Modeling the Angular Bispectrum in Photometric Galaxy Surveys

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Motivation

Future and on-going surveys will provide huge photometric datasets.

Harmonic power spectrum is already used to constraints cosmological parameters.

The harmonic bispectrum is the natural next step after the 2-point statistics, improving the constraining power of angular statistics.

A bit of math…

Once we define the density contrast and its Fourier counterpart, we can define the matter Power Spectrum and Bispectrum

$$\delta_g(\widehat{\mathbf{n}}, z) = \frac{n_g(\widehat{\mathbf{n}}, z) - \bar{n}(z)}{\bar{n}(z)}$$

$$\delta_{\mathbf{k}}(z) = \int d^3r\ \delta_g(\widehat{\mathbf{n}}, z) e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$\langle \delta_{\mathbf{k}} \delta_{\mathbf{k}'} \rangle \equiv (2\pi)^3 \delta^D(\mathbf{k} + \mathbf{k}') P(k)$$

$$\langle \delta_{\mathbf{k}_1} \delta_{\mathbf{k}_2} \delta_{\mathbf{k}_3} \rangle \equiv (2\pi)^3 \delta^D(\mathbf{k}_{123}) B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$$

A bit of math… on the sphere

Let’s integrate the density contrast along the line of sight (i.e., z in redshift space):

$$\delta(\widehat{\mathbf{n}}) = \int dz\, \phi(z) \delta_g(\widehat{\mathbf{n}}, z),$$

Radial selection function:

$$\phi(z) = \frac{dN_g}{dz} W(z)$$

A bit of math… on the sphere

Every spherical function can be decomposed by means of the Spherical Harmonics, i.e., the orthonormal basis on the sphere.

$$f(\widehat{\mathbf{n}}) = \sum_{\ell=0}^{+\infty} \sum_{m=-\ell}^{m=\ell} a_{\ell m} Y_{\ell m}(\widehat{\mathbf{n}}),$$

$$a_{\ell m} = \int_{S^2} d\Omega\, f(\widehat{\mathbf{n}}) Y^*_{\ell m}(\widehat{\mathbf{n}})$$

Harmonic coefficients contain all the information of the original distribution.

Spherical statistics

We can than define, as in Fourier space, power spectrum and bispectrum:

$$\langle a_{\ell m} a^*_{\ell' m'} \rangle \equiv \delta_l^{\ell'} \delta_m^{m'} C_\ell$$

$$\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = \mathcal{G}_{l_1 l_2 l_3}^{m_1 m_2 m_3} b_{l_1 l_2 l_3}$$

The Gaunt integral ensures that the three ell's form a closed triangle.

Matter spherical distribution statistics

Putting all together and take advantage of the *plane-wave expansion:*

$$a_{lm} = 4\pi i^l \int dz \phi(z) \int \frac{d^3 k}{(2\pi)^3} \delta_{\mathbf{k}}(z) j_l(kr(z)) Y^*_{lm}(\hat{\mathbf{k}})$$

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$$C_l = \frac{2}{\pi} \int k^2 dk \int dz_1 dz_2 \, \phi(z_1)\phi(z_2) \times j_l(kr_1) j_l(kr_2) P(k, z_1, z_2)$$

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$$b_{l_1 l_2 l_3} = \frac{8}{\pi^3} \int dx x^2 \prod_{i=1}^{3} \left\{ \int dk_i k_i^2 dz_i \phi(z_i) j_{l_i}(k_i r_i) j_{l_i}(k_i x) \right\} \times B(k_1, k_2, k_3, z_1, z_2, z_3)$$

Limber approximation

Allows to get rid of integrals of Bessel functions exploiting their properties

$$\int dk\ k^2 j_l(kr_1) j_l(kr_2) f(k) \simeq \frac{\pi}{2} \frac{\delta^D(r_1 - r_2)}{r_1^2} f\left(\frac{l + 1/2}{r_1}\right)$$

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$$C_l^{limb} = \int dz\, \frac{\phi^2(z)}{r^2(z)|r'(z)|} P\left(\frac{l + 1/2}{r(z)}, z\right)$$

$$b_{l_1 l_2 l_3} = \int dz \phi(z)^3 \frac{1}{r(z)^4 |r(z)'|^2} B\left(\frac{l_1 + \frac{1}{2}}{r(z)}, \frac{l_2 + \frac{1}{2}}{r(z)}, \frac{l_3 + \frac{1}{2}}{r(z)}; z\right)$$

Dataset

Photometric mocks extracted from N-body simulations produced by MICE collaboration (Crocce et al. 2011).

LambdaCDM, z_center=0.5, 125 mocks covering 1/8th of the sky.

Case	$\Delta z/(1+\bar{z})$	σ_z
1	0.03	0.03
2	0.05	0.03
3	0.03	0.06
4	0.05	0.06

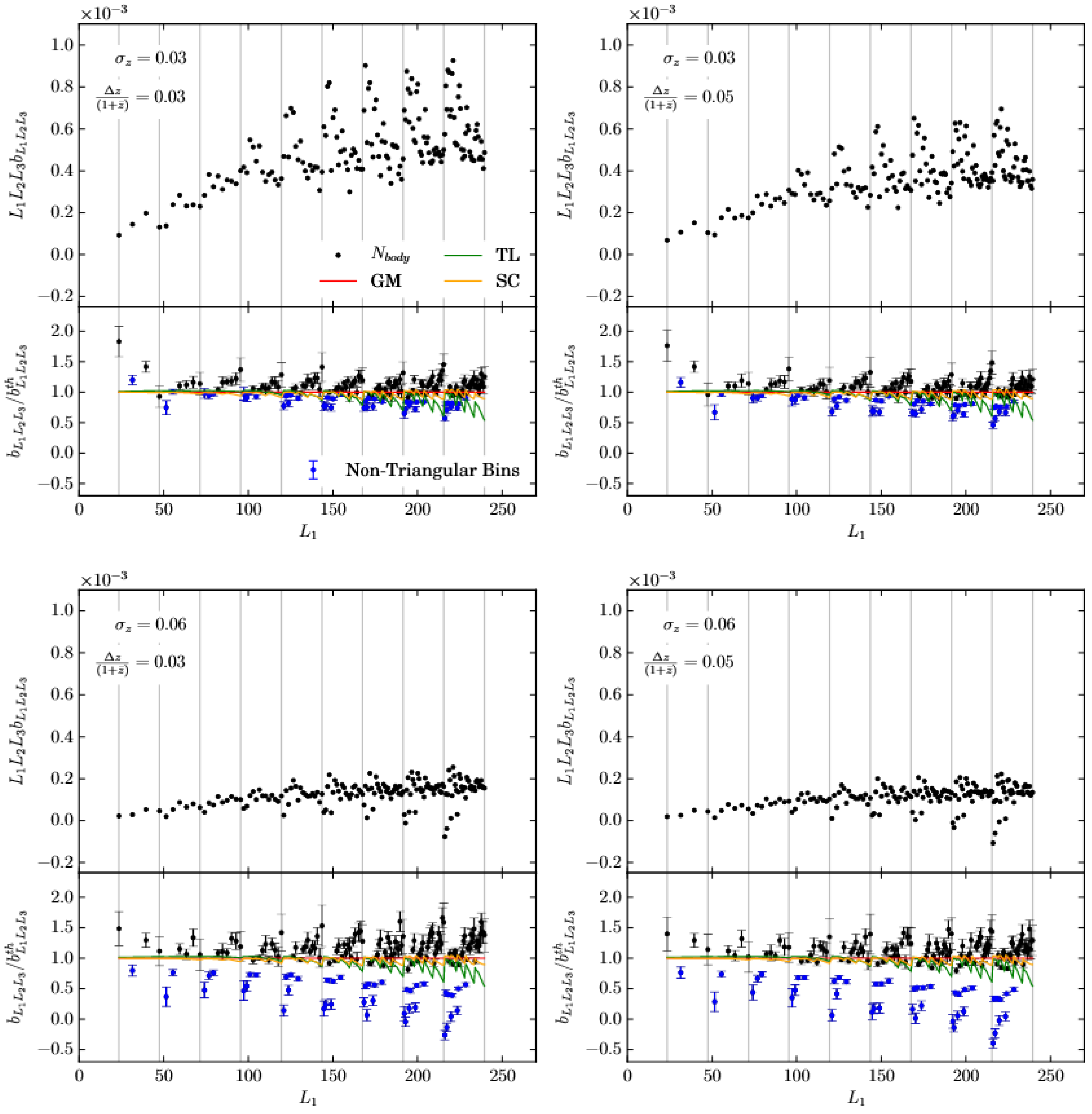
Results

Tree-level bispectrum model was used. Corrections from Scoccimarro, Couchman (2001) and Gil-Marin (2011) where also used.

L= [l_min, l_max]

Features are lost when z-bin size or photo-z error increase.

GM correction consistent with the measurements.



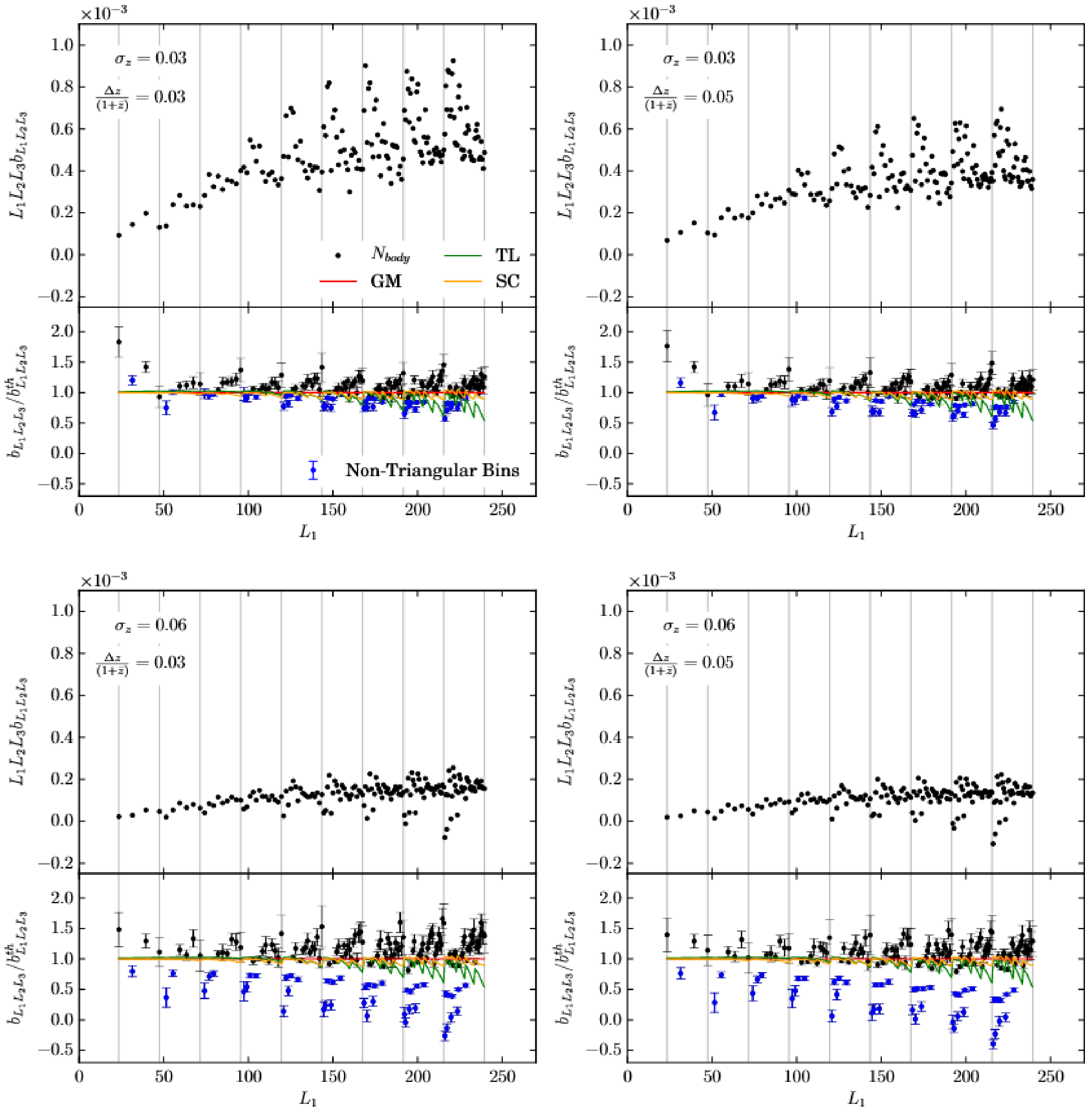
Results

Non-Triangular bins effect

When the three L does not form a closed triangle, BUT at least one triplet contained in them does.

Usually, these triplets form a flat triangle.

Homogenizing the sample destroys the flat configurations (i.e., the filaments).



Next step: Beyond Limber

Limber approximation performs better at high ell's (l>80).

Being able to compute the bispectrum integral fast comes at the price of information loss at the large scales.

We need a fast way to compute exact bispectrum at the largest scales…

PSICo: Power Spectrum Integration Code

PSICo performs FFTlog of the bispectrum in harmonic space.

The Bessel functions integral are evaluated exactly using Hankel transform.

Hybrid Python-C++ code make the computation fast (less than 0.01 sec per bisp configuration)

Currently under validation (Paper in prep.)

Conclusion and future perspectives

We recover bispectrum measurements using Limber integrated tree-level bispectrum.

Non-Limber code (PSICo) is ready to be used to get fast and exact bispectra.

Next step:

- Evaluation of forecast for future surveys
- Parameter estimation on real data
- Improve the B(k1,k2,k3) model (for example, by adding 1-loop correction)