Non-Gaussianities from primordial quantum diffusion

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2022 Workshops and Programs

Instituto de Física Teórica UAM-CSIC Madrid

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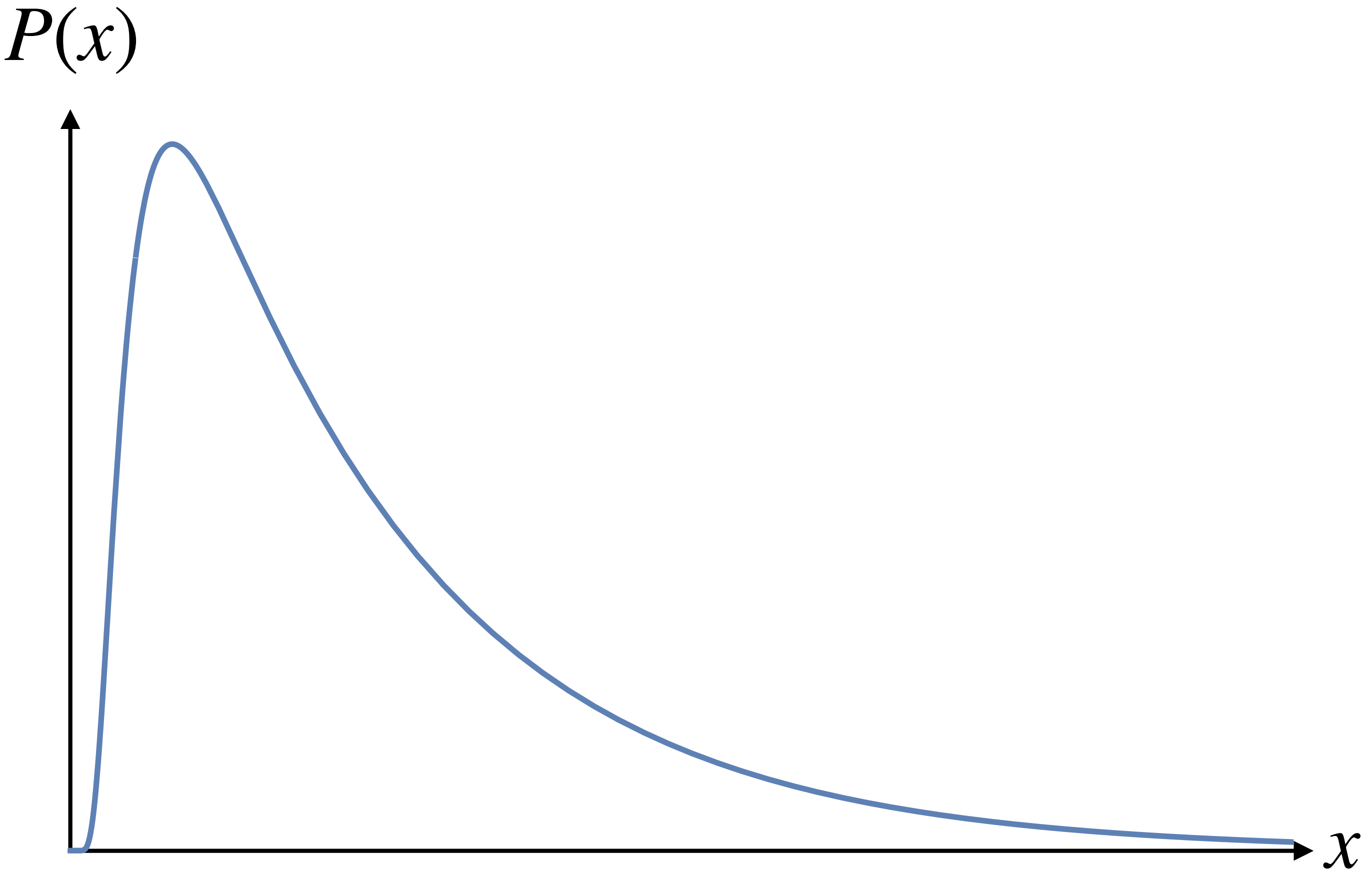
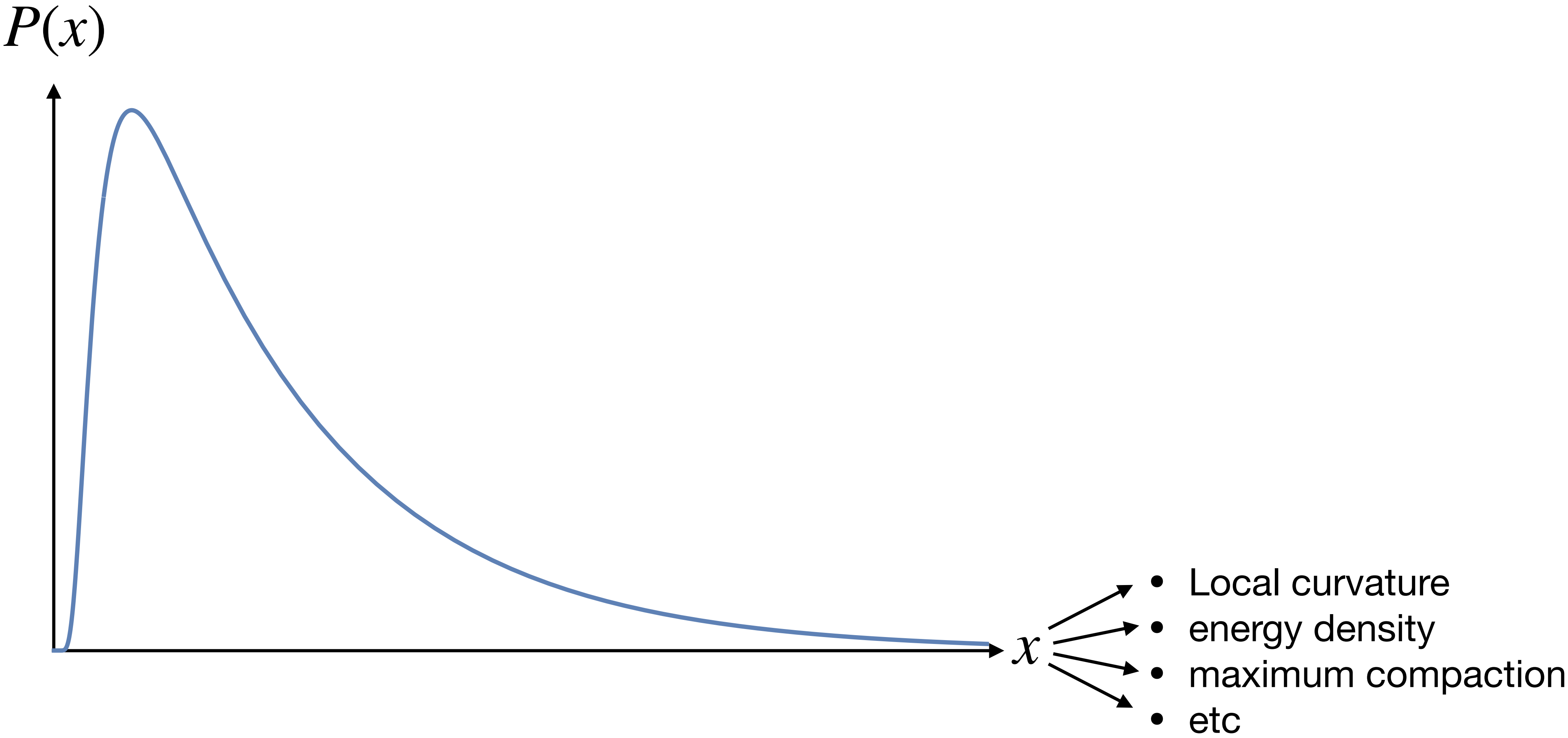
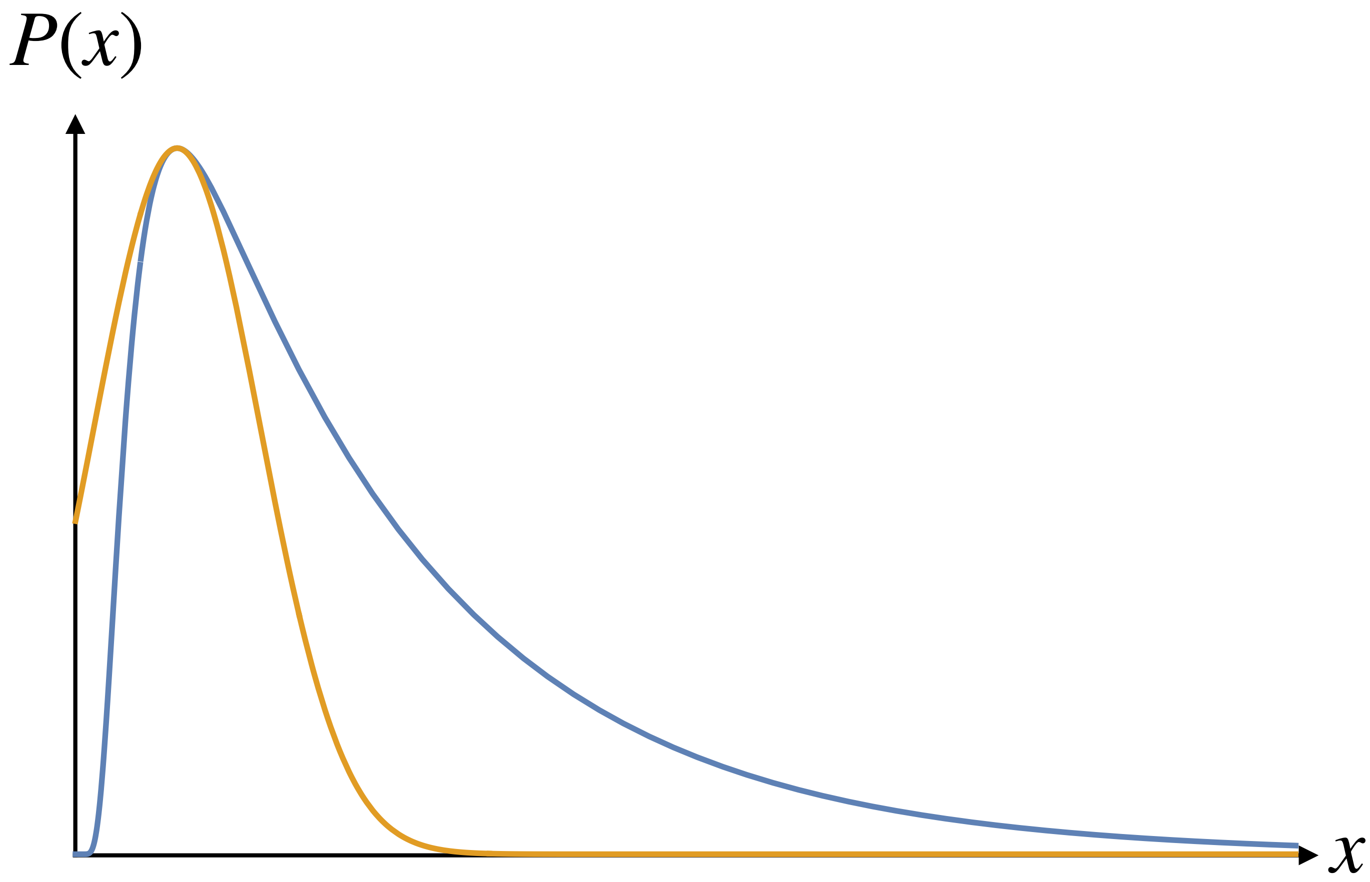
PNG workshop, Madrid


$P(x)$
x


$P(x)$
x
• Local curvature
• energy density
• maximum compaction
• etc



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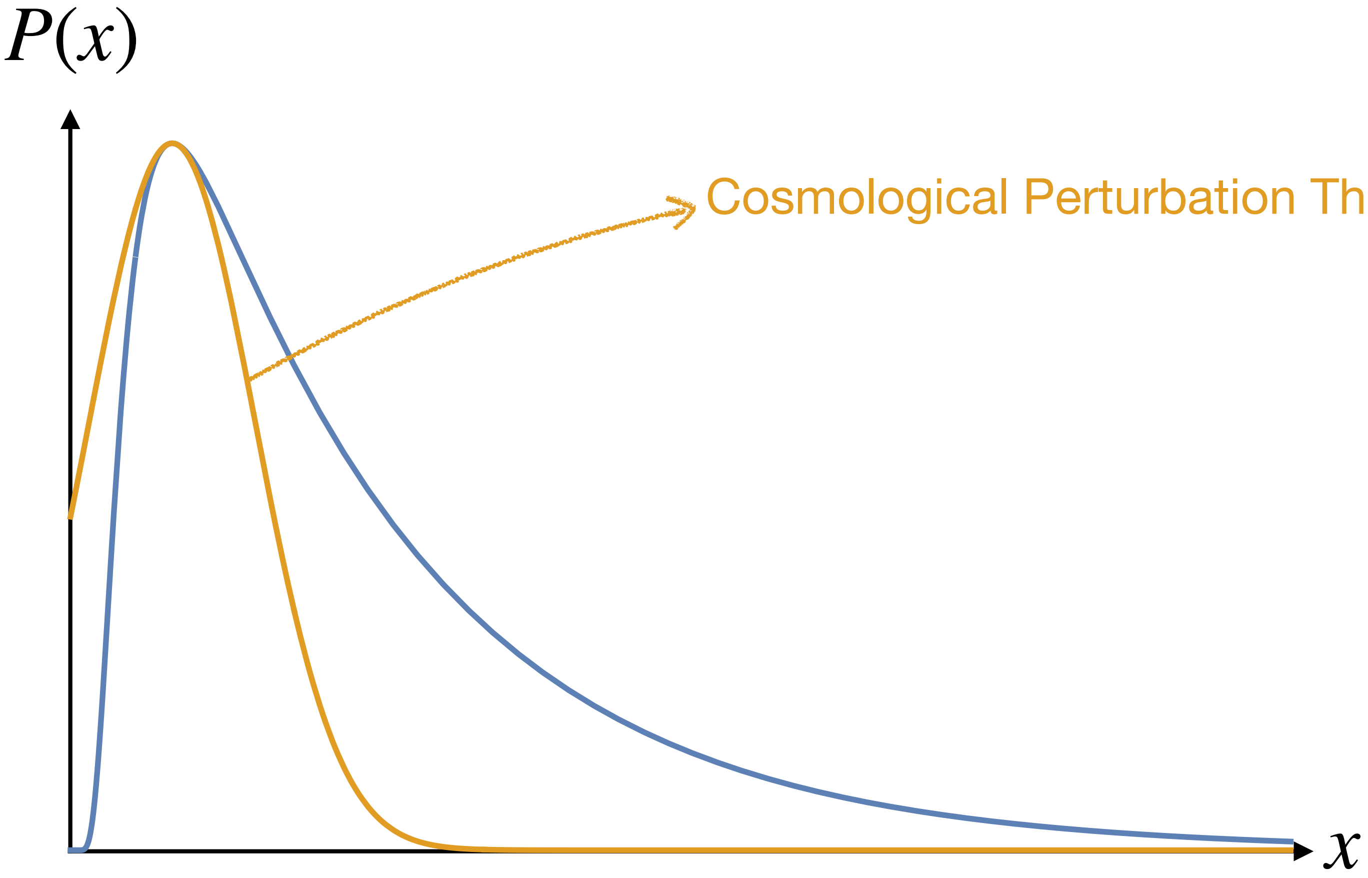
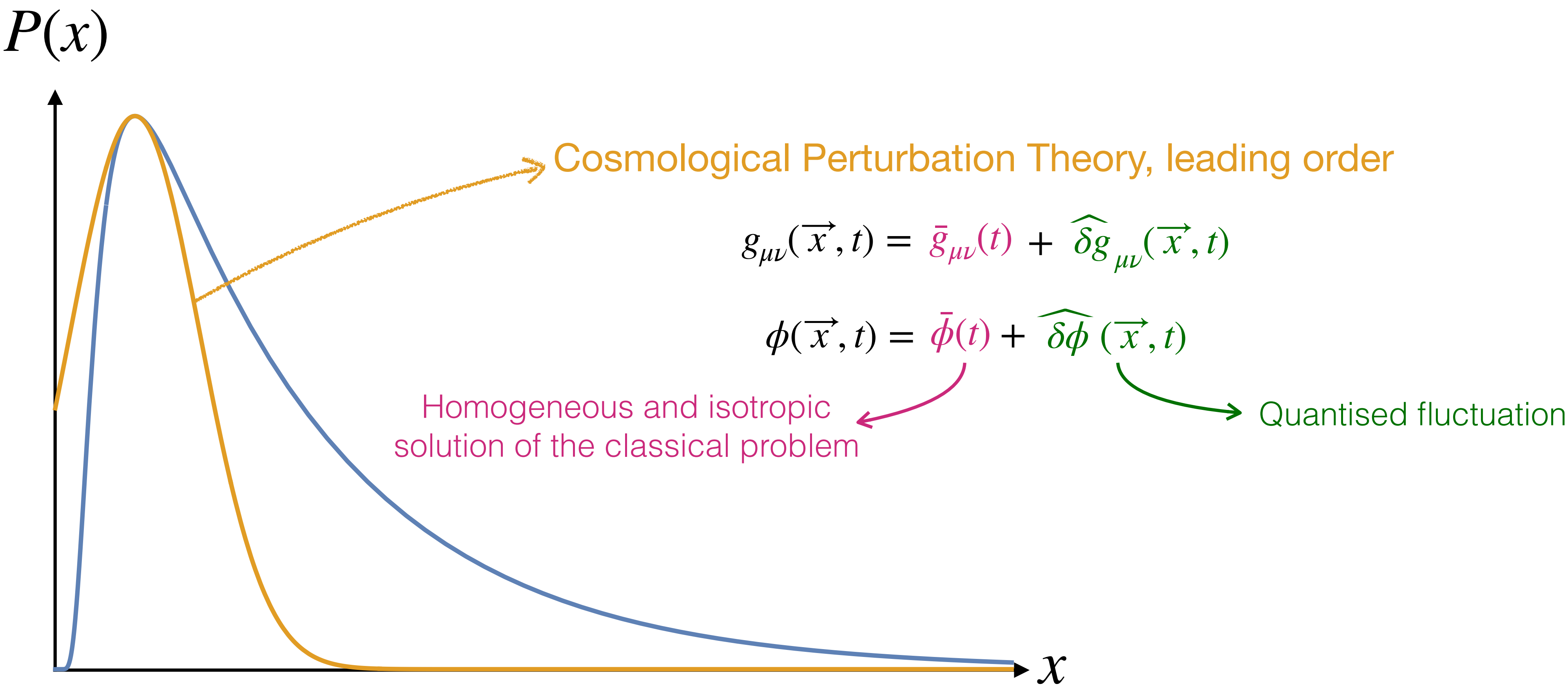
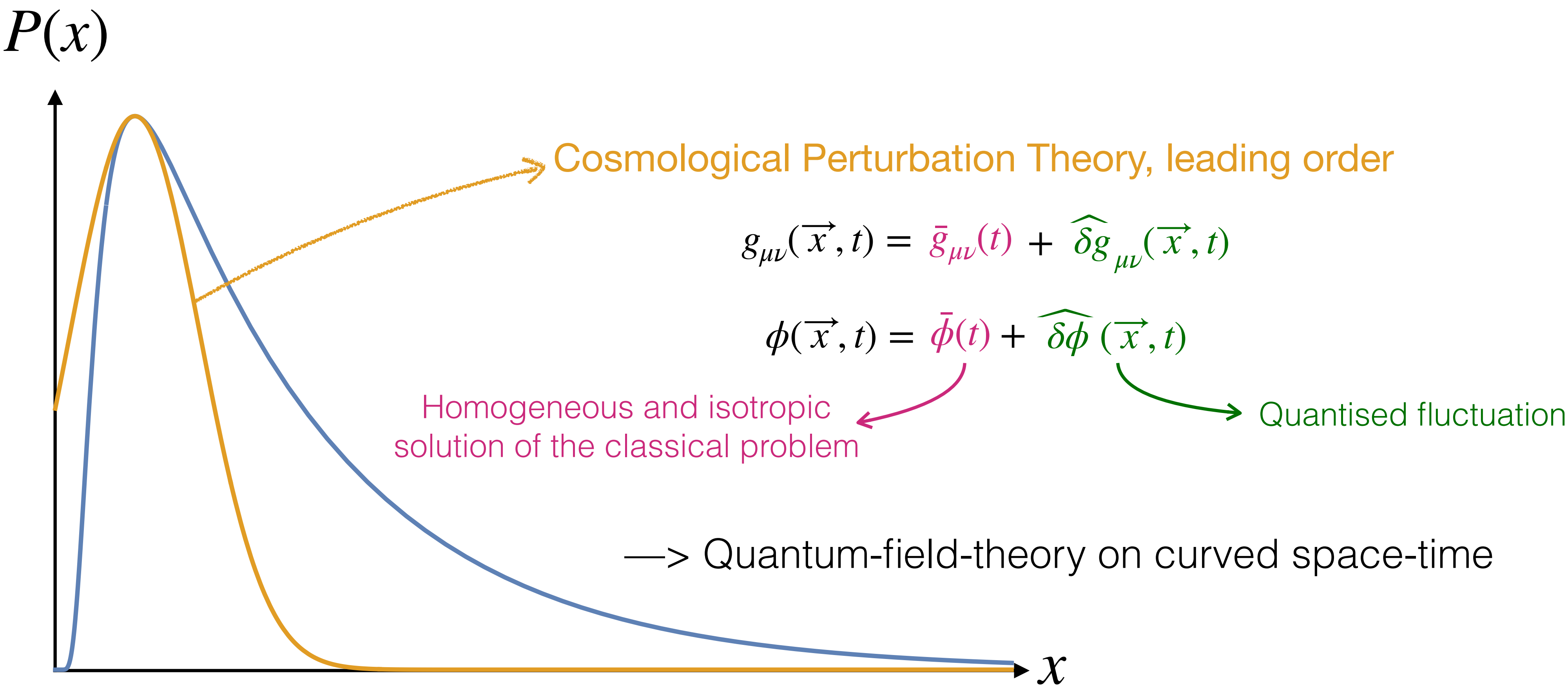


$P(x)$
Cosmological Perturbation Theory, leading order
x


$P(x)$
x
Cosmological Perturbation Theory, leading order
$g_{\mu\nu}(\vec{x}, t) = \bar{g}_{\mu\nu}(t) + \widehat{\delta g}_{\mu\nu}(\vec{x}, t)$
$\phi(\vec{x}, t) = \bar{\phi}(t) + \widehat{\delta\phi}(\vec{x}, t)$
Homogeneous and isotropic solution of the classical problem
Quantised fluctuation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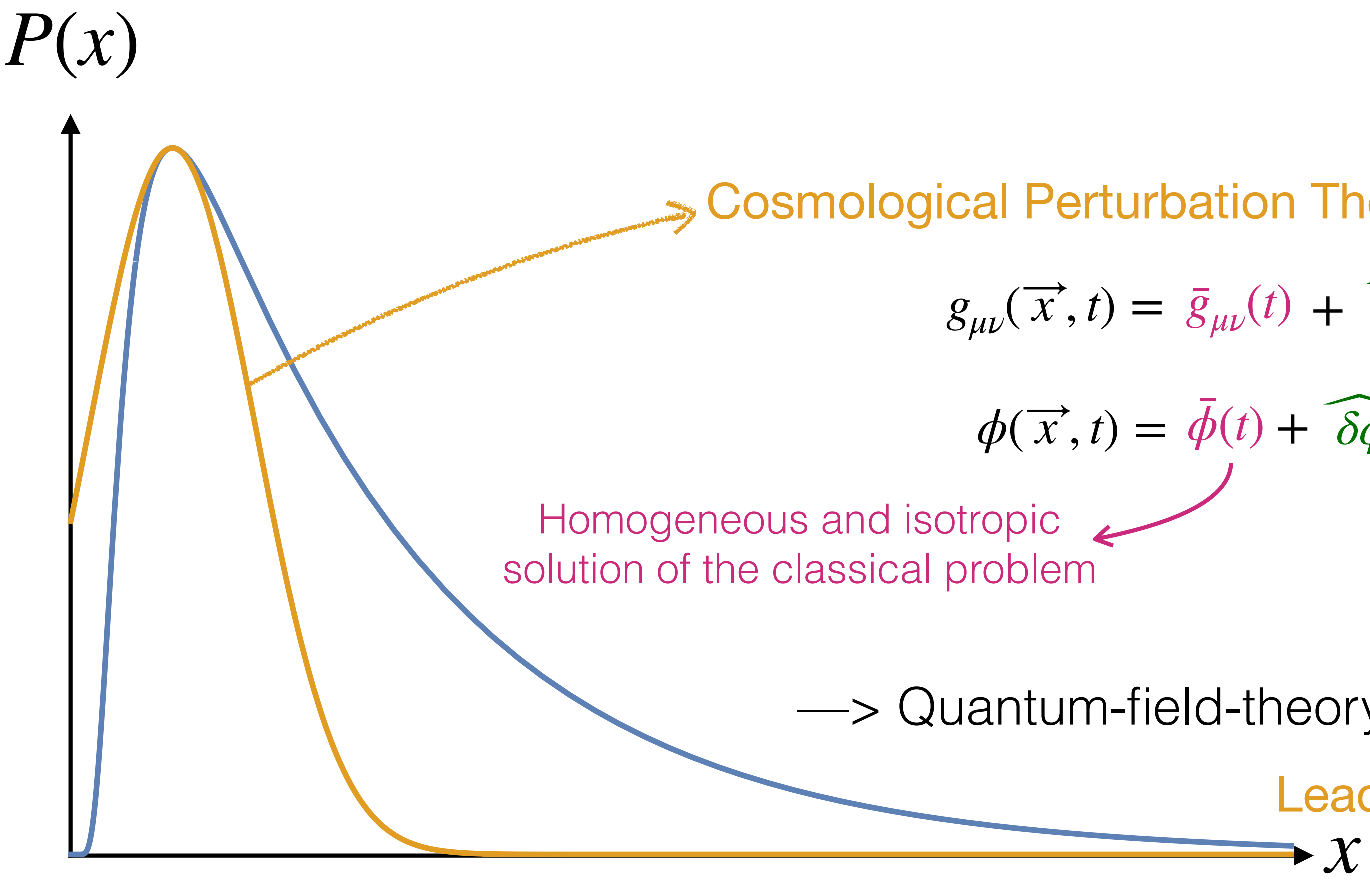
Quantised fluctuation
—> Quantum-field-theory on curved space-time



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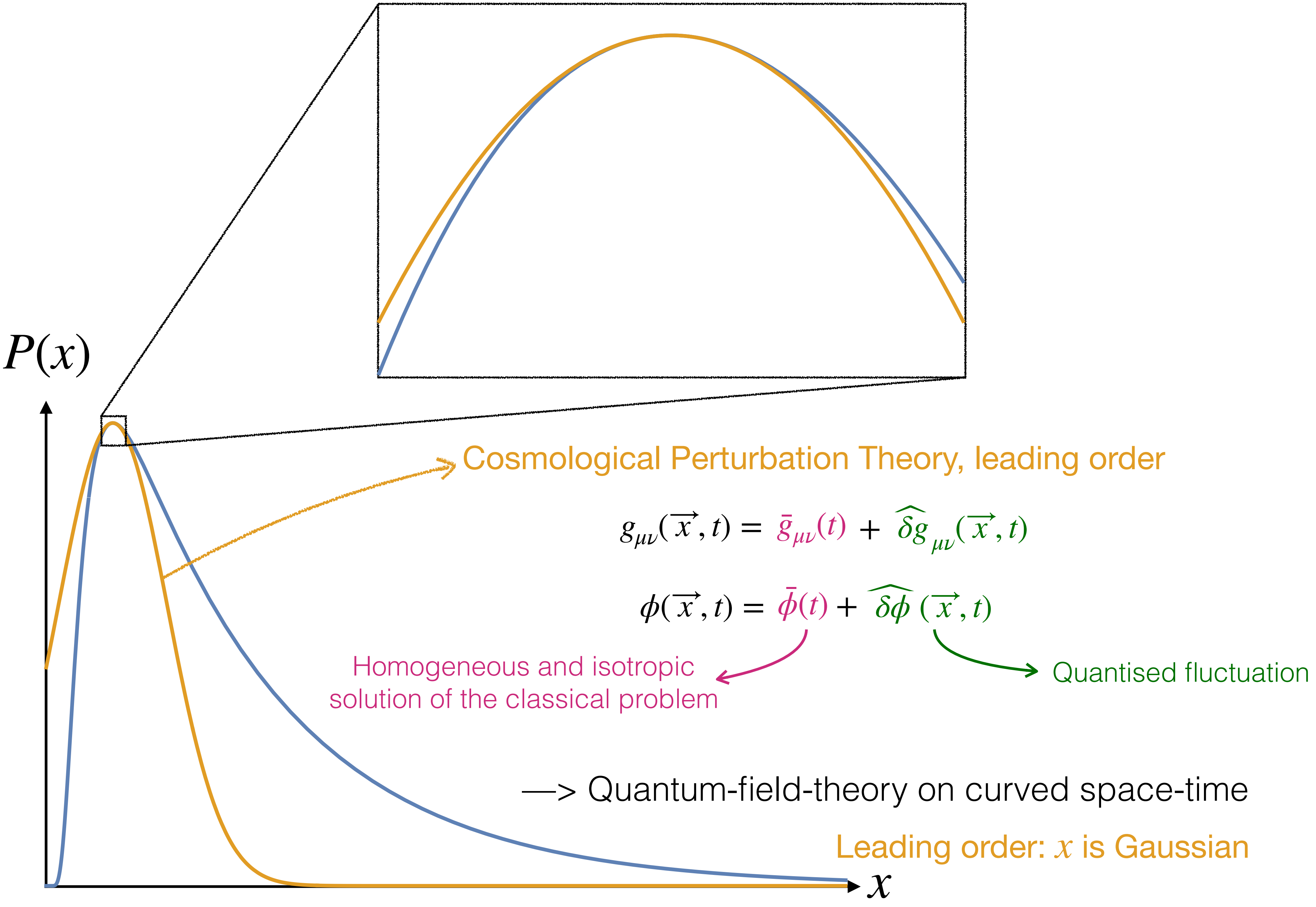
Leading order: x is Gaussian



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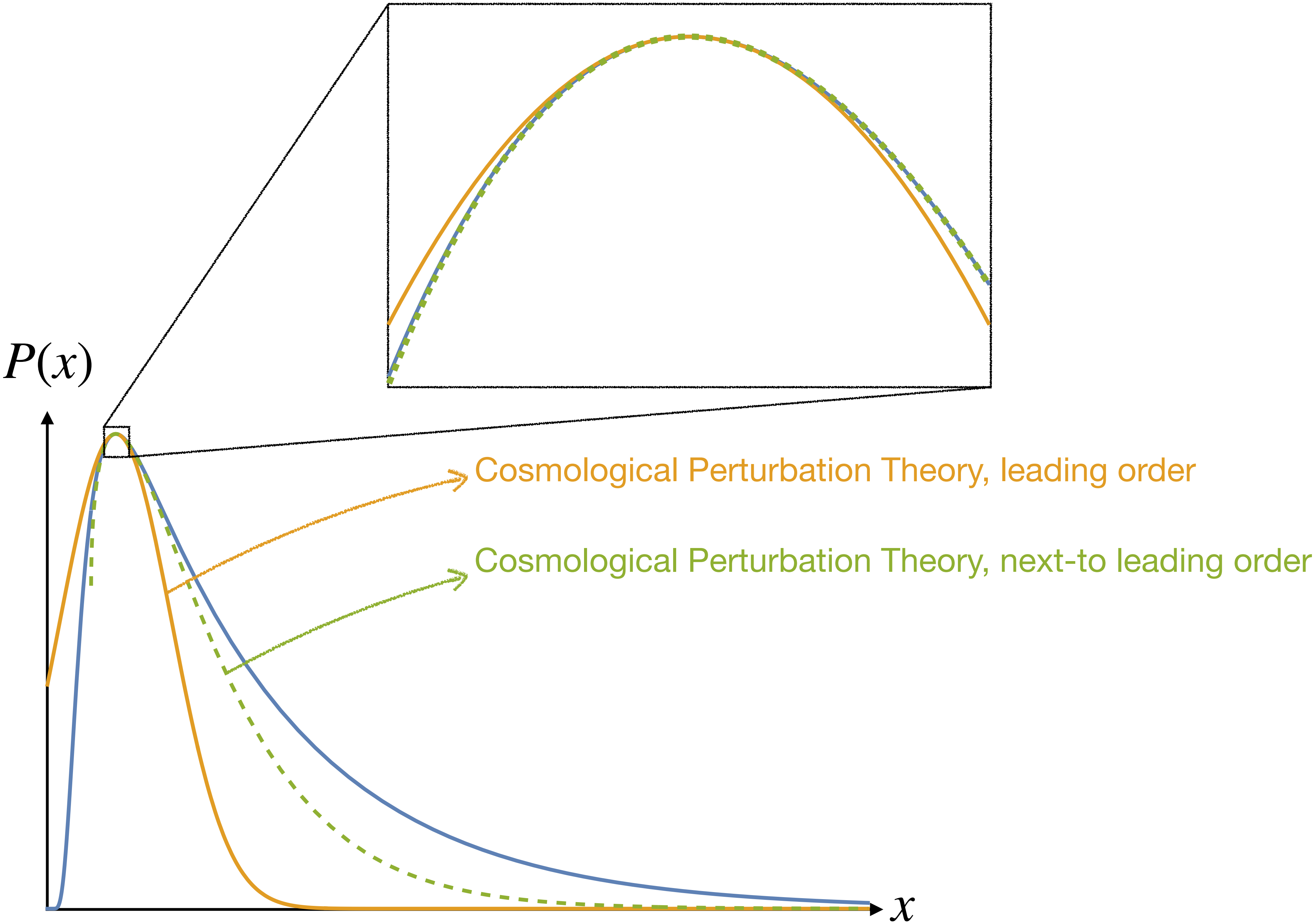
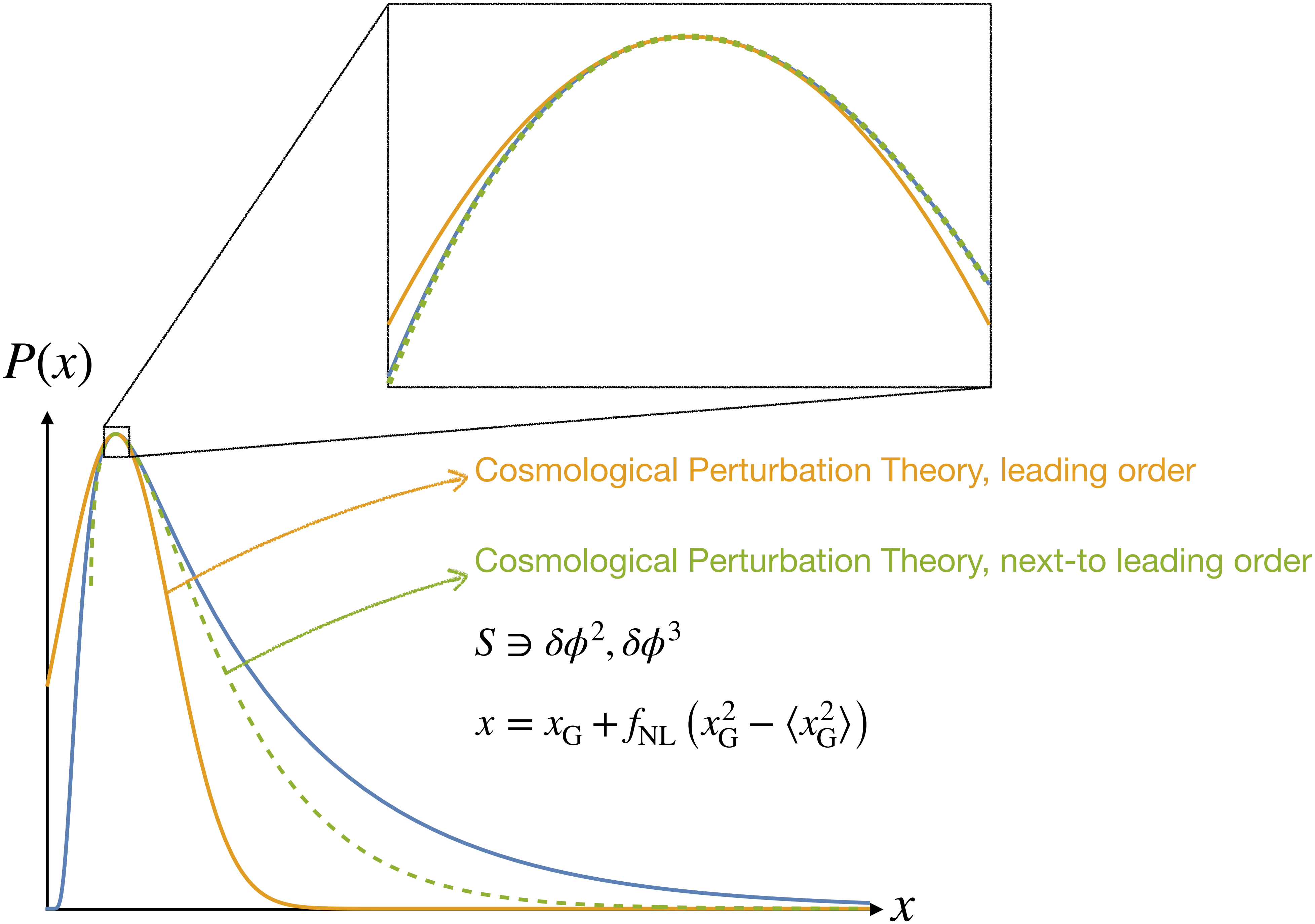
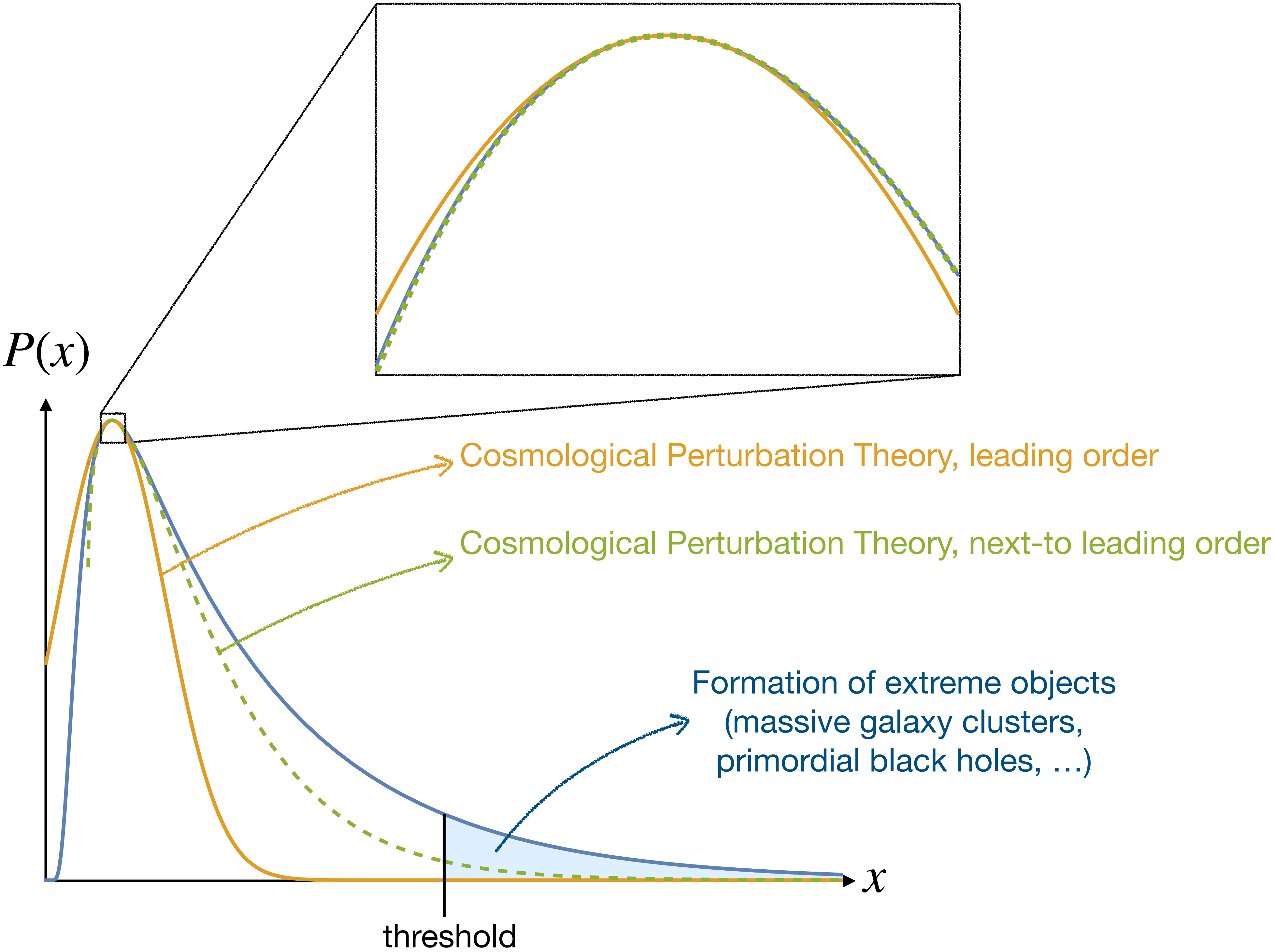
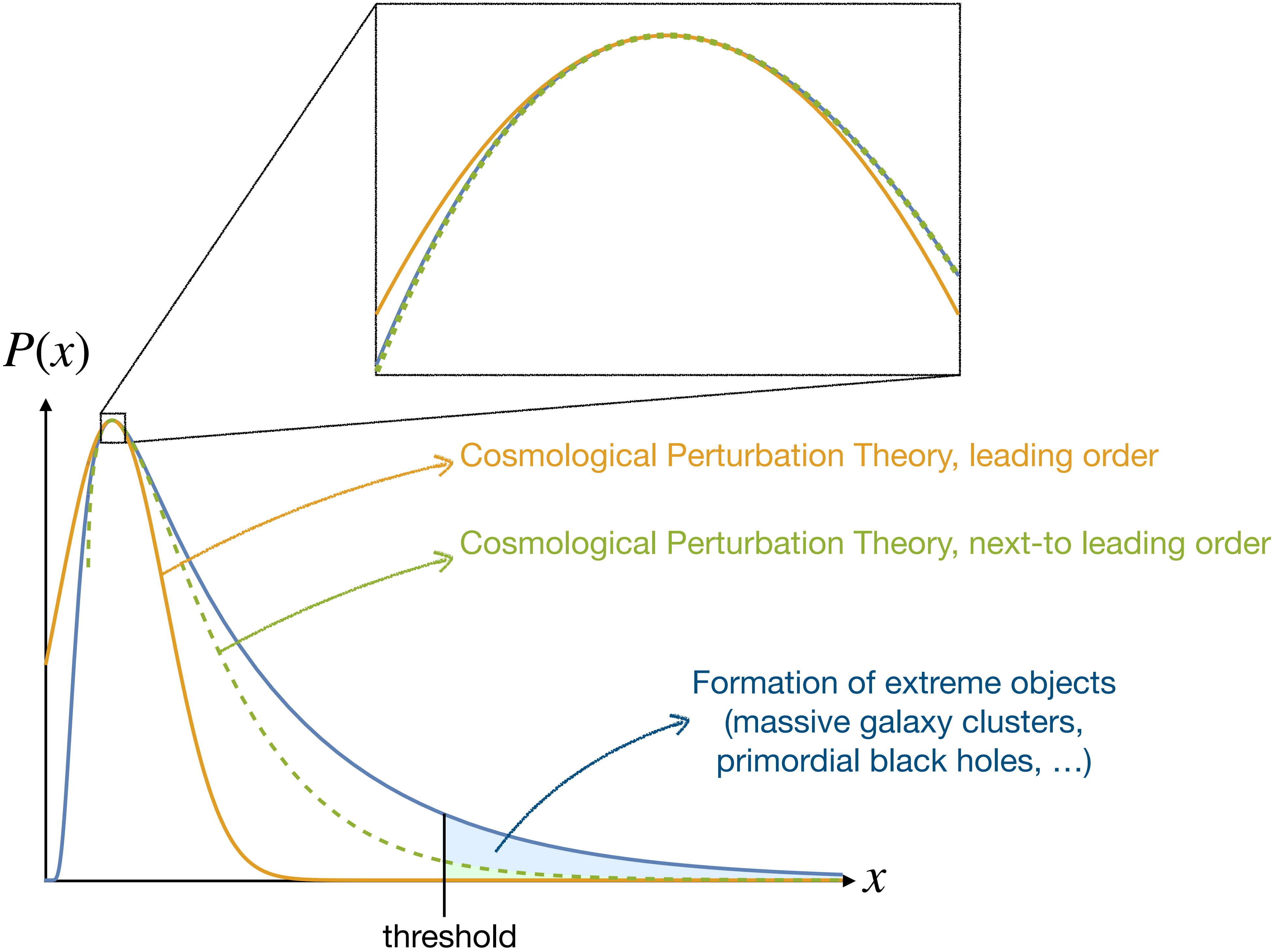


$P(x)$
Cosmological Perturbation Theory, leading order
Cosmological Perturbation Theory, next-to leading order
x



$P(x)$
x
Cosmological Perturbation Theory, leading order
Cosmological Perturbation Theory, next-to leading order
$S \ni \delta\phi^2, \delta\phi^3$
$x = x_G + f_{NL}\left(x_G^2 - \langle x_G^2 \rangle\right)$


$P(x)$
Cosmological Perturbation Theory, leading order
Cosmological Perturbation Theory, next-to leading order
Formation of extreme objects
(massive galaxy clusters,
primordial black holes, …)
x
threshold



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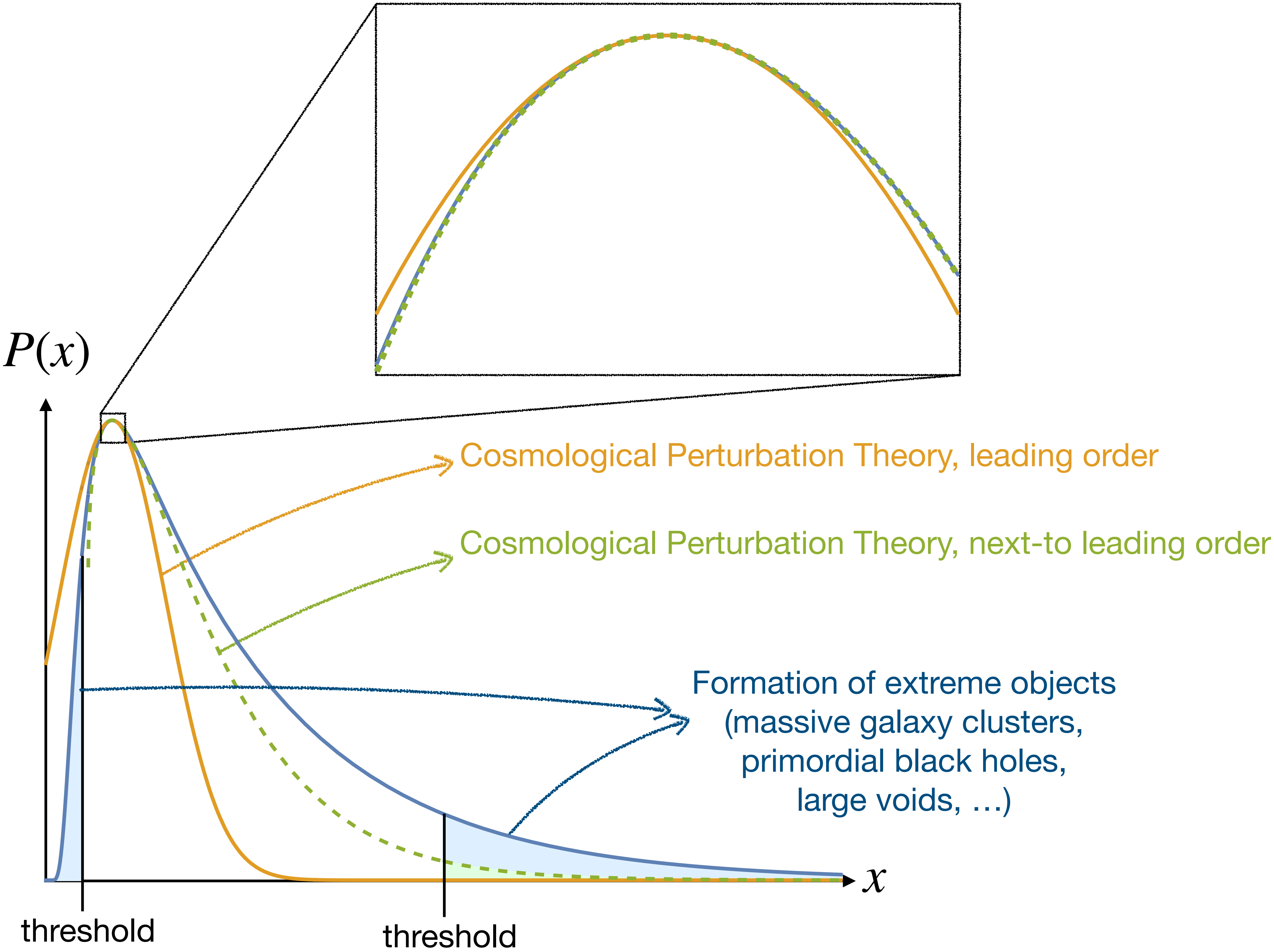
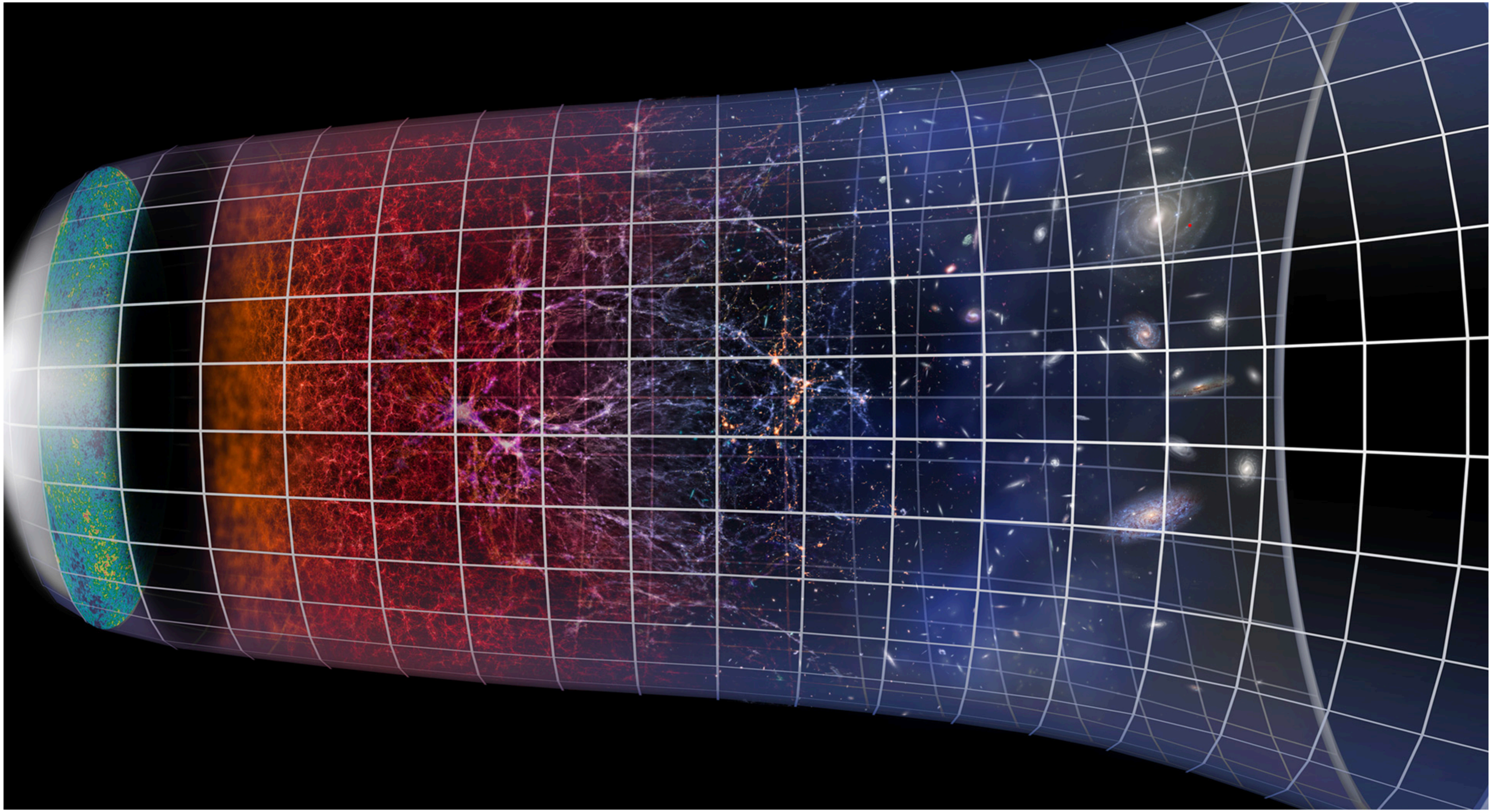


$P(x)$
Cosmological Perturbation Theory, leading order
Cosmological Perturbation Theory, next-to leading order
Formation of extreme objects
(massive galaxy clusters,
primordial black holes,
large voids, …)
x
threshold
threshold

Cosmic Inflation

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\vec{x}^2 \quad \text{with} \quad \ddot{a} > 0 \quad \text{and} \quad (10\,\mathrm{MeV})^4 < \rho < \left(10^{16}\,\mathrm{GeV}\right)^4$$



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Hubble parameter $H = \dot{a}/a$

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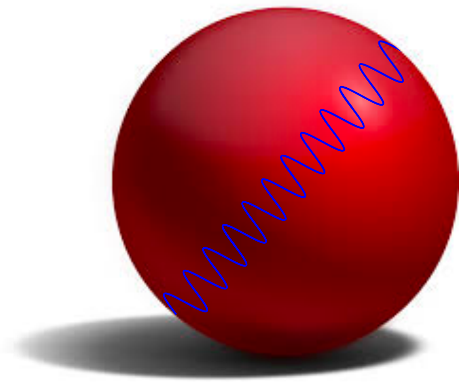
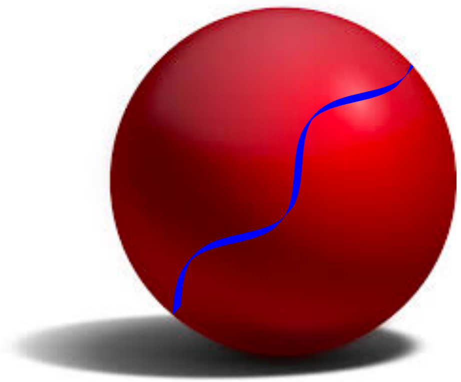


$\lambda \ll H^{-1}$

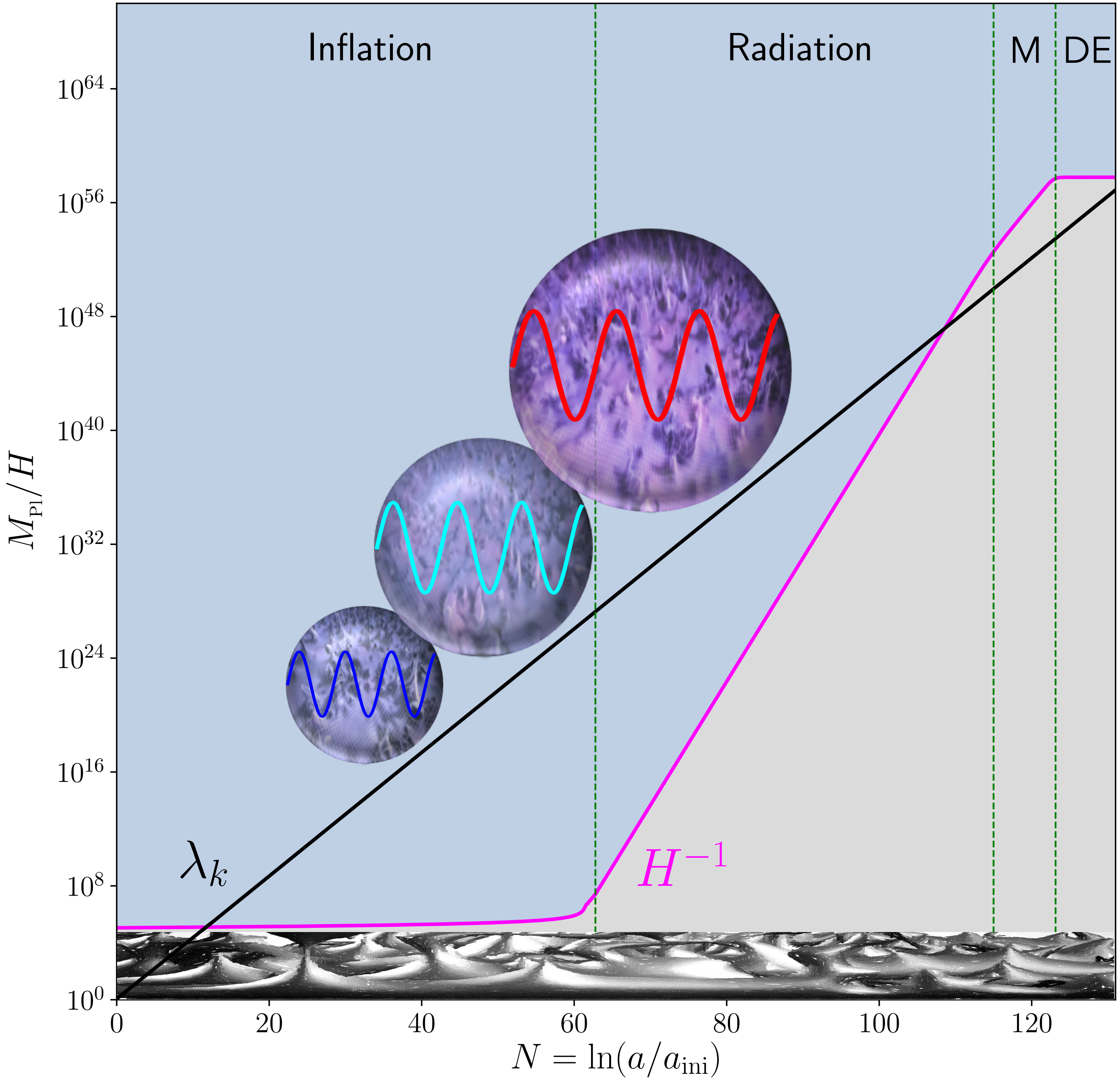
Insensitive to space-time curvature



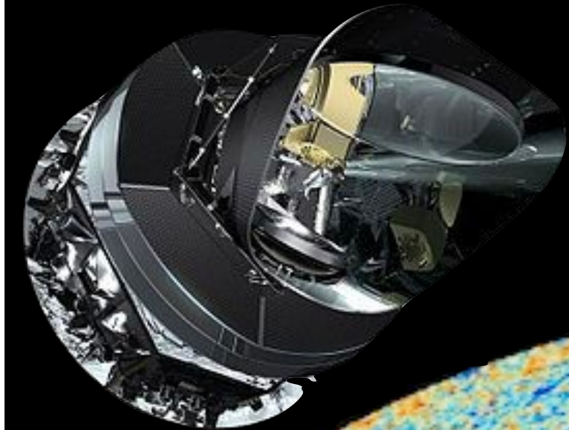
$\lambda \gtrsim H^{-1}$

Feels space-time curvature

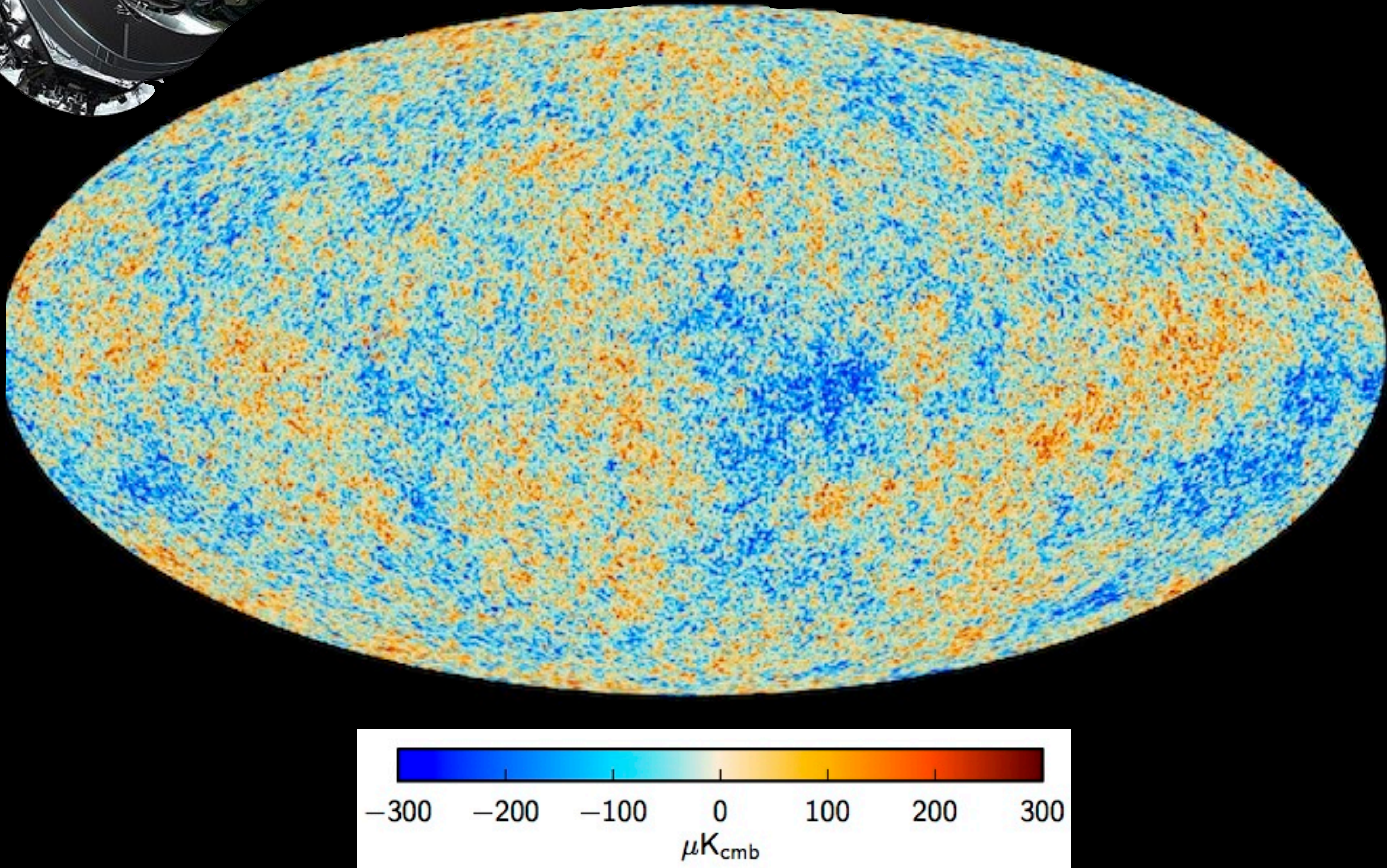
Cosmic Inflation



Planck satellite



$$\frac{\delta T}{T} \sim 10^{-5} \ll 1$$



Separate Universe

$$\mathrm{d}s^2 = a^2\left(-\mathrm{d}\eta^2 + \mathrm{d}\vec{x}^2\right)$$

de-Sitter universe: $a = -1/(H\eta), \quad -\infty < \eta < 0$

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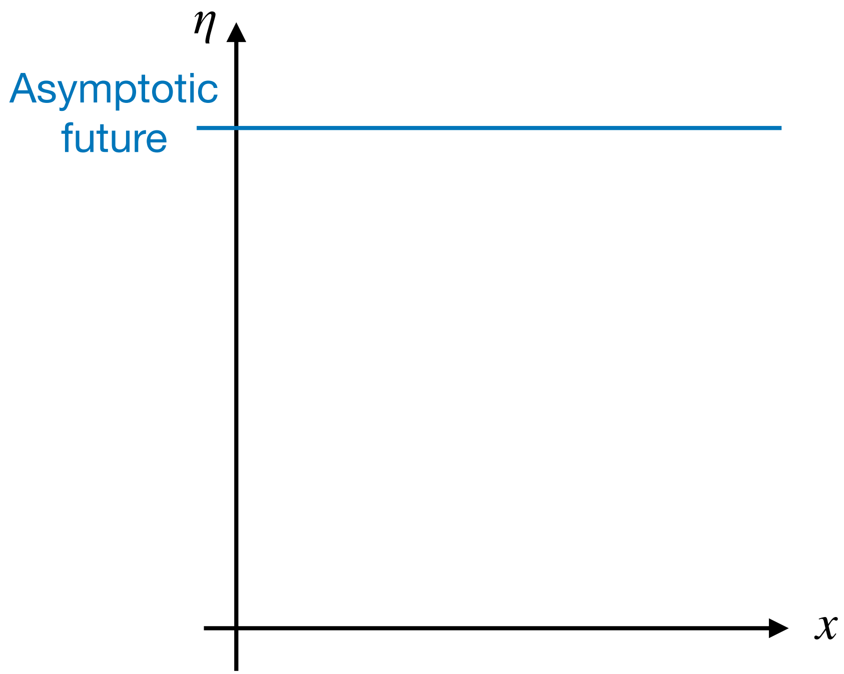

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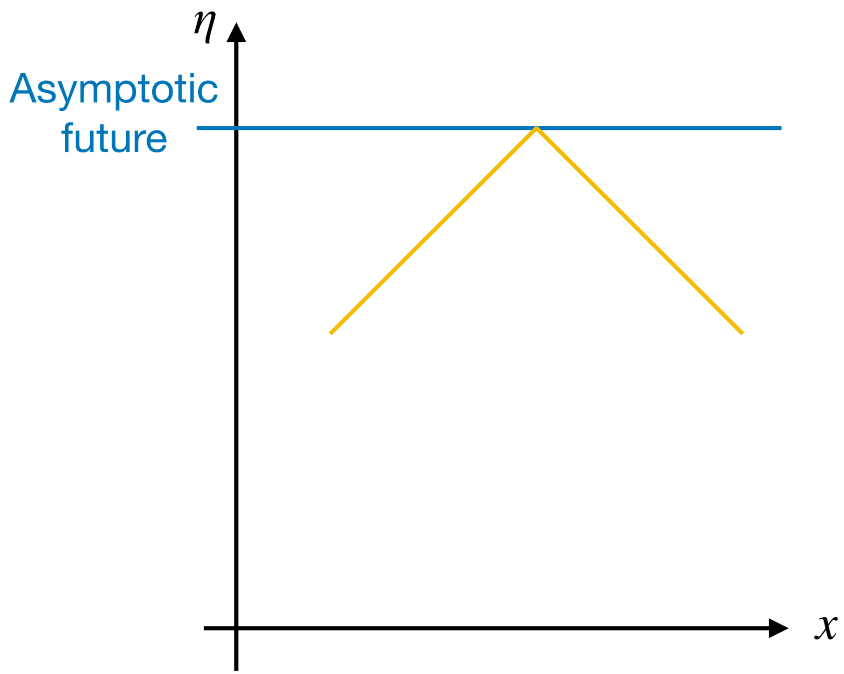

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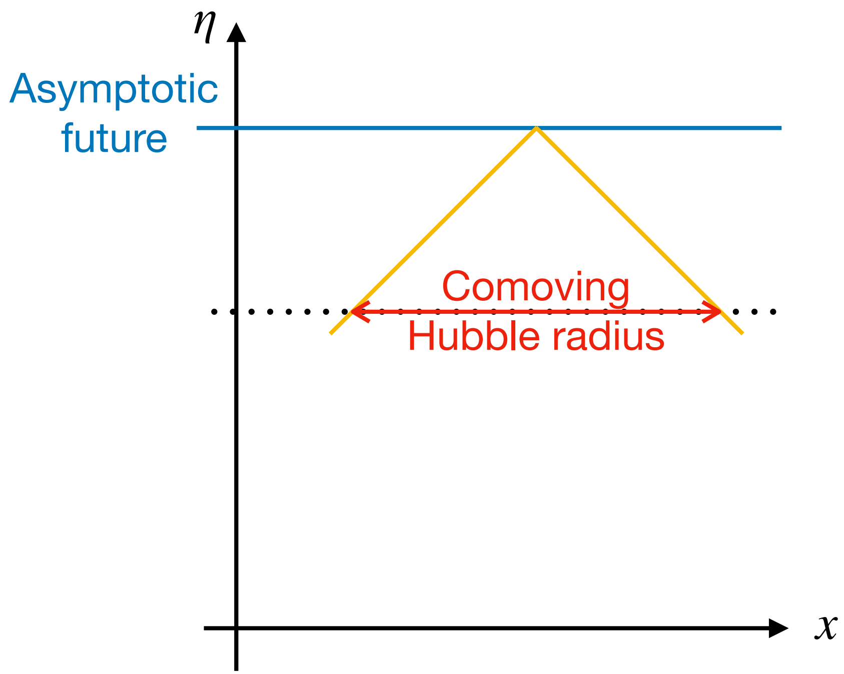

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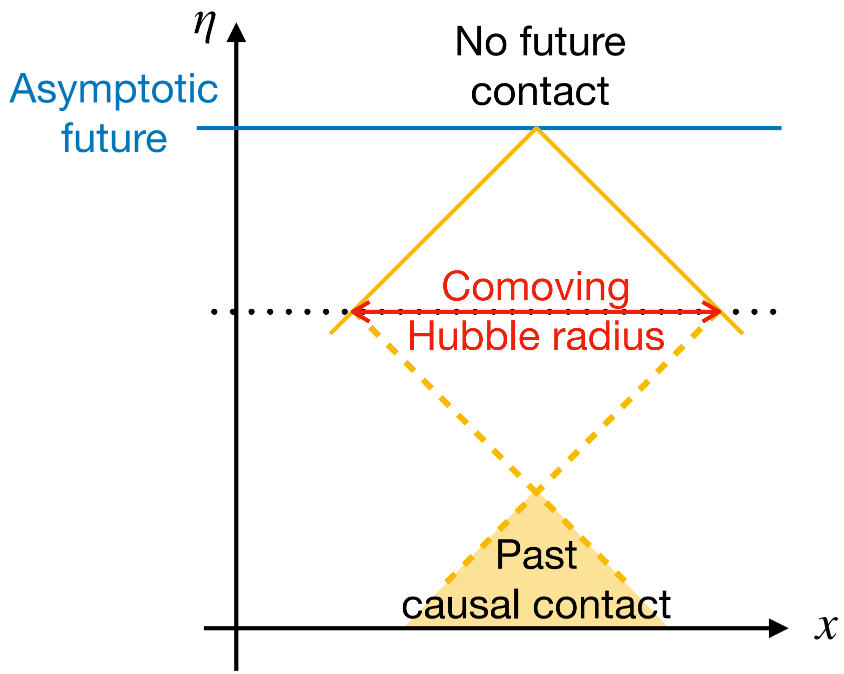

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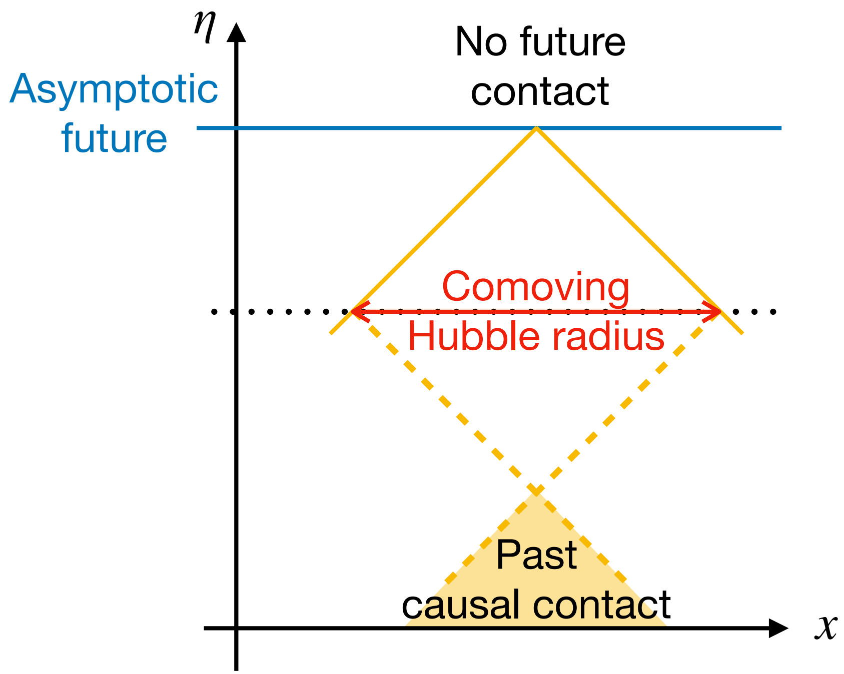

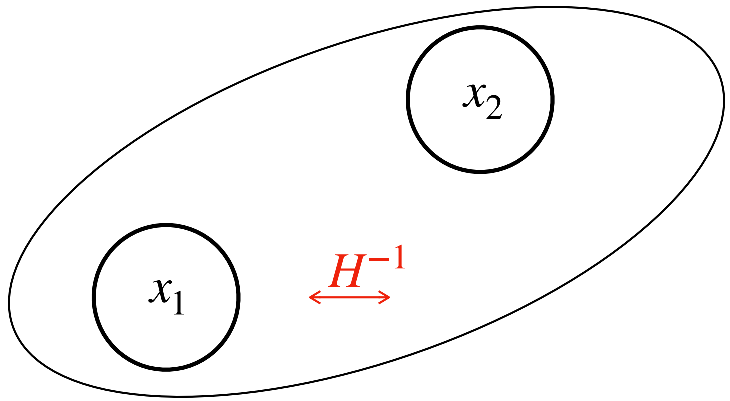

If a large fluctuation develops at $\boldsymbol{x}_1$, this cannot affect the local geometry at $\boldsymbol{x}_2$

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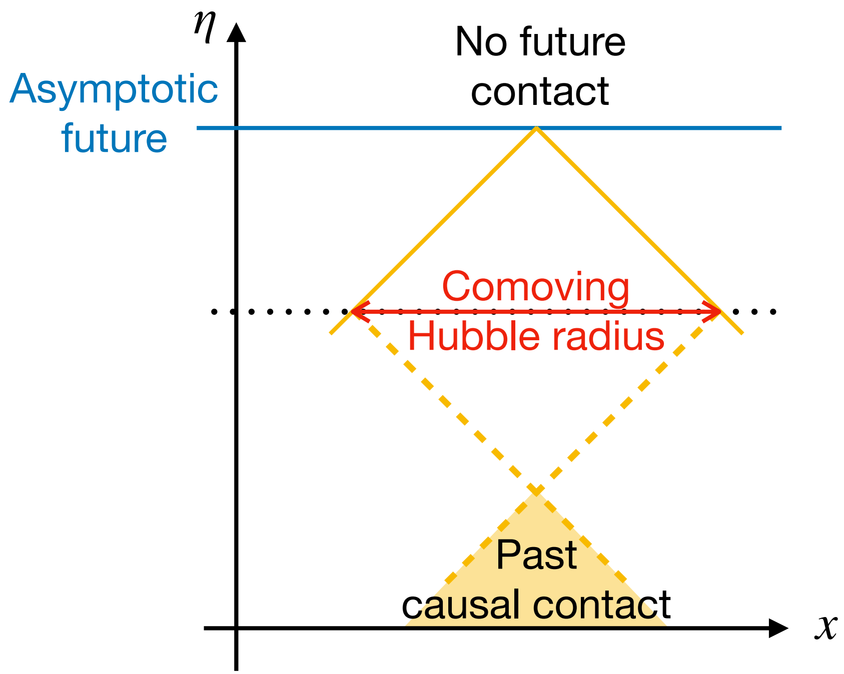
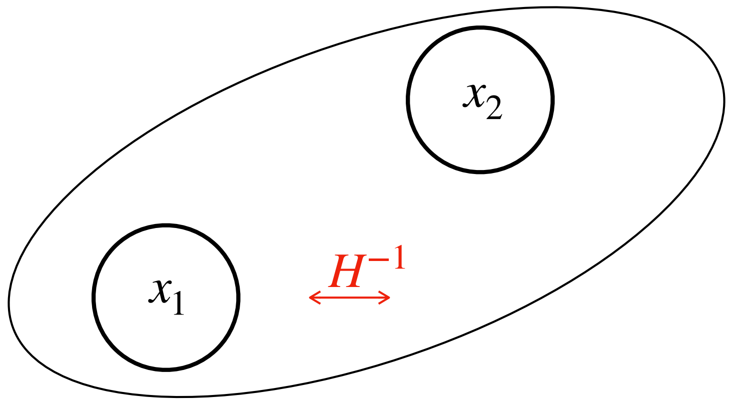


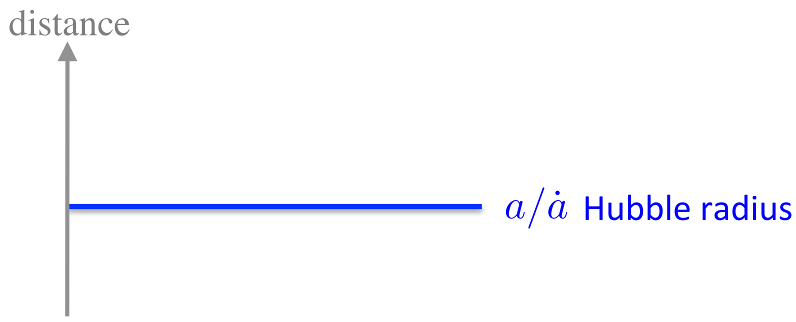


If a large fluctuation develops at $\boldsymbol{x}_1$, this cannot affect the local geometry at $\boldsymbol{x}_2$

Separate universe: On large scales, the universe can be described by an ensemble of independent, locally homogeneous and isotropic patches

Salopek & Bond; Sasaki & Stewart; Wands, Malik, Lyth & Liddle

Stochastic Inflation



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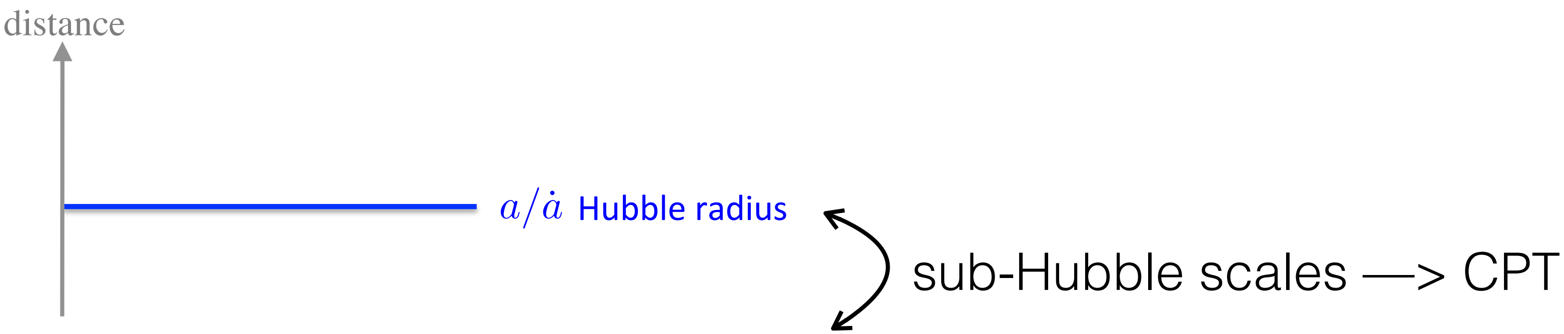

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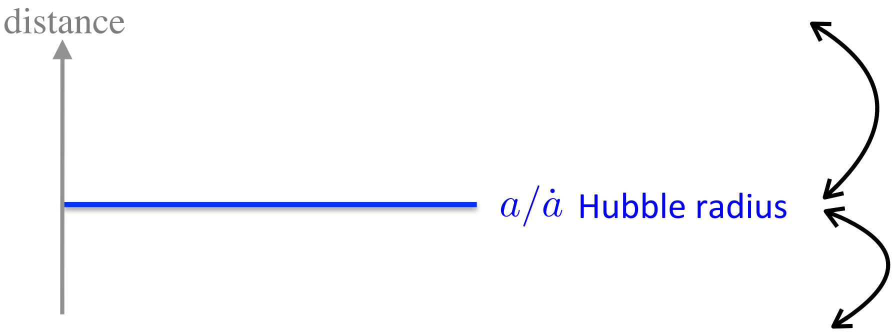

super-Hubble scales —> stochastic inflation

sub-Hubble scales —> CPT

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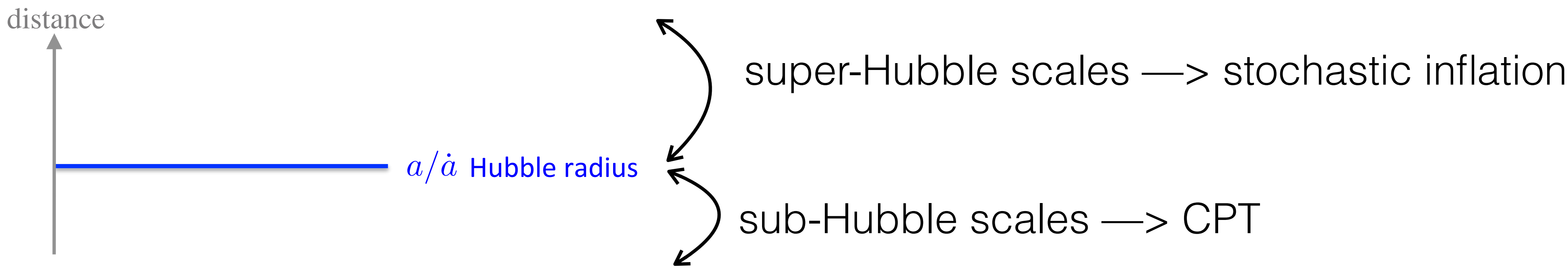

Coarse-grained field

$N = \ln(a)$

$$\hat{\mathbf{\Phi}}_{\rm cg}(N, \overrightarrow{x}) = \int_{k<\sigma H a(N)} \mathrm{d}\overrightarrow{k} \left[\mathbf{\Phi}_{\overrightarrow{k}}(N) e^{-i\overrightarrow{k}\cdot\overrightarrow{x}} \hat{a}_{\overrightarrow{k}} + \mathbf{\Phi}^{\star}_{\overrightarrow{k}}(N) e^{i\overrightarrow{k}\cdot\overrightarrow{x}} \hat{a}^{\dagger}_{\overrightarrow{k}} \right]$$

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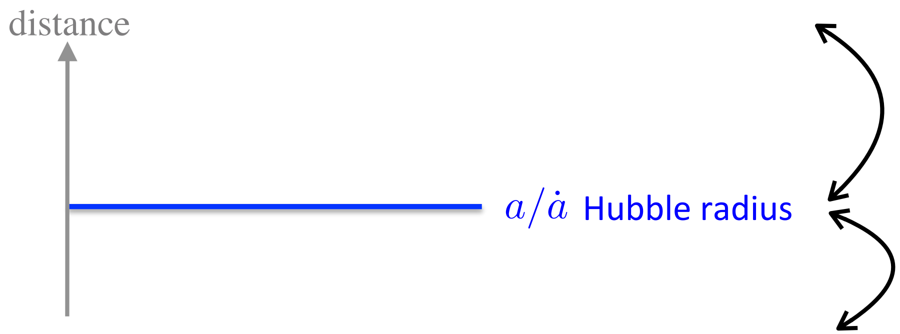

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Equation of motion

$$\frac{\mathrm{d}}{\mathrm{d}N}\mathbf{\Phi}_{\mathrm{cg}} = \mathscr{D}_{\mathrm{background}}(\mathbf{\Phi}_{\mathrm{cg}}) + \xi$$

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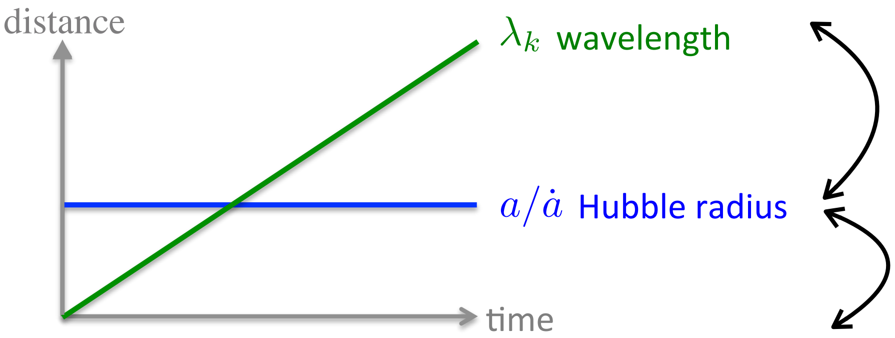

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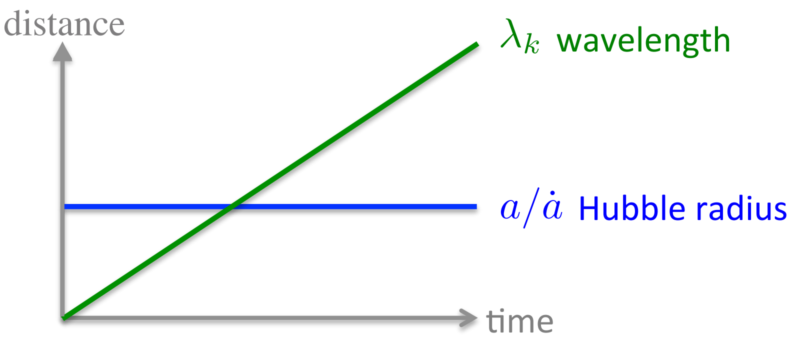

super-Hubble scales —> stochastic inflation

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Quantum fluctuations source the **background**

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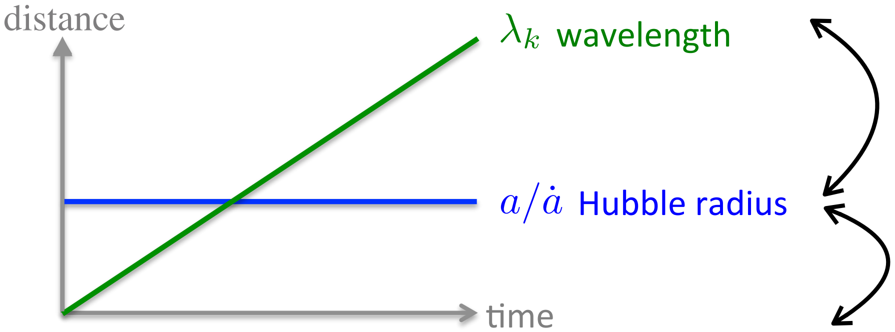

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Starobinsky, (1982) 1986

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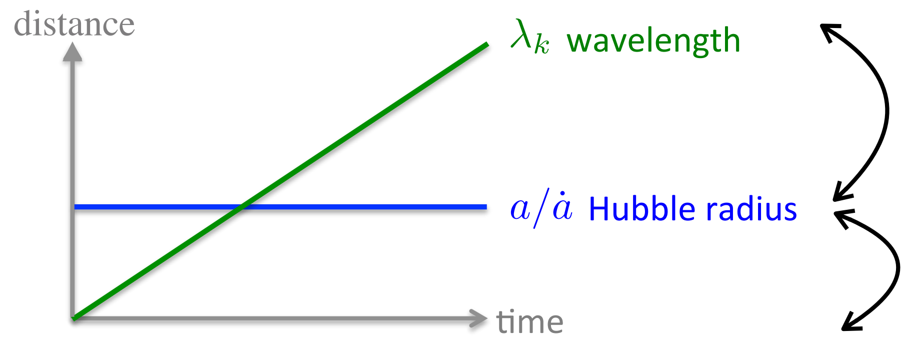

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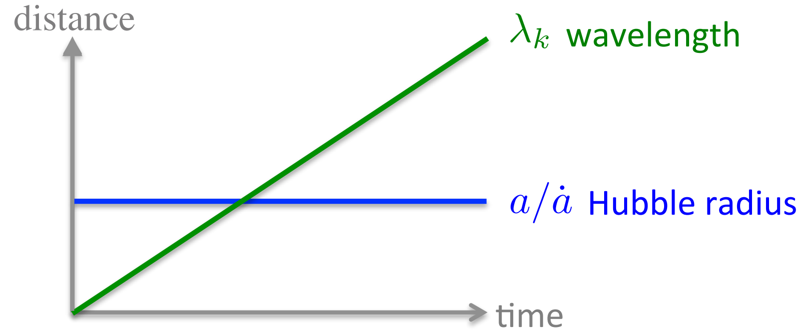
Over one e-fold: $$\frac{\Delta\phi_{\mathrm{quant}}}{\Delta\phi_{\mathrm{classical}}} \sim \zeta_{\mathrm{classical}}$$

Stochastic Inflation



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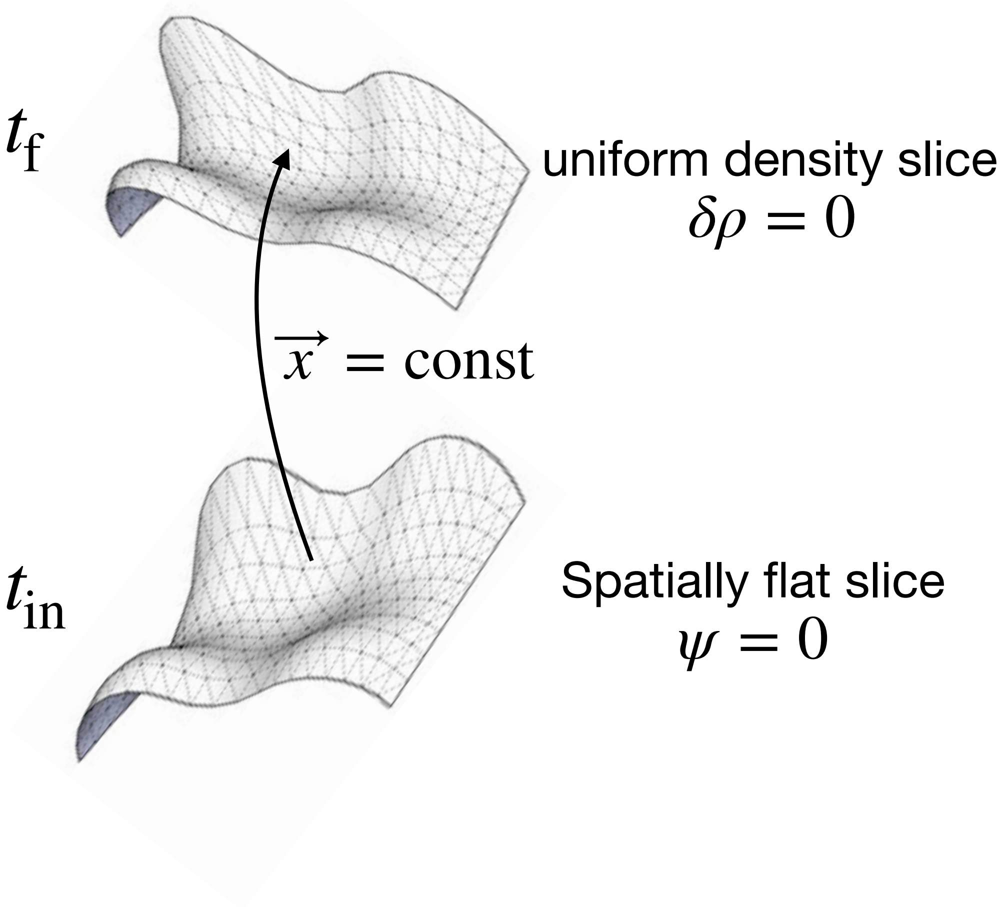
Over one e-fold: $\frac{\Delta\phi_{\rm quant}}{\Delta\phi_{\rm classical}} \sim \zeta_{\rm classical}$

What about far from the classical regime?

What about tail effects?

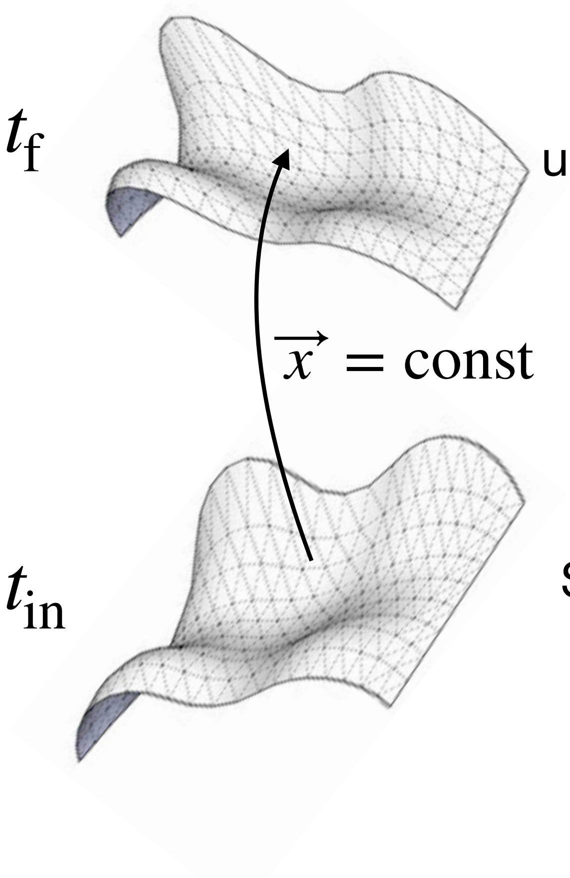
Stochastic-δN formalism



$$\zeta(t, x) = N(t, x) - N_0(t) \equiv \delta N$$

Lifshitz, Khalatnikov (1960)
Starobinsky (1983)
Wands, Malik, Lyth, Liddle (2000)

Stochastic-δN formalism



uniform density slice
$\delta\rho = 0$

Spatially flat slice
$\psi = 0$

$$\zeta(t, x) = N(t, x) - N_0(t) \equiv \delta N$$

Lifshitz, Khalatnikov (1960)
Starobinsky (1983)
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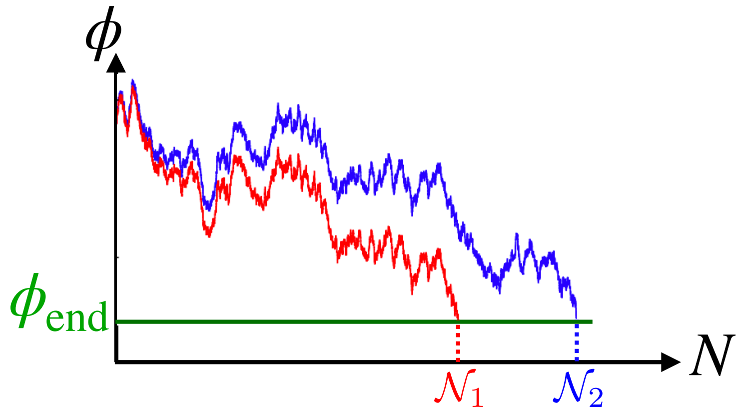
The realised number of e-folds
is a stochastic quantity:

$$\zeta_{\text{coarse grained}} = \mathcal{N} - \langle \mathcal{N} \rangle$$



Enqvist, Nurmi, Podolsky, Rigopoulos (2008) ; Fujita, Kawasaki, Tada, Takesako (2014); VV, Starobinsky (2015)

Stochastic-δN formalism

Stochastic-δ*N* formalism

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi(N)$$

Langevin equation

Stochastic-δ*N* formalism

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi(N) \longrightarrow \frac{\mathrm{d}}{\mathrm{d}N}P(\phi; N) = \frac{\partial}{\partial\phi}\left(\frac{V'}{3H^2}P\right) + \frac{\partial^2}{\partial\phi^2}\left(\frac{H^2}{8\pi^2}P\right)$$

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Equation for the PDF of the first passage time

VV, Starobinsky (2015)
Pattison, VV, Assadullahi, Wands (2017)

$$\frac{\mathrm{d}}{\mathrm{d}\mathcal{N}}P_{\mathrm{FPT}}(\mathcal{N};\phi) = \mathscr{L}_\phi^\dagger \cdot P_{\mathrm{FPT}}$$

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- Solve the first-passage-time problem → **See talk by Joe Jackson**
- This gives the one-point PDF of curvature perturbation coarse-grained at H_{end}

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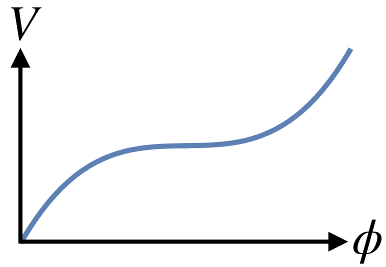
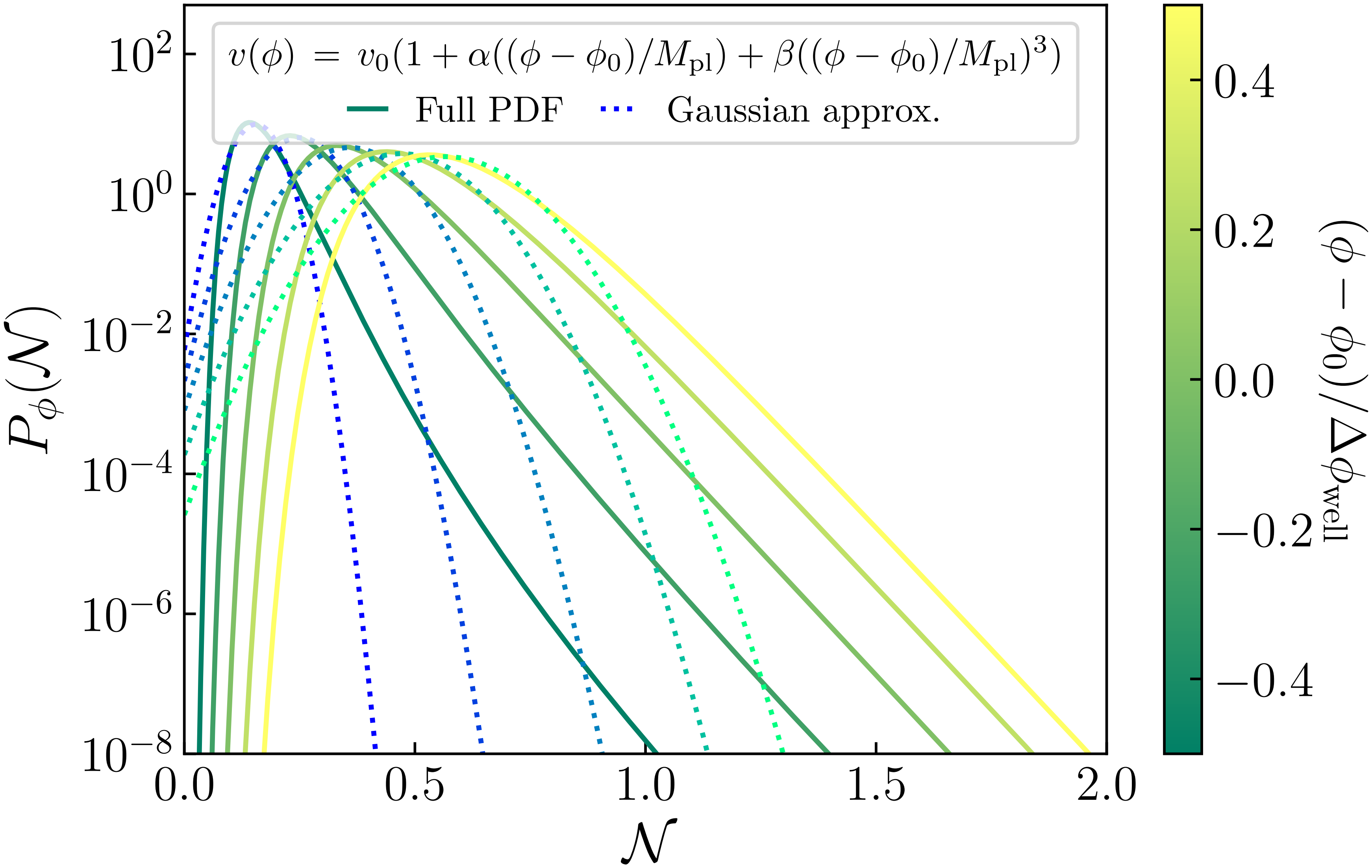
Exponential tails

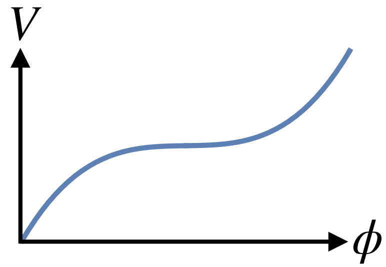
Pattison, VV, Assadullahi, Wands (2017)
Ezquiaga, Garcia-Bellido, VV (2020)

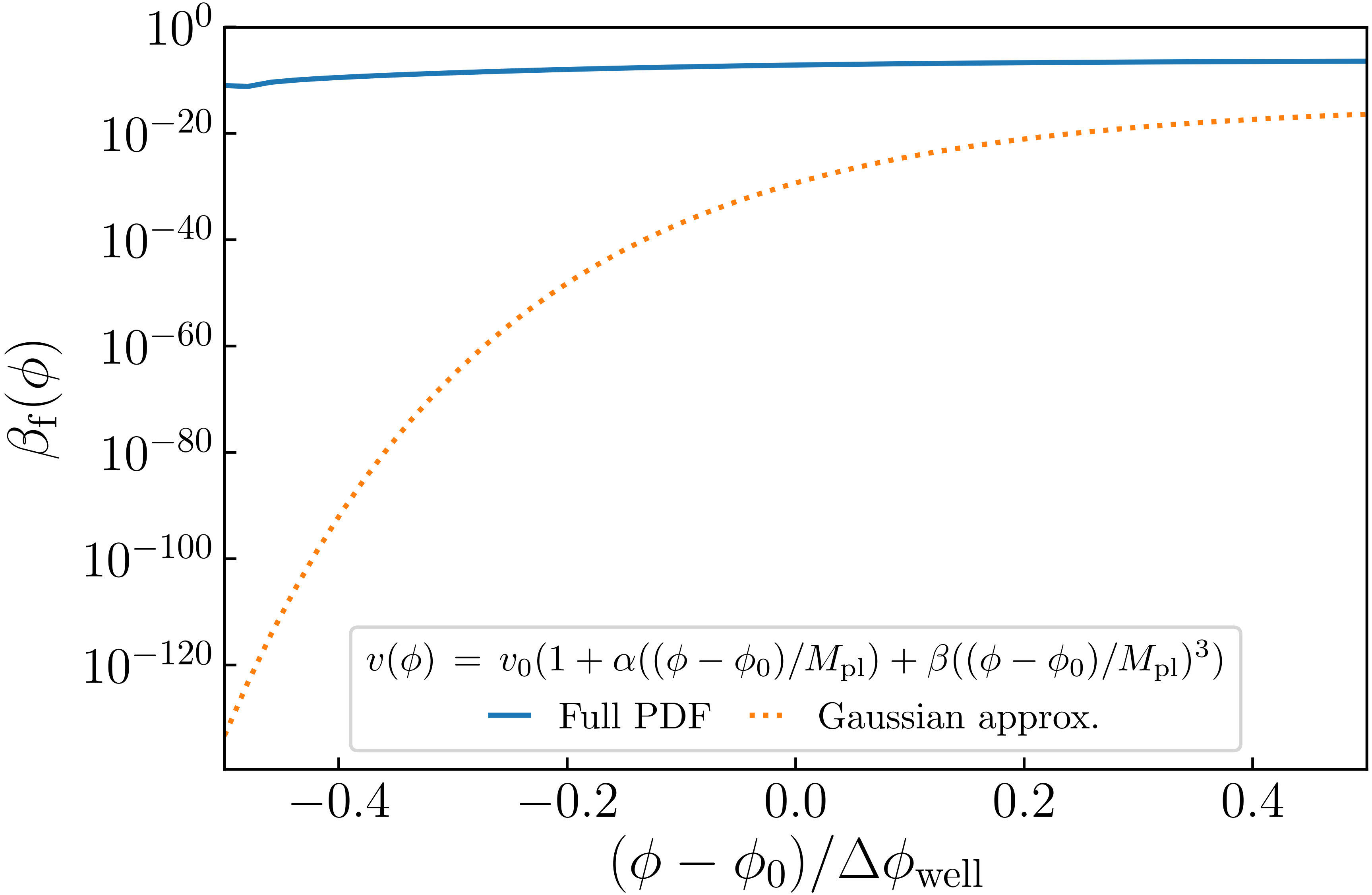




Impact on PBHs

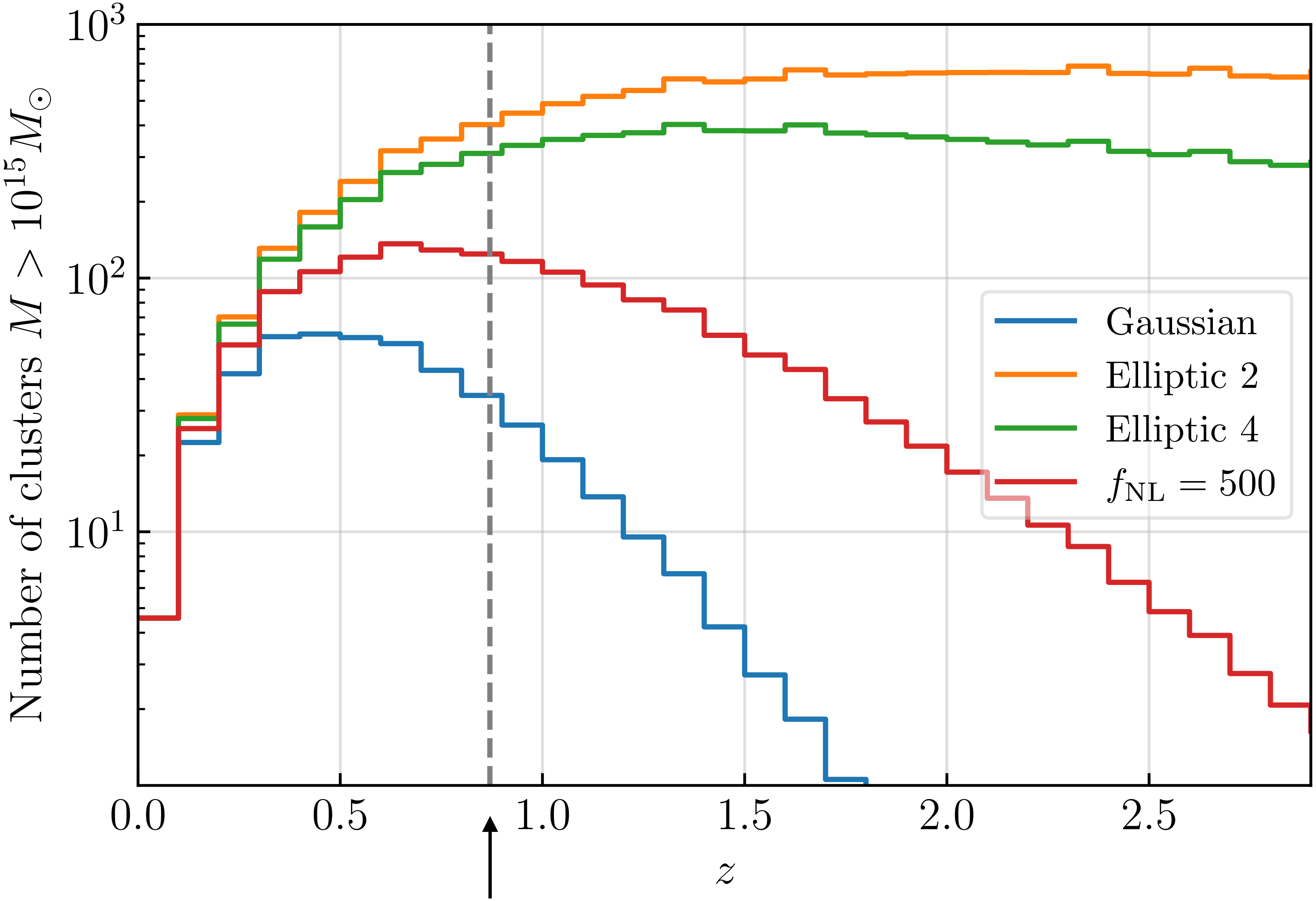
Pattison, VV, Assadullahi, Wands (2017)
Ezquiaga, Garcia-Bellido, VV (2020)





Impact on LSS

Ezquiaga, Garcia-Bellido, VV (2022)



El gordo

Extracting cosmological observables

Scale k ⟶ Hubble-crossing time ⟶ Hubble-crossing field ϕ_*

$$N_* = N_{\rm end} - N_{\rm bw}(k)$$

$$N_{\rm bw}(k) = \ln(a_{\rm end} H / k)$$

Extracting cosmological observables

Scale k → Hubble-crossing time → Hubble-crossing field ϕ_*

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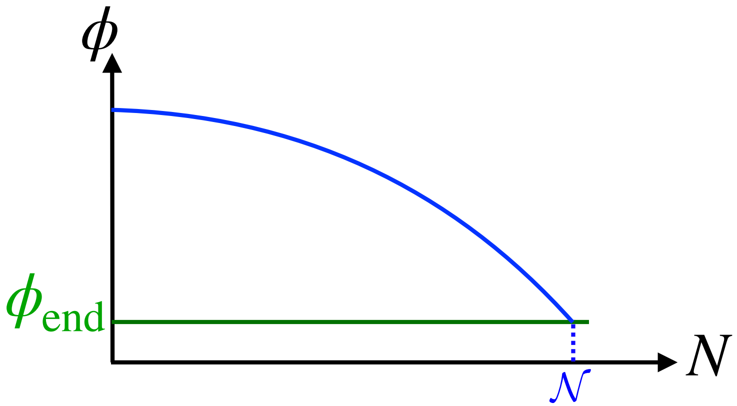
<u>Classical picture</u>



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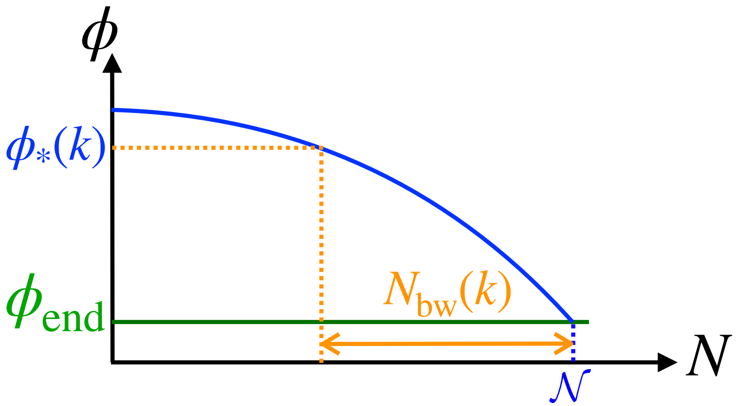

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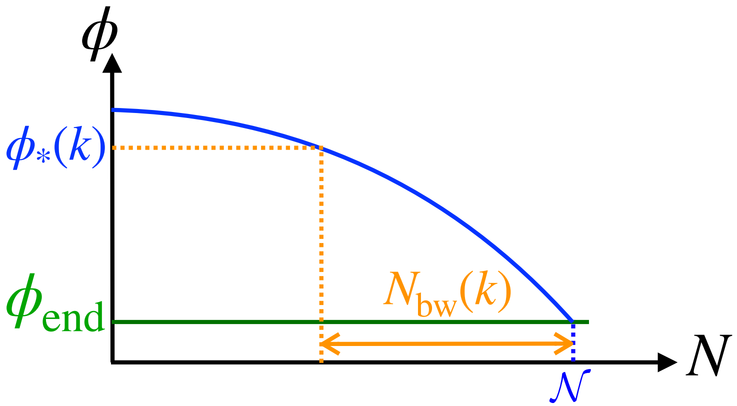

Observables (power spectrum etc) at scale k depend on <u>**local properties**</u> of the potential at location $\phi_*(k)$

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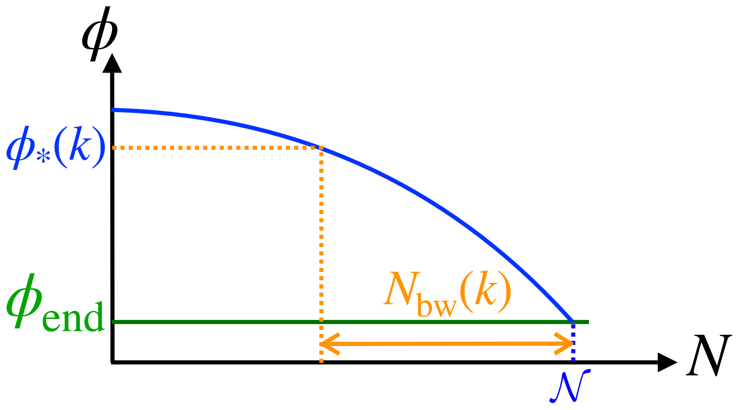

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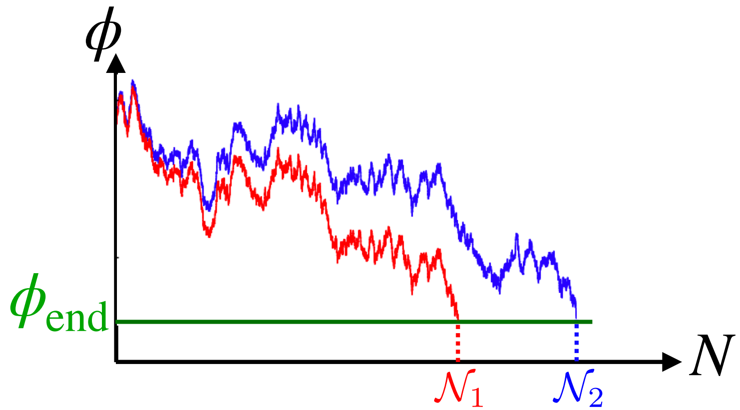
<u>Stochastic picture</u>



Extracting cosmological observables

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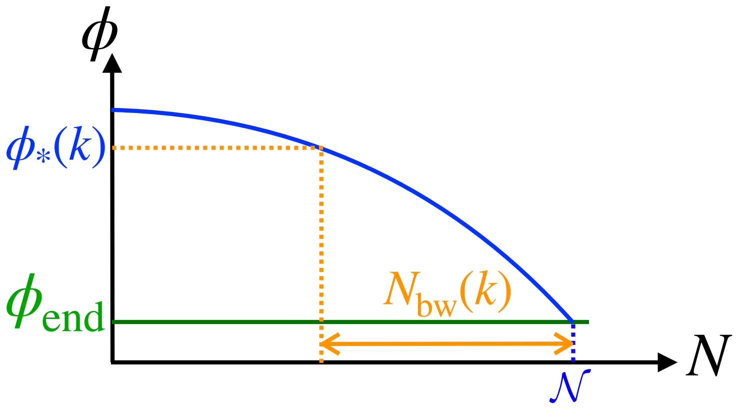

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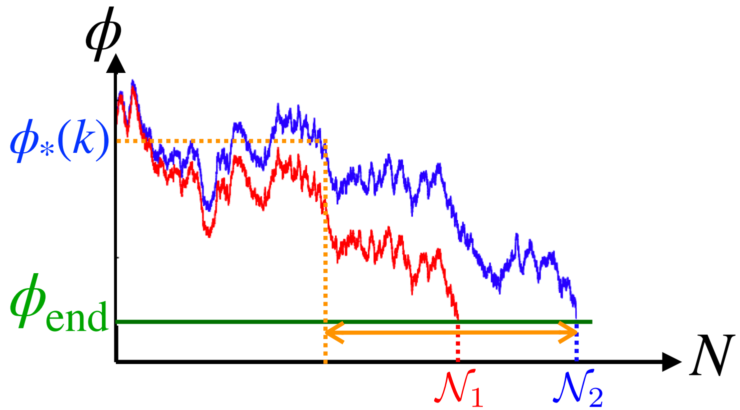

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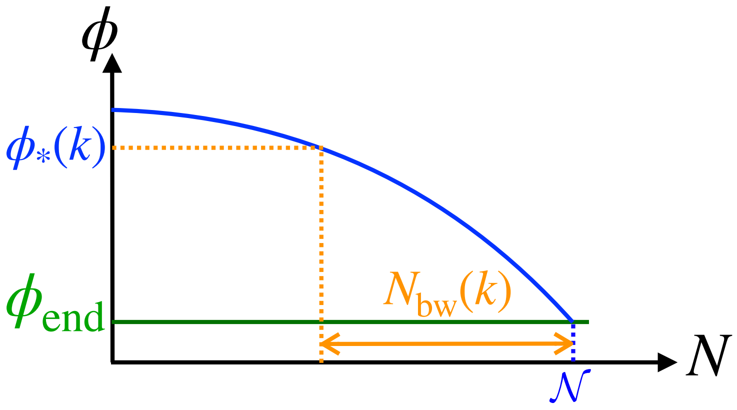

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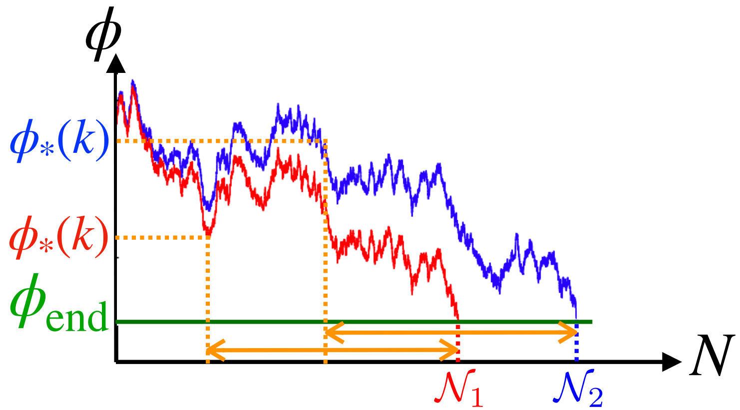

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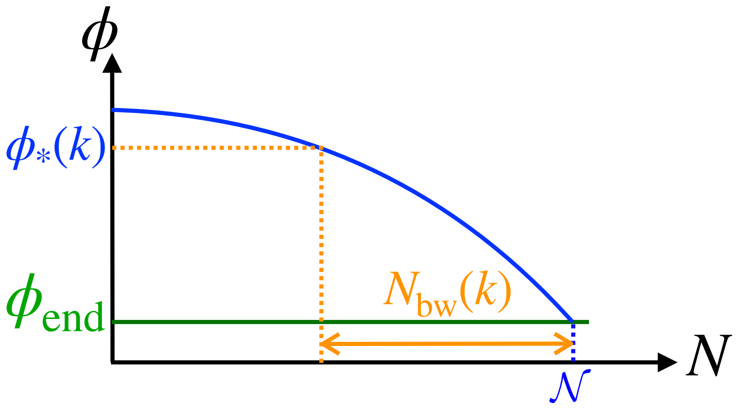

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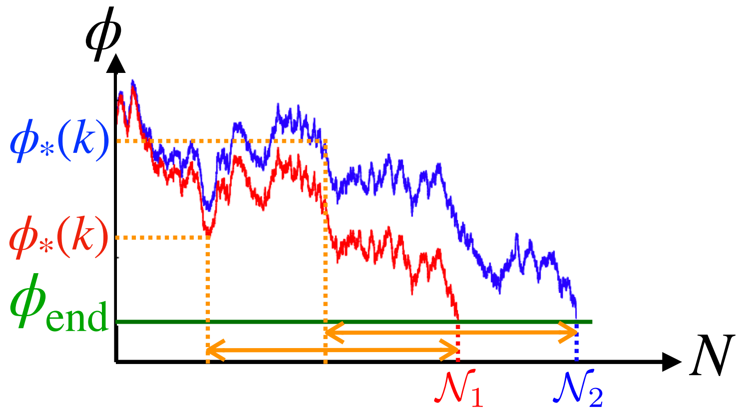

$$P_{\rm bw}(\phi_*; k) = P_{\rm FPT}\left[N_{\rm bw}(k); \phi_*\right] \frac{\int_0^\infty P(\phi_*; N)\,{\rm d}N}{\int_{N_{\rm bw}(k)}^\infty P_{\rm FPT}(\mathcal{N}; \phi_{\rm in})\,{\rm d}\mathcal{N}}$$

Kenta Ando, VV (2020)

Extracting cosmological observables

Scale k ⟶ Hubble-crossing time ⟶ Hubble-crossing field ϕ_*

$$N_* = N_{\mathrm{end}} - N_{\mathrm{bw}}(k)$$

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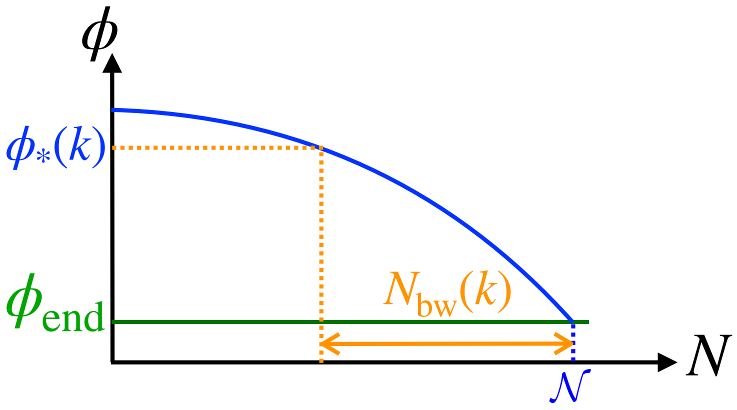

Observables (power spectrum etc) at scale k depend on **<u>local properties</u>** of the potential at location $\phi_*(k)$

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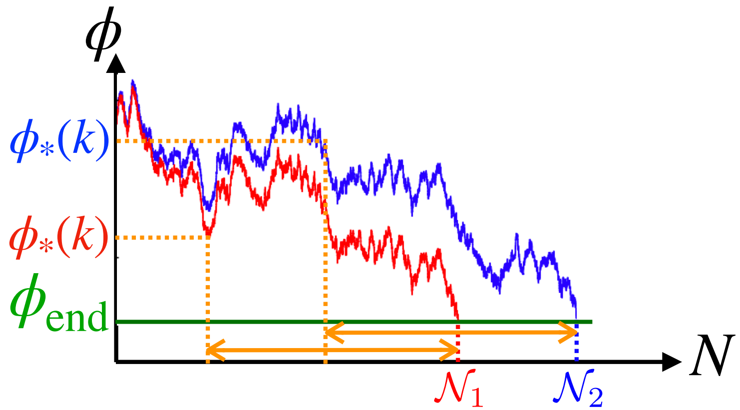

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Kenta Ando, VV (2020)

Observables at scale k depend on the **<u>whole potential</u>** and on the **<u>initial condition</u>**

Extracting cosmological observables

Power Spectrum

Kenta Ando, VV (2020)

$$\mathscr{P}_\zeta(k) = -\int_\Omega \mathrm{d}\mathbf{\Phi}_* \left.\frac{\partial P_{\mathrm{bw}}(\mathbf{\Phi}_*; N_{\mathrm{bw}})}{\partial N_{\mathrm{bw}}}\right|_{N_{\mathrm{bw}}(k)} \left\langle \delta\mathcal{N}^2(\mathbf{\Phi}_0 \to \mathbf{\Phi}_*)\right\rangle$$

Extracting cosmological observables

Power Spectrum

Kenta Ando, VV (2020)

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Integration over the full inflating domain

Extracting cosmological observables

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Kenta Ando, VV (2020)

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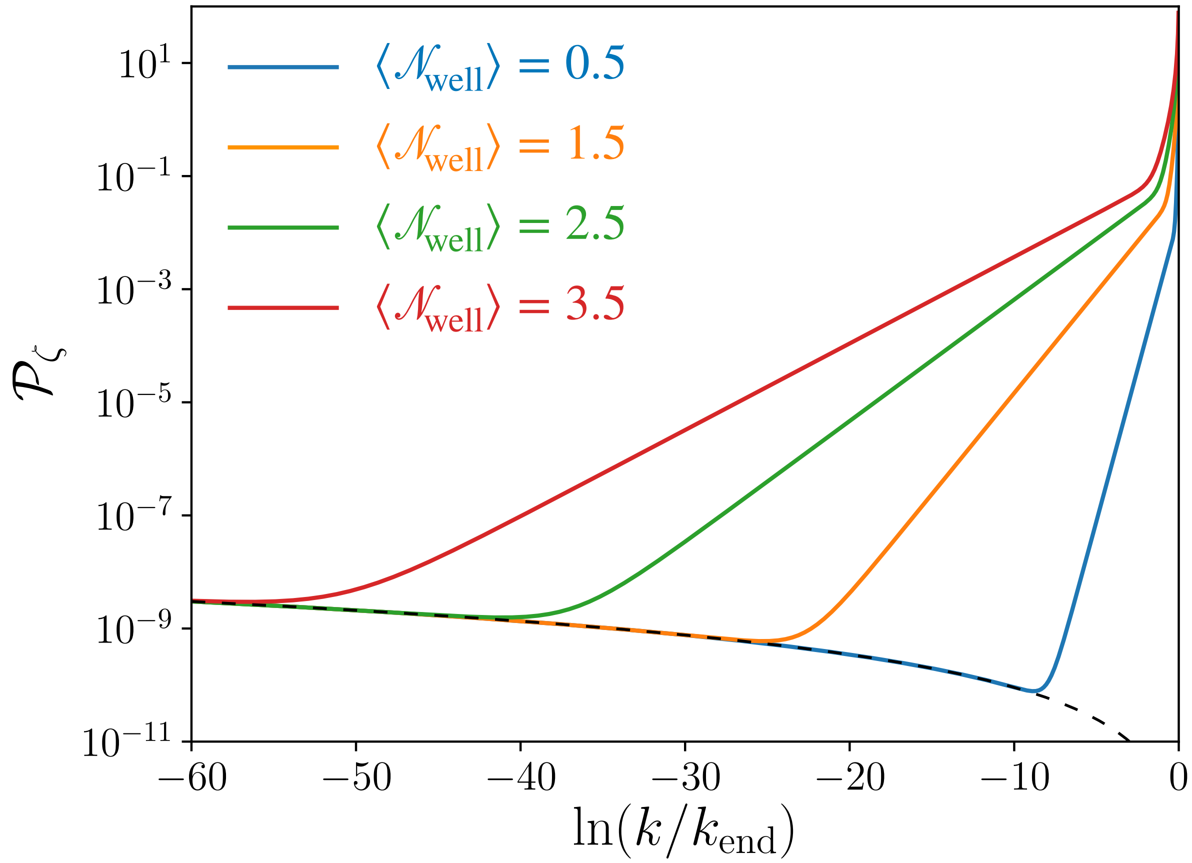

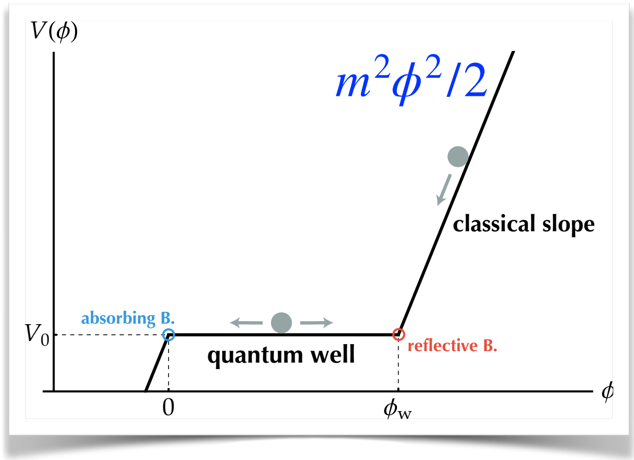

Extracting cosmological observables

One-point function at arbitrary scale

Yuichiro Tada, VV (2021)

Extracting cosmological observables

One-point function at arbitrary scale

Yuichiro Tada, VV (2021)

$$P\left(\zeta_R\right)=\int_\Omega \mathrm{d}\boldsymbol{\Phi}_* P_{\mathrm{bw}}\left[\boldsymbol{\Phi}_* \mid N_{\mathrm{bw}}(R)\right] P_{\mathrm{FPT}, \boldsymbol{\Phi}_0 \rightarrow \boldsymbol{\Phi}_*}\left[\zeta_R-\left\langle\mathcal{N}\left(\boldsymbol{\Phi}_*\right)\right\rangle+\left\langle\mathcal{N}\left(\boldsymbol{\Phi}_0\right)\right\rangle\right]$$

R

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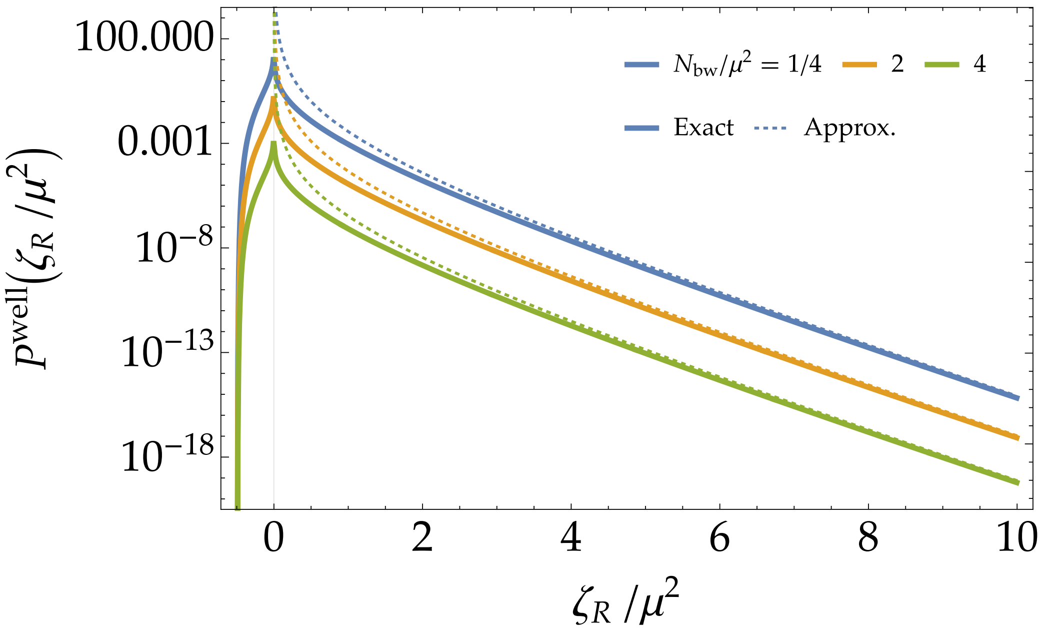

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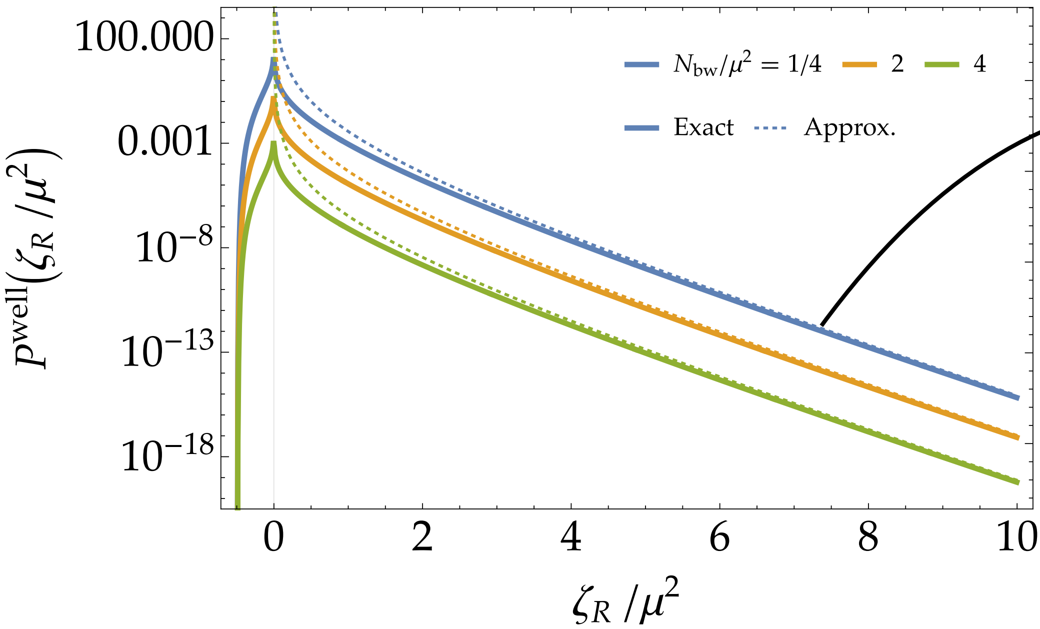


$$P\left(\zeta_R\right) \propto \frac{e^{-\frac{\pi^2}{4}\frac{\zeta_R}{\mu^2}}}{\left(\zeta_R/\mu^2\right)^3}$$

Quasi-exponential tail

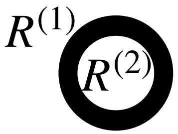
Extracting cosmological observables

One-point function at arbitrary scale

Yuichiro Tada, VV (2021)

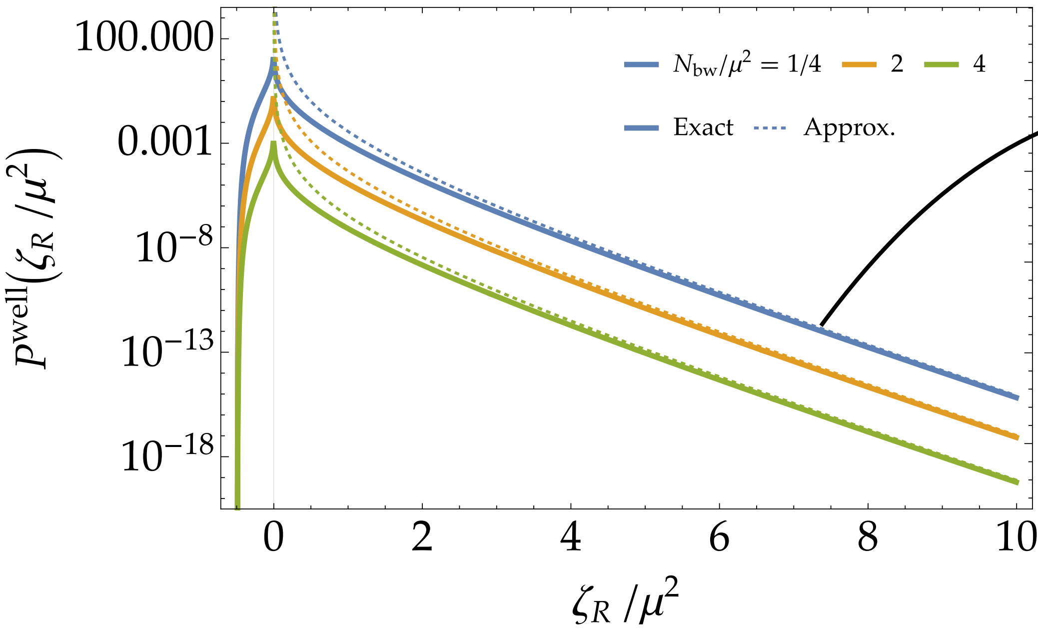
$$P\left(\zeta_R\right)=\int_\Omega \mathrm{d}\boldsymbol{\Phi}_* P_{\mathrm{bw}}\left[\boldsymbol{\Phi}_* \mid N_{\mathrm{bw}}(R)\right] P_{\mathrm{FPT}, \boldsymbol{\Phi}_0 \rightarrow \boldsymbol{\Phi}_*}\left[\zeta_R-\left\langle\mathcal{N}\left(\boldsymbol{\Phi}_*\right)\right\rangle+\left\langle\mathcal{N}\left(\boldsymbol{\Phi}_0\right)\right\rangle\right]$$

$$P(\Delta\zeta)=\int_\Omega \mathrm{d}\boldsymbol{\Phi}_*^{(1)} \mathrm{d}\boldsymbol{\Phi}_*^{(2)} P_{\mathrm{bw}}\left(\boldsymbol{\Phi}_*^{(1)}, \boldsymbol{\Phi}_*^{(2)} \middle| N_{\mathrm{bw}}^{(1)}, N_{\mathrm{bw}}^{(2)}\right) \delta\left[\Delta\zeta+\left\langle\mathcal{N}\left(\boldsymbol{\Phi}_*^{(1)}\right)\right\rangle-\left\langle\mathcal{N}\left(\boldsymbol{\Phi}_*^{(2)}\right)\right\rangle-\ln(1+\beta)\right]$$



→ Comoving density contrast

→ Compaction function



$$P\left(\zeta_R\right) \propto \frac{e^{-\frac{\pi^2}{4} \frac{\zeta_R}{\mu^2}}}{\left(\zeta_R/\mu^2\right)^3}$$

Quasi-exponential tail

Extracting cosmological observables

One-point function at arbitrary scale

Yuichiro Tada, VV (2021)

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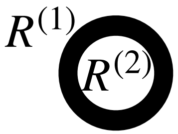

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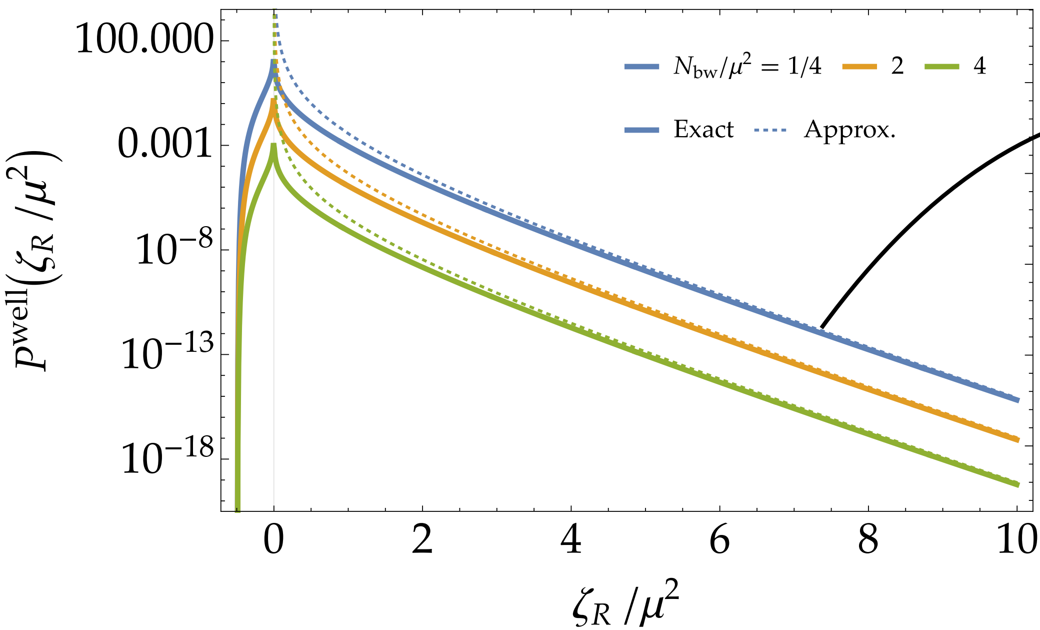

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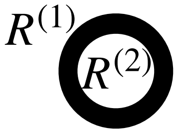
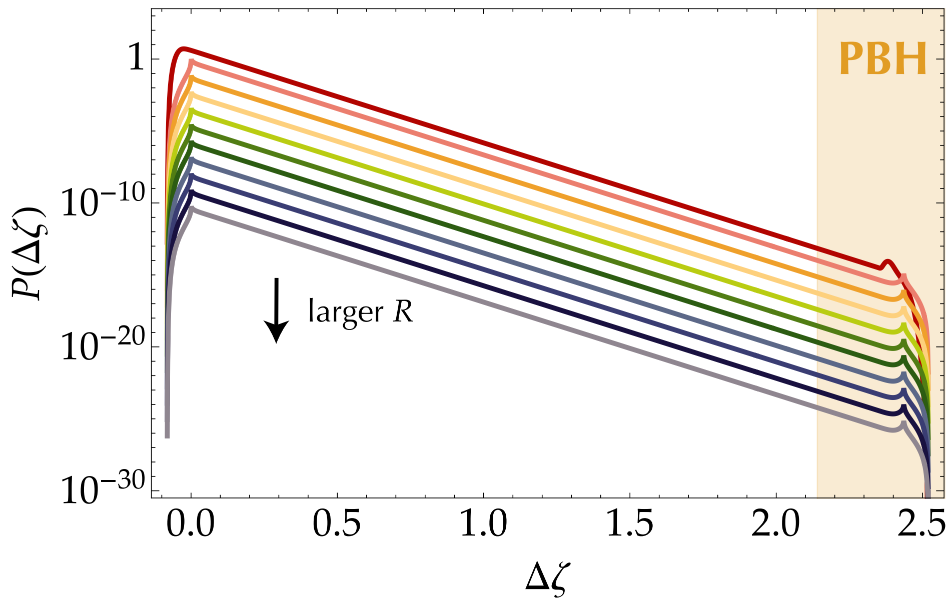
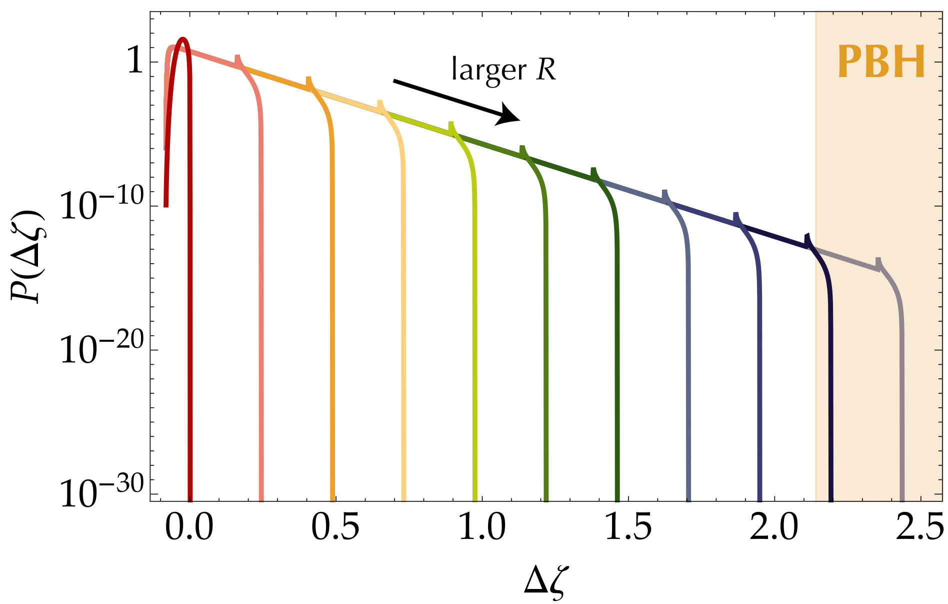


$\mu = \frac{1}{\sqrt{6}}, N_{\mathrm{bw}}^{(2)} > 0$



R_2 exits within the quantum well

$\mu = \frac{1}{\sqrt{6}}, N_{\mathrm{bw}}^{(2)} < 0$



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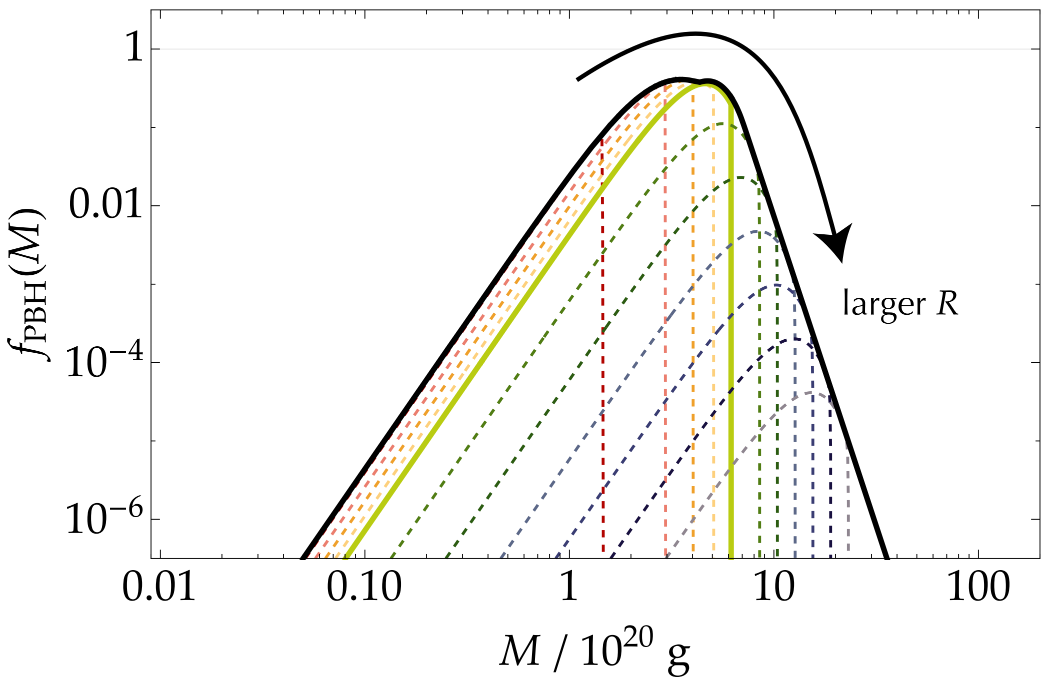
$$\mu=\frac{1}{\sqrt{6}}$$



Conclusions

- The back-reaction of vacuum quantum fluctuations on the background dynamics can be incorporated within the formalism of stochastic inflation

- This is necessary to describe regimes leading to large fluctuations, such as those yielding primordial black holes

- Quantum diffusion leads to exponential tails: non-perturbative breakdown of Gaussian statistics

- Most cosmological observables can be reconstructed from first-passage time analysis (power spectrum, mass functions, n-point functions?)

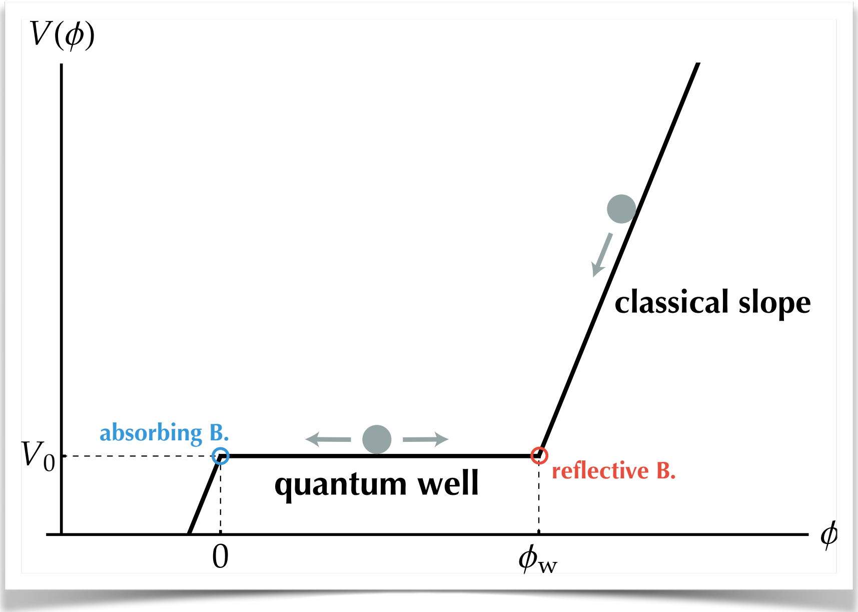
- Quantum diffusion makes the CMB probe the whole potential: models leading to PBHs are constrained by the CMB, even if those two sets of scales are well separated

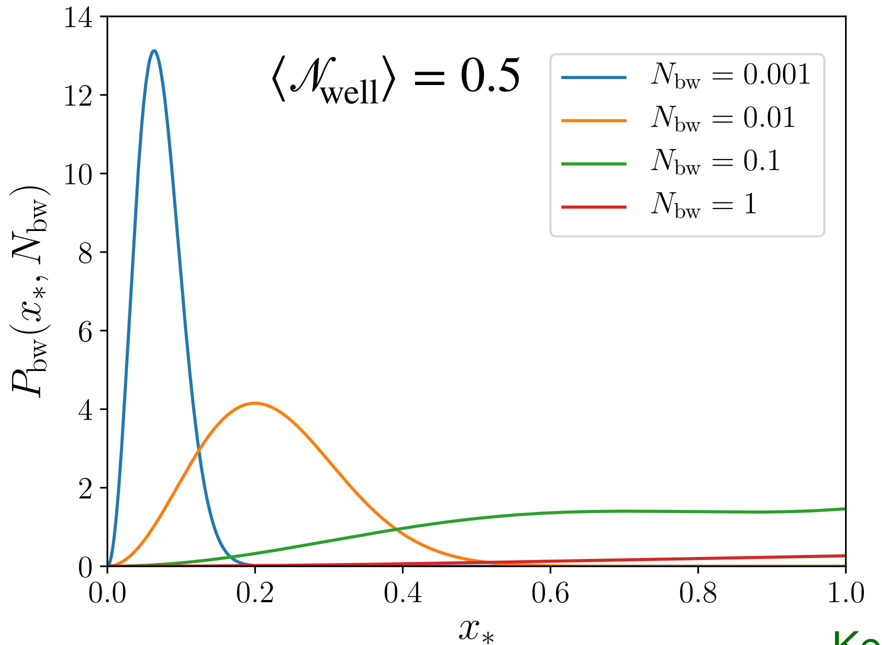
- **What is the best strategy to look for exponential tails in the data?**

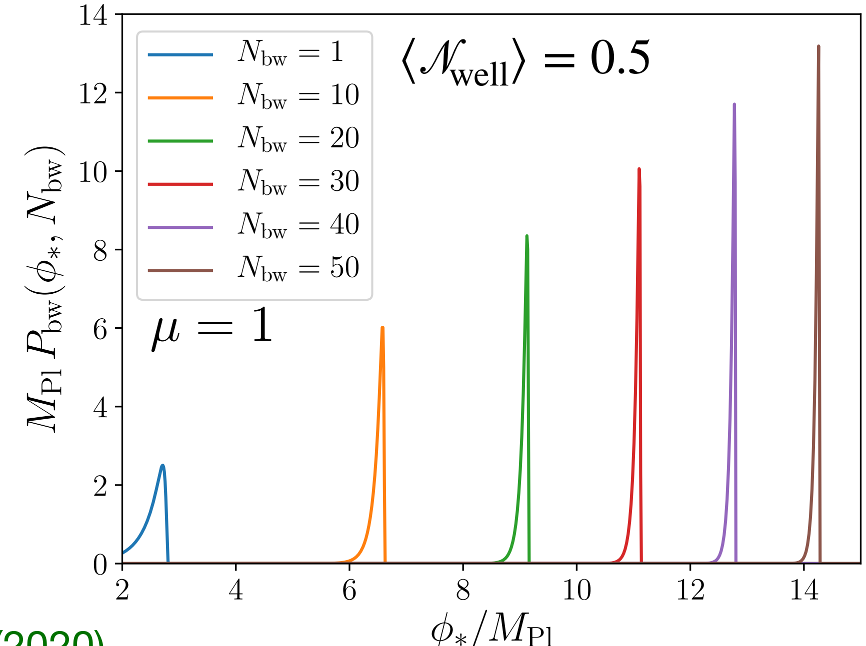
Thank you for your attention

Back-up slides

CMB probes the full potential







Kenta Ando, VV (2020)

Stochastic-δ*N* formalism

Moments obey an interactive equation VV, Starobinsky (2015) $v = V/(24\pi^2 M_{\rm Pl}^4)$

$$\left\langle \mathcal{N}^n \right\rangle''(\phi) - \frac{v'}{v^2}\left\langle \mathcal{N}^n \right\rangle'(\phi) = -\frac{n}{vM_{\rm Pl}^2}\left\langle \mathcal{N}^{n-1}\right\rangle(\phi)$$

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Mean number of e-folds $\langle \mathcal{N} \rangle(\phi) = \displaystyle\int_{\phi_{\mathrm{end}}}^{\phi} \frac{\mathrm{d}x}{M_{\mathrm{Pl}}} \int_{x}^{\phi_{\mathrm{UV}}} \frac{\mathrm{d}y}{M_{\mathrm{Pl}}} \frac{e^{\frac{1}{v(y)} - \frac{1}{v(x)}}}{v(y)}$

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Involves the full inflationary domain

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$v \ll 1, |v^2 v''/v'^2| \ll 1$

$$\simeq \int_{\phi_{\rm end}}^{\phi} \frac{\mathrm{d}x}{M_{\rm Pl}^2} \frac{v(x)}{v'(x)} \left[1 + v(x) - \frac{v''(x) v^2(x)}{v'^2(x)} + \cdots \right]$$

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classical result

first-order correction

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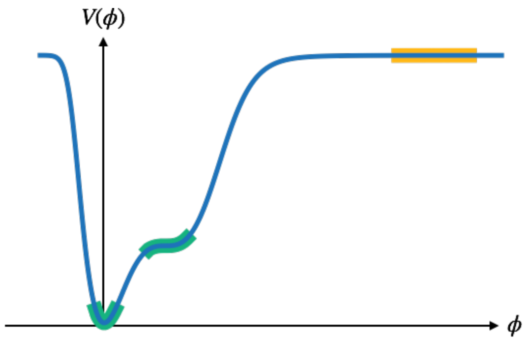

Stochastic-δ*N* formalism

Stochastic-δN formalism

Second moment and power spectrum VV, Starobinsky (2015)

$$\mathscr{P}_\zeta\left(\phi\right) = 2\frac{\int_\phi^{\phi_{\mathrm{UV}}}\frac{\mathrm{d}x}{M_{\mathrm{Pl}}}\left\{\int_x^{\phi_{\mathrm{UV}}}\frac{\mathrm{d}y}{M_{\mathrm{Pl}}}\frac{1}{v\left(y\right)}\exp\left[\frac{1}{v\left(y\right)}-\frac{1}{v(x)}\right]\right\}^2\exp\left[\frac{1}{v(x)}-\frac{1}{v\left(\phi\right)}\right]}{\int_\phi^{\phi_{\mathrm{UV}}}\frac{\mathrm{d}x}{M_{\mathrm{Pl}}}\frac{1}{v(x)}\exp\left[\frac{1}{v(x)}-\frac{1}{v\left(\phi\right)}\right]}$$

Stochastic-δN formalism

Second moment and power spectrum   VV, Starobinsky (2015)

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Third moment and local non-Gaussianity

$$f_{\mathrm{NL}} = \frac{5}{24}M_{\mathrm{Pl}}^2\left[6\frac{v'^2}{v^2} - 4\frac{v''}{v} + v\left(11\frac{v'^2}{v^2} - 158\frac{v''}{v} - 9\frac{v'''}{v'} + 118\frac{v''^2}{v'^2}\right) + \cdots\right]$$