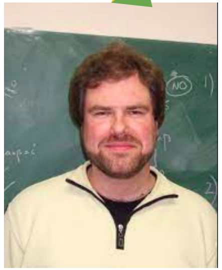
Observing relativistic effects in the bispectrum of large-scale structure

universität wien

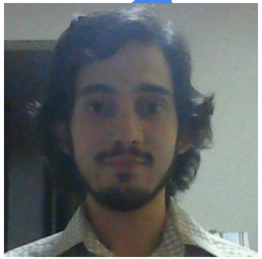
Thomas Montandon

- Bispectrum estimator
- Inflation/PNG
- Relativistic effects
- Simulation

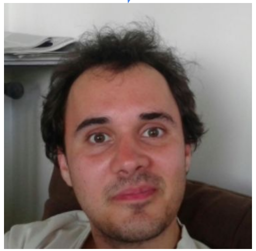


Bartjan van Tent

Jorge Noreña



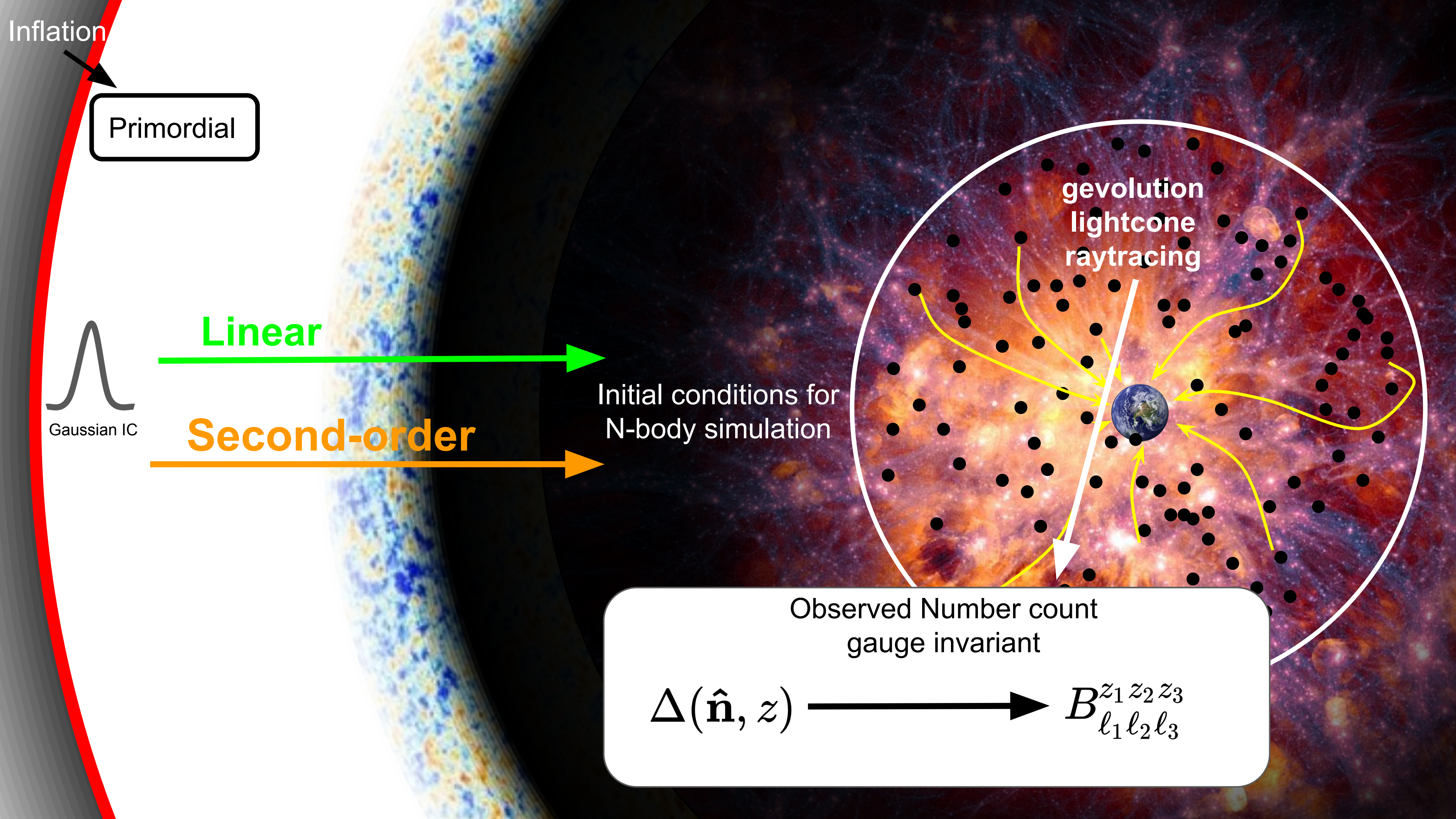
Juan Calles



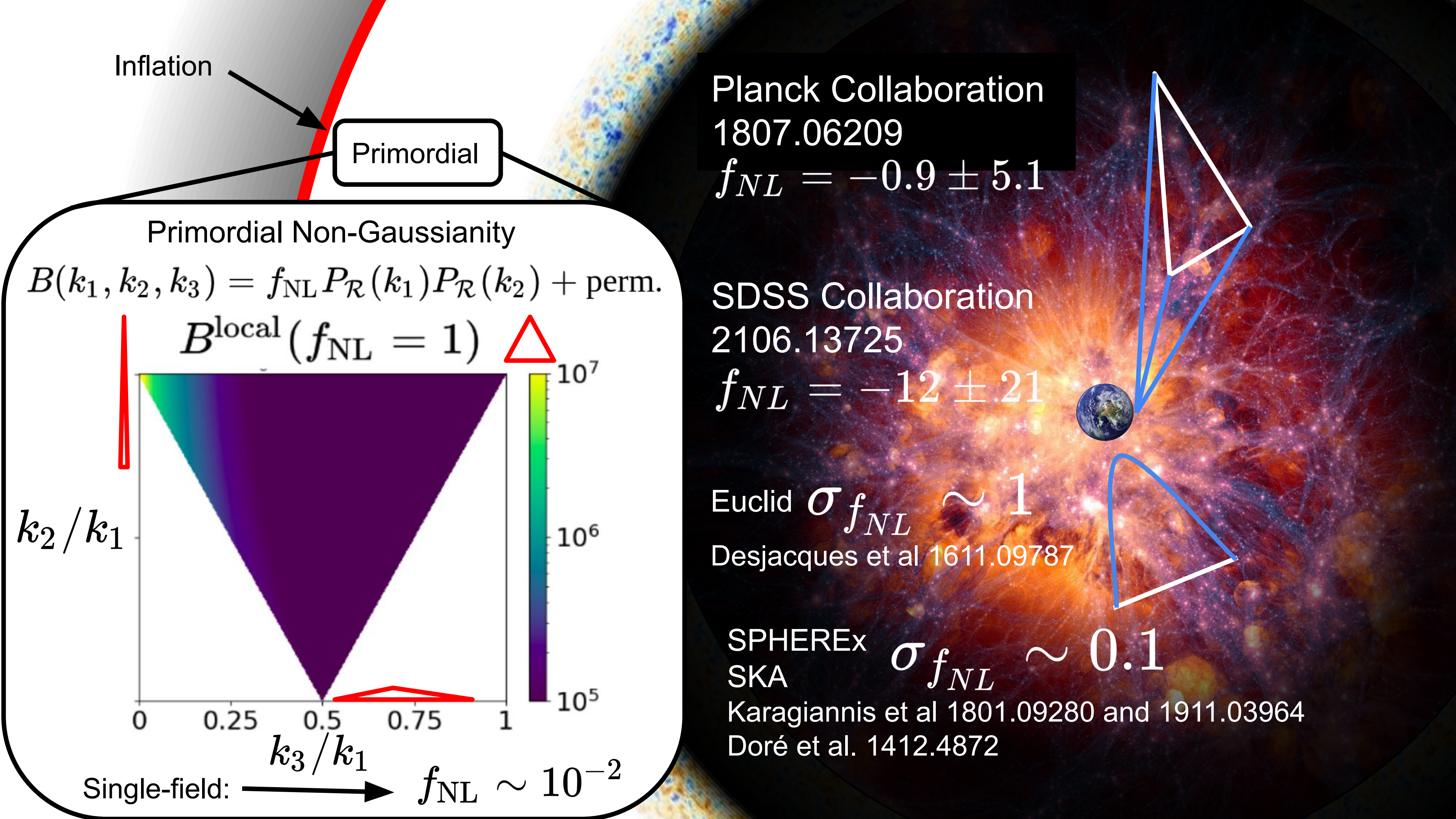
Clément Stahl

Julian Adamek

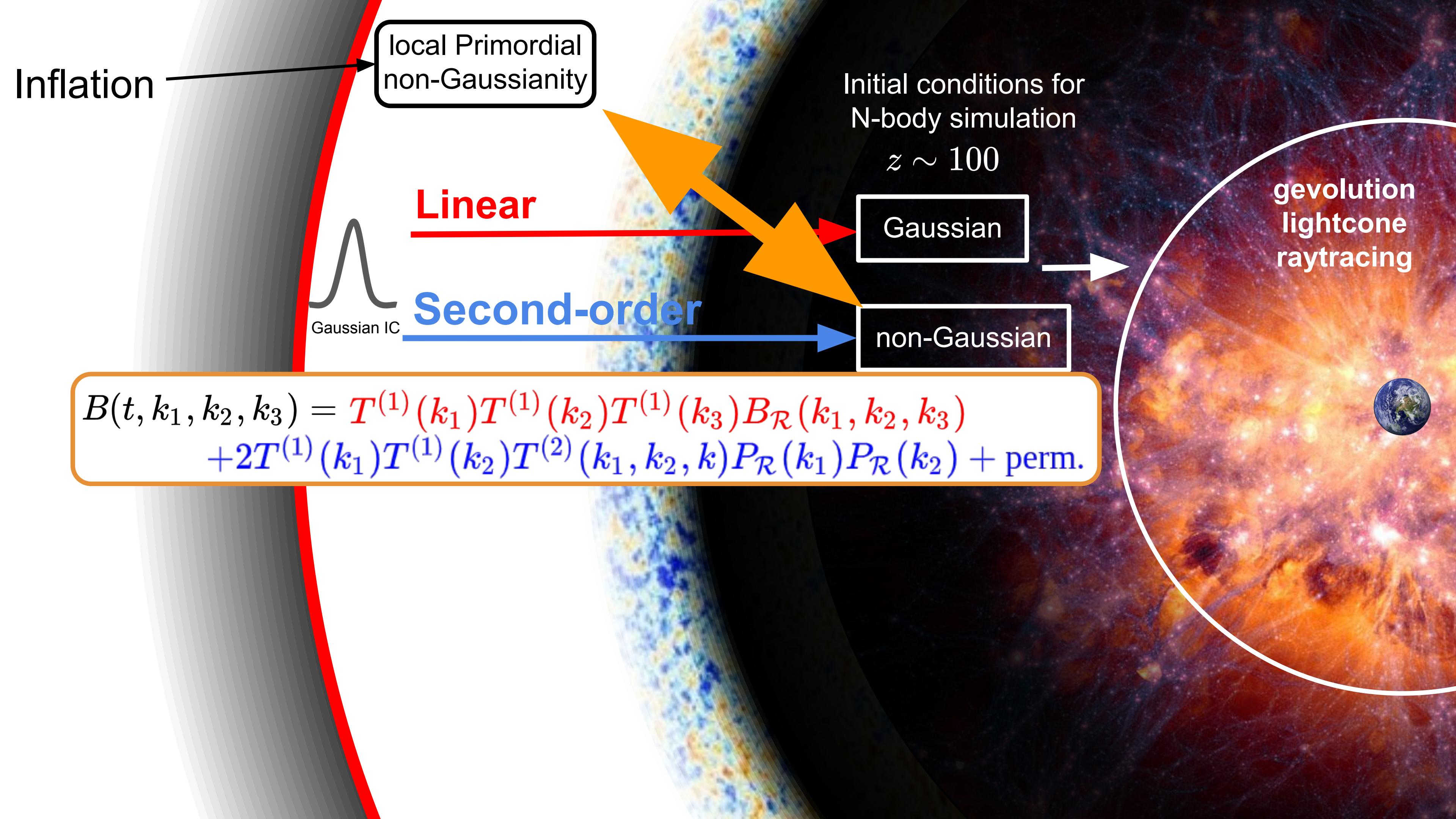
Oliver Hahn

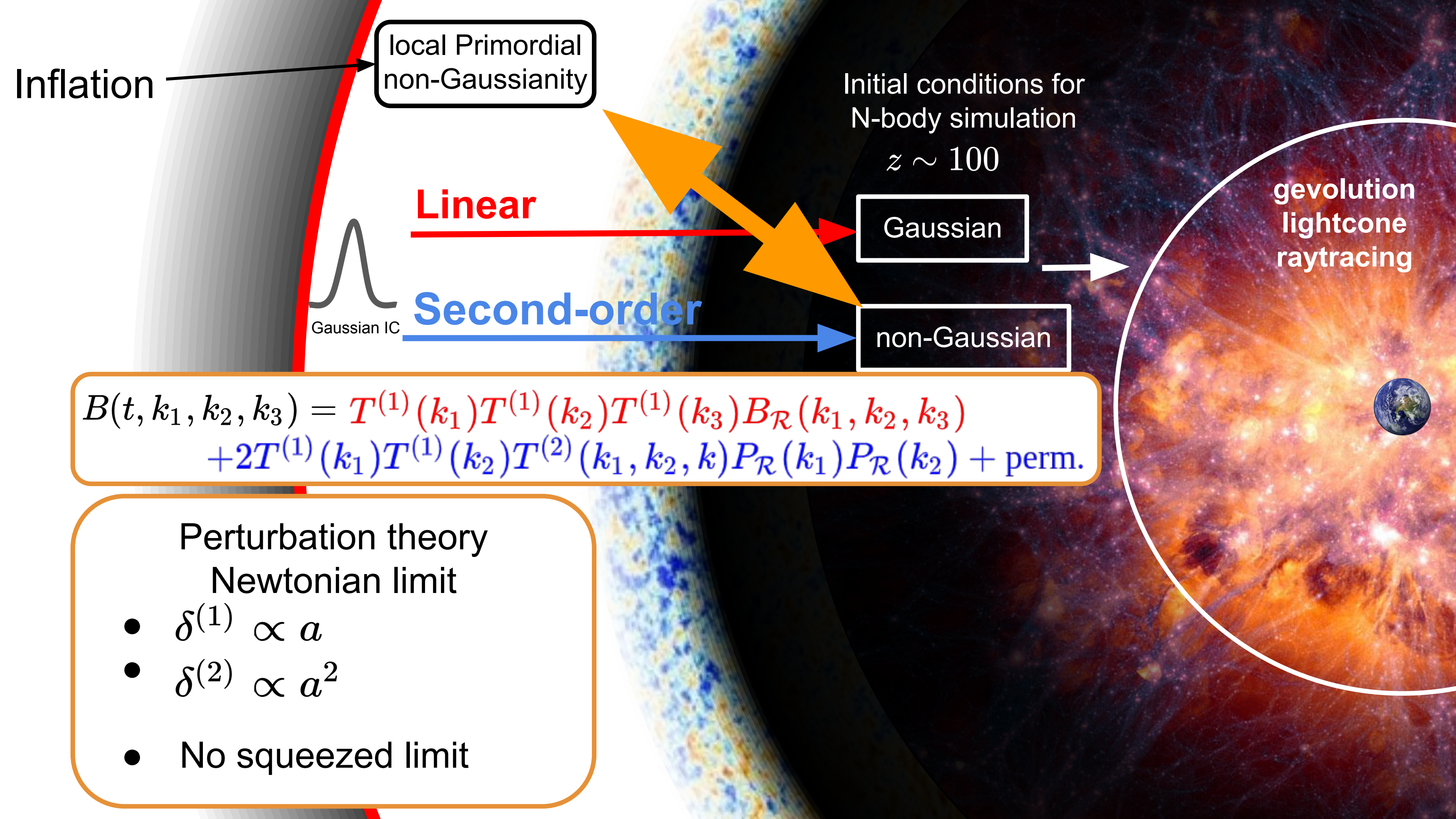


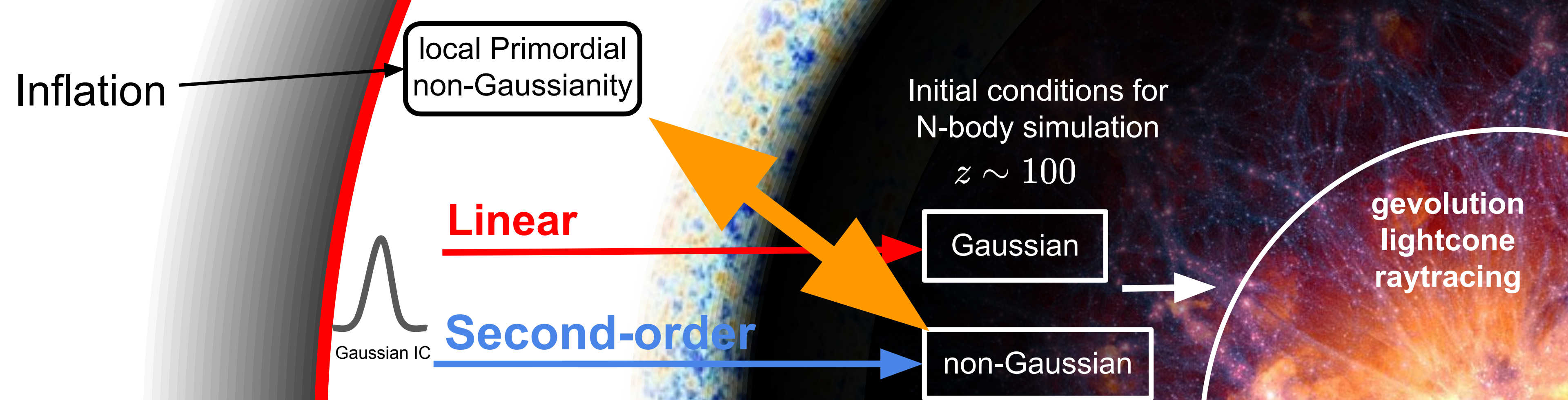
Inflation
Primordial
Linear
Second-order
Gaussian IC
Initial conditions for
N-body simulation
gevolution
lightcone
raytracing
Observed Number count
gauge invariant
$\Delta(\hat{\mathbf{n}}, z) \longrightarrow B^{z_1 z_2 z_3}_{\ell_1 \ell_2 \ell_3}$



Inflation
Primordial
Primordial Non-Gaussianity
$B(k_1,k_2,k_3) = f_{\rm NL} P_{\mathcal{R}}(k_1) P_{\mathcal{R}}(k_2) + \text{perm.}$
$B^{\rm local}(f_{\rm NL}=1)$
k_2/k_1
k_3/k_1
0
0.25
0.5
0.75
1
10^7
10^6
10^5
Single-field:
$f_{\rm NL} \sim 10^{-2}$
Planck Collaboration
1807.06209
$f_{NL} = -0.9 \pm 5.1$
SDSS Collaboration
2106.13725
$f_{NL} = -12 \pm 21$
Euclid $\sigma_{f_{NL}} \sim 1$
Desjacques et al 1611.09787
SPHEREx
SKA
$\sigma_{f_{NL}} \sim 0.1$
Karagiannis et al 1801.09280 and 1911.03964
Doré et al. 1412.4872


Inflation
local Primordial non-Gaussianity
Initial conditions for N-body simulation
$z \sim 100$
Linear
Gaussian
Gaussian IC
Second-order
non-Gaussian
gevolution
lightcone
raytracing
$B(t, k_1, k_2, k_3) = T^{(1)}(k_1)T^{(1)}(k_2)T^{(1)}(k_3)B_{\mathcal{R}}(k_1, k_2, k_3)$
$+2T^{(1)}(k_1)T^{(1)}(k_2)T^{(2)}(k_1, k_2, k)P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2) + \text{perm.}$


Inflation
local Primordial non-Gaussianity
Initial conditions for N-body simulation
$z \sim 100$
Linear
Gaussian
Gaussian IC
Second-order
non-Gaussian
gevolution lightcone raytracing
$B(t, k_1, k_2, k_3) = T^{(1)}(k_1)T^{(1)}(k_2)T^{(1)}(k_3)B_{\mathcal{R}}(k_1, k_2, k_3) + 2T^{(1)}(k_1)T^{(1)}(k_2)T^{(2)}(k_1, k_2, k)P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2) + \text{perm.}$
Perturbation theory
Newtonian limit
$\delta^{(1)} \propto a$
$\delta^{(2)} \propto a^2$
No squeezed limit



$$B(t,k_1,k_2,k_3) = T^{(1)}(k_1)T^{(1)}(k_2)T^{(1)}(k_3)B_{\mathcal{R}}(k_1,k_2,k_3) + 2T^{(1)}(k_1)T^{(1)}(k_2)T^{(2)}(k_1,k_2,k)P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2) + \text{perm.}$$

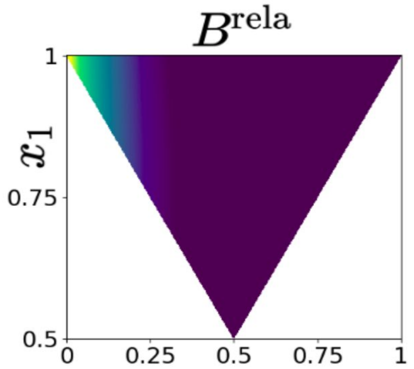
Perturbation theory
Newtonian limit

- $\delta^{(1)} \propto a$
- $\delta^{(2)} \propto a^2$
- No squeezed limit

Relativistic effects

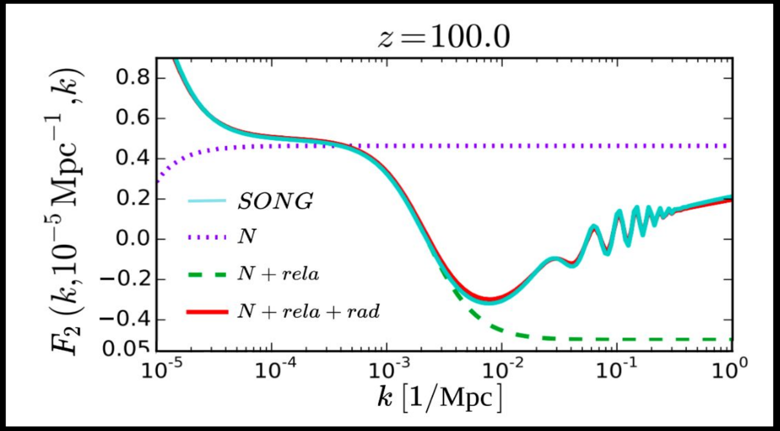
$$\propto \mathcal{H}^2/k^2$$

$$\delta_R^{(2)} \propto a^2\mathcal{H}^2/k^2 \propto a$$




Gaussian IC

Initial conditions for N-body simulation



1995 Matsubara (9510137)
1997 Matarrese (9707278)
2006 Bartolo, (0512481)
2010 Bartolo et al (1002.3759)
2015 Villa and Rampf (1505.04782)
2016 Tram et al (1602.05933)
2021 Adamek et al (2110.11249)
… and many more

$$\left(\frac{2}{3\mathcal{H}^2\Omega_m}\right)^{-2}\delta_{\rm fl}^{(2)} = \mathcal{H}^4\left[\frac{9}{4}uf^2 - 9\Omega_m f + \frac{3}{2}(1+2a_{\rm nl})(f+\frac{3}{2}\Omega_m)\right]\phi^2$$

$$-27\mathcal{H}^4\Omega_m f\Theta + \mathcal{H}^2\left[\frac{9}{4}\Omega_m - 2a_{\rm nl}(f+\frac{3}{2}\Omega_m)\right](\nabla\phi)^2$$

$$+\mathcal{H}^2\frac{18}{7}w\Psi + \mathcal{H}^2\left[\frac{9}{2}\Omega_m + (f+\frac{3}{2}\Omega_m)(1-2a_{\rm nl}) + f^2\right]\phi\Delta\phi$$

Relativistic effects

$$+\frac{1}{2}(1+\frac{3}{7}v)(\Delta\phi)^2 + \phi^{,l}\Delta\phi_{,l} + \frac{1}{2}(1-\frac{3}{7}v)\phi^{,lm}\phi_{,lm}$$

Newtonian

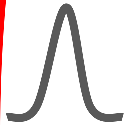
$$-\frac{1}{2}\left(f+\frac{3}{2}\Omega_m\right)\mathcal{F}_{\mathbf{x}}^{-1}\left[\left(\frac{\mathcal{H}^2}{k^2} + 3f\frac{\mathcal{H}^4}{k^4}\right)\frac{\partial\log T_\phi^{(1)}}{\partial\log k}\mathcal{F}_{\mathbf{k}}\left[2\phi\Delta^2\phi - 2(\Delta\phi)^2\right]\right]$$

Radiation effects

and

$v^{(2)}$

$\phi^{(2)}$



Gaussian IC

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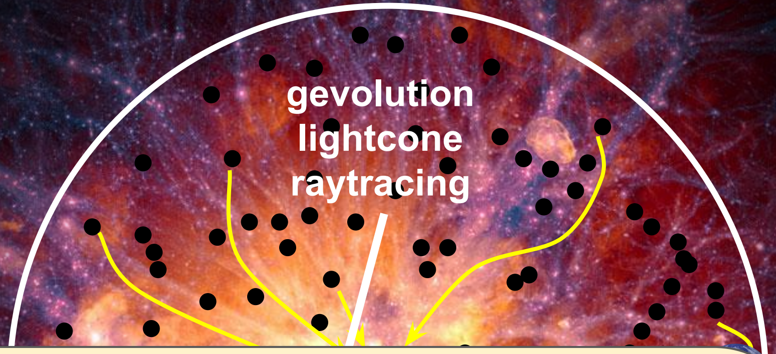

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2021 Adamek et al (2110.11249)
… and many more

MonofonIC (MUSIC2):

https://bitbucket.org/ohahn/monofonic/src/master/

Michaux, Hahn, Rampf, Angulo, 2008.09588

$\delta^{(2)}$

$v^{(2)}$

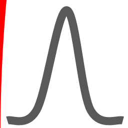
$\phi^{(2)}$

A relativistic hack

https://bitbucket.org/tomamtd/monofonic/src/relic/

- N-body/Poisson gauge (CLASS)
- Linear and quadratic growth factor solver (ODE)
- Eulerian perturbation theory up to second-order
- **De-aliasing**
- MPI+OpenMP/threads
- Only CDM

MonofonIC



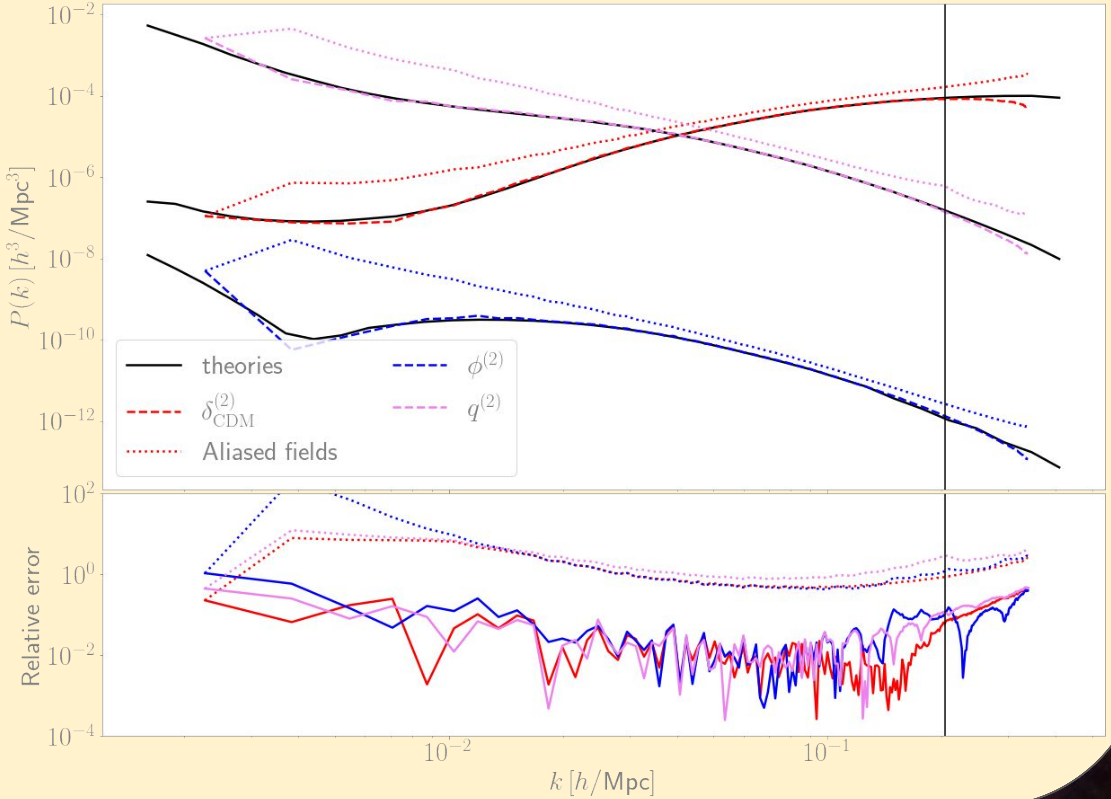
Gaussian IC

Box Size:

$3926\ \mathrm{Mpc}/h$

Computational time:

$1:30$ min



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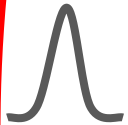
MonofonIC



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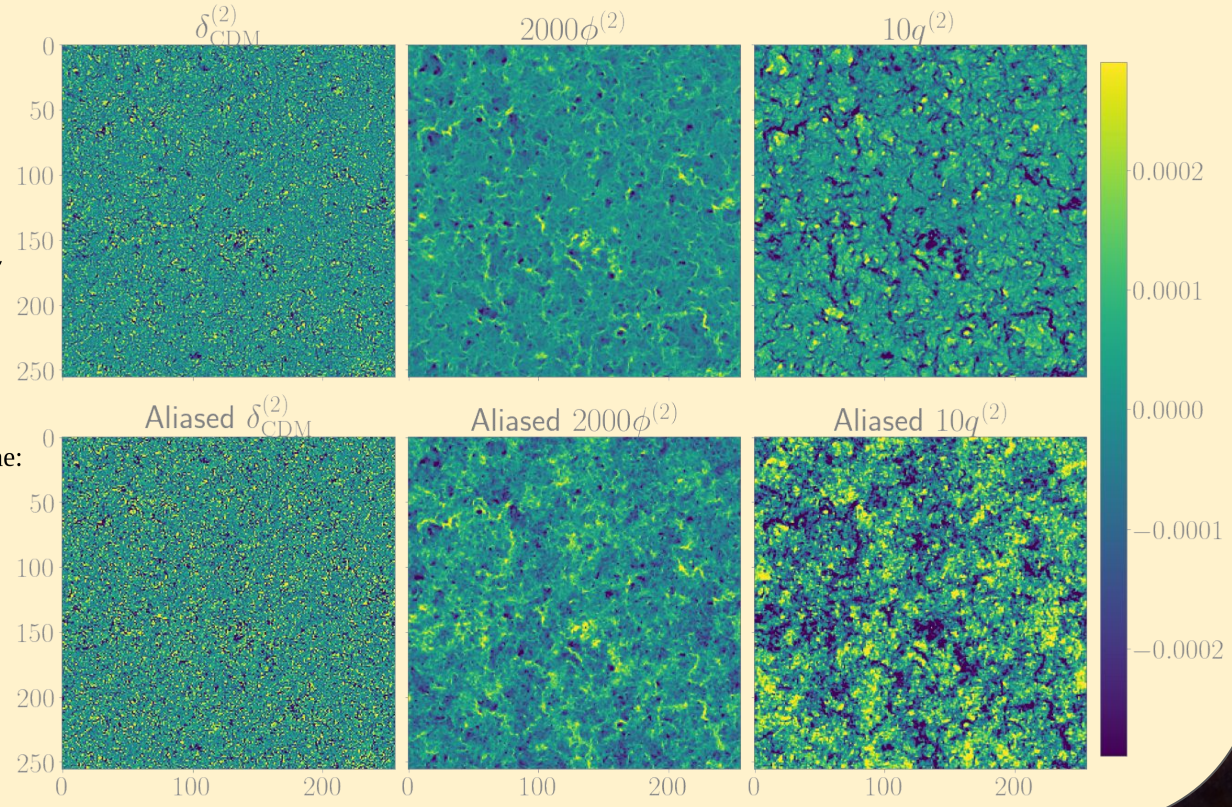


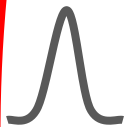
Initial conditions for N-body simulation

$z \sim 100$

gevolution

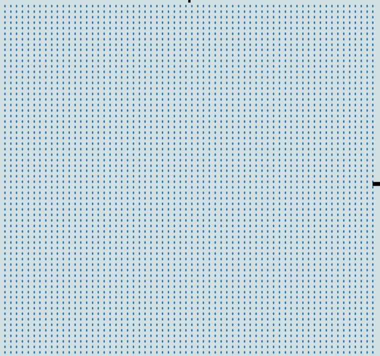
2021 Adamek et al (2110.11249)

gevolution
Discrete LPT: Iterative method



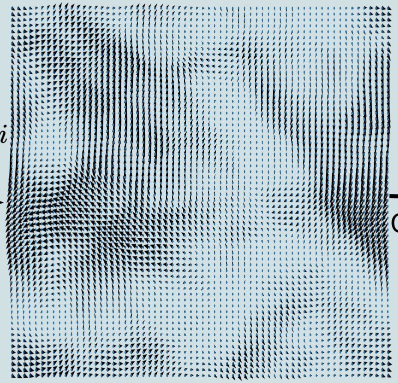
MonofonIC

Homogeneous template

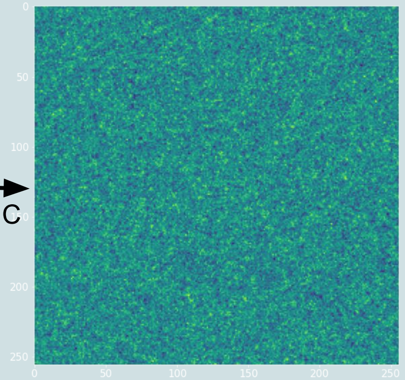


$x^i = y^i + \xi^i$

Displace particles



CIC



$$\rho(\mathbf{x}_g) = -\sum_{x'_g} \xi(\mathbf{x}'_g) \sum_{\rm p} \frac{m_p}{a^3} w_{\rm grad}(\mathbf{x}'_g - \mathbf{x}^0_p) \nabla \omega_{\rm CIC}(\mathbf{x}'_g - \mathbf{x}^0_p)$$

$\xi^{(1)}$ → **NON LINEAR** CIC → $\hat{\rho} = (1) + (1)^2 + \ldots$

Initial conditions for N-body simulation

$z \sim 100$

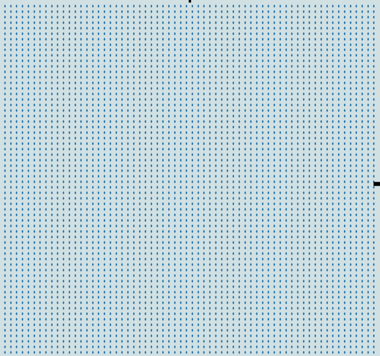
gevolution

2021 Adamek et al (2110.11249)

gevolution
Discrete LPT: Iterative method

Gaussian IC

Homogeneous template



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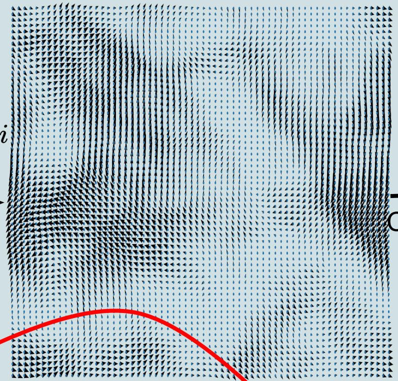

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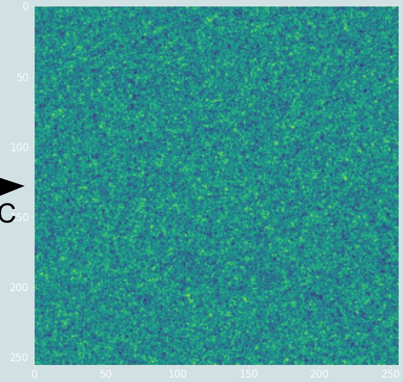


MonofonIC

$$\rho(\mathbf{x}_g) - \hat{\rho}(\mathbf{x}_g) = -\sum_{x'_g} \xi(\mathbf{x}'_g) \sum_{\mathrm{p}} \frac{m_p}{a^3} w_{\mathrm{grad}}(\mathbf{x}'_g - \mathbf{x}^0_p) \nabla \omega_{\mathrm{CIC}}(\mathbf{x}'_g - \mathbf{x}^0_p)$$

$\xi^{(1)} + \xi^{(2)}$ → **NON LINEAR** CIC → $\hat{\rho} = (1) + (1)^2 + \ldots$

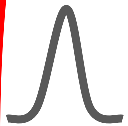
Initial conditions for N-body simulation

$z \sim 100$

gevolution

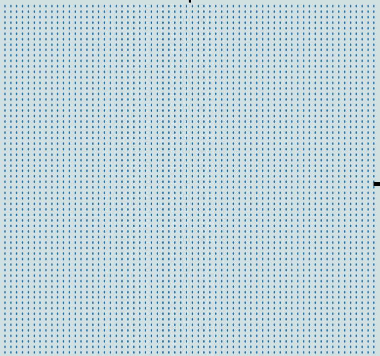
2021 Adamek et al (2110.11249)

gevolution
Discrete LPT: Iterative method



MonofonIC

Homogeneous template



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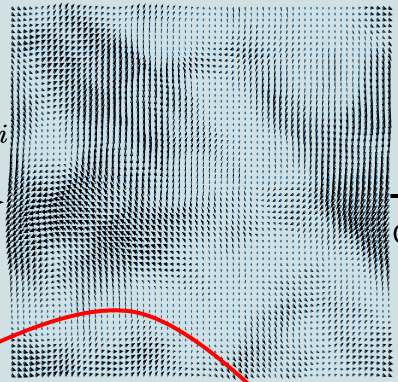
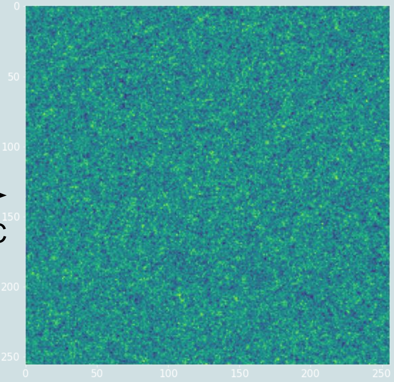
Displace particles



CIC



$$\rho(\mathbf{x}_g) - \hat{\rho}(\mathbf{x}_g) = -\sum_{x'_g} \xi(\mathbf{x}'_g) \sum_{\mathrm{p}} \frac{m_p}{a^3} w_{\mathrm{grad}}(\mathbf{x}'_g - \mathbf{x}^0_p) \nabla \omega_{\mathrm{CIC}}(\mathbf{x}'_g - \mathbf{x}^0_p)$$

$$\xi^{(1)} + \xi^{(2)} \rightarrow \textbf{NON LINEAR CIC} \rightarrow \hat{\rho} = (1) + (2) + (1)^2 + (2)^2 + \ldots$$

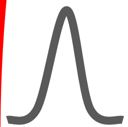
Initial conditions for N-body simulation

$z \simeq 100$

gevolution

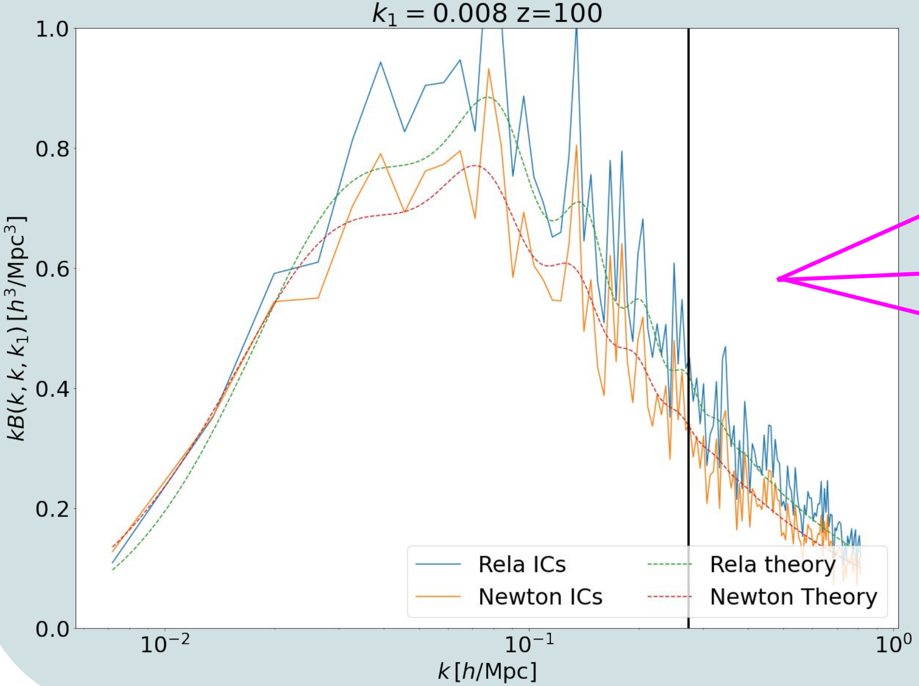
2016 Angulo and Pontzen (1603.05253)
Santiago Avila's talk

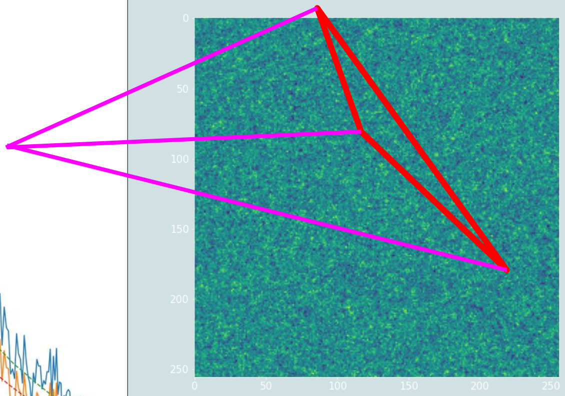
Pairing method



Gaussian IC

MonofonIC





Initial conditions for
N-body simulation

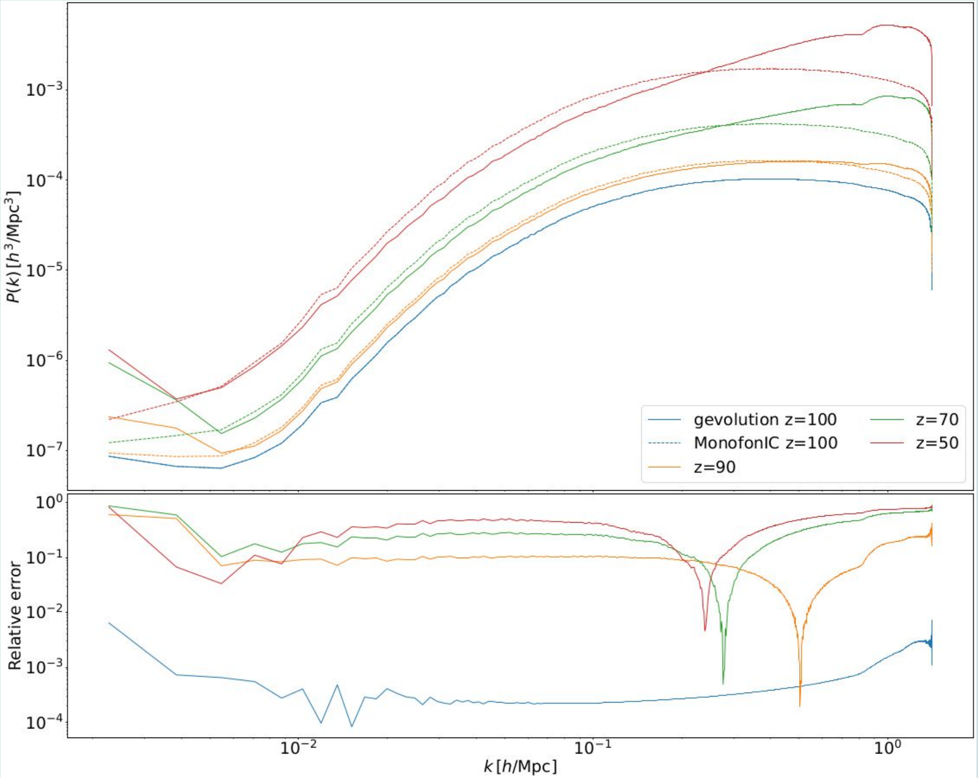
$z \simeq 100$

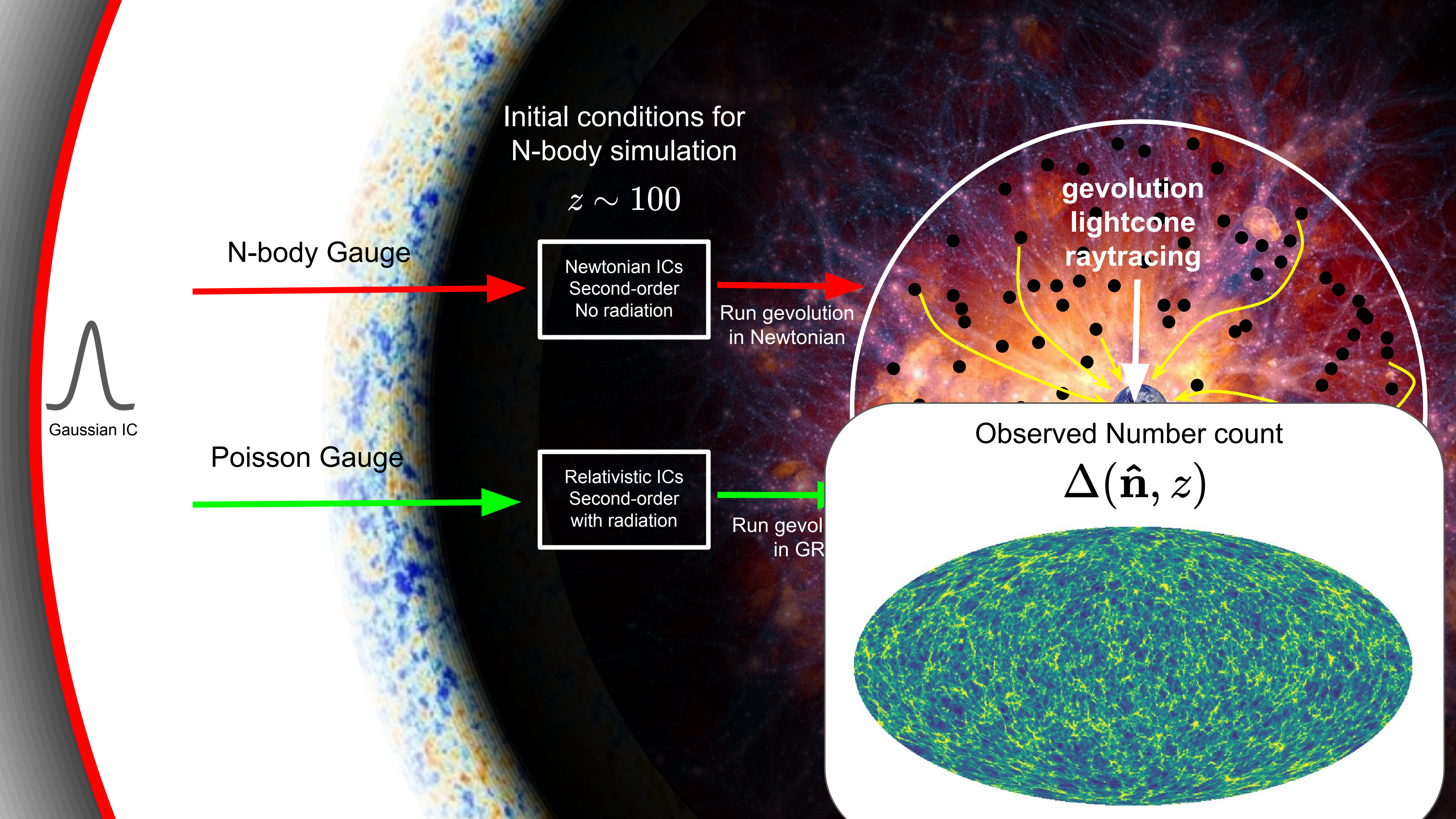
gevolution

Gaussian IC

MonofonIC

Second-order
evolution

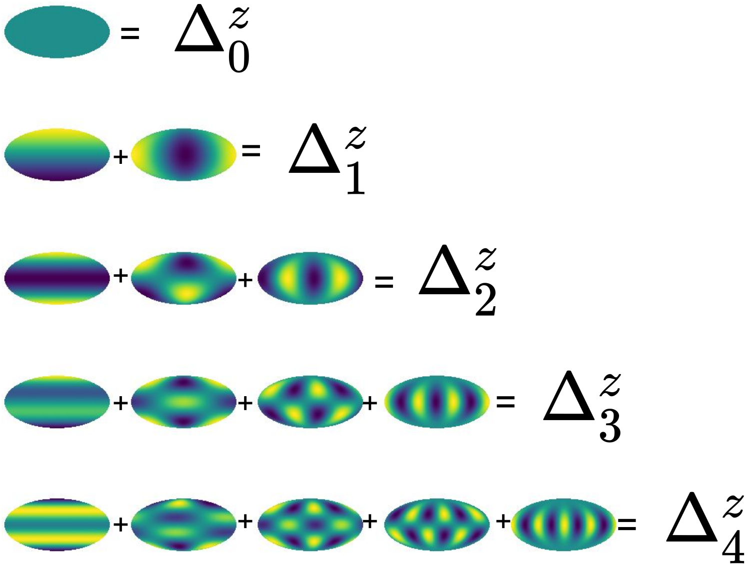




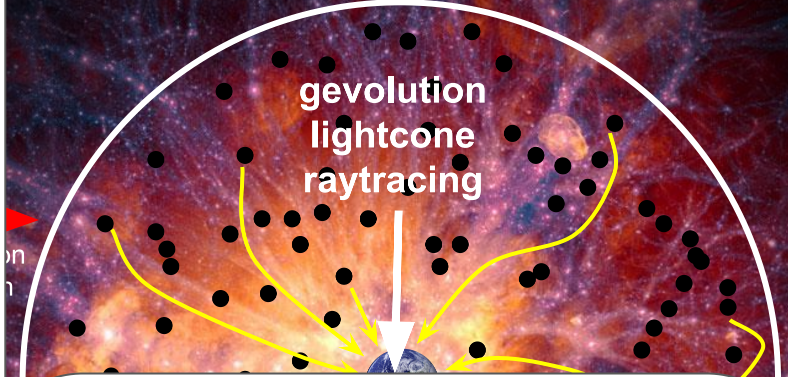
Initial conditions for
N-body simulation
$z \sim 100$
N-body Gauge
Newtonian ICs
Second-order
No radiation
Run gevolution
in Newtonian
gevolution
lightcone
raytracing
Gaussian IC
Poisson Gauge
Relativistic ICs
Second-order
with radiation
Run gevol
in GR
Observed Number count
$\Delta(\hat{\mathbf{n}}, z)$

The binned bispectrum estimator

$$\Delta_\ell^z(\hat{\mathbf{n}}) = \sum_{m=-\ell}^{\ell} a_{\ell m}^z Y_{\ell m}^*(\hat{\mathbf{n}})$$

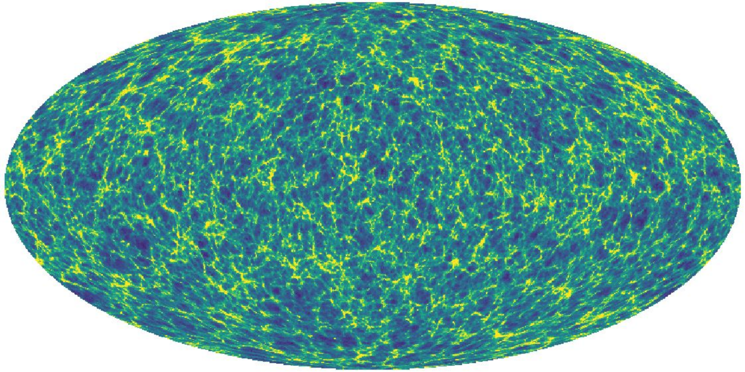


$$B_{\ell_1\ell_2\ell_3}^{z_1 z_2 z_3} = \left\langle \int d\hat{\mathbf{n}}\, \Delta_{\ell_1}^{z_1} \Delta_{\ell_2}^{z_2} \Delta_{\ell_3}^{z_3} \right\rangle$$



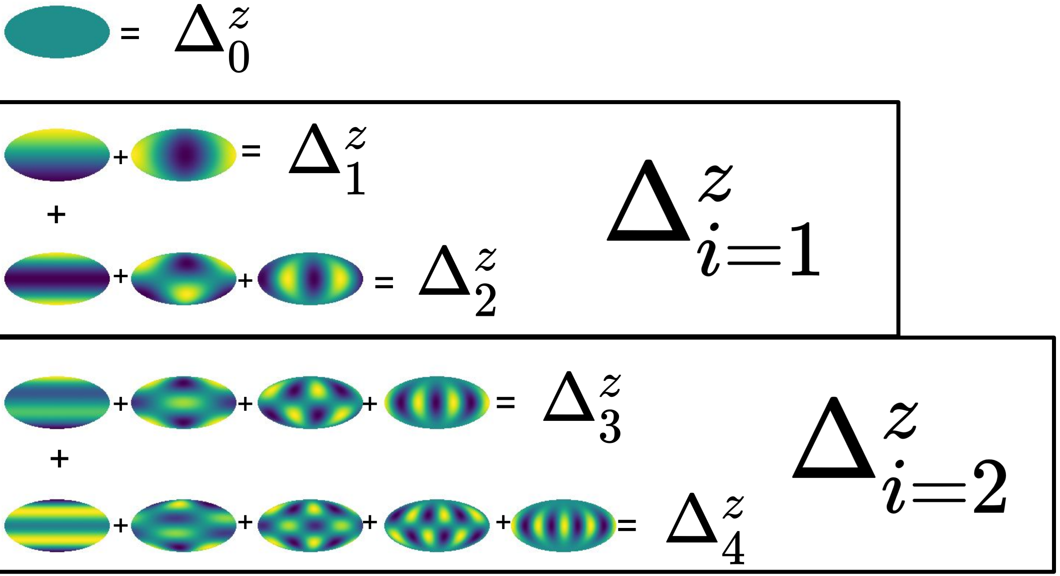
Observed Number count

$$\Delta(\hat{\mathbf{n}}, z)$$



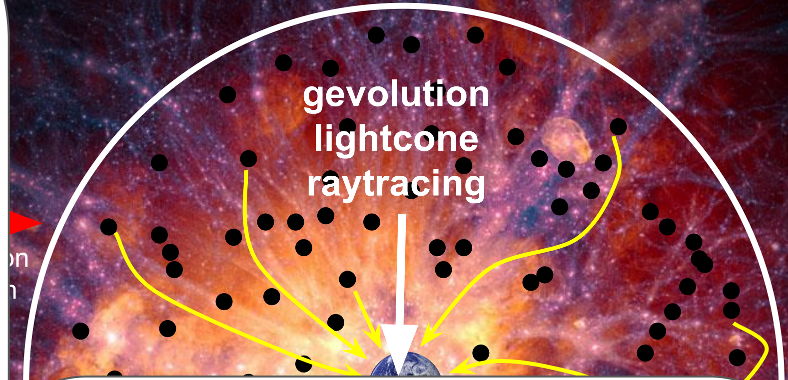
The binned bispectrum estimator

$$\Delta_i^z(\hat{\mathbf{n}}) = \sum_{\ell\in\Delta\ell} \sum_{m=-\ell}^{\ell} a_{\ell m}^z Y_{\ell m}^*(\hat{\mathbf{n}})$$



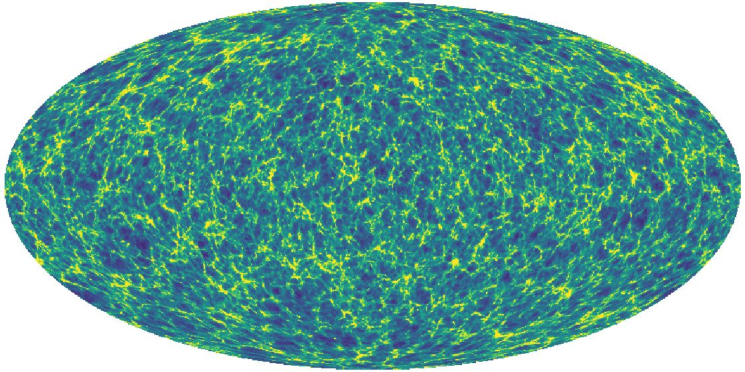
$$B_{i_1 i_2 i_3}^{z_1 z_2 z_3} \propto \left\langle \int d\hat{\mathbf{n}}\, \Delta_{i_1}^{z_1} \Delta_{i_2}^{z_2} \Delta_{i_3}^{z_3} \right\rangle$$

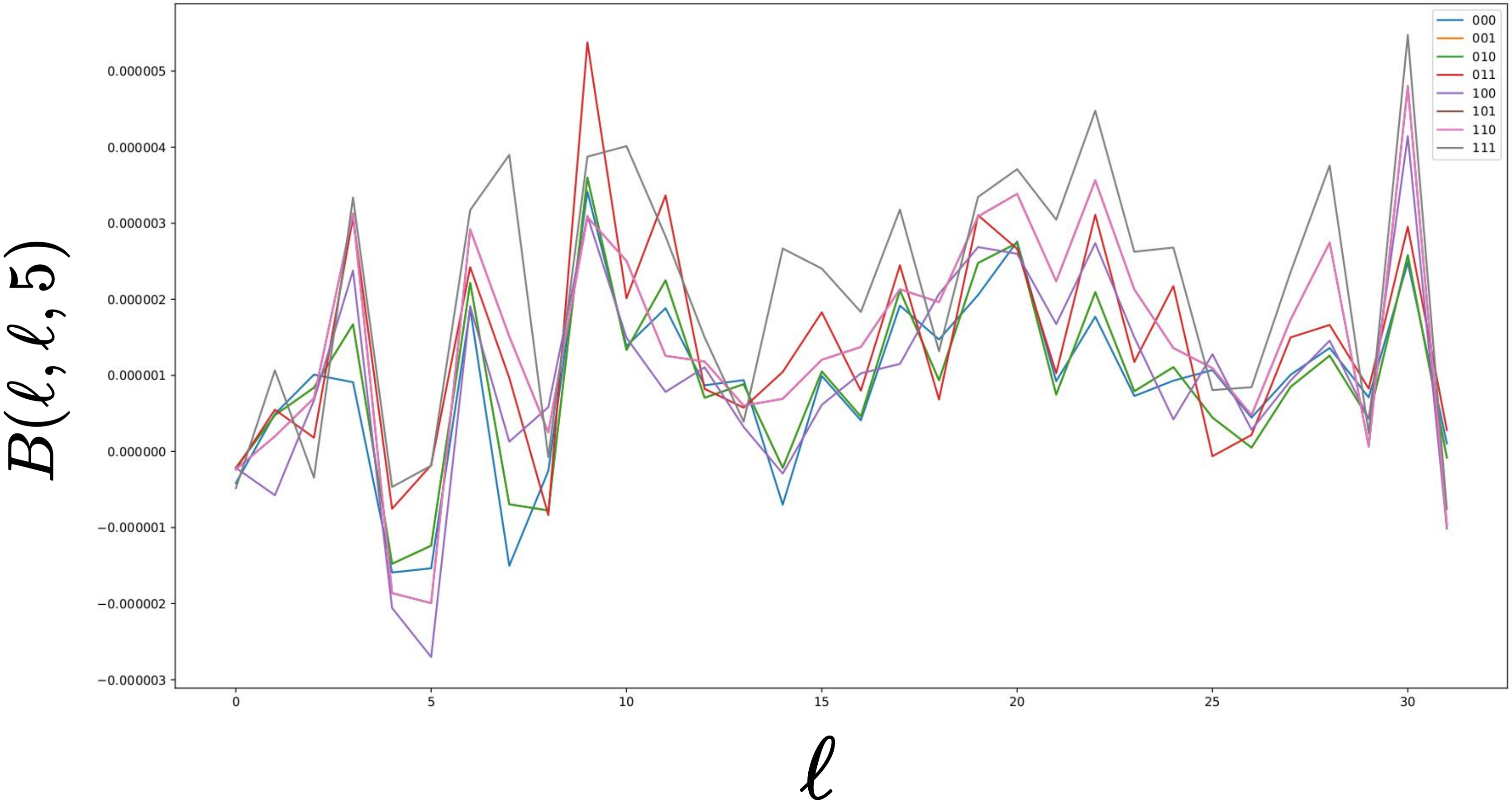
Planck: 99% of the information with 50 bins!
2015 Bucher et al 1509.08107



Observed Number count

$$\Delta(\hat{\mathbf{n}}, z)$$





$B(\ell, \ell, 5)$
ℓ
000
001
010
011
100
101
110
111
0.000005
0.000004
0.000003
0.000002
0.000001
0.000000
−0.000001
−0.000002
−0.000003
0
5
10
15
20
25
30

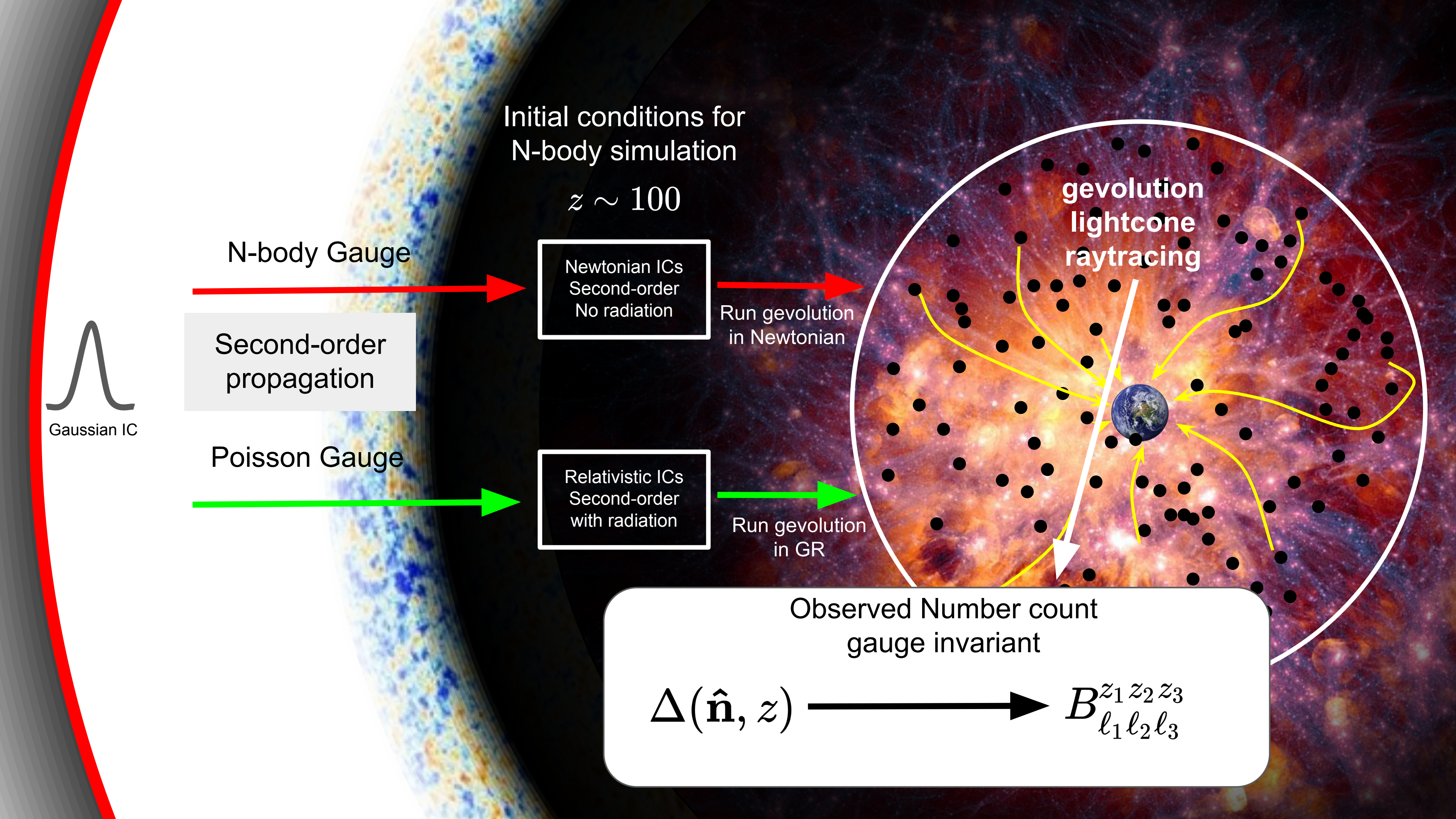
- Conclusion

$f_{\rm NL}$

$$f_{\rm NL}^{\rm Rela} = ?$$

- Local primordial non-Gaussianity and relativistic effects are degenerated
  - We must initialize N-body simulations at second-order
- Relativistic/Newtonian pipeline
  - Relativistic perturbation theory with MonofonIC
  - Discrete LPT with gevolution
  - Non-linear Raytracing
- Bispectrum estimator
  - First application of the CMB binned bispectrum estimator
  - Compare Newtonian/Relativistic


f_{NL}
Thank you !



Initial conditions for
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N-body Gauge
Second-order
propagation
Gaussian IC
Poisson Gauge
Newtonian ICs
Second-order
No radiation
Relativistic ICs
Second-order
with radiation
Run gevolution
in Newtonian
Run gevolution
in GR
gevolution
lightcone
raytracing
Observed Number count
gauge invariant
$\Delta(\hat{\mathbf{n}}, z) \longrightarrow B^{z_1 z_2 z_3}_{\ell_1 \ell_2 \ell_3}$

MonofonIC (MUSIC2):

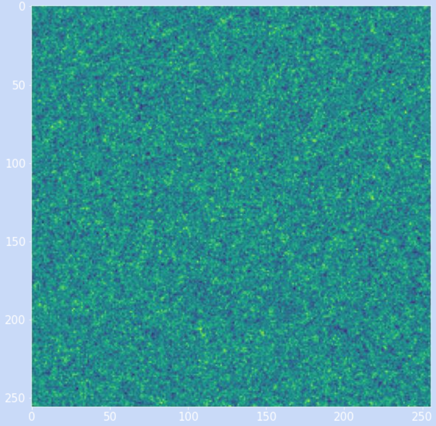
https://bitbucket.org/ohahn/monofonic/src/master/
Michaux, Hahn, Rampf, Angulo, 2008.09588

- Synchronous gauge (CLASS)
- Linear growth factor solver (ODE)
- Lagrangian perturbation theory up to third-order
- De-aliasing
- MPI+OpenMP/threads
- CDM+Baryon NL isocurvature modes

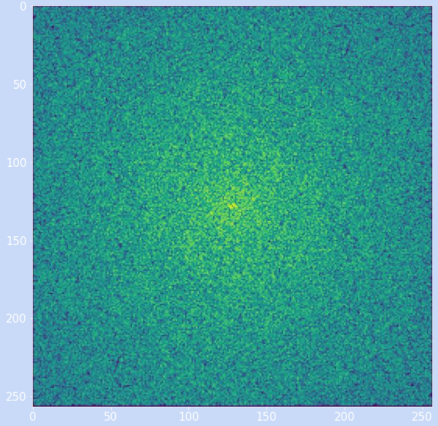
Aliasing? $f^2(\mathbf{x}) \neq f^2(\mathbf{x})$

$$(f^2)(\mathbf{k}) = \int \frac{d^3 k_1}{(2\pi)^3} f(\mathbf{k}_1) f(\mathbf{k} - \mathbf{k}_1)$$

$f(\mathbf{x})$



$f(\mathbf{k})$



finite resolution → Nyquist, FT periodic

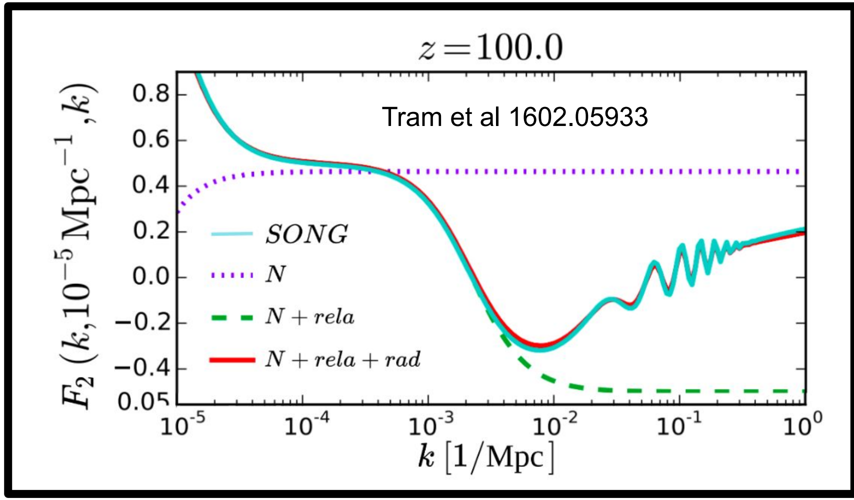
De-Aliasing: Orszag's 3/2 rule (Orszag 1971)

Initial conditions

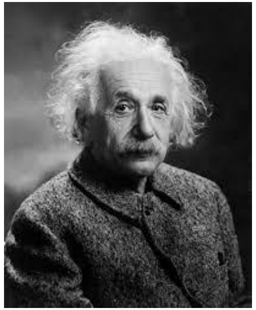
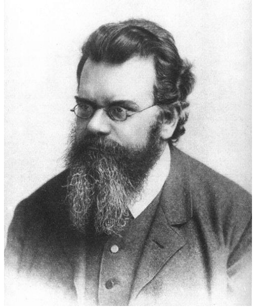
Second-order Einstein-Boltzmann system

1995 Matsubara (9510137)
1997 Matarrese, Mollerach, Bruni (9707278)
2006 Bartolo, Matarrese, Riotto (0512481)
2010 Bartolo et al (1002.3759)
2015 Villa and Rampf (1505.04782)
2016 Tram et al (1602.05933)
2021 Adamek et al (2110.11249)

… and many more



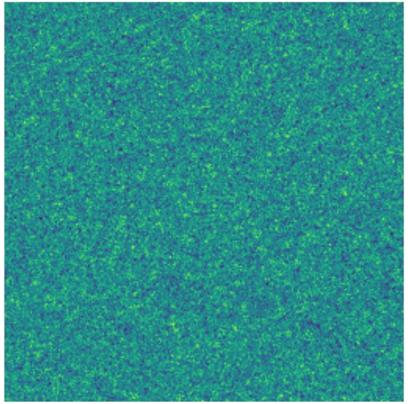
$$\mathcal{I}^{(2)}(\mathbf{k}_3) = \int \frac{d^3 k_1}{(2\pi)^3} T_{\mathcal{I}}^{(2)}(k_1, |\mathbf{k}_3 - \mathbf{k}_1|, k_3)\, \mathcal{R}(\mathbf{k}_1) \mathcal{R}(\mathbf{k}_3 - \mathbf{k}_1)$$

 $+$  $=$ 

SONG

Pettinari 1405.2280 $T^{(2)}$

$\delta^{(2)}$



Einstein equations

$G^0_0 = T^0_0 \;\to\; \phi^{(2)} = f(\delta^{(2)}, \chi^{(2)}, \phi'_{(2)})$

$(\hat{G}^i_j = \hat{T}^i_j)^{,j}_{,i} \;\to\; \chi^{(2)} = f((1)^2)$

$G^i_i = T^i_i \;\to\; \phi'_{(2)} = f(\chi^{(2)})$

$(G^i_0 = T^i_0)_{,i} \;\to\; v^{(2)} = f(\phi^{(2)}, \chi^{(2)}, \phi'_{(2)})$

Vanishing vorticy $\to$ curl $v^i_{(2)} = 0$

$\delta^{(2)}$

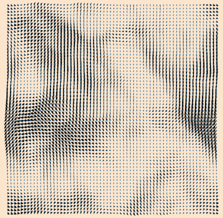
$v^{(2)}$

$\phi^{(2)}$

gevolution

$$\rho^{(1)}(\mathbf{x}_g) + \rho^{(2)}(\mathbf{x}_g) = -\sum_{x'_g} \xi(\mathbf{x}'_g) \sum_{\mathrm{p}} \frac{m_p}{a^3} w_{\mathrm{grad}}(\mathbf{x}'_g - \mathbf{x}^0_p) \nabla \omega_{\mathrm{CIC}}(\mathbf{x}'_g - \mathbf{x}^0_p)$$

$\xi^{(1)}$



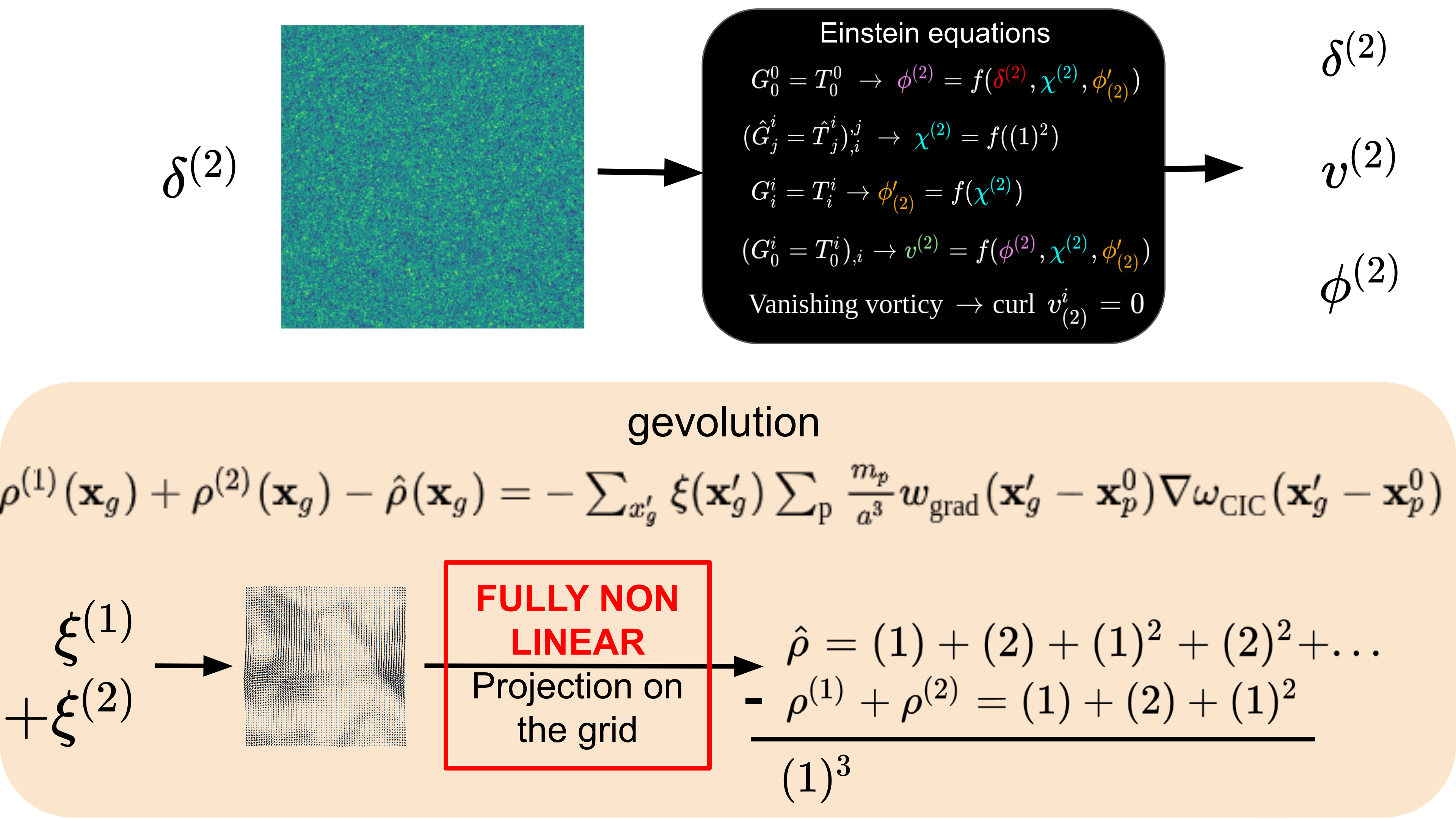
FULLY NON LINEAR

Projection on the grid

$$\hat{\rho} = (1) + (1)^2 + \dots$$

$$-\ \rho^{(1)} + \rho^{(2)} = (1) + (2) + (1)^2$$

$$(2)$$

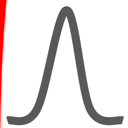

$\delta^{(2)}$
Einstein equations
$G^0_0 = T^0_0 \rightarrow \phi^{(2)} = f(\delta^{(2)}, \chi^{(2)}, \phi'_{(2)})$
$(\hat{G}^i_j = \hat{T}^i_j)^{,j}_{,i} \rightarrow \chi^{(2)} = f((1)^2)$
$G^i_i = T^i_i \rightarrow \phi'_{(2)} = f(\chi^{(2)})$
$(G^i_0 = T^i_0)_{,i} \rightarrow v^{(2)} = f(\phi^{(2)}, \chi^{(2)}, \phi'_{(2)})$
Vanishing vorticy $\rightarrow$ curl $v^i_{(2)} = 0$
$\delta^{(2)}$
$v^{(2)}$
$\phi^{(2)}$
gevolution
$\rho^{(1)}(\mathbf{x}_g) + \rho^{(2)}(\mathbf{x}_g) - \hat{\rho}(\mathbf{x}_g) = -\sum_{x'_g} \xi(\mathbf{x}'_g) \sum_p \frac{m_p}{a^3} w_{\rm grad}(\mathbf{x}'_g - \mathbf{x}^0_p) \nabla \omega_{\rm CIC}(\mathbf{x}'_g - \mathbf{x}^0_p)$
$\xi^{(1)}$
$+\xi^{(2)}$
FULLY NON LINEAR
Projection on the grid
$\hat{\rho} = (1) + (2) + (1)^2 + (2)^2 + \dots$
$- \rho^{(1)} + \rho^{(2)} = (1) + (2) + (1)^2$
$(1)^3$

Initial conditions for
N-body simulation

$z \simeq 100$

gevolution

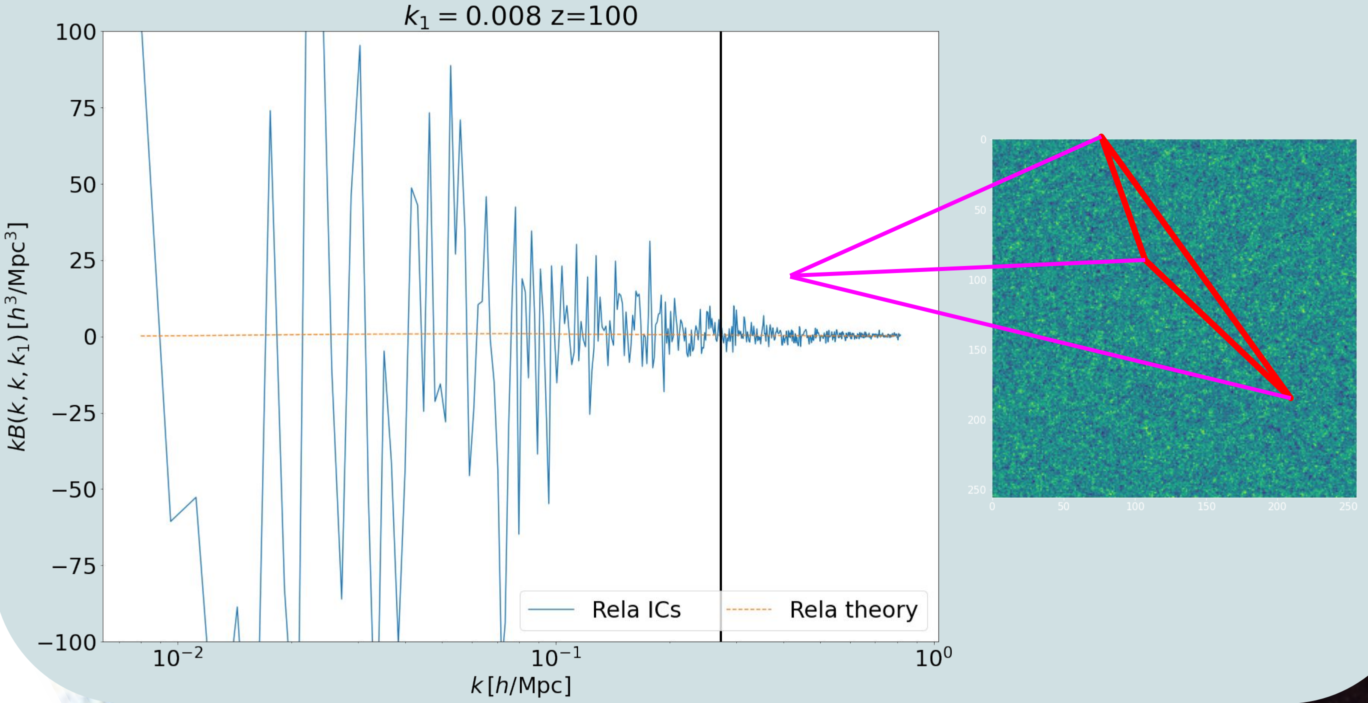
N-body Gaug



Poisson Gau

MonofonIC

Pairing method



Initial conditions for
N-body simulation

$z \simeq 100$

gevolution

N-body Gau

Gaussian IC

Poisson Gau

MonofonIC

Pairing method