# Bispectrum and finite volume effects: window-convolution

with F. Rizzo, M. Biagetti, E. Castorina, E. Sefusatti, P. Monaco

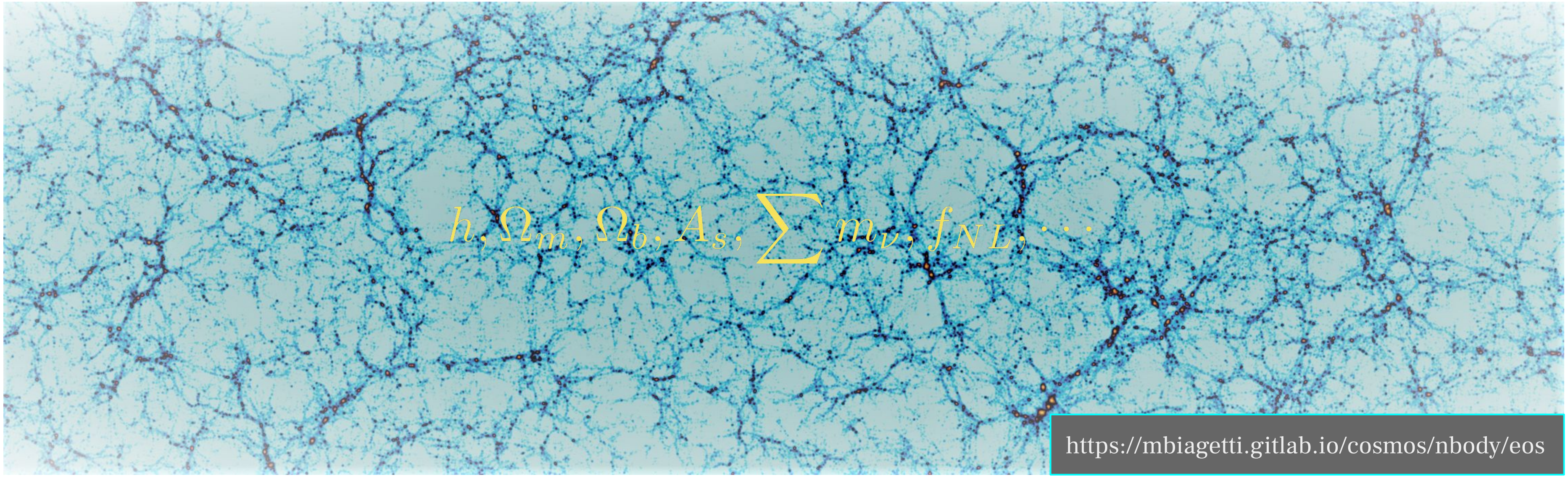
– PNG workshop, Madrid 2022 –

**Kevin Pardede**

**PhD Student @ AstroParticle Physics, SISSA**

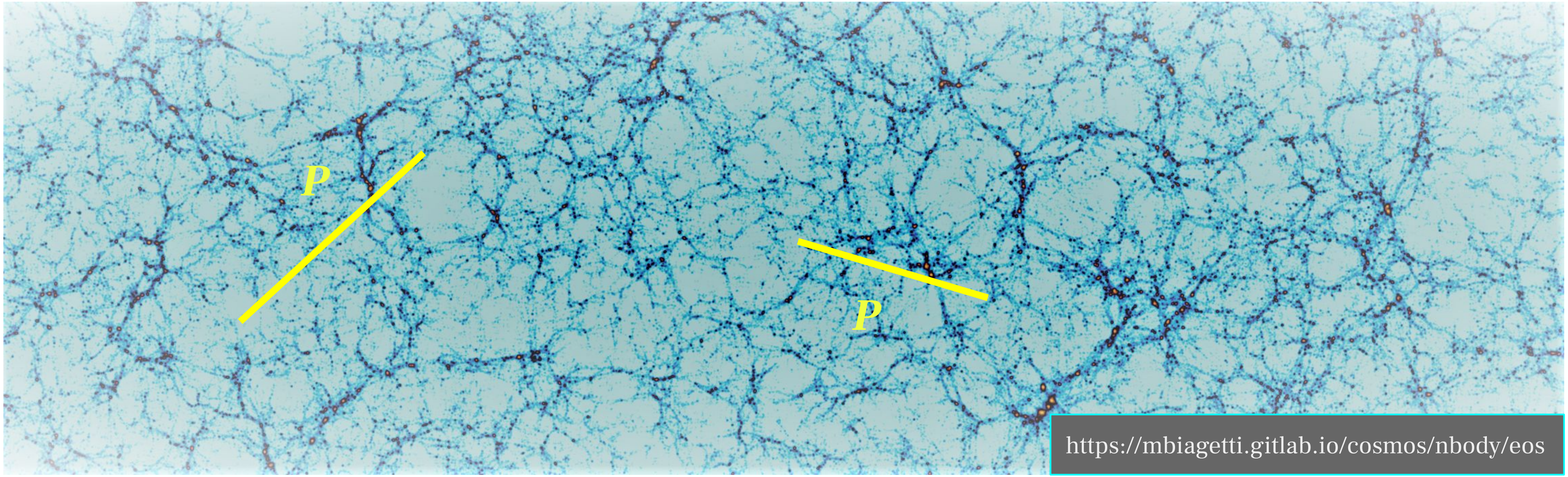
# Galaxy clustering

Galaxy distribution as an independent probe for the cosmological parameters



# Characterized by its summary statistics

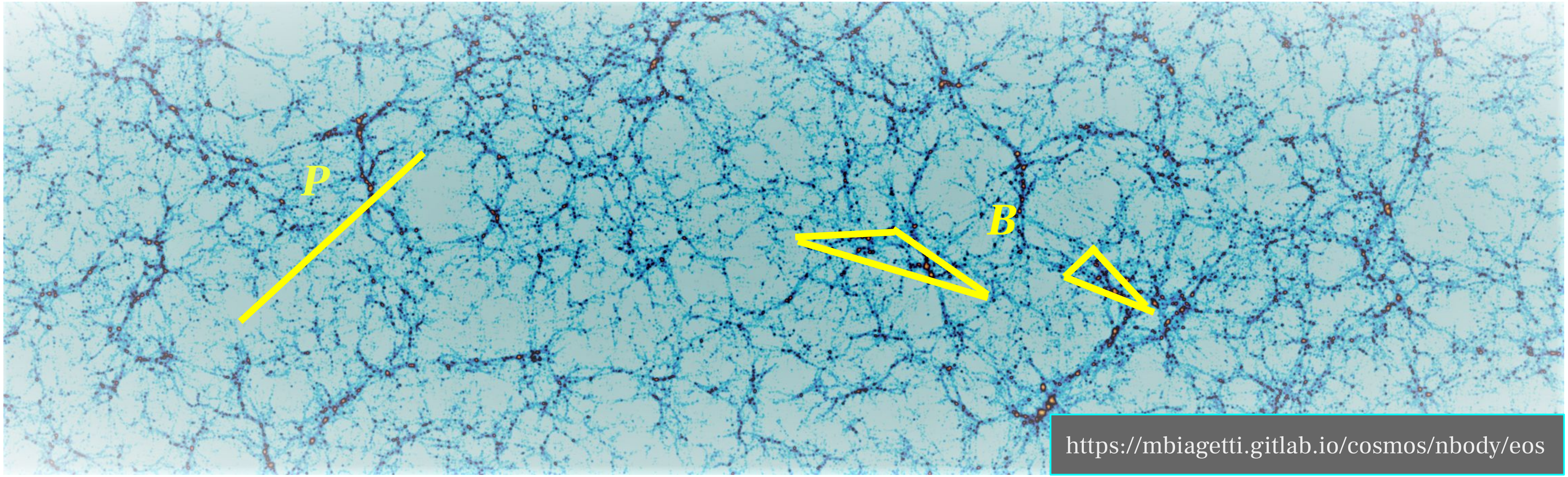
Power Spectrum (P) + Bispectrum (B) + …




$$\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\rangle = (2\pi)^3\delta_D(\mathbf{k}_1+\mathbf{k}_2)P(\mathbf{k}_1)$$

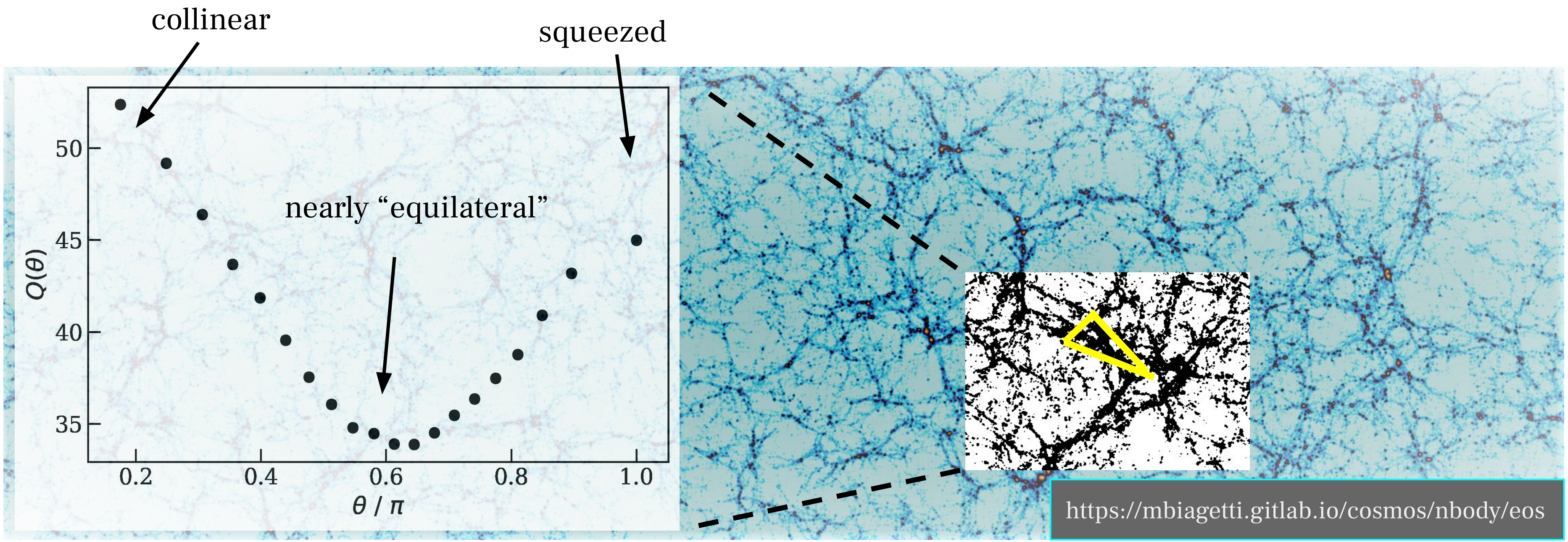
# Bispectrum captures non-Gaussianity

Power Spectrum (P) + **Bispectrum** (B) + …




$$\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle = (2\pi)^3 \delta_D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B(\mathbf{k}_1, \mathbf{k}_2)$$

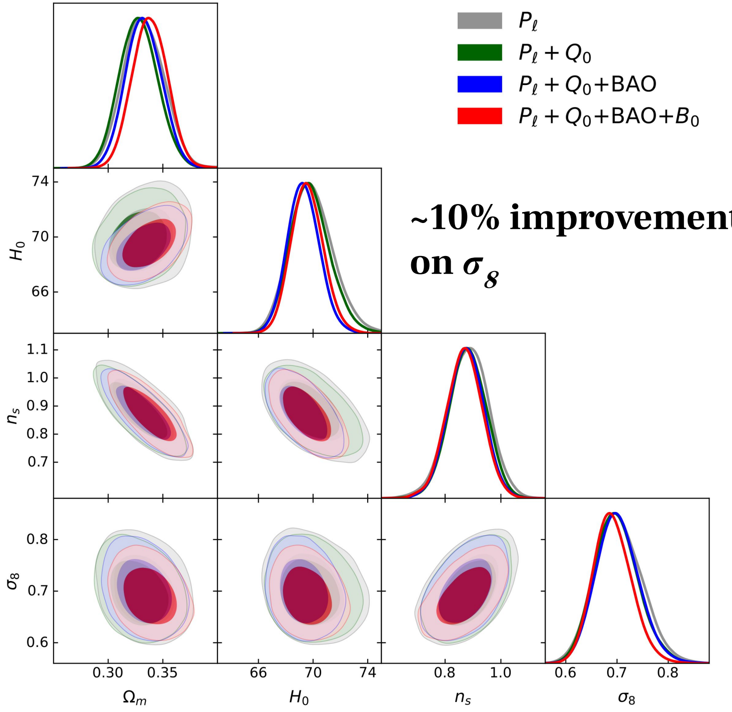
# … the filamentary structure




$$Q(k_1, k_2, k_3) \equiv \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + P(k_2)P(k_3) + P(k_3)P(k_1)}$$

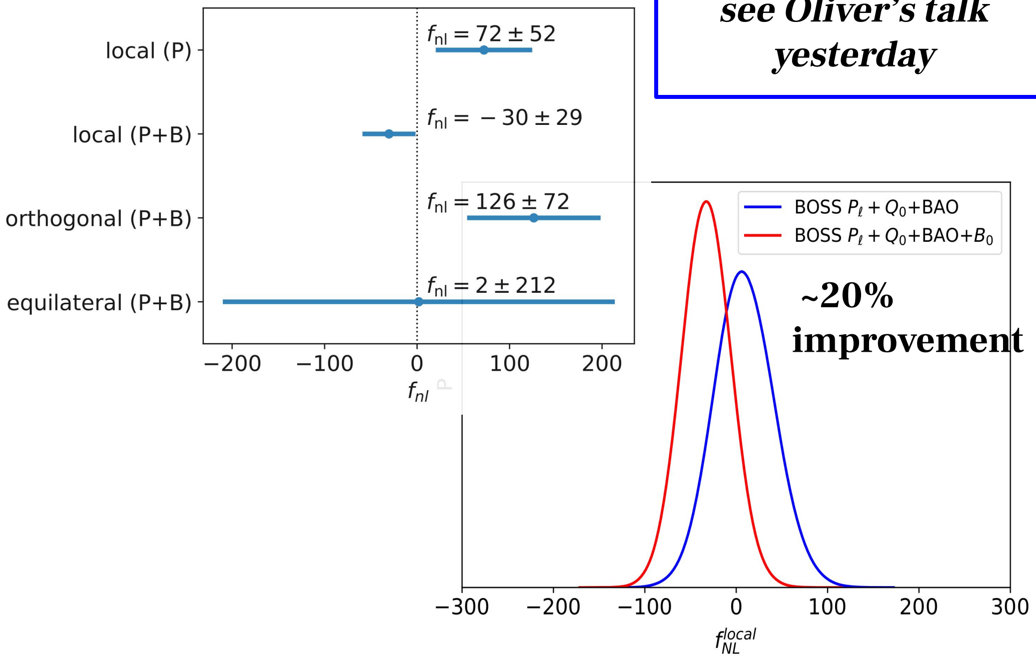
# Including bispectrum monopole (BOSS DR12)

**Constraint on cosmological params:**




**~10% improvement on $\sigma_8$**

**Constraint on primordial non-Gaussianity:**




***see Oliver's talk yesterday***

**~20% improvement**

Philcox&Ivanov21 (also: D'Amico+19)

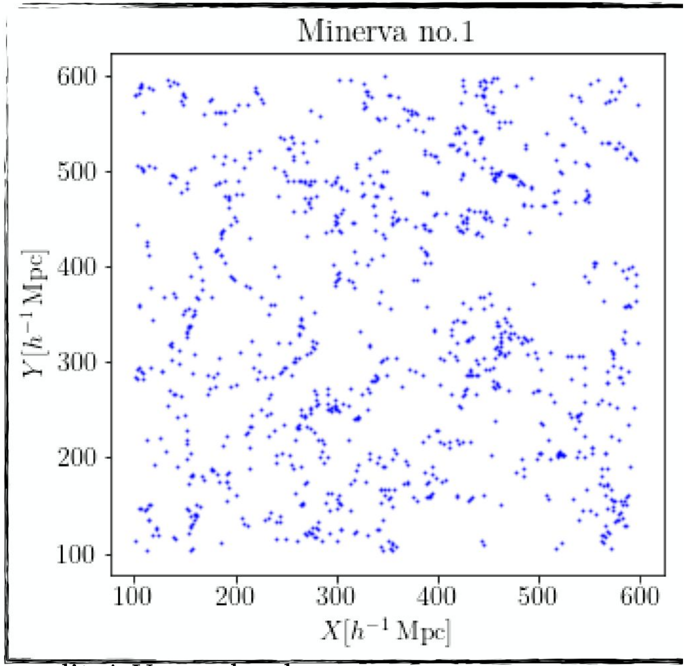
also one-loop bispectrum: Philcox+22, D'Amico+22b

Cabass+22b, D'Amico+22a

non-local PNG: Cabass+22a, bispectrum is **necessary**

# The bispectrum multipoles: test on simulations

1. 298 **Minerva** (N-body) Grieb+16
2. 10000 **Pinocchio** (3LPT) Monaco+02




credit: A.Veropalumbo

Rizzo, Moretti, Pardede+ (arXiv: 2204.13628)

**@**$z = 1$

ΛCDM cosmology

$L_{box} = 1500$ Mpc/$h$

$V_{eff} \simeq 1000\ (\mathrm{Gpc}/h)^3$

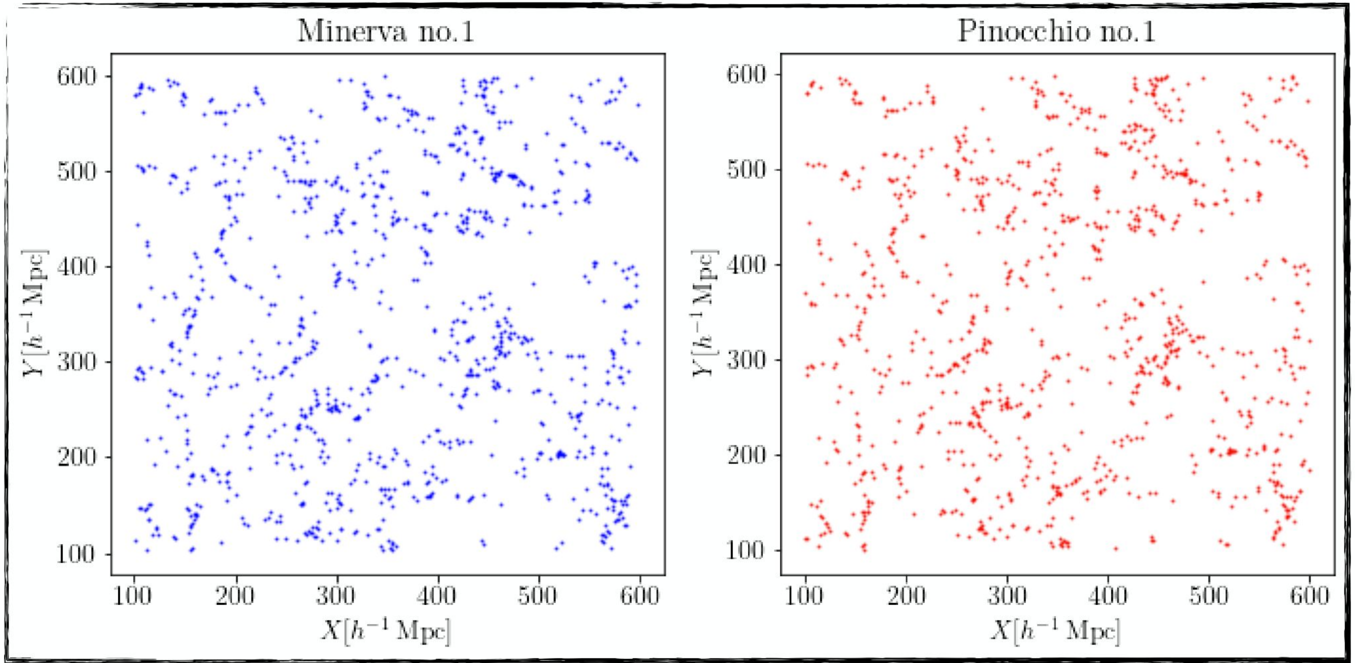
$\simeq$ 2x volume in PT-challenge Nishimichi+20

# … the numerical covariance

1. 298 **Minerva** (N-body) Grieb+16
2. 10000 **Pinocchio** (3LPT) Monaco+02

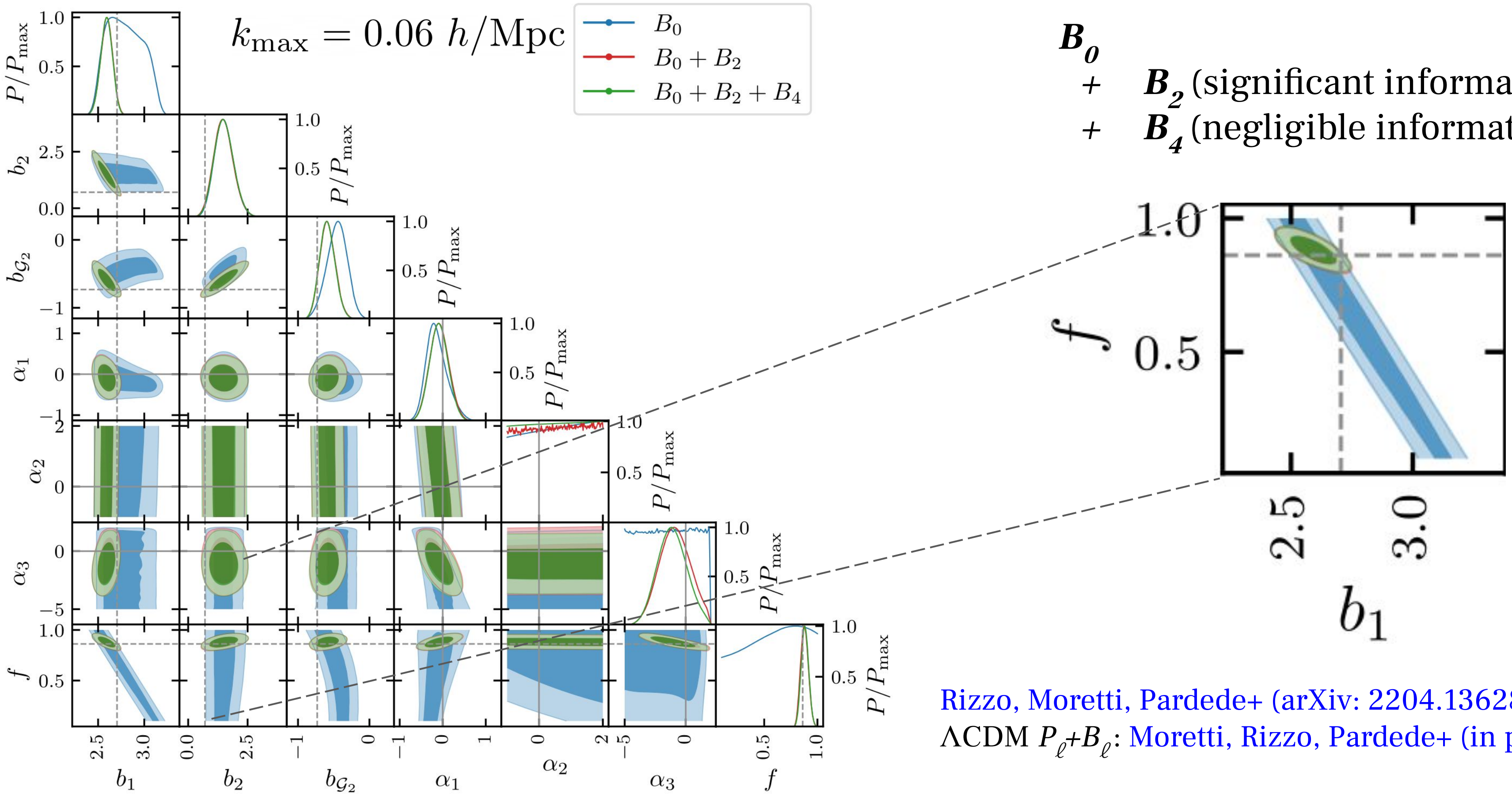
- approx. based on Lagrangian pert. theory
- relatively fast and accurate

provide a robust estimate of the covariance




credit: A.Veropalumbo

# Inclusion of the bispectrum multipoles




$\boldsymbol{B_0}$

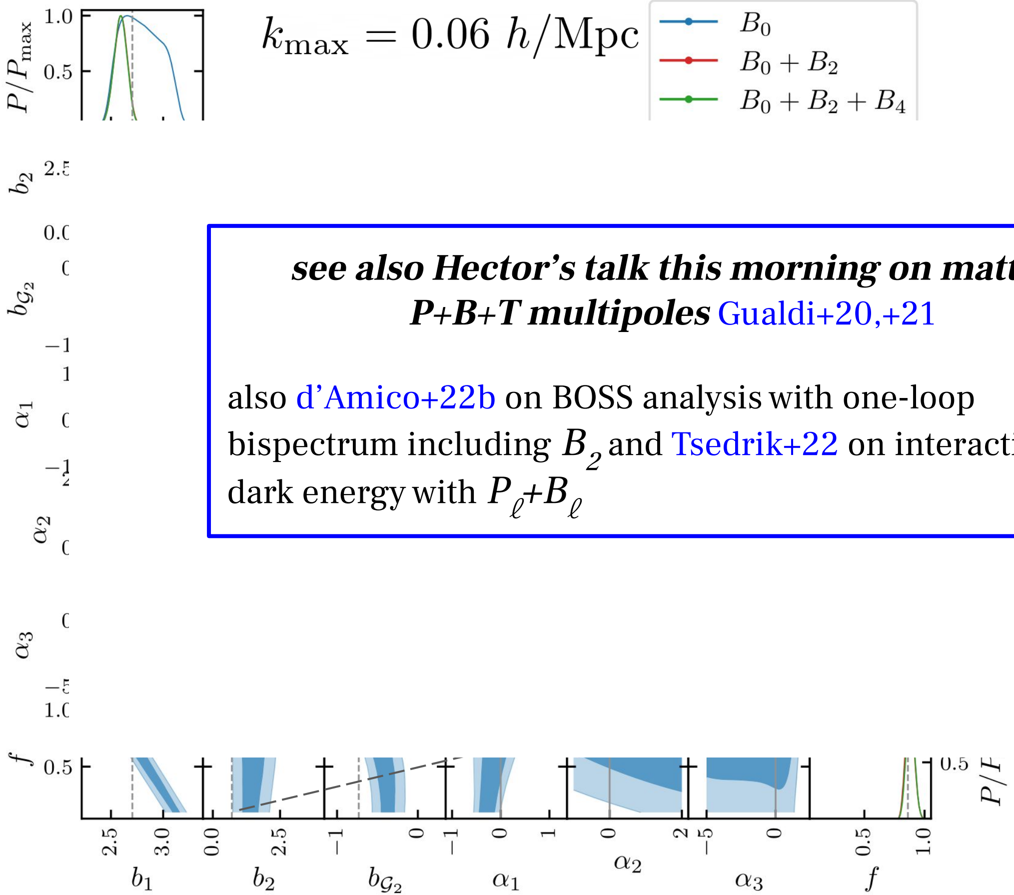
\+ $\boldsymbol{B_2}$ (significant information)

\+ $\boldsymbol{B_4}$ (negligible information)

Rizzo, Moretti, Pardede+ (arXiv: 2204.13628)

ΛCDM $P_\ell + B_\ell$: Moretti, Rizzo, Pardede+ (in prep.)

# Inclusion of the bispectrum multipoles



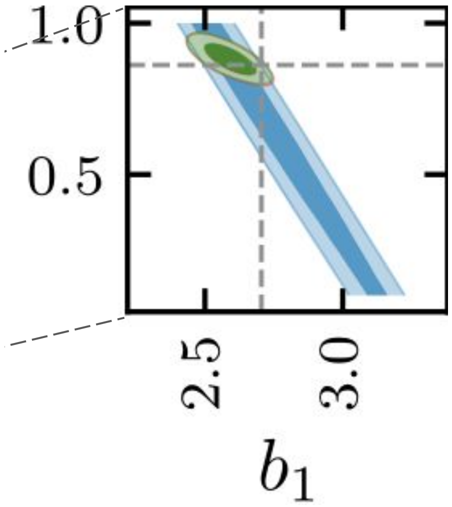

***see also Hector's talk this morning on matter P+B+T multipoles*** Gualdi+20,+21

also d'Amico+22b on BOSS analysis with one-loop bispectrum including $B_2$ and Tsedrik+22 on interacting dark energy with $P_\ell$+$B_\ell$

***$B_0$***

\+ ***$B_2$*** (significant information)

***$B_4$*** (negligible information)




Rizzo, Moretti, Pardede+ (arXiv: 2204.13628)

ΛCDM $P_\ell$+$B_\ell$: Moretti, Rizzo, Pardede+ (in prep.)

# Measurement vs. theory: large-scale systematics

Scoccimarro estimator* Scoccimarro +15

**FFT-based, optimal on small-scale**

$$\hat{B}_L(k_1,k_2,k_3)=\frac{2L+1}{V_B}\int_{k_1}d^3q_1\int_{k_2}d^3q_2\int_{k_3}d^3q_3\delta_D(\mathbf{q}_{123})\tilde{\delta}_L(\mathbf{q}_1)\tilde{\delta}(\mathbf{q}_2)\tilde{\delta}(\mathbf{q}_3)$$

**binning operator**

choose one side as the **LOS** $\tilde{\delta}_L(\mathbf{q})\equiv\int d^3x\;\tilde{\delta}(\mathbf{x})\mathcal{L}_L(\mathbf{q}\cdot\mathbf{x})e^{-i\mathbf{q}\cdot\mathbf{x}}$

**window function** $\tilde{\delta}(\mathbf{x})=W(\mathbf{x})\delta(\mathbf{x})$

$$\int_{k_1}d^3q_1\equiv\int_{|k_1-\Delta k/2|\le k_1\le|k_1+\Delta k/2|}d^3q_1\quad V_B\equiv\int_{k_1}d^3q_1\int_{k_2}d^3q_2\int_{k_3}d^3q_3\delta_D(\mathbf{q}_{123})$$

# Measurement vs. theory: large-scale systematics

Scoccimarro estimator* Scoccimarro +15

*window-free estimator Tegmark+97, has been revived recently: Philcox20, Philcox21

***see Oliver's talk yesterday***
***also Mike's talk tomorrow on hybrid basis approach***

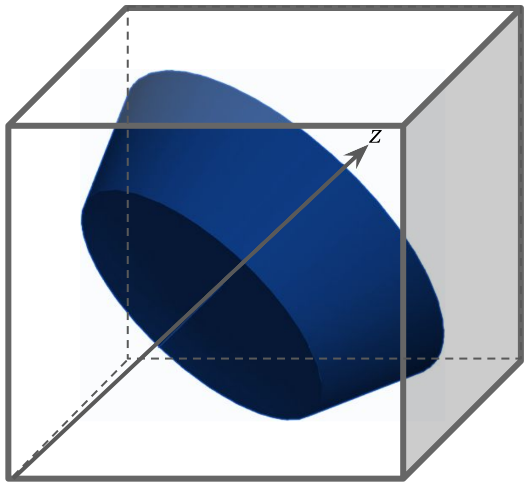
**window function** $\tilde{\delta}(\mathbf{x}) = W(\mathbf{x})\delta(\mathbf{x})$

# Survey window effects in bispectrum

Pardede, Rizzo, Biagetti, Castorina, Sefusatti, Monaco (arXiv: 2203.04174)

# Estimator is biased by window function

We need to Fourier transform $\delta(\mathbf{x})$



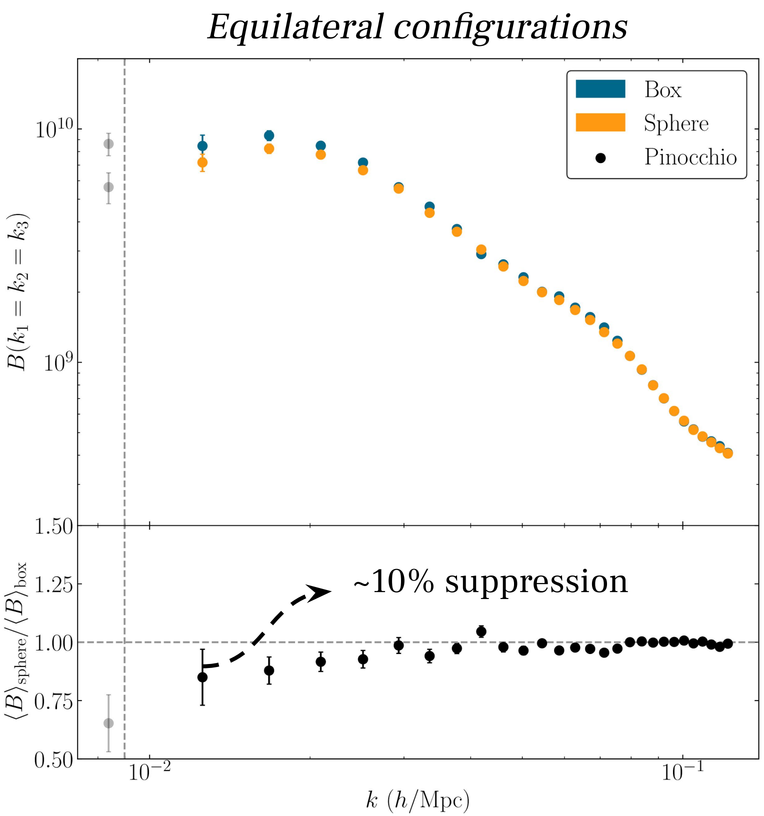

Baumgart, Fry 1991

Defined over a periodic box

$$\tilde{\delta}(\mathbf{x}) = \underbrace{W(\mathbf{x})}_{\text{window function}}\delta(\mathbf{x})$$

$$\Longrightarrow \tilde{\delta}(\mathbf{k}) = \int \frac{d^3k'}{(2\pi)^3} W(\mathbf{k}-\mathbf{k}')\delta(\mathbf{k}')$$

# In bispectrum …




window convolution will mix modes

**10000 Pinocchio sphere catalogue**

*Note*: this is a huge volume ≈ 3500 [Gpc/$h$]$^3$

… main effect is on large scale

# To include window effect in bispectrum

Schematically (monopole with no binning operator)

$$\tilde{B}(\vec{k}_1, \vec{k}_2) = \int \frac{d^3p_1}{(2\pi)^3} \int \frac{d^3p_2}{(2\pi)^3} B_W(\vec{k}_1 - \vec{p}_1, \vec{k}_2 - \vec{p}_2) B(\vec{p}_1, \vec{p}_2)$$

- 6D integral
- Time ~ **hours**/evaluation
- Not feasible for likelihood analysis

# An approximation

$$\tilde{B}[P_L] \simeq B[\tilde{P}_L]$$

**1DFFTLog-approx**

$$\tilde{B}(\mathbf{k}_1, \mathbf{k}_2) \simeq Z(\mathbf{k}_1, \mathbf{k}_2)\tilde{P}_L(k_1)\tilde{P}_L(k_2) + \text{cyc.}$$

- Reduced to power spectrum-window convolution
  see e.g. Wilson+15, Castorina+17, d'Amico+19
- BOSS DR 11/12 Gil-Marin+14a, b and +16a, b
- Recently used in d'Amico +19,+22
- Doesn't work for squeezed triangles

Computed via (1D) FFTLog

# Exact bispectrum window convolution

Taking $\langle \hat{B}_L \rangle = \tilde{B}_L$

$$\tilde{B}_L(k_1, k_2, k_3) = \frac{2L+1}{V_B} \int_{k_1} d^3q_1 \int_{k_2} d^3q_2 \int_{k_3} d^3q_3 \, \delta_D(\vec{q}_{123})$$
$$\times \int d^3x_3 \int d^3x_{13} \int d^3x_{23} \, e^{-i\vec{q}_1 \cdot \vec{x}_{13}} e^{-i\vec{q}_2 \cdot \vec{x}_{23}} \zeta(\vec{x}_{13}, \vec{x}_{23}, \hat{x}_3)$$
$$\times W(\vec{x}_1) \, W(\vec{x}_2) \, W(\vec{x}_3) \, \mathcal{L}_L(\hat{q}_1 \cdot \hat{x}_3)$$

Strategy: separate out the angular part

# As a matrix multiplication

We showed that bispectrum-window convolution
can be casted into a 1D integral

**2DFFTLog**

$$\tilde{B}_\ell[T_i] = \sum_{j,\ell'} \mathcal{M}_{\ell\ell'}[T_i, T'_j] B_{\ell'}[T'_j]$$

**Mixing matrix**

Computable via (2D) FFTLog

e.g. `2D-FFTLog` (Fang+20)

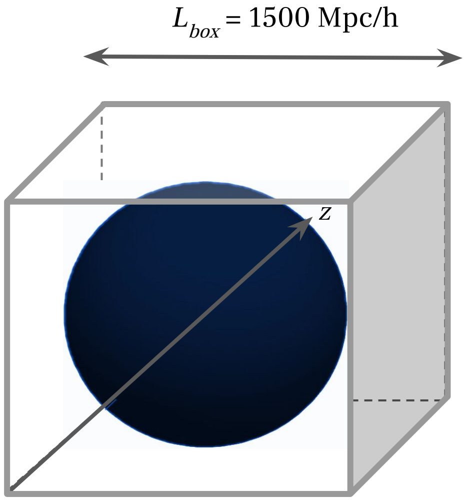
of the window 3PCF multipoles

**Bispectrum**

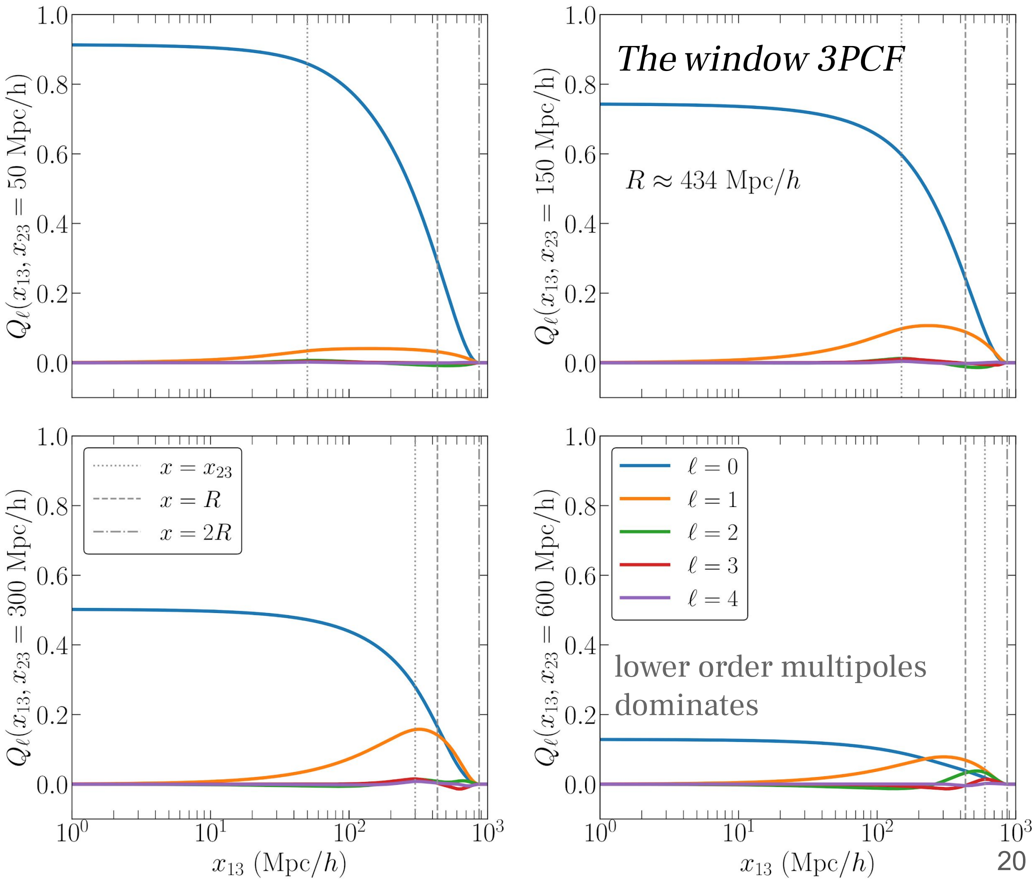
Function of three sides ($k_1$, $k_2$, $k_3$)

# Spherical window convolution in real-space

*Sphere catalogue:*
**Minerva/Pinocchio** carved on a sphere of $R \sim 434$ Mpc/$h$



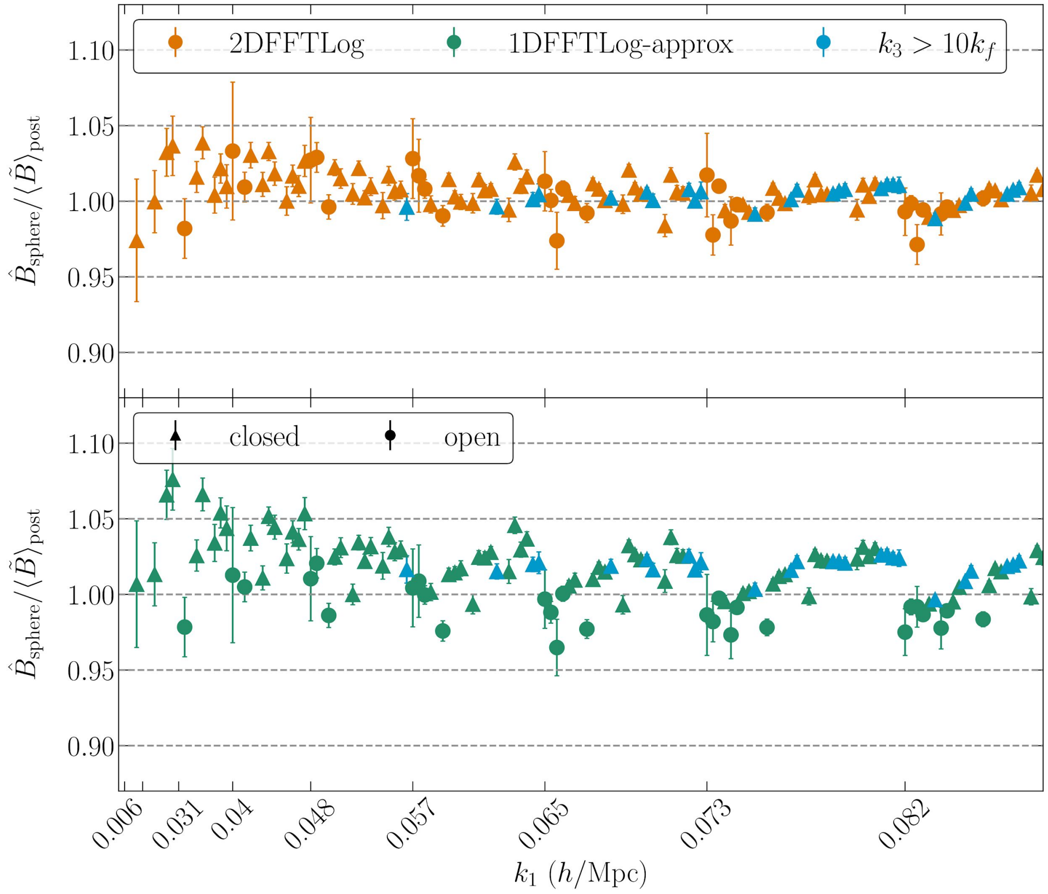

Total vol = $700^3$ (Mpc/$h$)]$^3$



# Full-set triangles

Fit on **Pinocchio** mocks

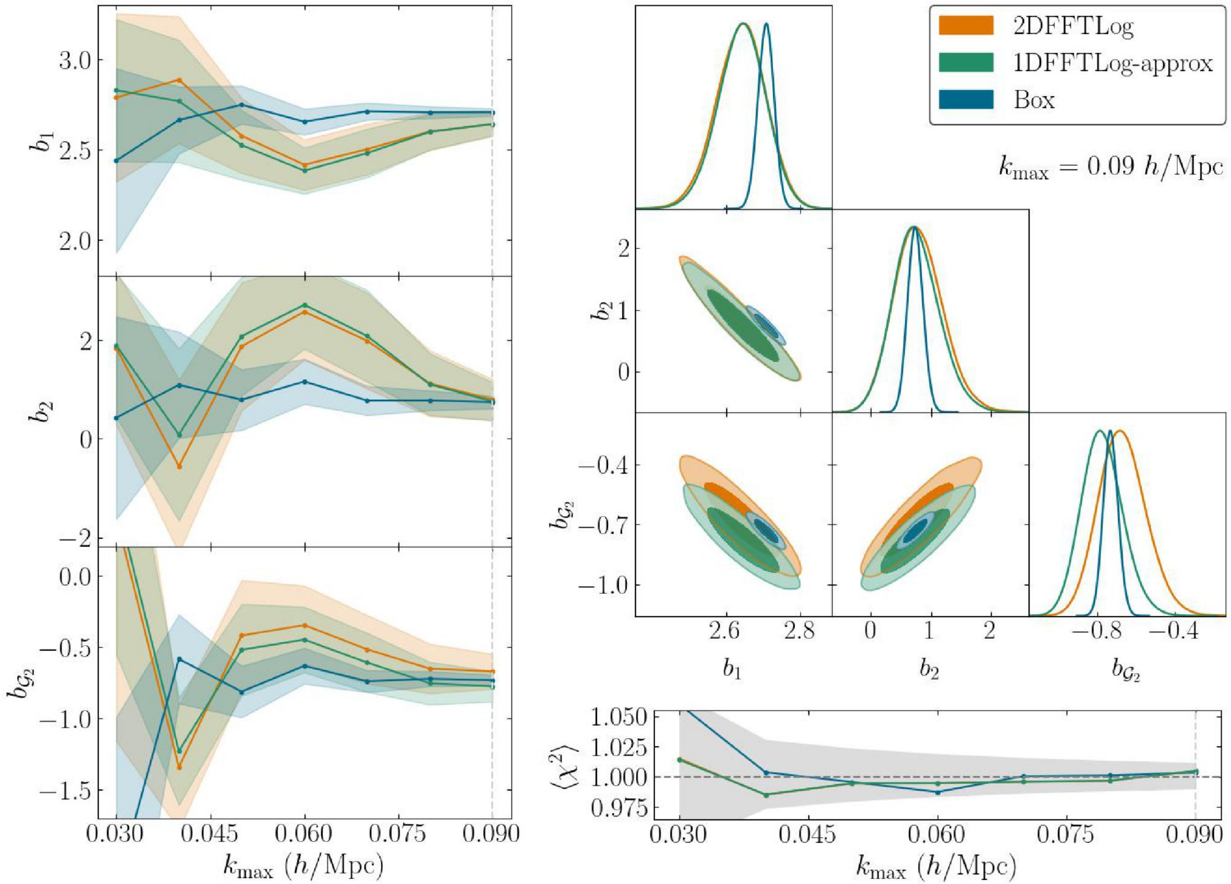
volume ≈ 3500 [Gpc/$h$]$^3$



# Recovering bias parameters

Analysis on **Minerva** data

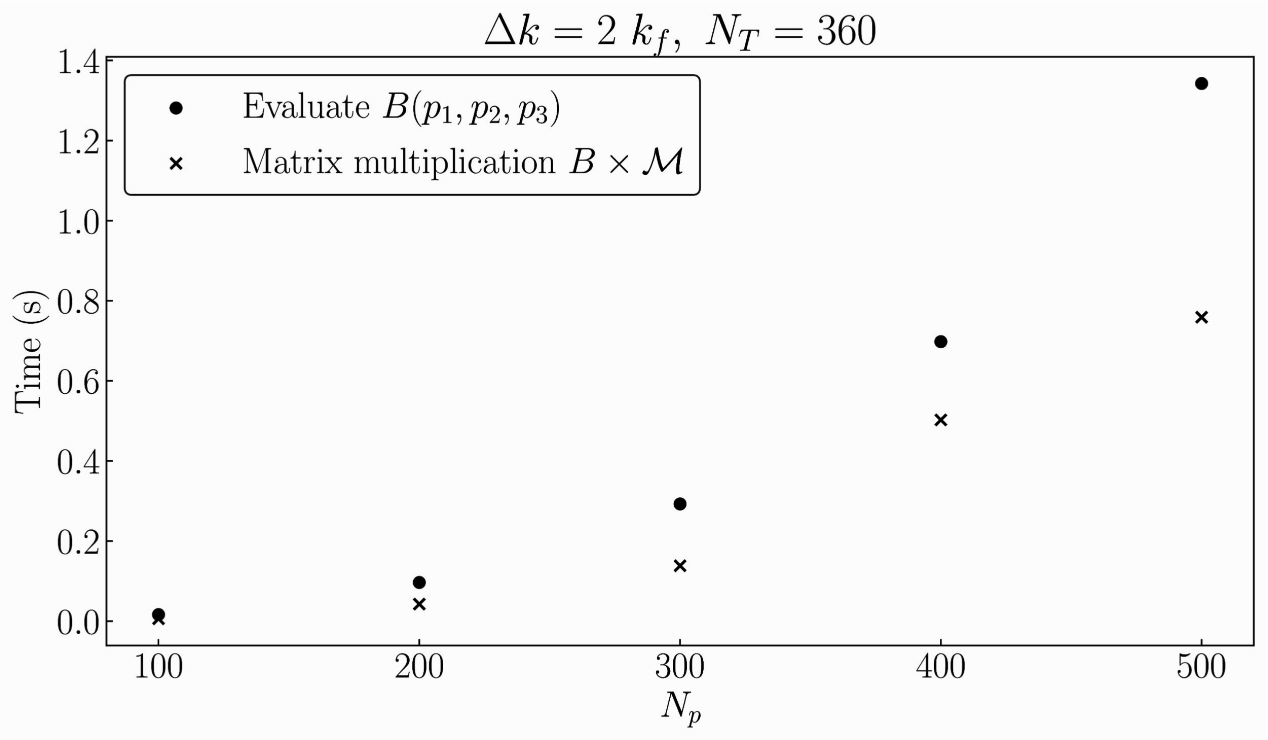
≈ ¼ times volume in Nishimichi+20
≈ 10 times $z \in [1.5, 1.8]$ *Euclid* volume



# Window convolution computation time

Takes ~ **2 seconds**

⇒ comparable to a typical Boltzmann solver call



# Finally: wide-angle effect in bispectrum

$$\hat{B}_L(k_1,k_2,k_3) = \frac{2L+1}{V_B}\int_{k_1} d^3q_1 \int_{k_2} d^3q_2 \int_{k_3} d^3q_3 \delta_D(\mathbf{q}_{123}) \boxed{\tilde{\delta}_L(\mathbf{q}_1)} \tilde{\delta}(\mathbf{q}_2)\tilde{\delta}(\mathbf{q}_3)$$

choice of **LOS** $\tilde{\delta}_L(\mathbf{q}) \equiv \int d^3x\, \tilde{\delta}(\mathbf{x})\mathcal{L}_L(\mathbf{q}\cdot\mathbf{x})e^{-i\mathbf{q}\cdot\mathbf{x}}$

- correction beyond plane-parallel approximation (scale as $k^{-2}$ for monopole)
- coupled to the window function
- interesting (and simpler) target would be the squeezed configurations

work in progress with E. Di Dio and E. Castorina

see also Noorikuhani & Scoccimarro 2022

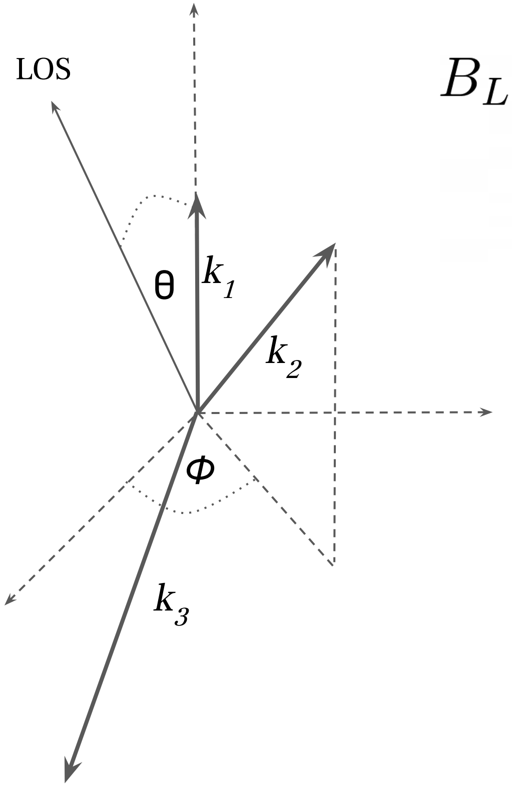
# Summary

1. Including bispectrum multipoles analysis is important but come with extra modelling complexity, ex: survey window effects
2. We gave an efficient formulation for bispectrum window convolution
3. We tested the formulation in ideal case of spherical window convolution in real space
4. Useful in future surveys when you want to extract signal, free from large scale systematic effects

**Thank you!**

# -Extras-

# The bispectrum multipoles




$$B_L(k_1, k_2, k_3) = \frac{2L+1}{4\pi} \int d\cos\theta \int d\phi B(k_1, k_2, k_3, \theta, \phi) \, \mathcal{L}_L(\cos\theta) \, .$$

PT (perturbation theory) model

angles w.r.t line of sight

- galaxies are not in their rest frame
- *m*≠0 contains negligible information Gagrani+16
- **tree-level: only even multipoles exist *B₀*, *B₂*, *B₄*, …**

# Tree-level bispectrum

$$B(\mathbf{k}_1, \mathbf{k}_2, \hat{\mathbf{x}}) = B^{(\mathrm{det})}(\mathbf{k}_1, \mathbf{k}_2, \hat{\mathbf{x}}) + B^{(\mathrm{stoch})}(\mathbf{k}_1, \mathbf{k}_2, \hat{\mathbf{x}})$$

$$B^{(\mathrm{det})}(\mathbf{k}_1, \mathbf{k}_2, \hat{\mathbf{x}}) = 2Z_2(\mathbf{k}_1, \mathbf{k}_2, \hat{\mathbf{x}}) Z_1(\mathbf{k}_1, \hat{\mathbf{x}}) Z_1(\mathbf{k}_2, \hat{\mathbf{x}}) P_L(k_1) P_L(k_2) + \mathrm{cyc.}$$

$$B^{(\mathrm{stoch})}(\mathbf{k}_1, \mathbf{k}_2, \hat{\mathbf{x}}) = \frac{1}{\bar{n}}[(1+\alpha_1)b_1 + (1+\alpha_3) f(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{x}})^2] Z_1(\mathbf{k}_1, \hat{\mathbf{x}}) P_L(k_1) + \mathrm{cyc.} + \frac{1+\alpha_2}{\bar{n}^2}$$

$$Z_1(\mathbf{k}, \hat{\mathbf{x}}) = b_1 + f(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{x}})^2$$

$$Z_2(\mathbf{k}_1, \mathbf{k}_2, \hat{\mathbf{x}}) = \frac{b_2}{2} + b_1 F_2(\mathbf{k}_1, \mathbf{k}_2) + b_{\mathcal{G}_2} S(\mathbf{k}_1, \mathbf{k}_2)$$

$$f(\hat{\mathbf{k}}_{12} \cdot \hat{\mathbf{x}})^2 G(\mathbf{k}_1, \mathbf{k}_2) + \frac{f(\mathbf{k}_{12} \cdot \hat{\mathbf{x}})}{2} \left[ \frac{\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{x}}}{k_1} Z_1(\mathbf{k}_2, \hat{\mathbf{x}}) + \frac{\hat{\mathbf{k}}_2 \cdot \hat{\mathbf{x}}}{k_2} Z_1(\mathbf{k}_1, \hat{\mathbf{x}}) \right]$$

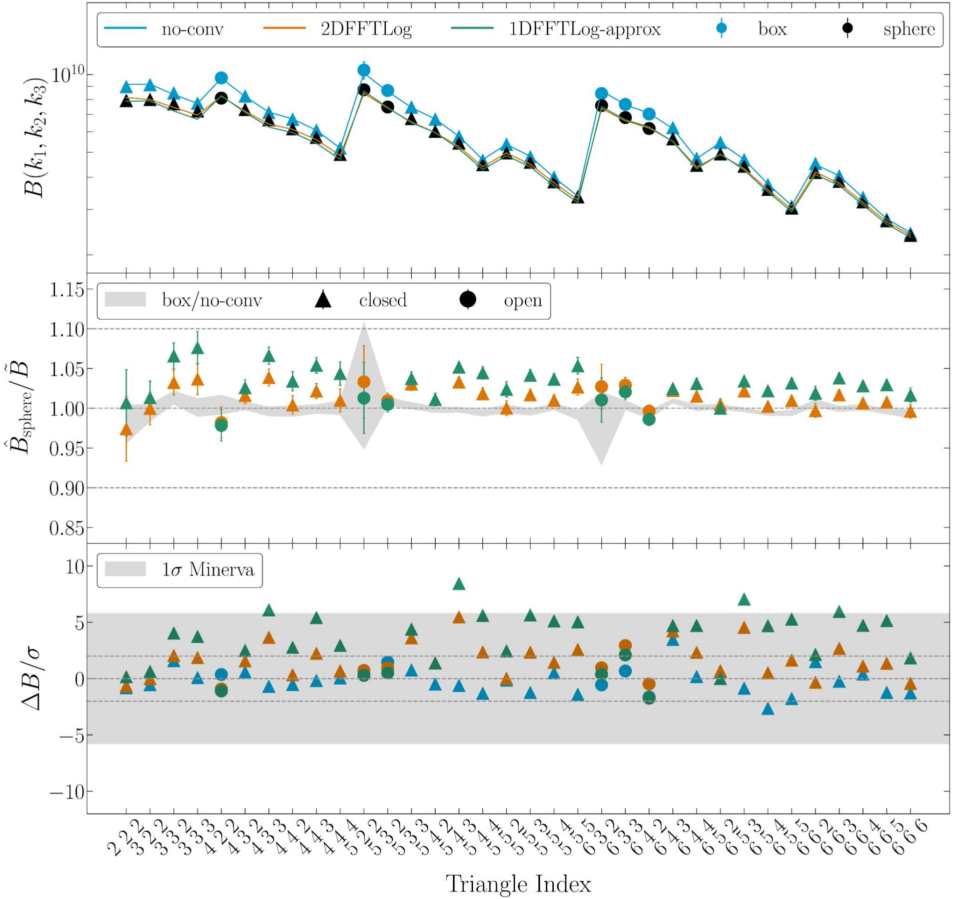
$$\mathbf{k}_{12} \equiv \mathbf{k}_1 + \mathbf{k}_2$$

# First few triangles

Fit on **Pinocchio** mocks

volume $\approx 3500\ [\mathrm{Gpc}/h]^3$

Minerva volume:
consistent within 1-sigma



# Exact bispectrum window convolution

Taking $\langle \hat{B}_L \rangle = \tilde{B}_L$

$$\tilde{B}_L(k_1, k_2, k_3) = \frac{2L+1}{V_B} \int_{k_1} d^3 q_1 \int_{k_2} d^3 q_2 \int_{k_3} d^3 q_3 \, \delta_D(\vec{q}_{123})$$
$$\times \int d^3 x_3 \int d^3 x_{13} \int d^3 x_{23} \, e^{-i\vec{q}_1 \cdot \vec{x}_{13}} e^{-i\vec{q}_2 \cdot \vec{x}_{23}} \zeta(\vec{x}_{13}, \vec{x}_{23}, \hat{x}_3)$$
$$\times W(\vec{x}_1) \, W(\vec{x}_2) \, W(\vec{x}_3) \, \mathcal{L}_L(\hat{q}_1 \cdot \hat{x}_3)$$

Need to: systematically reduce the angular integration

# … the final expression

Some form of integral between

the unconvolved bisp. and the mixing matrix

$$\tilde{B}_L(k_1,k_2,k_3) = \int \frac{dp_1}{2\pi^2}p_1^2 \int \frac{dp_2}{2\pi^2}p_2^2 \int \frac{dp_3}{2\pi^2}p_3^2 \sum_{L'M'} B_{L'M'}(p_1,p_2,p_3)$$

$$\times \sum_{\ell} \underbrace{I_{\ell\ell 0}(p_1,p_2,p_3)\mathcal{Q}^L_{L',-M',\ell}(k_1,k_2,k_3;p_1,p_2)}_{\text{the mixing matrix}}$$

# Window convolution ~ matrix mult.

One part of the mixing matrix is a known function

$$\tilde{B}_L(k_1, k_2, k_3) = \int \frac{dp_1}{2\pi^2} p_1^2 \int \frac{dp_2}{2\pi^2} p_2^2 \int \frac{dp_3}{2\pi^2} p_3^2 \sum_{L'M'} B_{L'M'}(p_1, p_2, p_3)$$

$$\times \sum_{\ell} I_{\ell\ell 0}(p_1, p_2, p_3) \mathcal{Q}^L_{L',-M',\ell}(k_1, k_2, k_3; p_1, p_2)$$

enforce the triangle condition

$$\tilde{B}_\ell[T_i] = \sum_{j,\ell'} \mathcal{M}_{\ell\ell'}[T_i, T'_j] B_{\ell'}[T'_j]$$

$$I_{\ell\ell 0}(x, y, z) = (-1)^\ell \frac{\pi^2}{xyz} \theta(1 - \hat{x} \cdot \hat{y}) \theta(1 + \hat{x} \cdot \hat{y}) \mathcal{L}_\ell(\hat{x} \cdot \hat{y})$$

# Contribution from the the window 3PCF

The other contribution is coming from the random catalogue

$$\tilde{B}_L(k_1,k_2,k_3) = \int \frac{dp_1}{2\pi^2}p_1^2 \int \frac{dp_2}{2\pi^2}p_2^2 \int \frac{dp_3}{2\pi^2}p_3^2 \sum_{L'M'} B_{L'M'}(p_1,p_2,p_3)$$
$$\times \sum_{\ell} I_{\ell\ell 0}(p_1,p_2,p_3) \boxed{\mathcal{Q}^L_{L',-M',\ell}(k_1,k_2,k_3;p_1,p_2)}$$

require several steps of computations

# How to compute the 3PCF contribution?

$W(\mathbf{x})$ → the random catalogue

3PCF of the window e.g. by direct-counting

$$Q^{L}_{L'M'\ell\ell'\ell_1\ell_2}(x_{13}, x_{23})$$

(2D) Hankel transform
e.g. 2D-FFTLog (Fang+20)

$$\mathcal{W}^{L}_{L'M'\ell\ell'\ell_1\ell_2}(q_1, q_2, p_1, p_2)$$

linear combination

$$\mathcal{Q}^{L}_{L'M'\ell}(k_1, k_2, k_3; p_1, p_2)$$

# The window 3PCF - **measurement**

$$Q^{L}_{L'M'\ell\ell'\ell_1\ell_2}(x_{13},x_{23}) \equiv (-1)^{M'} \sum_{\tilde{\ell}_1,\tilde{\ell}_2} \sum_{\substack{M,m_1,m_2\\ m,m',\tilde{m}_1,\tilde{m}_2}} 4\pi i^{\ell'-\ell+\ell_2-\ell_1}\, \mathcal{G}^{Mm_1m_2}_{L\ell_1\ell_2}\, \mathcal{G}^{M'mm'}_{L'\ell\ell'}\, \mathcal{G}^{m_1m'\tilde{m}_1}_{\ell_1\ell'\tilde{\ell}_1}\, \mathcal{G}^{m_2m\tilde{m}_2}_{\ell_2\ell\tilde{\ell}_2}$$

$$\times \int d^3x_3 \int \frac{d^2\hat{x}_{13}}{4\pi} \int \frac{d^2\hat{x}_{23}}{4\pi} Y^*_{LM}(\hat{x}_3) Y_{\tilde{\ell}_1\tilde{m}_1}(\hat{x}_{13}) Y_{\tilde{\ell}_2\tilde{m}_2}(\hat{x}_{23})$$
$$\times W(\vec{x}_3+\vec{x}_{13}) W(\vec{x}_3+\vec{x}_{23}) W(\vec{x}_3).$$

Computed via e.g. direct counting, FFT-based, etc.

# The window 3PCF – **Hankel transf.**

Combination of two dimensional Hankel transforms

$$\mathcal{W}^{L}_{L'M'\ell\ell'\ell_1\ell_2}(q_1,q_2;p_1,p_2) \equiv (4\pi)^2 \int dx_{13}\, x_{13}^2 \int dx_{23}\, x_{23}^2\, j_{\ell'}(p_1 x_{13})\, j_{\ell}(p_2 x_{23}) \times \left[ j_{\ell_1}(q_1 x_{13})\, j_{\ell_2}(q_2 x_{23})\, Q^{L}_{L'M'\ell\ell'\ell_1\ell_2}(x_{13},x_{23}) \right],$$

A two dimensional Hankel transform
e.g. `2DFFTLog` Fang+20

Window 3PCF multipoles

# The window 3PCF – **binning**

How to handle the binning operator?

$$\mathcal{Q}^{L}_{L'M'\ell}(k_1,k_2,k_3;p_1,p_2) \simeq \sum_{\ell_1,\ell_2,\ell'} 16\pi^2 \frac{I_{\ell_2\ell_2 0}(k_1,k_2,k_3)}{I_{000}(k_1,k_2,k_3)} \mathcal{W}^{L}_{L'M'\ell\ell'\ell_1\ell_2}(k_1,k_2;p_1,p_2)\,.$$

- Evaluated at the center of the bin
- Bin numerically later

# The window 3PCF - **FFT-based**

$$Q^{L}_{L'M'\ell\ell'\ell_1\ell_2}(x_{13},x_{23}) = (-1)^{M'} \sum_{\substack{M,m_1,m_2\\ m,m'}} \sum_{\substack{\tilde{\ell}_1,\tilde{\ell}_2\\ \tilde{m}_1,\tilde{m}_2}} 4\pi i^{\ell'-\ell+\ell_2-\ell_1}\, \mathcal{G}^{Mm_1m_2}_{L\ell_1\ell_2}\mathcal{G}^{M'mm'}_{L'\ell\ell'}\,\mathcal{G}^{\tilde{m}_1m_1m'}_{\tilde{\ell}_1\ell_1\ell'}\,\mathcal{G}^{\tilde{m}_2m_2m}_{\tilde{\ell}_2\ell_2\ell}$$

$$\times \int d^3x_3\; W_{\tilde{\ell}_1\tilde{m}_1}(\vec{x}_3;x_{13})W_{\tilde{\ell}_2\tilde{m}_2}(\vec{x}_3;x_{23})W_{LM}(\vec{x}_3)$$

$$W_{\ell m}(\vec{x}_3;x_{ij}) \equiv\; i^{\ell}\int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{x}_3} j_\ell(qx_{ij})Y_{\ell m}(\hat{q})W(\vec{q})$$

$$W_{LM}(\vec{x}_3) \equiv\; W(\vec{x_3})Y^{*}_{LM}(\vec{x}_3)\,.$$

# Default parameters

| $N_p$ | $\ell_{max}$ | $\ell'_{max}$ | $P_{min}$ [$h$ Mpc$^{-1}$] | $p_{max}$ [$h$ Mpc$^{-1}$] |
|---|---|---|---|---|
| 512 | 30 | 2 | $10^{-5}$ | 0.5 |