PNG and beyond
September 2022, 23rd, Madrid

Based on:

[Dimastrogiovanni, Fasiello, LP 2022]
JCAP 09 (2022) 031

[Garcia-Saenz, LP, Renaux-Petel, Werth 2022]
ArXiv: 2207.14267

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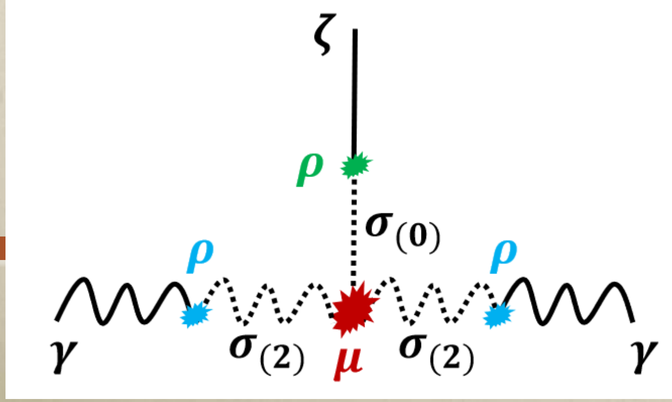
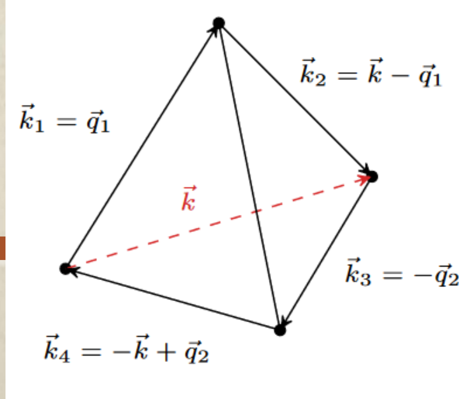
PRIMORDIAL NON-GAUSSIANITIES AND GRAVITATIONAL WAVES
AN INTERTWINED STORY



Mixed bispectrum (scalar-tensor-tensor) inducing GW anisotropies



Tetrahedron shape for the scalar trispectrum inducing GW

PNG and beyond
September 2022, 23rd, Madrid

Based on:

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JCAP 09 (2022) 031

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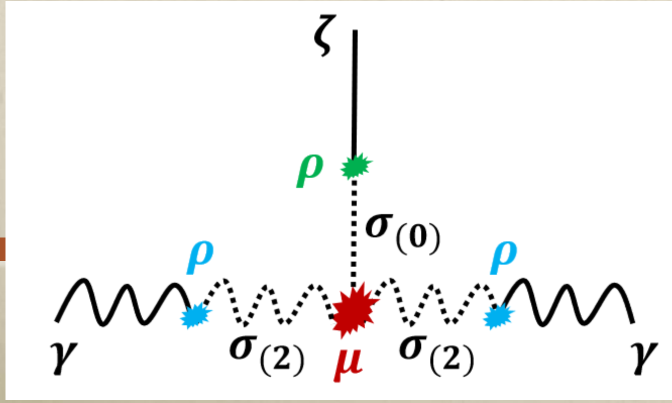
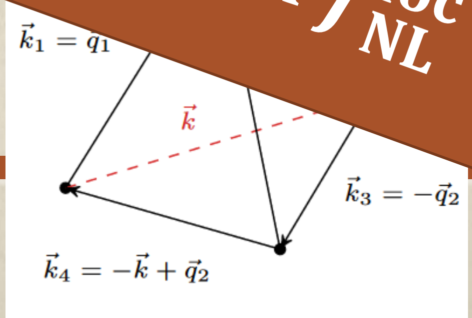
...rica (IFT) UAM-CSIC

Actually, a glimpse into the rich physics of PNG and why we should look beyond f_{NL}^{loc}

PRIMORDIAL N... ...AVITATIONAL WAVES
AN INTERTWINED STORY



Mixed bispectrum (scalar-tensor-tensor) inducing GW anisotropies



...pe
...ispectrum

WHAT I WILL NOT TALK ABOUT

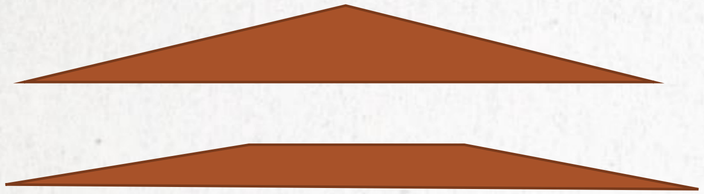
Scalar primordial non-Gaussianities

in multifield inflation

[Fumagalli *et al.*, LP 2019]

Phys. Rev. Lett. *123, 201302*

A multifield instability in curved field space

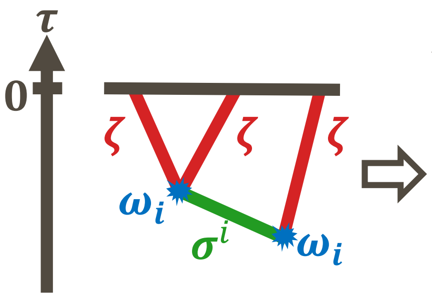


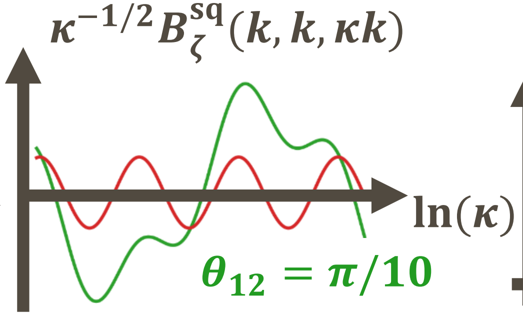
$$f_{\rm NL}^{\rm flat} = \mathcal{O}(50)$$

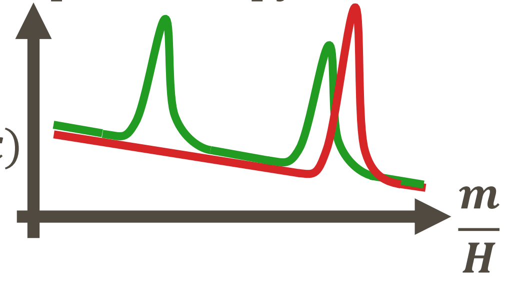
$$g_{\rm NL}^{\rm flat} = \mathcal{O}(10^5) \quad \text{etc.}$$

[LP, Aoki, Renaux-Petel, Yamaguchi 2021]

ArXiv:2112.05710







Cubic action for perturbations in multifield inflation with non-derivative interactions

- $N_{\rm field} = 2$

[Garcia-Saenz, LP, Renaux-Petel 2020]

J. High Energ. Phys. 2020, 73 (2020)

$$f_{\rm nl}^{\rm eq} \simeq \left(\frac{1}{c_s^2} - 1\right)\left(-\frac{85}{324} + \frac{15}{243} A\right)$$

- $N_{\rm field}$ generic

[LP 2020]

J. Cosm. & Astro. Phys. 04(2021)048

$$\begin{aligned} A = & -\frac{1}{2}(1+c_s^2) + \frac{4}{3}(1+2c_s^2)\epsilon H^2 M_p^2 \left(m^{-2}\right)_{11} R_{m\sigma m\sigma} \\ & - \frac{\kappa}{6}(1-c_s^2)\left(m^{-2}\right)_{11}\Big[V_{;mmm} + 2\epsilon H^2 M_p^2 R_{m\sigma m\sigma;m} \\ & + 4\sqrt{2\epsilon} H M_p \left(\Omega^\alpha{}_m + \frac{1}{(m^{-2})_{11}} \frac{\mathrm{d}\left(m^{-2}\right)^\alpha{}_1}{\mathrm{d}t}\right) R_{m\alpha m\sigma}\Big], \end{aligned}$$

Important remark: can you see $f_{\rm NL}^{\rm loc}$ in this slide? → need to look for more theoretically motivated templates!

WHAT I WILL NOT TALK ABOUT

Scalar primordial non-Gaussianities

in multifield inflation

[Fumagal …0]

Phys. Rev. Le…

A multifield instability in curved field space

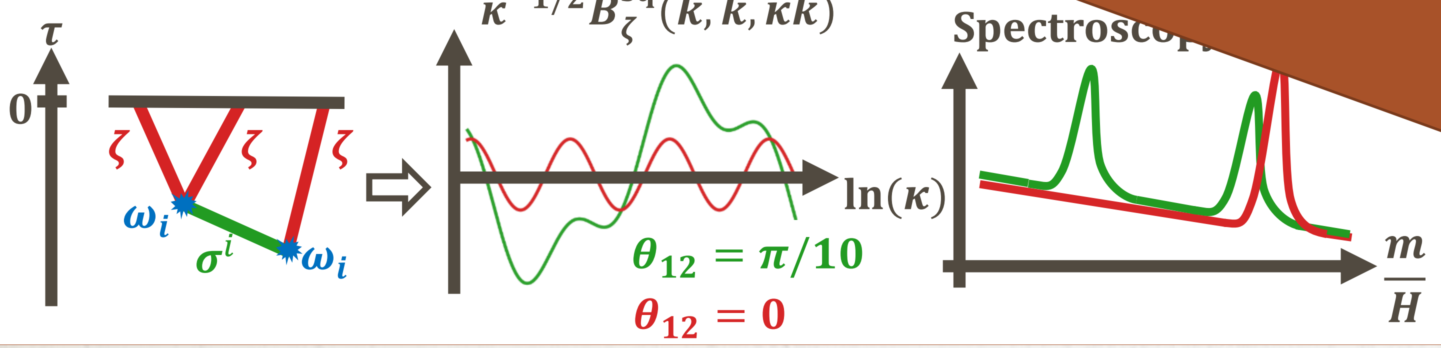
…0)

Multifield inflation DOES NOT IMPLY $f_{NL}^{loc} \gg 1$

(not even $f_{NL}^{loc} > 1$)

[LP, Aoki, Renaux-Petel, Yamaguch…

ArXiv:2112.05710



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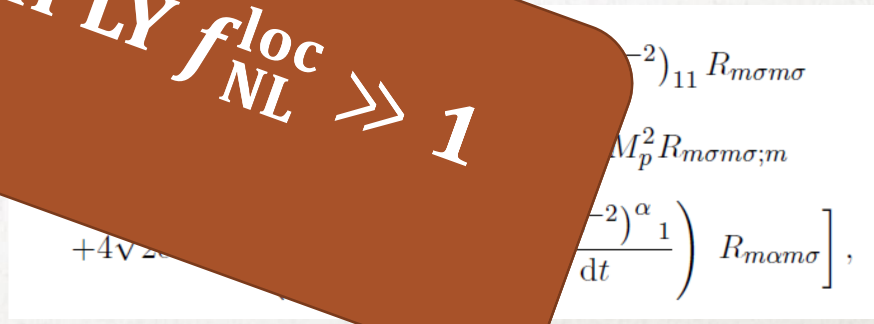
…20]

…04(2021)048



Important remark: can you see f_{NL}^{loc} in this slide? → need to look for more theoretically motivated templates!

OTHER KINDS OF PNG

Trispectrum

- Higher-order correlation functions:

SSSS $$\left\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\zeta_{\vec{k}_4}\right\rangle = (2\pi)^3\,\delta^{(3)}\left(\overrightarrow{k_1}+\overrightarrow{k_2}+\overrightarrow{k_3}+\overrightarrow{k_4}\right)\times T_\zeta\left(\overrightarrow{k_1},\overrightarrow{k_2},\overrightarrow{k_3},\overrightarrow{k_4}\right)$$

etc.

- Tensor and mixed scalar-tensor PNG

SST $$\left\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\gamma_{\vec{k}_3}\right\rangle = (2\pi)^3\,\delta^{(3)}\left(\overrightarrow{k_1}+\overrightarrow{k_2}+\overrightarrow{k_3}\right)\times B_{\zeta\zeta\gamma}(k_1,k_2,k_3)$$

STT $$\left\langle \zeta_{\vec{k}_1}\gamma_{\vec{k}_2}\gamma_{\vec{k}_3}\right\rangle = (2\pi)^3\,\delta^{(3)}\left(\overrightarrow{k_1}+\overrightarrow{k_2}+\overrightarrow{k_3}\right)\times B_{\zeta\gamma\gamma}(k_1,k_2,k_3)$$

TTT $$\left\langle \gamma_{\vec{k}_1}\gamma_{\vec{k}_2}\gamma_{\vec{k}_3}\right\rangle = (2\pi)^3\,\delta^{(3)}\left(\overrightarrow{k_1}+\overrightarrow{k_2}+\overrightarrow{k_3}\right)\times B_{\gamma\gamma\gamma}(k_1,k_2,k_3)$$

All these correlators are observable and contain information about high-energy physics and inflation

OTHER KINDS OF PNG: CONSTRAINTS

Bounds at CMB scales

- Higher-order correlation functions:

$$\text{SSSS} \xrightarrow{\text{squeezed}} g_{\rm NL} = (-5.8 \pm 6.5)10^4$$ **[Planck 2018]**

$$\text{SSSS} \xrightarrow{\text{collapsed}} \tau_{\rm NL} = \;???$$

from theory…

$\tau_{\rm NL} \geq \left(\frac{6}{5} f_{\rm NL}^{\rm loc}\right)^2$ **with equality in single-field only**

[Suyama, Yamaguchi 2007]
[Smith, LoVerde, Zaldarriaga 2011]

- Tensor and mixed scalar-tensor PNG

$$\text{SST} \xrightarrow{\text{squeezed}} f_{\rm NL,loc}^{\zeta\zeta\gamma} = -48 \pm 28$$ **[Shiraishi, Liguori, Fergusson 2017]**

$$\text{STT} \xrightarrow{\text{squeezed}} f_{\rm NL,loc}^{\zeta\gamma\gamma} = \;???$$

$$\text{TTT} \xrightarrow{\text{squeezed}} f_{\rm NL,loc}^{\gamma\gamma\gamma} = 220 \pm 170$$ **[WMAP 2013]**

… and nothing else…

Please tell me if these are outdated

OTHER KINDS OF PNG: CONSTRAINTS

- Higher-order correlation functions:

$$\text{SSSS} \xrightarrow{\text{squeezed}} g_{\text{NL}} = (-5.8 \pm 6.5)10^4$$ **[Planck 2018]**

$$\text{SSSS} \xrightarrow{\text{collapsed}} \tau_{\text{NL}} = \ ???$$

THIS TALK

- Tensor and mixed scalar-tensor PNG

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$$\text{STT} \xrightarrow{\text{squeezed}} f_{\text{NL,loc}}^{\zeta\gamma\gamma} = \ ???$$

$$\text{TTT} \xrightarrow{\text{squeezed}} f_{\text{NL,loc}}^{\gamma\gamma\gamma} = 220 \pm 170$$ **[WMAP 2013]**

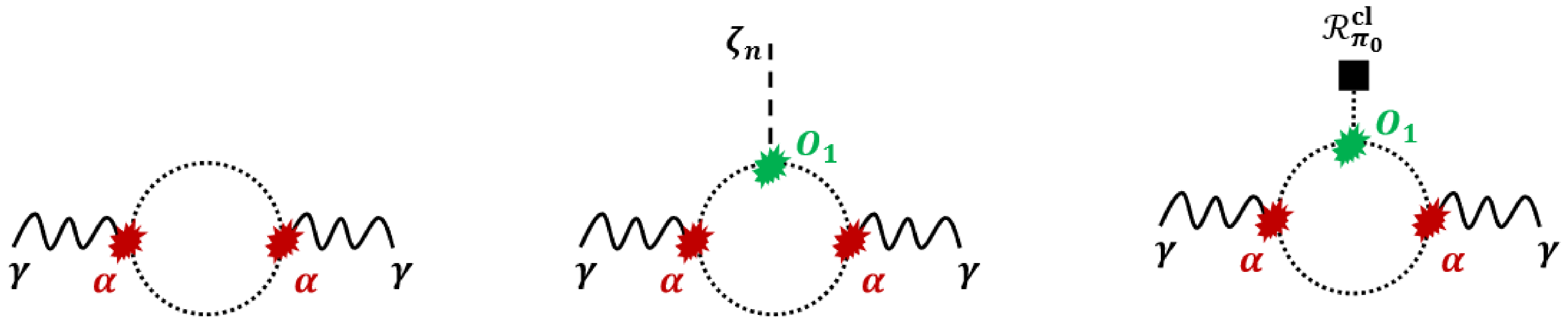
Please tell me if these are outdated

[Dimastrogiovanni, Fasiello, LP 2022]
JCAP 09 (2022) 031

I. STT and TTT squeezed PNG:

Induced anisotropies in the SGWB

Stochastic Gravitational Wave Background



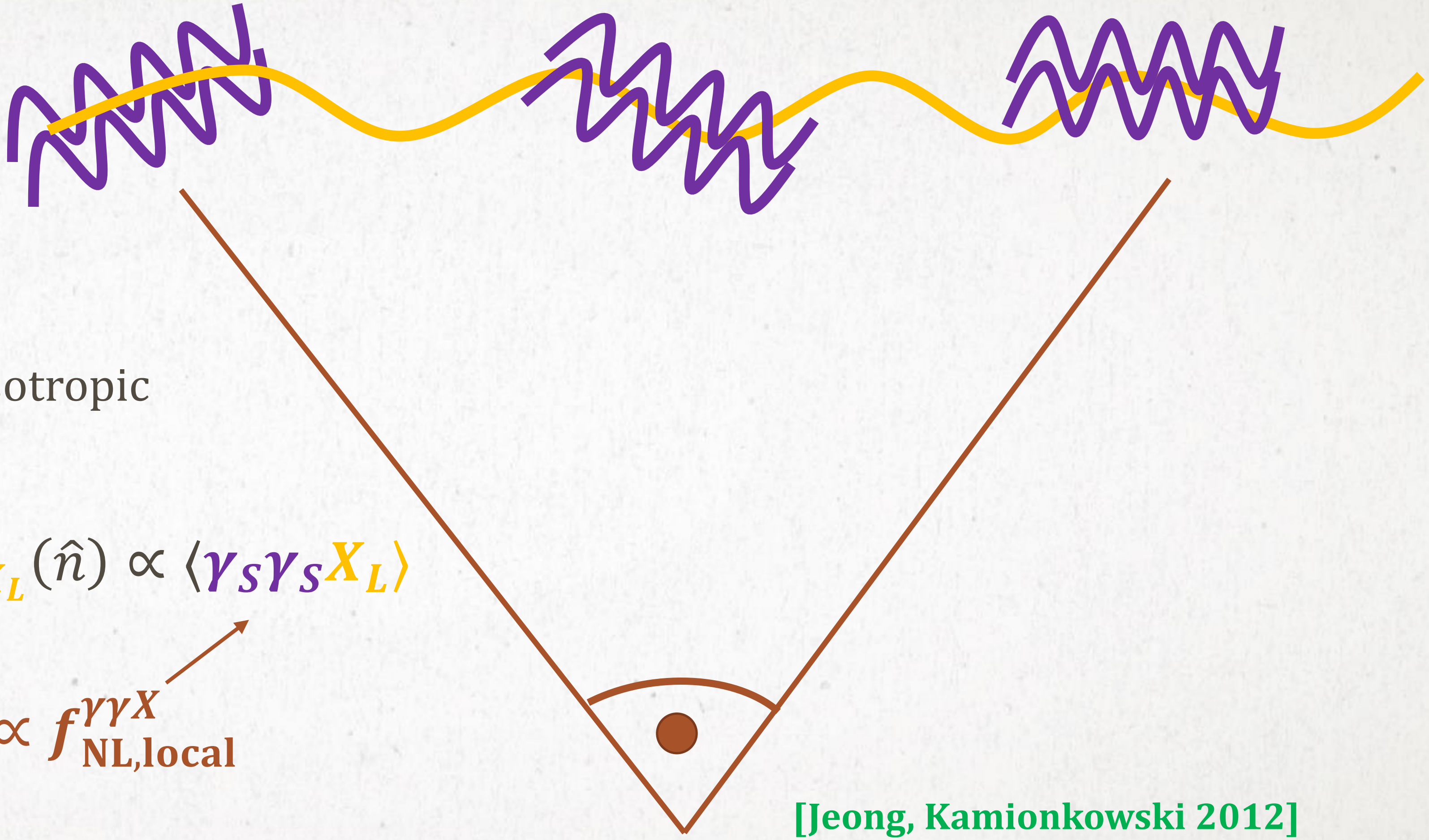
(a) One-loop tensor power spectrum

(b) One-loop scalar-tensor-tensor bispectrum

(c) One-loop tensor two-point function in the presence of a classical scalar source.

PNG-INDUCED ANISOTROPIES IN THE SGWB

- The idea:



Consider the modulation of
two short modes by a **long one**:
seen from far away the signal is anisotropic

$$\langle\gamma_s\gamma_s\rangle \to \delta_{\mathrm{GW}}(\hat{n}, f_s) \propto \langle\gamma_s\gamma_s\rangle_{X_L}(\hat{n}) \propto \langle\gamma_s\gamma_s X_L\rangle$$

$$\frac{\Omega_{\mathrm{GW}}(\hat{n}, f)}{\bar{\Omega}_{\mathrm{GW}}(f)} - 1$$

$$\propto f_{\mathrm{NL,local}}^{\gamma\gamma X}$$

[Jeong, Kamionkowski 2012]

Here γ_s is a tensor (anisotropies of the SGWB) but first introduced for scalars (anisotropies of LSS)

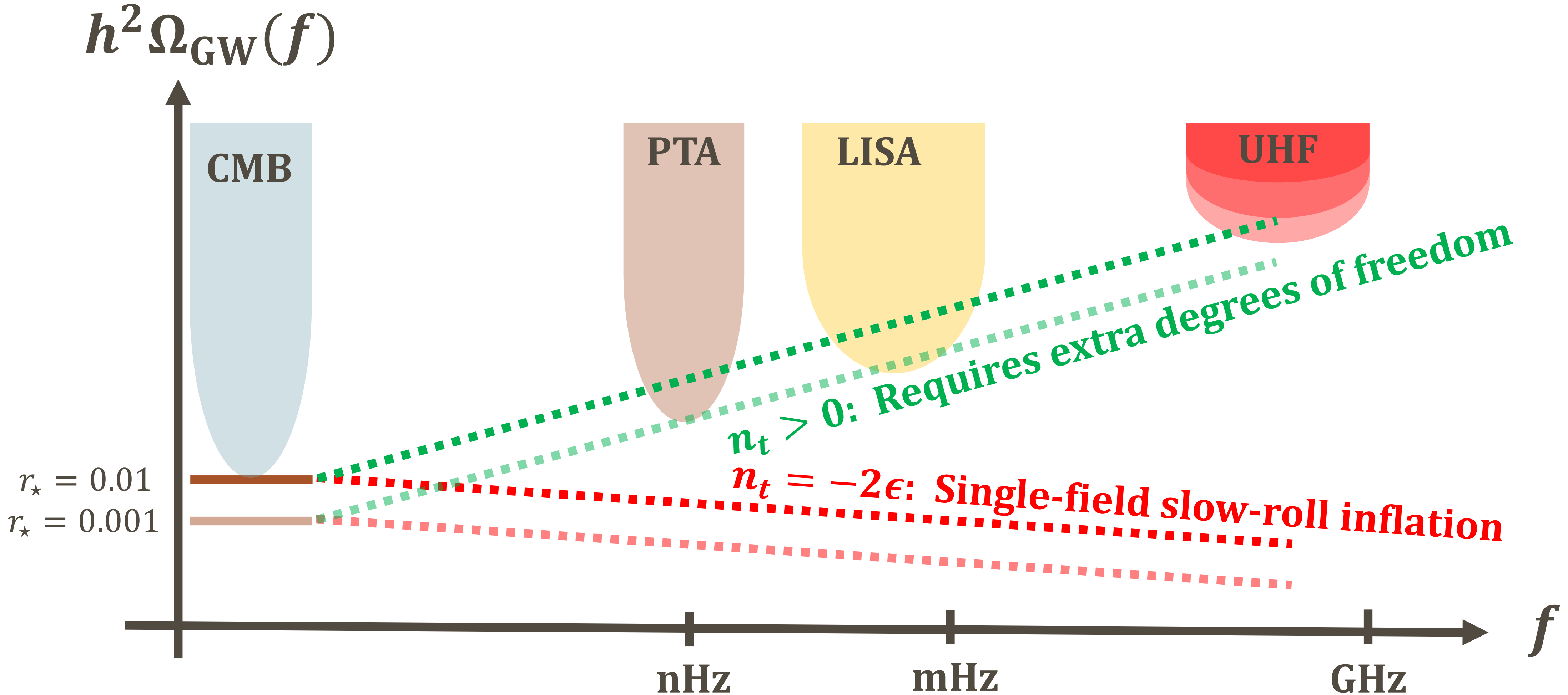
Also X_L can be ζ_L (modulation by a soft scalar mode) or γ_L (modulation by a soft tensor mode)

INGREDIENTS FOR OBSERVABLE GW ANISOTROPIES

- Having an observable monopole signal

INGREDIENTS FOR OBSERVABLE GW ANISOTROPIES

- Having an observable monopole signal → smaller scales, requires a blue tilt: $n_t > 0$



INGREDIENTS FOR OBSERVABLE GW ANISOTROPIES

- Having an observable monopole signal: $n_t > 0$

- Having large STT or TTT bispectra in the (ultra) squeezed limit

INGREDIENTS FOR OBSERVABLE GW ANISOTROPIES

- Having an observable monopole signal: $n_t > 0$

- Having large STT or TTT bispectra in the (ultra) squeezed limit: $f_{\mathrm{NL,sq}}^{\zeta\gamma\gamma}$, $f_{\mathrm{NL,sq}}^{\gamma\gamma\gamma} \gg 1$

$$\left\langle \gamma_{\vec{k}_1}^{\lambda_1} \gamma_{\vec{k}_2}^{\lambda_2} \right\rangle_{\gamma_{\vec{q}_L}} = \sum_{\lambda_3} \int_{|\vec{q}|<q_L} d^3q \, \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{q}) \gamma_{\mathbf{q}}^{*\lambda_3} \frac{B_\gamma^{\lambda_1\lambda_2\lambda_3}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{q})}{P_\gamma^{\lambda_3}(q)}$$

"heuristic" formula of the literature

[Ricciardone, Tasinato 2017]

[Dimastrogiovanni, Fasiello, Tasinato 2019]

INGREDIENTS FOR OBSERVABLE GW ANISOTROPIES

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- That this squeezed limit is not due to spurious gauge artifacts (consistency relation discussed yesterday)

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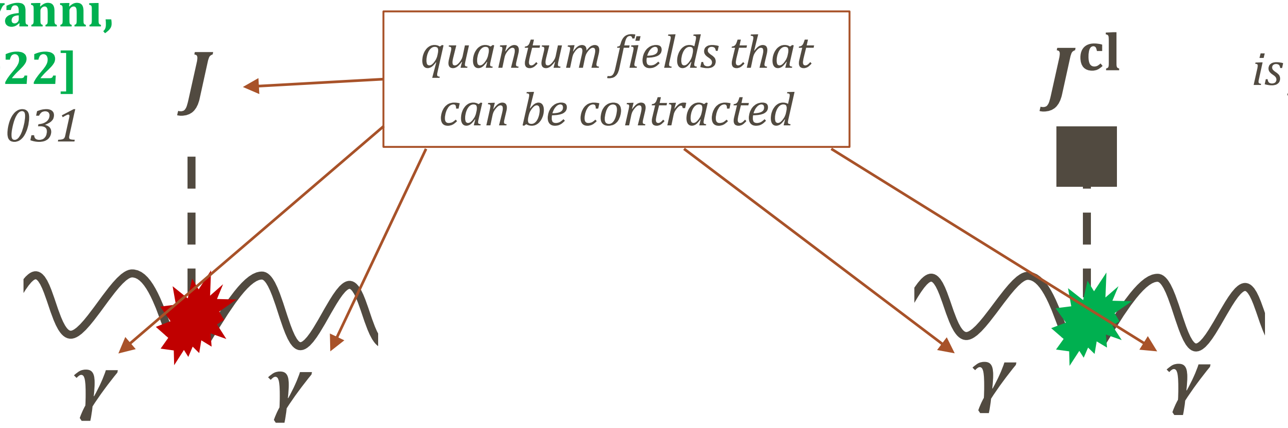
This work: ❖ Go beyond the heuristic approach and **compute** the two-point function with a classical source

[Dimastrogiovanni, Fasiello, LP 2022]
JCAP 09 (2022) 031



classical field that is factored out of vevs

For a QED example see
[Peskin, Schroeder 1995]

INGREDIENTS FOR OBSERVABLE GW ANISOTROPIES

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[Dimastrogiovanni, Fasiello, LP 2022]
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This work: ❖ Go beyond the heuristic approach and **compute** the two-point function with a classical source

$$\boxed{\langle \gamma_{\vec{k}_1} \gamma_{\vec{k}_2} \rangle_{J^{\mathrm{cl}}} \underset{|\vec{k}_1 + \vec{k}_2| \ll k_{1,2}}{=} \int \mathrm{d}^3\vec{q}\ \delta^{(3)}(\vec{q} + \vec{k}_1 + \vec{k}_2) P_\gamma(k_1) f_{\mathrm{NL,sq}}^{J\gamma\gamma}(\vec{k}_1, \vec{k}_2, \vec{q}) J^{\mathrm{cl}}(\vec{q})}$$

Non-diagonal part, $\vec{k}_1 + \vec{k}_2 \neq \vec{0}$, of the 2-pt function does not vanish → anisotropies

$J^{\mathrm{cl}}(\vec{q})$ is a statistical quantity → so is $\langle \gamma_{\vec{k}_1} \gamma_{\vec{k}_2} \rangle_{J^{\mathrm{cl}}}$ → $\langle \delta(\hat{n}_1) \delta(\hat{n}_2) \rangle \propto \langle J^{\mathrm{cl}}(\vec{q}) J^{\mathrm{cl}}(\vec{q}\,') \rangle \neq 0$

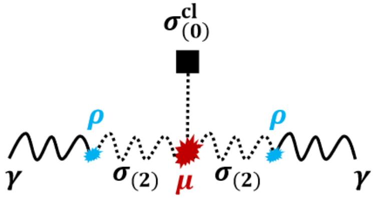
INGREDIENTS FOR OBSERVABLE GW ANISOTROPIES

- Having an observable monopole signal: $\boldsymbol{n_t > 0}$ 1
- Having large STT or TTT bispectra in the (ultra) squeezed limit: $f_{\mathrm{NL,sq}}^{\zeta\gamma\gamma},\ f_{\mathrm{NL,sq}}^{\gamma\gamma\gamma} \gg 1$ 2
- That this **squeezed limit is not due to spurious gauge artifacts** 3 (consistency relation discussed yesterday)

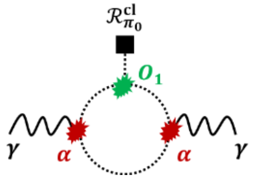
[Dimastrogiovanni, Fasiello, LP 2022]
JCAP 09 (2022) 031

This work:
- Go beyond the heuristic approach and compute the two-point function with a classical source
- Prove that some already existing inflationary models verify **all 3 requirements above**

Example 1: EFT of spin-2 field $\sigma_{ij} = \partial_i\partial_j\sigma^{(0)} + \sigma_{ij}^{(2)}$ →
[Bordin *et al.* 2018]



Example 2: Supersolid inflation Two scalars (ζ_n, R_{π_0}) →
[Celoria *et al.* 2021]
adiabatic entropic



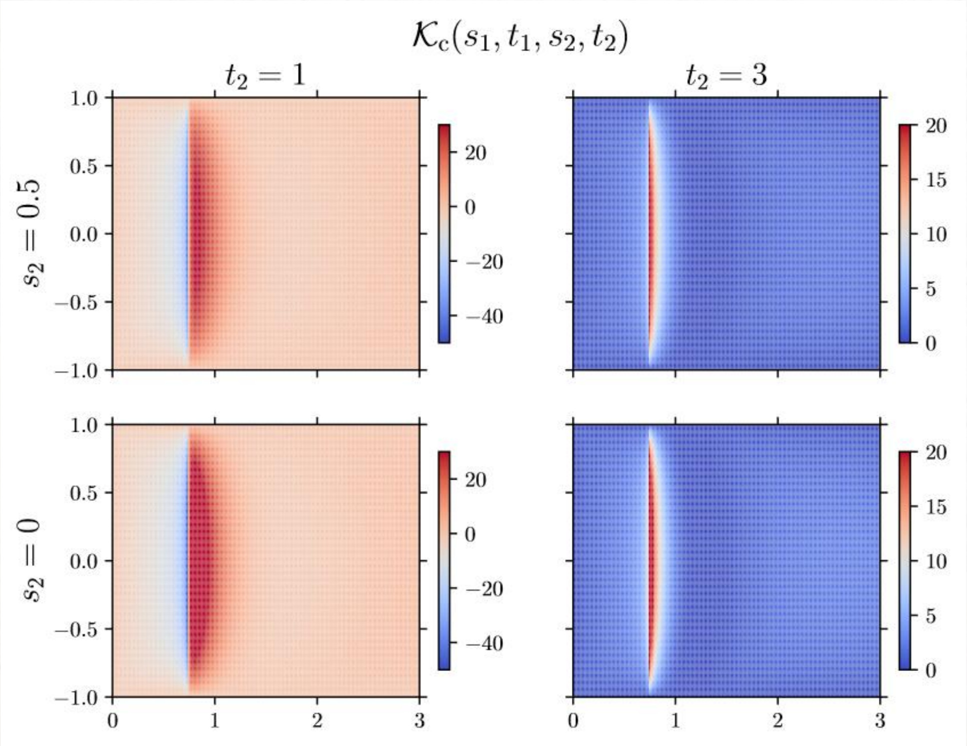
$\boldsymbol{\delta_{\mathrm{GW}}(\hat{n}) \geq 10\%}$

[Garcia-Saenz, LP, Renaux-Petel, Werth 2022]

ArXiv: 2207.14267

II. Scalar-trispectrum

-induced gravitational waves



Kernel of integration over scalar trispectrum shapes

SCALAR-INDUCED GW

- At horizon re-entry in the radiation era:

$$\gamma_k'' + 2\mathcal{H}\gamma_k' + k^2\gamma_k = \mathcal{S}_k$$

Source term including scalar perturbations at quadratic order

$$\propto \int \mathrm{d}^3\vec{q}\ \ (\dots)\zeta_{\vec{q}}\zeta_{\vec{k}-\vec{q}}$$

SCALAR-INDUCED GW

- At horizon re-entry in the radiation era:

$$\gamma_k'' + 2\mathcal{H}\gamma_k' + k^2\gamma_k = \mathcal{S}_k \left\{ \begin{array}{l} \text{Source term including scalar perturbations at quadratic order} \\ \\ \propto \int \mathrm{d}^3\vec{q}\ (\dots)\zeta_{\vec{q}}\zeta_{\vec{k}-\vec{q}} \end{array} \right.$$

- The tensor two-point function is proportional to the scalar four-point function:

$$P_\gamma(k) = \int \mathrm{d}^3\vec{q}_1 \int \mathrm{d}^3\vec{q}_2\ \mathcal{K}(\vec{q}_1, \vec{q}_2) \times \langle \zeta_{\vec{q}_1}\zeta_{\vec{k}-\vec{q}_1}\zeta_{-\vec{q}_2}\zeta_{-\vec{k}+\vec{q}_2} \rangle$$

general kernel

[Adshead, Lozanov, Weiner 2021]

[Garcia-Saenz, LP, Renaux-Petel, Werth 2022] *we discuss its symmetries, etc.*

ArXiv: 2207.14267

SCALAR-INDUCED GW

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Disconnected (Gaussian) piece:

$$(2\pi)^3\delta^{(3)}(\vec{q}_1 - \vec{q}_2)P_\zeta(q_1)P_\zeta\left(\left|\vec{k} - \vec{q}_1\right|\right)$$

$$+ \text{perm.}$$

[Many works]

Connected (non-Gaussian) piece:

$$T_\zeta\left(\vec{q}_1, \vec{k} - \vec{q}_1, -\vec{q}_2, -\vec{k} + \vec{q}_2\right)$$

[Garcia-Saenz, LP, Renaux-Petel, Werth 2022]

ArXiv: 2207.14267

SCALAR-TRISPECTRUM-INDUCED GW

- Only a few recent works working out *some* scalar trispectrum effects:

[Garcia-Bellido, Peloso, Unal 2017]
[Unal 2018]
[Atal, Domenech 2021]
[Adshead, Lozanov, Weiner 2021]

- But *all* limited themselves to **local non-linearities:** $\zeta = \zeta_G + f_{\rm NL}^{\rm loc}\zeta_G^2$

renormalizes the power spectrum: $P_\zeta = P_{\zeta_G} + 3f_{\rm NL}^2\left(P_{\zeta_G}\right)^2$

induces NGs: $\langle\zeta^4\rangle_{\rm connected} \propto f_{\rm NL}^2 P_{\zeta_G}^3 + \mathcal{O}\left(f_{\rm NL}^3\right)$

… and did not check perturbative control → large effects from NGs

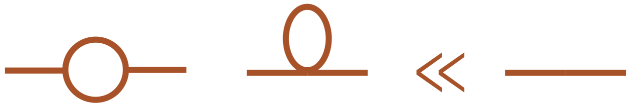
NO-GO THEOREM FOR SCALAR-TRISPECTRUM-INDUCED GW

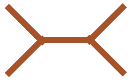
Lemma. Given real symmetric matrices $\boldsymbol{A}$ and $\boldsymbol{B}$, with $\boldsymbol{A}$ positive definite, then $\boldsymbol{C} \equiv \boldsymbol{AB}$ is diagonalizable (over the complex numbers) and has real eigenvalues.

This work:

[Garcia-Saenz, LP, Renaux-Petel, Werth 2022]

ArXiv: 2207.14267

- we investigate motivated scalar trispectrum shapes
- **AND** we check perturbative control → **small loops:** 

	Shape	$\Omega^{\rm GW}_{\rm connected}/\Omega^{\rm GW}_{\rm disconnected}$	Perturbativity bound
Local shapes	$g_{\rm NL}$	0	...
	$\tau_{\rm NL}$	$\mathbf{4} \times \tau_{\rm NL}\mathcal{P}_\zeta \log(kL)$	$\tau_{\rm NL}\mathcal{P}_\zeta \log(kL) \ll 1$
"Equilateral" shapes: interactions from EFToI	$t_{\rm NL}^{\dot\zeta^4}, t_{\rm NL}^{\dot\zeta^2(\partial\zeta)^2}, t_{\rm NL}^{(\partial\zeta)^4}$ 	0 or negligible	...
	$t_{\rm NL}^{[\dot\zeta^3]^2}, t_{\rm NL}^{[\dot\zeta(\partial\zeta)^2]^2}, t_{\rm NL}^{\dot\zeta^3\times\zeta(\partial\zeta)^2}$ 	$\mathcal{O}(\mathbf{10^{-1}}) \times (H/\Lambda_\star)^4$ **Numerically computed coefficient**	$H/\Lambda_\star \ll 1$
"Cosmo. collider" shapes: exchange of massive and spinning fields	$\tau_{\rm NL}^{\rm exchange}(\Delta, S)$ 	$\mathbf{4}\boldsymbol{f}(\boldsymbol{\Delta},\boldsymbol{S}) \times \tau_{\rm NL}\mathcal{P}_\zeta \log(kL)$	?

$\boldsymbol{f}(\boldsymbol{\Delta},\boldsymbol{S}) < \mathbf{1}$

L is the size of the Universe (IR cutoff)

$\Lambda_\star$ is the smallest strong coupling scale

CONCLUSION

- Primordial NGs contain much more information than a single number $f_{\mathrm{NL}}^{\mathrm{local}}$

- Depending on the mass spectrum and interactions of primordial field content, scalar and tensor PNGs are of different **amplitudes** and **shapes**

- Small-scale ultra-squeezed STT and TTT PNGs survive in the form of induced anisotropies in the SGWB

- The scalar trispectrum sources GWs at horizon re-entry but its relative contribution must remain small IN SCALE-INVARIANT MODELS

Warning for scale-dependent models: compute perturbativity bounds!

Formidable opportunity to use the non-linear Universe as a particle detector

BACK UP SLIDES

ANISOTROPIES

SEVERAL SOURCES OF ANISOTROPIES

- GWs signal from astrophysical sources expected to be anisotropic

[Cusin *et al*. 2017, 2018, 2019]

[Bertacca *et al*. 2019]

[Bellomo *et al*. 2021]

- Cosmological background propagates through structures → anisotropic

[Alba, Maldacena 2015]

[Contaldi *et al*. 2016]

[Bartolo *et al*. 2018, 2019] *These anisotropies inherit a non-Gaussian statistics from propagation*

[Domcke, Jinno, Rubira 2020]

- Primordial NGs also induce anisotropies:

[Jeong, Kamionkowski 2012] *Anisotropies of the LSS from the same effect*

[Brahma, Nelson, Chandera 2013]

[Dimastrogiovanni *et al*. 2014, 2015, 2021]

PNG-INDUCED ANISOTROPIES

[Dimastrogiovanni, Fasiello, LP 2022]
ArXiv:2203.17192

- Formal derivations with the in-in formalism:
  - ❖ We look for interactions between small and large scales → $f^{\mathrm{sq}}_{\mathrm{NL},\gamma\gamma\gamma}$ and $f^{\mathrm{sq}}_{\mathrm{NL},\gamma\gamma\zeta}$
  - ❖ A long-wavelength mode J_L can be treated classicaly and has negligible derivatives:

$$\hat{J}_L = J_L(\tau)\hat{a}_{\vec{k}} + J_L^*(\tau)\hat{a}^\dagger_{-\vec{k}} \rightarrow J_L^{\mathrm{cl}}(\tau)\underbrace{\left(\hat{a}_{\vec{k}} + \hat{a}^\dagger_{-\vec{k}}\right)}_{\boldsymbol{b}_{\vec{k}}} \quad ; \quad \left(\partial_i J_L^{\mathrm{cl}}, \partial_t J_L^{\mathrm{cl}}\right) \text{ are negligible}$$

$$\left[\hat{a}_{\vec{k}}, \hat{a}^\dagger_{\vec{k}'}\right] = (2\pi)^3\delta^{(3)}(\vec{k} - \vec{k}') \qquad \left[b_{\vec{k}}, b^\dagger_{\vec{k}'}\right] = 0$$

<u>Ex</u>: massless scalar perturbation $\quad Q_k(\tau) = \dfrac{e^{-ik\tau}}{\sqrt{2k^3}}(1 + ik\tau) \underset{-k\tau \to 0}{\rightarrow} \dfrac{1}{\sqrt{2k^3}}$ purely real

PNG-INDUCED ANISOTROPIES

[Dimastrogiovanni, Fasiello, LP 2022]
ArXiv:2203.17192

- Formal derivations with the in-in formalism:
  - We look for interactions between small and large scales → $f^{\rm sq}_{{\rm NL},\gamma\gamma\gamma}$ and $f^{\rm sq}_{{\rm NL},\gamma\gamma\zeta}$
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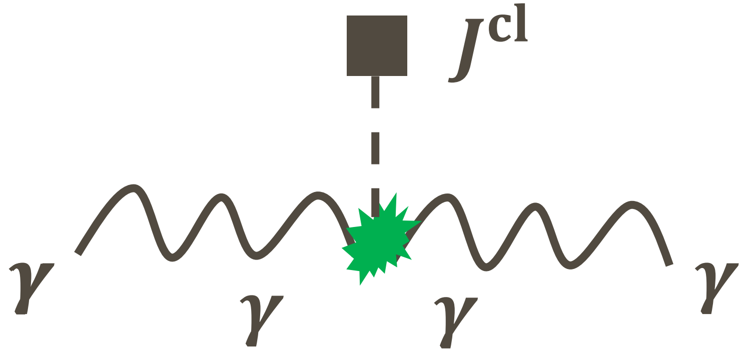
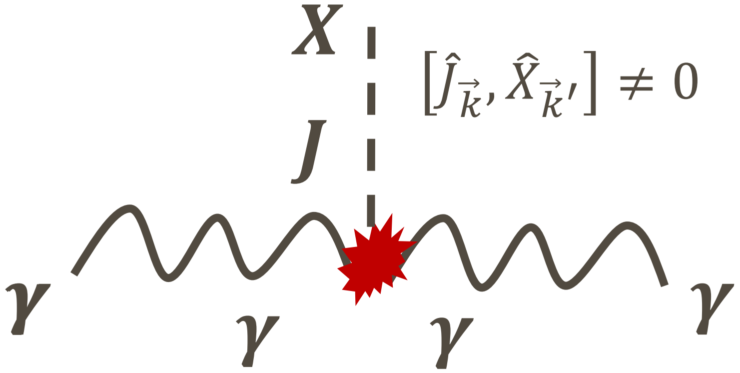
  - A **3-pt interaction** involving J_L becomes a **2-pt interaction** times a classical source $J_L^{\rm cl}$
  - 2-pt functions in the presence of a classical source are now defined:

$$< \gamma_{\vec{k}_1}\gamma_{\vec{k}_2} >_{J^{\rm cl}}$$

$$\langle \gamma_{\vec{k}_1}\gamma_{\vec{k}_2}X_{\vec{k}_3}\rangle = (2\pi)^3\,\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)B^{\gamma\gamma X}$$





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  - A **3-pt interaction** involving J_L becomes a **2-pt interaction** times a classical source $J_L^{\rm cl}$
  - 2-pt functions in the presence of a classical source are now defined
  - We compute both diagrams with the in-in formalism and are therefore able to relate them:

$$\boxed{\langle \gamma_{\vec{k}_1}\gamma_{\vec{k}_2}\rangle_{X^{\rm cl}} \underset{|\vec{k}_1+\vec{k}_2| \ll k_{1,2}}{=} \int {\rm d}^3\vec{q}\ \delta^{(3)}(\vec{q}+\vec{k}_1+\vec{k}_2)P_\gamma(k_1) f^{\gamma\gamma X}_{\rm NL,sq}(\vec{k}_1,\vec{k}_2,\vec{q})X^{\rm cl}(\vec{q})}$$

Derivation makes clear that the non-diagonal part of the 2-pt function does not vanish → anisotropies

J can be X (the formula reduces then to the one in the literature), or not but you need $[\hat{J},\hat{X}] \neq 0$

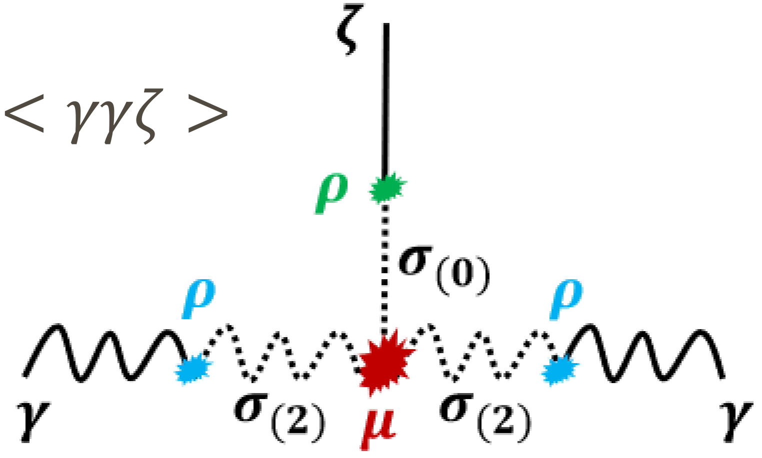
MULTIFIELD MODELS WITH LARGE ANISOTROPIES

- Spin-2 EFT of inflation: $\sigma_{ij} = \partial_i \partial_j \sigma^{(0)} + \sigma_{ij}^{(2)}$ **[Bordin *et al.* 2018]**

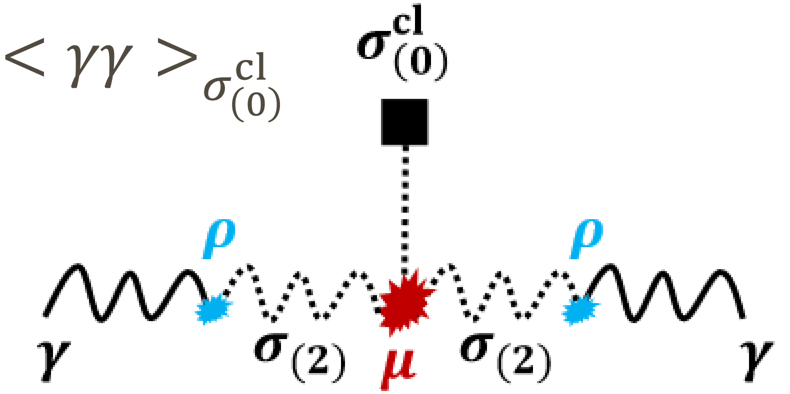
$\rightarrow$ $\sigma^{(2)}$ couples linearly to γ and can enhance the tensor power spectrum: $A_t / \dfrac{2H^2}{M_{Pl}^2} \sim \dfrac{\rho^2}{c_2^3}$

make the tilt blue: $n_t \sim -3\, \partial_t c_2 / (H\, c_2)$

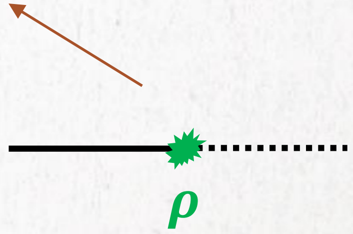
We compute anisotropies explicitly and find: $\sqrt{< \delta_{GW}^2(k_S, \hat{n}) >} \sim \dfrac{< \gamma\gamma\zeta > (k_S, k_S, k_L)}{P_\gamma(k_S) P_{\zeta\sigma^{(0)}}(k_L)} \sqrt{\mathcal{P}_{\sigma^{(0)}}(k_L)}$



(a) Mixed scalar-tensor-tensor bispectrum.



(b) Tensor two-point function in the presence of a classical scalar source.



[Dimastrogiovanni, Fasiello, LP 2022]
ArXiv:2203.17192

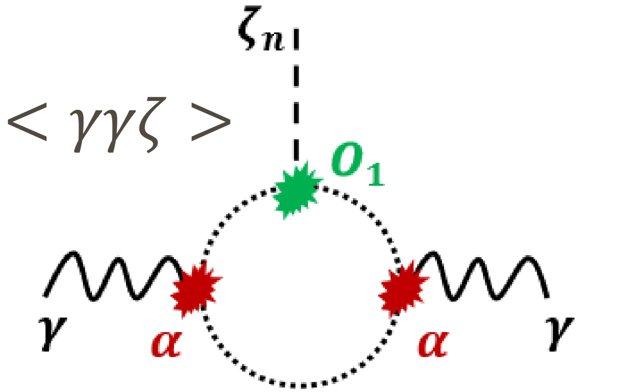
MULTIFIELD MODELS WITH LARGE ANISOTROPIES

- Supersolid inflation: two fundamental scalar fluctuations $(\overset{\text{adiabatic}}{\zeta_n}, \overset{\text{entropic}}{R_{\pi_0}})$ **[Celoria *et al.* 2021]**

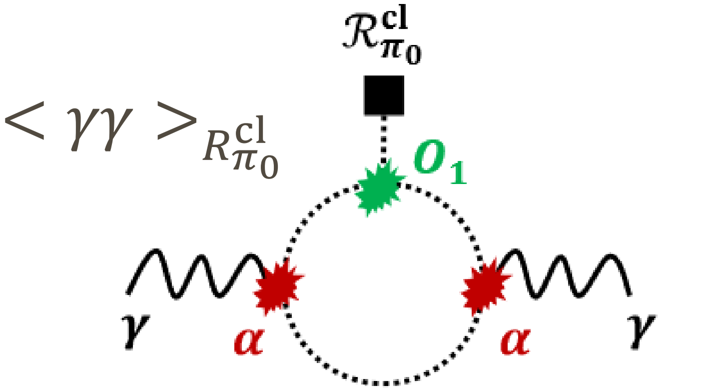
→ R_{π_0} couples **quadratically** to γ and can enhance the tensor power spectrum: $A_t / \frac{2H^2}{M_{Pl}^2} > 1$

make the tilt blue: $n_t = 2(n_s^{\text{en}} - 1) > 0$

We compute anisotropies explicitly and find: $\sqrt{< \delta^2_{GW}(k_S, \hat{n}) >} \sim \underbrace{f_{\text{NL,sq}}^{\gamma\gamma\zeta_n}(k_S, k_S, k_L)}_{\gg 1} \underbrace{\left(\sqrt{\frac{\mathcal{P}_{\zeta_n} \mathcal{P}_{R_{\pi_0}}}{\mathcal{P}_{\zeta_n R_{\pi_0}}}} \right)_{k_L}}_{\mathcal{O}(1)} \underbrace{A_s^{1/2}}_{4 \times 10^{-5}}$



(b) One-loop scalar-tensor-tensor bispectrum



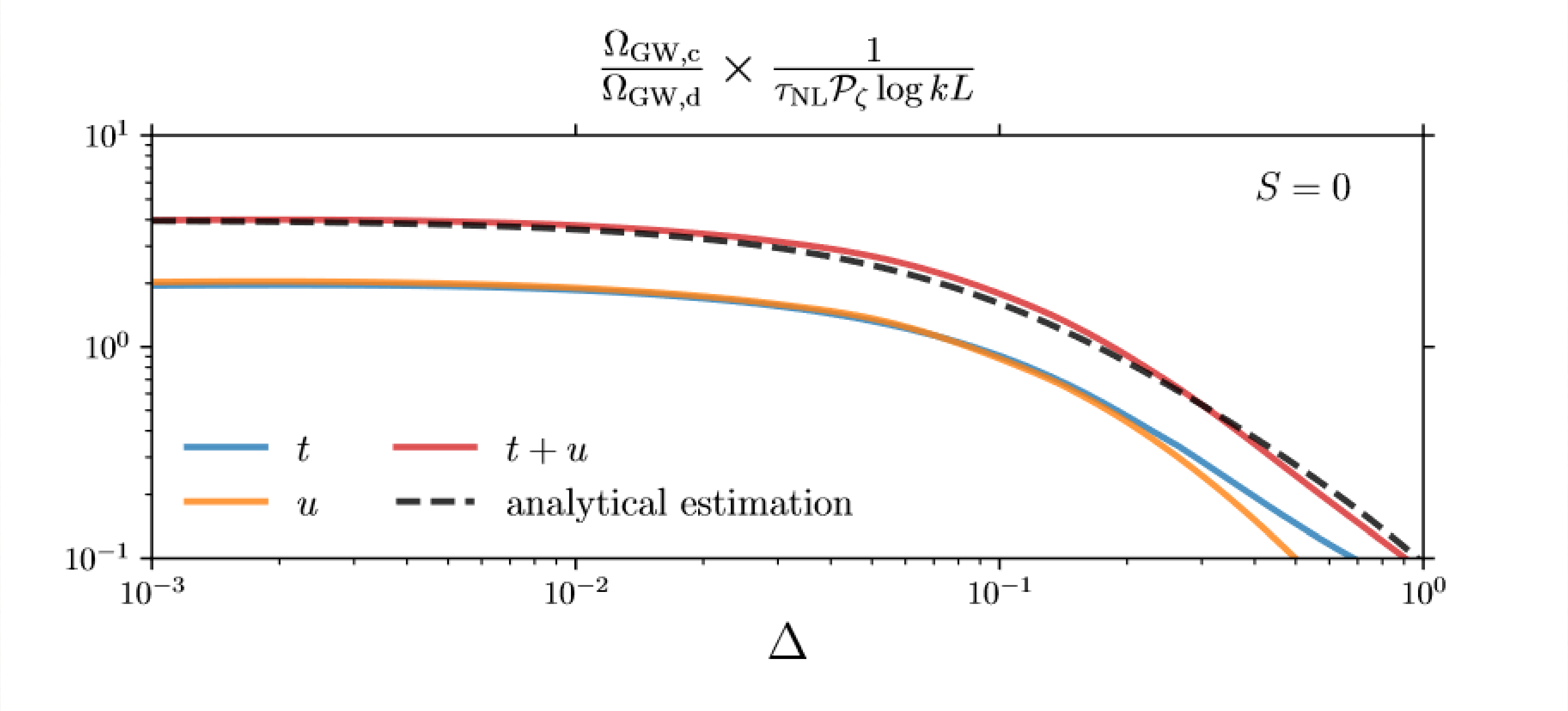
(c) One-loop tensor two-point function in the presence of a classical scalar source.

[Dimastrogiovanni, Fasiello, LP 2022]
ArXiv:2203.17192

BACK UP SLIDES

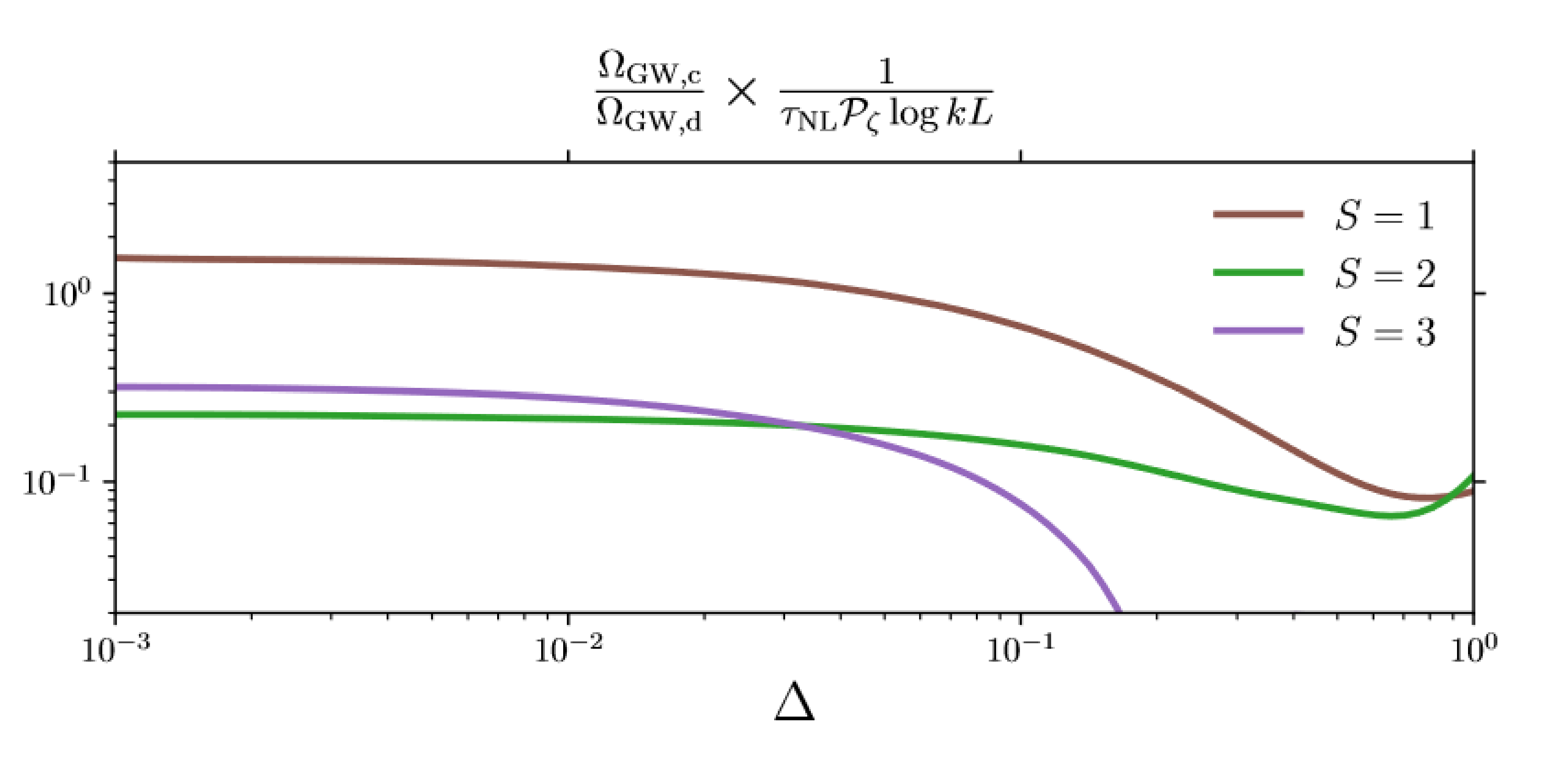
TRISPECTRUM INDUCED

EXCHANGE OF A MASSIVE SCALAR FIELD



Analytical estimation: $$\frac{\Omega_{\rm GW,c}}{\Omega_{\rm GW,d}} \simeq 4\,\tau_{\rm NL}\mathcal{P}_\zeta \,\frac{\alpha^{2\Delta}}{2\Delta}\left[1 - e^{-2\Delta \log kL}\right]$$

EXCHANGE OF A MASSIVE SPINNING FIELD



BACK UP SLIDES

SCALAR PNG

BISPECTRUM IN MULTIFIELD INFLATION

The squeezed limit as a cosmological collider

Remember the single-field result: $\boxed{f_{\mathrm{NL}}^{\mathrm{squeezed}} \propto n_s - 1 \ll 1}$ *universal relation*

Two-field result:

[Chen, Wang 2009]

[Noumi, Yamaguchi, Yokoyama 2013] (one extra heavy field $m_s > 3H/2$, perturbatively coupled)

[Arkani-Hamed, Maldacena 2015]

[Arkani-Hamed, Baumann, Lee, Pimentel 2018]

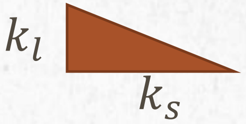
BISPECTRUM IN MULTIFIELD INFLATION

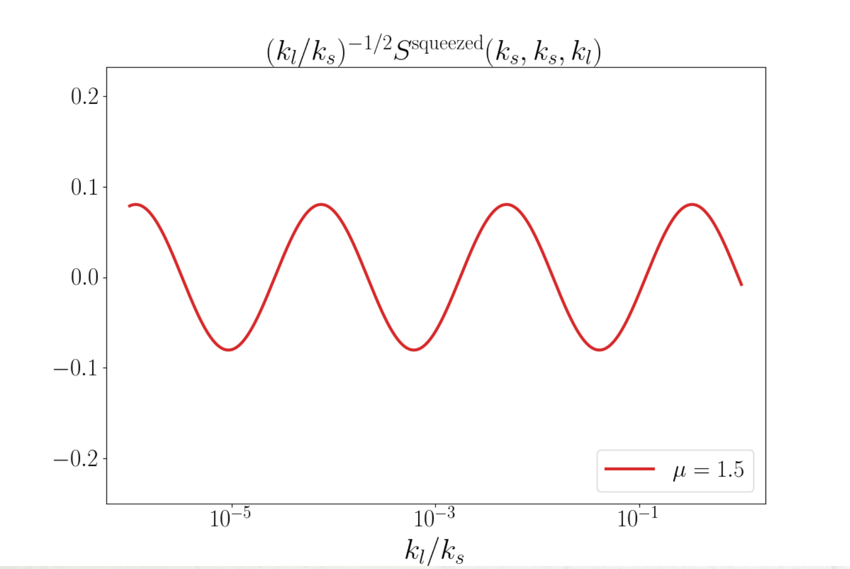
The squeezed limit as a cosmological collider

[Noumi, Yamaguchi, Yokoyama 2013]

Small coupling between the two fields : $\boldsymbol{\eta_\perp \ll 1}$

Two-field result: $$f_{\mathrm{NL}}^{\text{squeezed}} \underset{k_l \ll k_s}{\simeq} \boldsymbol{\eta_\perp^2 e^{-\pi\mu}} \cos\left[\boldsymbol{\mu} \log\left(\frac{k_l}{k_s}\right)\right]$$





Oscillatory pattern: massive particle

$\boldsymbol{\mu = \sqrt{\frac{m_s^2}{H^2} - \frac{9}{4}}}$ **the reduced mass**

Boltzmann suppression for heavy particles

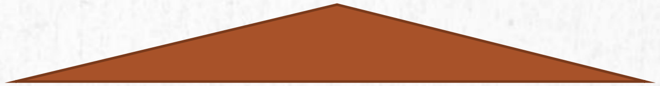
$\boldsymbol{H/2\pi \sim T}$ **so** $\boldsymbol{\mathrm{e}^{-\pi\mu} \sim e^{-\frac{m_s}{T}}}$

BISPECTRUM IN MULTIFIELD INFLATION

Probing other regimes

- Large coupling, $\eta_\perp \gg 1$ $\rightarrow$ Multifield instability $\rightarrow$ Large flattened NGs:

[Fumagalli, Garcia-Saenz, Lucas Pinol, Renaux-Petel, Ronayne 2019]
Phys. Rev. Lett. 123, 201302

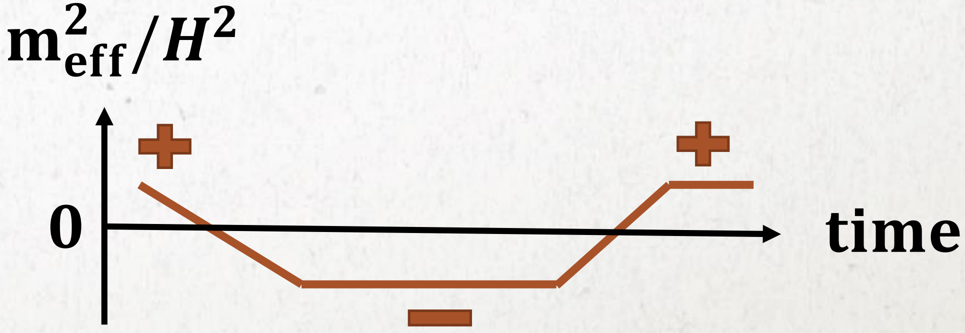


$f_{\mathrm{NL}}^{\mathrm{flat}} = \mathcal{O}(50)$

Higher-order correlation functions are boosted in similar configurations

$g_{\mathrm{NL}}^{\mathrm{flat}} = \mathcal{O}(10^5)$ etc.

Clear sign of transiently unstable degrees of freedom:



BISPECTRUM IN MULTIFIELD INFLATION

Probing other regimes

- Large mass, $|m_s^2| \gg H^2 \;\rightarrow$ Single-field effective theory for ζ
  (including the instability with $m_s^2 < 0$)

$$\hookrightarrow \quad f_{\mathrm{nl}}^{\mathrm{eq}} \simeq \left(\frac{1}{c_s^2} - 1\right)\left(-\frac{85}{324} + \frac{15}{243}\boldsymbol{A}\right)$$

Speed of sound:
Dictated by the bilinear coupling $\eta_\perp$
[Achucarro, Gong, Hardeman, Palma, Patil 2012]

Single-field effective interactions
Dictated by the multifield cubic interactions
[Garcia-Saenz, Lucas Pinol, Renaux-Petel 2019]
J. High Energ. Phys. **2020,** 73 (2020)

Later extended to any number of heavy fields: [Lucas Pinol 2020] *J. Cosm. & Astro. Phys.* 04(**2021**)048

THE EFT OF INFLATION

REVISITED...

Bottom-up approach: unknown natural values of the coefficients

[Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore 2009]

$$S_3^{\mathrm{EFT}}[\zeta] = \int d\tau d^3x a^2 M_p^2 \frac{\epsilon}{\mathcal{H}}\left(\frac{1}{c_s^2} - 1\right)\left(\zeta'(\partial_i\zeta)^2 + \frac{\boldsymbol{A}}{c_s^2}\zeta'^3\right)$$

with $\boldsymbol{A} = \mathcal{O}(1)$ but **undetermined**

THE EFT OF INFLATION

REVISITED...

In our top-down approach we DERIVE those coefficients

$$S_3^{\mathrm{EFT}}[\zeta] = \int d\tau d^3x a^2 M_p^2 \frac{\epsilon}{\mathcal{H}}\left(\frac{1}{c_s^2}-1\right)\left(\zeta'(\partial_i\zeta)^2 + \frac{\boldsymbol{A}}{c_s^2}\zeta'^3\right)$$

$$\text{with } \boldsymbol{A} = \underbrace{-\frac{1}{2}(1+c_s^2)}_{\text{Previously known}} + \underbrace{\frac{2}{3}(1+2c_s^2)\frac{\epsilon R_{\mathrm{fs}} H^2 M_p^2}{m_s^2}}_{\text{Scalar curvature of the field space}} - \frac{1}{6}(1-c_s^2)\left(\underbrace{\frac{\kappa V_{;sss}}{m_s^2}}_{3^{\mathrm{rd}}\text{ derivative of the potential}} + \underbrace{\frac{\kappa\epsilon H^2 M_p^2 R_{\mathrm{fs},s}}{m_s^2}}_{\text{Derivative of the scalar curvature}}\right)$$

Previously known

Scalar curvature of the field space

3^{rd} derivative of the potential

Derivative of the scalar curvature

[Garcia-Saenz, Lucas Pinol, Renaux-Petel 2019]
J. High Energ. Phys. **2020,** 73 (2020)

Bending radius of the trajectory: $\kappa = \sqrt{2\epsilon}M_p/\eta_\perp$

BISPECTRUM IN MULTIFIELD INFLATION

Probing more than one extra field

[Lucas Pinol 2020]
J. Cosm. & Astro. Phys. 04(**2021**)048

➢ I extended previous works for any number N_{field} of kinetically coupled scalars:

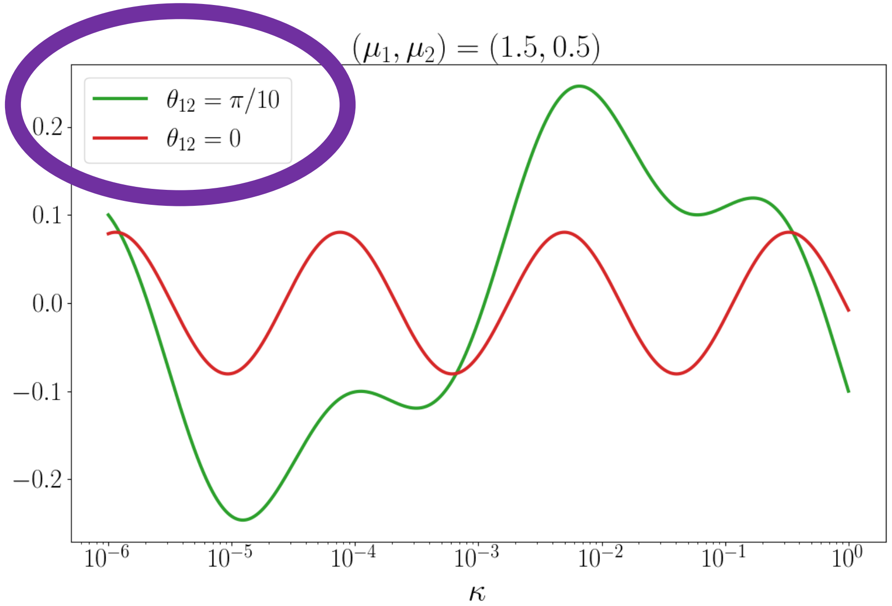
- Most generic cubic action for perturbations at lowest order in derivatives

$$
\begin{aligned}
\mathcal{L}^{(3)} = {} & \underbrace{M_p^2 a^3 \left[\epsilon(\epsilon-\eta)\dot{\zeta}^2\zeta + \epsilon(\epsilon+\eta)\zeta\frac{(\partial\zeta)^2}{a^2} + \left(\frac{\epsilon}{2}-2\right)\frac{1}{a^4}(\partial\zeta)(\partial\chi)\partial^2\chi + \frac{\epsilon}{4a^4}\partial^2\zeta(\partial\chi)^2\right]}_{\text{single-field (recovering Maldacena)}} \\
& \underbrace{\begin{aligned}
& + a^3 \left\{ \sqrt{2\epsilon}\,\omega_1 M_{\rm Pl} \left[\frac{\mathcal{F}^1}{H}\left(\frac{(\partial\zeta)^2}{a^2} - \dot{\zeta}^2 - \dot{\zeta}\zeta H\left(\eta + 2u_1\right)\right) + 2\frac{\Omega_{1\alpha}}{H}\dot{\zeta}\zeta\mathcal{F}^\alpha\right]\right. \\
& + \left[\frac{\epsilon}{2}m^2_{\alpha\beta} + \frac{\dot{\left(m^2_{\alpha\beta}\right)}}{2H} + \Omega_{\gamma\beta}\left(\epsilon\Omega^\gamma{}_\alpha + \frac{\dot{\Omega}^\gamma{}_\alpha}{H} - \frac{m^2_{\gamma\alpha}}{H}\right)\right]\zeta\mathcal{F}^\alpha\mathcal{F}^\beta + \epsilon\Omega_{\alpha\beta}\zeta\dot{\mathcal{F}}^\alpha\mathcal{F}^\beta \\
& + \left(2\epsilon H^2 M_{\rm Pl}^2 R_{\alpha\sigma\beta\sigma} - \omega_1^2\delta_{\alpha 1}\delta_{\beta 1}\right)\frac{\dot{\zeta}}{H}\mathcal{F}^\alpha\mathcal{F}^\beta + \frac{1}{2}\epsilon\zeta\left(\left(\dot{\mathcal{F}}^\alpha\right)^2 + \frac{(\partial\mathcal{F}^\alpha)^2}{a^2}\right) \qquad (3.10) \\
& - \frac{1}{a^2}\left(\partial\mathcal{F}^\alpha\right)\left(\partial\chi\right)\left(\dot{\mathcal{F}}^\alpha + \Omega_{\alpha\beta}\mathcal{F}^\beta\right) + \frac{2}{3}\sqrt{2\epsilon}\,H M_{\rm Pl} R_{\alpha\beta\gamma\sigma}\dot{\mathcal{F}}^\alpha\mathcal{F}^\beta\mathcal{F}^\gamma \\
& \left. - \frac{1}{6}\left(V_{;\alpha\beta\gamma} - 4\sqrt{2\epsilon}\,H M_{\rm Pl}\left(\omega_1\delta_{\alpha 1}R_{\beta\sigma\gamma\sigma} + \Omega^\delta{}_\alpha R_{\delta\beta\gamma\sigma}\right) + 2\epsilon H^2 M_{\rm Pl}^2 R_{\alpha\sigma\beta\sigma;\gamma}\right)\mathcal{F}^\alpha\mathcal{F}^\beta\mathcal{F}^\gamma \right\}
\end{aligned}}_{\geq \text{ two fields}} \\
& + \mathcal{D}, \longrightarrow \geq \text{ boundary terms (they contribute!)}
\end{aligned}
$$

BISPECTRUM IN MULTIFIELD INFLATION

Probing more than one extra field **[Lucas Pinol 2020]**
J. Cosm. & Astro. Phys. 04(**2021**)048

- I extended previous works for any number N_{field} of kinetically coupled scalars:
  - Most generic cubic action for perturbations at lowest order in derivatives
  - In the case of heavy fields, integrating out procedure still possible
  - We can probe many-fields interactions in the squeezed limit



[LP, Aoki, Renaux-Petel, Yamaguchi 2021]
ArXiv:2112.05710

Interesting features:

- ❖ Several extra massive fields lead to modulated oscillations
- ❖ Light fields or light and heavy also lead to characteristic signals
- ❖ Theory described with **mixing angles** for flavour and mass eigenstates

Inflationary flavour oscillations