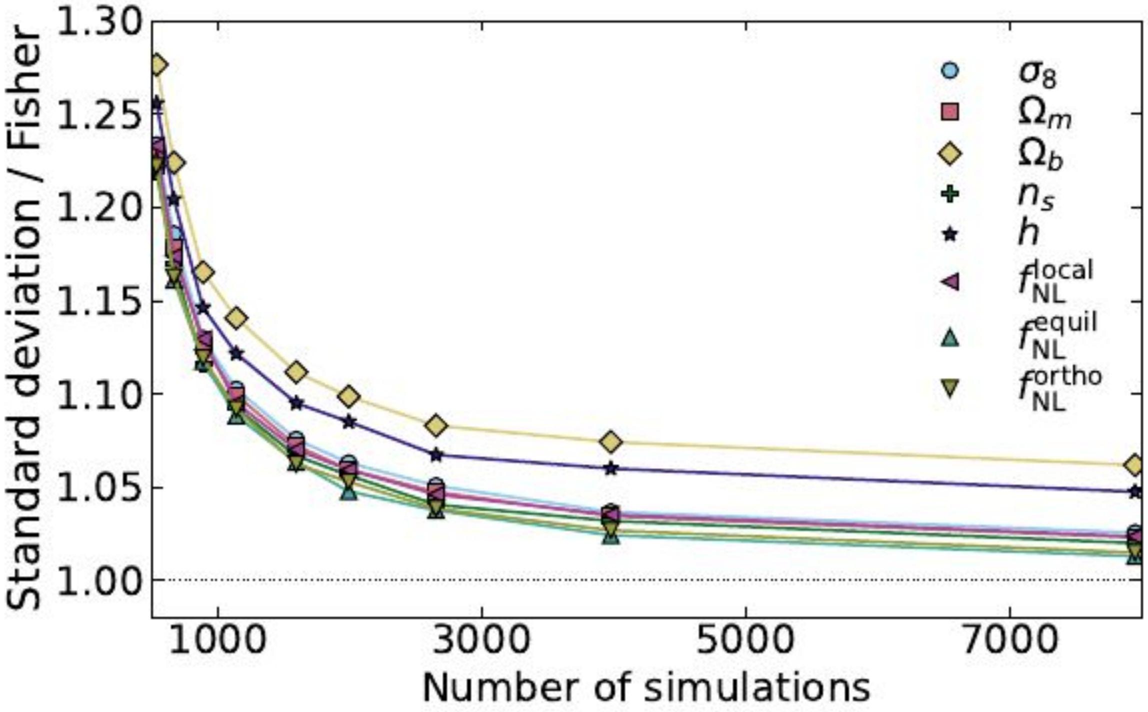
Independent constraints:

$f_{\mathrm{NL}}^{\mathrm{local}}$	$f_{\mathrm{NL}}^{\mathrm{equil}}$	$f_{\mathrm{NL}}^{\mathrm{ortho}}$
±16	±77	±40

Quijote-PNG: modal bispectrum convergence



Quasi-optimality of the estimator



Summary

Conclusions:

- We prove the capability of our approach to optimally extract PNG information on non-linear scales beyond the perturbative regime!!
- The overall estimation procedure is unbiased and fast.
- The Gaussian likelihood assumption only leads to sub-optimality. This can be improved with neural networks.
- The dark matter power spectrum itself contains negligible PNG information, as expected, including it as an ancillary statistic increases the PNG information content extracted from the bispectrum by a factor of order 2.

Future work:

- Extend the pipeline to include: biased tracers (dark matter halos and galaxies).
- This has already been done in Coulton et al 2022 (arXiv:2206.15450), while the accompanying paper with modal estimator and score function is coming soon (Jung et al 2022, in prep.)
- The halo PNG Quijote catalogues are public now (https://quijote-simulations.readthedocs.io)!!
- Account for redshift space and incomplete sky-coverage