Coordinate choice and galaxy bias in the presence of general relativistic effects and primordial non-Gaussianity = GR effects

Obinna Umeh

A Cosmic Window to Fundamental Physics: Primordial Non-Gaussianity and Beyond

September 23, 2022

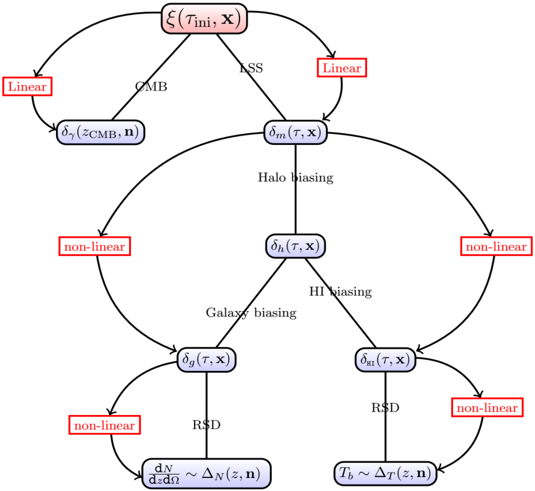
UNIVERSITY OF PORTSMOUTH

ICG Portsmouth

Collaborators

- Sheean Jolicoeur, Roy Maartens (UWC, South Africa)
- Alireza Allahyari (Kharazmi University, Tehran, Iran)
- Daniele Bertacca (Padova), Stefano Camera(Torino, Italy)
- Chris Clarkson, Eline M. De Weerd (Queen Mary, UK)
- Julien Larena (Montpellier, France)
- Robert Crittenden, Kazuya Koyama (ICG, Portsmouth, UK)
- Fabian Schmidt (Max–Planck–Institut, Garching, Germany)

A Cosmic window to fundamental(primordial) physics:PNG



- Newtonian theory(Cosmic tension)
- Einstein theory (correct theory of gravitation)

We should always ask what is the position of GR on this?

General Relativity

- $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$
- $\nabla^{\mu} T_{\mu\nu} = 0$
- EMT: $T_{\mu\nu} = \rho u_{\mu} u_{\nu} + p h_{\mu\nu} + 2q_{(\mu} u_{\nu)} + \pi_{\mu\nu}$
- Non-linear scalar perturbation
- Vector and Tensor perturbations (even if it does not exist at $\tau_{\rm ini}$)
- Horizons: gravitational disturbances travel at a finite speed.

Newtonian limit

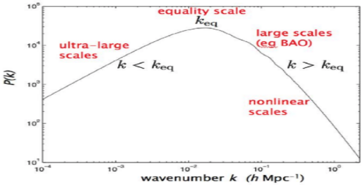
- $R_{00} = 4\pi\rho = \nabla^2\Phi(\text{linear})$
- Non-relativistic fluid, $u^{\mu}_{\rm NR}$: $\nabla^{\mu}\left(\rho u_{\mu} u_{\nu}\right) = 0$
- Linear scalar perturbation
- Instantaneous gravitational effect on all matter.

GR effects in the number count of sources

- Historical classifications of the local number count fluctuations

$$\Delta_g^{(r)} = \Delta_{g\,\mathrm{N}}^{(r)} + \Delta_{g\,\mathrm{GR}}^{(r)}$$

- $\Delta_{g\,\mathrm{N}}^{(r)}$ dominant on sub-Horizon scales: $d \ll d_H \equiv 1/\mathcal{H}$



- Contribution on sub-Horizon scales

$$\Delta_{g\,\mathrm{N}}^{(r)} = \delta_{g,\mathrm{N}}^{(r)} - \frac{1}{\mathcal{H}}\partial_\parallel^2 v_{\mathrm{N}}^{(r)} + \sum_{i,j}^{r-1} \Delta_{g\,\mathrm{N}}^{(i-1)} \Delta_{g\,\mathrm{N}}^{(j-1)}$$

- In the fluid limit, $\delta_{m,N}$ and $\partial_\parallel v_N$ are described by $\nabla^\mu \left(\rho u_\mu u_\nu\right) = 0$ (*F. Bernardeau, S. Colombi, E. Gaztanaga and R. Scoccimarro,, Phys. Rept.* **367** *(2002), 1-248*)

In full GR: Dynamical effects emerge

- Non-linear scalar perturbations: Bias model: $\delta_g^{(r)} \propto \delta_m^{(r)}$

$$\delta_m^{(r)} = \delta_{m,\mathrm{N}}^{(r)} + \delta_{m,\mathrm{d,GR}}^{(r)}, \qquad v^{i\,(r)} = v^{i\,(r)}_{\mathrm{N}} + v^{i\,(r)}_{\mathrm{d,GR}}$$

$$\Phi^{(r)} = \Phi_{\mathrm{N}}^{(r)} + \Phi_{\mathrm{d,GR}}^{(r)}, \qquad \Psi^{(r)} = \Psi_{\mathrm{N}}^{(r)} + \Psi_{\mathrm{d,GR}}^{(r)}$$

- At linear order for dust, the dynamical GR effects vanish, i.e $\delta_{m,\mathrm{d,GR}}^{(r)} = 0 = v^{i\,(r)}_{\mathrm{d,GR}} = \Phi_{\mathrm{d,GR}}^{(r)} = \Psi_{\mathrm{d,GR}}^{(r)}$.

- GR effects: $\Delta_{g\,\mathrm{GR}}^{(r)} \equiv \Delta_{g\,\mathrm{GR,loc}}^{(r)} + \Delta_{g\,\mathrm{GR,dyn}}^{(r)} + \Delta_{g\,\mathrm{GR,V/T}}^{(r)}$

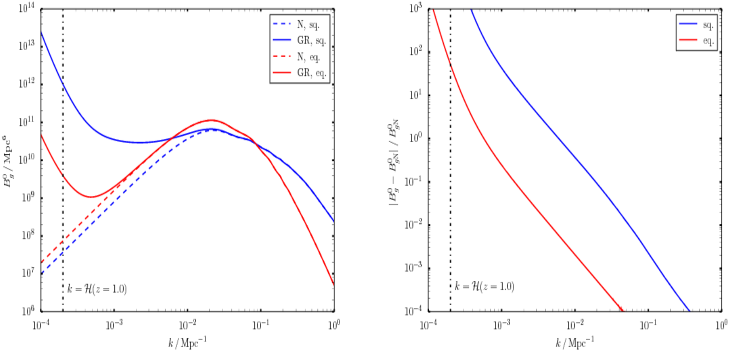
$$\begin{aligned}\Delta_{g\,\mathrm{GR}}^{(r)} = & \left[b_e - 2\mathcal{Q} + \frac{2(\mathcal{Q}-1)}{\chi\mathcal{H}} - \frac{\mathcal{H}'}{\mathcal{H}^2}\right]\left[\partial_{|} v^{(r)} - \Phi^{(r)}\right] \\ & -2(1-\mathcal{Q})\Psi^{(r)} + \Phi^{(r)} + \frac{1}{\mathcal{H}}\Psi^{(r)\prime} + (3-b_e)\,\mathcal{H} v^{(r)} \\ & + \sum_{i,j}^{r-1} \Delta_{g\,\mathrm{GR}}^{(i-1)} \Delta_{g\,\mathrm{N}}^{(j-1)} + \sum_{i,j}^{r-1} \Delta_{g\,\mathrm{GR}}^{(i-1)} \Delta_{g\,\mathrm{GR}}^{(j-1)}\end{aligned}$$

Full expression for the galaxy bispectrum(tree level)

$$
\begin{aligned}
B_g(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \quad = \quad & \Big[\mathcal{K}_{\rm N}^{(1)}(\boldsymbol{k}_1)\mathcal{K}_{\rm N}^{(1)}(\boldsymbol{k}_2)\mathcal{K}_{\rm N}^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \\
& + \mathcal{K}_{\rm GR}^{(1)}(\boldsymbol{k}_1)\mathcal{K}_{\rm GR}^{(1)}(\boldsymbol{k}_2)\mathcal{K}_{\rm GR}^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \\
& + \mathcal{K}_{\rm N}^{(1)}(\boldsymbol{k}_1)\mathcal{K}_{\rm N}^{(1)}(\boldsymbol{k}_2)\mathcal{K}_{\rm GR}^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \\
& + \mathcal{K}_{\rm GR}^{(1)}(k_1)\mathcal{K}_{\rm GR}^{(1)}(k_2)\mathcal{K}_{\rm N}^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \\
& + 2\mathcal{K}_{\rm N}^{(1)}(\boldsymbol{k}_1)\mathcal{K}_{\rm GR}^{(1)}(\boldsymbol{k}_2)\Big\{\mathcal{K}_{\rm N}^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \\
& + \mathcal{K}_{\rm GR}^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)\Big\}\Big] P(k_1)P(k_2) \\
& + 2 \text{ cyc. perm.}
\end{aligned}
$$

Bispectrum: Local GR effect vs Newtonian limit

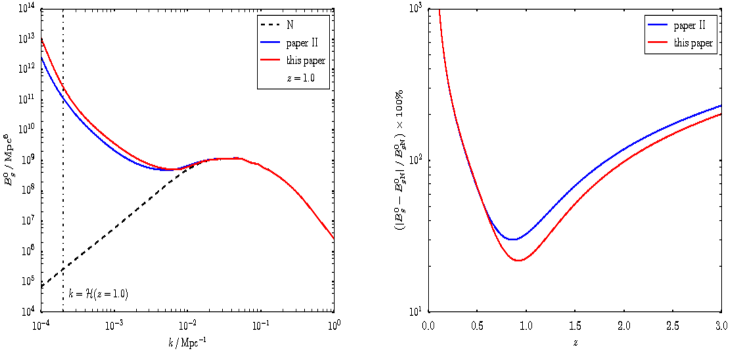
$$\Delta^{(r)}_{g\,\mathrm{GR}} \equiv \Delta^{(r)}_{g\,\mathrm{GR,loc}} + \cancel{\Delta^{(r)}_{g\,\mathrm{GR,dyn}}} + \cancel{\Delta^{(r)}_{g\,\mathrm{GR,V/T}}}$$



- **O. Umeh**, *S. Jolicoeur, R. Maartens and C. Clarkson, JCAP* **03** *(2017), 034*
- *S. Jolicoeur,* **O. Umeh**, *R. Maartens and C. Clarkson, JCAP* **09** *(2017), 040*

Bispectrum: Local and dynamical GR effects vs Newtonian limit

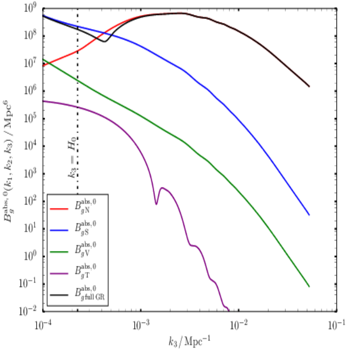
$$\Delta^{(r)}_{g\,\mathrm{GR}} \equiv \Delta^{(r)}_{g\,\mathrm{GR,loc}} + \Delta^{(r)}_{g\,\mathrm{GR,dyn}} + \cancel{\Delta^{(r)}_{g\,\mathrm{GR,V/T}}}$$



- *S. Jolicoeur,* **O. Umeh**, *R. Maartens and C. Clarkson, JCAP* **03** *(2018), 036*

Local + dynamical GR effects + V/T vs Newtonian limit

$$\Delta^{(r)}_{g\,\mathrm{GR}} \equiv \Delta^{(r)}_{g\,\mathrm{GR,loc}} + \Delta^{(r)}_{g\,\mathrm{GR,dyn}} + \Delta^{(r)}_{g\,\mathrm{GR,V/T}}$$



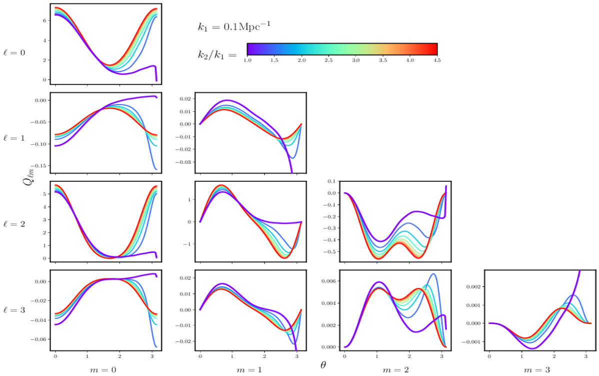
$$\begin{aligned}
B_{g\mathrm{V}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) &= \mathcal{K}^{(1)}(\boldsymbol{k}_1)\mathcal{K}(\boldsymbol{k}_2)^{(1)}\mathcal{K}^{(2)}_{\mathrm{V}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)P(k_2)P(k_1) + 2\text{ cyc. perm.}\,, \\
B_{g\mathrm{T}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) &= \mathcal{K}^{(1)}(\boldsymbol{k}_1)\mathcal{K}^{(1)}(\boldsymbol{k}_2)\mathcal{K}^{(2)}_{\mathrm{T}}(\boldsymbol{k}_1)P(k_2)P(k_1) + 2\text{ cyc. perm.}\,. \\
B_{g\mathrm{S}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) &= \mathcal{K}^{(1)}(\boldsymbol{k}_1)\mathcal{K}^{(1)}(\boldsymbol{k}_2)\mathcal{K}^{(2)}_{\mathrm{S}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)P(k_2)P(k_1) + 2\text{ cyc. perm.}\,. \\
B_g^{\mathrm{full,GR}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) &= \mathcal{K}^{(1)}(\boldsymbol{k}_1)\mathcal{K}^{(1)}(\boldsymbol{k}_2)\mathcal{K}^{(2)}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)P(k_1)P(k_2) + 2\text{ cycl. perm.}\,.
\end{aligned}$$

where $\mathcal{K}^{(2)} = \mathcal{K}^{(2)}_{\mathrm{S}} + \mathcal{K}^{(2)}_{\mathrm{V}} + \mathcal{K}^{(2)}_{\mathrm{T}}$.

- *S. Jolicoeur, A. Allahyari, C. Clarkson, J. Larena,* **O. Umeh** *and R. Maartens, JCAP* **03** *(2019), 004*

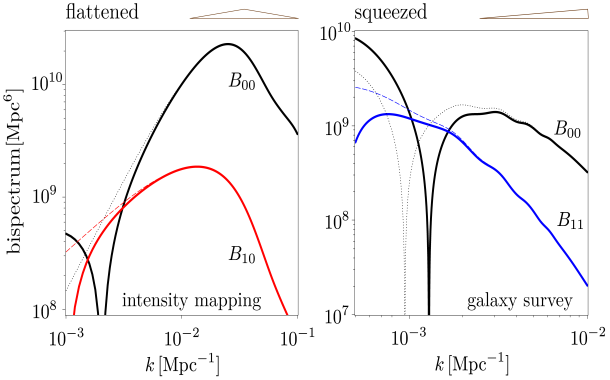
Bispectrum: GR effects and odd multipoles (Kelvin's talk)

$$B_g^{\ell m}(k_1,k_2,k_3) = \frac{(2\ell+1)}{4\pi}\int_0^{2\pi}\mathrm{d}\phi\int_{-1}^{1}\mathrm{d}\mu_1 \; B_g(k_1,k_2,k_3,\mu_1,\phi)\,Y^*_{\ell m}(\mu_1,\phi)\,,$$



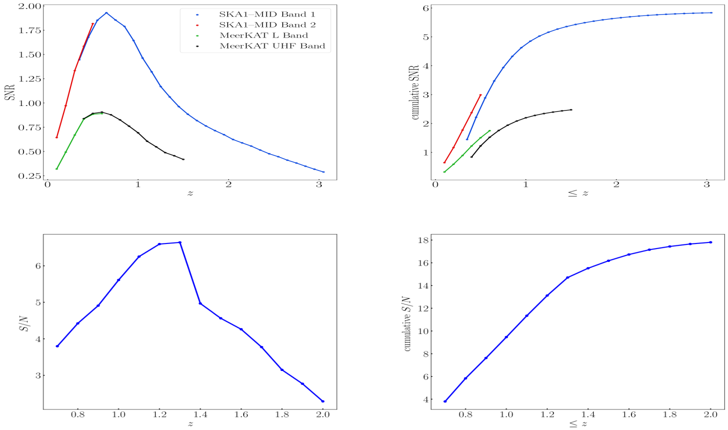
- *E. M. de Weerd, C. Clarkson, S. Jolicoeur, R. Maartens and* **O. Umeh**, *JCAP* **05** *(2020), 018*

Size of the bispectrum dipole(parity odd)



- *C. Clarkson, E. M. de Weerd, S. Jolicoeur, R. Maartens and* **O. Umeh**, *Mon. Not. Roy. Astron. Soc.* **486** *(2019) no.1, L101-L104*
- *D. Jeong and F. Schmidt, Phys. Rev. D* **102** *(2020) no.2, 023530*

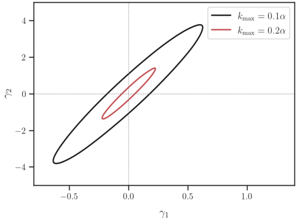
Detectability of bisp. dipole with Hα, MeerKAT and SKA

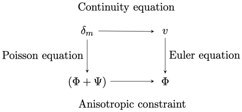


- *S. Jolicoeur, R. Maartens, E. M. De Weerd,* **O. Umeh**, *C. Clarkson and S. Camera, JCAP* **06** *(2021), 039*
- *R. Maartens, S. Jolicoeur,* **O. Umeh**, *E. M. De Weerd, C. Clarkson and S. Camera, JCAP* **03** *(2020), 065*

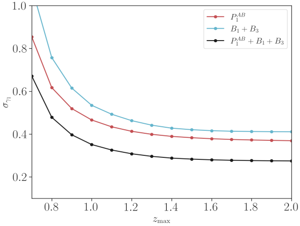
Odd multipoles and testing of the equivalence principle on cosmological scales

$$
\begin{aligned}
\Delta_{\mathrm{D}}^{(2)} = & \left[b_e - 2\mathcal{Q} - \frac{2(1-\mathcal{Q})}{\chi\mathcal{H}} - \frac{\mathcal{H}'}{\mathcal{H}^2}\right]\partial_\parallel v_g{}^{(2)} + \left(\partial_\parallel v_g{}^{(2)} - \partial_i v_b^{(2)}\right) + \frac{1}{\mathcal{H}}\left(\partial_\parallel v_g{}^{(2)\prime} - \partial_i v_b^{(2)\prime}\right) \\
& - \frac{2}{\mathcal{H}}\partial_\parallel \partial_j v_b^{(1)} \partial^j v_b^{(1)} + \frac{2}{\mathcal{H}}\left[\delta_g^{(1)} - \frac{2}{\mathcal{H}}\partial_\parallel^2 v_g{}^{(1)}\right]\left[\partial_\parallel v_g{}^{(1)\prime} - \partial_\parallel v_b^{(1)\prime} - \mathcal{H}\partial_\parallel v_b^{(1)}\right] \\
& - \frac{2}{\mathcal{H}}\left[v_b^{(1)\prime} + \mathcal{H} v_b^{(1)}\right]\left[\partial_\parallel \delta_g^{(1)} - \frac{1}{\mathcal{H}}\partial_\parallel^3 v_g{}^{(1)}\right] \\
& + 4\partial_\parallel v_g{}^{(1)}\left(1 - \frac{1}{\chi\mathcal{H}}\right)\frac{\partial \delta_g^{(1)}}{\partial \ln L} + 2\partial_\parallel v_g{}^{(1)}\delta_g^{(1)}\left[1 + b_e - 2\mathcal{Q} - \frac{2(1-\mathcal{Q})}{\chi\mathcal{H}} - \frac{\mathcal{H}'}{\mathcal{H}^2}\right] \\
& + \frac{2}{\mathcal{H}}\partial_\parallel v_g{}^{(1)}\left[\delta_g^{(1)\prime} - \frac{2}{\mathcal{H}}\partial_\parallel^2 v_g{}^{(1)\prime} + \frac{1}{\mathcal{H}}\partial_\parallel^2 v_b^{(1)\prime} + \partial_\parallel^2 v_b^{(1)}\right] - \frac{2}{\mathcal{H}}\nabla_{\perp i} v_g{}^{(1)} \nabla_\perp^i \partial_\parallel v_g{}^{(1)} \\
& + \frac{2}{\mathcal{H}}\partial_\parallel v_g{}^{(1)}\partial_\parallel^2 v_g{}^{(1)}\left[-2 - 2b_e + 4\mathcal{Q} + \frac{4(1-\mathcal{Q})}{\chi\mathcal{H}} + 3\frac{\mathcal{H}'}{\mathcal{H}^2}\right] .
\end{aligned}
\tag{2.16}
$$





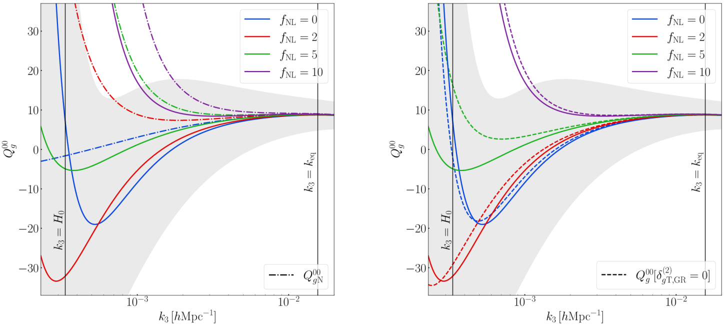
$$\partial^i v_b^{(1)\prime} + \mathcal{H}\partial^i v_b^{(1)} + \partial^i \Phi^{(1)} = 0\,,$$
$$\partial^i v_c^{(1)\prime} + \mathcal{H}\left[1 + \Theta_1\right]\partial^i v_c^{(1)} + \left[1 + \Theta_2\right]\partial^i \Phi^{(1)} = 0\,,$$



- **O. Umeh**, *K. Koyama and R. Crittenden, JCAP* ***08*** *(2021), 049*,
- *C. Bonvin and P. Fleury, JCAP* ***05*** *(2018), 061*

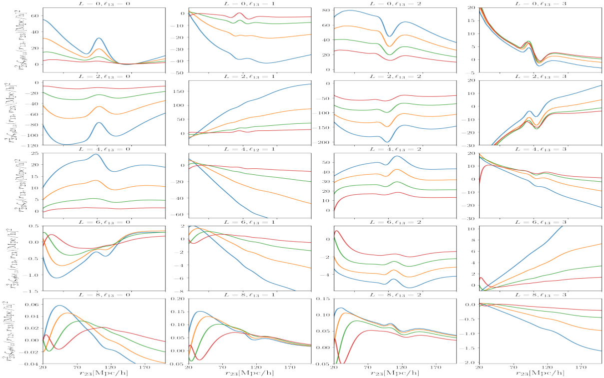
Contamination of the Local Primordial non-Gaussianity signal

$$\Delta^{(r)}_{g\,\mathrm{GR}} \equiv \Delta^{(r)}_{g\,\mathrm{GR,loc}} + \Delta^{(r)}_{g\,\mathrm{GR,dyn}} + \cancel{\Delta^{(r)}_{g\,\mathrm{GR,V/T}}}$$



Computational difficulty: 2DFFTLOG(Fang+, 2020)

$$\zeta_{gL\ell_{12}}(r_{13}, r_{23}) \propto \int_0^\infty \frac{\mathrm{d}k_1}{k_1} \int_0^\infty \frac{\mathrm{d}k_2}{k_2} \left[\frac{k_1^3 k_2^3 B^{\phi_n}_{gL\ell_{12}}(k_1, k_2)}{(2\pi^2)^2} \right] j_{\ell_{12}+L}(k_1 r_{13}) j_{\ell_{12}}(k_2 r_{23})$$



- **O. Umeh**, *JCAP* **05** *(2021), 035*, (Walter's talk)

Technical session: How to define galaxy bias in GR

The rest of the talk will be very technical.

- **O. Umeh**, *K. Koyama, R. Maartens, F. Schmidt and C. Clarkson, JCAP* **05** *(2019), 020*
- **O. Umeh** *and K. Koyama, JCAP* **12** *(2019), 048*
- **O. Umeh** + *others, (2022)*

Galaxy bias in GR effect in the matter density field

- The current galaxy bias model is expressed in terms of $\delta^{(r)}_{m,\mathrm{N}}$

$$\delta_g = b_1\delta_{m,\mathrm{N}} + \frac{1}{2}\left[b_2\left((\delta_{m,\mathrm{N}})^2 - \sigma^2_\Lambda\right)\right] + \frac{1}{3!}\left[b_3(\delta_{m,\mathrm{N}})^3\right]$$

- Note that matter density in GR

$$\delta^{(r)}_m = \delta^{(r)}_{m,\mathrm{N}} + \delta^{(r)}_{m,\mathrm{d,GR}}$$

- In GR the form of $\delta^{(r)}_{m,\mathrm{d,GR}}$ is gauge dependent but in in SC gauge

$$\delta^{(2)}_{m,\mathrm{d,GR}} = \frac{1}{2}\partial\zeta_{\mathrm{ini}}\partial^i\nabla^{-2}\delta^{(1)}_m - 2\zeta_{\mathrm{ini}}\delta^{(1)}_m$$

- How do you define the galaxy bias model in the GR, earlier attempts argued $\delta_{m,\mathrm{N}} \to \delta_m$

$$\delta_g = b_1\delta_m + \frac{1}{2}\left[b_2\left((\delta_m)^2 - \sigma^2_\Lambda\right)\right] + \frac{1}{3!}\left[b_3(\delta_m)^3\right]$$

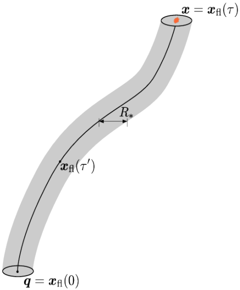
Review of galaxy bias formalism

- Galaxies reside in massive, gravitationally bound structures.
- The potential well of gravitationally bound structures is dominated by dark matter.
- Perturbation theory holds
- Proper number density of galaxy

$$n_g(\tau, \boldsymbol{x}) = F_g[\Phi(\boldsymbol{q})](\tau, \boldsymbol{x})$$

- Expand around $\Phi(\tau_{\rm ini}, \boldsymbol{q}) = \Phi_0$

$$\Phi(\mathbf{x}) = \Phi_0 + (\partial_i \Phi_0) x^i + \frac{1}{2}(\partial_i \partial_j \Phi_0) x^i x^j + \mathcal{O}(\mathbf{x})^3$$



- *V. Desjacques, D. Jeong and F. Schmidt, Phys. Rept.* **733** *(2018), 1-193*

Newtonian galaxy bias model: Equivalence principle

- Find a set coordinate transformation to remove gradient terms: $x^i = q^i + \xi^i$.
- The remaining terms are physical quantities

$$\partial_i \partial_j \Phi_0 = \frac{1}{3} \nabla^2 \Phi_0 \delta_{ij} + D_{ij} \Phi_0 r$$

- Use the Poisson equation to relate to the matter density

$$\nabla^2 \Phi(\boldsymbol{q}) = \frac{3}{2} \Omega_\mathrm{m} \mathcal{H}^2 \delta_\mathrm{mN}(\boldsymbol{q})$$

- If you approach it this way in GR,

$$\begin{aligned} \nabla^2 \Phi(\boldsymbol{q}) \quad &+ \quad \left[2\Phi(\boldsymbol{q}) \nabla^2 \Phi(\boldsymbol{q}) - \frac{1}{2} \partial_i \Phi(\boldsymbol{q}) \partial^i \Phi(\boldsymbol{q}) \right] \left(1 + \frac{2}{3} \frac{f}{\Omega_m} \right) \\ &= \quad \frac{3}{2} \Omega_\mathrm{m} \mathcal{H}^2 \delta_\mathrm{m,N}(\boldsymbol{q}) \,. \end{aligned}$$

First principle

- In GR the fundamental building block is the metric

$$n_g(\tau, \boldsymbol{x}) = F_g[a^2(\tau)\gamma_{ij}(\boldsymbol{q})\mathrm{dq^i dq^j}](\tau, \mathrm{x})$$

- EMT $T_{\mu\nu} = \sum_I \rho_I u_\mu u_\nu$, u^a is the 4-velocity attached to coordinate grid.
- Hamiltonian constraint (all the matter species are

$$R_{\mu\nu}u^\mu u^\nu \quad = \quad \frac{1}{2}\sum_I \rho_I = \frac{1}{2}\left[\rho_c + \rho_g + \cdots + \rho_N\right]$$

- Conservation equation $\nabla^\mu T_{I\mu\nu} = J_I^\mu = \dot{\rho}_I + \rho_I \Theta = U_I$
- In the limit of zero rate of energy transfer

$$\frac{\dot{\rho}_g}{\rho_g} \stackrel{?}{=} -\Theta \stackrel{?}{=} \frac{\dot{\rho}_c}{\rho_c}$$

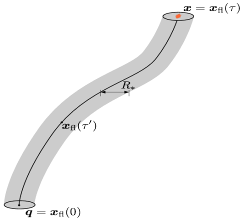
- Θ is the expansion of the time like geodesics. Its evolution is crucial to how the galaxy is formulated in GR.

Gravitational singularity

- The determinant of the metric relate to Θ according to

$$\det[\gamma_{\rm ij}](\tau) = \det[\gamma_{\rm ij}](\tau_{\rm ini}) \exp\left[\int \mathrm{d}\tau' \Theta(\tau')\right]$$

- Any small perturbation of γ_{ij} at $\Theta = 0$ leads to $\det[\gamma_{\rm ij}](\tau) \to 0$.
- This indicates a break down of the expanding coordinate system. Using halo model, you can show that $\Theta(\tau_\star, R) = 0$ at the Lagrangian radius (*E. Witten, Rev. Mod. Phys.* **92** *(2020) no.4, 045004.*)
- Thus we need another coordinate system, $\tilde{q}^i$ for $r < R$.



Recent paper

Large gauge transformations, local coordinates and cosmological observables

Ermis Mitsou, Jaiyul Yoo

In recent years new types of coordinate transformations have appeared in cosmology on top of the standard gauge transformations, such as the dilatations and special conformal transformations, or the ones leading to (conformal) Fermi coordinates. Some of these can remove effects that are invariant under the standard gauge transformations and also affect asymptotic boundary conditions, thus introducing a non-trivial ambiguity in our cosmological modeling. In this short note we point out that this ambiguity is irrelevant for the quantities we use to compare our model with observations -- the cosmological observable relations -- as they are invariant under all of these transformations. Importantly, this invariance holds only if one takes into account all the relativistic contributions to an observable, which is not the case in the literature in general. We finally also show that the practically-relevant property of conformal Fermi coordinates (a FLRW metric up to second order in distance) can be achieved through a globally-defined standard gauge transformation.

Local coordinate system

- We require that both coordinate system are related according to

$$\tilde{q}^i = q^i + \xi^i .$$

- The metric transforms on both sides transfoms

$$\tilde{\gamma}_{ij} - \gamma_{ij} = \xi^k \partial_k \gamma_{ij} + \gamma_{kj} \partial_i \xi^k + \gamma_{ik} \partial_j \xi^k$$

- We then seek ξ^i that solves the following conformal Killing equation in order to preserve the determinant

$$\mathcal{L}_\xi \gamma_{ij} = (\mathcal{A} - 1)\, \gamma_{ij} \,,$$

- So that the metric transforms as $\tilde{\gamma}_{ij} = \mathcal{A}\gamma_{ij}$, where $\mathcal{A}$ is an effective conformal factor.
- The valid coordinate system small scale becomes

$$\begin{aligned} \tilde{\tau} &= \tau \,, \\ \tilde{q}^i &= q^i \left(1 - \frac{5}{3}\Phi_0\right) - \frac{5}{3} q^i q^j \partial_j \Phi_0 + \frac{5}{6} q_j q^j \partial^i \Phi_0 \,. \end{aligned}$$

Galaxy bias in GR

- Number density in initial coordinates becomes

$$n_g(\tau, \boldsymbol{x}) = F_g[(a_{\rm eff}^2(\tau)\tilde{\gamma}_{ij}(\tilde{\boldsymbol{q}})\mathrm{d}\tilde{q}^i\mathrm{d}\tilde{q}^j](\tau, \boldsymbol{x})$$

- Changing to local coordinates

$$\tilde{\gamma}_{ij} = \tilde{a}^2(\varphi_{\rm ini})\left[\delta_{ij} - \frac{2}{3}\delta_{m,\rm N}(\boldsymbol{q})\delta_{ij} + 2D_{ij}\delta_{m,\rm N}(\boldsymbol{q})\right]$$

- Specify the galaxy bias

$$\begin{aligned} 1 + \delta_{{\rm gC}l}(\boldsymbol{q}) &= \left[1 + \delta_{\rm g}^{\sf L}(\tilde{\boldsymbol{q}})\right][1 + \delta_{\rm mC}(\tau, \boldsymbol{q})] \\ &= 1 + b_1^{\rm E}\left[\delta_l^{(1)}(\boldsymbol{q}) + \frac{1}{2}\delta_{{\rm CN}l}^{(2)}(\boldsymbol{q})\right] \\ &\quad + \frac{1}{2}\left[b_2^{\rm E}\left(\delta_l^{(1)}(\boldsymbol{q})\right)^2 + b_{K^2}^{\rm E}K_l^2(\boldsymbol{q}) + \delta_{{\rm mGR,C}l}^{(2)}(\boldsymbol{q})\right] \end{aligned}$$

- Sanity check $b_1^{\rm E} \to 1$, $b_2^{\rm E} \to 0$, $b_{K^2}^{\rm E} \to 0$ lead to dark matter density field.

Conclusion

- GR light-cone perturbations must be understood on horizon scales.
- Non-linear GR contamination of the primordial non-Gaussianity signal is a real danger.
- In real space, we need to understand higher order tracer bias correction within GR.
- Tensor and vector perturbations may not be negligible in synchronous gauge (*J. C. Hwang, D. Jeong and H. Noh, Astrophys. J.* **842** *(2017) no.1, 46.*).