Model independent approach to galaxy clustering

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TECHNION
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Standard perturbation theory

Dark matter: non relativistic, non interacting, subject to gravitational interaction

$$\frac{\partial \delta(\mathbf{x},\tau)}{\partial \tau} + \nabla_{\mathbf{x}}\Big[\big(1+\delta(\mathbf{x},\tau)\big)\,\mathbf{v}(\mathbf{x},\tau)\Big] = 0$$ Continuity equation

$$\frac{\partial \mathbf{v}(\mathbf{x},\tau)}{\partial \tau} + \mathscr{H}(\tau)\mathbf{v}(\mathbf{x},\tau) + \big(\mathbf{v}\cdot\nabla_{\mathbf{x}}\big)\,\mathbf{v}(\mathbf{x},\tau) + \nabla_{\mathbf{x}}\phi(\mathbf{x},\tau) = \frac{1}{\rho}\nabla_{\mathbf{x}}\tau$$

Euler equation

$$\nabla^2_{\mathbf{x}}\phi(\mathbf{x},\tau) = \frac{3}{2}\Omega_m(\tau)\mathscr{H}^2(\tau)\delta(\mathbf{x},\tau)$$ Poisson equation

Standard perturbation theory

Observational effects

From dark matter to galaxies: bias, redshift space, small scale effects

$$\delta(\mathbf{k},\tau) \longrightarrow \delta_g^{\mathrm{rs}}(\mathbf{k},\tau) = \mathscr{F}[\delta, \partial_i\partial_j\phi, \ldots](\mathbf{k},\tau)$$

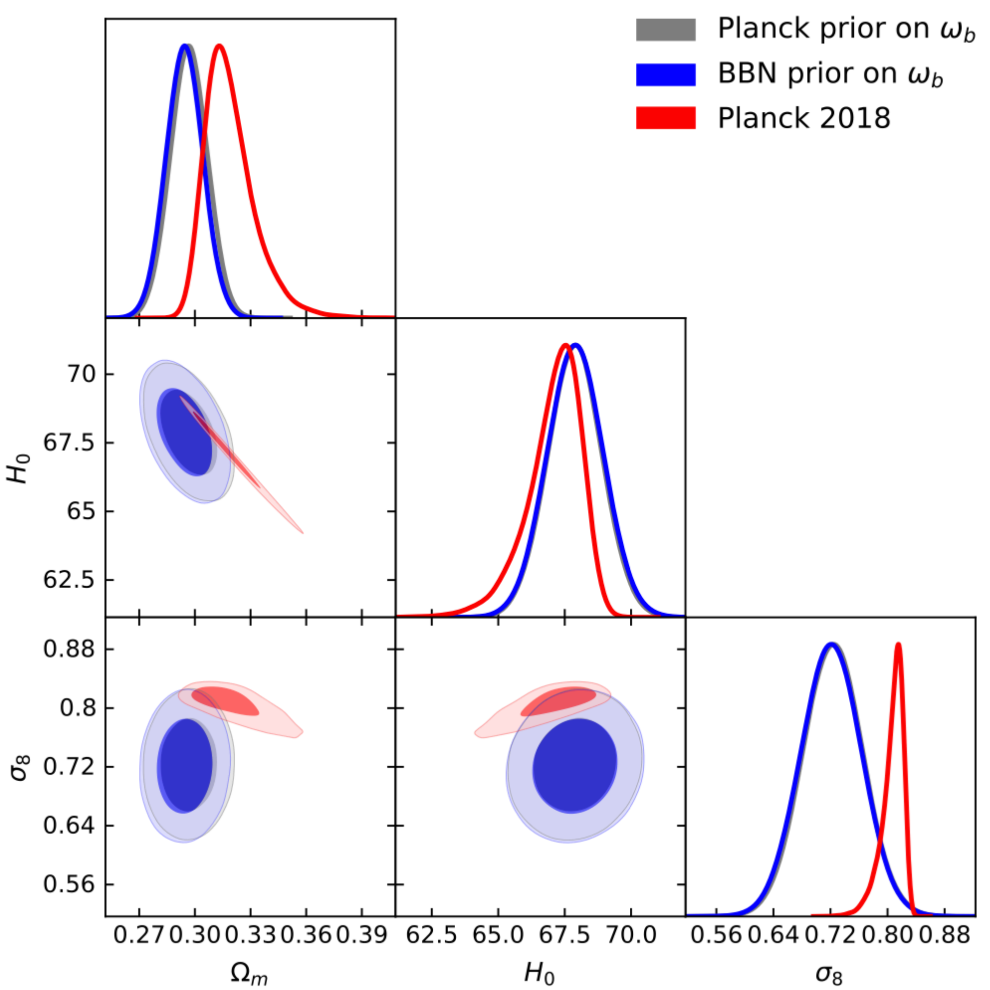
Perturbative solution

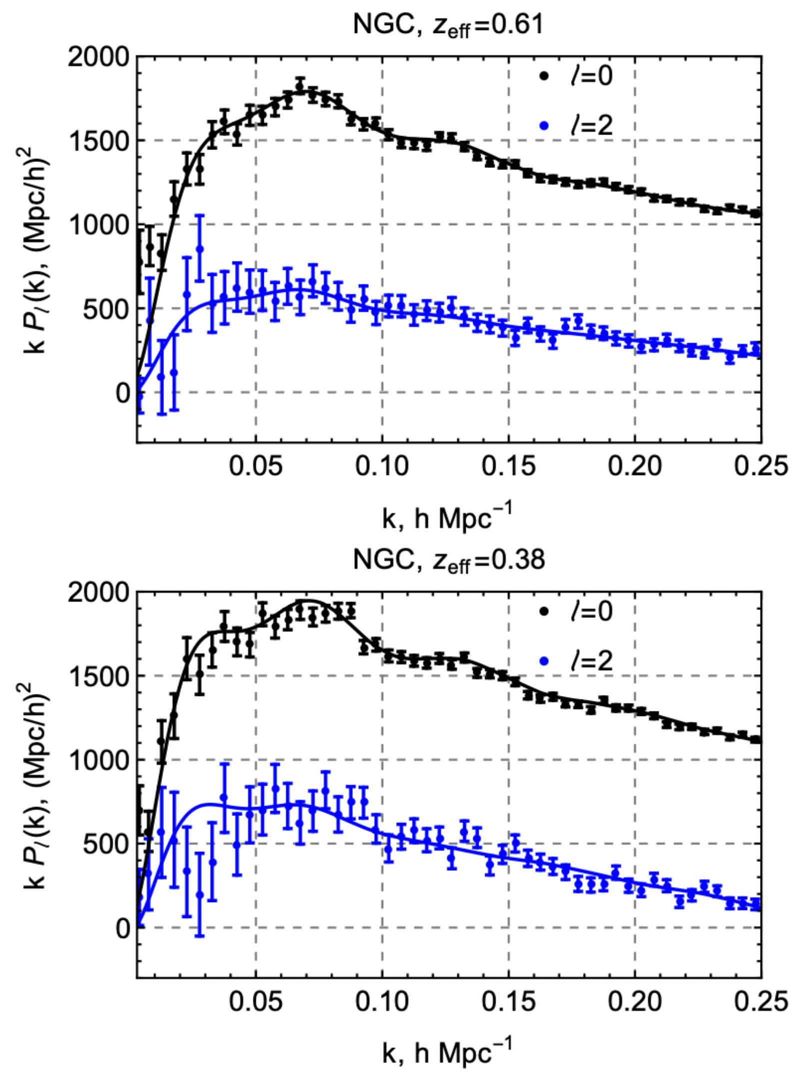
$$\delta_g^{\mathrm{rs},(n)}(\mathbf{k}) = \int \frac{d^3q_1}{(2\pi)^3}\ldots\int \frac{d^3q_n}{(2\pi)^3}\delta_D(\mathbf{k}-\mathbf{q}_{1\ldots n})Z_n\left(\mathbf{q}_1,\ldots,\mathbf{q}_n\right)\delta^{(1)}(\mathbf{q_1})\ldots\delta^{(1)}(\mathbf{q_n})$$

$$\delta_g^{\mathrm{rs}}(\mathbf{k}) = \delta_g^{\mathrm{rs,\,PT}}(\mathbf{k}) + \delta_g^{\mathrm{rs,\,SN}}(\mathbf{k}) + \delta_g^{\mathrm{rs,\,CT}}(\mathbf{k})$$

Standard perturbation theory

Results

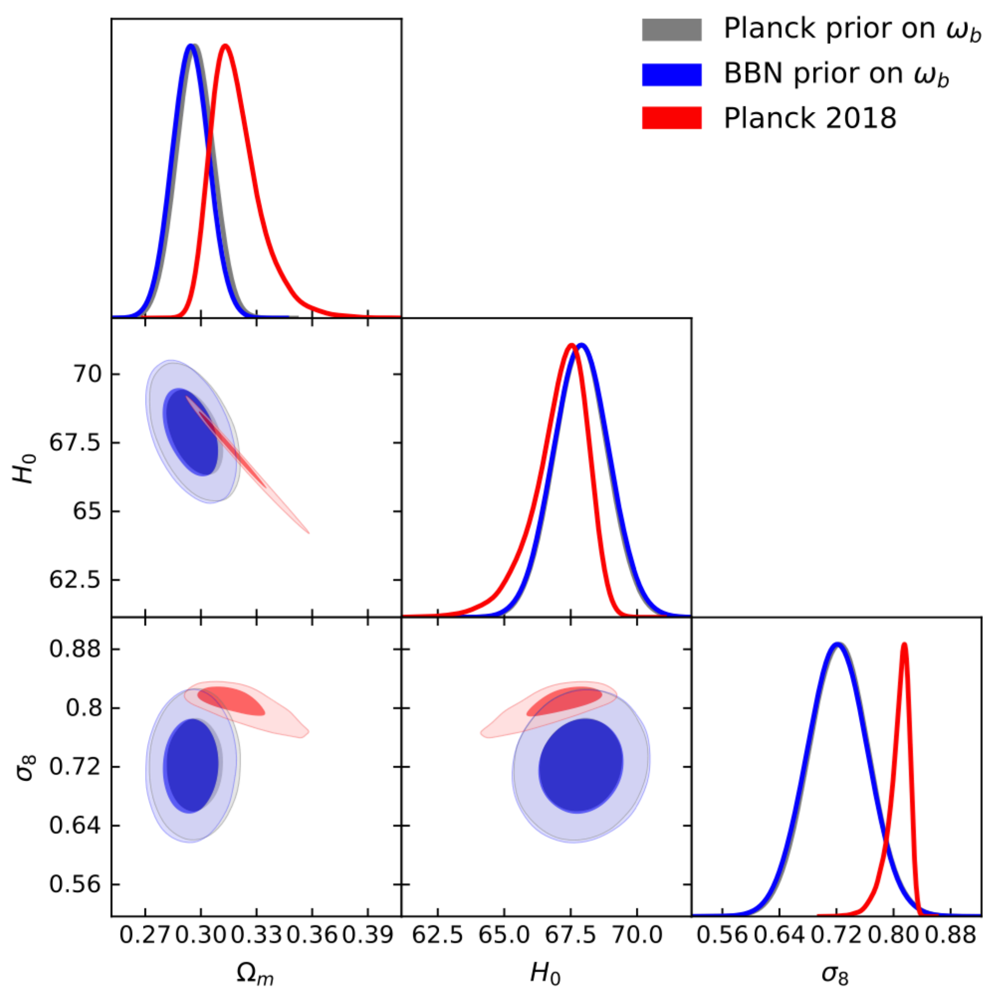


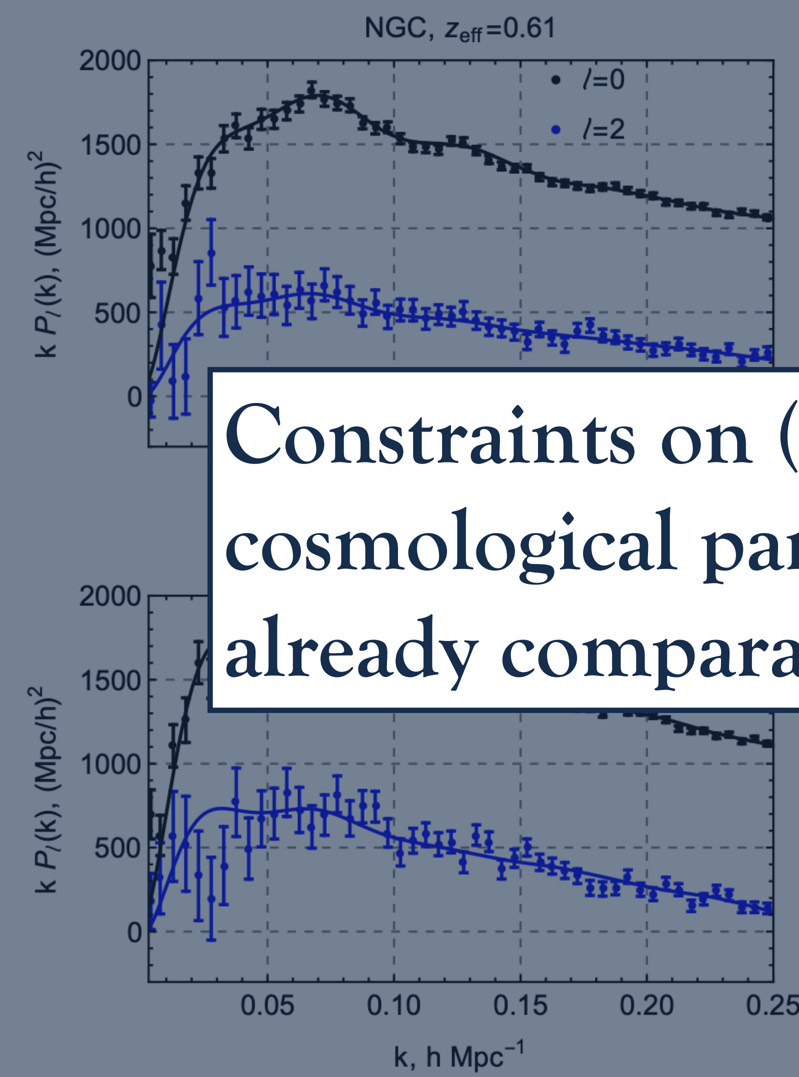


D'Amico et al. 1909.05271
Ivanov et al. 1909.05277
Colas et al. 1909.07951
Chen et al. 2110.05530

Standard perturbation theory

Results





D'Amico et al. 1909.05271

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Chen et al. 2110.05530

Constraints on (some) cosmological parameters already comparable with Planck

Role of symmetries in cosmology

Role of symmetries in cosmology

Equivalence principle

Model independent measurement using the EP in the BAO range

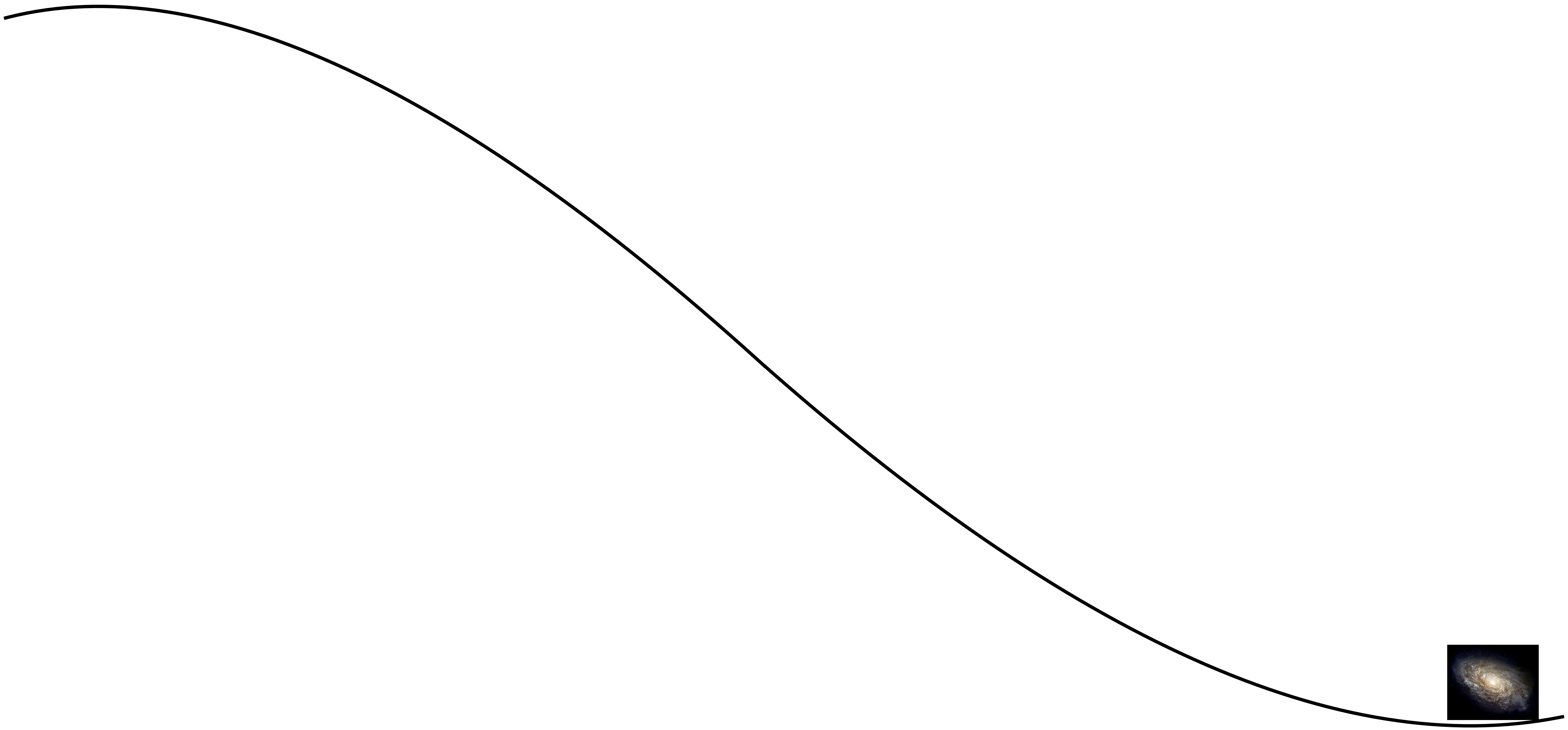
- **MM**, Nishimichi T., Pietroni M., Phys. Rev. D100 (2019)
- **MM**, Nishimichi T., Pietroni M., JCAP 07 (2020)

Constraints the perturbative kernels in a model independent way: **Large Scale Structure Bootstrap**!

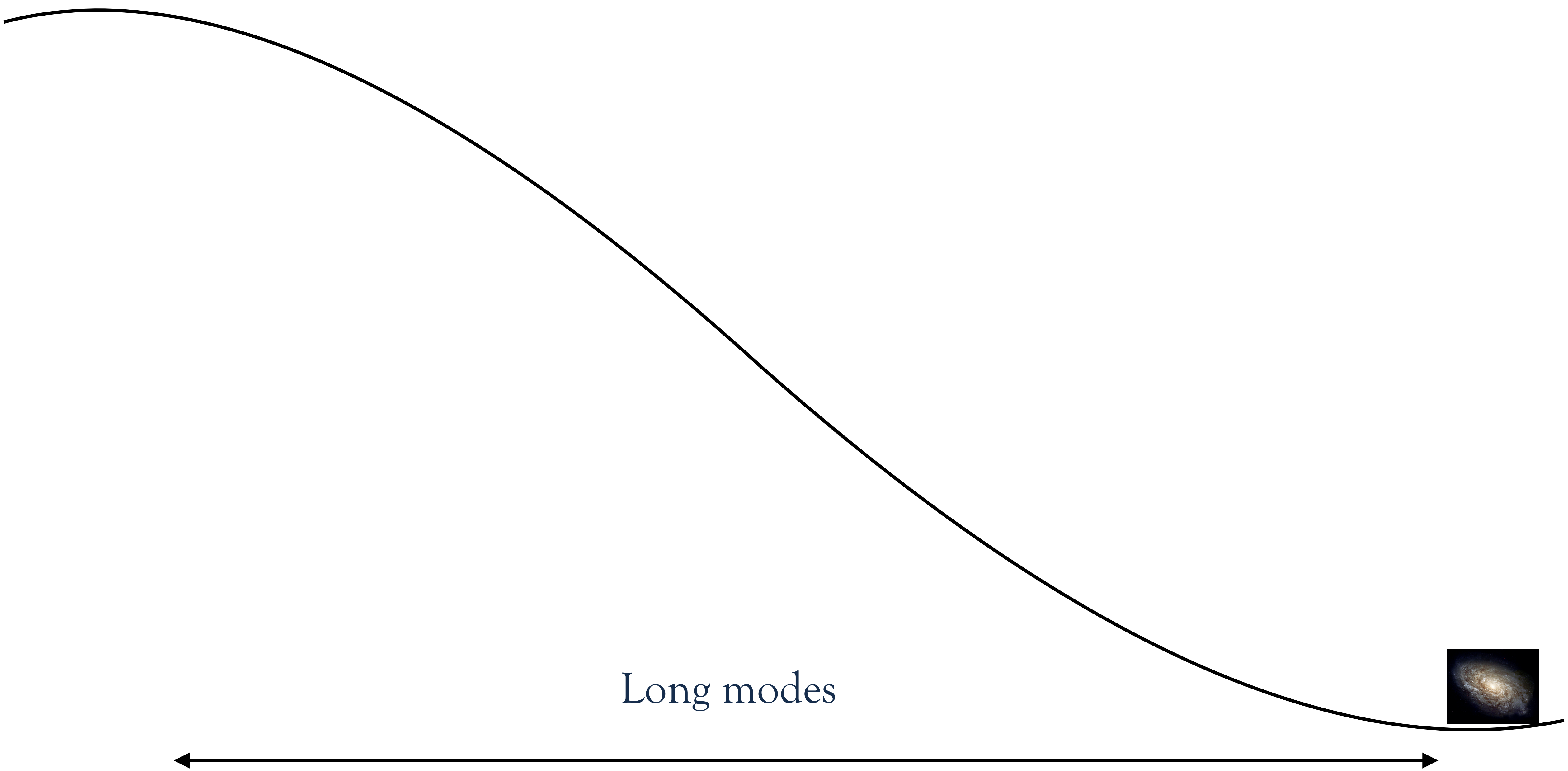
- D'Amico G., **MM**, Pietroni M., Vernizzi F., JCAP 10 (2021)

Equivalence principle

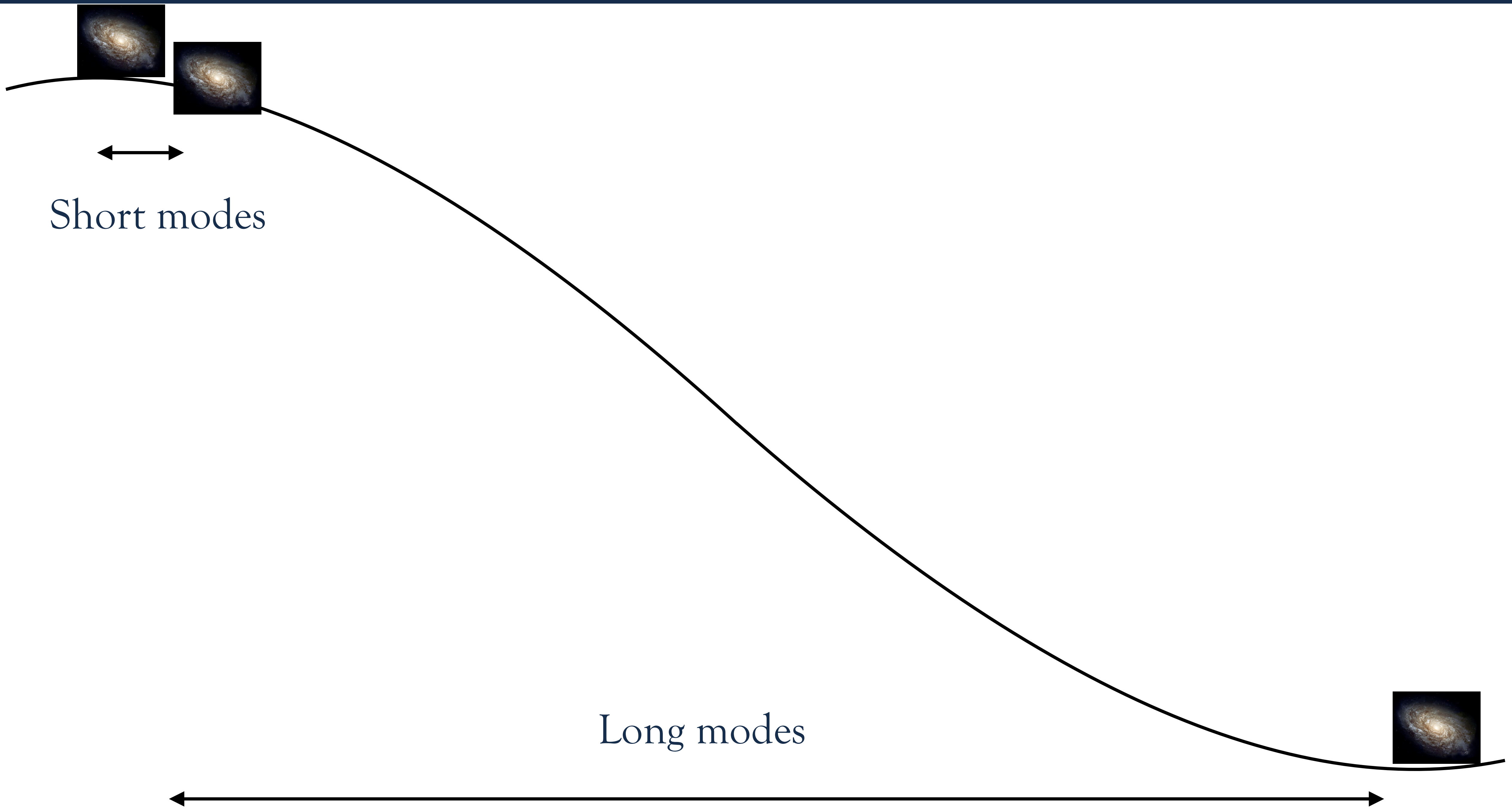
Equivalence principle



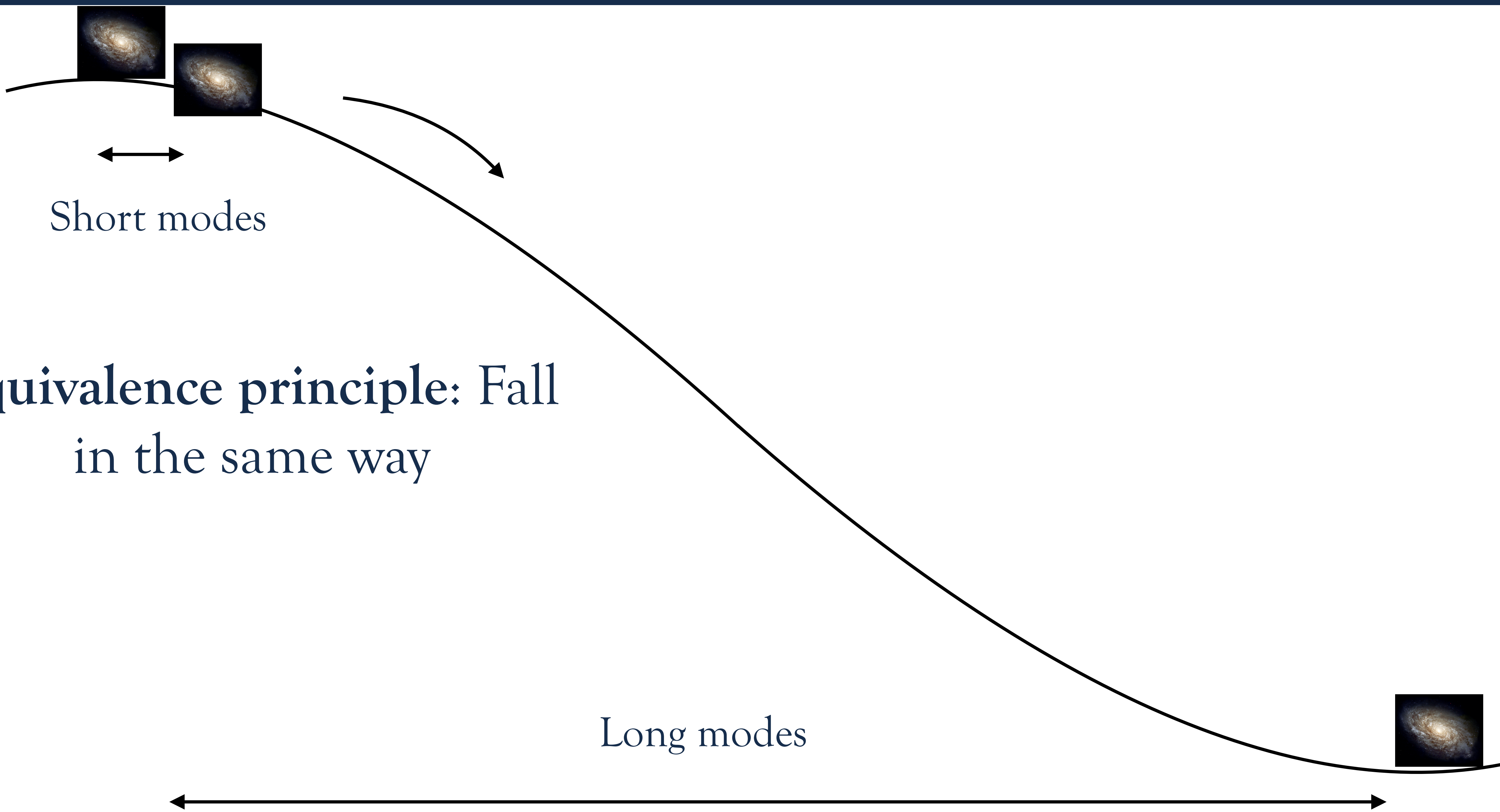
Equivalence principle



Equivalence principle



Equivalence principle



Equivalence principle: Fall in the same way

Equivalence principle

Consistency relations

Exact equalities among correlation functions of different order
(Kehagias A., Riotto A., Nucl.Phys. 2013,
Peloso M., Pietroni M., JCAP 2013,
Creminelli P., Noreña J., Simonović M., Vernizzi F., JCAP 2013)

$$\langle \delta(\mathbf{x}_1, \tau_1) \dots \delta(\mathbf{x}_n, \tau_n) \,|\, \Phi_L \rangle = \langle \delta(\tilde{\mathbf{x}}_1, \tilde{\tau}_1) \dots \delta(\tilde{\mathbf{x}}_n, \tilde{\tau}_n) \rangle \,.$$

Effect of long modes on linear scales
$\longrightarrow$ **Equivalence Principle** (or Galilean Invariance)

$$\langle \delta_m(\mathbf{q}, \tau) \delta_g(\mathbf{k}_1, \tau_1) \dots \delta_g(\mathbf{k}_n, \tau_n) \rangle'_{\mathbf{q}\to 0} = - P_L(q, \tau) \sum_{i=1}^{n} \frac{D_+(\tau_i)}{D(\tau)} \frac{\mathbf{q}\cdot\mathbf{k}_i}{q^2} \langle \delta_g(\mathbf{k}_1, \tau_1) \dots \delta_g(\mathbf{k}_n, \tau_n) \rangle'$$

LSS Bootstrap

- D'Amico G., **MM**, Pietroni M., Vernizzi F., JCAP 10 (2021)

LSS Bootstrap

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$$\delta_g^{(n)}(\mathbf{k}) = \int \frac{d^3 q_1}{(2\pi)^3} \dots \int \frac{d^3 q_n}{(2\pi)^3} K_n\left(\mathbf{q}_1, \dots, \mathbf{q}_n\right) \delta^{(1)}(\mathbf{q_1}) \dots \delta^{(1)}(\mathbf{q_n})$$

LSS Bootstrap

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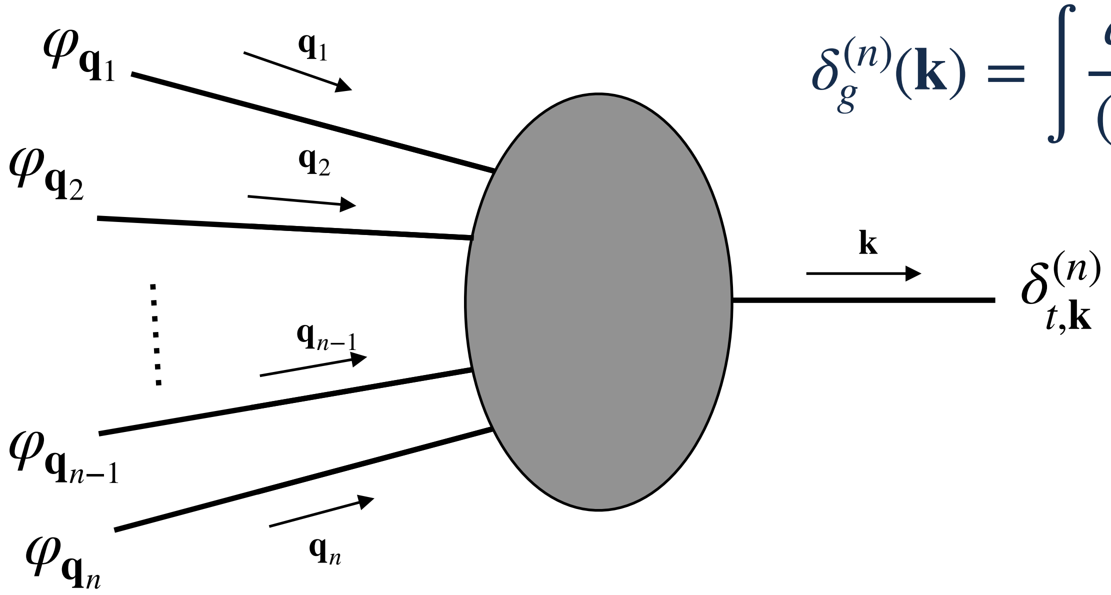

$$\delta_g^{(n)}(\mathbf{k}) = \int \frac{d^3q_1}{(2\pi)^3} \ldots \int \frac{d^3q_n}{(2\pi)^3} K_n\left(\mathbf{q}_1, \ldots, \mathbf{q}_n\right) \delta^{(1)}(\mathbf{q_1}) \ldots \delta^{(1)}(\mathbf{q_n})$$

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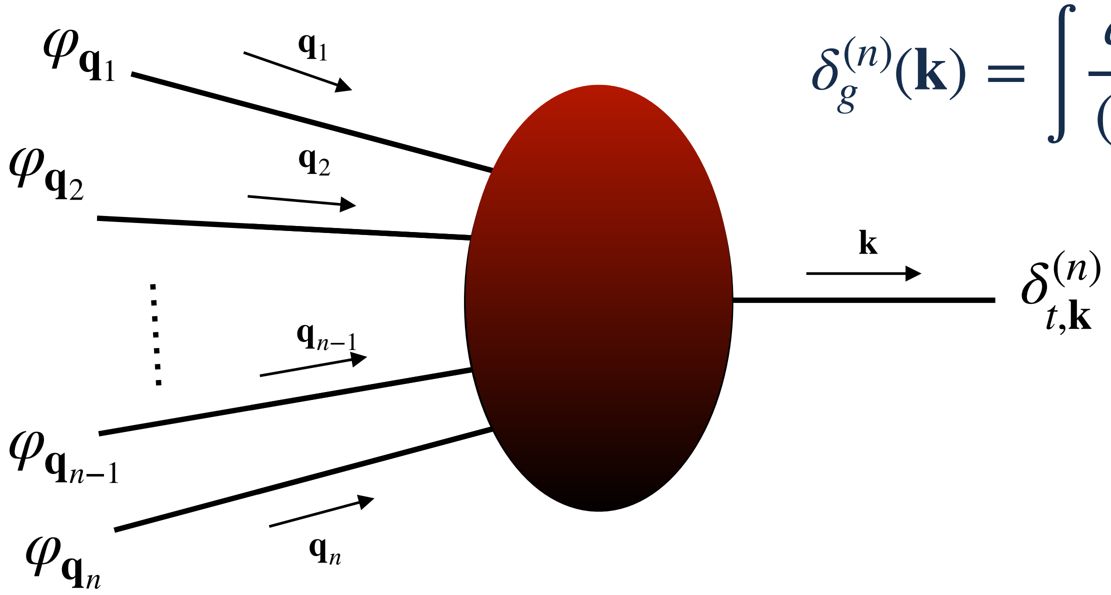

$$\delta_g^{(n)}(\mathbf{k}) = \int \frac{d^3q_1}{(2\pi)^3}\dots\int \frac{d^3q_n}{(2\pi)^3} K_n\left(\mathbf{q}_1,\dots,\mathbf{q}_n\right)\delta^{(1)}(\mathbf{q_1})\dots\delta^{(1)}(\mathbf{q_n})$$

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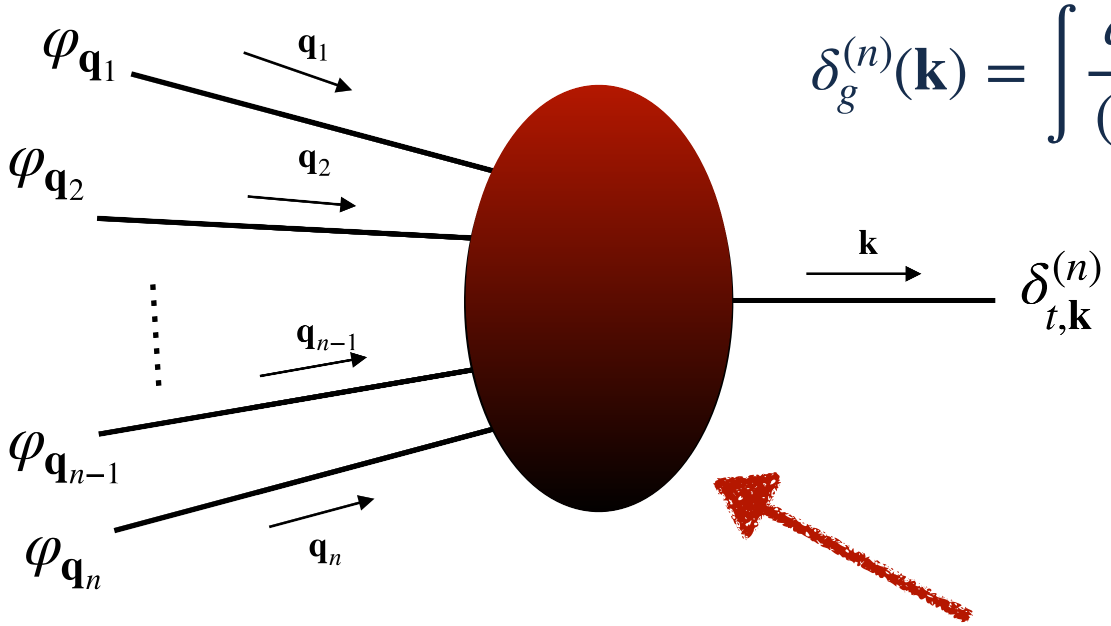


SYMMETRIES

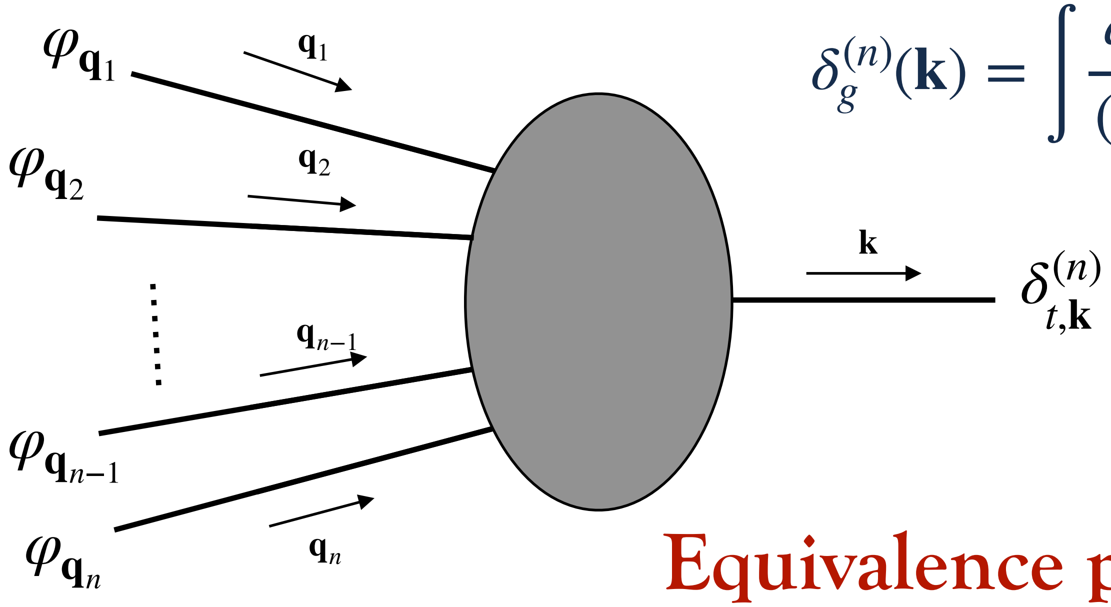
Homogeneity and isotropy

Mass and momentum conservation (only for dark matter)

Equivalence principle

LSS Bootstrap

- D'Amico G., **MM**, Pietroni M., Vernizzi F., JCAP 10 (2021)



$$\delta_g^{(n)}(\mathbf{k}) = \int \frac{d^3q_1}{(2\pi)^3}\dots\int\frac{d^3q_n}{(2\pi)^3}K_n\left(\mathbf{q}_1,\dots,\mathbf{q}_n\right)\delta^{(1)}(\mathbf{q_1})\dots\delta^{(1)}(\mathbf{q_n})$$

Equivalence principle

Leading Order: single momentum going $\to 0$

LSS Bootstrap

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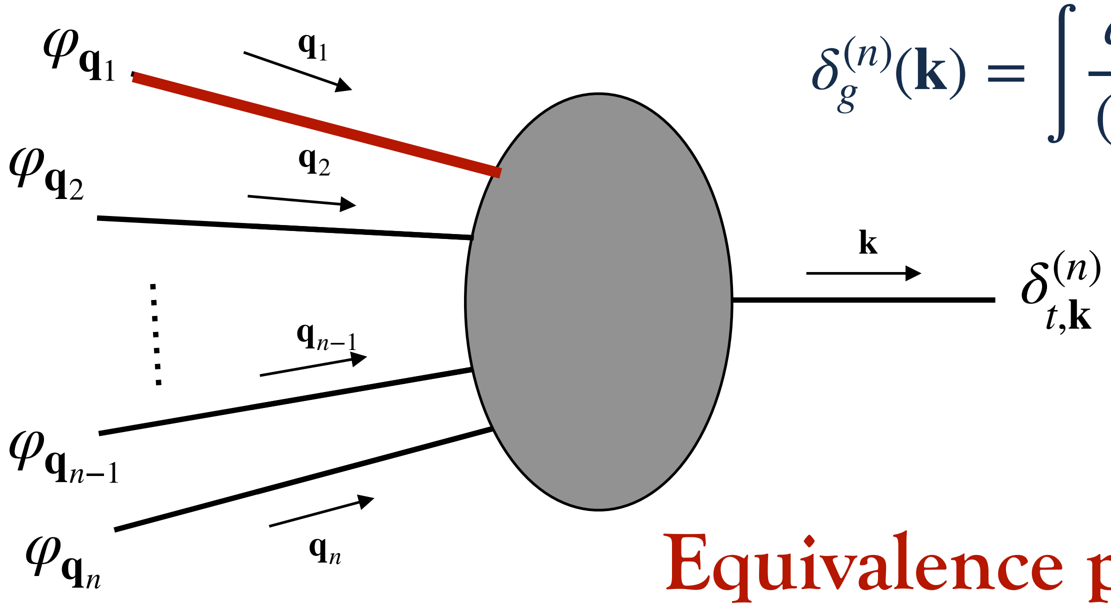

$$\delta^{(n)}_g(\mathbf{k}) = \int \frac{d^3q_1}{(2\pi)^3} \dots \int \frac{d^3q_n}{(2\pi)^3} K_n\left(\mathbf{q}_1, \dots, \mathbf{q}_n\right) \delta^{(1)}(\mathbf{q_1}) \dots \delta^{(1)}(\mathbf{q_n})$$

Equivalence principle

Leading Order: single momentum going $\to 0$

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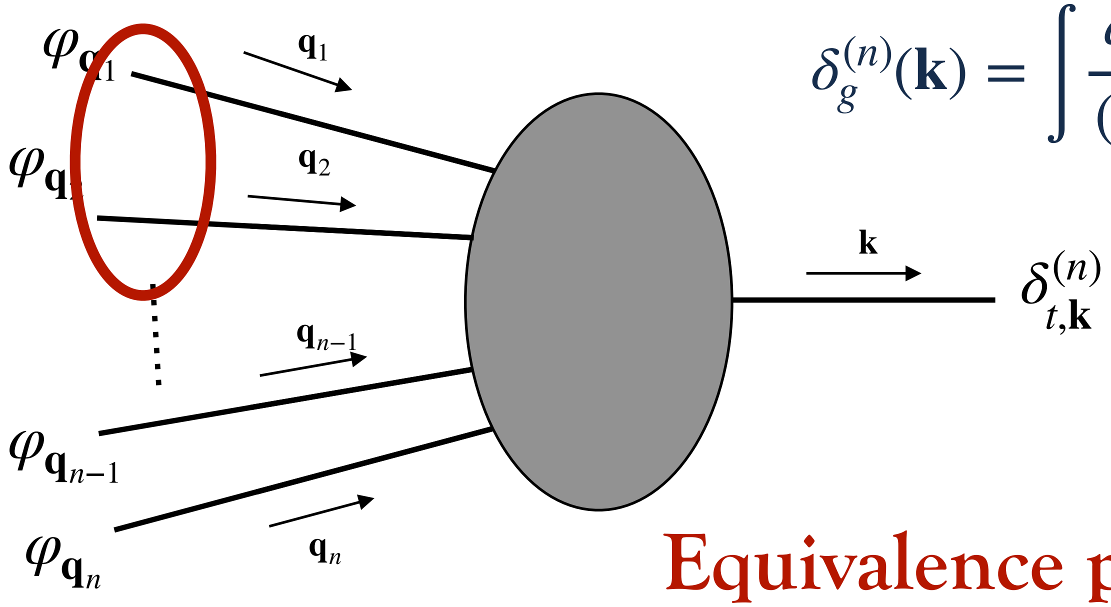

$$\delta_g^{(n)}(\mathbf{k}) = \int \frac{d^3q_1}{(2\pi)^3} \dots \int \frac{d^3q_n}{(2\pi)^3} K_n\left(\mathbf{q}_1, \dots, \mathbf{q}_n\right) \delta^{(1)}(\mathbf{q_1}) \dots \delta^{(1)}(\mathbf{q_n})$$

Equivalence principle

Leading Order: single momentum going $\rightarrow 0$

Next-to-Leading Order: sum of two momenta going $\rightarrow 0$

LSS Bootstrap

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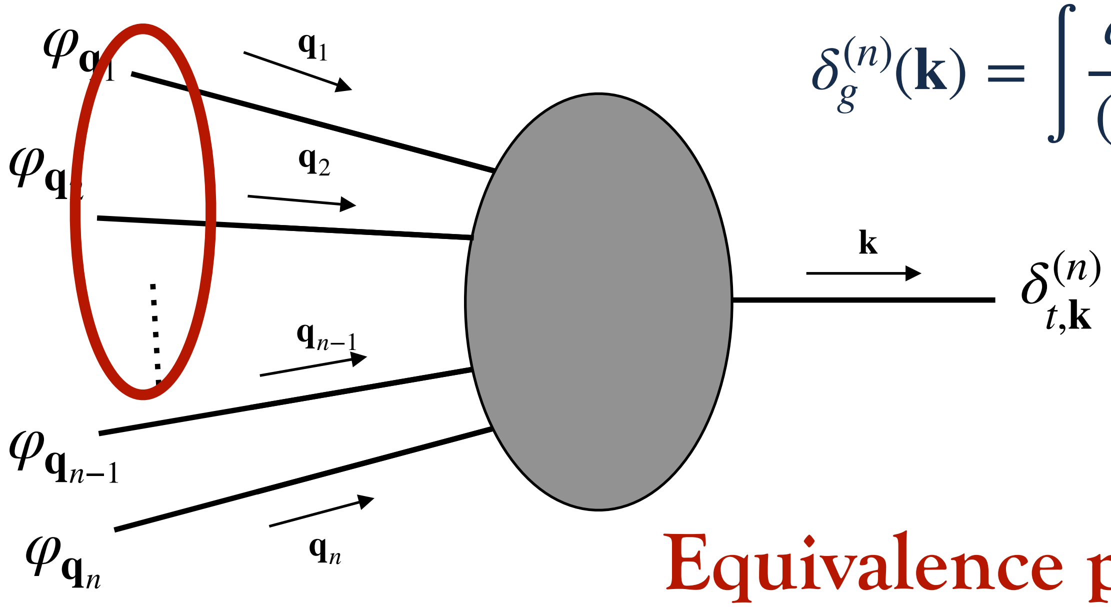

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Equivalence principle

Leading Order: single momentum going $\rightarrow 0$

Next-to-Leading Order: sum of two momenta going $\rightarrow 0$

Next-to-Next-to-Leading Order: sum of three momenta going $\rightarrow 0$

LSS Bootstrap

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Equivalence principle

Leading Order: single momentum going $\rightarrow 0$

Next-to-Leading Order: sum of two momenta going $\rightarrow 0$

Next-to-Next-to-Leading Order: sum of three momenta going $\rightarrow 0$

LSS Bootstrap

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Equivalence principle

Leading Order: single momentum going $\to 0$

Next-to-Leading Order: sum of two momenta going $\to 0$

N.$^{l-1}$-to-Leading Order: sum of $l-1$ momenta going $\to 0$

LSS Bootstrap

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Equivalence principle

Leading Order: single momentum going $\rightarrow 0$

Next-to-Leading Order: sum of two momenta going $\rightarrow 0$

N.$^{l-1}$-to-Leading Order: sum of $l-1$ momenta going $\rightarrow 0$

$$\lim_{\mathbf{q}_1,\ldots,\mathbf{q}_m \to 0} K_n(\mathbf{q}_1,\ldots,\mathbf{q}_m,\mathbf{q}_{m+1},\ldots,\mathbf{q}_n) = \frac{\mathbf{q}_1 \cdot \mathbf{Q}_{n,m}}{q_1^2} \cdots \frac{\mathbf{q}_m \cdot \mathbf{Q}_{n,m}}{q_m^2} K_{n-m}(\mathbf{q}_{m+1},\ldots,\mathbf{q}_n) + O\left(\left(\frac{1}{q}\right)^{m-1}\right)$$

LSS Bootstrap

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Equivalence principle

Leading Order: single momentum going $\rightarrow 0$

Next-to-Leading Order: sum of two momenta going $\rightarrow 0$

N.$^{l-1}$-to-Leading Order: sum of $l-1$ momenta going $\rightarrow 0$

$$\lim_{\mathbf{q}_1+\mathbf{q}_2\to 0} K_n(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \ldots, \mathbf{q}_n) \supset \frac{\mathbf{k}\cdot\mathbf{q}_{12}}{q_{12}^2} K_{n-2}(\mathbf{q}_3, \ldots, \mathbf{q}_n) \int^{\eta} d\eta'\, f_+(\eta') \frac{D_+(\eta')^2}{D_+(\eta)^2} G_2(\mathbf{q}_1, \mathbf{q}_2; \eta')$$

LSS Bootstrap

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Equivalence principle

Leading Order: single momentum going $\to 0$

Next-to-Leading Order: sum of two momenta going $\to 0$

N.$^{l-1}$-to-Leading Order: sum of l momenta going $\to 0$

$$\lim_{\mathbf{Q}_{l,0}\to 0} K_n(\mathbf{q}_1,\ldots,\mathbf{q}_l,\mathbf{q}_{l+1},\ldots,\mathbf{q}_n) \supset \frac{\mathbf{k}\cdot\mathbf{Q}_{l,0}}{Q_{l,0}^2}\int^{\eta} d\eta'\, f_+(\eta')\left(\frac{D_+(\eta')}{D_+(\eta)}\right)^l G_l(\mathbf{q}_1,\ldots,\mathbf{q}_l;\eta')K_{n-l}(\mathbf{q}_{l+1},\ldots,\mathbf{q}_n;\eta)$$

LSS Bootstrap

Kernel at second order

$$K_1(\mathbf{q_1}) = c_0$$

$$K_2(\mathbf{q_1}, \mathbf{q_2}) = c_1 + c_\beta \beta(\mathbf{q}_1, \mathbf{q}_2) + c_\gamma \gamma(\mathbf{q}_1, \mathbf{q}_2)$$

$$\beta(\mathbf{q}_1, \mathbf{q}_2) = \frac{|\mathbf{q}_1 + \mathbf{q}_2|^2 \mathbf{q}_1 \cdot \mathbf{q}_2}{2 q_1^2 q_2^2}$$

$$\gamma(\mathbf{q}_1, \mathbf{q}_2) = 1 - \left(\frac{\mathbf{q}_1 \cdot \mathbf{q}_2}{q_1 \, q_2} \right)^2$$

$$\alpha_a(\mathbf{q}_1, \mathbf{q}_2) = \frac{\mathbf{q}_1 \cdot \mathbf{q}_2}{q_1^2} - \frac{\mathbf{q}_1 \cdot \mathbf{q}_2}{q_2^2}$$

LSS Bootstrap

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$$\{ c_0, c_1, c_\beta, c_\gamma \}$$

LSS Bootstrap

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$$\{c_0, c_1, \cancel{c_\beta}, c_\gamma\}$$

Leading Order

LSS Bootstrap

Kernel at second order

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$$\{c_0, c_1, \not{c_\beta}, c_\gamma\}$$

Leading Order

Only 3 parameters left!
(tracers)

LSS Bootstrap

Kernel at second order

$$K_1(\mathbf{q_1}) = c_0$$

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$$\{\cancel{c_0}, \cancel{c_1}, \cancel{c_\beta}, c_\gamma\}$$

Leading Order

Mass+momentum conservation (matter)

Only 3 parameters left!
(tracers)

LSS Bootstrap

Kernel at second order

$$K_1(\mathbf{q_1}) = c_0$$

$$K_2(\mathbf{q_1}, \mathbf{q_2}) = c_1 + c_\beta \beta(\mathbf{q}_1, \mathbf{q}_2) + c_\gamma \gamma(\mathbf{q}_1, \mathbf{q}_2)$$

$$\beta(\mathbf{q}_1, \mathbf{q}_2) = \frac{|\mathbf{q}_1 + \mathbf{q}_2|^2 \mathbf{q}_1 \cdot \mathbf{q}_2}{2 q_1^2 q_2^2}$$

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$$\alpha_a(\mathbf{q}_1, \mathbf{q}_2) = \frac{\mathbf{q}_1 \cdot \mathbf{q}_2}{q_1^2} - \frac{\mathbf{q}_1 \cdot \mathbf{q}_2}{q_2^2}$$

$$\{ \cancel{c_0}, \cancel{c_1}, \cancel{c_\beta}, c_\gamma \}$$

Leading Order

Mass+momentum conservation (matter)

Only 3 parameters left! (tracers)

Only 1 parameter left! (matter)

LSS Bootstrap

Kernel at third order

$$K_3(\mathbf{q_1}, \mathbf{q_2}, \mathbf{q_3}) = c_2 + c_{\gamma 1}\gamma(\mathbf{q}_1, \mathbf{q}_2) + c_{\gamma 2}\gamma(\mathbf{q}_{12}, \mathbf{q}_3) + c_{\beta 1}\beta(\mathbf{q}_1, \mathbf{q}_2) + c_{\beta 2}\beta(\mathbf{q}_{12}, \mathbf{q}_3)$$

$$+c_{\gamma\gamma}\gamma(\mathbf{q}_1, \mathbf{q}_2)\gamma(\mathbf{q}_{12}, \mathbf{q}_3) + c_{\beta\beta}\beta(\mathbf{q}_1, \mathbf{q}_2)\beta(\mathbf{q}_{12}, \mathbf{q}_3) + c_{\gamma\beta}\gamma(\mathbf{q}_1, \mathbf{q}_2)\beta(\mathbf{q}_{12}, \mathbf{q}_3)$$

$$+c_{\beta\gamma}\beta(\mathbf{q}_1, \mathbf{q}_2)\gamma(\mathbf{q}_{12}, \mathbf{q}_3) + \left(c_\alpha + c_{\gamma\alpha}\gamma(\mathbf{q}_1, \mathbf{q}_2) + c_{\beta\alpha}\beta(\mathbf{q}_1, \mathbf{q}_2)\right)\alpha_a(\mathbf{q}_{12}, \mathbf{q}_3)$$

LSS Bootstrap

Kernel at third order

$$K_3(\mathbf{q_1}, \mathbf{q_2}, \mathbf{q_3}) = c_2 + c_{\gamma 1}\gamma(\mathbf{q}_1, \mathbf{q}_2) + c_{\gamma 2}\gamma(\mathbf{q}_{12}, \mathbf{q}_3) + c_{\beta 1}\beta(\mathbf{q}_1, \mathbf{q}_2) + c_{\beta 2}\beta(\mathbf{q}_{12}, \mathbf{q}_3)$$

$$+c_{\gamma\gamma}\gamma(\mathbf{q}_1, \mathbf{q}_2)\gamma(\mathbf{q}_{12}, \mathbf{q}_3) + c_{\beta\beta}\beta(\mathbf{q}_1, \mathbf{q}_2)\beta(\mathbf{q}_{12}, \mathbf{q}_3) + c_{\gamma\beta}\gamma(\mathbf{q}_1, \mathbf{q}_2)\beta(\mathbf{q}_{12}, \mathbf{q}_3)$$

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$$\{c_2, c_{\gamma 1}, c_{\gamma 2}, c_{\beta 1}, c_{\beta 2}, c_{\gamma\gamma}, c_{\beta\beta}, c_{\gamma\beta}, c_{\beta\gamma}, c_\alpha, c_{\gamma\alpha}, c_{\beta\alpha}\}$$

LSS Bootstrap

Kernel at third order

$$K_3(\mathbf{q_1},\mathbf{q_2},\mathbf{q_3}) = c_2 + c_{\gamma1}\gamma(\mathbf{q}_1,\mathbf{q}_2) + c_{\gamma2}\gamma(\mathbf{q}_{12},\mathbf{q}_3) + c_{\beta1}\beta(\mathbf{q}_1,\mathbf{q}_2) + c_{\beta2}\beta(\mathbf{q}_{12},\mathbf{q}_3)$$

$$+c_{\gamma\gamma}\gamma(\mathbf{q}_1,\mathbf{q}_2)\gamma(\mathbf{q}_{12},\mathbf{q}_3) + c_{\beta\beta}\beta(\mathbf{q}_1,\mathbf{q}_2)\beta(\mathbf{q}_{12},\mathbf{q}_3) + c_{\gamma\beta}\gamma(\mathbf{q}_1,\mathbf{q}_2)\beta(\mathbf{q}_{12},\mathbf{q}_3)$$

$$+c_{\beta\gamma}\beta(\mathbf{q}_1,\mathbf{q}_2)\gamma(\mathbf{q}_{12},\mathbf{q}_3) + \left(c_\alpha + c_{\gamma\alpha}\gamma(\mathbf{q}_1,\mathbf{q}_2) + c_{\beta\alpha}\beta(\mathbf{q}_1,\mathbf{q}_2)\right)\alpha_a(\mathbf{q}_{12},\mathbf{q}_3)$$

$$\{c_2, c_{\gamma1}, \cancel{c_{\gamma2}}, \cancel{c_{\beta1}}, \cancel{c_{\beta2}}, c_{\gamma\gamma}, \cancel{c_{\beta\beta}}, c_{\gamma\beta}, c_{\beta\gamma}, \cancel{c_\alpha}, c_{\gamma\alpha}, \cancel{c_{\beta\alpha}}\}$$

Leading Order

LSS Bootstrap

Kernel at third order

$$K_3(\mathbf{q_1},\mathbf{q_2},\mathbf{q_3}) = c_2 + c_{\gamma 1}\gamma(\mathbf{q}_1,\mathbf{q}_2) + c_{\gamma 2}\gamma(\mathbf{q}_{12},\mathbf{q}_3) + c_{\beta 1}\beta(\mathbf{q}_1,\mathbf{q}_2) + c_{\beta 2}\beta(\mathbf{q}_{12},\mathbf{q}_3)$$

$$+c_{\gamma\gamma}\gamma(\mathbf{q}_1,\mathbf{q}_2)\gamma(\mathbf{q}_{12},\mathbf{q}_3) + c_{\beta\beta}\beta(\mathbf{q}_1,\mathbf{q}_2)\beta(\mathbf{q}_{12},\mathbf{q}_3) + c_{\gamma\beta}\gamma(\mathbf{q}_1,\mathbf{q}_2)\beta(\mathbf{q}_{12},\mathbf{q}_3)$$

$$+c_{\beta\gamma}\beta(\mathbf{q}_1,\mathbf{q}_2)\gamma(\mathbf{q}_{12},\mathbf{q}_3) + \left(c_\alpha + c_{\gamma\alpha}\gamma(\mathbf{q}_1,\mathbf{q}_2) + c_{\beta\alpha}\beta(\mathbf{q}_1,\mathbf{q}_2)\right)\alpha_a(\mathbf{q}_{12},\mathbf{q}_3)$$

$$\{c_2, c_{\gamma 1}, \cancel{c_{\gamma 2}}, \cancel{c_{\beta 1}}, \cancel{c_{\beta 2}}, c_{\gamma\gamma}, \cancel{c_{\beta\beta}}, \cancel{c_{\gamma\beta}}, \cancel{c_{\beta\gamma}}, \cancel{c_\alpha}, c_{\gamma\alpha}, \cancel{c_{\beta\alpha}}\}$$

Leading Order

Next-to-Leading Order

LSS Bootstrap

Kernel at third order

$$K_3(\mathbf{q_1}, \mathbf{q_2}, \mathbf{q_3}) = c_2 + c_{\gamma 1}\gamma(\mathbf{q}_1, \mathbf{q}_2) + c_{\gamma 2}\gamma(\mathbf{q}_{12}, \mathbf{q}_3) + c_{\beta 1}\beta(\mathbf{q}_1, \mathbf{q}_2) + c_{\beta 2}\beta(\mathbf{q}_{12}, \mathbf{q}_3)$$

$$+c_{\gamma\gamma}\gamma(\mathbf{q}_1, \mathbf{q}_2)\gamma(\mathbf{q}_{12}, \mathbf{q}_3) + c_{\beta\beta}\beta(\mathbf{q}_1, \mathbf{q}_2)\beta(\mathbf{q}_{12}, \mathbf{q}_3) + c_{\gamma\beta}\gamma(\mathbf{q}_1, \mathbf{q}_2)\beta(\mathbf{q}_{12}, \mathbf{q}_3)$$

$$+c_{\beta\gamma}\beta(\mathbf{q}_1, \mathbf{q}_2)\gamma(\mathbf{q}_{12}, \mathbf{q}_3) + \left(c_\alpha + c_{\gamma\alpha}\gamma(\mathbf{q}_1, \mathbf{q}_2) + c_{\beta\alpha}\beta(\mathbf{q}_1, \mathbf{q}_2)\right)\alpha_a(\mathbf{q}_{12}, \mathbf{q}_3)$$

$$\{c_2, c_{\gamma 1}, \cancel{c_{\gamma 2}}, \cancel{c_{\beta 1}}, \cancel{c_{\beta 2}}, c_{\gamma\gamma}, \cancel{c_{\beta\beta}}, \cancel{c_{\gamma\beta}}, \cancel{c_{\beta\gamma}}, \cancel{c_\alpha}, c_{\gamma\alpha}, \cancel{c_{\beta\alpha}}\}$$

Leading Order (red strikes: $c_{\gamma 2}, c_{\beta 1}, c_{\beta 2}, c_{\beta\beta}, c_\alpha, c_{\beta\alpha}$)

Next-to-Leading Order (green strikes: $c_{\gamma\beta}, c_{\beta\gamma}$)

Only 4 parameters left!
(tracers)

LSS Bootstrap

Kernel at third order

$$K_3(\mathbf{q_1}, \mathbf{q_2}, \mathbf{q_3}) = c_2 + c_{\gamma 1}\gamma(\mathbf{q}_1, \mathbf{q}_2) + c_{\gamma 2}\gamma(\mathbf{q}_{12}, \mathbf{q}_3) + c_{\beta 1}\beta(\mathbf{q}_1, \mathbf{q}_2) + c_{\beta 2}\beta(\mathbf{q}_{12}, \mathbf{q}_3)$$

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$$+c_{\beta\gamma}\beta(\mathbf{q}_1, \mathbf{q}_2)\gamma(\mathbf{q}_{12}, \mathbf{q}_3) + \left(c_\alpha + c_{\gamma\alpha}\gamma(\mathbf{q}_1, \mathbf{q}_2) + c_{\beta\alpha}\beta(\mathbf{q}_1, \mathbf{q}_2)\right)\alpha_a(\mathbf{q}_{12}, \mathbf{q}_3)$$

$$\{c_2, c_{\gamma 1}, c_{\gamma 2}, c_{\beta 1}, c_{\beta 2}, c_{\gamma\gamma}, c_{\beta\beta}, c_{\gamma\beta}, c_{\beta\gamma}, c_\alpha, c_{\gamma\alpha}, c_{\beta\alpha}\}$$

Leading Order Next-to-Leading Order Mass+mom. conservation

Only 4 parameters left!
(tracers)

LSS Bootstrap

Kernel at third order

$$K_3(\mathbf{q_1},\mathbf{q_2},\mathbf{q_3}) = c_2 + c_{\gamma 1}\gamma(\mathbf{q}_1,\mathbf{q}_2) + c_{\gamma 2}\gamma(\mathbf{q}_{12},\mathbf{q}_3) + c_{\beta 1}\beta(\mathbf{q}_1,\mathbf{q}_2) + c_{\beta 2}\beta(\mathbf{q}_{12},\mathbf{q}_3)$$

$$+c_{\gamma\gamma}\gamma(\mathbf{q}_1,\mathbf{q}_2)\gamma(\mathbf{q}_{12},\mathbf{q}_3) + c_{\beta\beta}\beta(\mathbf{q}_1,\mathbf{q}_2)\beta(\mathbf{q}_{12},\mathbf{q}_3) + c_{\gamma\beta}\gamma(\mathbf{q}_1,\mathbf{q}_2)\beta(\mathbf{q}_{12},\mathbf{q}_3)$$

$$+c_{\beta\gamma}\beta(\mathbf{q}_1,\mathbf{q}_2)\gamma(\mathbf{q}_{12},\mathbf{q}_3) + \left(c_\alpha + c_{\gamma\alpha}\gamma(\mathbf{q}_1,\mathbf{q}_2) + c_{\beta\alpha}\beta(\mathbf{q}_1,\mathbf{q}_2)\right)\alpha_a(\mathbf{q}_{12},\mathbf{q}_3)$$

$$\{\cancel{c_2}, \cancel{c_{\gamma 1}}, \cancel{c_{\gamma 2}}, \cancel{c_{\beta 1}}, \cancel{c_{\beta 2}}, c_{\gamma\gamma}, \cancel{c_{\beta\beta}}, \cancel{c_{\gamma\beta}}, \cancel{c_{\beta\gamma}}, \cancel{c_\alpha}, c_{\gamma\alpha}, \cancel{c_{\beta\alpha}}\}$$

Leading Order — Next-to-Leading Order — Mass+mom. conservation

Only 4 parameters left!
(tracers)

Only 2 parameters left!
(matter)

LSS Bootstrap

General time dependence

Bootstrap expansion:
valid for every scale-independent model

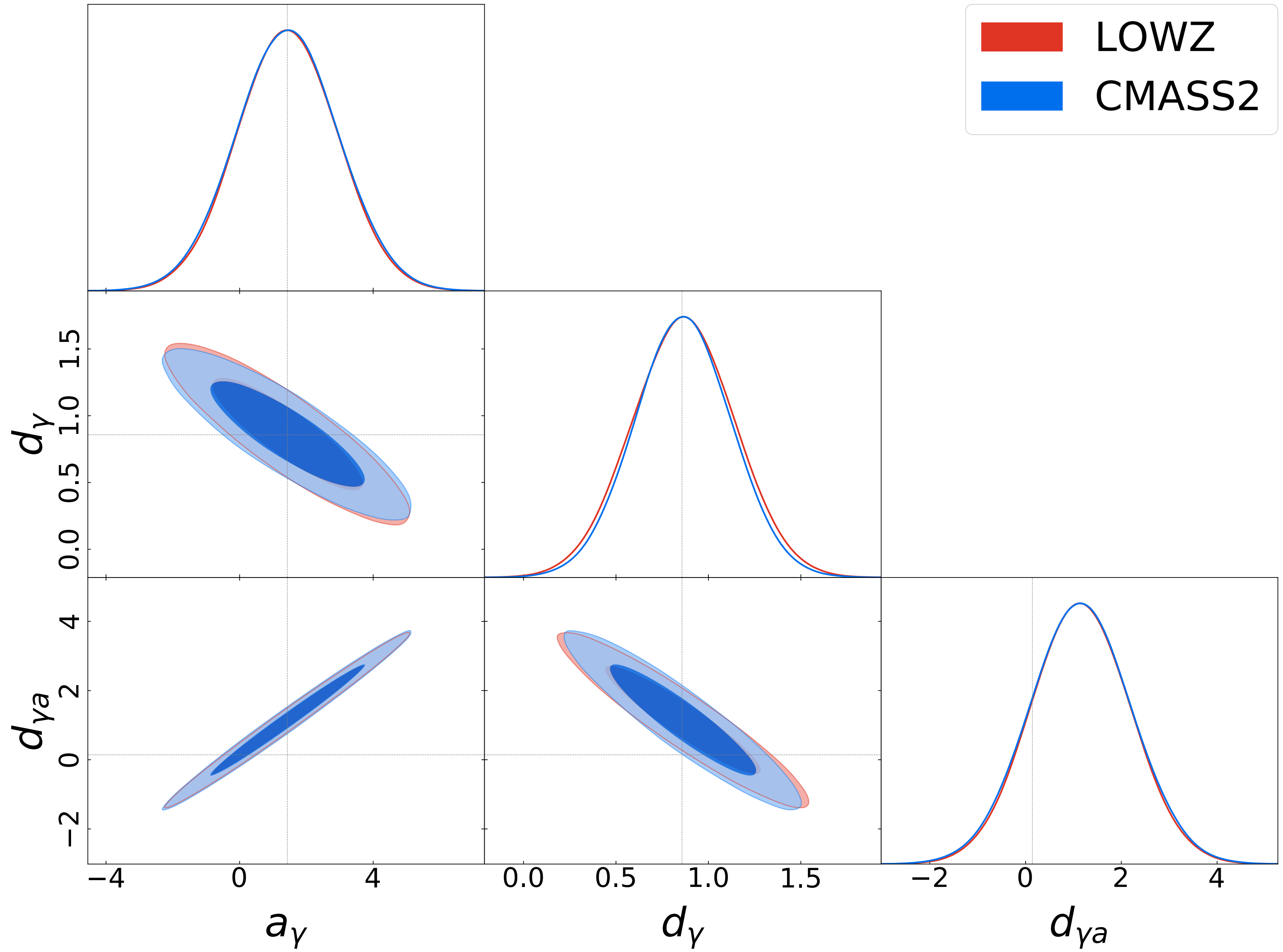
$$-\frac{k^2}{\mathcal{H}^2}\phi(\mathbf{k},\eta) = \frac{3}{2}\Omega_m(\eta)\mu(\eta)\delta(\mathbf{k},\eta) + S(\mathbf{k},\eta)$$

ΛCDM (exact time dep., Donath Y., Senatore L., 2020)
nDGP (Dvali et al. 2000, Schmidt F. 2009)
k-mouflage (Babichev et al. 2009, Brax and Valaegias 2014)
JBD (Brans C., Dicke R., 1961)

D'Amico G., **MM**, Piga L., Pietroni M., Vernizzi F., Wright B., 2209.XXXX

LSS Bootstrap

Matter and velocity kernels



In the 1-loop PS the only time dependent functions are $a_\gamma,\ d_\gamma, d_{\gamma a} \equiv d_{\gamma\alpha} - d_{\gamma\gamma}/2$

Constrain (possible) deviations from ΛCDM with a model-independent approach

LSS Bootstrap & PNG

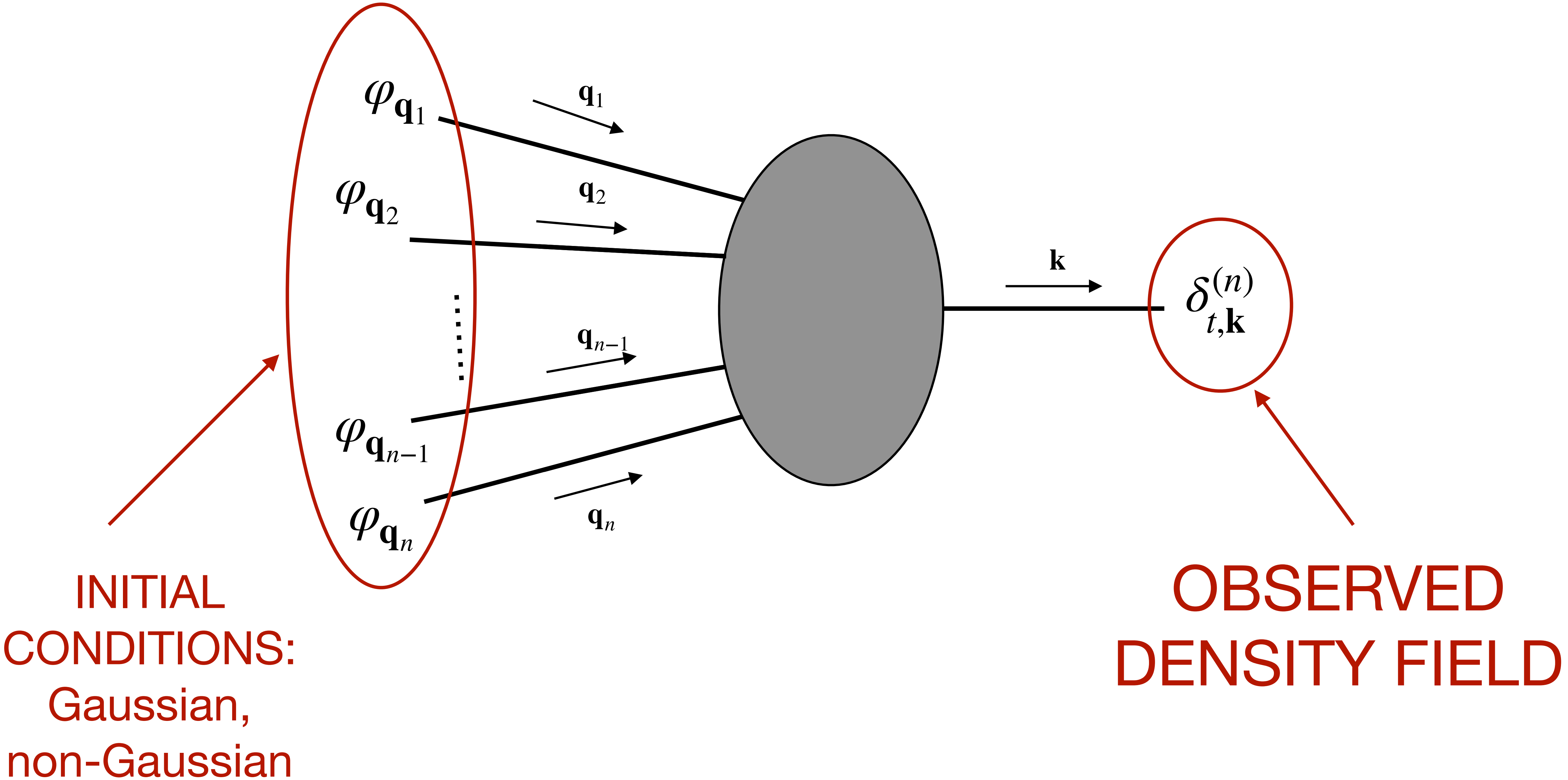
Consistency relations

$$\lim_{k\to 0} \frac{B_\delta(k, q, |\mathbf{k}+\mathbf{q}|; \tau, \tau', \tau'')}{P_\delta(q; \tau', \tau'')P_\delta(k; \tau, \tau)} = -\frac{\mathbf{q}\cdot\mathbf{k}}{k^2}\frac{D_+(\tau') - D_+(\tau'')}{D_+(\tau)}$$

$$+\frac{6f_{NL}\Omega_{m,0}H_0^2}{k^2T(k)}\frac{D_+(\tau_0)}{D_+(\tau)} + O\left(k^0, f_{NL}^2\right)$$

Peloso M., Pietroni M., *JCAP* 05 (2013) 031
Goldstein S., et al., 2209.06228

LSS Bootstrap & PNG



Conclusions

- Importance of Equivalence Principle in Cosmology
- Perturbative EP: LSS bootstrap
- Easily generalized to NG IC
- First step for a novel approach to Galaxy Clustering

Thanks for your attention

Backup slides

In this thesis

Equivalence principle

Model independent measurement using the EP in the BAO range

- **MM**, Nishimichi T., Pietroni M., Phys. Rev. D100 (2019)
- **MM**, Nishimichi T., Pietroni M., JCAP 07 (2020)

Constraints the perturbative kernels in a model independent way: **Large Scale Structure Bootstrap**!

- D'Amico G., **MM**, Pietroni M., Vernizzi F., JCAP 10 (2021)

CR and BAO

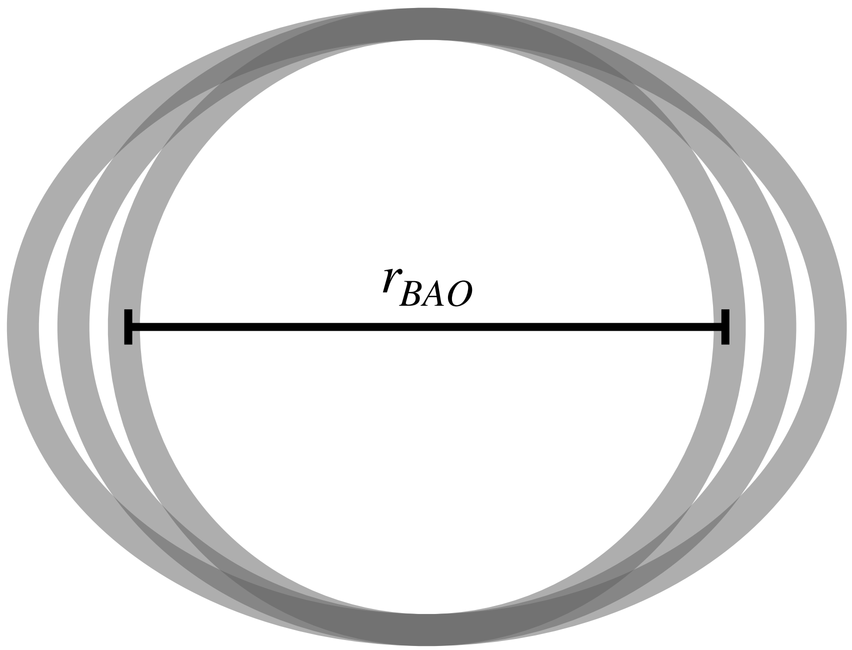
- **MM**, Nishimichi T., Pietroni M., Phys. Rev. D100 (2019)
- **MM**, Nishimichi T., Pietroni M., JCAP 07 (2020)

Squeezed limit of **Bispectrum** (real space)

$$\lim_{q/k\to 0} B_{\alpha\beta\gamma}(q,k_+,k_-;t_\alpha,t_\beta,t_\gamma) \simeq \frac{\mathbf{k}\cdot\mathbf{q}}{q^2}P_{\alpha m}(q;t_\alpha)\left[\frac{D(t_\beta)}{D(t_\alpha)}P_{\beta\gamma}(k_-;t_\beta,t_\gamma) - \frac{D(t_\gamma)}{D(t_\alpha)}P_{\beta\gamma}(k_+;t_\beta,t_\gamma)\right] + O\left(\left(\frac{q}{k}\right)^0\right)$$

Peloso M. and Pietroni M., JCAP (2013)
Kehagias A. and Riotto A., Nucl. Phys. (2013)

In configuration space $\quad \propto \mu^2 \dfrac{d\log\xi(r)}{d\log r} \qquad \mu = \hat{\mathbf{k}}\cdot\hat{\mathbf{q}}$



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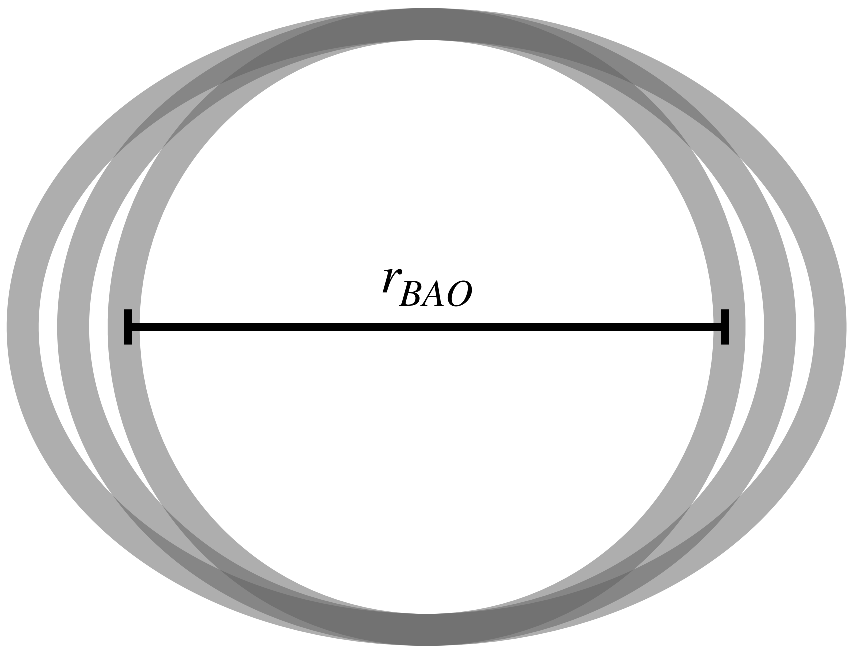

Equal-time squeezed limit

CR and BAO

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- **MM**, Nishimichi T., Pietroni M., JCAP 07 (2020)

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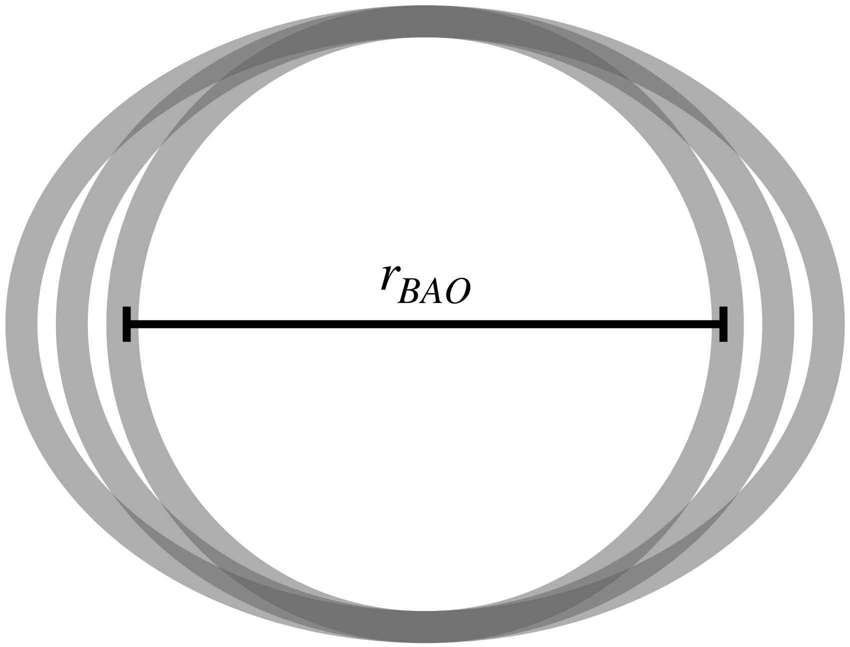

Equal-time squeezed limit

$$\lim_{q/k\to 0} \frac{B_{\alpha\alpha\alpha}(q,k_+.k_-)}{P_{\alpha\alpha}(q)P_{\alpha\alpha}(k)} = -\frac{\mu^2}{b_\alpha(q)}\frac{d\log P_{\alpha\alpha}(k)}{d\log k} + O\left(\left(\frac{q}{k}\right)^0\right)$$

Unchanged by nonlinearities!

Baldauf T. et al., Phys.Rev.D (2015)

CR and BAO

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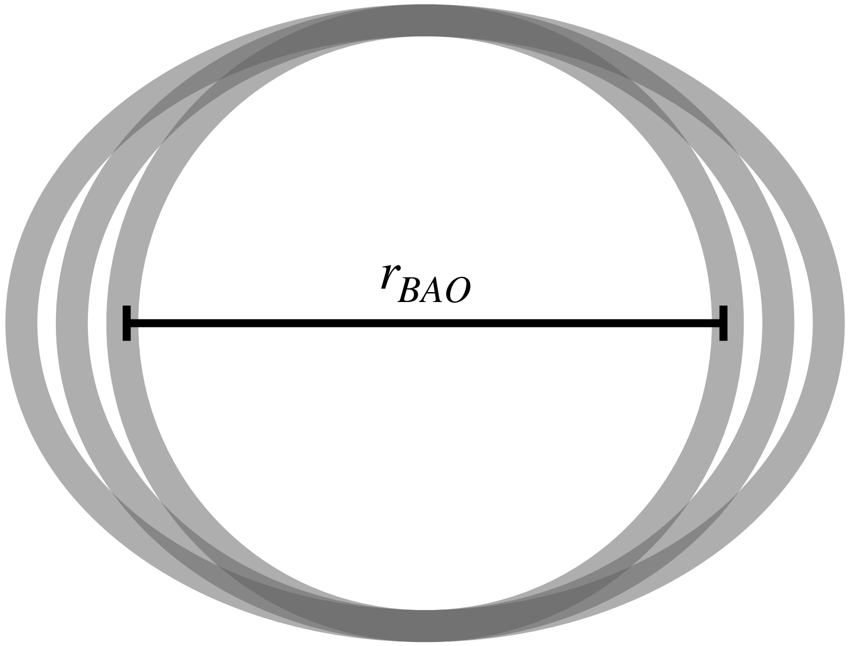

Equal-time squeezed limit

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In presence of a scale like the BAO, the oscillating part of the derivative is enhanced by a $\sim k\, r_{BAO}$ factor, we can isolate it to verify CR and measure bias

CR and BAO

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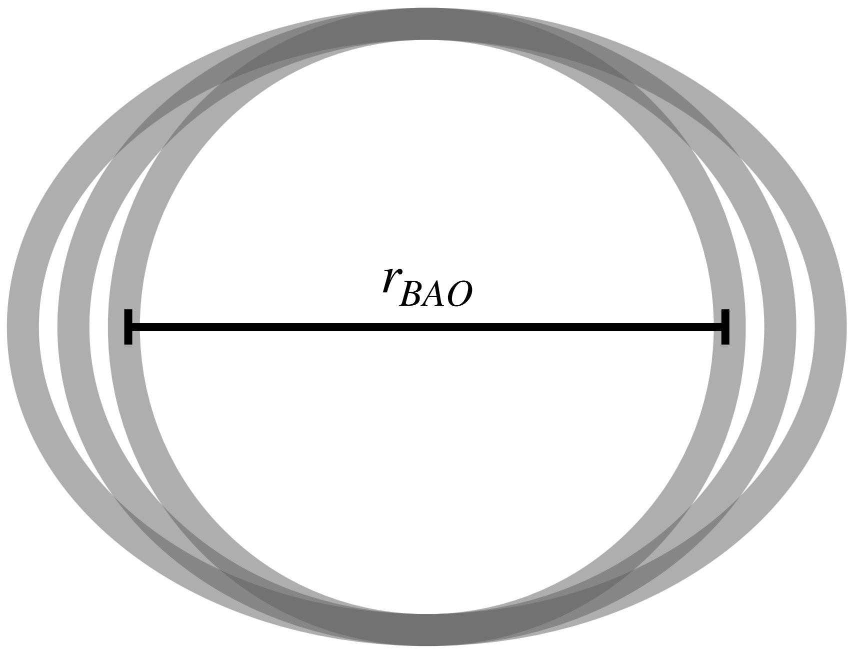

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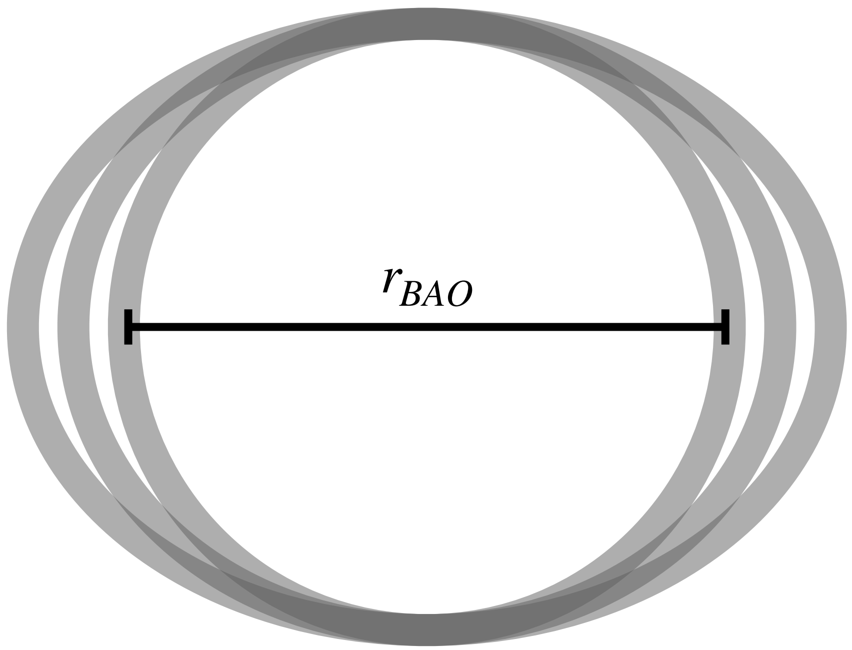
Equal-time squeezed limit



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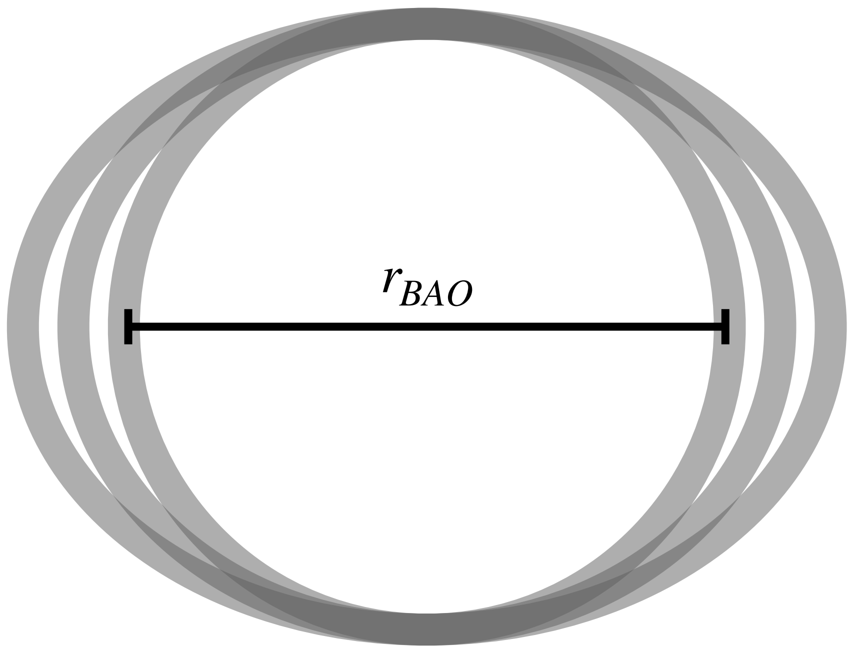

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Unchanged by nonlinearities!

Baldauf T. et al., Phys.Rev.D (2015)

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Peloso M. and Pietroni M., JCAP (2013)
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In configura… $\frac{d\log\xi(r)}{}$

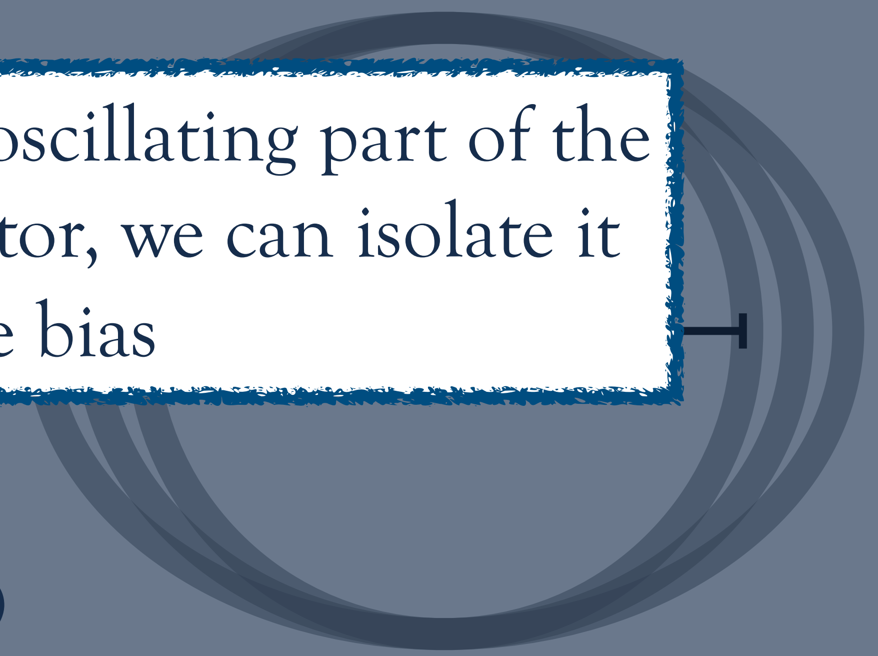
Equal-time…

In presence of a scale like the BAO, the oscillating part of the derivative is enhanced by a $\sim k\, r_{BAO}$ factor, we can isolate it to verify CR and measure bias

$$\lim_{q/k\to 0} \frac{B_{\alpha\alpha\alpha}(q, \ldots)}{P_{\alpha\alpha}(q)P_{\alpha\alpha}(k)} = - \frac{\ldots}{b_\alpha(q)} \frac{\ldots}{d\log k} + O\left(\left(\frac{q}{k}\right)\right)$$

Unchanged by nonlinearities!

Baldauf T. et al., Phys.Rev.D (2015)

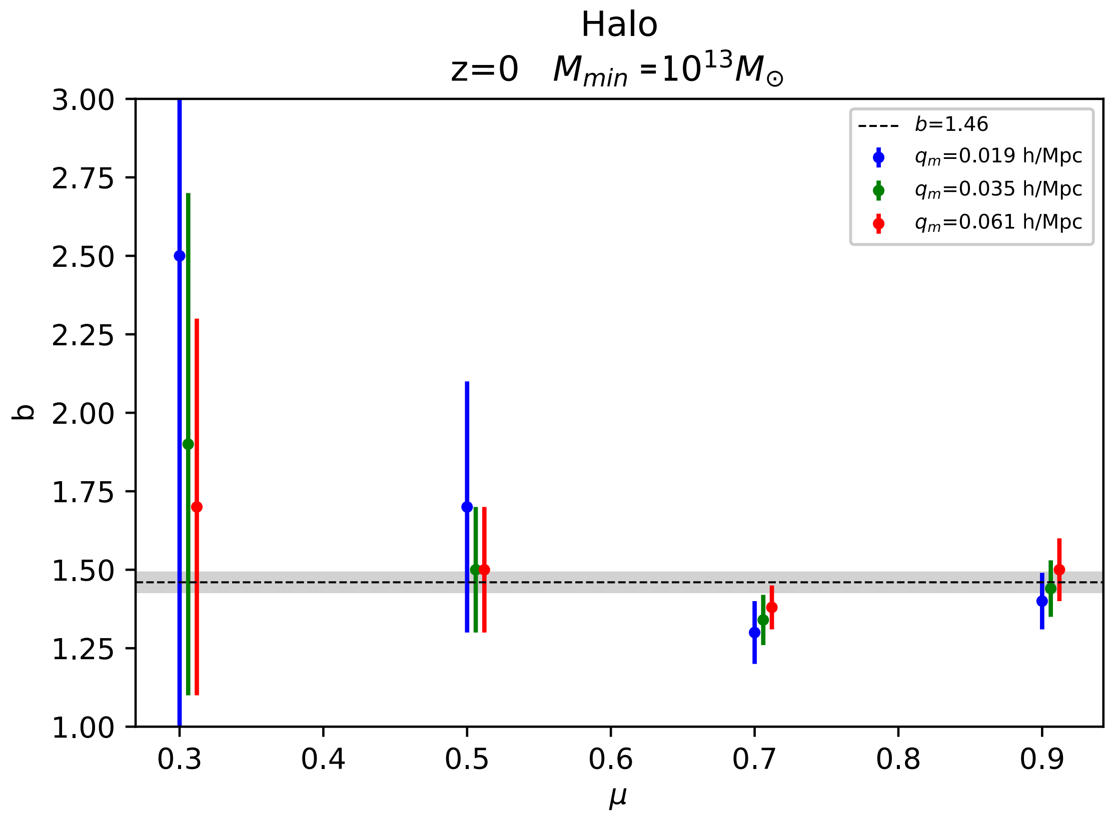


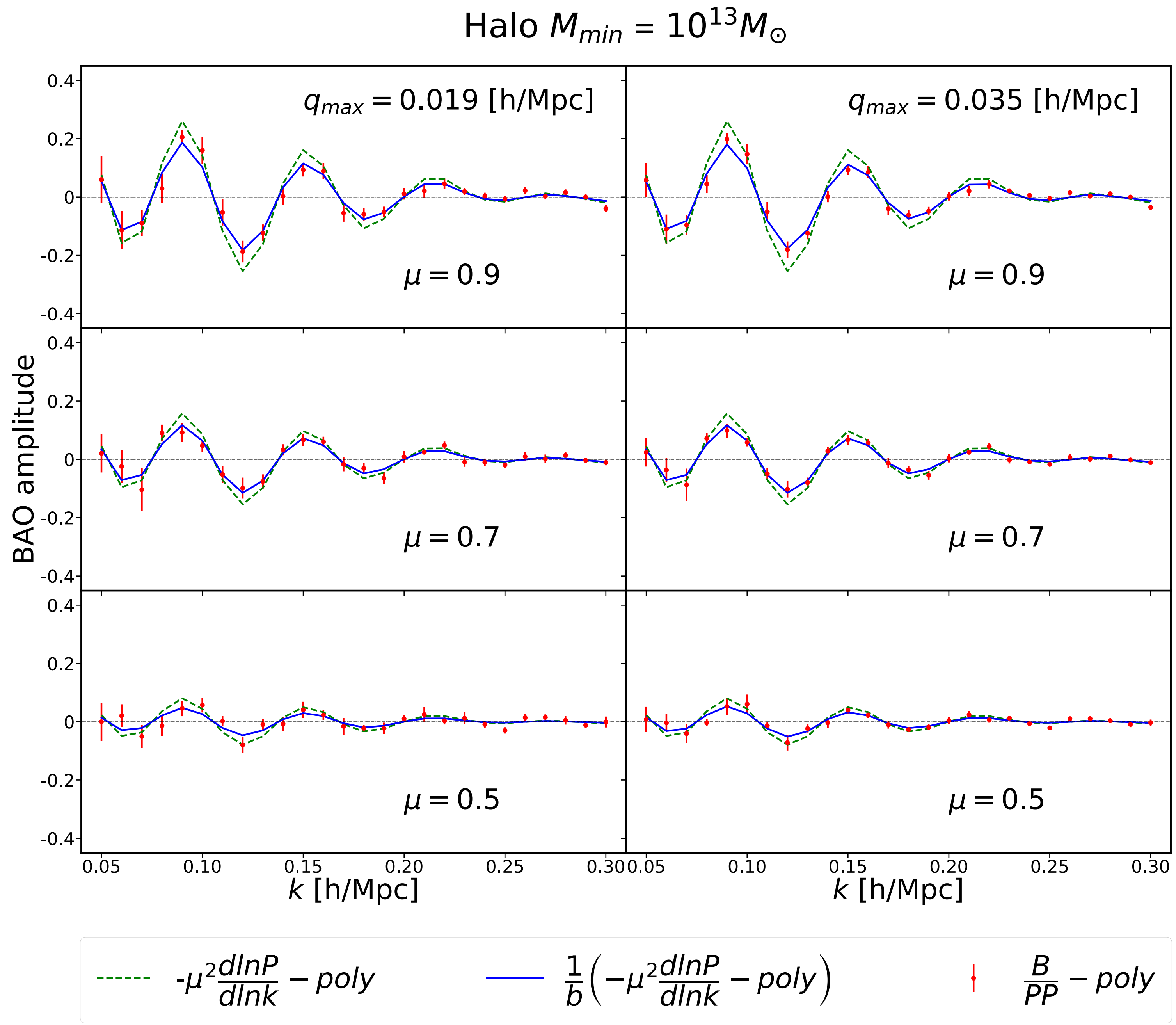
CR and BAO

N-body simulations: **real space w/ biased tracers**, MM+ (2019)

$$\lim_{q/k\to 0} \frac{B_t(q,k_+,k_-)}{P_t(q)P_t(k)} = -\frac{\mu^2}{b_t}\frac{d\log P(k)}{d\log k} + \dots$$

Bias parameter $\quad b_t = \lim_{q\to 0} \frac{P_{tt}(q)}{P_{tm}(q)}$





CR and BAO

N-body simulations: **redshift space w/ biased tracers,** MM+ (2020)

Multipoles + Kaiser

$$\lim_{q/k\to 0} \frac{B_t^{(0)}(q,k)}{P_t^{(0)}(q)P_t^{(0)}(k)} = -\left[\frac{1}{3b_t} + \frac{b_t-1}{9b_t}\beta_t \frac{1+\frac{3}{5}\beta_t}{1+\frac{2}{3}\beta_t+\frac{1}{5}\beta_t^2}\right]\frac{\partial \log P_t^{(0)}(k)}{\partial \log k} + ..$$

$$\lim_{q/k\to 0} \frac{B_t^{(l_k=2)}(q,k)}{P_t^{(0)}(q)P_t^{(0)}(k)} = -\frac{2\beta_t}{45 b_t}\frac{2+b_t(5+3\beta_t)}{1+\frac{2}{3}\beta_t+\frac{1}{5}\beta_t^2}\frac{\partial \log P_t^{(0)}(k)}{\partial \log k} + \ldots$$

$$\lim_{k\to 0} \frac{P^{(2)}(k)}{P_t^{(0)}(k)} = \frac{4\beta_t}{21}\frac{7+3\beta_t}{1+\frac{2}{3}\beta_t+\frac{1}{5}\beta_t^2}$$

Growth rate $$f \equiv \frac{d\log D}{d \log a}$$

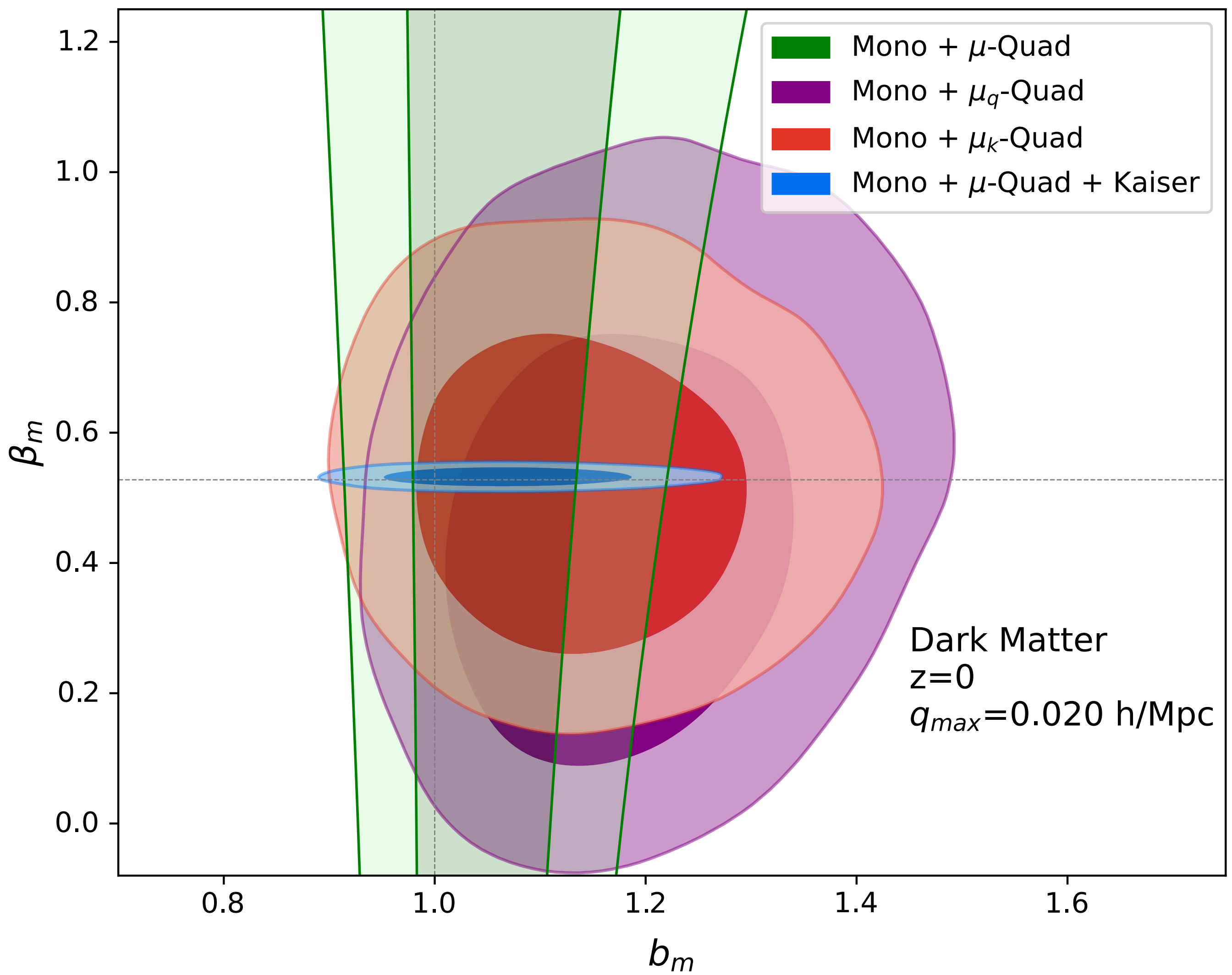
$$\beta_t \equiv \frac{f}{b_t}$$

Angles

$$\mu \equiv \hat{\mathbf{k}}\cdot\hat{\mathbf{q}}$$

$$\mu_k \equiv \hat{\mathbf{k}}\cdot\hat{\mathbf{z}}$$

$$\mu_q \equiv \hat{\mathbf{q}}\cdot\hat{\mathbf{z}}$$



CR and BAO

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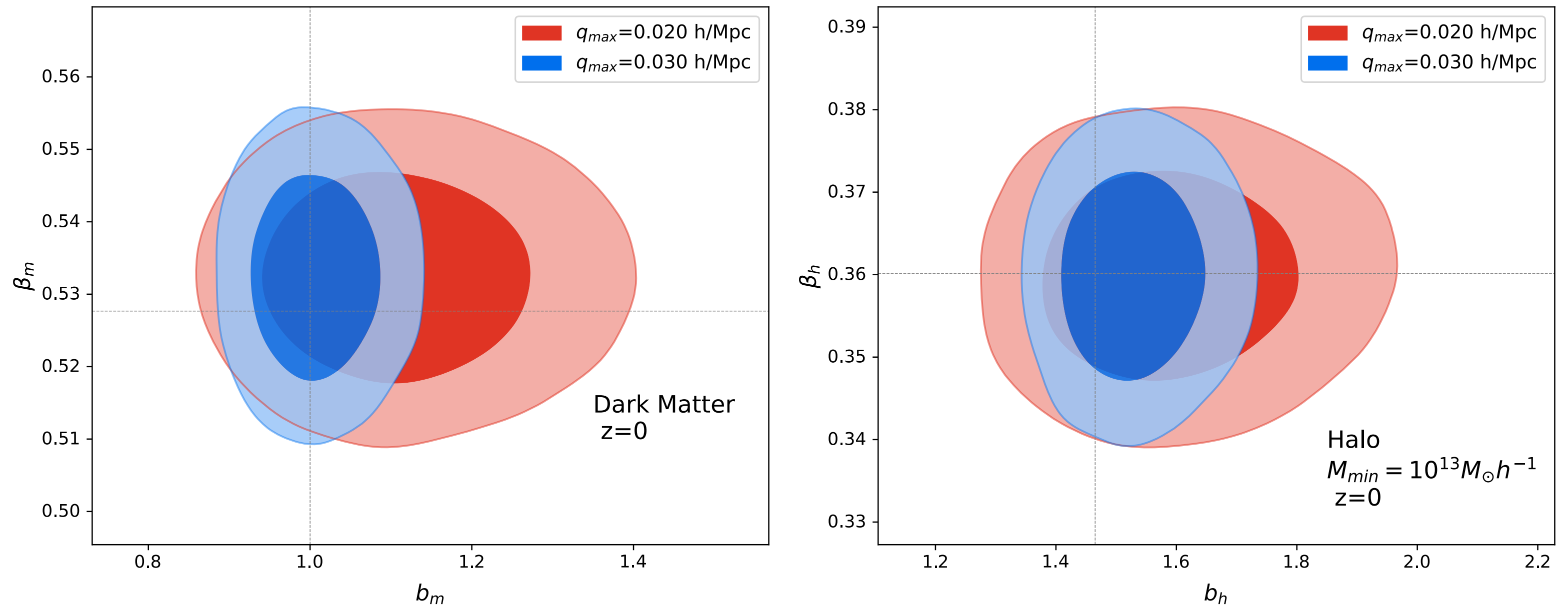
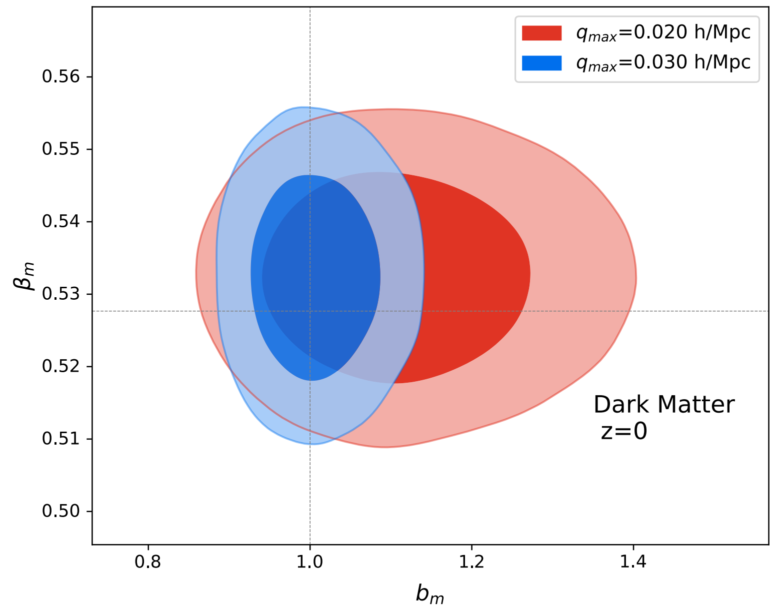


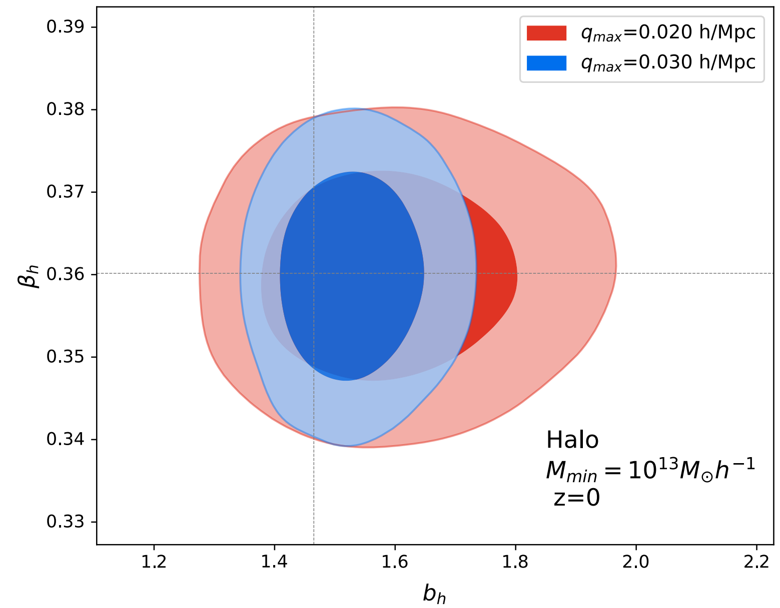
$M_{\rm min} = 10^{13} h^{-1} M_\odot$ $z=0$				
q_{max} (h/Mpc)	b_h	b_h^{fid}	$f = \beta_h b_h$	f^{fid}
0.020	$1.58^{+0.15}_{-0.13}$	1.47	$0.57^{+0.06}_{-0.06}$	0.528
0.030	$1.53^{+0.08}_{-0.08}$	1.47	$0.55^{+0.04}_{-0.04}$	0.538
$M_{\rm min} = 10^{13} h^{-1} M_\odot$ $z=1$				
q_{max} (h/Mpc)	b_h	b_h^{fid}	$f = \beta_h b_h$	f^{fid}
0.020	$2.85^{+0.39}_{-0.32}$	2.686	$0.93^{+0.14}_{-0.14}$	0.877
0.030	$2.58^{+0.17}_{-0.16}$	2.686	$0.82^{+0.07}_{-0.07}$	0.877
$M_{\rm min} = 10^{14} h^{-1} M_\odot$ $z=0$				
q_{max} (h/Mpc)	b_h	b_h^{fid}	$f = \beta_h b_h$	f^{fid}
0.020	$3.40^{+0.83}_{-0.61}$	2.446	$0.73^{+0.18}_{-0.18}$	0.528
0.030	$2.29^{+0.21}_{-0.18}$	2.446	$0.49^{+0.06}_{-0.06}$	0.528

Model independent measurement of $f(z)$ and b_t at 10%!

CR and BAO

N-body simulations: **redshift space w/ biased tracers,** MM+ (2020)





$M_{\rm min} = 10^{13} h^{-1}\, M_{\odot}$ $\quad z = 0$				
q_{max} (h/Mpc)	b_h	b_h^{fid}	$f = \beta_h b_h$	f^{fid}
0.020	$1.58^{+0.15}_{-0.13}$	1.47	$0.57^{+0.06}_{-0.06}$	0.528
0.030	$1.53^{+0.08}_{-0.08}$	1.47	$0.55^{+0.04}_{-0.04}$	0.538
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q_{max} (h/Mpc)	b_h	b_h^{fid}	$f = \beta_h b_h$	f^{fid}
0.020	$2.85^{+0.39}_{-0.32}$	2.686	$0.93^{+0.14}_{-0.14}$	0.877
0.030	$2.58^{+0.17}_{-0.16}$	2.686	$0.82^{+0.07}_{-0.07}$	0.877
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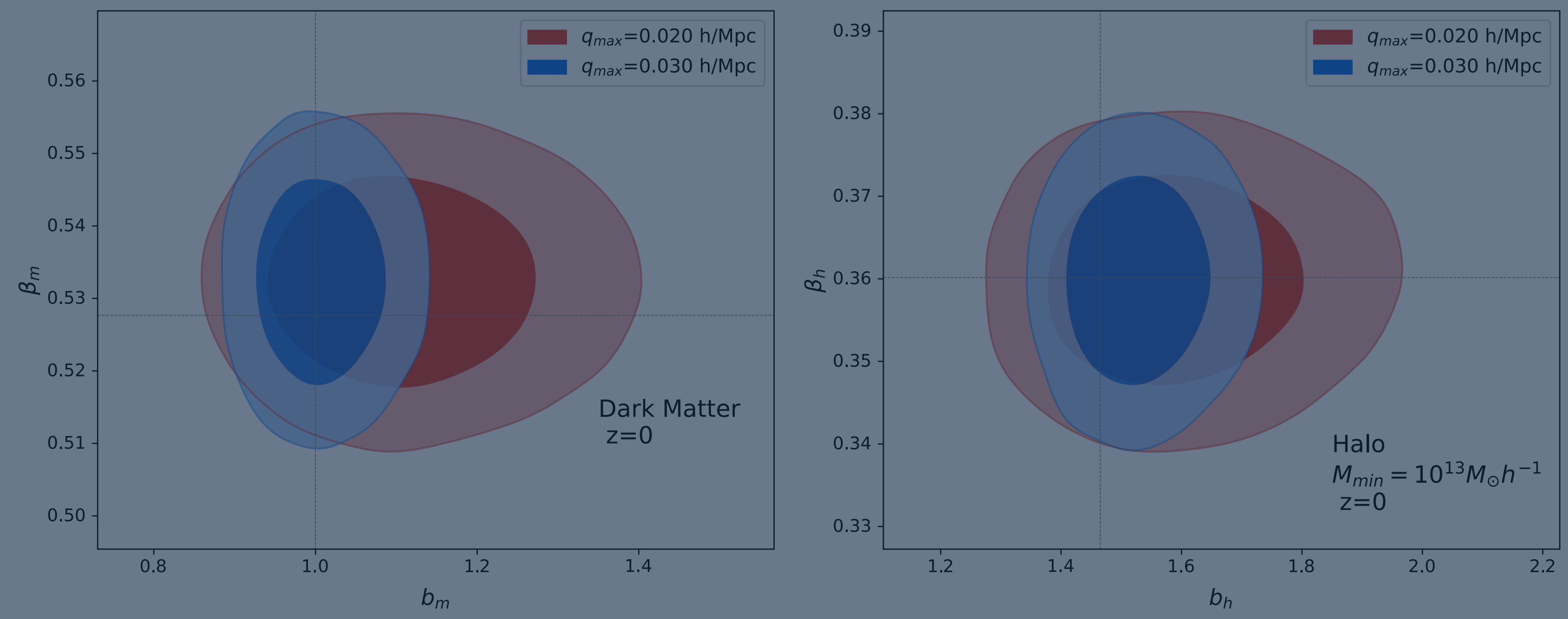
q_{max} (h/Mpc)	b_h	b_h^{fid}	$f = \beta_h b_h$	f^{fid}
0.020	$3.40^{+0.83}_{-0.61}$	2.446	$0.73^{+0.18}_{-0.18}$	0.528
0.030	$2.29^{+0.21}_{-0.18}$	2.446	$0.49^{+0.06}_{-0.06}$	0.528

CR and BAO

N-body simulations: redshift space w/ biased tracers, MM+ (2020)



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