Challenges & Reconsiderations in

Primordial Non-Gaussianity Constraints from Galaxy Clustering

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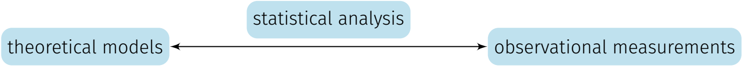
A Cosmic Window to Fundamental Physics: PNG and Beyond · IFT UAM · Madrid, 2022

Challenges & Reconsiderations

Confronting theory with observations



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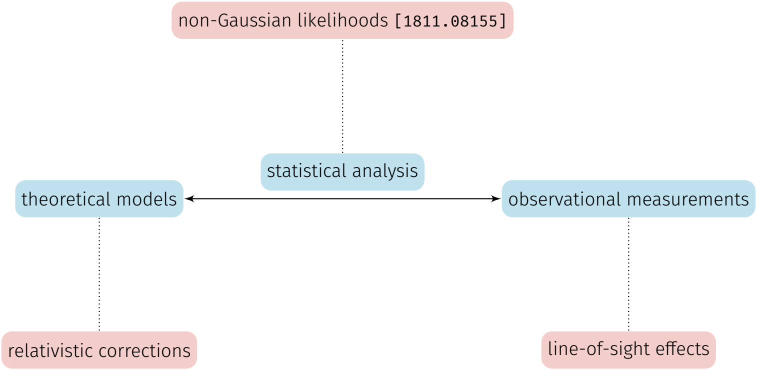

Impact of Relativistic Corrections for Local PNG

With F Beutler & D Bacon [2007.01802]

Relativistic corrections

$$\delta(\mathbf{r}, z) = b_1 \delta_{\rm m} - \mathcal{H}^{-1} \hat{\mathbf{r}} \cdot \partial_r \mathbf{v}$$

Relativistic corrections

$$\begin{aligned}\delta(\mathbf{r}, z) = b_1 \delta_\mathrm{m} &- \mathcal{H}^{-1} \hat{\mathbf{r}} \cdot \partial_r \mathbf{v} \\ &- g_1 \hat{\mathbf{r}} \cdot \mathbf{v} - (b_\mathrm{e} - 3) \mathcal{H} \nabla^{-2} \boldsymbol{\nabla} \cdot \mathbf{v} \\ &+ \mathcal{H}^{-1} \Phi' - (2 - 5s) \Phi + \Psi + g_1 \Psi + \cdots\end{aligned}$$

where

$$b_\mathrm{e}(z) = -\frac{\partial \ln \bar{n}(z; <\bar{m})}{\partial \ln(1 + z)}\,, \qquad s(z) = \left.\frac{\partial}{\partial m}\right|_{\bar{m}} \lg \bar{n}(z; <\bar{m})\,.$$

See Yoo (2009), Bonvin & Durrer (2011), Challinor & Lewis (2011).

Relativistic corrections

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See Yoo (2009), Bonvin & Durrer (2011), Challinor & Lewis (2011).

In Fourier space,

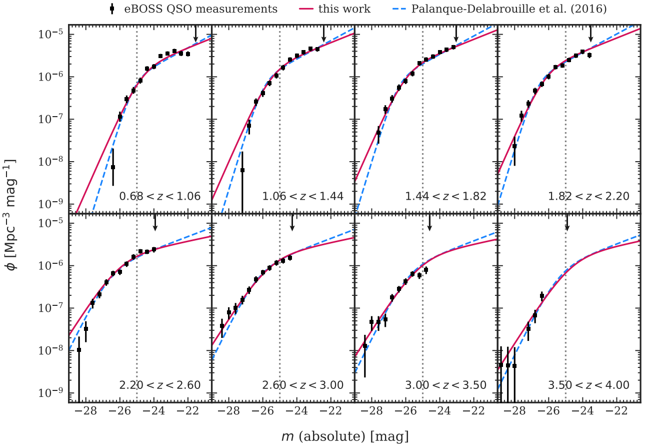
$$\delta(\boldsymbol{k}) = \left[b_1 + f\mu^2 + \mathrm{i}(\mathcal{H}/k)g_1 f\mu + (\mathcal{H}/k)^2 g_2\right]\delta_\mathrm{m}(\boldsymbol{k})\,,$$

where

$$\begin{aligned}g_1(z) &= \mathcal{H}'/\mathcal{H}^2 + (2-5s)/(\mathcal{H}\chi) + 5s - b_\mathrm{e}\,,\\ g_2(z) &= -(b_\mathrm{e}-3)f + \left(\mathcal{H}'/\mathcal{H}^2 - 1\right)\left[g_1 + f - (2-5s)\right].\end{aligned}$$

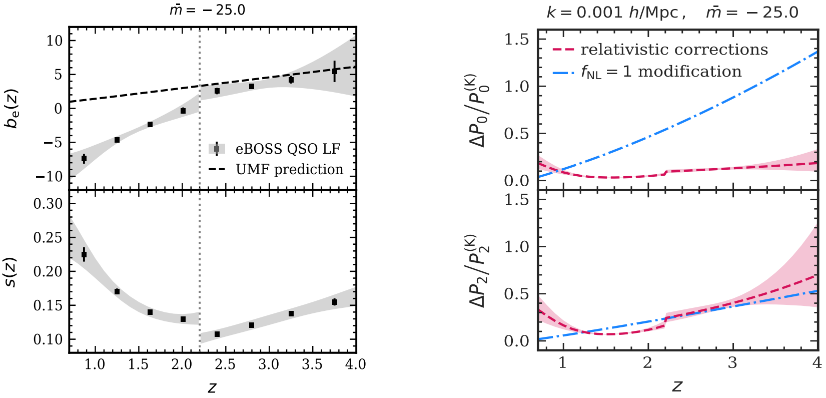
Refit the eBOSS QSO luminosity function to derive $\bar{n}(z; <\bar{m}) = \int_{-\infty}^{\bar{m}} \mathrm{d}m\, \phi(m, z)$.

See also Palanque-Delabrouille+ (2016) & Pozzetti+ (2016).



Impact of Relativistic Corrections for Local PNG

Results



Lesson

❗ Forward/joint modelling

Hybrid-Basis Fourier Analysis

With S Avila, D Bianchi, R Crittenden & W Percival [2007.14962]

Plane-parallel/distant-observer approximation(s)

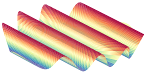
$$\delta_{\mathrm{s}}(\boldsymbol{k}, z) = b(z)\delta_{\mathrm{m}}(\boldsymbol{k}, z) + f(z) \int \frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3} R(\boldsymbol{q}, \boldsymbol{k}) \delta_{\mathrm{m}}(\boldsymbol{q}, z) \,,$$

with $R(\boldsymbol{q}, \boldsymbol{k}) \to \mu^2 \delta^{(\mathrm{D})}(\boldsymbol{q} - \boldsymbol{k})$.

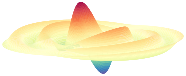
See Kaiser (1987), Zaroubi & Hoffman (1996).

Rayleigh expansion—

$$\underbrace{\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}}_{\langle \boldsymbol{r} | \boldsymbol{k} \rangle} = 4\pi \sum_{\ell m} \mathrm{i}^{\ell} Y^{*}_{\ell m}(\hat{\boldsymbol{k}}) \underbrace{j_{\ell}(k_{\ell n} r) Y_{\ell m}(\hat{\boldsymbol{r}})}_{\langle \boldsymbol{r} | \ell m n \rangle} \,.$$



plane wave $|\boldsymbol{k}\rangle = |k_1, k_2, k_3\rangle$



'spherical wave' $|\mu\rangle = |\boldsymbol{\ell}, m, n\rangle$

Spherical Fourier–Bessel decomposition

$$D_\mu = \sum_\nu \underbrace{M_{\mu\nu}}_{\text{angular}} \Big(b \underbrace{\Phi_{\mu\nu}}_{\text{radial}} + f \underbrace{\Upsilon_{\mu\nu}}_{\text{distortion}} \Big) (D_\mathrm{m})_\nu$$

See also Fisher+ (1995), Heavens & Taylor (1995), Yoo (2013) & Alonso+ (2015).

All cosmological dependence is encoded in the covariance matrix (2-point statistics):

$$\langle D_\mu D_\nu \rangle = \sum_\sigma M_{\mu\sigma} M^*_{\nu\sigma} \left[b_*(k_\sigma) \boldsymbol{\Phi}_{\mu\sigma} + f_* \boldsymbol{\Upsilon}_{\mu\sigma} \right] \left[b_*(k_\sigma) \boldsymbol{\Phi}_{\nu\sigma} + f_* \boldsymbol{\Upsilon}_{\nu\sigma} \right] \boldsymbol{\kappa}_\sigma^{-1} P_{\mathrm{m}*}(k_\sigma) \,.$$

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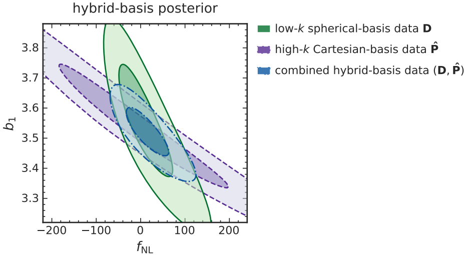
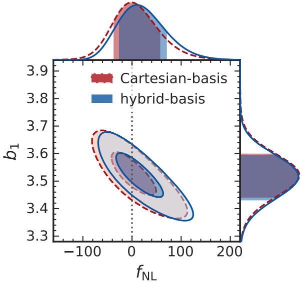
Hybrid-basis likelihoods

$$\mathcal{L}(\theta; \boldsymbol{D}) = \frac{1}{|\pi C(\theta)|^{1/2}} \exp\left[-\boldsymbol{D}^\dagger C(\theta)^{-1} \boldsymbol{D} \right] \qquad \left(C_{\mu\nu} = \langle D_\mu D_\nu^* \rangle \right)$$

$$\mathcal{L}\Big(\theta; \{\widehat{P}_\ell\}, \widehat{\Sigma}\Big) = \text{normal or modified-}t\text{ PDF} \,.$$

See Sellentin & Heavens (2015).





Lesson

Appropriate prescriptions for the **geometry** and **scale** of the problem

Tripolar Spherical-Harmonic Bispectrum Analysis

With R Neveux & F Beutler... (in prep)

Tripolar Bispectrum Analysis
FFT-based estimator

Tripolar spherical-harmonic decomposition

$$B(\boldsymbol{k}_1, \boldsymbol{k}_2, \hat{\boldsymbol{n}}) = \sum_{\ell_1 \ell_2 L} H^{-1}_{\ell_1 \ell_2 L} B_{\ell_1 \ell_2 L}(k_1, k_2) \sum_{m_1 m_2 M} \begin{pmatrix} \ell_1 & \ell_2 & L \\ m_1 & m_2 & M \end{pmatrix} y^{m_1}_{\ell_1}(\hat{\boldsymbol{k}}_1)\, y^{m_2}_{\ell_2}(\hat{\boldsymbol{k}}_2)\, y^{M}_{L}(\hat{\boldsymbol{n}})$$

FFT-based estimator

$$\widehat{B}_{\ell_1 \ell_2 L}(k_1, k_2) = \frac{N_{\ell_1 \ell_2 L} H_{\ell_1 \ell_2 L}}{I} \sum_{m_1 m_2 M} \begin{pmatrix} \ell_1 & \ell_2 & L \\ m_1 & m_2 & M \end{pmatrix} \int \mathrm{d}^3\boldsymbol{x}\, F^{m_1}_{\ell_1}(\boldsymbol{x}; k_1) F^{m_2}_{\ell_2}(\boldsymbol{x}; k_2) G^{M}_{L}(\boldsymbol{x})$$

where with $\delta n^m_\ell(\boldsymbol{k}) = \mathcal{F}\{y^m_\ell \delta n(\boldsymbol{x})\}$,

$$F^m_\ell(\boldsymbol{x}; k) = \int \frac{\mathrm{d}^2\hat{\boldsymbol{k}}}{4\pi} \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} y^{m*}_\ell(\hat{\boldsymbol{k}}) \delta n(\boldsymbol{k}), \quad G^m_\ell(\boldsymbol{x}) = \int \frac{\mathrm{d}^3\hat{\boldsymbol{k}}}{(2\pi)^3} \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} \delta n^m_\ell(\boldsymbol{k}) .$$

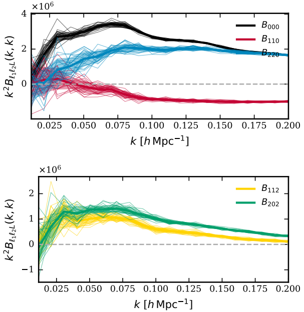
See Sugiyama+ (2019).

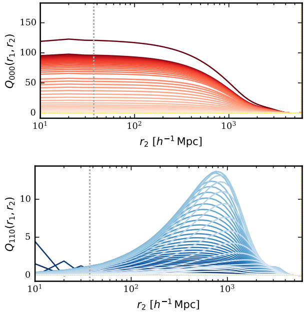
Features in comparison with $B_{LM}(k_1, k_2, k_3)$ (Scoccimarro 2015):

1) lower data vector dimensions;
2) no triangle counting;
3) inverse FFT involved.

Validation with DESI `FirstGen` mock catalogues:

$B_{\ell_1 \ell_2 L}$ & $Q_{\ell_1 \ell_2 L}$





Challenges:

- window convolution and wide-angle corrections;
- impact & mitigation of systematics incl. fibre collision;
- effective redshift;
- …
- which estimator for (which type of) PNG?

Question Time

HorizonGRound

MikeSWang/HorizonGRound

HARMONIA

MikeSWang/Harmonia

TRIUMVIRATE

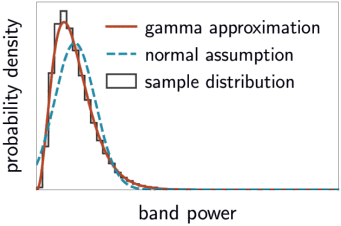
MikeSWang/Triumvirate

(Preview at `http://cuillin.roe.ac.uk/~swang/Triumvirate`; coming soon!)

Non-Gaussian Likelihoods

With W Percival, S Avila, R Crittenden & D Bianchi [1811.08155]

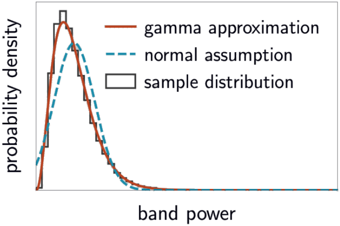
Non-Gaussian Likelihoods



Non-normal distribution

$$\underbrace{\begin{bmatrix}\delta(\boldsymbol{k}_1)\\ \vdots \\ \delta(\boldsymbol{k}_n)\end{bmatrix}}_{\sim\mathbb{C}N} \xrightarrow{\text{non-linear}} \underbrace{\begin{bmatrix}P(\boldsymbol{k}_1)\\ \vdots \\ P(\boldsymbol{k}_n)\end{bmatrix}}_{\sim\mathrm{Exp}} \xrightarrow{\text{mixing}} \underbrace{\begin{bmatrix}\widetilde{P}(k_1)\\ \vdots \\ \widetilde{P}(k_p)\end{bmatrix}}_{\dot\sim\Gamma}$$

Non-Gaussian Likelihoods



Non-normal distribution

$$\underbrace{\begin{bmatrix} \delta(\boldsymbol{k}_1) \\ \vdots \\ \delta(\boldsymbol{k}_n) \end{bmatrix}}_{\sim \mathbb{C}\mathrm{N}} \xrightarrow{\text{non-linear}} \underbrace{\begin{bmatrix} P(\boldsymbol{k}_1) \\ \vdots \\ P(\boldsymbol{k}_n) \end{bmatrix}}_{\sim \mathrm{Exp}} \xrightarrow{\text{mixing}} \underbrace{\begin{bmatrix} \widetilde{P}(k_1) \\ \vdots \\ \widetilde{P}(k_p) \end{bmatrix}}_{\dot\sim \Gamma}$$

Parameter dependence of the covariance matrix

$$\Sigma(\theta) = \underbrace{\begin{bmatrix} \sigma & & \\ & \ddots & \\ & & \sigma \end{bmatrix}}_{\Lambda} \underbrace{\begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} \sigma & & \\ & \ddots & \\ & & \sigma \end{bmatrix}}_{\Lambda}$$

Non-Gaussian Likelihoods

'Box–Cox' Gaussianisation

$$\widetilde{P} \mapsto Z := \widetilde{P}^{\,\nu}\,, \quad \boldsymbol{\nu} \approx 1/3\,.$$

Covariance decomposition

$$\widehat{\Sigma}(\theta) = \Lambda(\theta)\,\Lambda_{\mathrm{f}}^{-1}\,\widehat{\Sigma}_{\mathrm{f}}\,\Lambda_{\mathrm{f}}^{-1}\,\Lambda(\theta)\,.$$

Non-Gaussian Likelihoods

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Covariance decomposition

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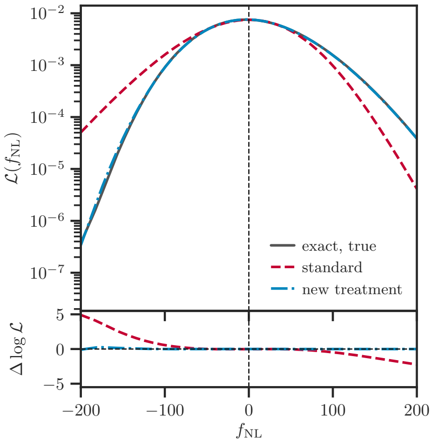
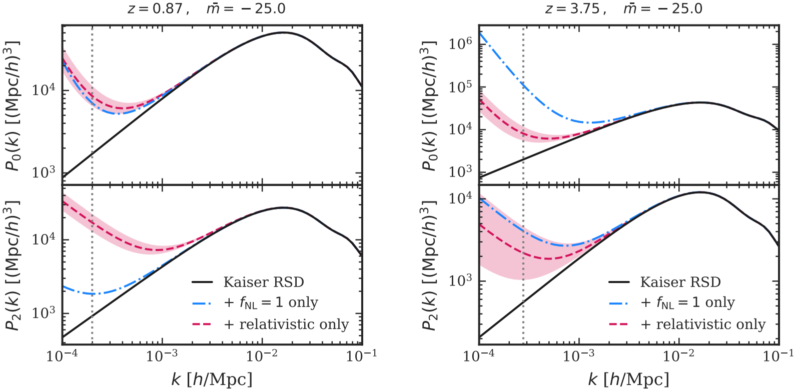
Lessons

- Range of validity
- First principles



Impact of Relativistic Corrections for Local PNG



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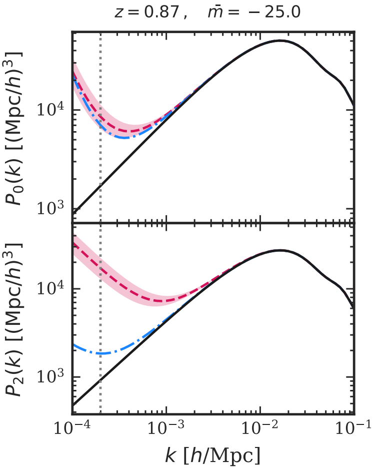
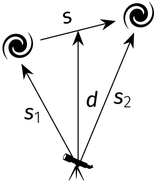


Hybrid-Basis Fourier Analysis



$$R(\mathbf{q}, \mathbf{k}) = \int \mathrm{d}^3\mathbf{r}\, \mathrm{e}^{\mathrm{i}(\mathbf{q}-\mathbf{k})\cdot\mathbf{r}} \left\{ (\hat{\mathbf{q}} \cdot \hat{\mathbf{r}})^2 - \mathrm{i} \left[2 + \frac{\partial \ln \bar{n}(\mathbf{r}, z)}{\partial \ln r} \right] \frac{\hat{\mathbf{q}} \cdot \hat{\mathbf{r}}}{qr} \right\}$$

$$M_{\mu\nu} = \int \mathrm{d}^2\hat{\mathbf{r}}\, Y^*_{\ell_\mu m_\mu}(\hat{\mathbf{r}}) M(\hat{\mathbf{r}}) Y_{\ell_\nu m_\nu}(\hat{\mathbf{r}}) \,,$$

$$\Phi_{\mu\nu} = \kappa_{\ell_\nu n_\nu} \int \mathrm{d}r\, r^2 w(\breve{r}) j_{\ell_\mu}(k_{\ell_\mu n_\mu}\breve{r}) j_{\ell_\nu}(k_{\ell_\nu n_\nu} r) G(z(r), k_{\ell_\nu n_\nu}) \boldsymbol{\phi}(r) \,,$$

$$\Upsilon_{\mu\nu} = \frac{\kappa_{\ell_\nu n_\nu}}{k_{\ell_\nu n_\nu}} \int \mathrm{d}r\, r^2 \frac{\mathrm{d}}{\mathrm{d}\breve{r}} \left[w(\breve{r}) j_{\ell_\mu}(k_{\ell_\mu n_\mu}\breve{r}) \right] j'_{\ell_\nu}(k_{\ell_\nu n_\nu} r) \boldsymbol{\gamma}(z(r)) F(z(r)) \boldsymbol{\phi}(r) \,,$$