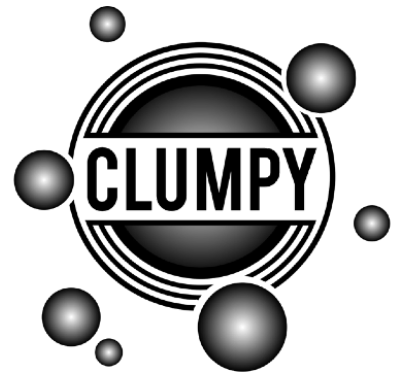


Dark matter modeling of astrophysical targets

From dark clumps to galaxy clusters and the extragalactic signal

Céline Combet, LPSC Grenoble
Moritz Hütten, MPP Munich

After this lecture, you will (hopefully)

1. know the basics of dark matter halo modeling
2. know what are the targets for DM indirect detection and how to model their DM content, based on 1. but also on their specific properties
3. become aware of the limitations and uncertainties linked to the DM modeling of these targets
4. feel like exploring how things can be improved!
5. have all you need for the  tutorial/hands on this afternoon

Outline

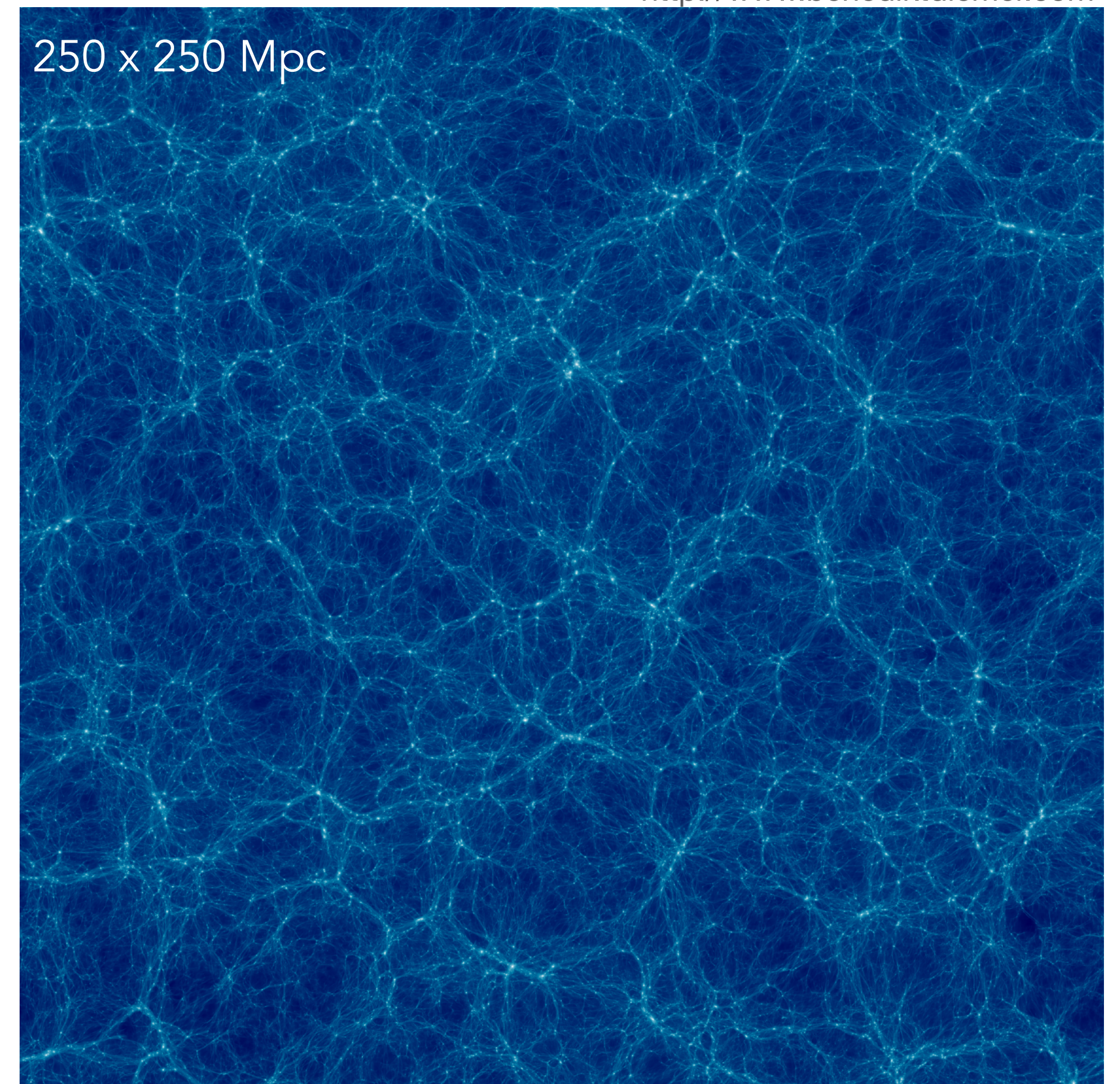
1. Reminder: structure formation
2. Dark matter halos: basic modeling
3. The gamma-ray signal from DM annihilation/decay
 - Derivation of the exotic signal
 - General considerations to compute the "astrophysical" factor
4. DM modeling and J-factor estimation of astrophysical targets
 - Galactic targets
 - Extragalactic scale

Reminder: structure formation

Structure formation in Λ CDM

- structures in the universe form from fluctuations in the primordial density field
- structure growth depends on the underlying cosmology (expansion vs gravity)
- haloes form after the collapse of "high density" regions
- today, the universe is highly structured (voids, filaments and haloes)

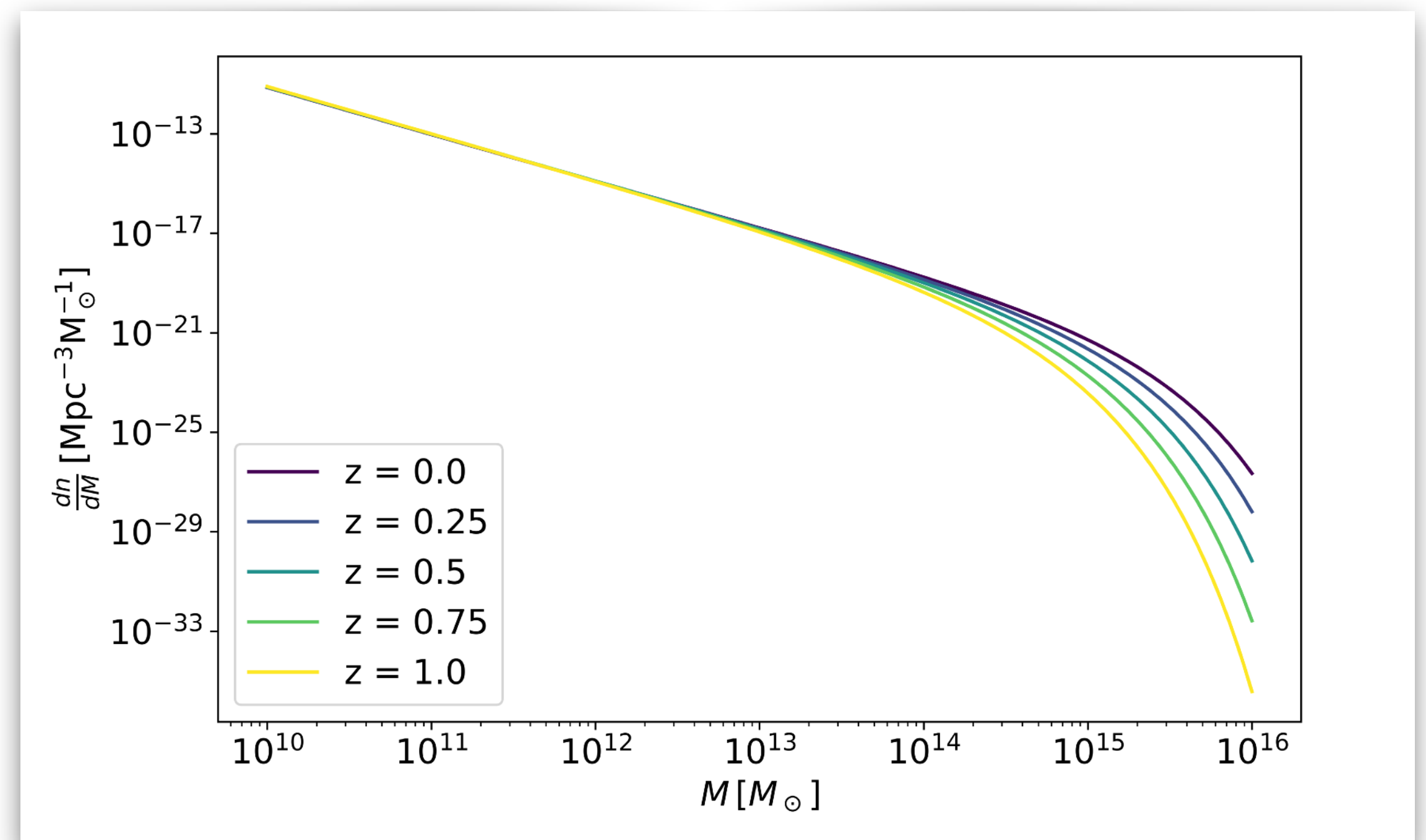
<http://www.benediktdiemer.com>



Structure formation in Λ CDM

The halo mass function

- Number density of halos as a function of mass and redshift; depends on the underlying cosmology
- General properties
 - The lower the mass, the more numerous the haloes
 - Forming high mass haloes takes time. Less massive haloes at high redshift than at low redshift
- First described by Press & Schechter (1974), later (semi-)analytical refinements (Sheth & Tormen 1999). Now generally fitted using numerical simulations (DM-only or hydro)

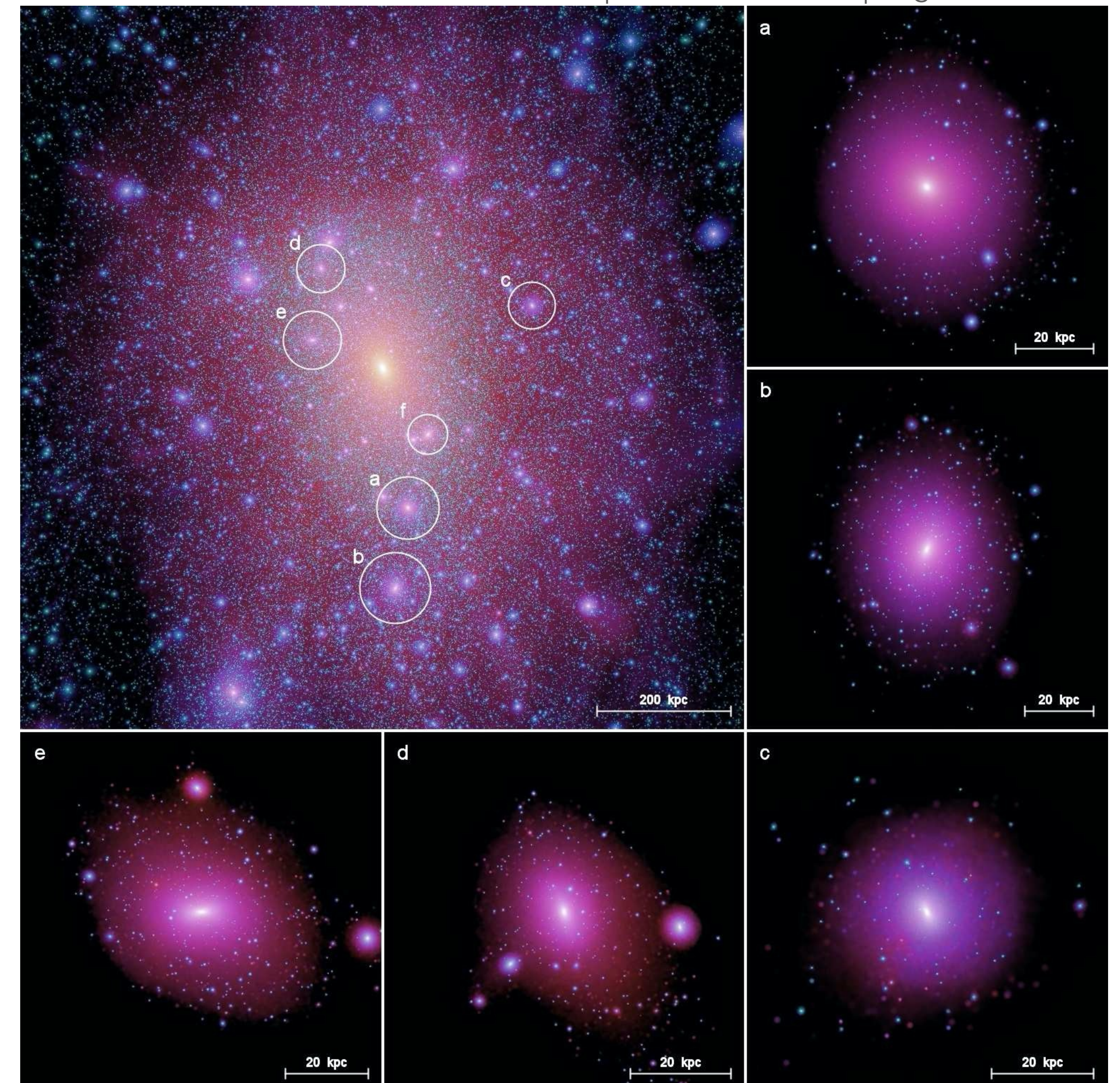


Structure formation in Λ CDM

Substructures, halo mass range

- bottom-up structure formation: smallest haloes form first and merge into larger structures
- substructures: haloes in haloes in haloes...
- halo mass range:
 - * Theoretical $M_{\min} \sim 10^{-12} - 10^{-6} M_{\text{sun}}$. Depends on the free-streaming scale
 - * [Simulated M_{\min} depends on simulation resolution $>$ theoretical M_{\min}]
 - * $M_{\max} \sim 10^{15} M_{\text{sun}}$ (galaxy clusters)

Galactic halo from the Aquarius Simulation, Springel et al (2008)

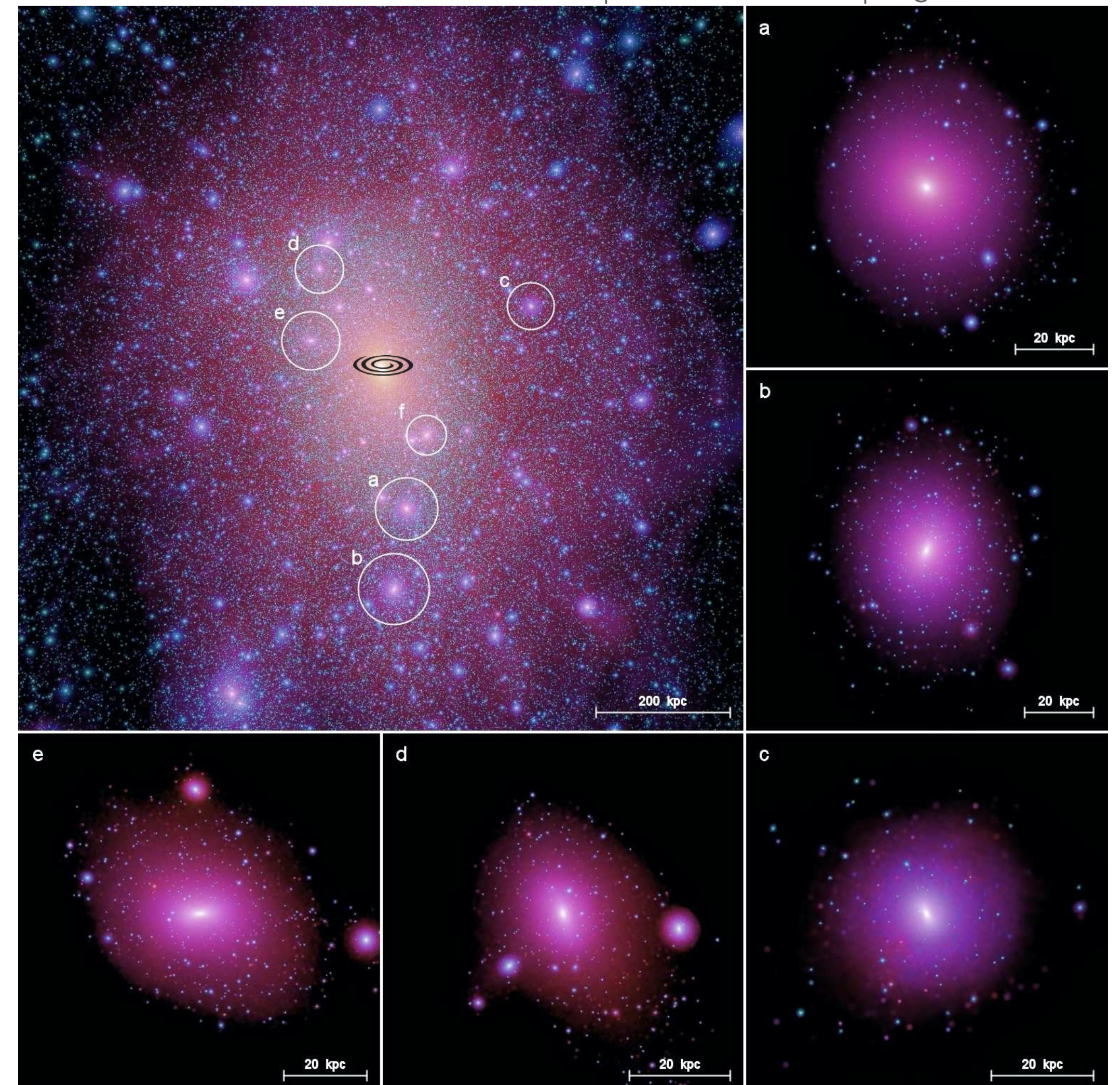


Structure formation in Λ CDM

Locations of DM haloes?

- Gas accumulates in massive haloes. If dense enough, trigger star formation.
- Locations of DM haloes we know about: galaxy clusters, galaxies...
- Plenty of low mass DM haloes that we don't "see"!
- DM haloes extend much beyond the visible part

Galactic halo from the Aquarius Simulation, Springel et al (2008)



Dark matter halo: basic modeling

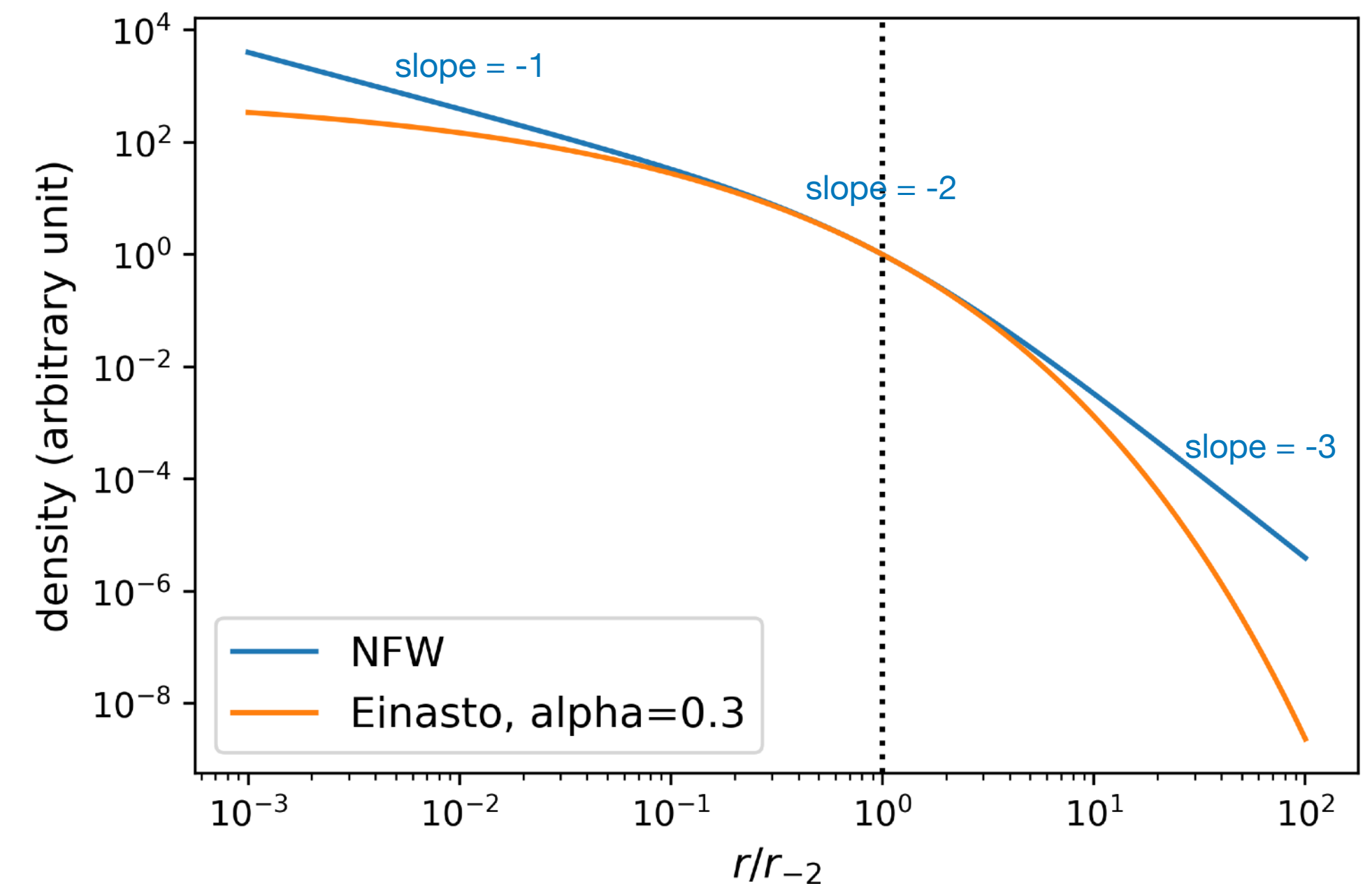
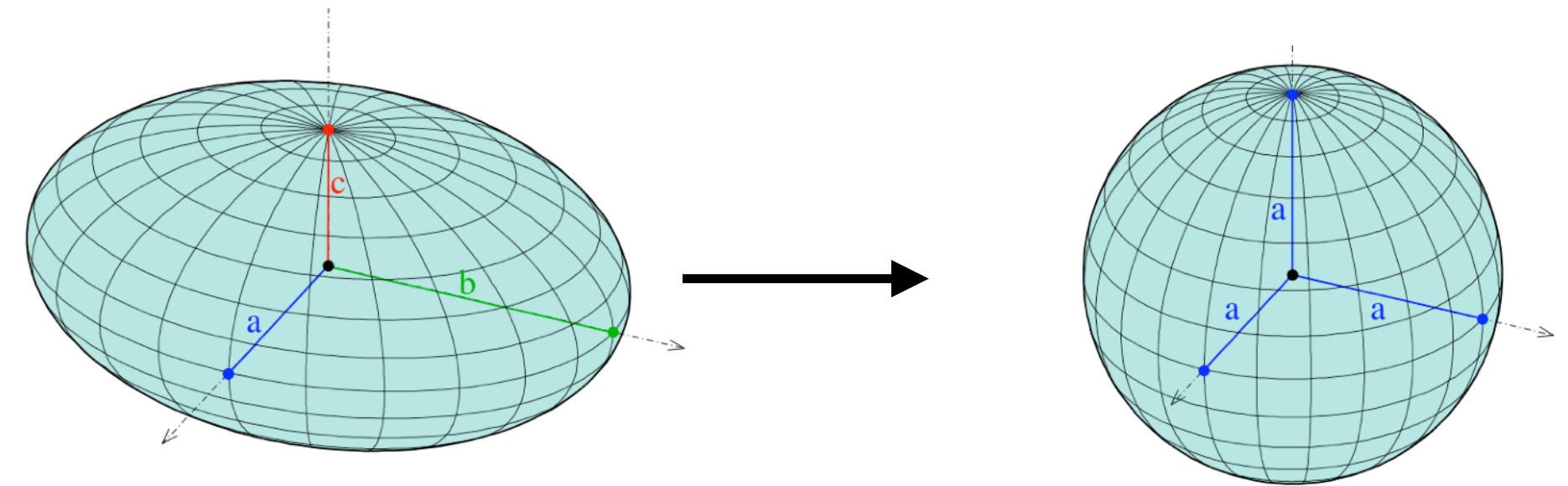
DM halo properties

General properties from DM-only simulations

- Haloes are triaxial, preferentially prolate
- In practice, generally measure spherical density profile
- Two main parametrisations are used to describe radial density profile

$$\rho(r) = \frac{\rho_s}{\left(\frac{r}{r_s}\right) \left(1 + \frac{r}{r_s}\right)^2} \quad \text{NFW (Navarro, Frenk \& White (1997))}$$

$$\rho(r) = \rho_{-2} \exp \left\{ -\frac{2}{\alpha} \left[\left(\frac{r}{r_{-2}}\right)^\alpha - 1 \right] \right\} \quad \text{Einasto (1965, stellar systems)}$$



DM halo properties

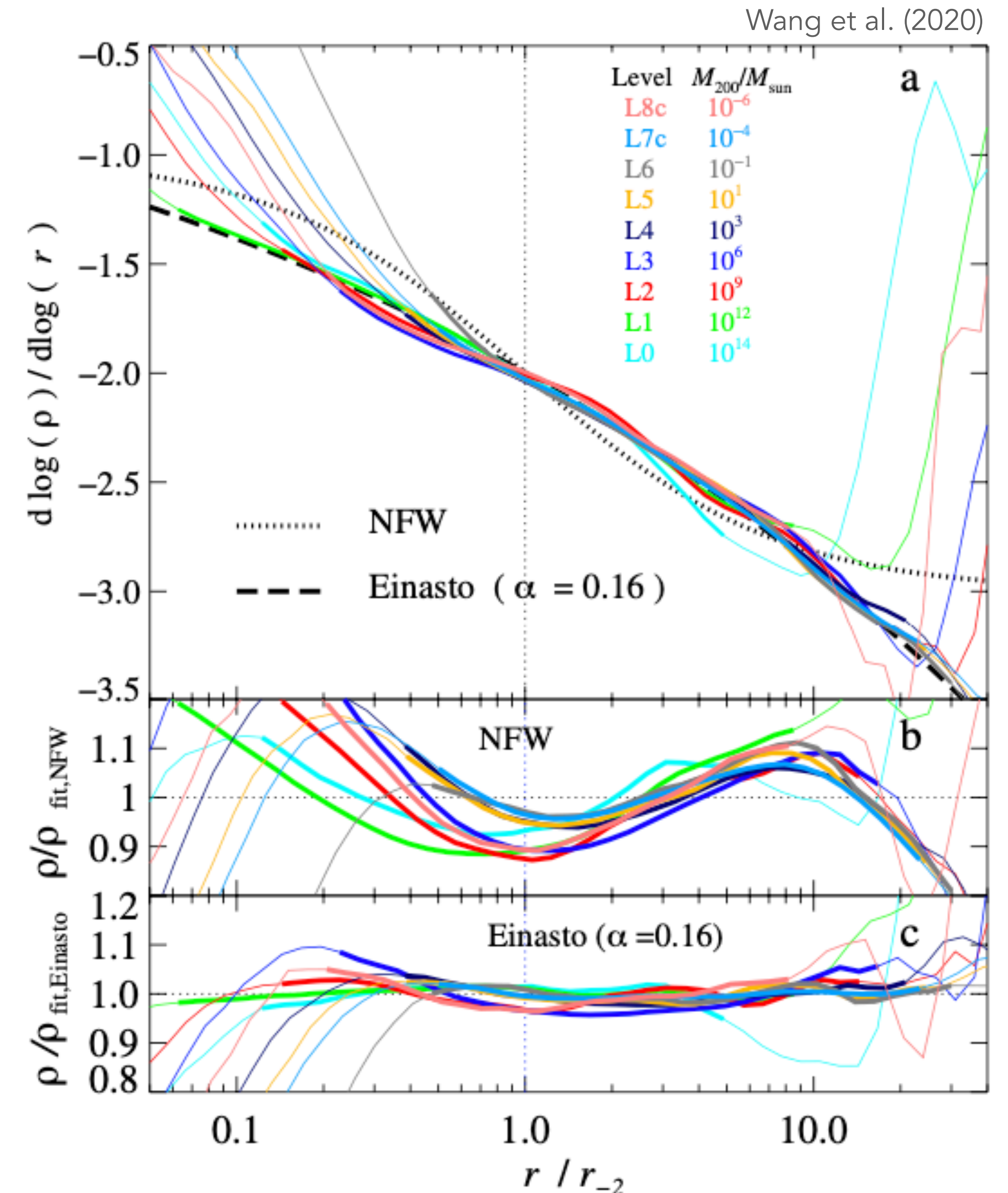
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DM halo properties

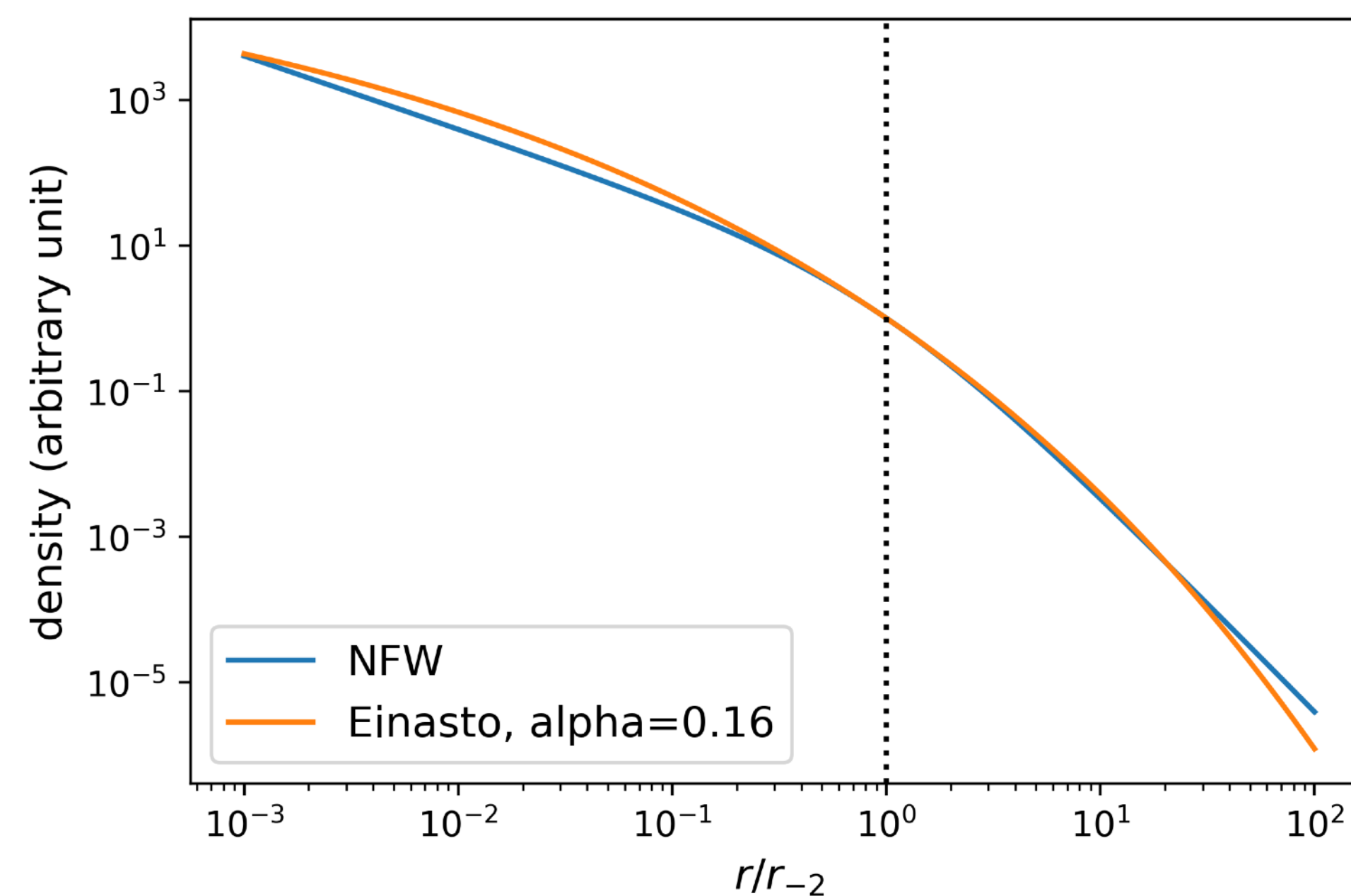
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DM halo properties

Mass, radius and concentration

- The spherical overdensity mass/radius definition

$$M_{\Delta, \text{bkg}} \equiv \frac{4}{3} \pi R_{\Delta}^3 \times \Delta \times \rho_{\text{bkg}}(z) \quad \rho_{\text{bkg}}(z) = \begin{cases} \rho_{\text{crit}}(z) = \frac{3H(z)^2}{8\pi G} \\ \rho_m(z) = \Omega_m(z) \rho_{\text{crit}}(z) \end{cases}$$

$M_{200,c}$, $M_{200,m}$, $M_{500,c}$, etc .

$\Delta = 200$ often used as "halo size"

$M_{200,c} = 10^{-6} M_{\text{sun}} \sim 1 M_{\text{earth}}$, $R_{200,c} \sim 65000 \text{ AU}$

$M_{200,c} = 10^{15} M_{\text{sun}}$, $R_{200,c} \sim 2 \text{ Mpc}$

- Which is the largest: $R_{200,c}$ or $R_{500,c}$?

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- The concentration parameter

$$c_{\Delta} = \frac{R_{\Delta}}{r_{-2}} \quad (\text{for NFW, } r_{-2} = r_s)$$

- Which is the largest: $R_{200,c}$ or $R_{500,c}$?
- Profile has 2 free parameters (ρ_{-2}, r_{-2}). Show that it can be equivalently determined for (M_{Δ}, c_{Δ})

DM halo properties

Mass, radius and concentration

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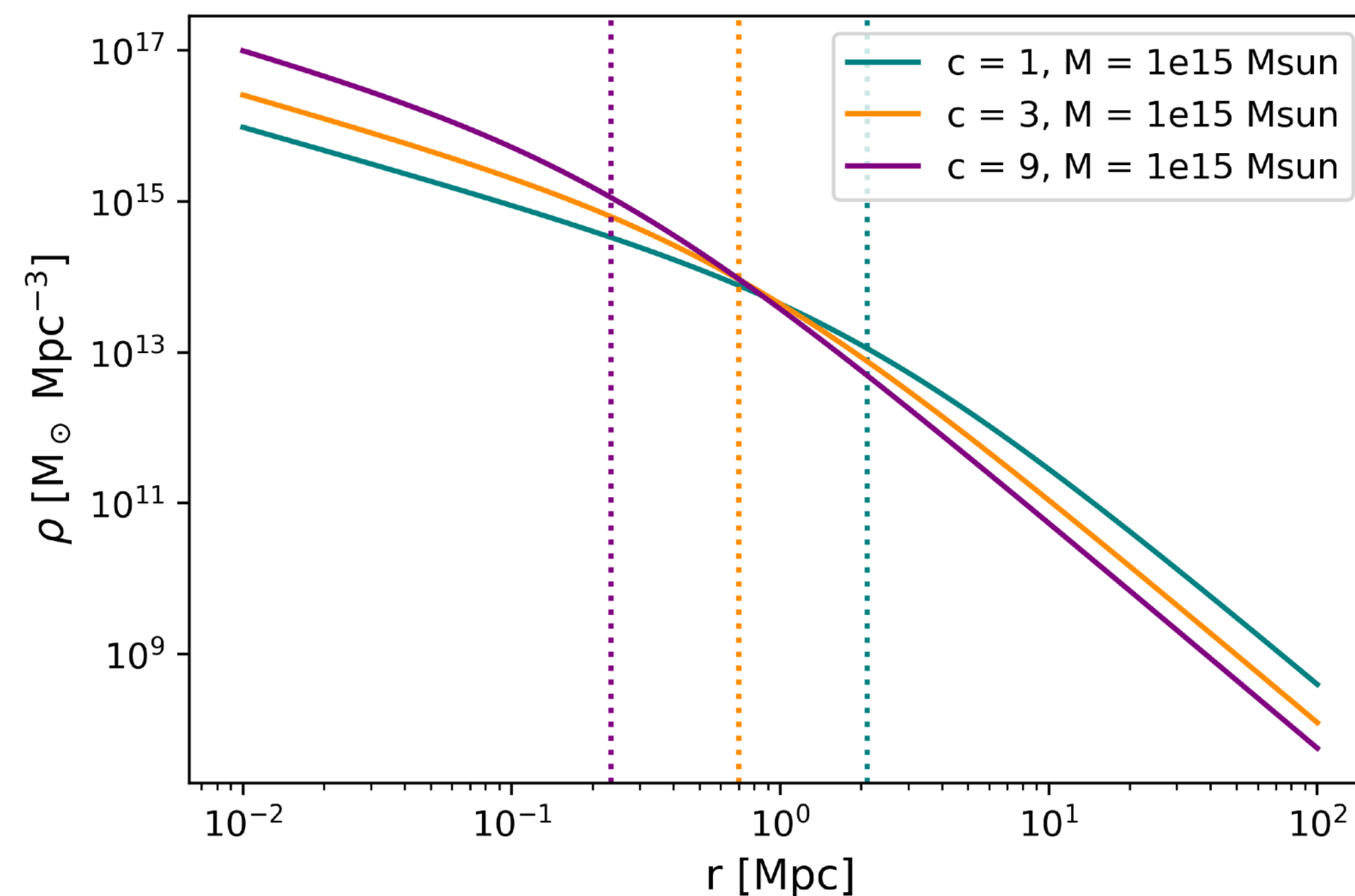
$$c_{\Delta} = \frac{R_{\Delta}}{r_{-2}} \quad (\text{for NFW, } r_{-2} = r_s)$$

At a given mass, the higher the concentration the denser the inner regions of the halo

- Which is the largest: $R_{200,c}$ or $R_{500,c}$?
- Profile has 2 free parameters (ρ_{-2}, r_{-2}). Show that it can be equivalently determined for (M_{Δ}, c_{Δ})

$$1. M_{\Delta} \rightarrow R_{\Delta} \rightarrow r_{-2} = \frac{R_{\Delta}}{c_{\Delta}}$$

$$2. M_{\Delta} = \int_0^{R_{\Delta}} 4\pi r^2 \rho(r) dr \rightarrow \rho_{-2}$$



DM halo properties

Mass, radius and concentration

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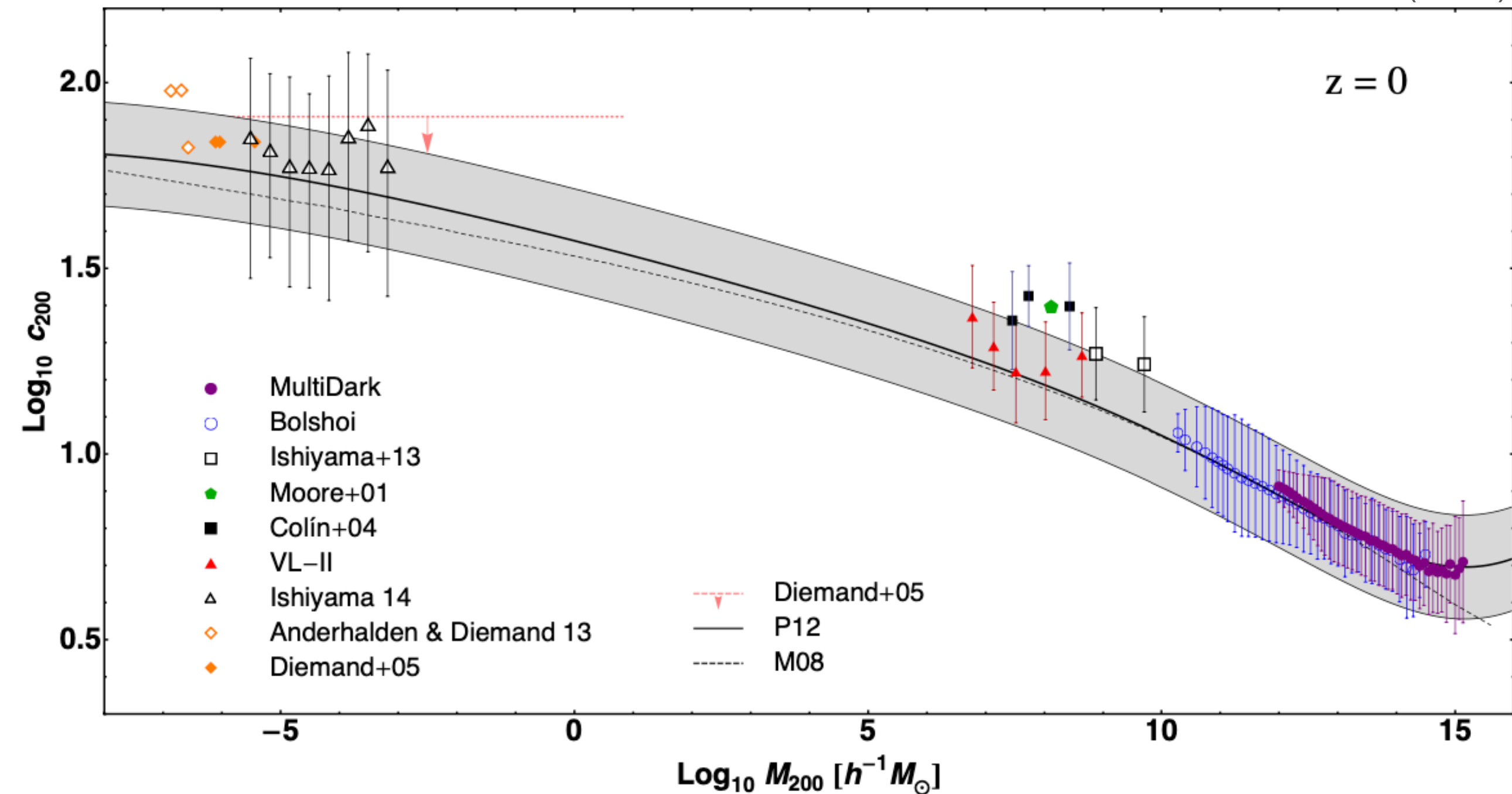
- The concentration parameter

$$c_{\Delta} = \frac{R_{\Delta}}{r_{-2}} \quad (\text{for NFW, } r_{-2} = r_s)$$

Concentration depends on mass and redshift:

- Low mass haloes are more concentrated than high mass counterparts
- Concentration decreases with increasing redshift as $\sim (1+z)^{-1}$

Sanchez-Conde et al. (2014)



DM halo properties

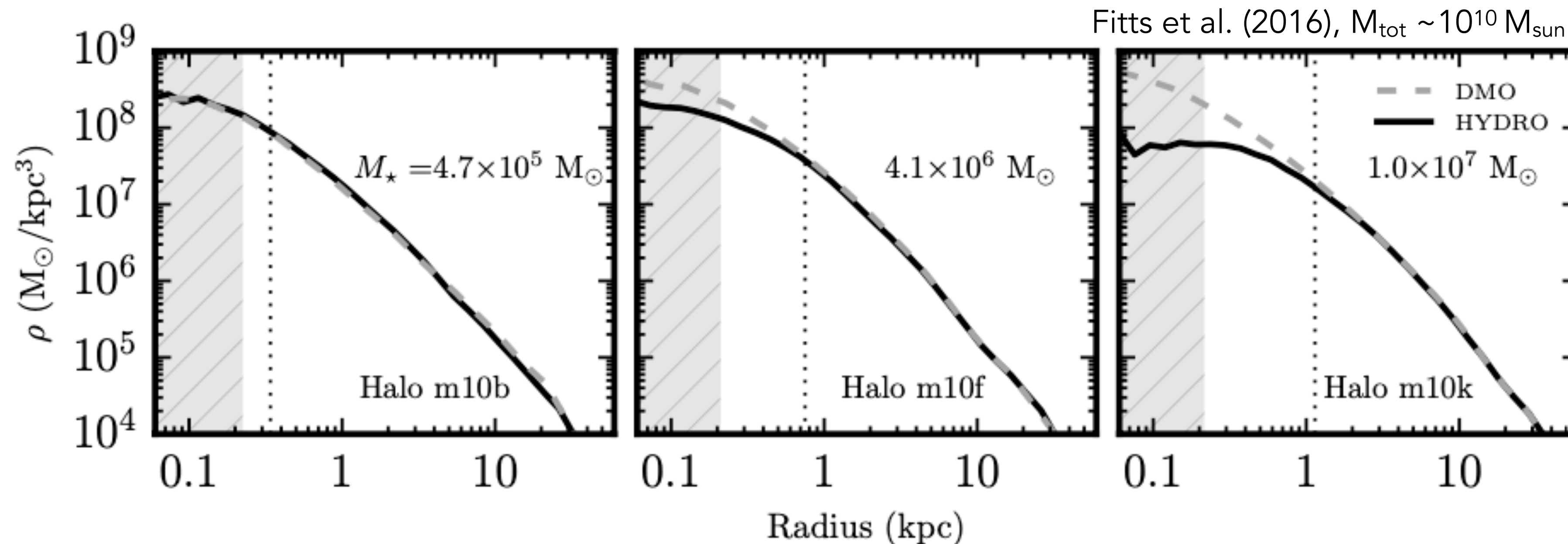
Cusps vs. cores

- DM-only simulations → DM halos are cuspy (steep inner slope, e.g. $\gamma = 1$ for NFW)
- Adiabatic contraction in the presence of baryon condensation/central BH can make the halo cuspier
- Observations of galaxies and dwarf galaxies suggests that the density can be 'cored' ($\gamma \rightarrow 0$). **Cusp vs. Core problem**
 - Baryonic feedback may produce core density profiles
 - Self-Interacting DM

Allow for more general parametrisation

- free α in the Einasto profile (not fixed to 0.17-0.16)
- Zhao parametrisation (= NFW for $(\alpha, \beta, \gamma) = (1, 3, 1)$)

$$\rho(r) = \frac{\rho_s}{(r/r_s)^\gamma [1 + (r/r_s)^\alpha]^{(\beta-\gamma)/\alpha}}$$



The γ -ray signal from DM annihilation (or decay)

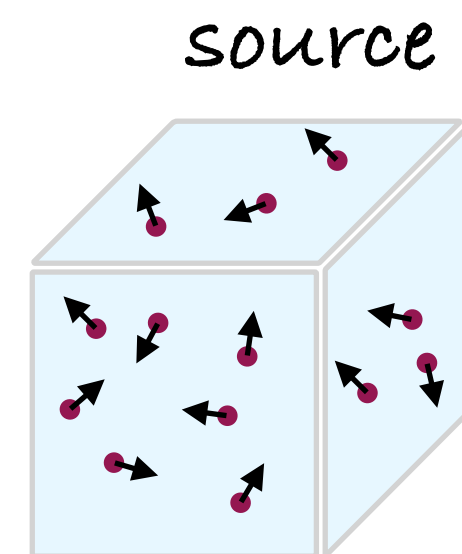
1. Derivation, definition of the J-factor
2. General considerations on estimating the J-factor

The exotic DM signal

The source emission

Consider a volume V containing N dark matter particles

- If Majorana DM, $\chi = \bar{\chi}$
- If not, $N/2 \chi$ and $N/2 \bar{\chi}$



1. For a pair of DM particles, express the annihilation probability in a time dt , given the annihilation cross section $\sigma_{\text{ann}}(v)$ and their relative velocity v

$$dp_{1\text{pair}} = \frac{dV}{V} = \frac{\sigma_{\text{ann}} v dt}{V}$$

2. Express the total annihilation rate ($\#/s$), in the volume V , considering the N particles

3. Transform the above per unit volume, as a function of the mass of the particle m_χ and mass density ρ

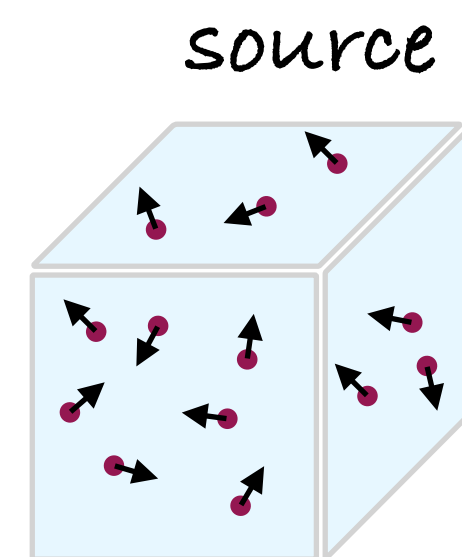
4. Given the differential photon spectrum dN/dE , find the source photon emission rate, per unit of volume

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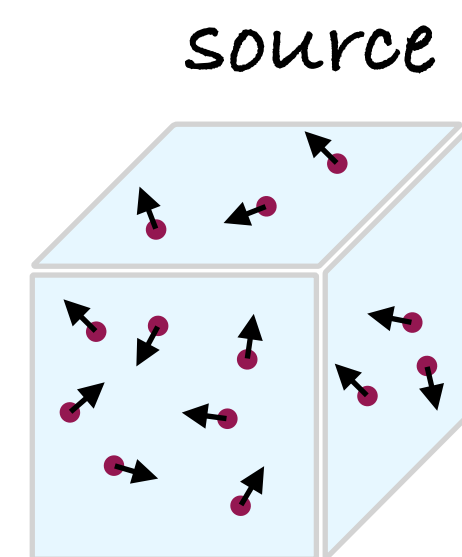
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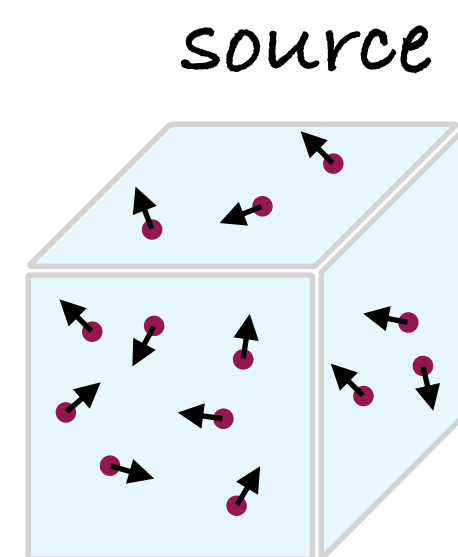
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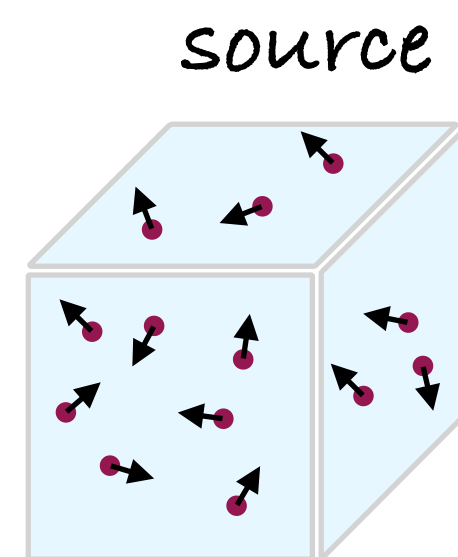
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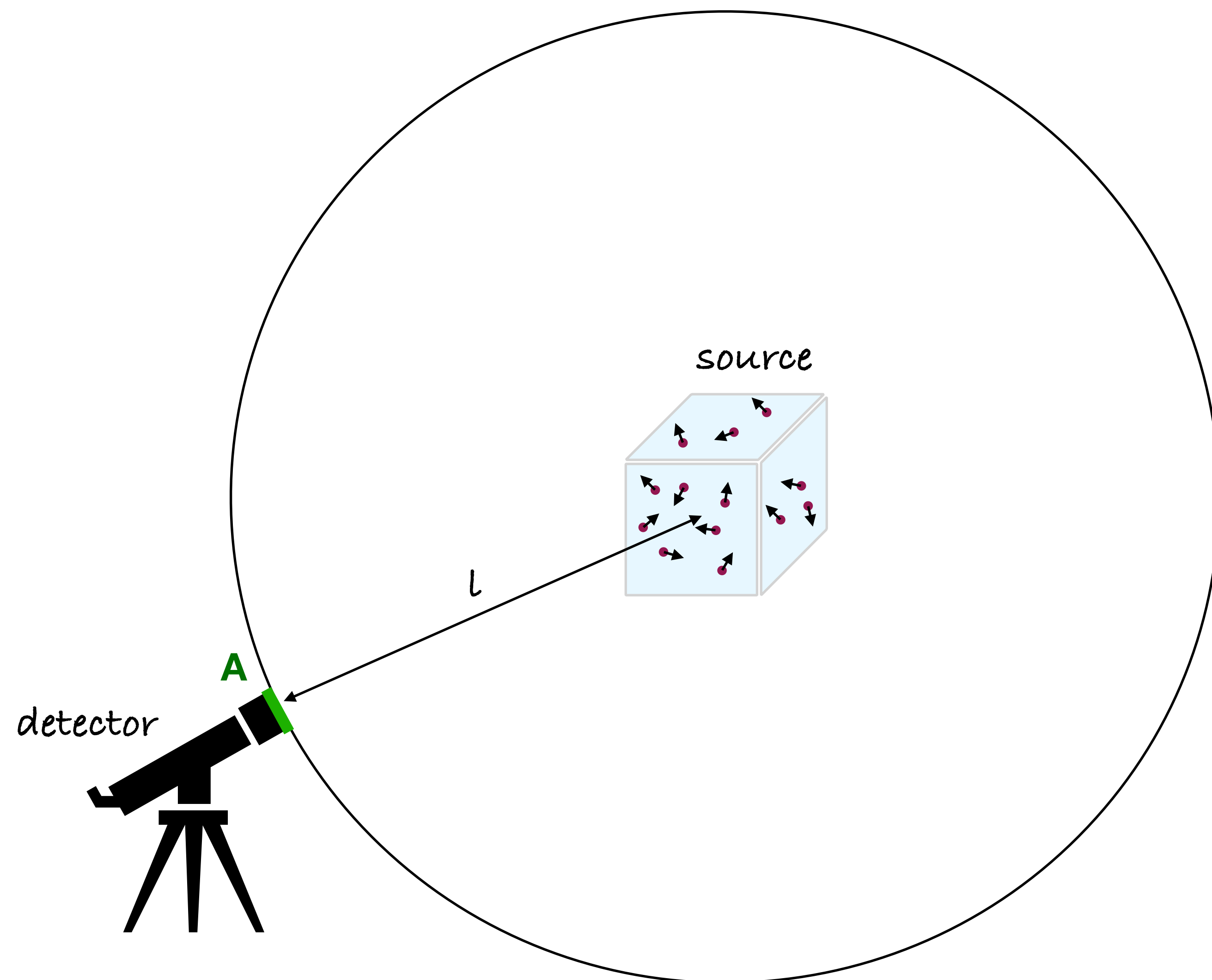
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4. Given the differential photon spectrum dN/dE , find the source photon emission rate, per unit of volume

$$\frac{d\Gamma_{\text{src}}}{dV dE} = \frac{d\Gamma_{\text{tot}}}{dV} \times \frac{dN_\gamma}{dE}$$

The exotic DM signal

The received flux

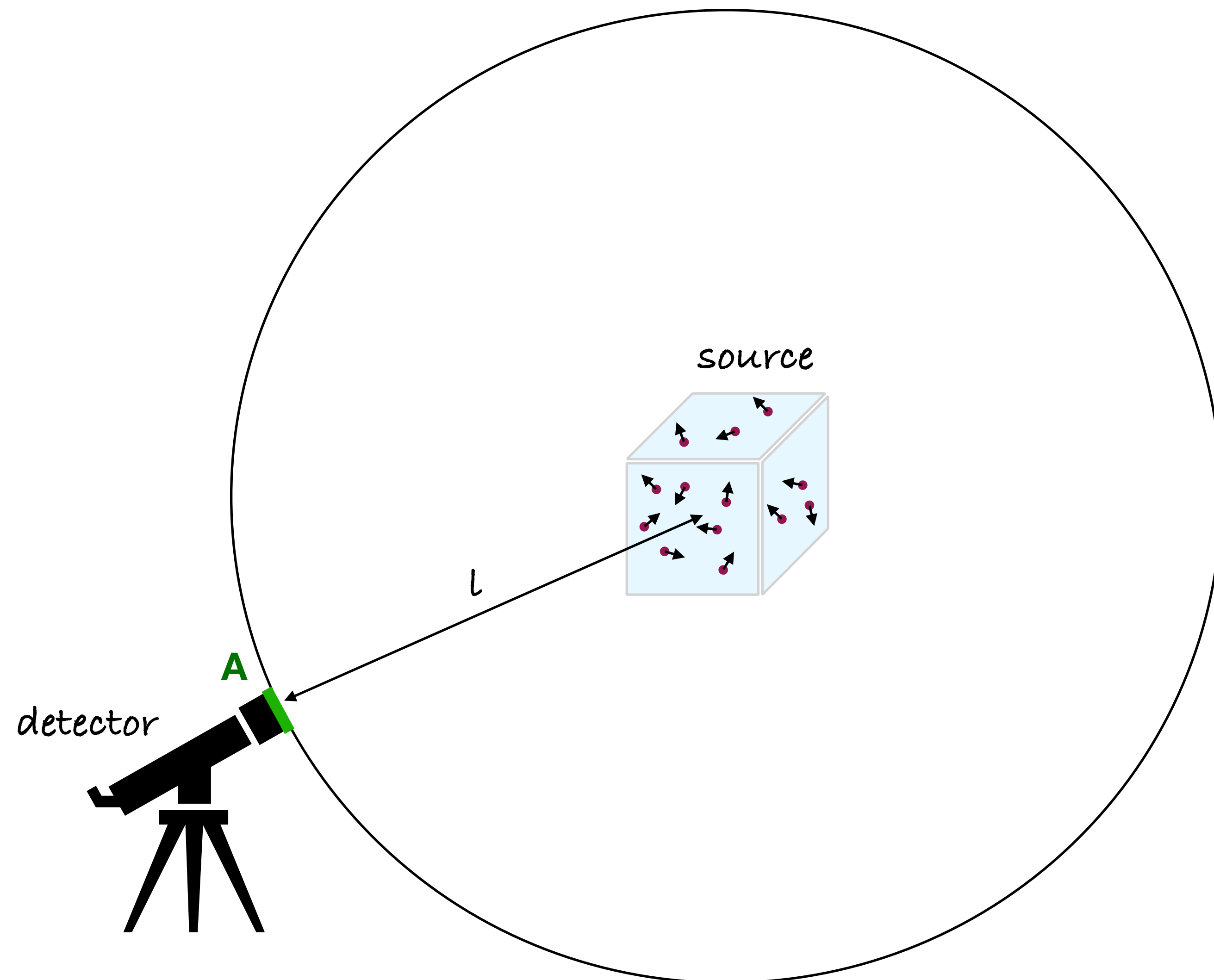


1. Differential photon rate received in the detector?

$$\frac{d\Gamma_{\text{det}}}{dVdE} =$$

The exotic DM signal

The received flux

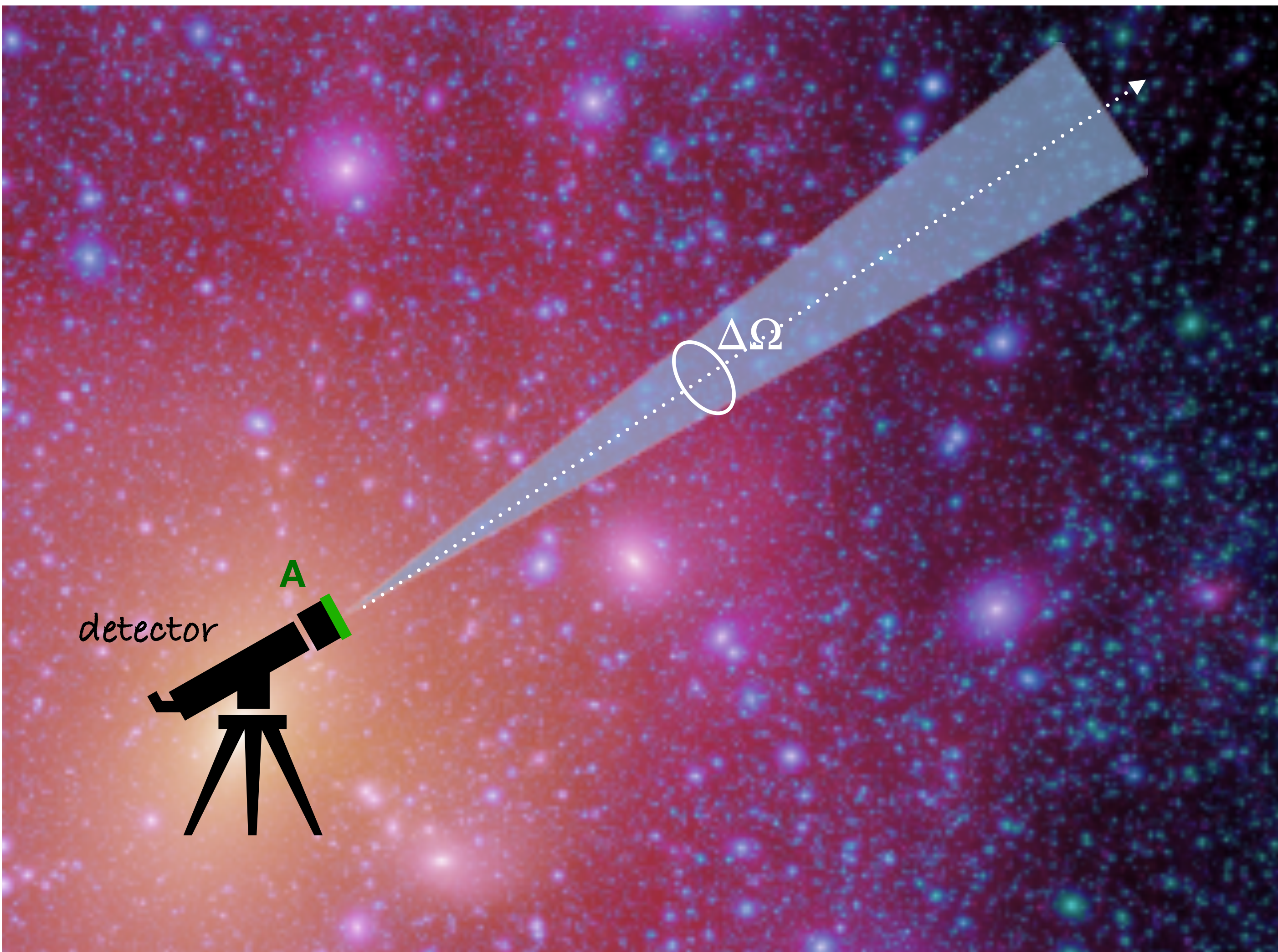


1. Differential photon rate received in the detector?

$$\frac{d\Gamma_{\text{det}}}{dVdE} = \frac{A}{4\pi l^2} \frac{d\Gamma_{\text{src}}}{dVdE}$$

The exotic DM signal

The received flux



1. Differential photon rate received in the detector?

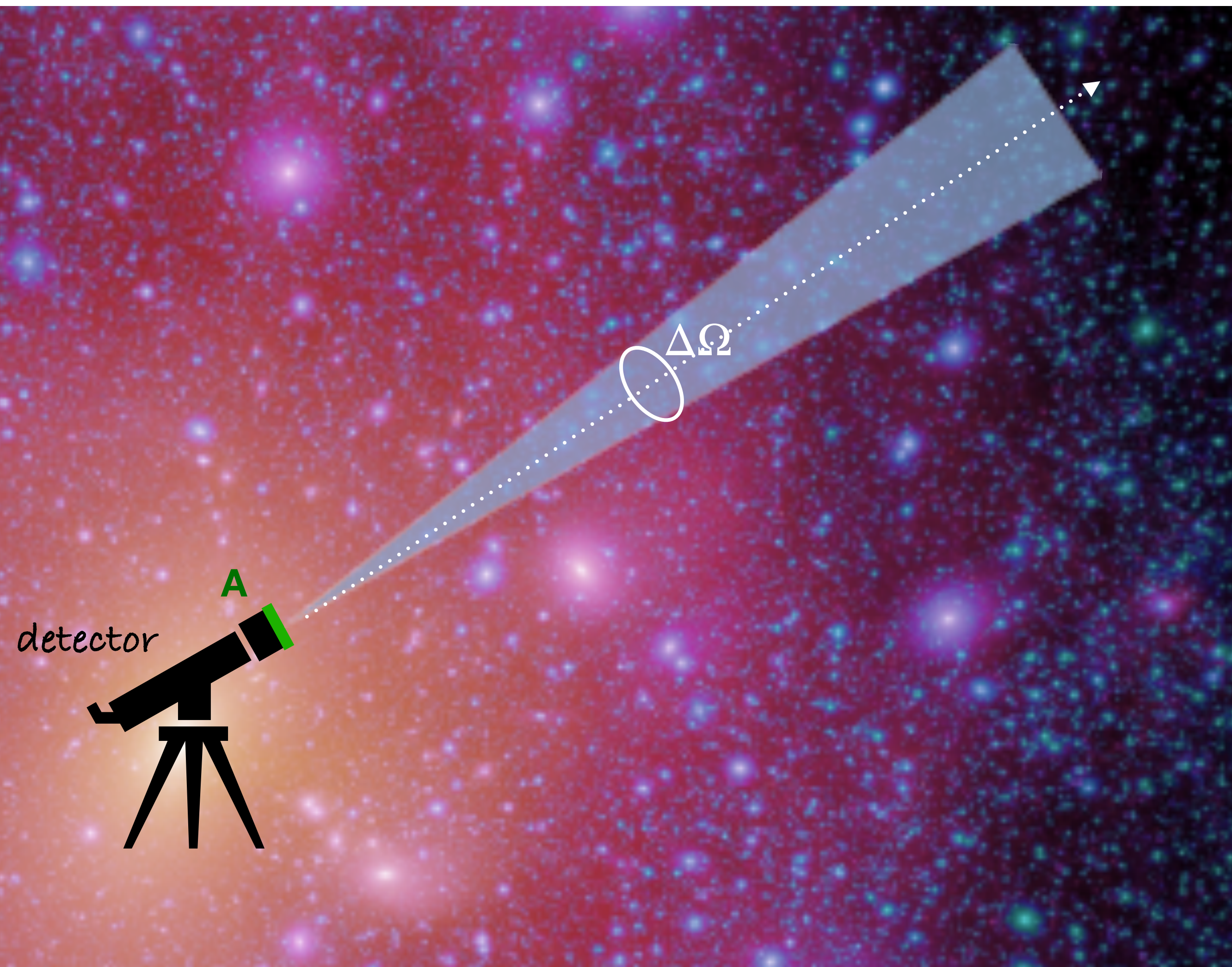
$$\frac{d\Gamma_{\text{det}}}{dVdE} = \frac{A}{4\pi l^2} \frac{d\Gamma_{\text{src}}}{dVdE}$$

2. What is the differential flux received, integrating over the entire observed volume (with $\sigma v = \text{cst}$)?

$$\frac{d\phi_{\text{det}}}{dE} = \int_V \frac{1}{A} \frac{d\Gamma_{\text{det}}}{dVdE} dV = \dots$$

The exotic DM signal

The received flux



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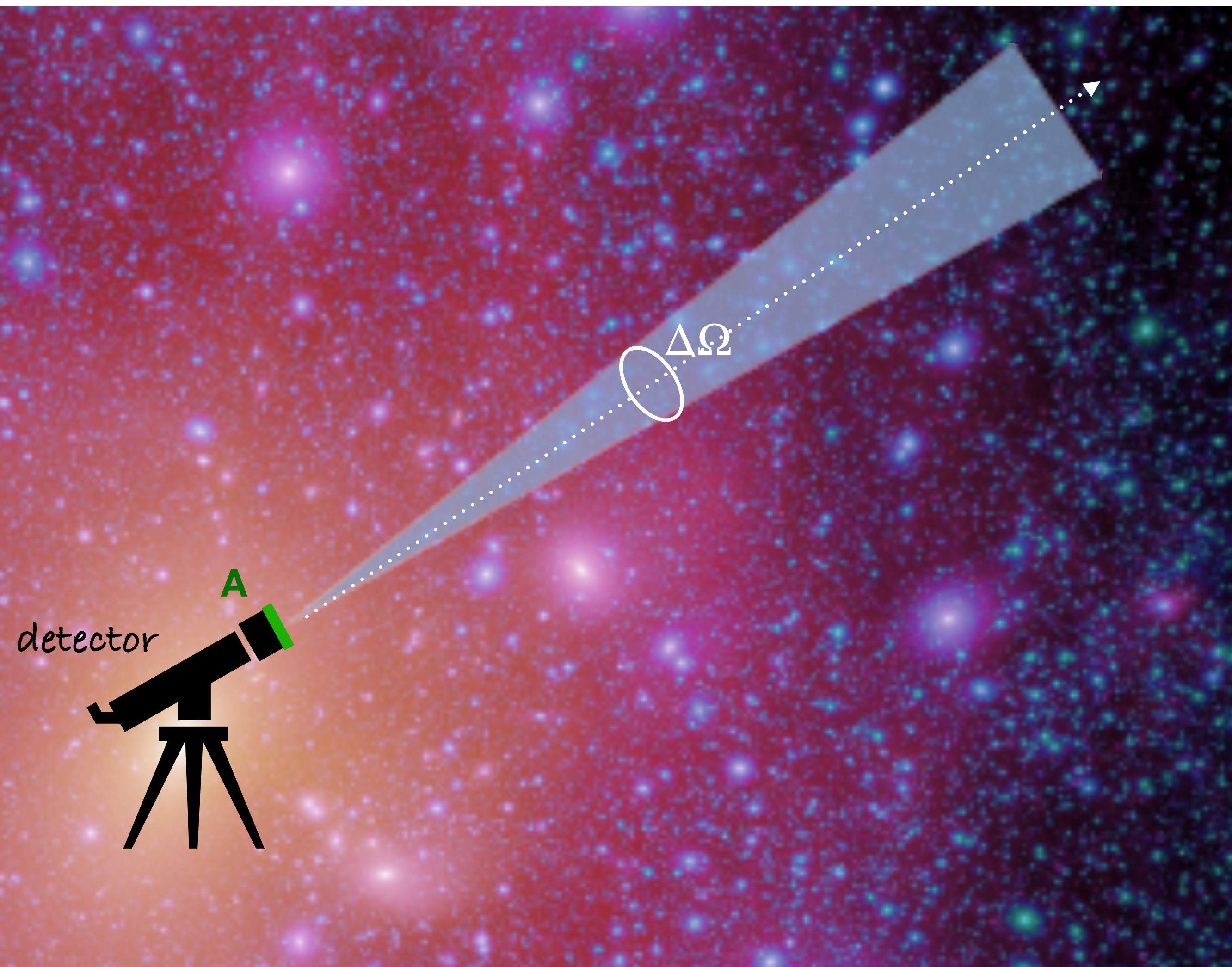
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using $dV = l^2 \sin \alpha dl d\alpha d\beta = l^2 dl d\Omega$

NB: if $\sigma v \neq \text{cst}$, cannot factor it out (e.g. Boddy et al. 2020)

The exotic DM signal

The received flux



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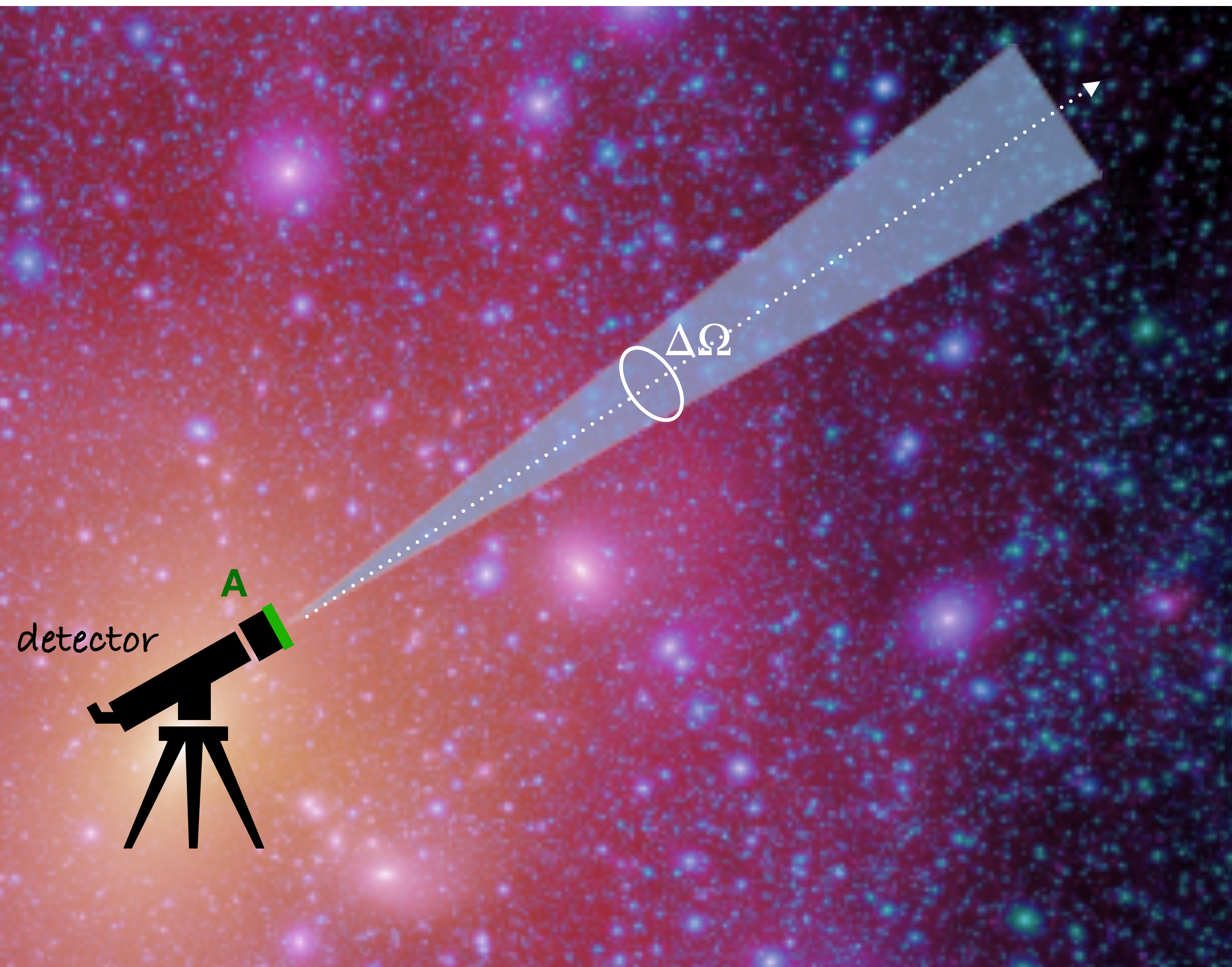
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using $dV = l^2 \sin \alpha dl d\alpha d\beta = l^2 dl d\Omega$

What does this become for decaying DM (single particle process) ?

The exotic DM signal

The received flux



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using $dV = l^2 \sin \alpha dl d\alpha d\beta = l^2 dl d\Omega$

Decaying DM = single particle process

- $\delta = 1$
- $\langle \sigma_{\text{ann}} v \rangle \rightarrow 1/\tau_\chi$, $\tau_\chi = \text{lifetime}$
- $\rho^2, m_\chi^2 \rightarrow \rho, m_\chi$

The exotic annihilation DM signal

Recap

$$\frac{d\Phi}{dE}(E, \vec{k}, \Delta\Omega(\alpha_{\text{int}})) = \underbrace{\frac{\langle\sigma v\rangle}{4\pi \delta m_{\text{DM}}^2} \sum_f \frac{dN^f}{dE} B_f}_{\text{particle physics}} \times \underbrace{\int_0^{2\pi} d\beta \int_0^{\alpha_{\text{int}}} \sin\alpha d\alpha \int_0^{l_{\text{max}}} \rho^2(\vec{k} l, \alpha, \beta) dl}_{\text{Astrophysical "J-factor": } [M_{\odot}^2 \text{ kpc}^{-5}] \text{ or } [\text{GeV}^2 \text{ cm}^{-5}]}$$

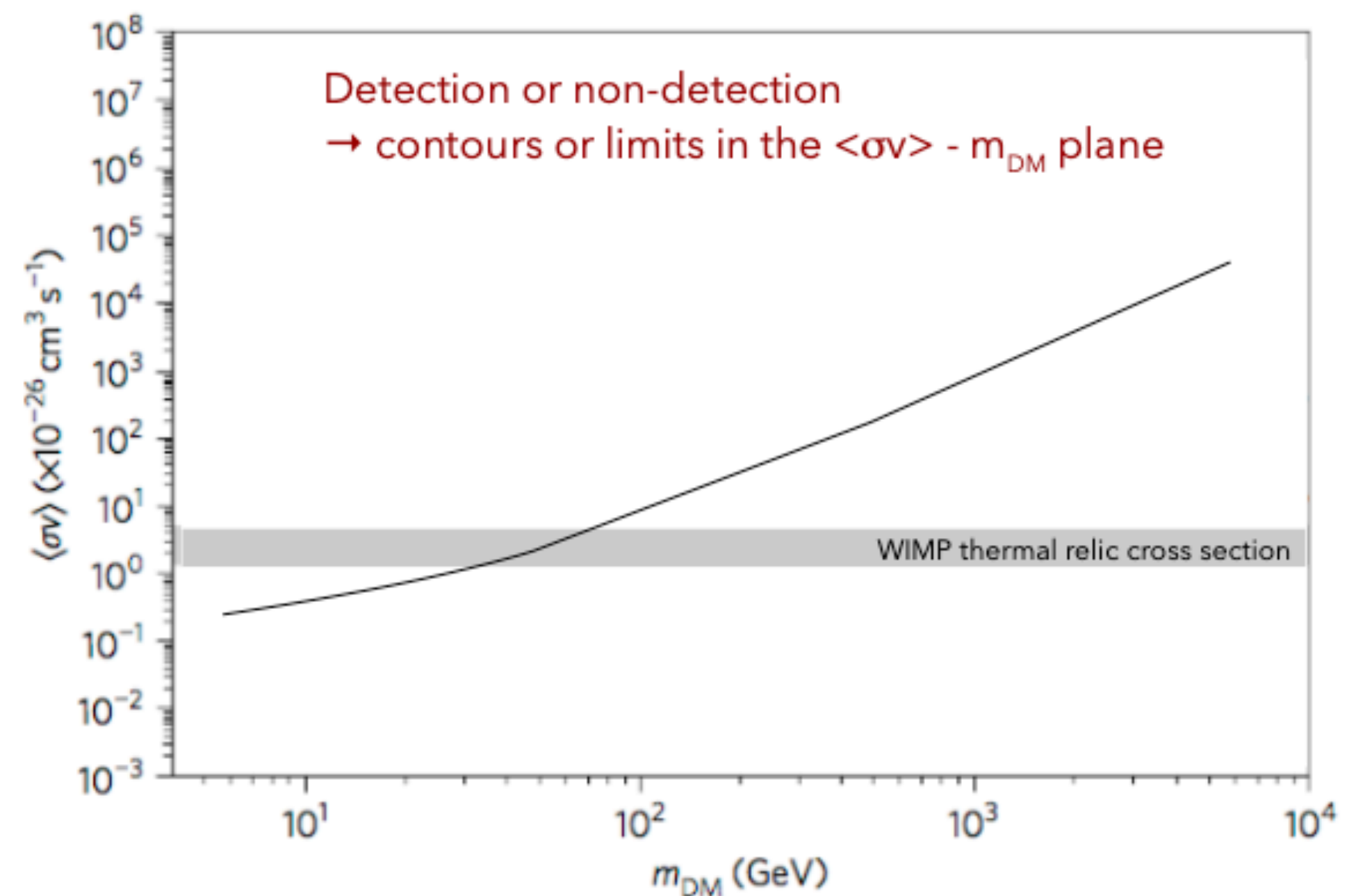
- Predicted flux depends on the DM mass, annihilation cross section, instrument resolution, and DM distribution
- An accurate estimation of the J-factor (i.e. of ρ) is required to place robust limit on the DM candidate properties.
- "Model uncertainties" should always be considered

The exotic annihilation DM signal

Recap

$$\frac{d\Phi}{dE}(E, \vec{k}, \Delta\Omega(\alpha_{\text{int}})) = \underbrace{\frac{\langle\sigma v\rangle}{4\pi \delta m_{\text{DM}}^2} \sum_f \frac{dN^f}{dE} B_f}_{\text{particle physics}} \times \underbrace{\int_0^{2\pi} d\beta \int_0^{\alpha_{\text{int}}} \sin\alpha d\alpha \int_0^{l_{\text{max}}} \rho^2(\vec{k} l, \alpha, \beta) dl}_{\text{Astrophysical "J-factor": } [M_{\odot}^2 \text{ kpc}^{-5}] \text{ or } [\text{GeV}^2 \text{ cm}^{-5}]}$$

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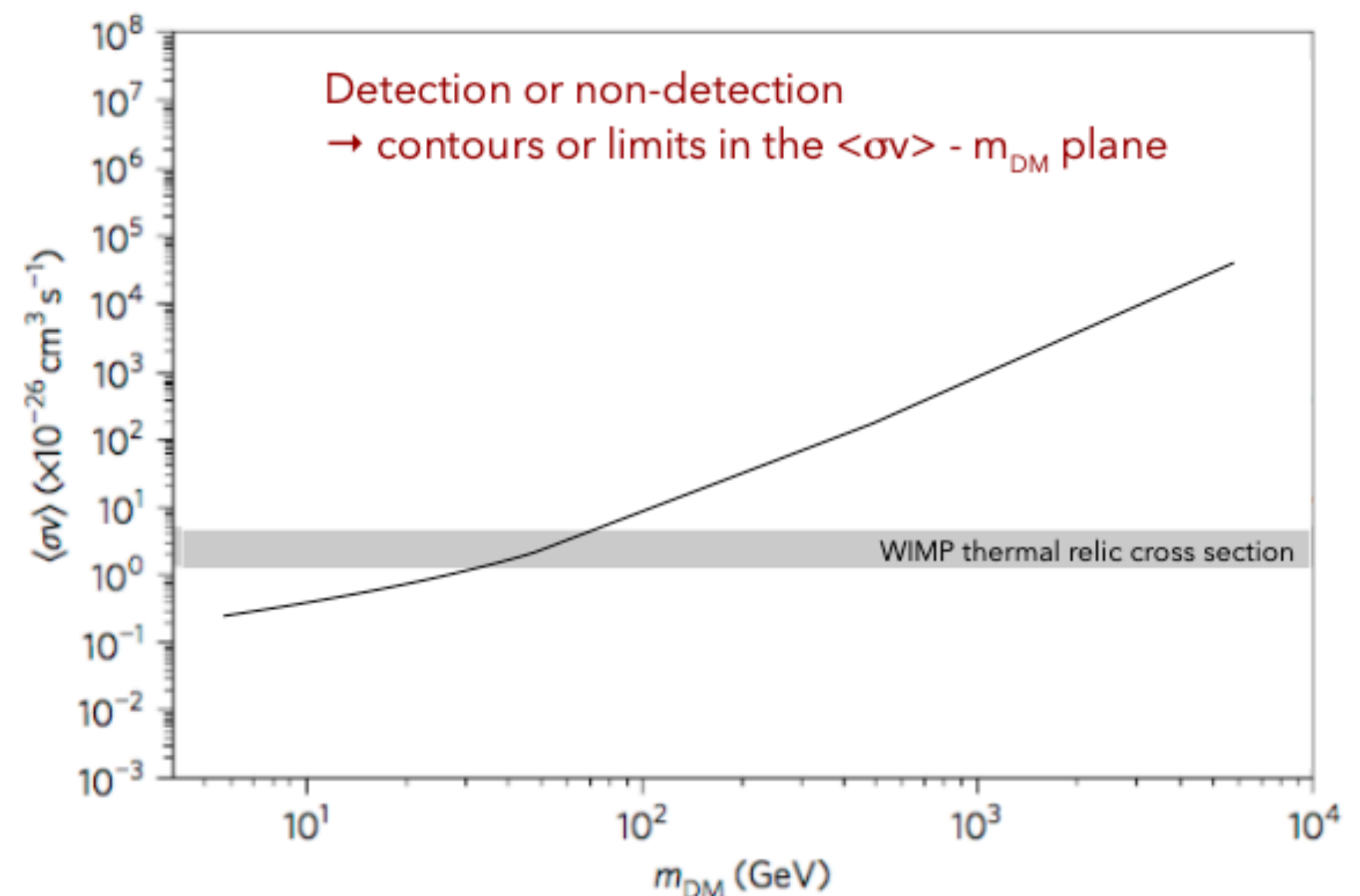
The exotic annihilation DM signal

Recap

$$\frac{d\Phi}{dE}(E, \vec{k}, \Delta\Omega(\alpha_{\text{int}})) = \underbrace{\frac{\langle\sigma v\rangle}{4\pi \delta m_{\text{DM}}^2} \sum_f \frac{dN^f}{dE} B_f}_{\text{particle physics}} \times \underbrace{\int_0^{2\pi} d\beta \int_0^{\alpha_{\text{int}}} \sin\alpha d\alpha \int_0^{l_{\text{max}}} \rho^2(\vec{k} l, \alpha, \beta) dl}_{\text{Astrophysical "J-factor": } [M_{\odot}^2 \text{ kpc}^{-5}] \text{ or } [\text{GeV}^2 \text{ cm}^{-5}]}$$

▲ Source of confusion: some authors have the 4pi in the J-factor

- Predicted flux depends on the DM mass, annihilation cross section, instrument resolution, and DM distribution
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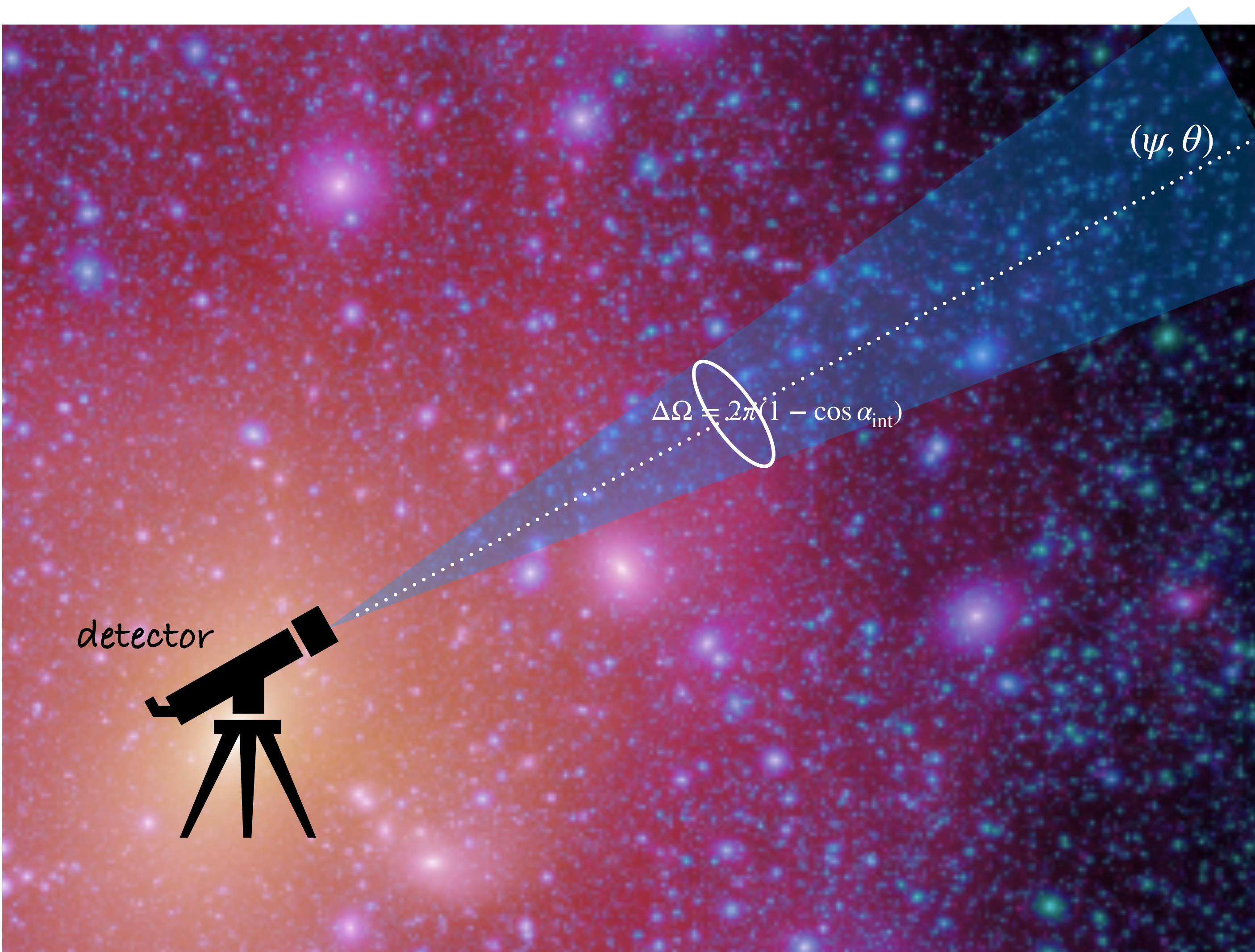


The γ -ray signal from DM annihilation (or decay)

1. Derivation, definition of the J-factor
2. General considerations regarding the J-factor

The J-factor

Global picture



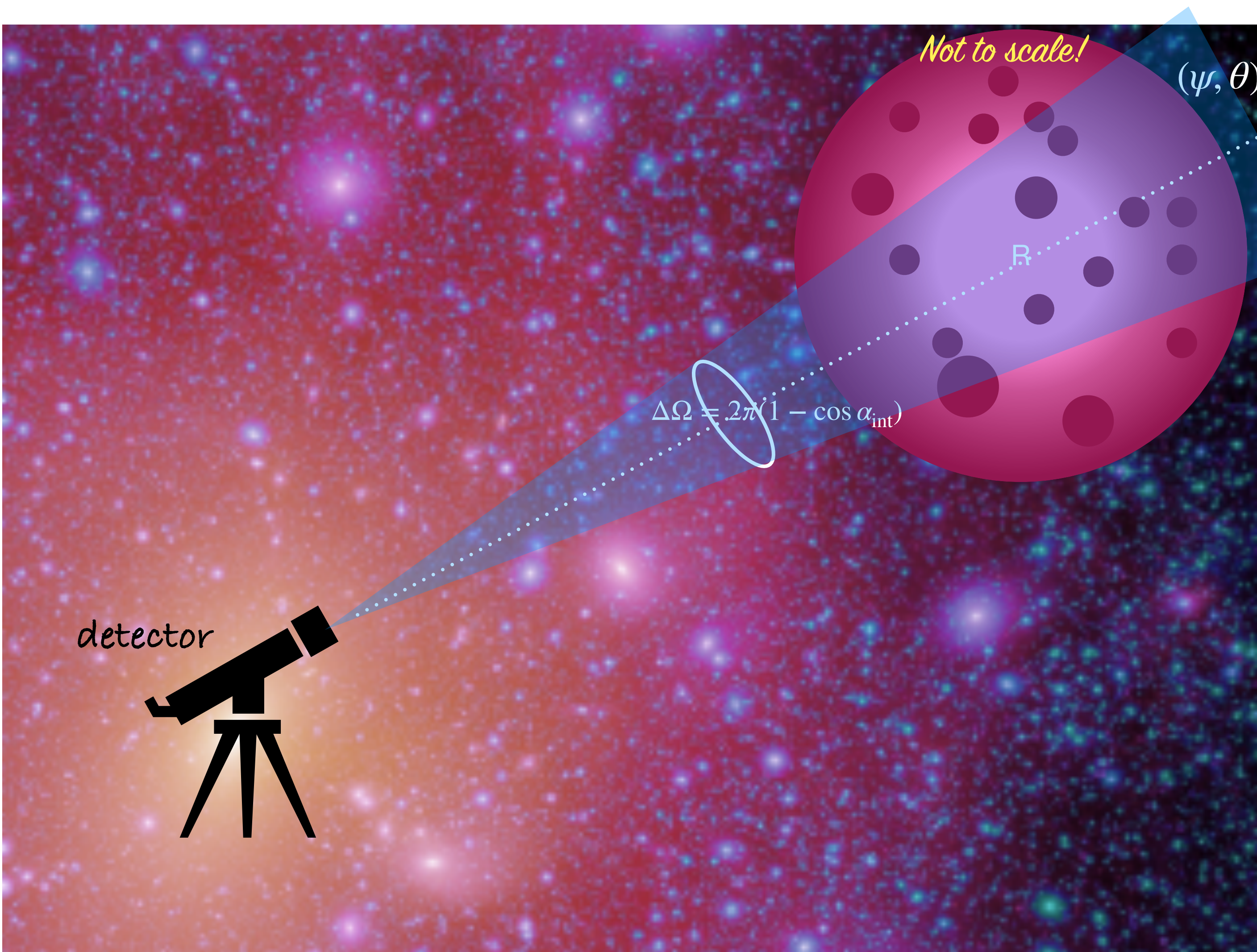
$$J(\psi, \theta, \alpha_{\text{int}}) = \int_0^{2\pi} d\beta \int_0^{\alpha_{\text{int}}} \sin \alpha d\alpha \int_{\text{los}} \rho^2(l, \alpha, \beta; \psi, \theta) dl$$

Without assumption, ρ is a complicated function

1. Wherever we look, we are looking through the MW halo
 - smooth component
 - substructure component, low mass dark haloes

The J-factor

Global picture



$$J(\psi, \theta, \alpha_{\text{int}}) = \int_0^{2\pi} d\beta \int_0^{\alpha_{\text{int}}} \sin \alpha d\alpha \int_{l_{\text{os}}} \rho^2(l, \alpha, \beta; \psi, \theta) dl$$

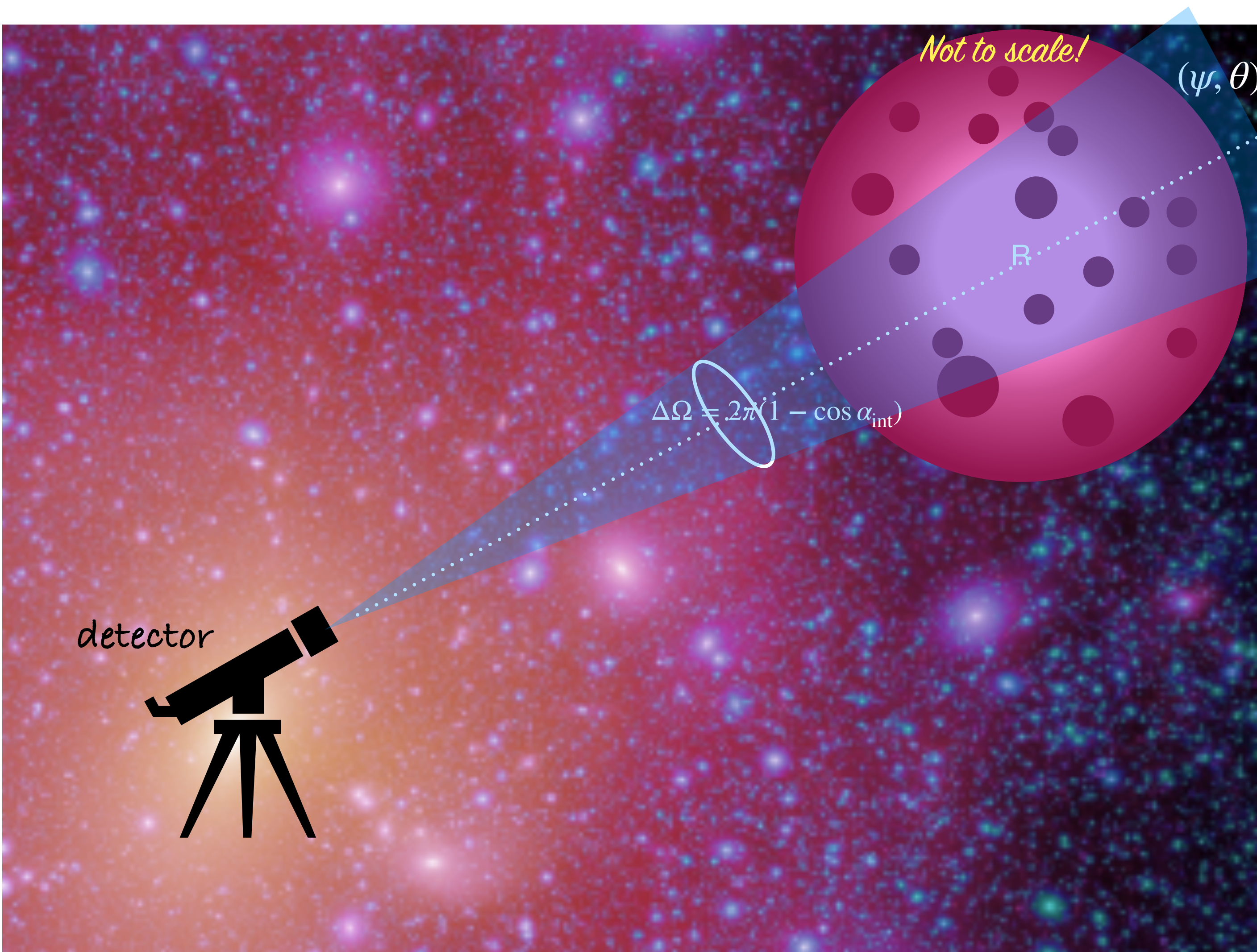
Without assumption, ρ is a complicated function

1. Wherever we look, we are looking through the MW halo
 - smooth component
 - substructure component, low mass dark haloes

2. If looking in the direction of a specific target, the target will have
 - a smooth component
 - a population of subhaloes

The J-factor

Global picture



$$J(\psi, \theta, \alpha_{\text{int}}) = \int_0^{2\pi} d\beta \int_0^{\alpha_{\text{int}}} \sin \alpha d\alpha \int_{\text{los}} \rho^2(l, \alpha, \beta; \psi, \theta) dl$$

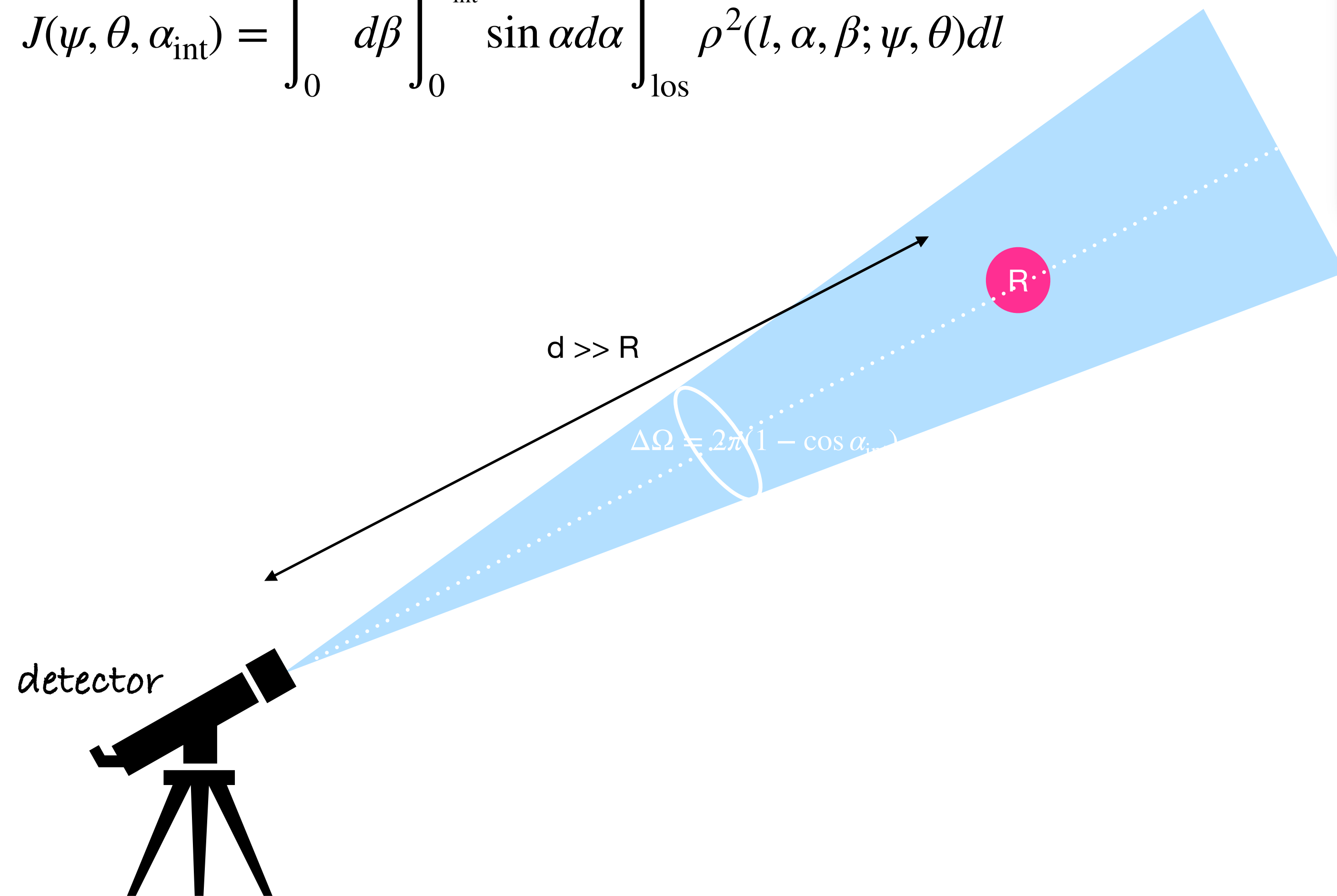
What ingredients are needed to predict the J-factor?

Effect of substructures on the J-factor?

The J-factor

Point-like approximation

$$J(\psi, \theta, \alpha_{\text{int}}) = \int_0^{2\pi} d\beta \int_0^{\alpha_{\text{int}}} \sin \alpha d\alpha \int_{\text{los}} \rho^2(l, \alpha, \beta; \psi, \theta) dl$$



Consider a DM halo

1. fully contained in the integration volume
2. with size \ll distance to the observer
3. ignore the contribution from the smooth MW

How is the J-factor expressed ?

The J-factor

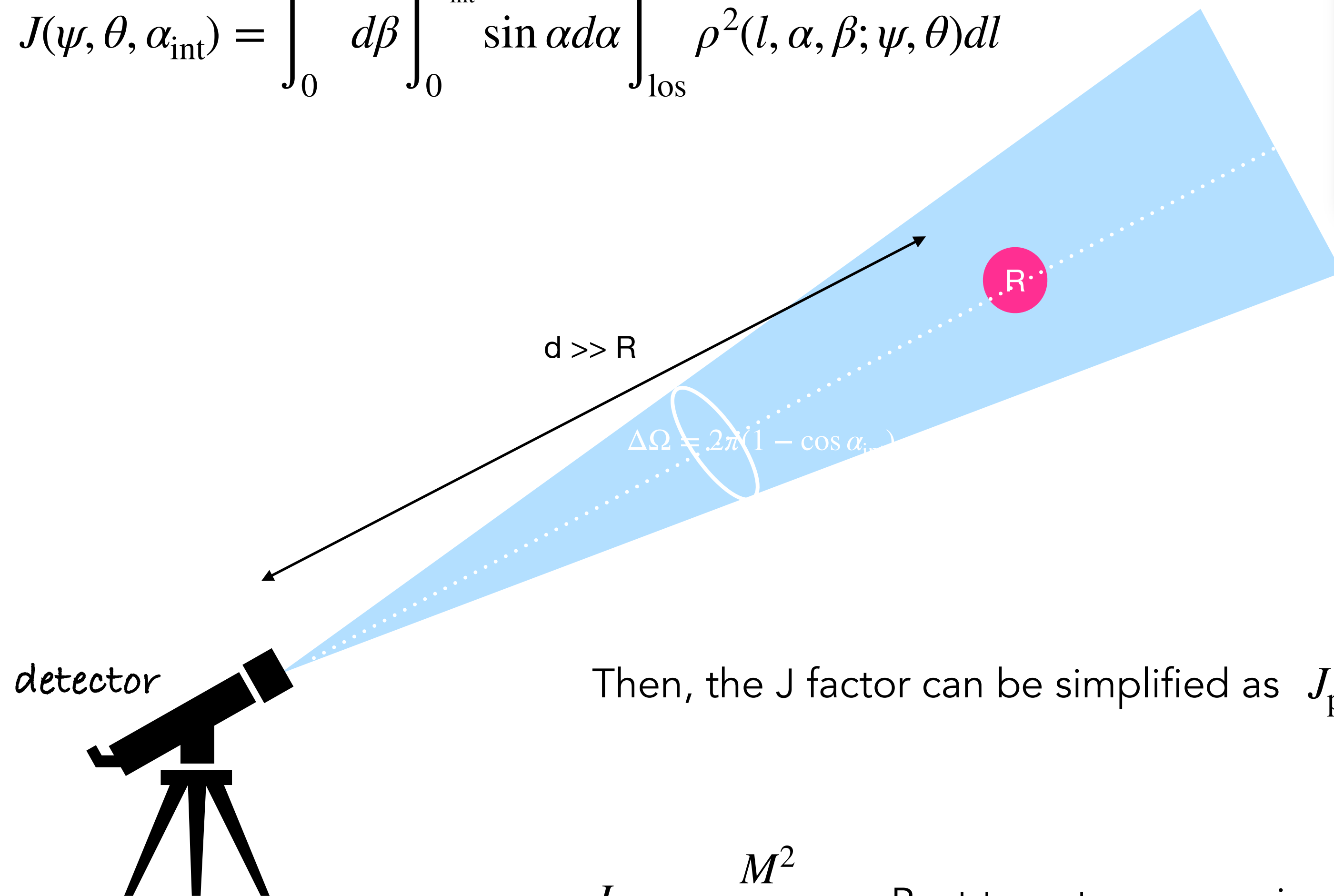
Point-like approximation

$$J(\psi, \theta, \alpha_{\text{int}}) = \int_0^{2\pi} d\beta \int_0^{\alpha_{\text{int}}} \sin \alpha d\alpha \int_{\text{los}} \rho^2(l, \alpha, \beta; \psi, \theta) dl$$

Consider a DM halo

1. fully contained in the integration volume
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3. ignore the contribution from the smooth MW

How is the J-factor expressed ?



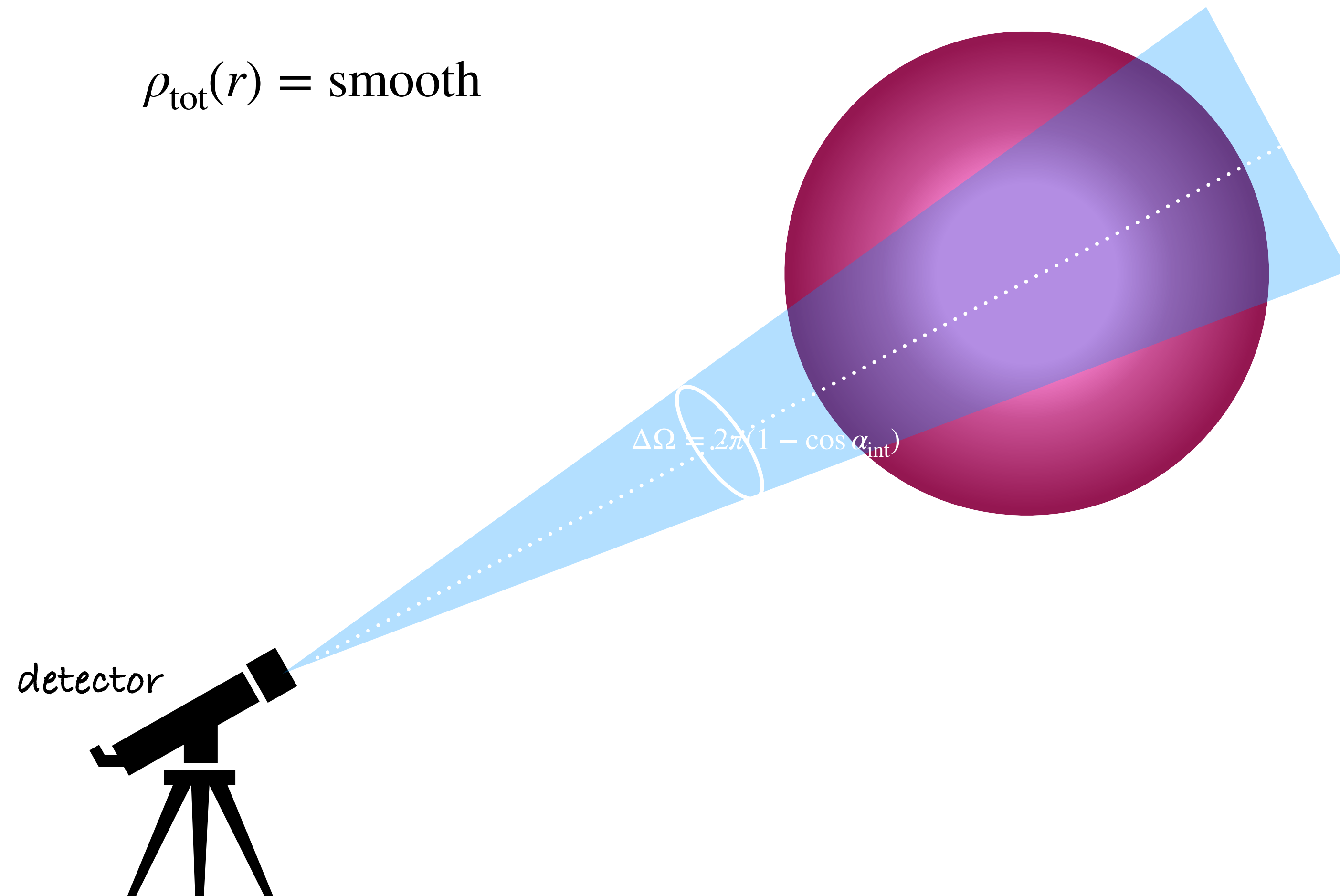
Then, the J factor can be simplified as $J_{\text{point}} = \frac{\mathcal{L}}{d^2}$ with $\mathcal{L} = \mathcal{L}(M, c) = \int_{V_{\text{halo}}} \rho^2(M, c) dV$
halo "luminosity"

$J_{\text{point}} \sim \frac{M^2}{d^2 V} \rightarrow$ Best targets are massive/dense and close

The J-factor

Extended halo

$$\rho_{\text{tot}}(r) = \text{smooth}$$



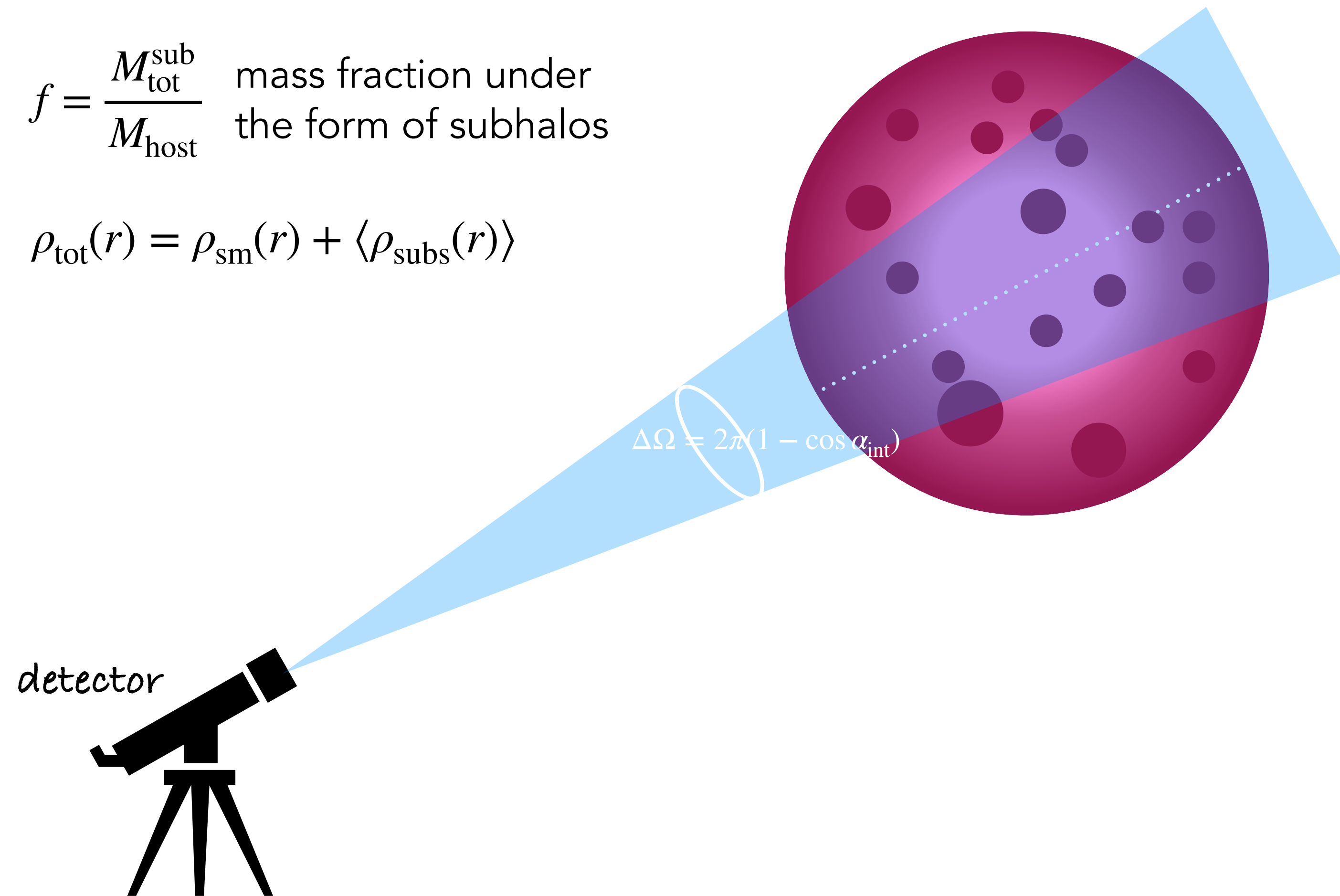
$$J = \int_0^{\Delta\Omega} \int_{l_{\text{os}}} \rho_{\text{tot}}^2 dl d\Omega$$

The J-factor

Extended halo with substructures

$$f = \frac{M_{\text{tot}}^{\text{sub}}}{M_{\text{host}}} \quad \text{mass fraction under the form of subhalos}$$

$$\rho_{\text{tot}}(r) = \rho_{\text{sm}}(r) + \langle \rho_{\text{subs}}(r) \rangle$$



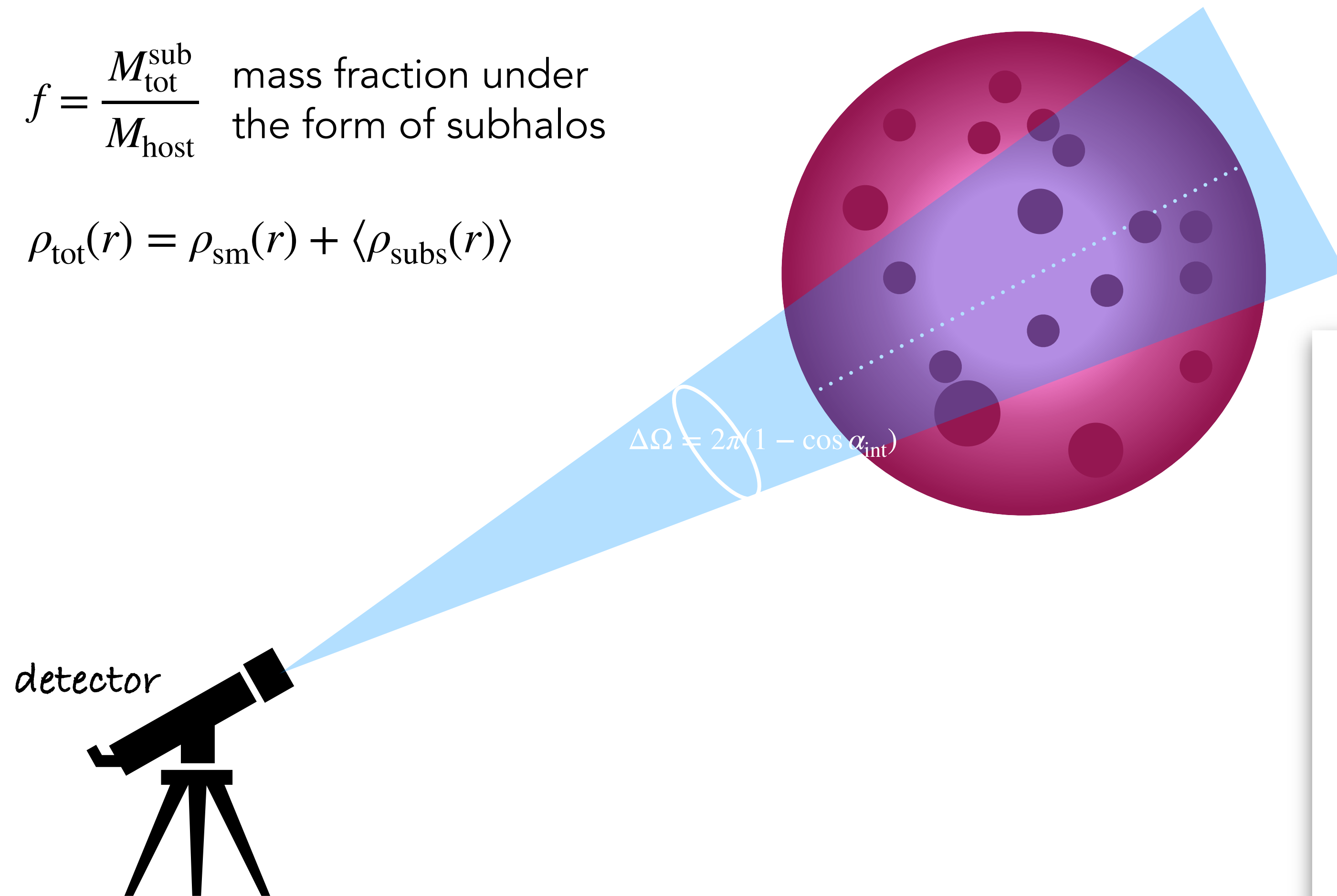
$$J = \int_0^{\Delta\Omega} \int_{\text{los}} \left(\rho_{\text{sm}} + \sum_i \rho_{\text{cl}}^i \right)^2 dl d\Omega$$

The J-factor

Extended halo with substructures

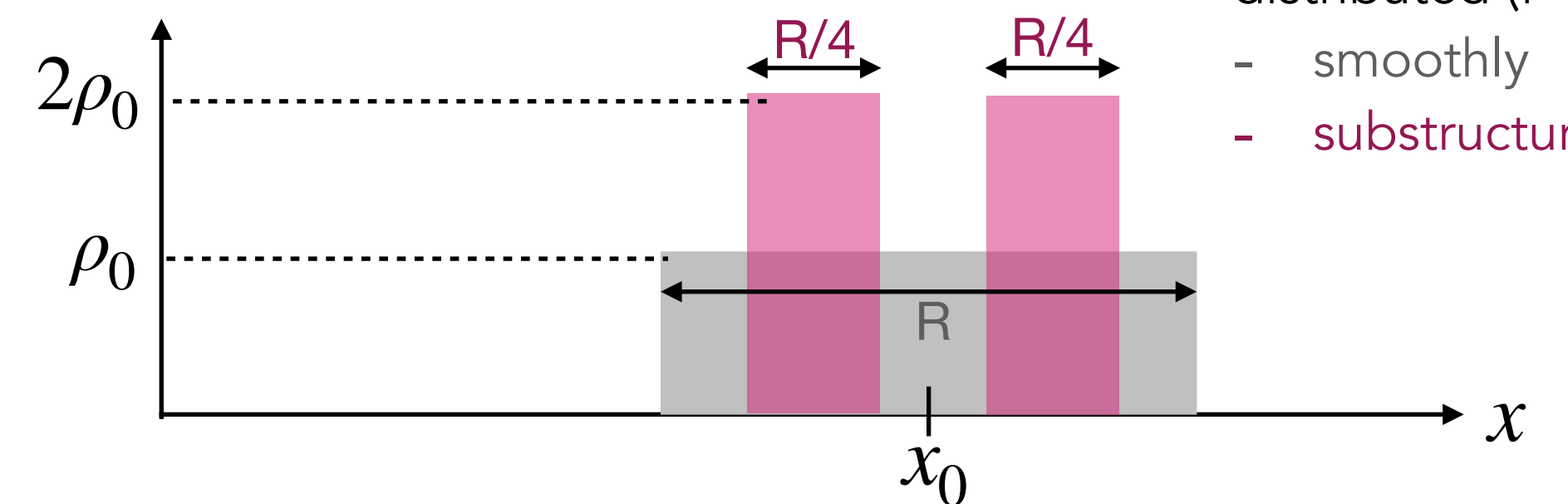
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$$J = \int_0^{\Delta\Omega} \int_{\text{los}} \left(\rho_{\text{sm}} + \sum_i \rho_{\text{cl}}^i \right)^2 dl d\Omega$$

1D toy model



$$J_{\text{no-sub}} = ?$$

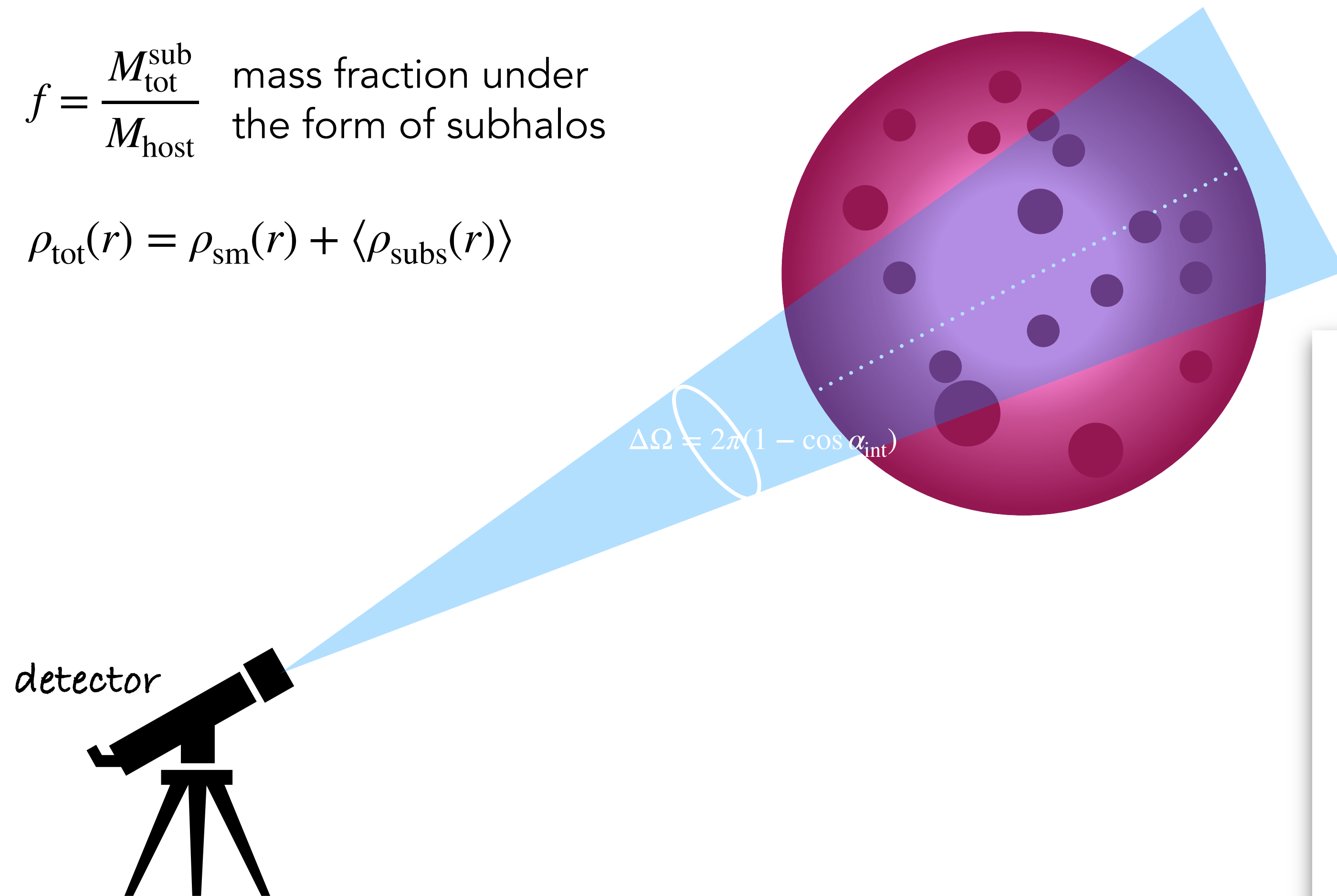
$$J_{\text{subs}} = ?$$

The J-factor

Extended halo with substructures

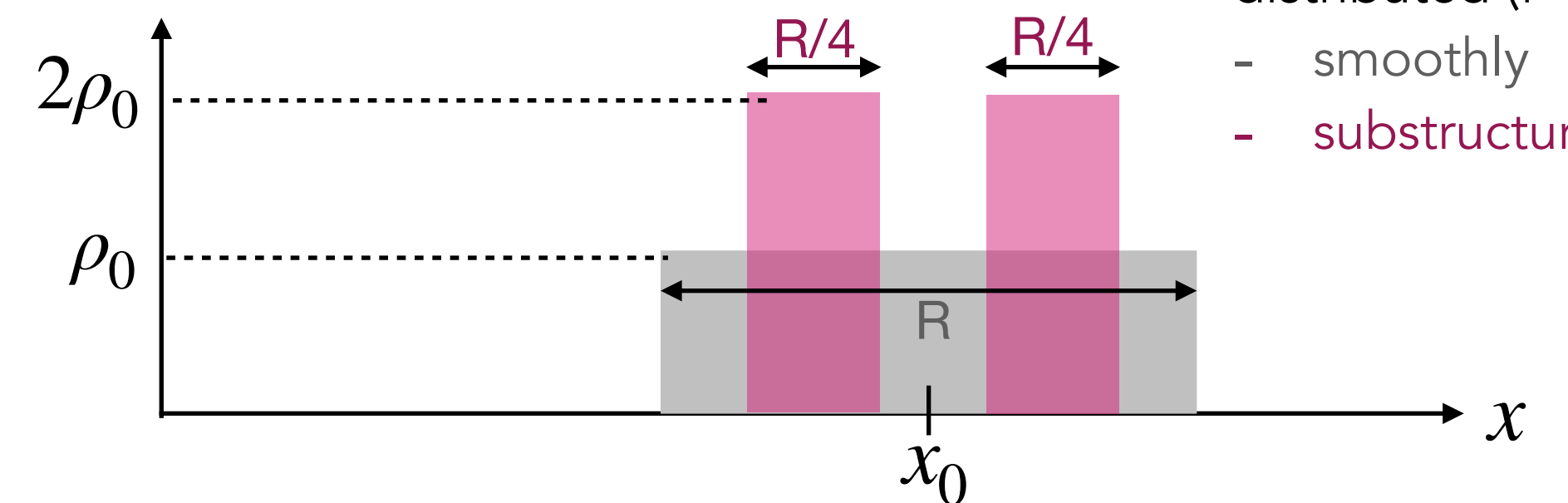
$$f = \frac{M_{\text{tot}}^{\text{sub}}}{M_{\text{host}}} \quad \text{mass fraction under the form of subhalos}$$

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$$J = \int_0^{\Delta\Omega} \int_{\text{los}} \left(\rho_{\text{sm}} + \sum_i \rho_{\text{cl}}^i \right)^2 dl d\Omega$$

1D toy model



Same total mass distributed ($f=1$)

- smoothly
- substructures

$$J_{\text{no-sub}} \propto \int_{x_0-R/2}^{x_0+R/2} \rho_0^2 dx = \rho_0^2 R$$

$$J_{\text{subs}} \propto \sum_i \int_{x_i-R/8}^{x_i+R/8} (2\rho_0)^2 dx = 2\rho_0^2 R$$

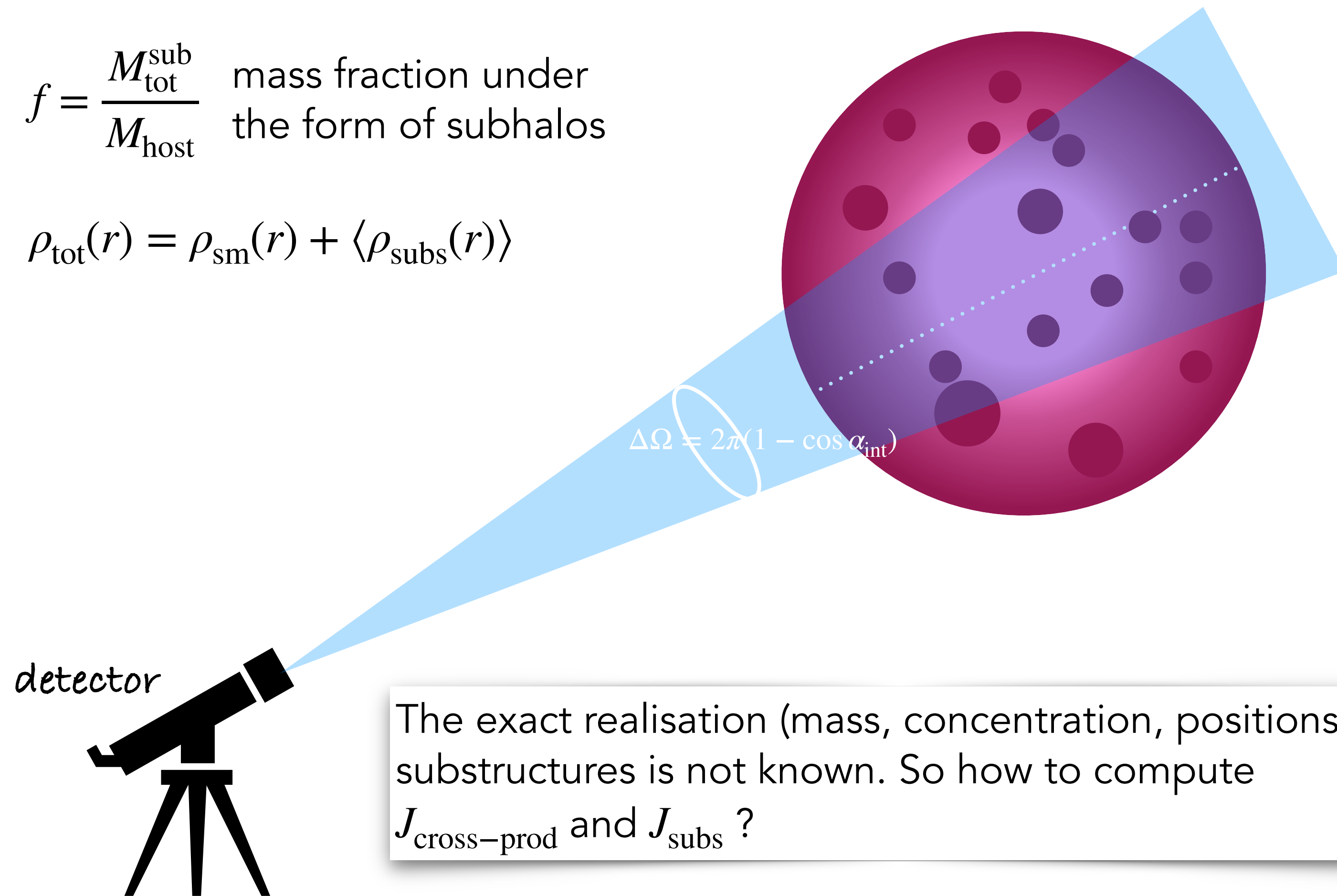
Substructures will boost the annihilation signal
(what about decay?)

The J-factor

Extended halo with substructures

$$f = \frac{M_{\text{tot}}^{\text{sub}}}{M_{\text{host}}} \quad \text{mass fraction under the form of subhalos}$$

$$\rho_{\text{tot}}(r) = \rho_{\text{sm}}(r) + \langle \rho_{\text{subs}}(r) \rangle$$



The exact realisation (mass, concentration, positions) of substructures is not known. So how to compute $J_{\text{cross-prod}}$ and J_{subs} ?

$$J = \int_0^{\Delta\Omega} \int_{\text{los}} \left(\rho_{\text{sm}} + \sum_i \rho_{\text{cl}}^i \right)^2 dl d\Omega$$

expands into 3 terms

$$J_{\text{sm}} \equiv \int_0^{\Delta\Omega} \int_{\text{los}} \rho_{\text{sm}}^2 dl d\Omega$$

$$J_{\text{cross-prod}} \equiv 2 \int_0^{\Delta\Omega} \int_{\text{los}} \rho_{\text{sm}} \sum_i \rho_{\text{cl}}^i dl d\Omega$$

$$J_{\text{subs}} = \int_0^{\Delta\Omega} \int_{\text{los}} \left(\sum_i \rho_{\text{cl}}^i \right)^2 dl d\Omega$$

The J-factor

Extended halo with substructures

Solution: go to the continuous limit, assuming substructure spatial, mass and concentration distributions.

$$\frac{d^3N}{dVdMdc} = N_{\text{tot}} \frac{d\mathcal{P}_V}{dV}(r) \cdot \frac{d\mathcal{P}_M}{dM}(M) \cdot \frac{d\mathcal{P}_c}{dc}(M, c)$$



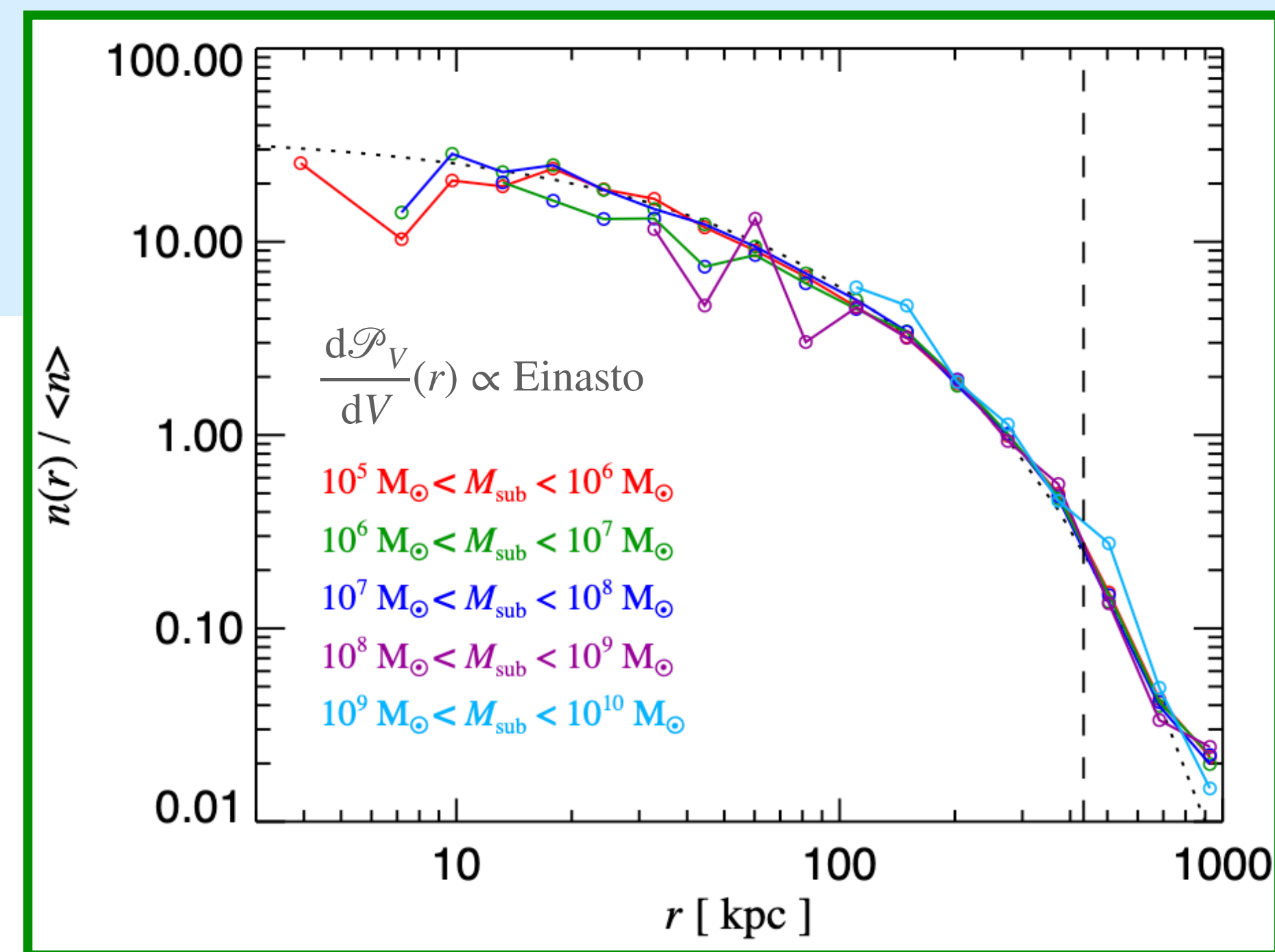
This assumes that spatial and mass/
concentration distributions are NOT correlated.
Does not hold when baryonic effects are
considered (see later)

The J-factor

Extended halo with substructures

Solution: go to the continuous limit, assuming substructure spatial, mass and concentration distributions.

$$\frac{d^3N}{dVdMdc} = N_{\text{tot}} \frac{d\mathcal{P}_V(r)}{dV} \cdot \frac{d\mathcal{P}_M(M)}{dM} \cdot \frac{d\mathcal{P}_c(M, c)}{dc}$$

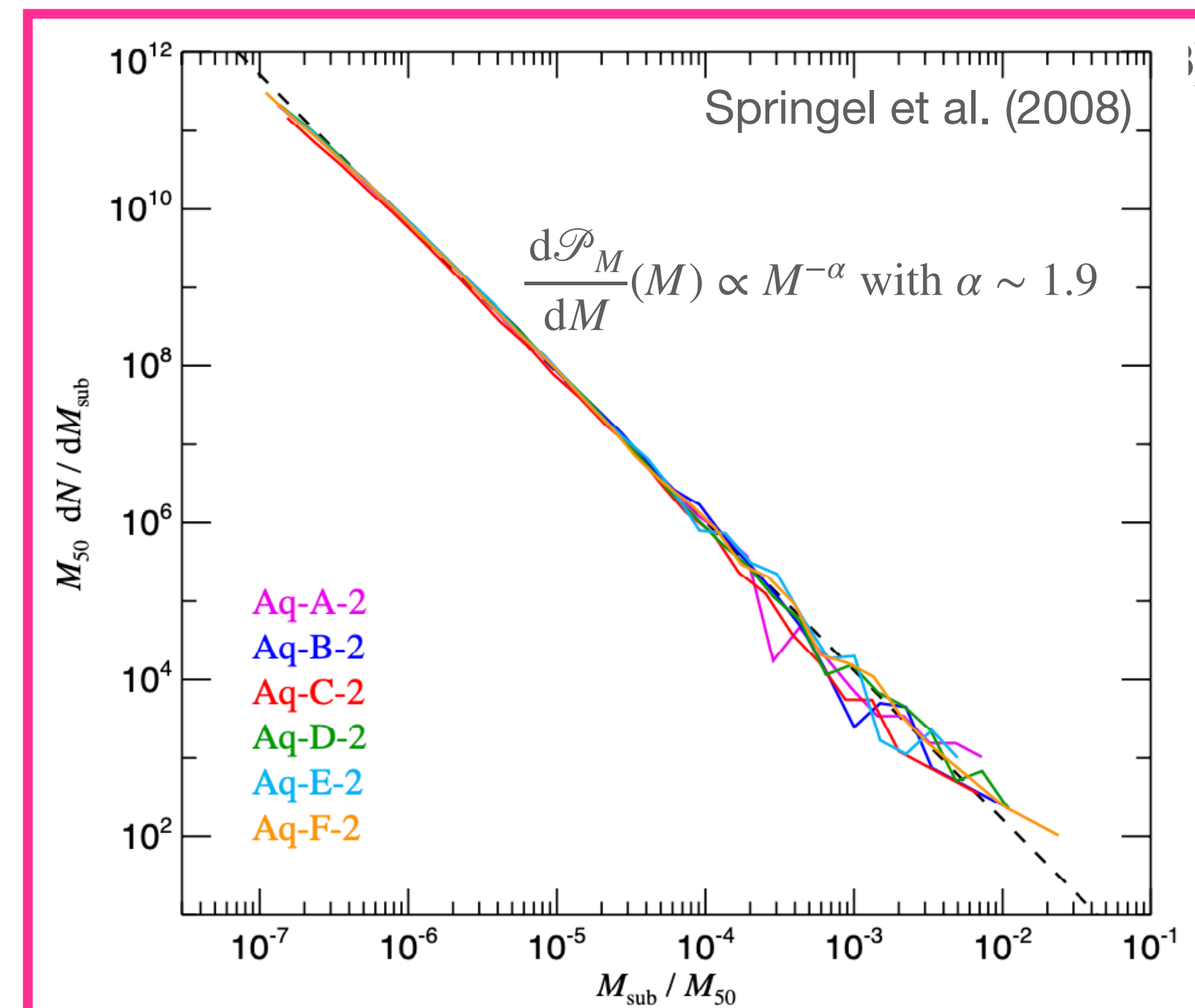
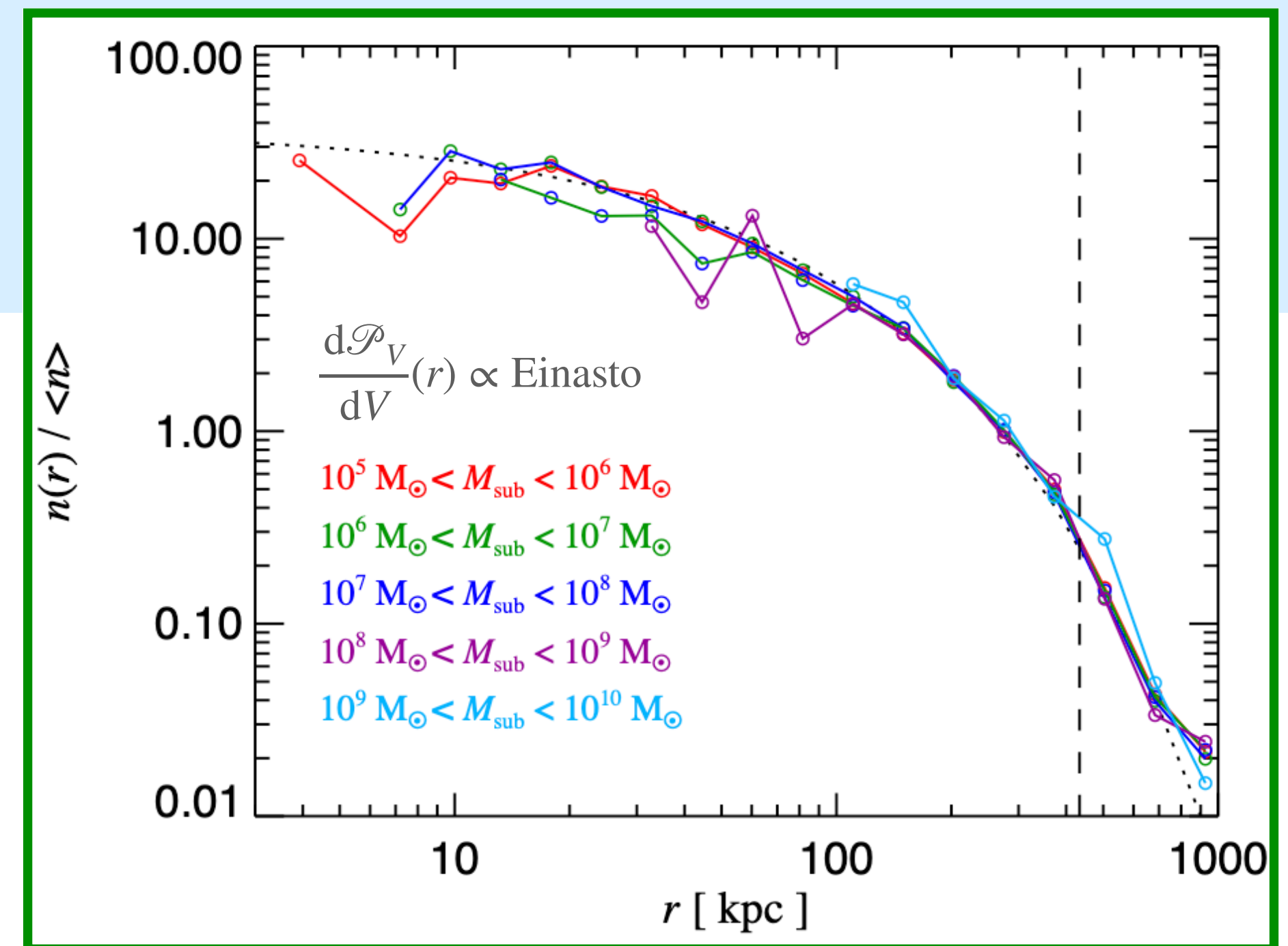


The J-factor

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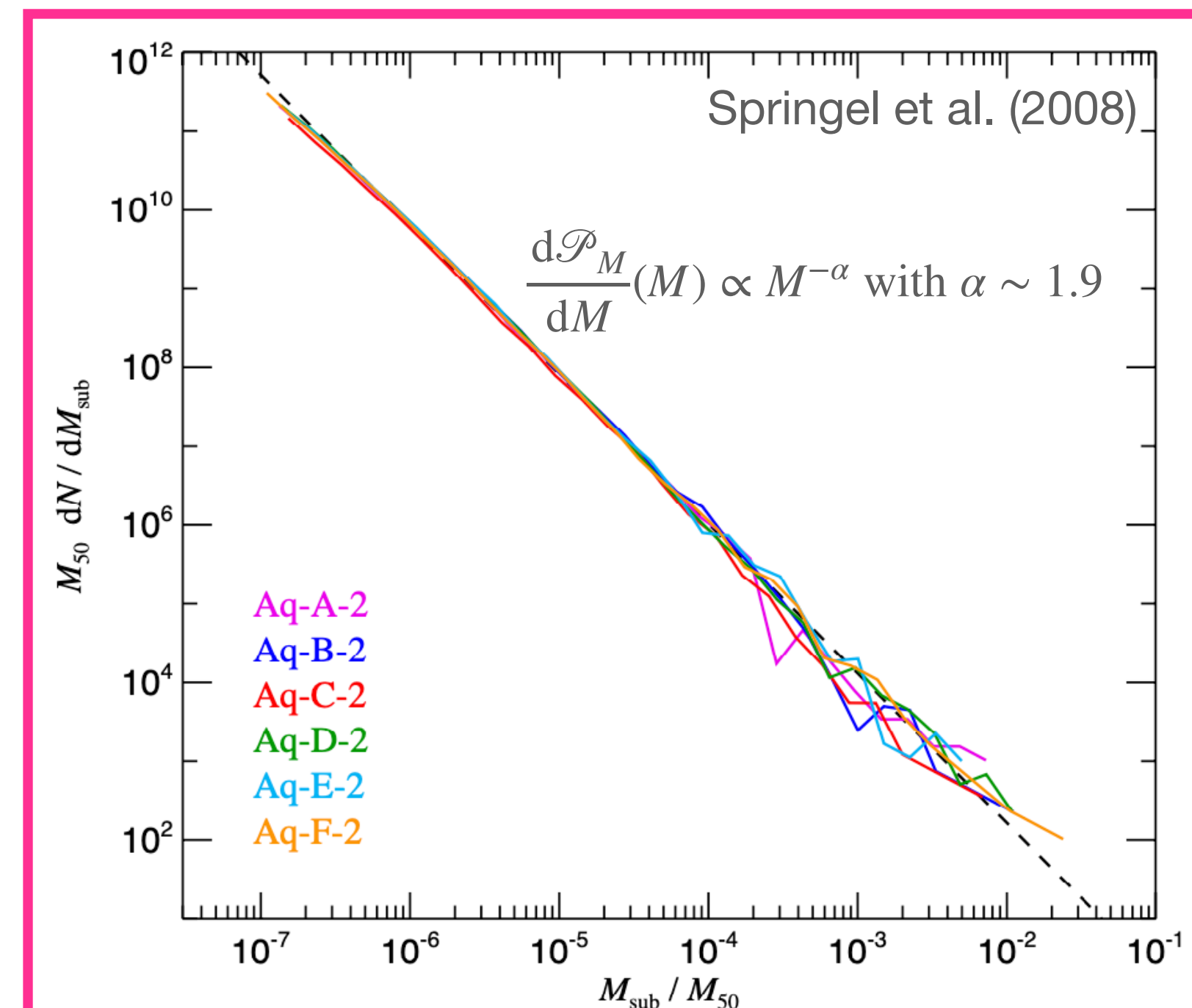
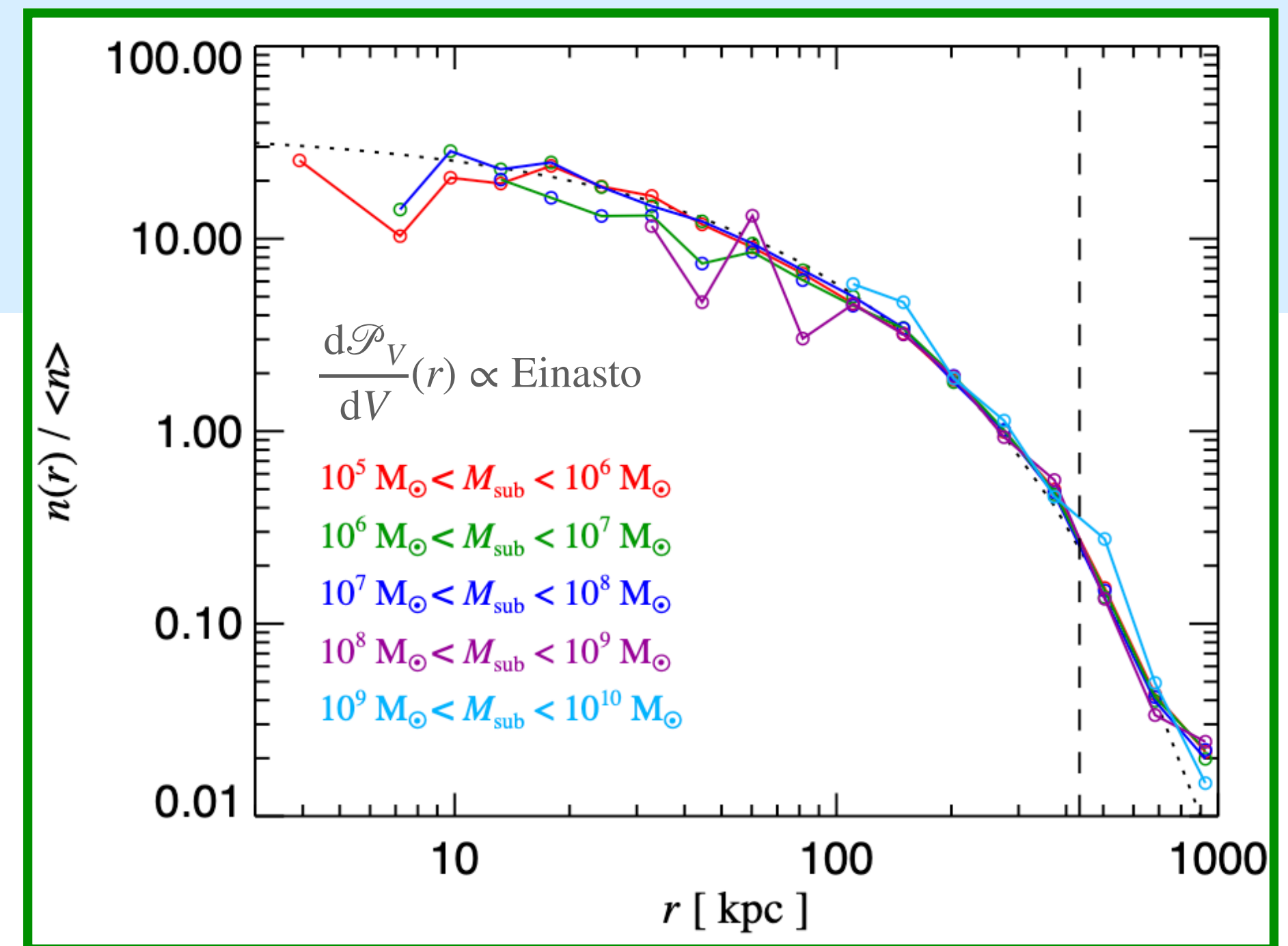
The J-factor

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DM-only simulations give ~ 100 clumps between 10^8 - $10^{10} M_{\text{sun}}$. How many subhalos pertain a MW-like halo if $M_{\text{min}} = 10^{-6} M_{\text{sun}}$?

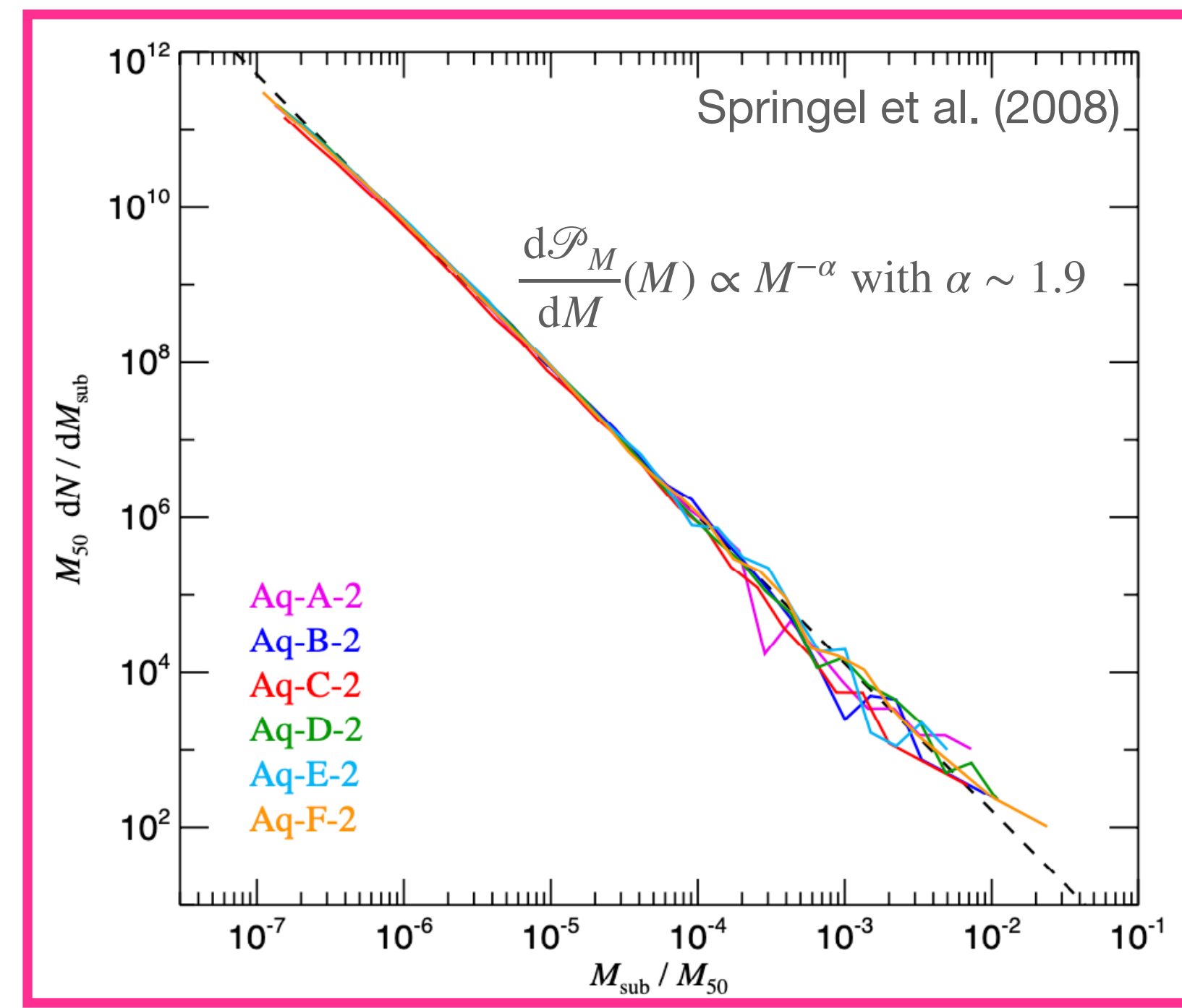
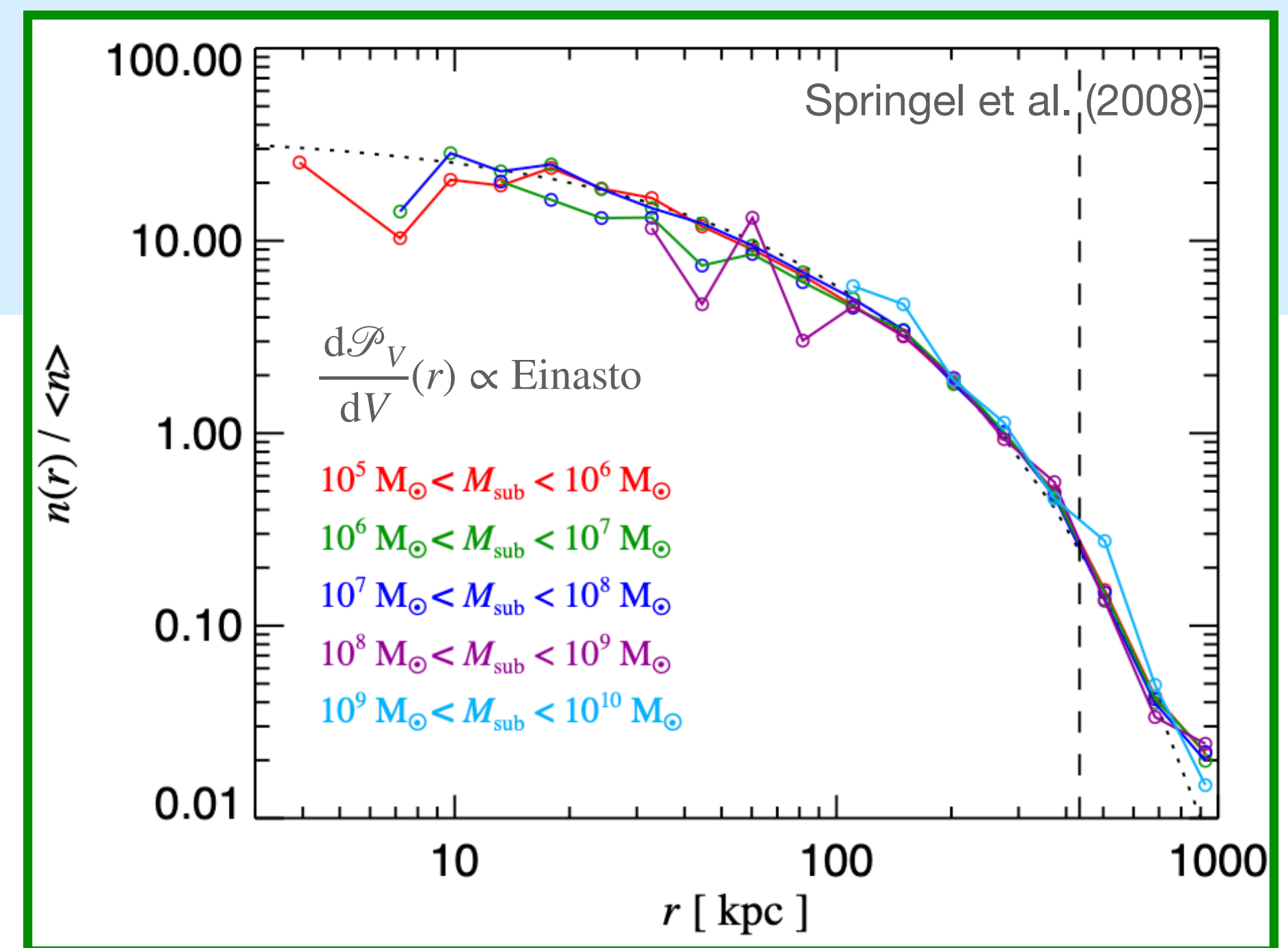
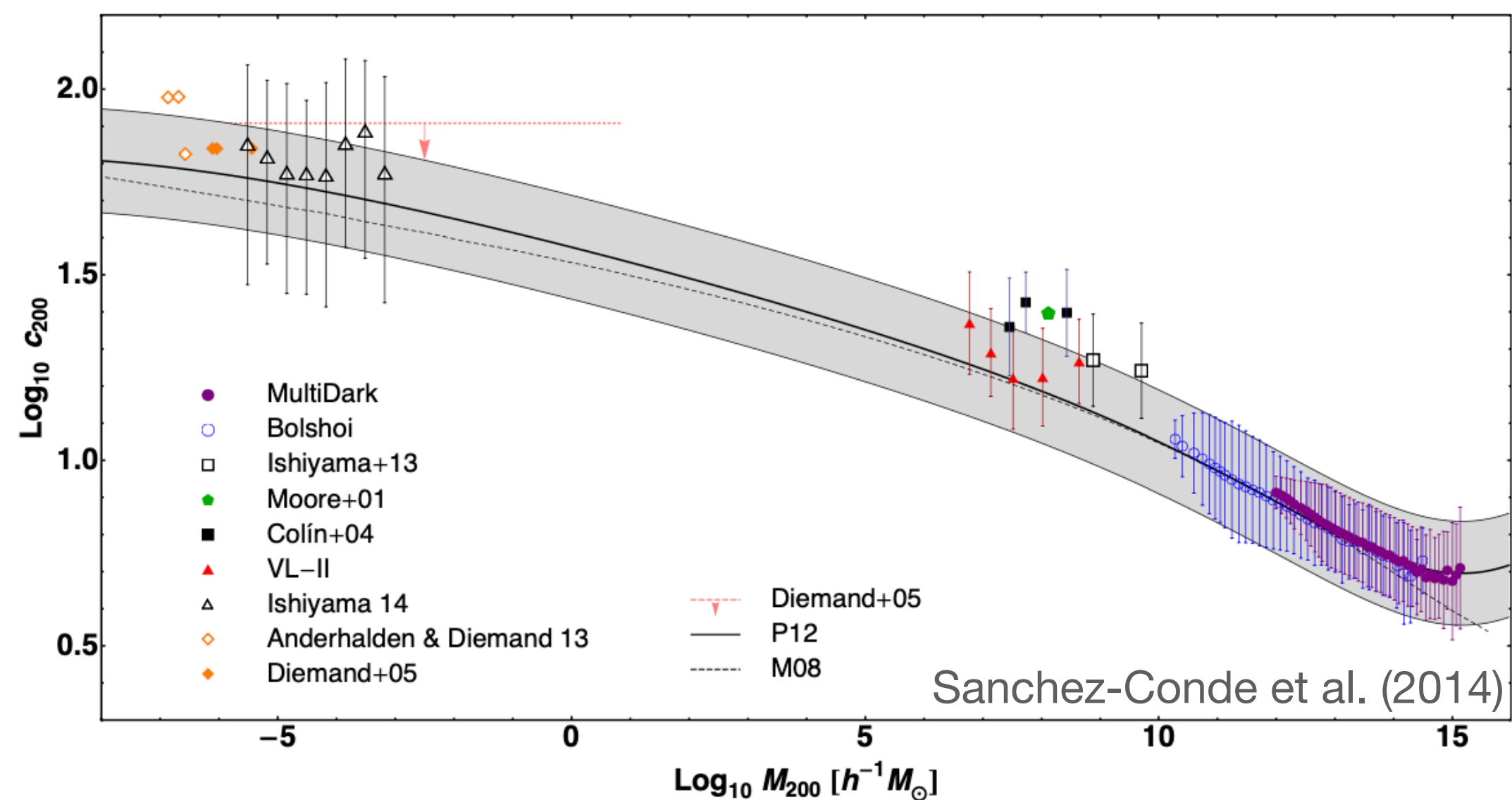


The J-factor

Extended halo with substructures

Solution: go to the continuous limit, assuming substructure spatial, mass and concentration distributions.

$$\frac{d^3N}{dVdMdc} = N_{\text{tot}} \frac{d\mathcal{P}_V(r)}{dV} \cdot \frac{d\mathcal{P}_M(M)}{dM} \cdot \frac{d\mathcal{P}_c(M, c)}{dc}$$

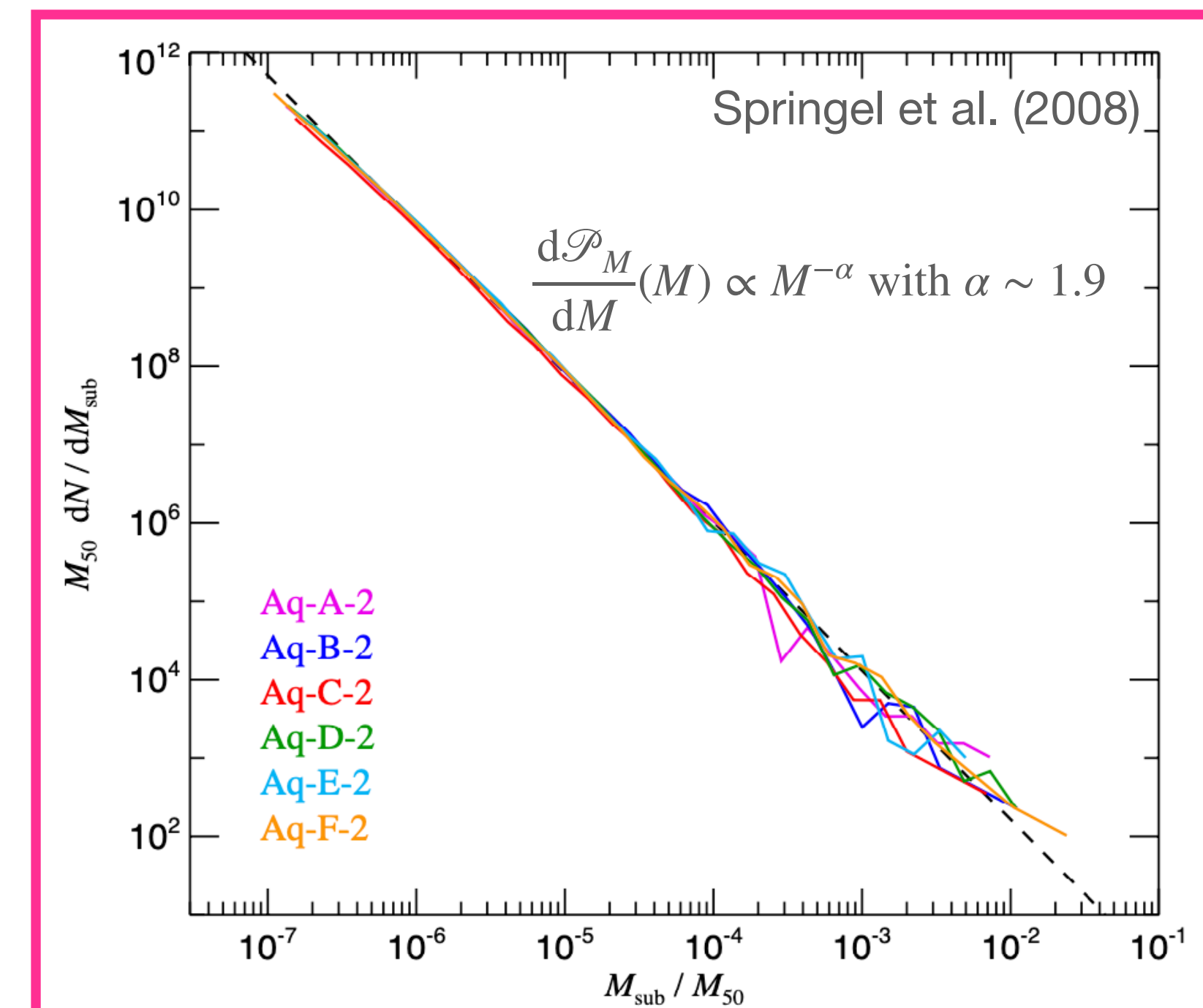
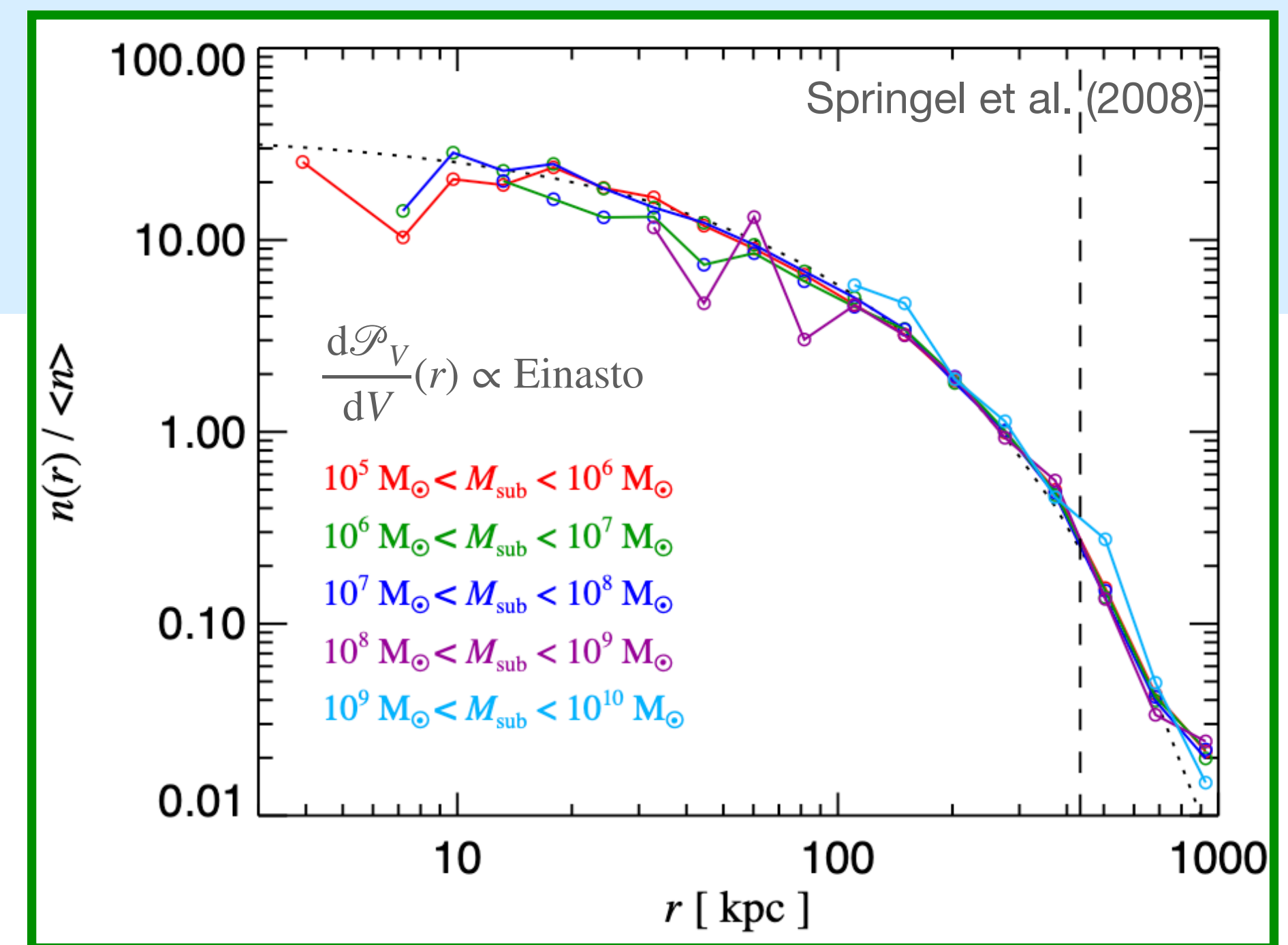
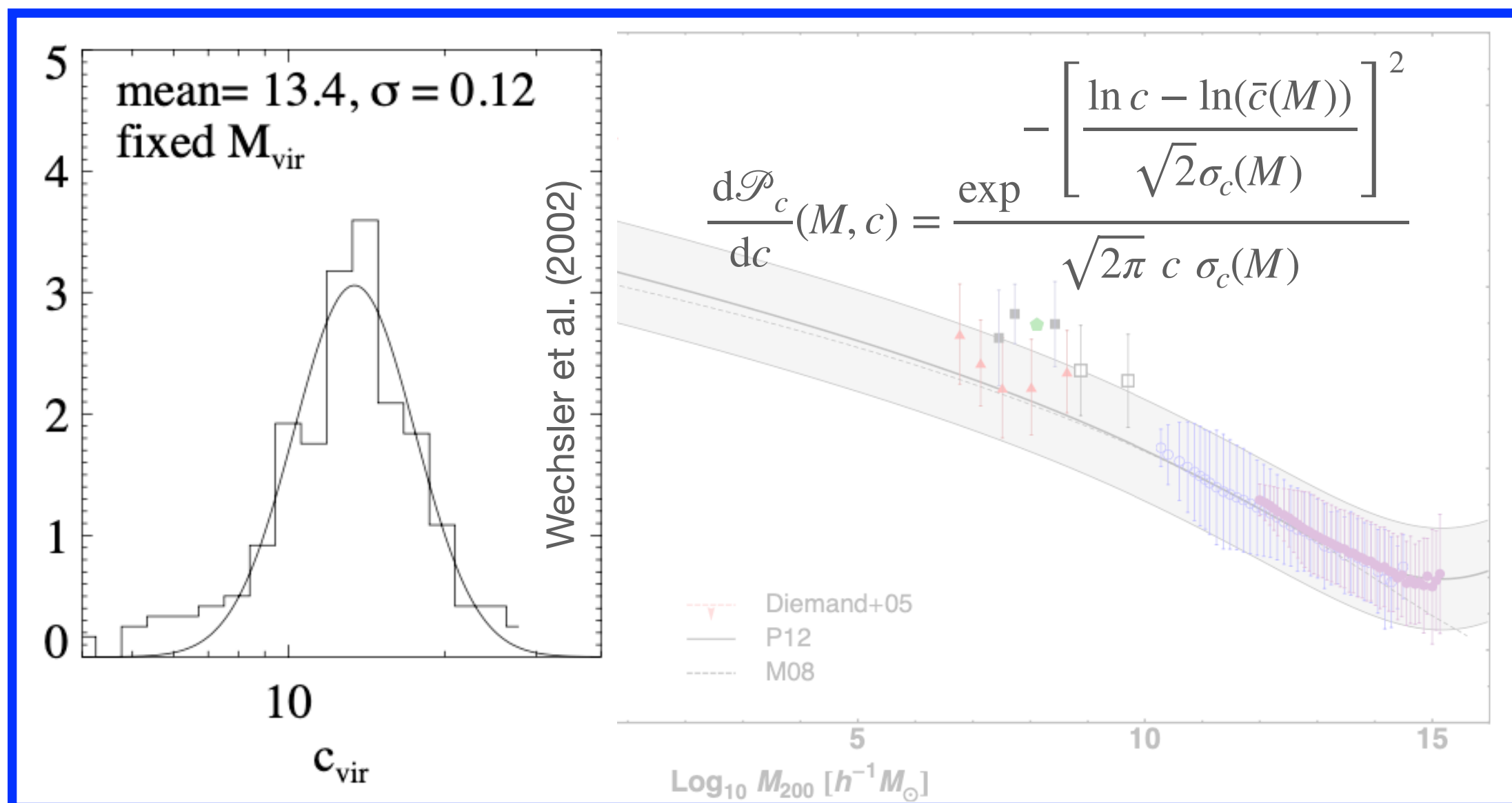


The J-factor

Extended halo with substructures

Solution: go to the continuous limit, assuming substructure spatial, mass and concentration distributions.

$$\frac{d^3N}{dVdMdc} = N_{\text{tot}} \frac{d\mathcal{P}_V(r)}{dV} \cdot \frac{d\mathcal{P}_M(M)}{dM} \cdot \frac{d\mathcal{P}_c(M, c)}{dc}$$

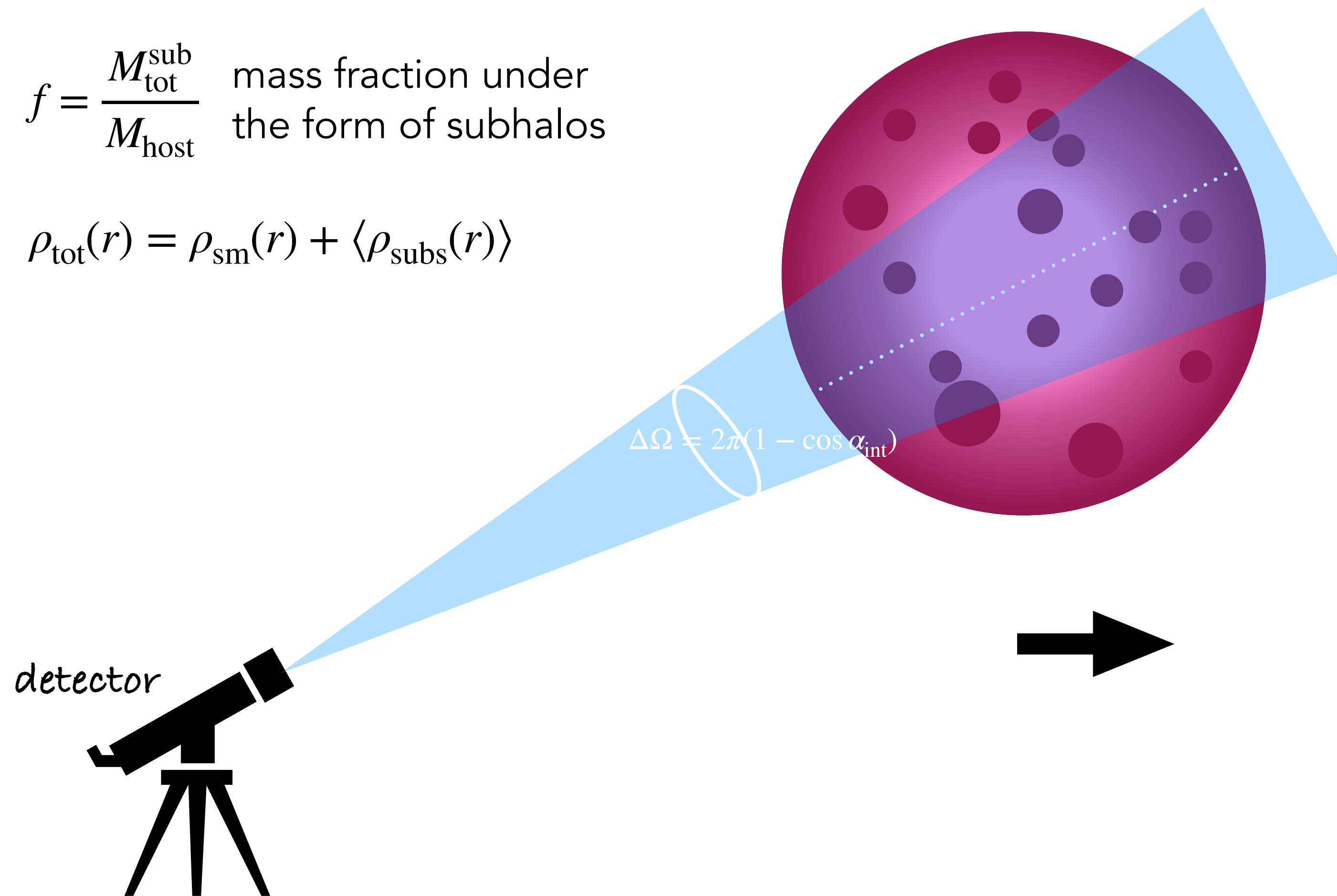


The J-factor

Extended halo with substructures

$$f = \frac{M_{\text{tot}}^{\text{sub}}}{M_{\text{host}}} \quad \text{mass fraction under the form of subhalos}$$

$$\rho_{\text{tot}}(r) = \rho_{\text{sm}}(r) + \langle \rho_{\text{subs}}(r) \rangle$$



$$J = \int_0^{\Delta\Omega} \int_{\text{los}} \left(\rho_{\text{sm}} + \sum_i \rho_{\text{cl}}^i \right)^2 dl d\Omega$$

expands into 3 terms

$$J_{\text{sm}} \equiv \int_0^{\Delta\Omega} \int_{\text{los}} \rho_{\text{sm}}^2 dl d\Omega$$

$$J_{\text{cross-prod}} \equiv 2 \int_0^{\Delta\Omega} \int_{\text{los}} \rho_{\text{sm}} \sum_i \rho_{\text{cl}}^i dl d\Omega$$

$$J_{\text{subs}} = \int_0^{\Delta\Omega} \int_{\text{los}} \left(\sum_i \rho_{\text{cl}}^i \right)^2 dl d\Omega$$

The J-factor

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$$\langle \rho_{\text{subs}}(r) \rangle = f M_{\text{host}} \frac{d\mathcal{P}_V(r)}{dV}$$

$$\Delta\Omega = 2\pi(1 - \cos\alpha_{\text{int}})$$

detector



$$J = \int_0^{\Delta\Omega} \int_{\text{los}} \left(\rho_{\text{sm}} + \sum_i \rho_{\text{cl}}^i \right)^2 dl d\Omega$$

expands into 3 terms

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The J-factor

Extended halo with substructures

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The J-factor

Extended halo with substructures

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$$\langle \rho_{\text{subs}}(r) \rangle = f M_{\text{host}} \frac{d\mathcal{P}_V(r)}{dV}$$

$$\Delta\Omega = 2\pi(1 - \cos\alpha_{\text{int}})$$



$$\langle J_{\text{subs}} \rangle = N_{\text{tot}} \int_0^{\Delta\Omega} \int_{l_{\text{min}}}^{l_{\text{max}}} \frac{d\mathcal{P}_V}{dV}(l, \Omega) dl d\Omega \int_{M_{\text{min}}}^{M_{\text{max}}} \frac{d\mathcal{P}_M}{dM}(M) \times \int_{c_{\text{min}}(M)}^{c_{\text{max}}(M)} \frac{d\mathcal{P}_c}{dc}(c, M) \mathcal{L}(M, c) dc dM$$

single halo luminosity

$$J = \int_0^{\Delta\Omega} \int_{\text{los}} \left(\rho_{\text{sm}} + \sum_i \rho_{\text{cl}}^i \right)^2 dl d\Omega$$

expands into 3 terms

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The J-factor

Extended halo with substructures

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single halo luminosity

The J-factor

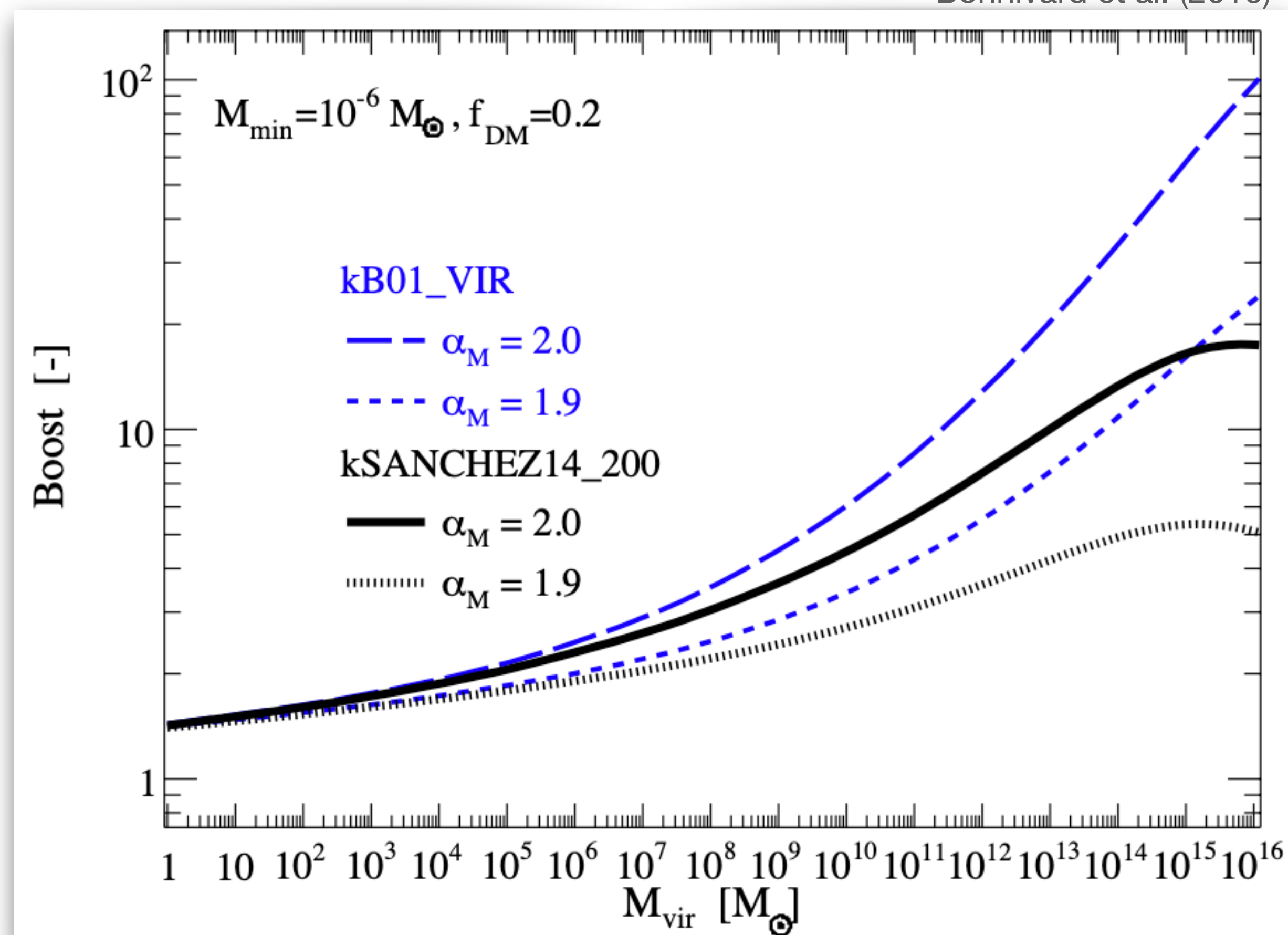
Substructure boost

$$\text{Boost} = \frac{J_{\text{sm}} + J_{\text{subs}} + J_{\text{crossprod}}}{J_{\text{no-sub}}}$$

Boost sensitive to subhalo

- spatial distribution
- mass distribution
- mass-concentration relation + distribution
- inner density profile
- mass range
- distance from halo centre, integration angle

Bonnivard et al. (2016)



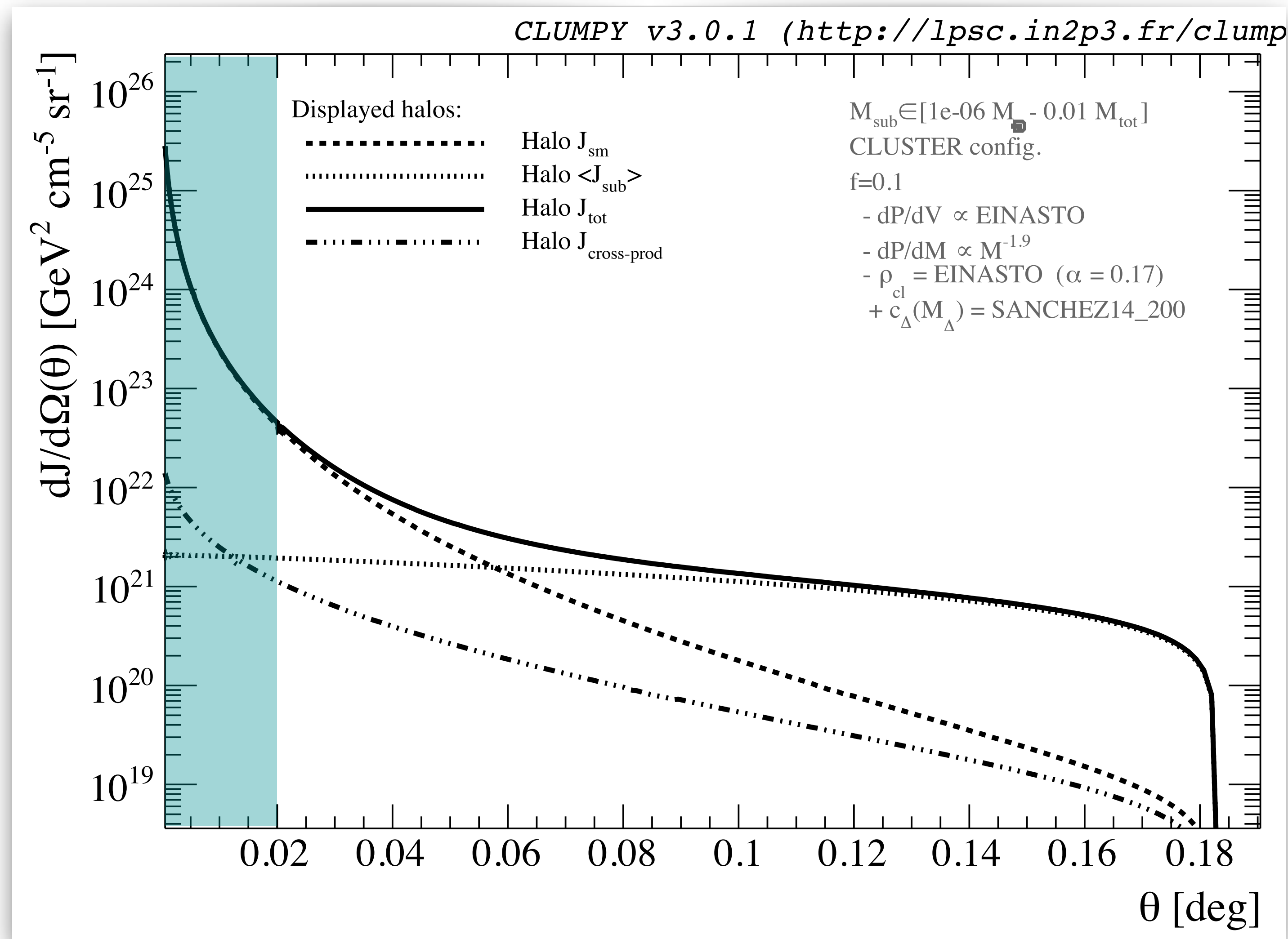
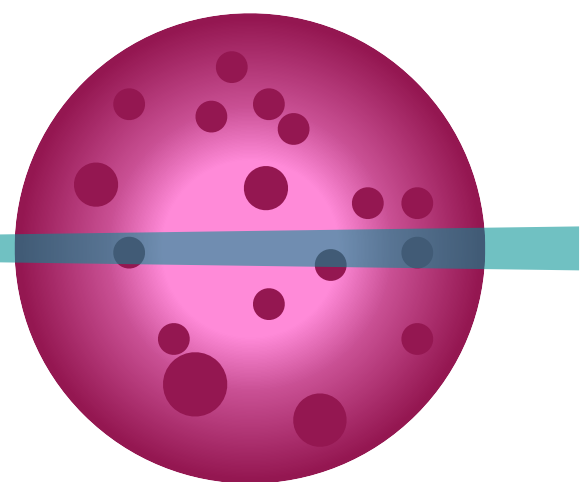
The J-factor

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Boost sensitive to subhalo

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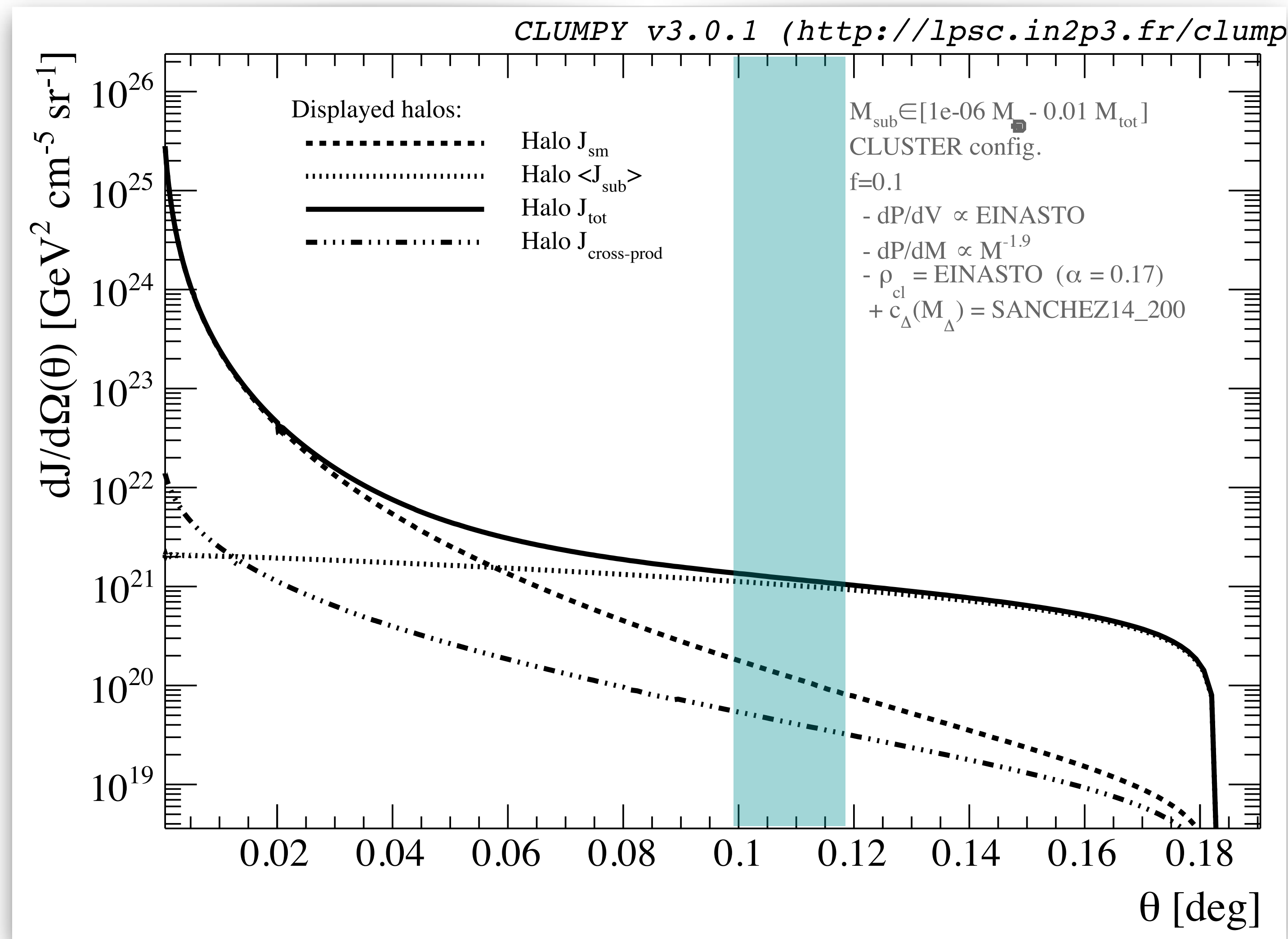
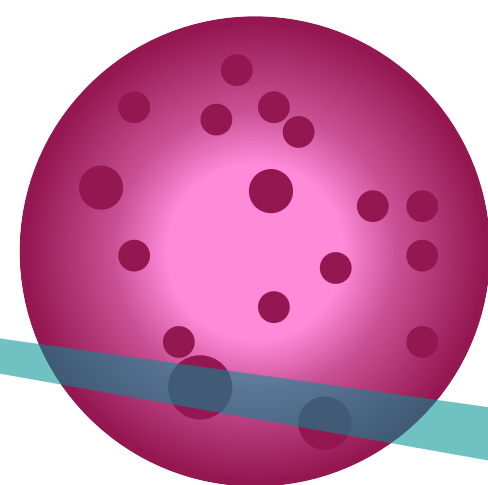
The J-factor

Substructure boost

$$\text{Boost} = \frac{J_{\text{sm}} + J_{\text{subs}} + J_{\text{crossprod}}}{J_{\text{no-sub}}}$$

Boost sensitive to subhalo

- spatial distribution
- mass distribution
- mass-concentration relation + distribution
- inner density profile
- mass range
- distance from halo centre, integration angle



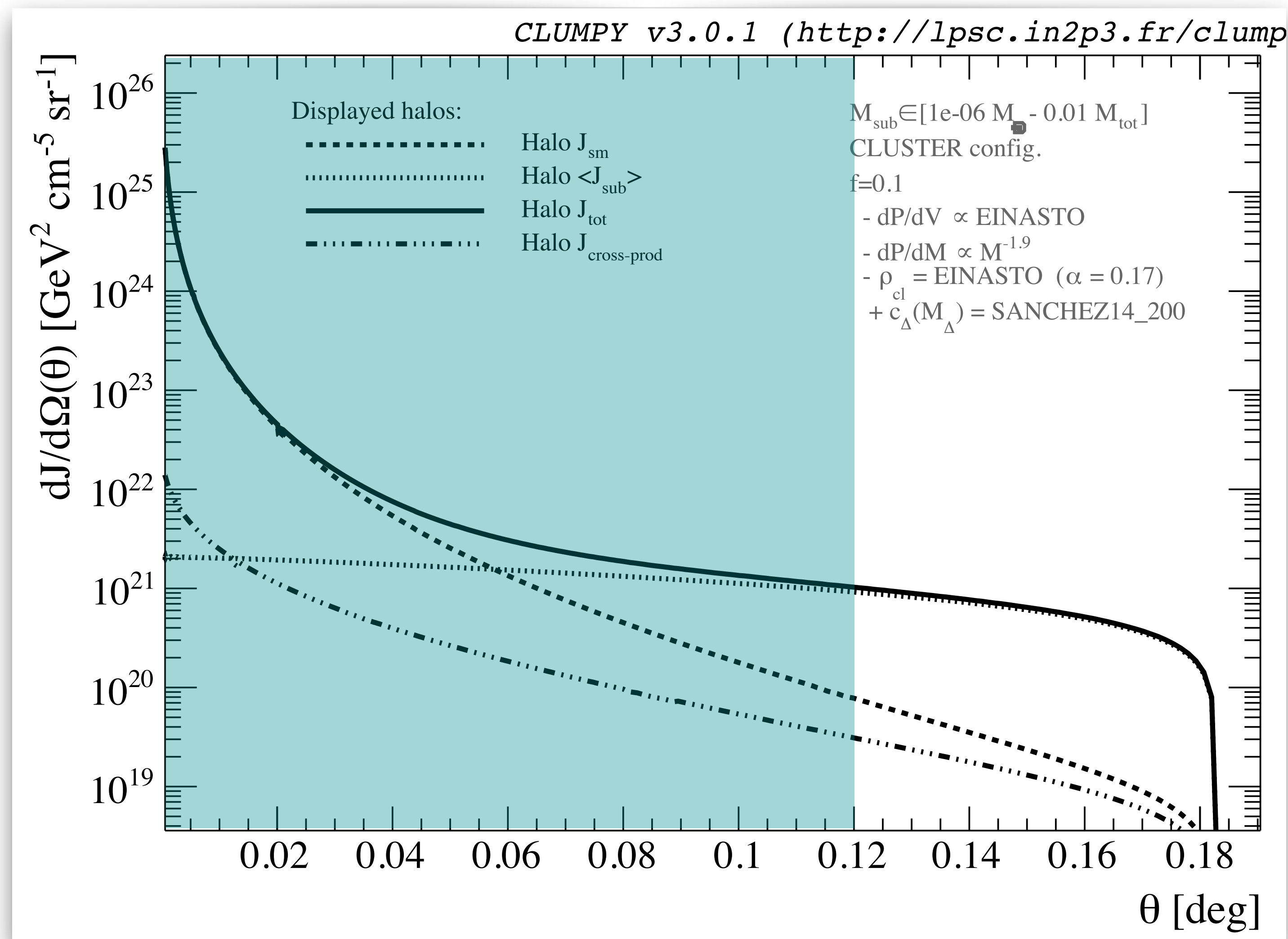
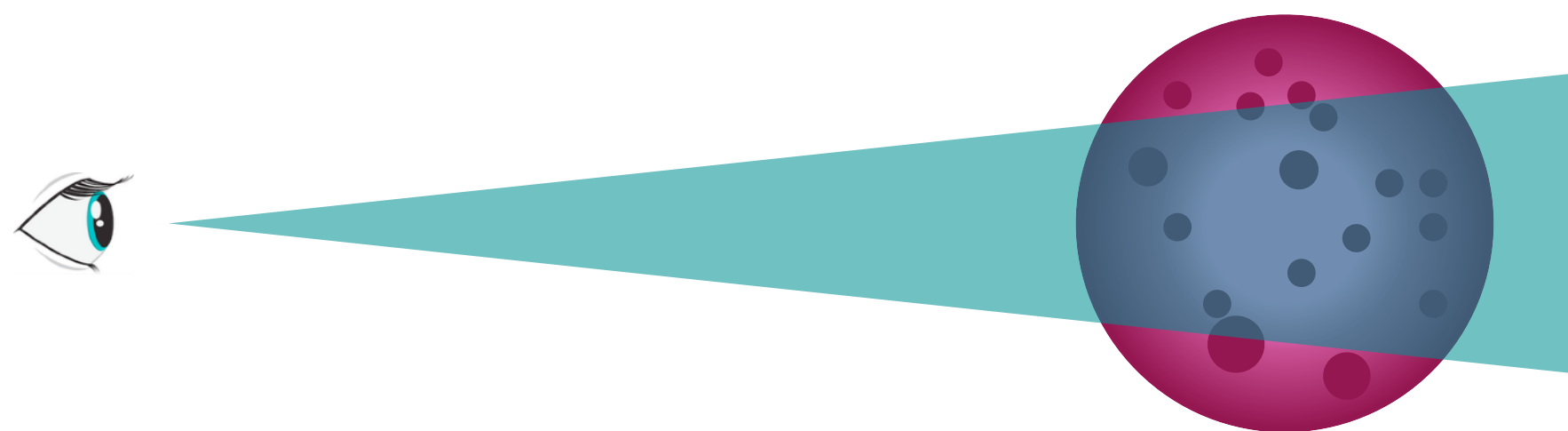
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The J-factor

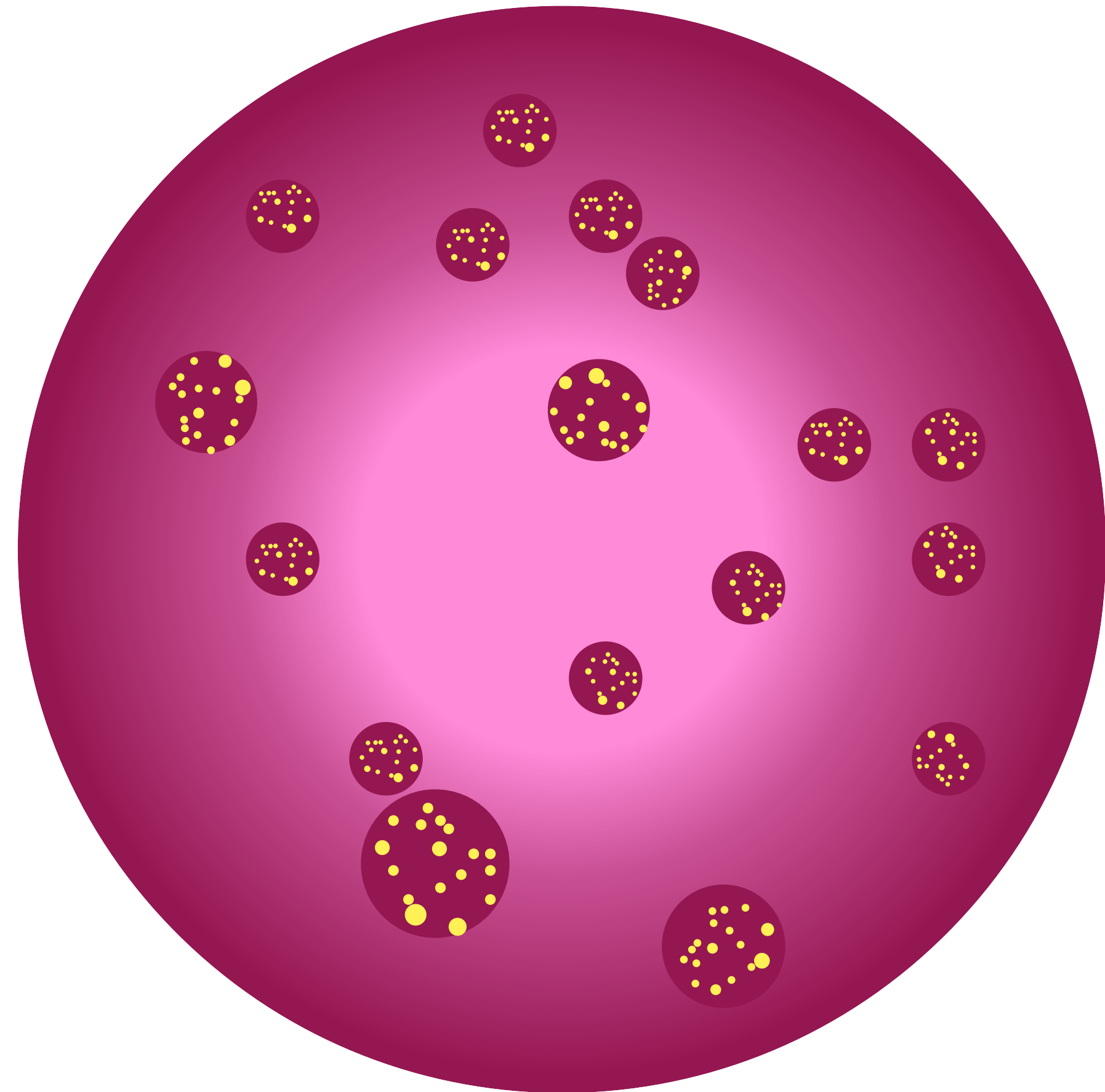
Multi-level boost factor?

So far, we considered one level of substructure within the parent halo. But hierarchical formation: haloes in haloes in haloes, etc...

Considering "point-like" subhalos, show that the 'boosted' luminosity for n levels of substructures can be recursively computed as

$$\mathcal{L}_n(M) = \mathcal{L}_{\text{sm}}(M) + \mathcal{L}_{\text{crossprod}}(M) + N_{\text{tot}}(M) \int_{M_{\text{min}}}^{M_{\text{max}}(M)} \mathcal{L}_{n-1}(M') \frac{d\mathcal{P}_M}{dM'}(M') dM'$$

with $\mathcal{L}_0(M, c) \equiv \int_{V_{\text{cl}}} [\rho_{\text{cl}}^{\text{tot}}(M, c)]^2 dV$



The J-factor

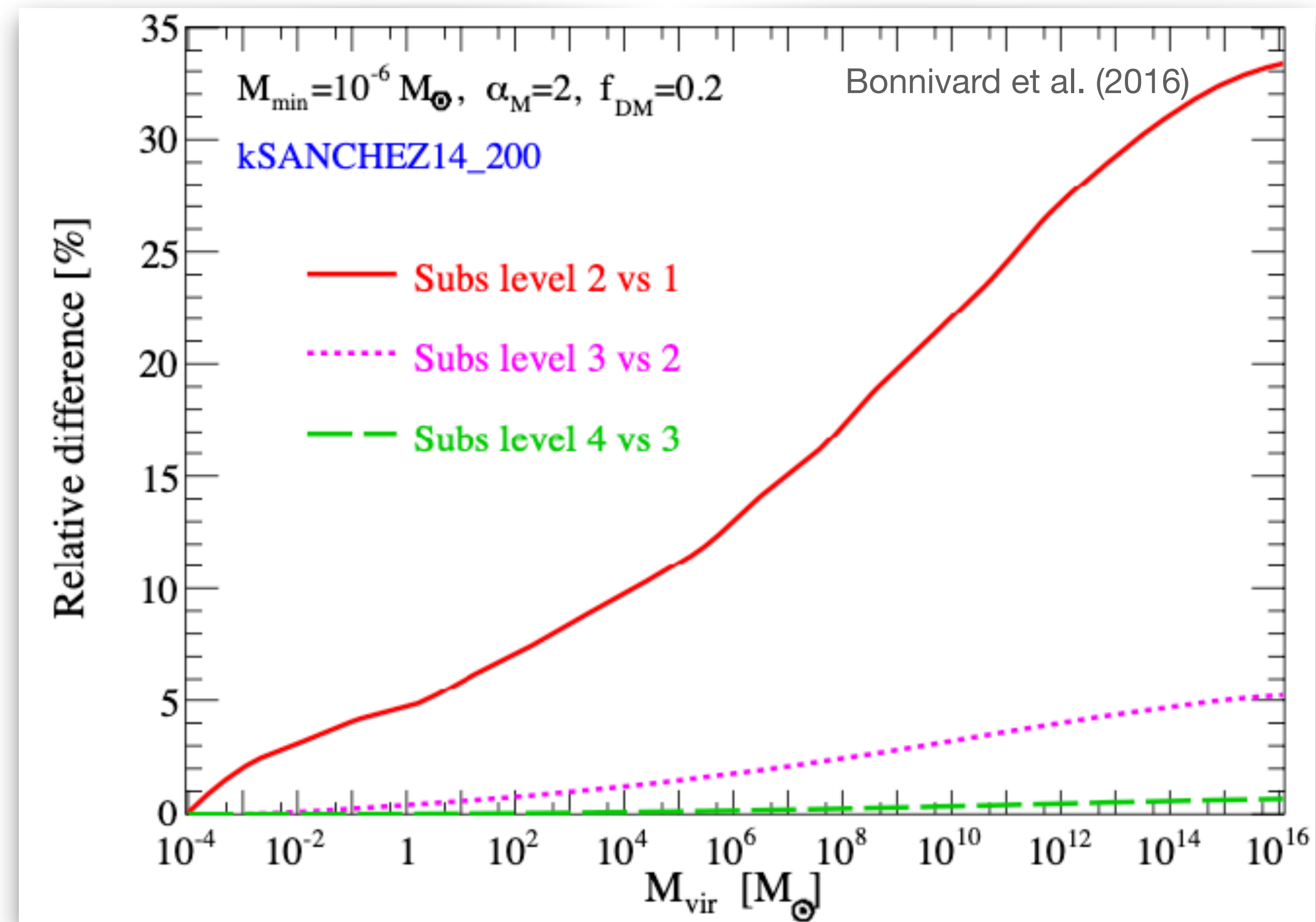
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$$\text{with } \mathcal{L}_0(M, c) \equiv \int_{V_{\text{cl}}} [\rho_{\text{cl}}^{\text{tot}}(M, c)]^2 dV$$



No much gain to go beyond $n=1$ or 2 and computationally expensive...

The J and D factors

Recap

$$J(\psi, \theta, \alpha_{\text{int}}) = \int_0^{2\pi} d\beta \int_0^{\alpha_{\text{int}}} \sin \alpha d\alpha \int_{\text{los}} \rho^2(l, \alpha, \beta; \psi, \theta) dl$$

- Need a robust estimation of the J-factor (or D-factor) to constrain the dark matter properties
 - * DM mass - $\langle \sigma v \rangle$ for annihilation
 - * DM mass - lifetime for decay
- Modeling of the DM distribution generally assumes spherical symmetry and
 - * a smooth DM component
 - * a subhalo population that may boost the annihilation signal
- Determination of the smooth and substructure component generally relies on
 - * results from numerical simulations (DM-only or hydro)
 - * and/or observational properties of the systems under scrutiny



Second half of the lecture



The J and D factors

Recap

$$D(\psi, \theta, \alpha_{\text{int}}) = \int_0^{2\pi} d\beta \int_0^{\alpha_{\text{int}}} \sin \alpha d\alpha \int_{\text{los}} \rho(l, \alpha, \beta; \psi, \theta) dl$$

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Second half of the lecture



Targets for indirect detection in
gamma-rays?

Targets

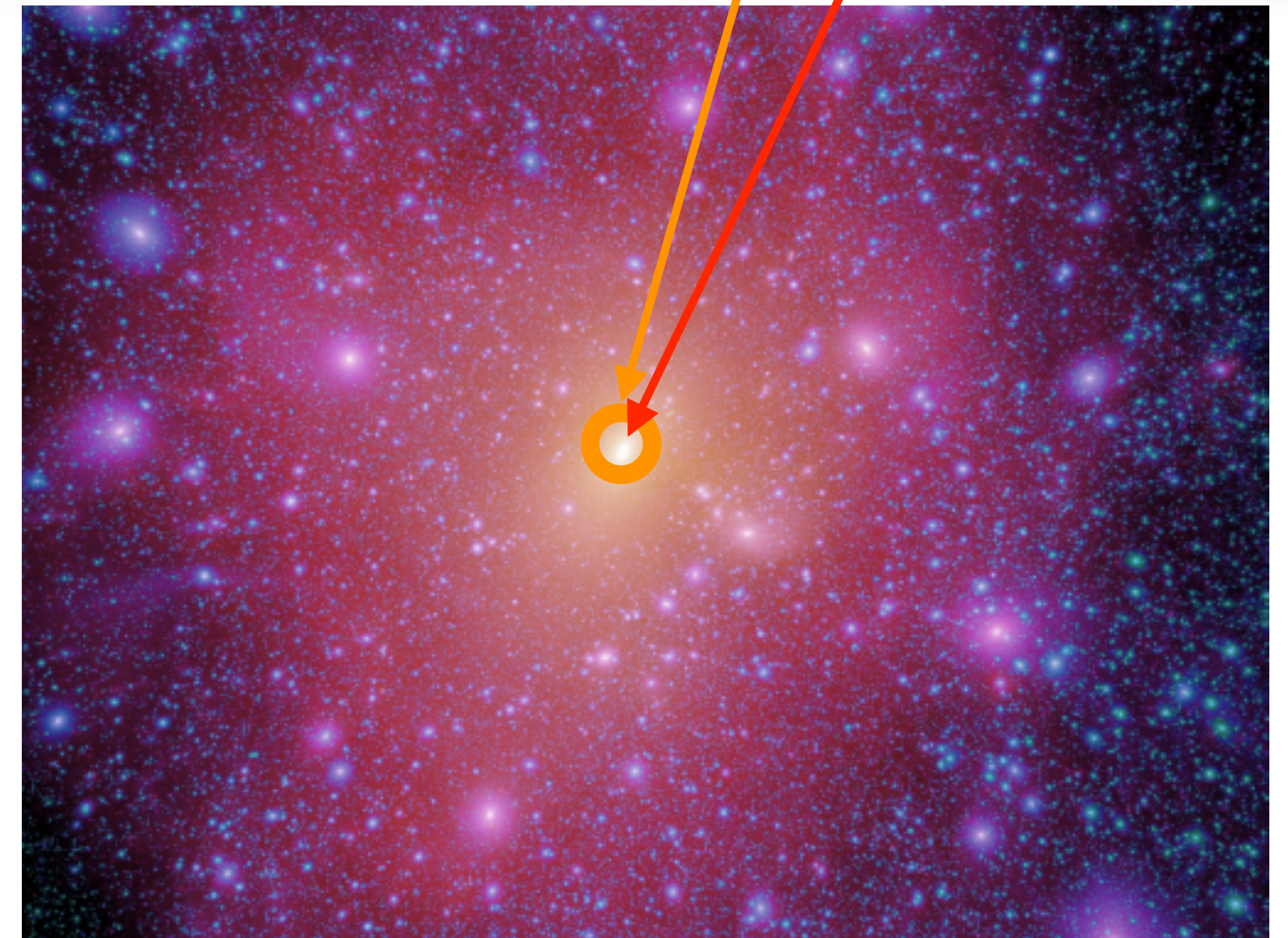
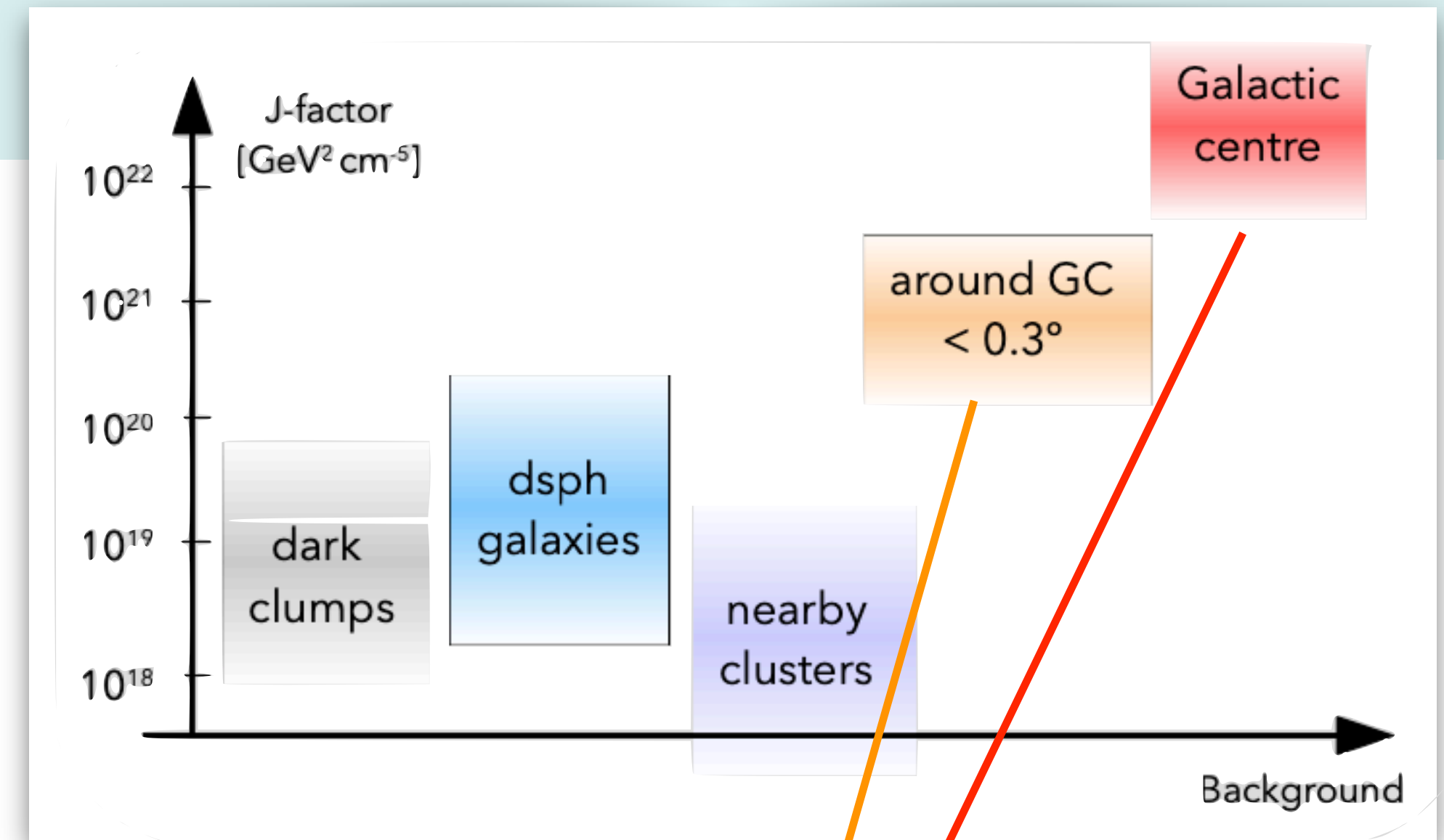
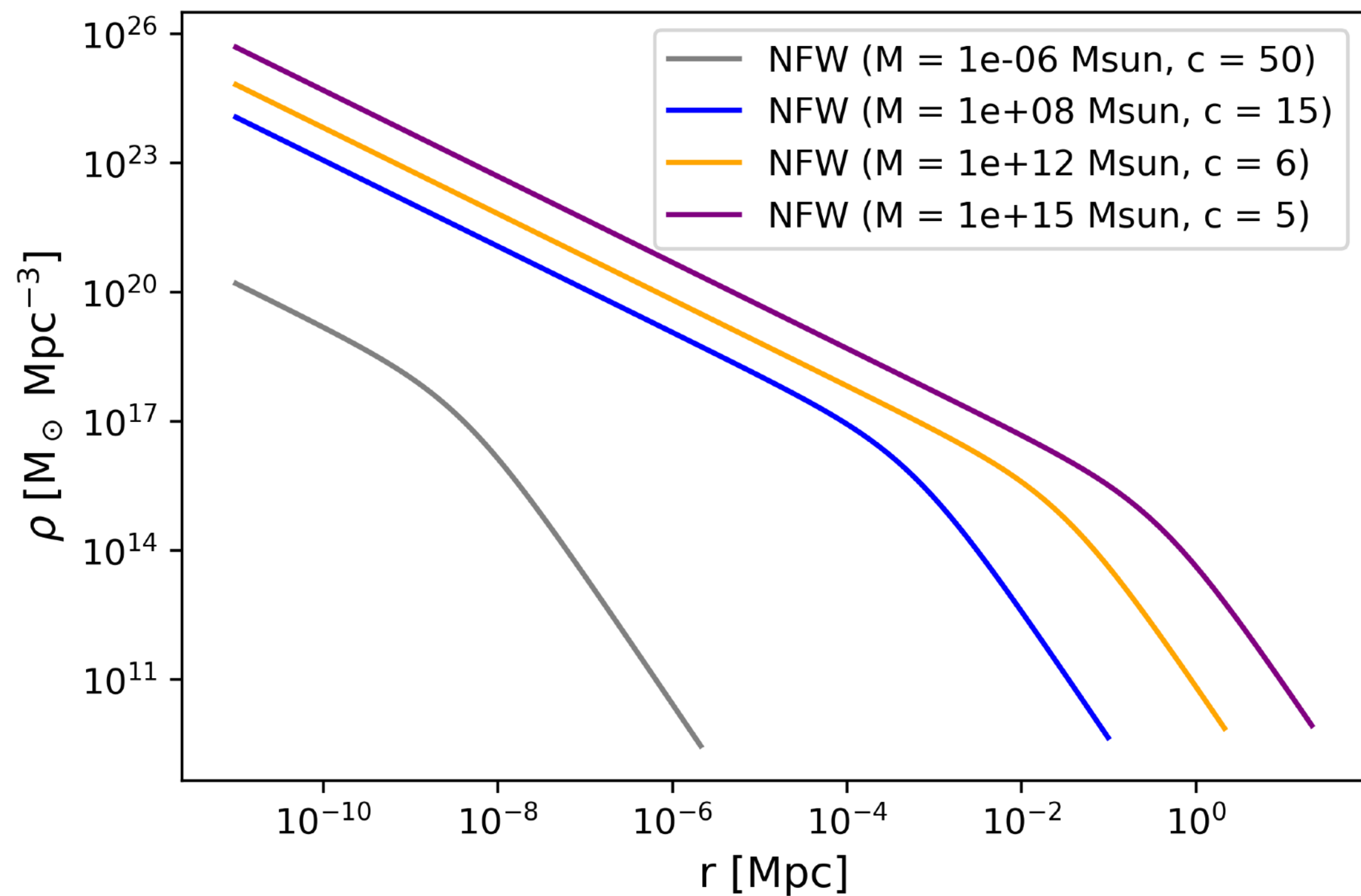
What makes a good target for indirect detection in γ -rays?

- it is massive/dense (and we have means to evaluate its density) [Remember $J_{\text{point}} \sim \frac{M^2}{d^2V}$]
- it is located close to us
- it has little astrophysical gamma-ray background
- optional: it is visible at other wavelengths [so we know where to look]

Targets

What makes a good target for indirect detection in γ -rays?

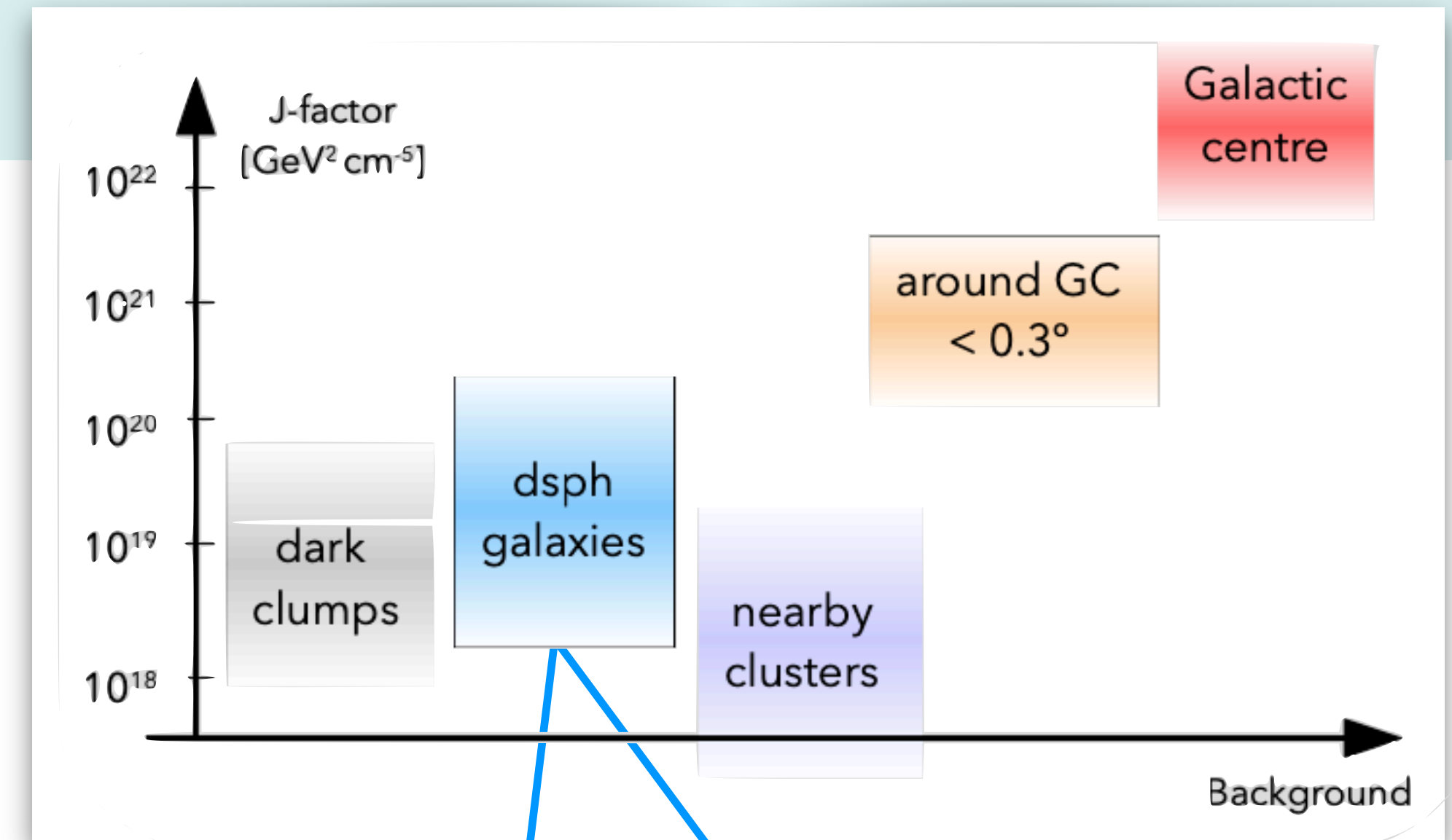
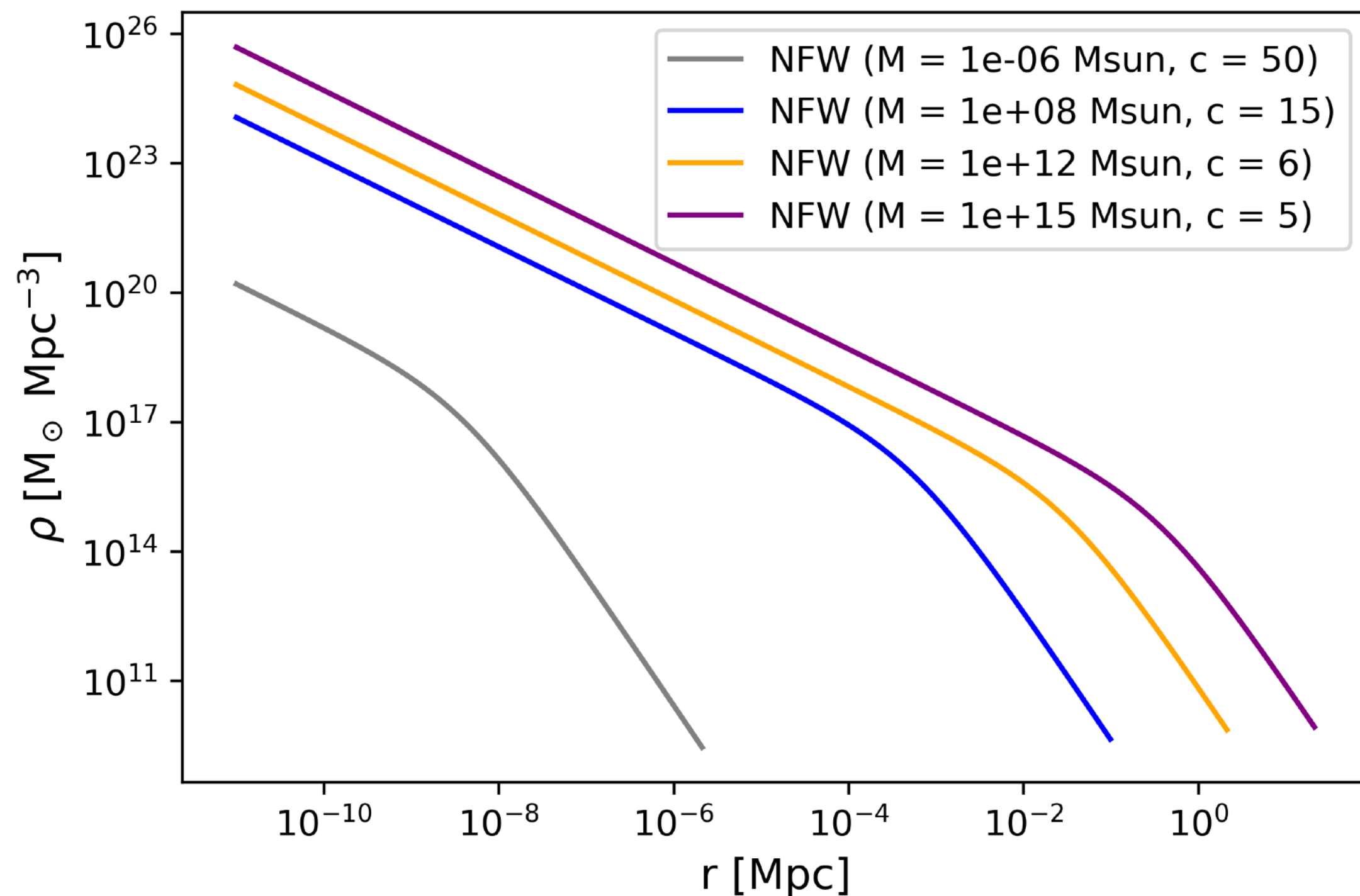
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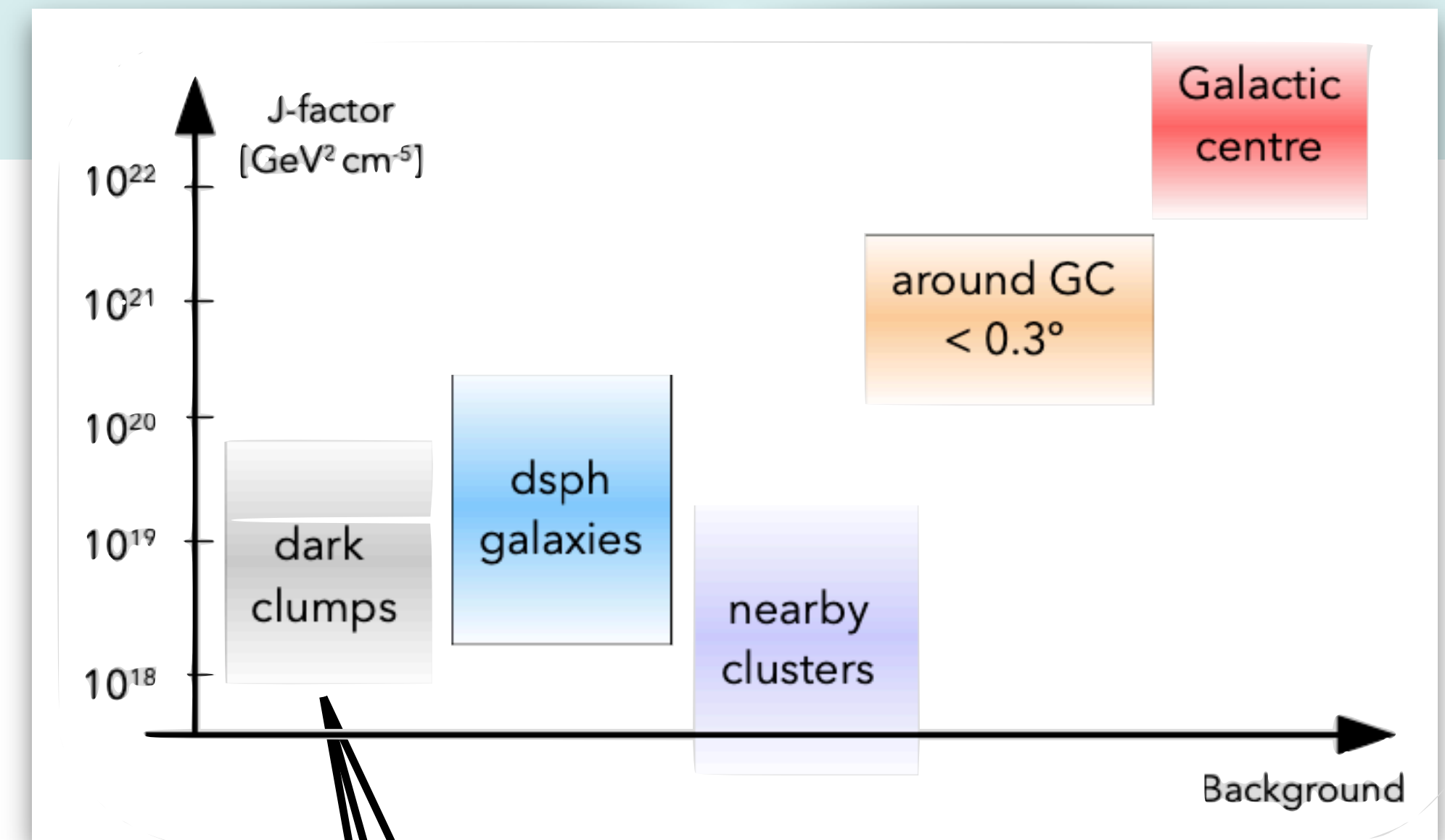
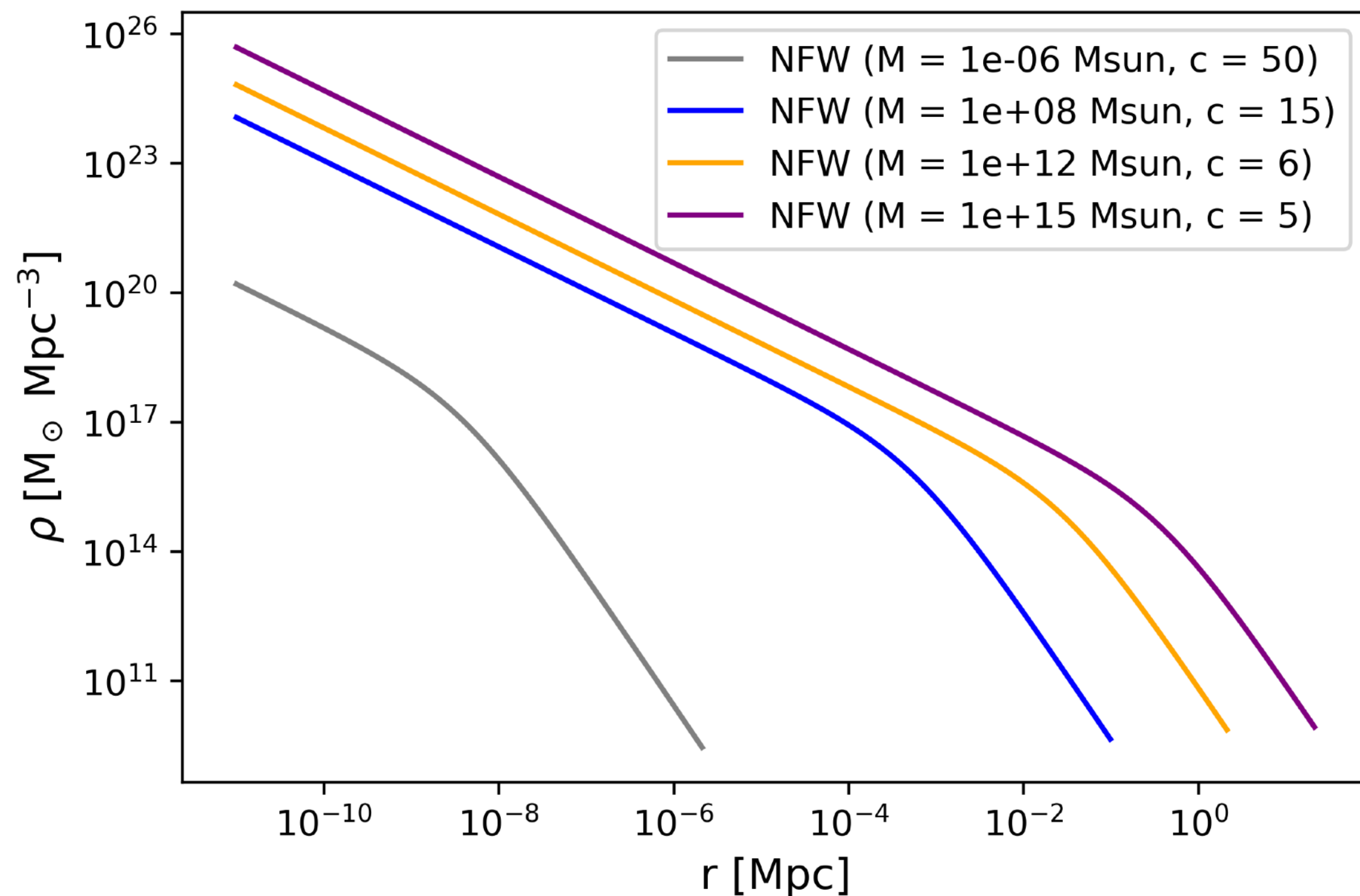
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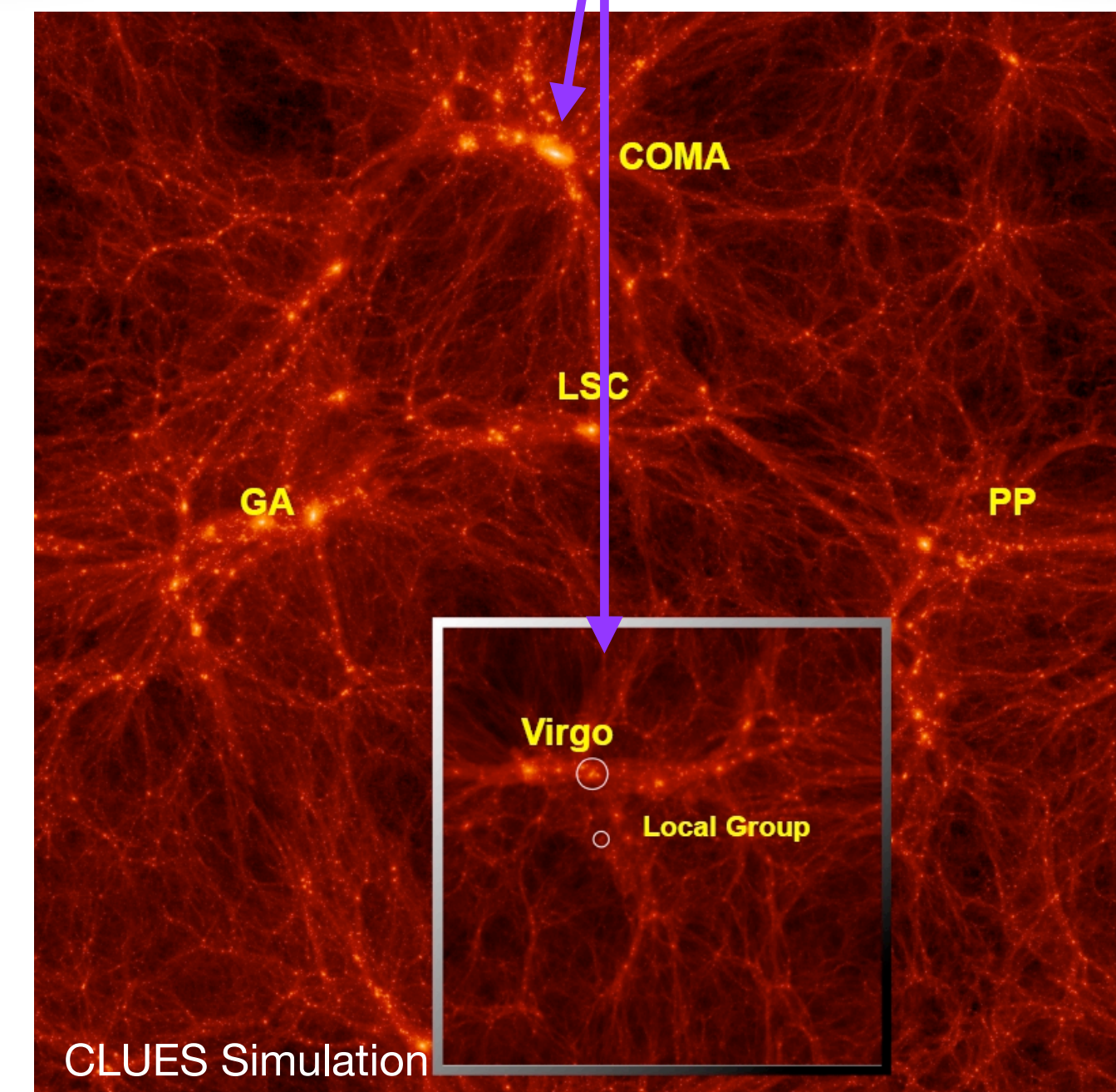
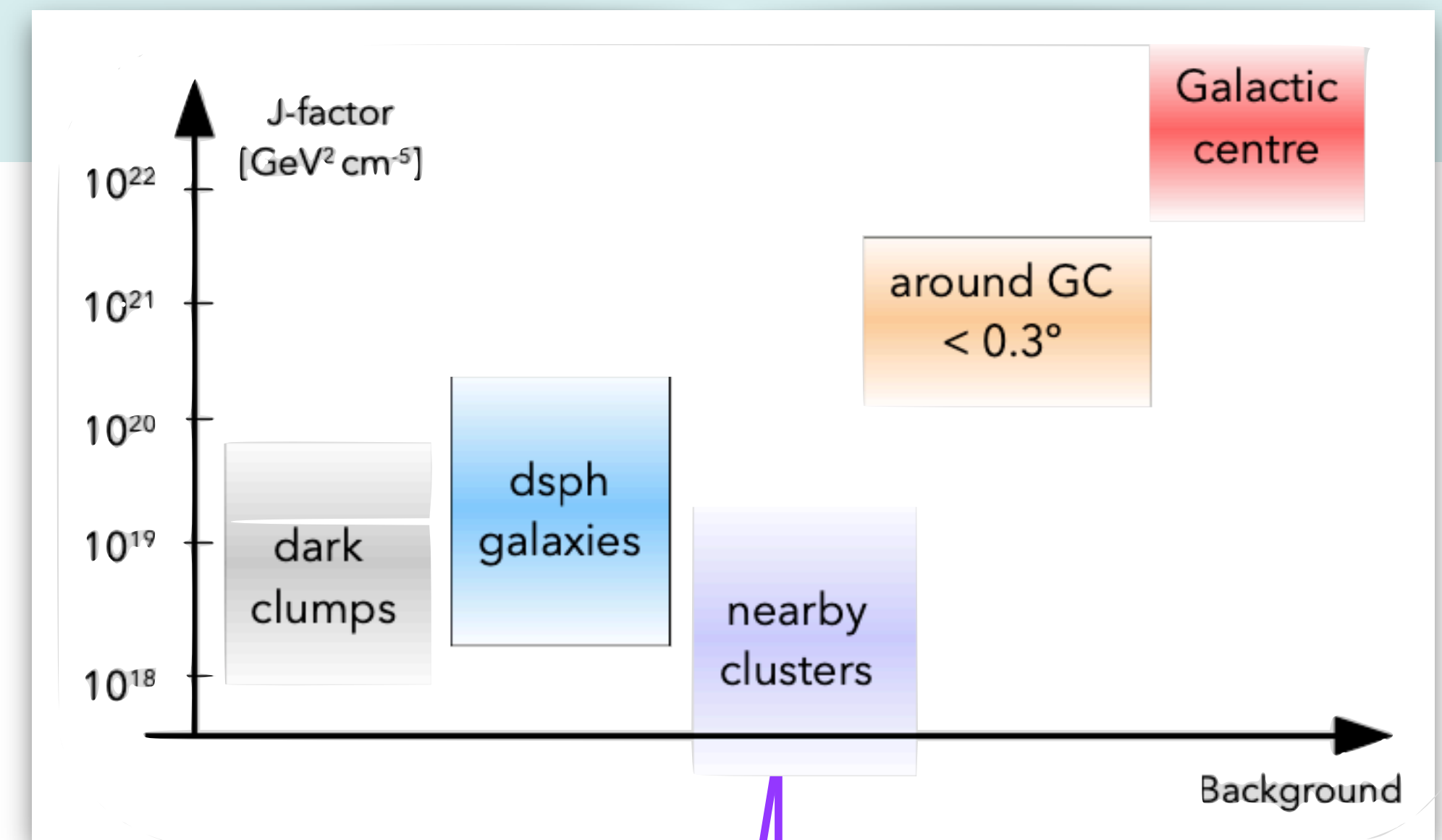
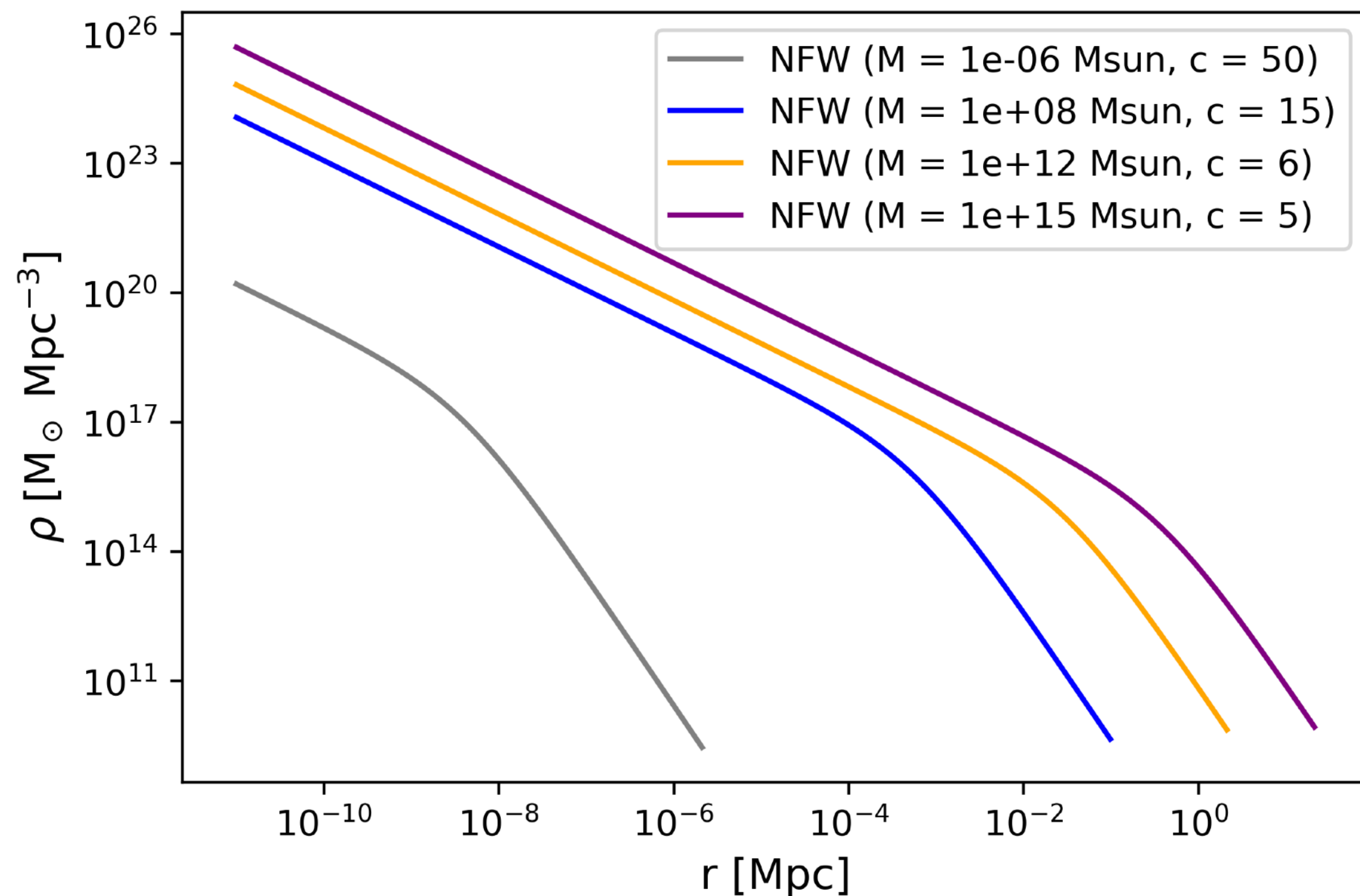
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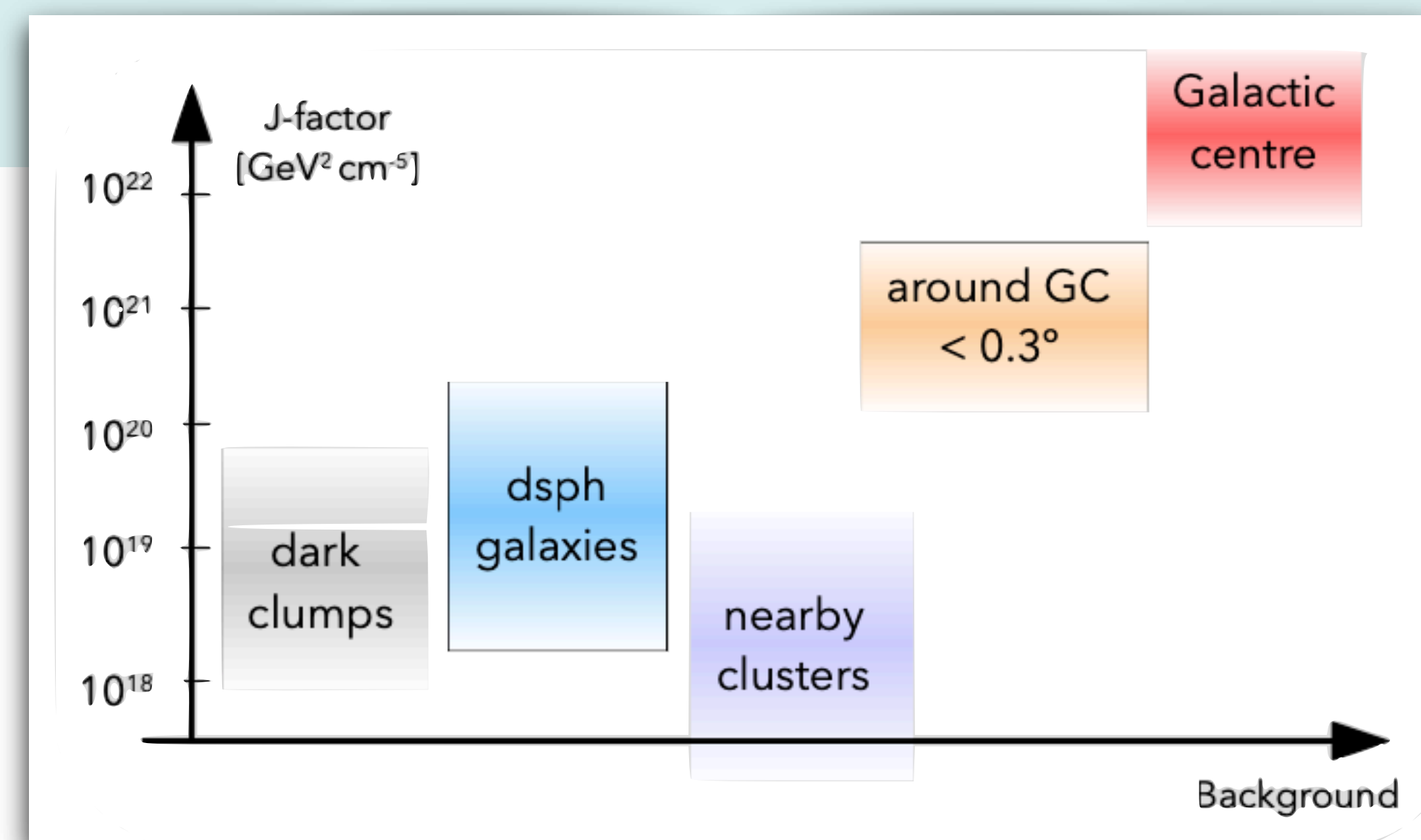
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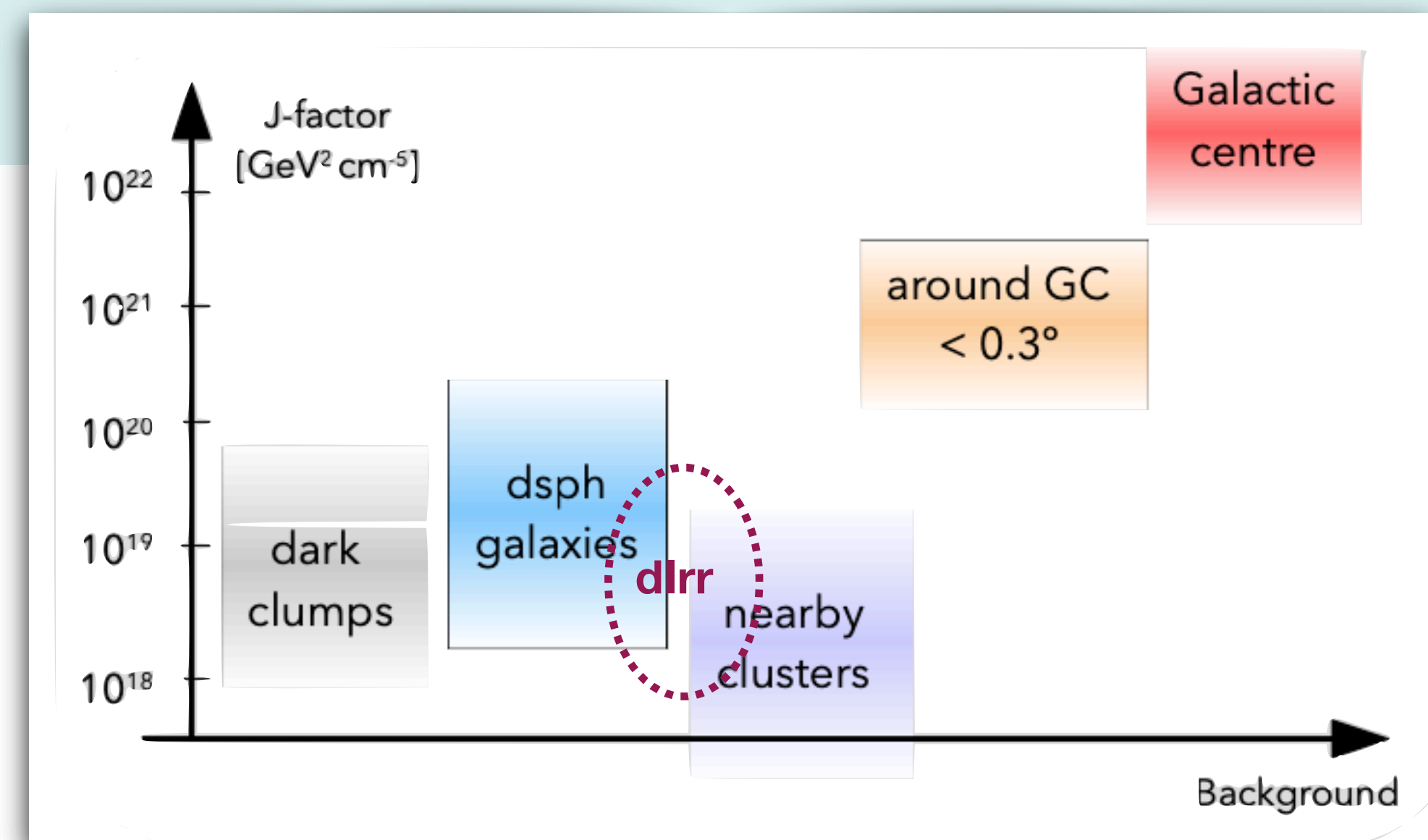


Now: go through the specifics of the DM modeling of each type of targets!

Targets

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Now: go through the specifics of the DM modeling of each type of targets!

NB - Not covered in this lecture: dwarf irregular galaxies have recently joined the list of possible targets

- $d\text{Sph} < M_{\text{irr}} < \text{MW-like galaxies}$
- relevant when part of the local group ($d \sim \text{Mpc}$)
- star forming regions, so may have gamma-ray background

DM modeling of galactic targets

1. Galactic center region
2. Dwarf spheroidal galaxies
3. Dark galactic clumps

The Galactic halo - central regions

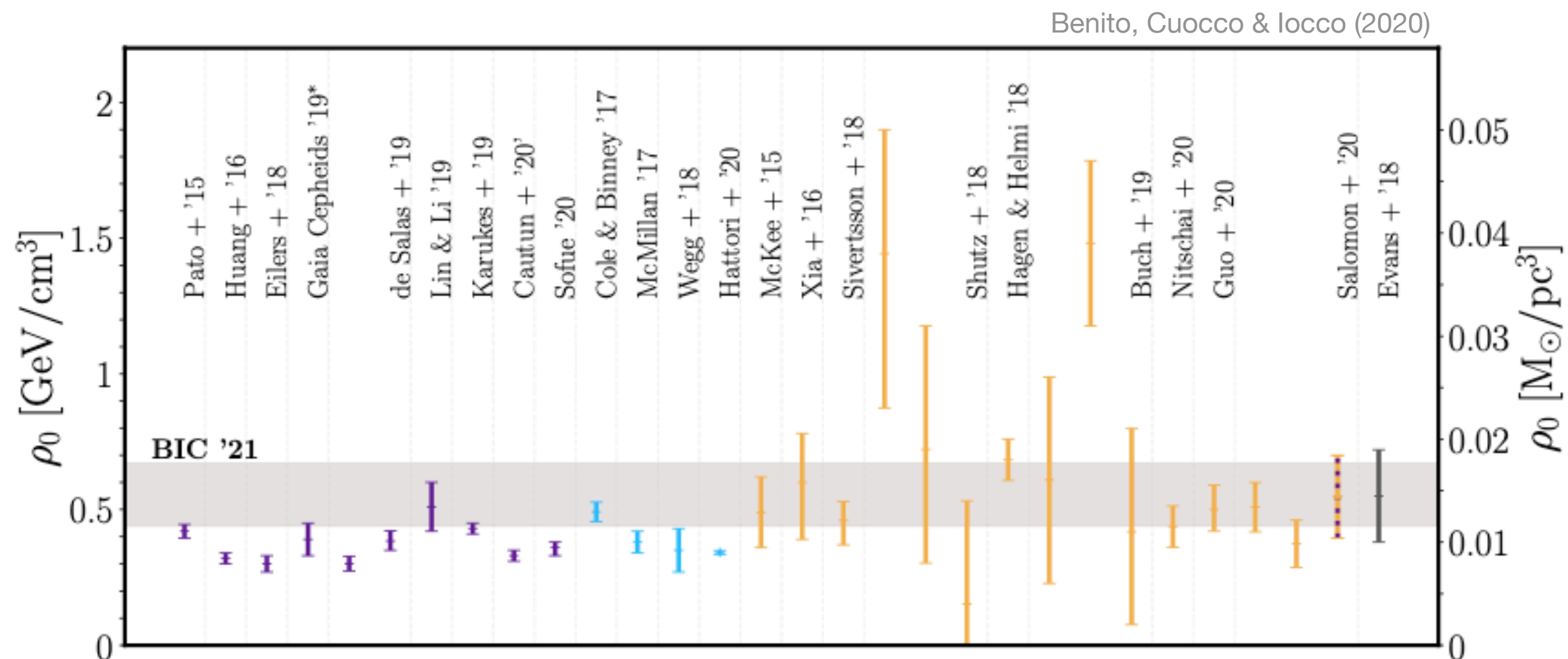
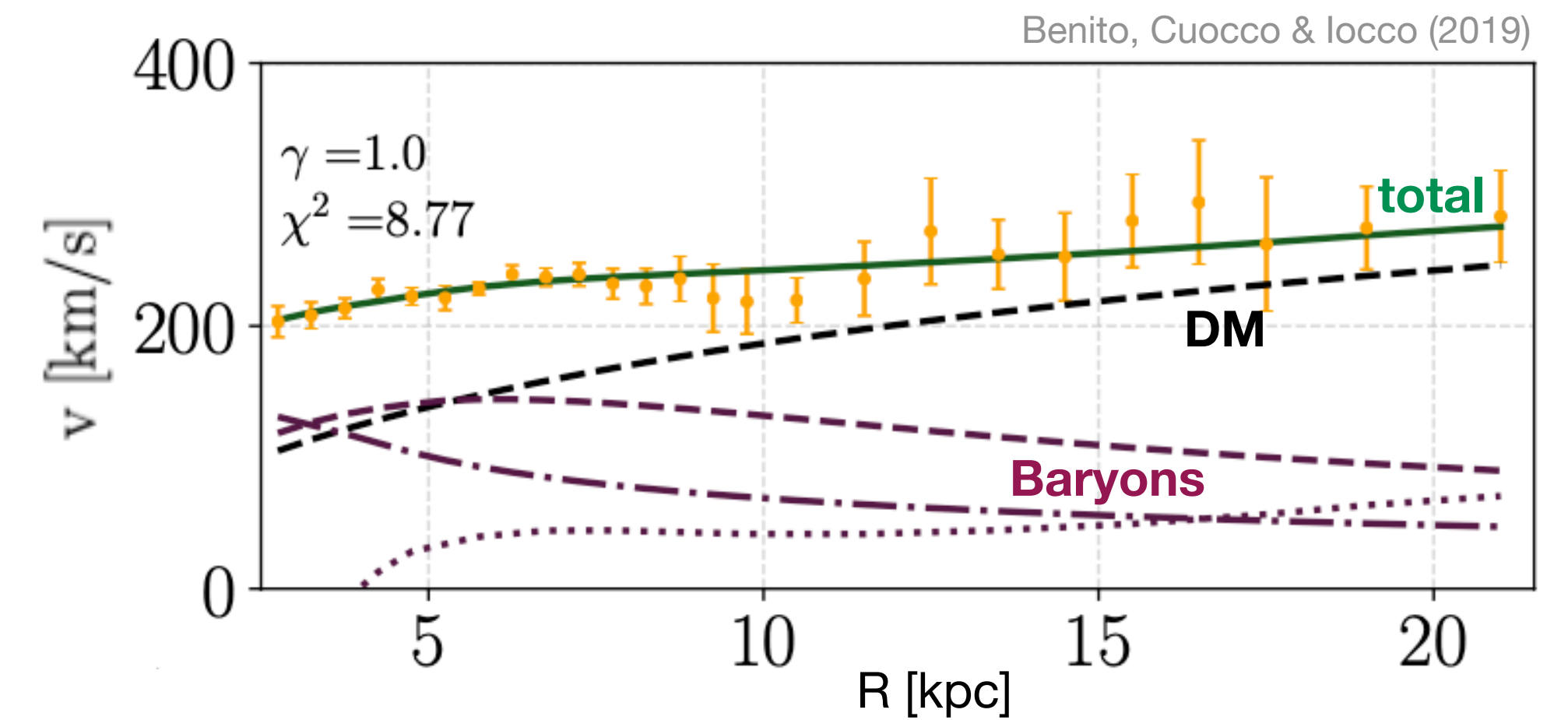
- The Milky Way sits in a $\sim 10^{12} M_{\text{sun}}$ DM halo and the Earth is located at 8.5 kpc from the center
→ suggests large J-factor, hence prime target for indirect detection
- However, large gamma-ray emission from astrophysical processes (see M. Doro's lecture)
→ Not ideal, and complex data analysis
- Need an estimation of the DM profile of the MW

Modeling of the Galactic halo

Adjust multi-component MW mass modeling on Galactic rotation curves

- dark matter halo component (NFW or free inner slope)
- stellar disk
- gas
- stellar bulge/bar

to a compilation of kinematic data (stars, masers, gas, K-stars)



Depending on analyses, determination of the **local DM density** but also parameters of the DM profile

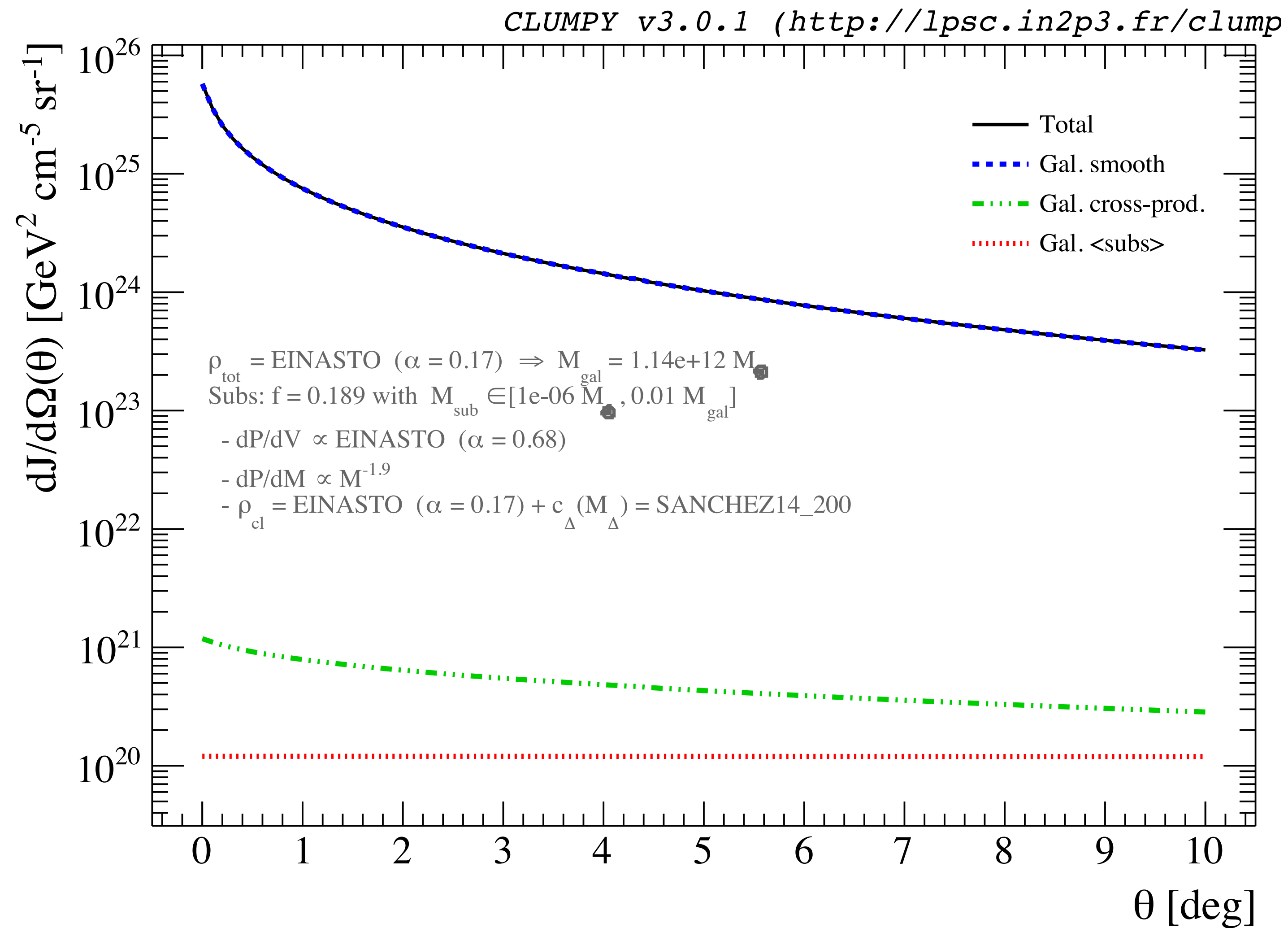
Inner slope of the profile is not well constrained

Modeling of the Galactic halo

Recipe

1. Find MW halo parametrisations/normalisation from the literature, either simulations and/or global MW modeling typically
 - NFW profile, with $r_s \sim 20$ kpc
 - Einasto, with $r_{-2} \sim 20$ kpc and $\alpha \sim 0.17$
 - $\rho_{\odot} = \rho(R_{\odot}) = 0.4 \text{ GeV cm}^{-3} \rightarrow \rho_s$
 - Alternatively, provide ρ_{\odot} and $M(< R)$
2. Indirect detection towards the Galactic centre (or close to the GC)
 - Explore various inner slopes as not well constrained (e.g. H.E.S.S or Fermi-LAT galactic centre analyses)

The Galactic halo - central regions



Towards the Galactic centre, substructures may be neglected and only need to model the smooth component

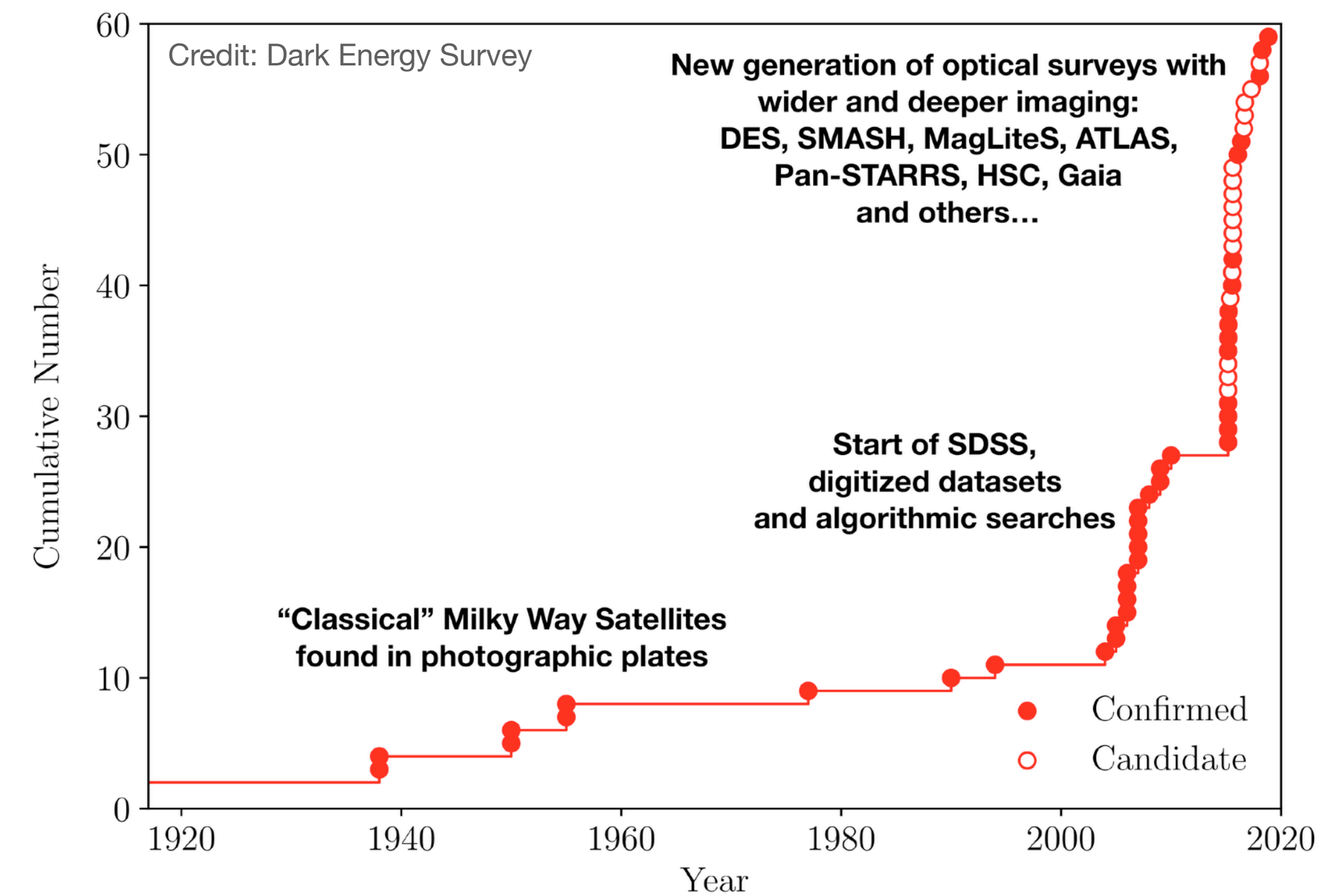
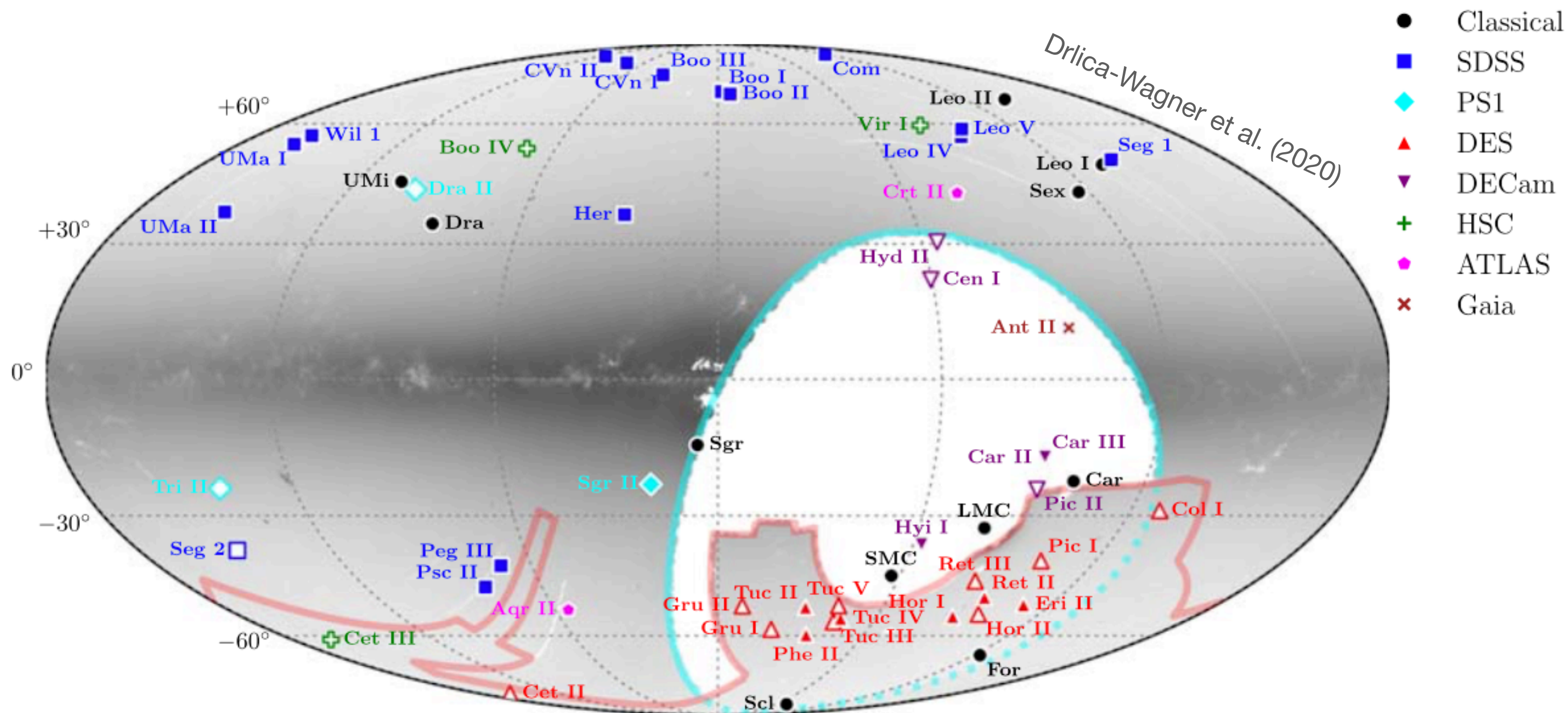
DM modeling of galactic targets

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Satellite galaxies of the MW

Overview

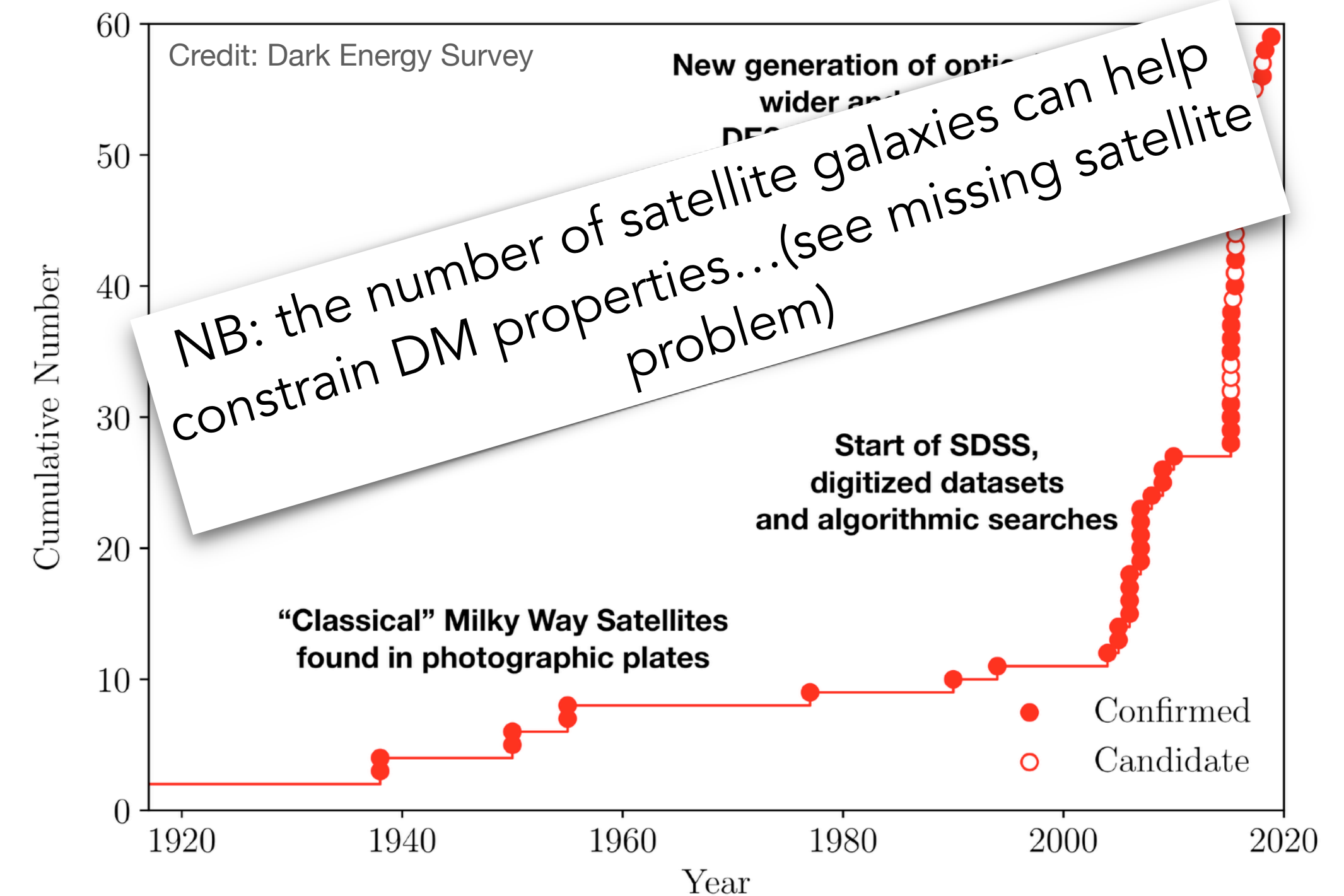
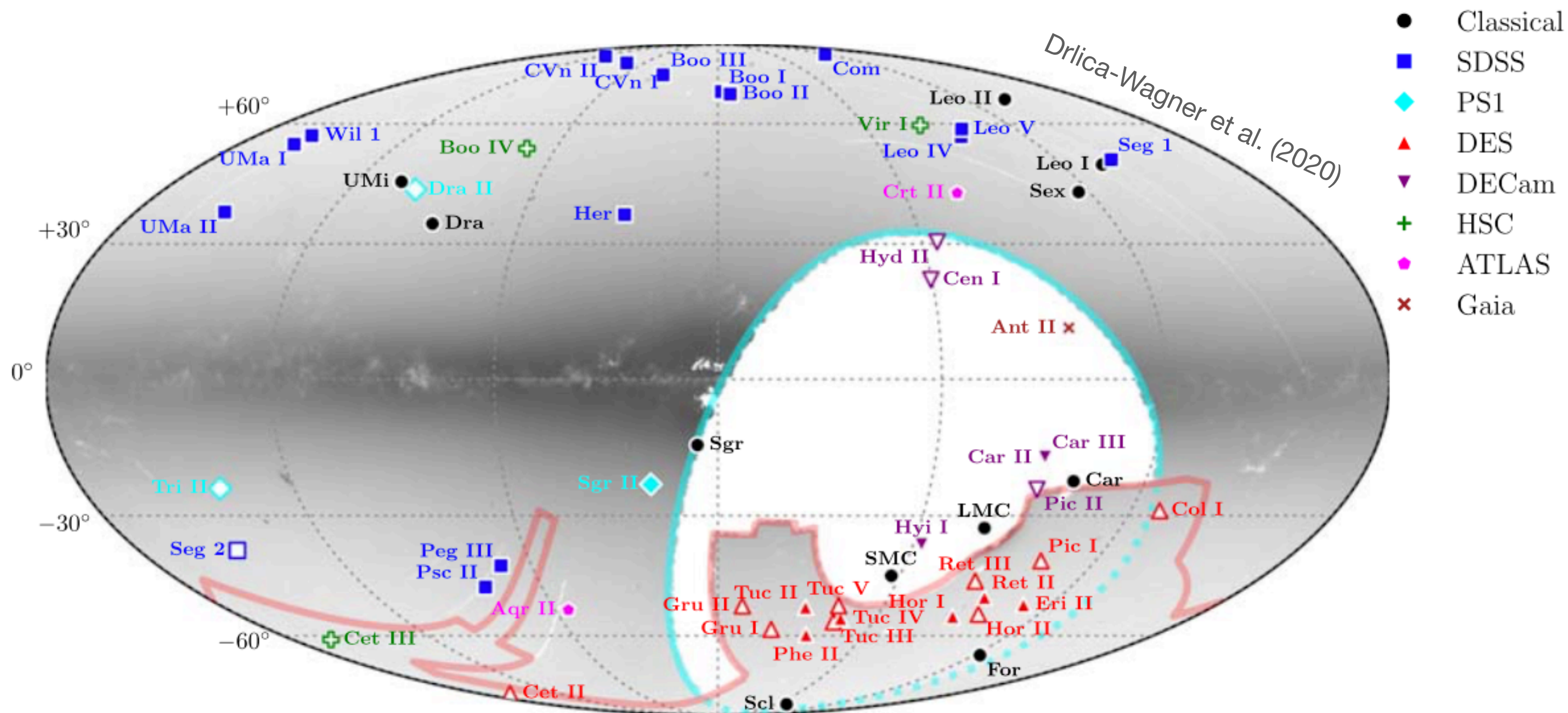
- Population of faint galaxies orbiting the MW
- No gamma-ray background
- Distance: $\sim 20 - 300$ kpc
- Size $\sim 10^{-2}$ size of spiral galaxies



Satellite galaxies of the MW

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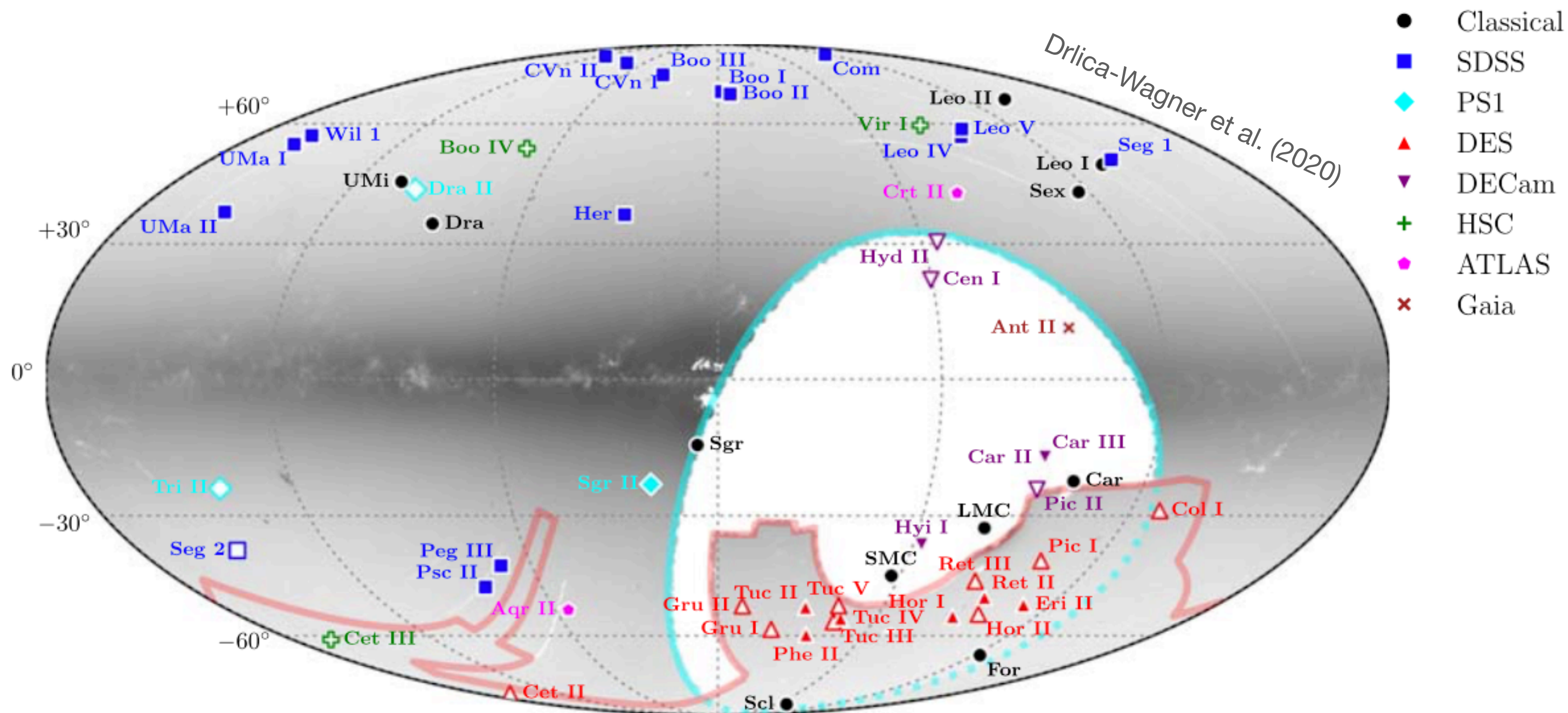
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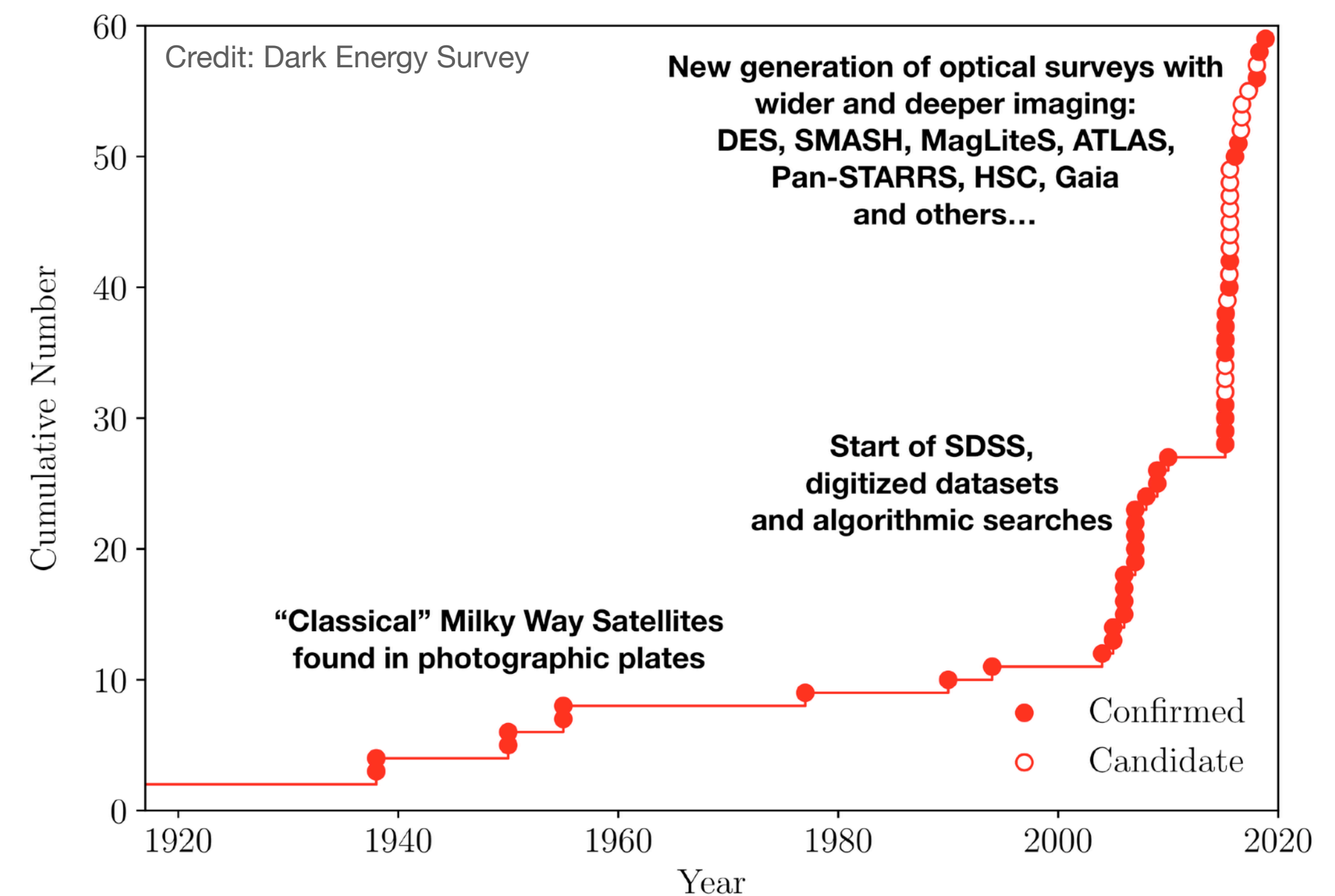
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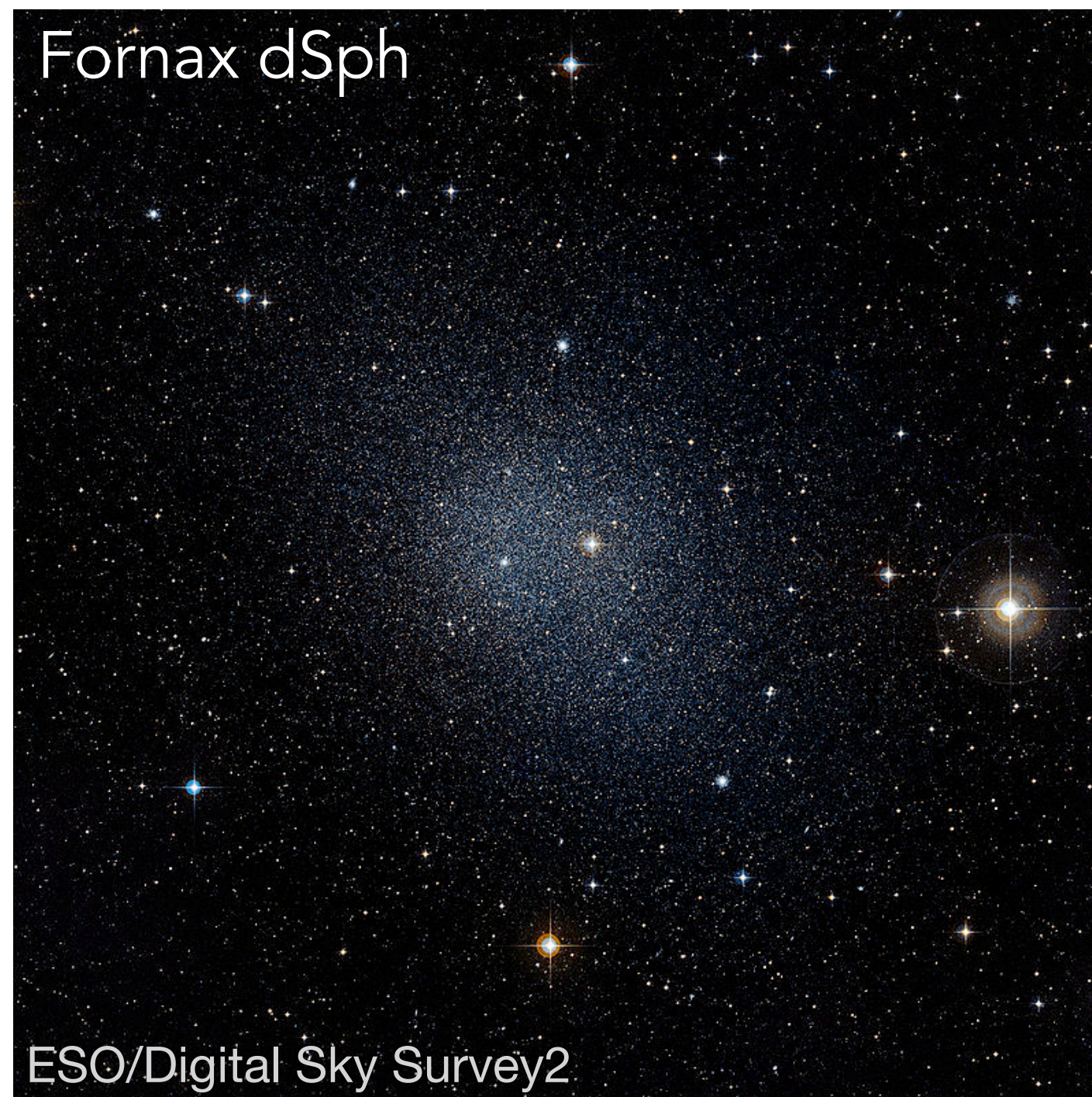
Strongly DM-dominated systems!
Great targets for indirect detection



Dwarf spheroidal galaxies

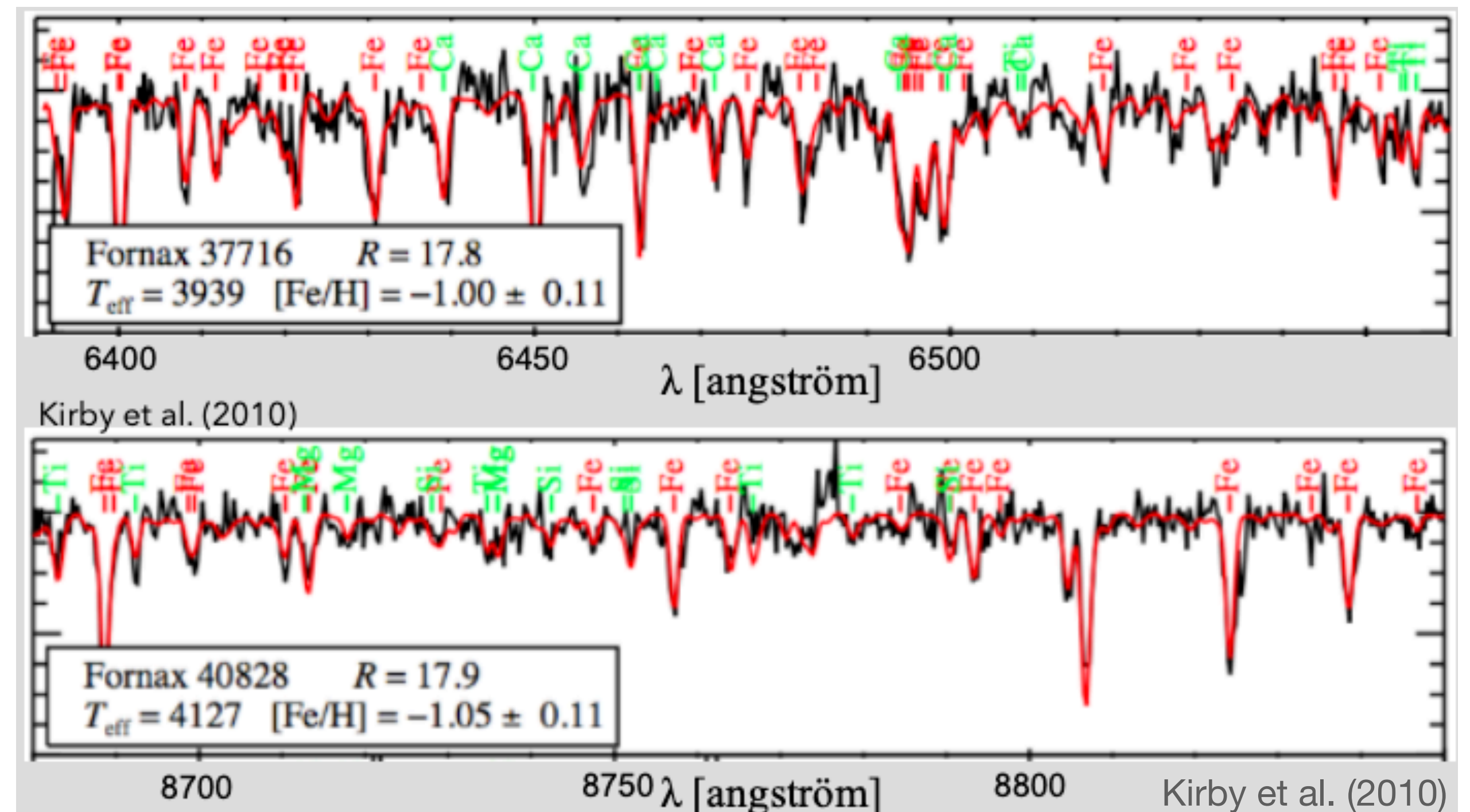
Observables

Photometry



"Light profile" → dSph candidate

Spectroscopy of individual stars in the object



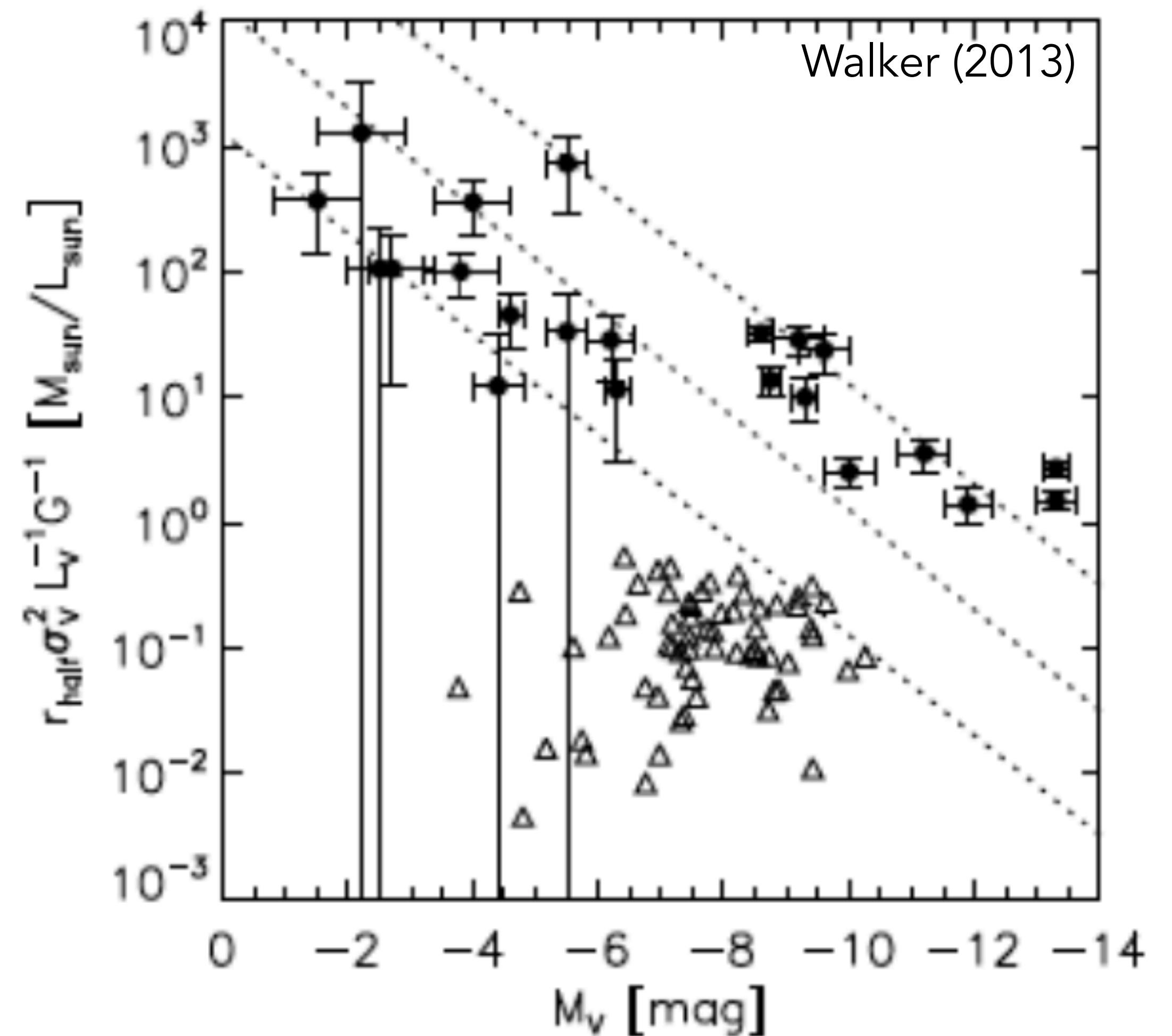
Dynamics - velocity dispersion σ^2 → dSph status confirmation

Dwarf spheroidal galaxies

DM-dominated systems

Virial theorem $\sigma^2 \sim \frac{GM}{R}$

Mass-to-light ratio $\frac{M}{L} \sim \frac{R\sigma^2}{L_V G}$



Dwarf spheroidal galaxies

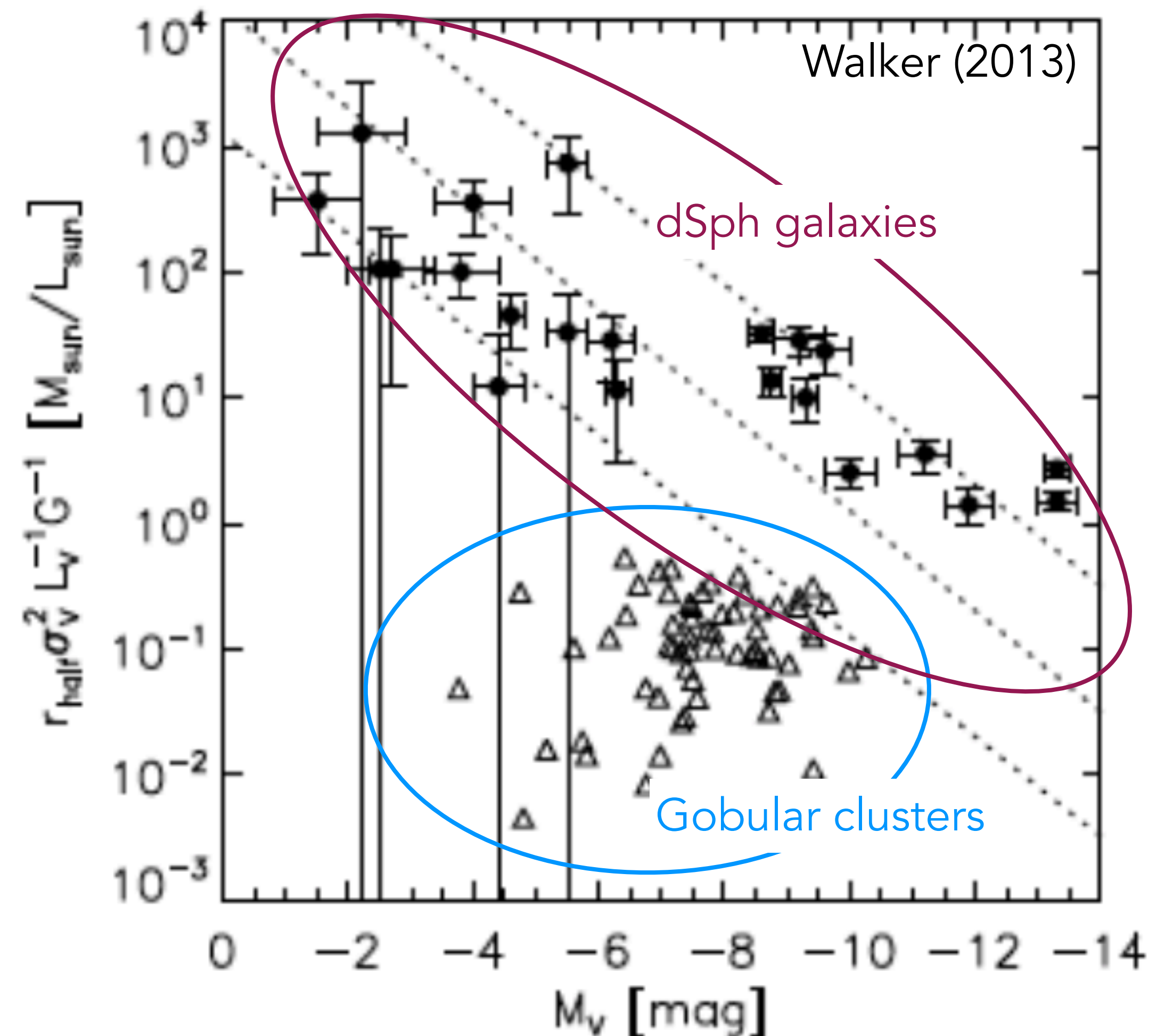
DM-dominated systems

Virial theorem $\sigma^2 \sim \frac{GM}{R}$

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Dsph galaxies have large M/L ratios indicating DM-dominated systems

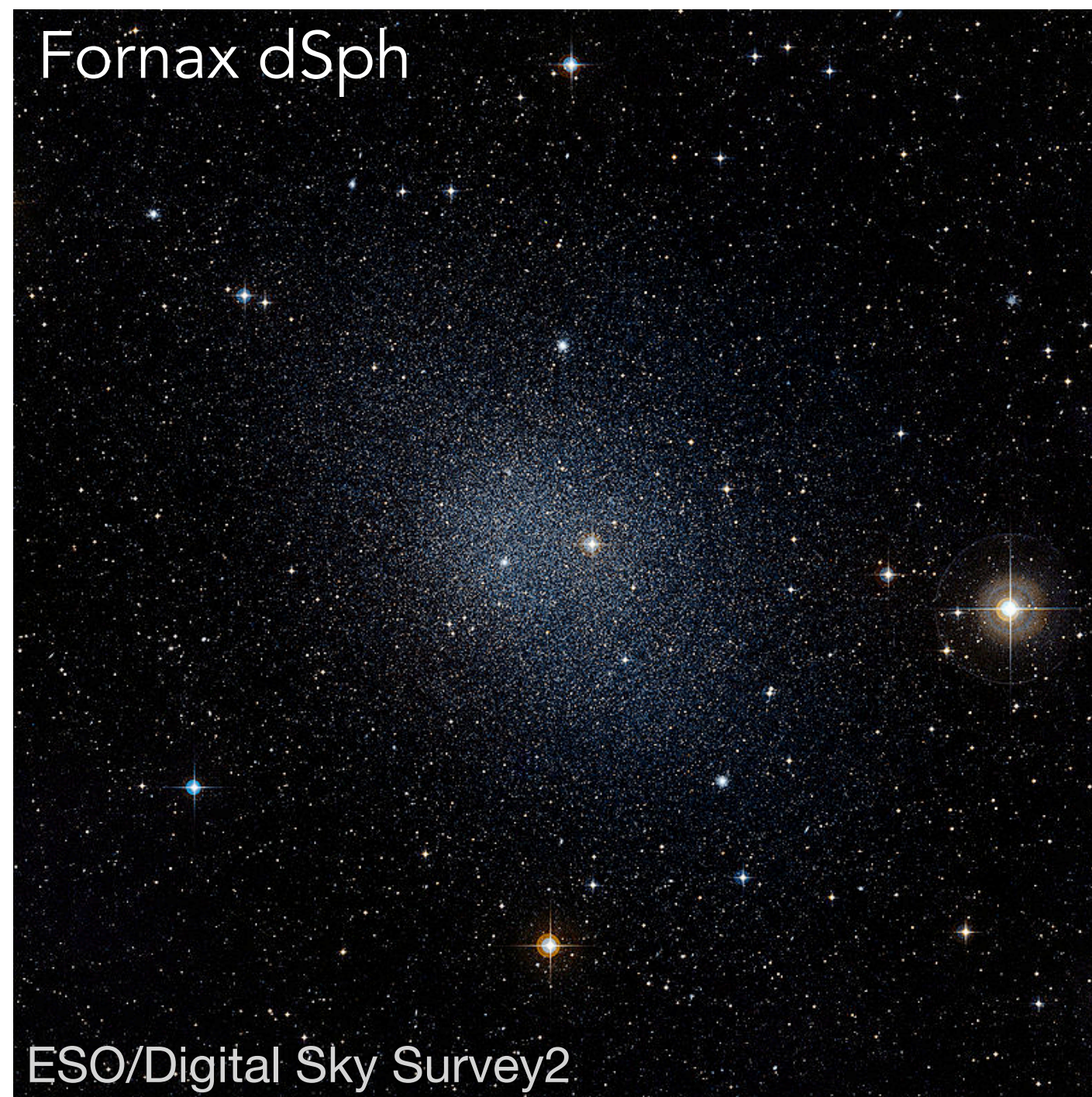
How can we constrain the DM profile in those object for robust estimation of the J-factor?



Dwarf spheroidal galaxies

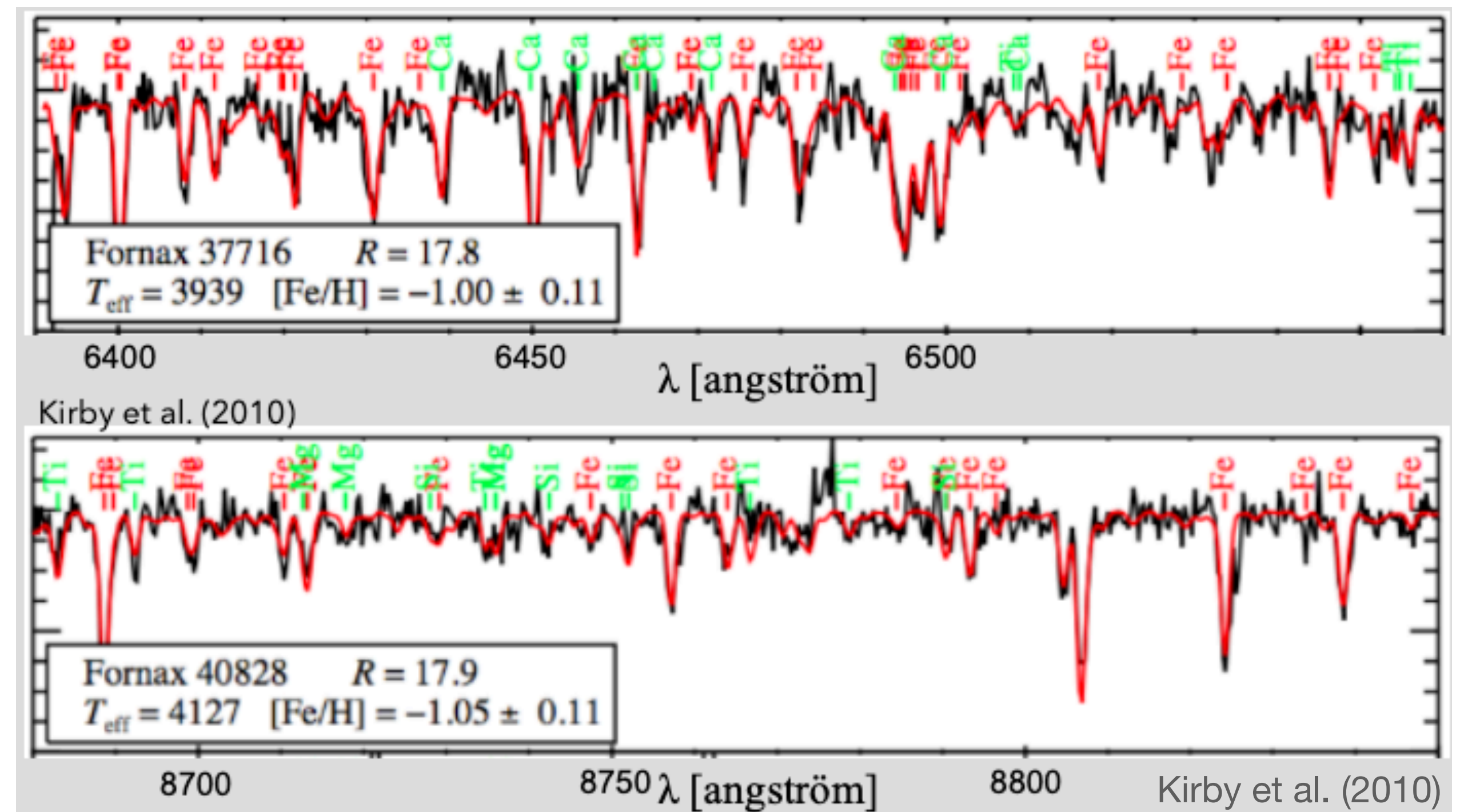
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"Light profile" $\rightarrow I(R)$

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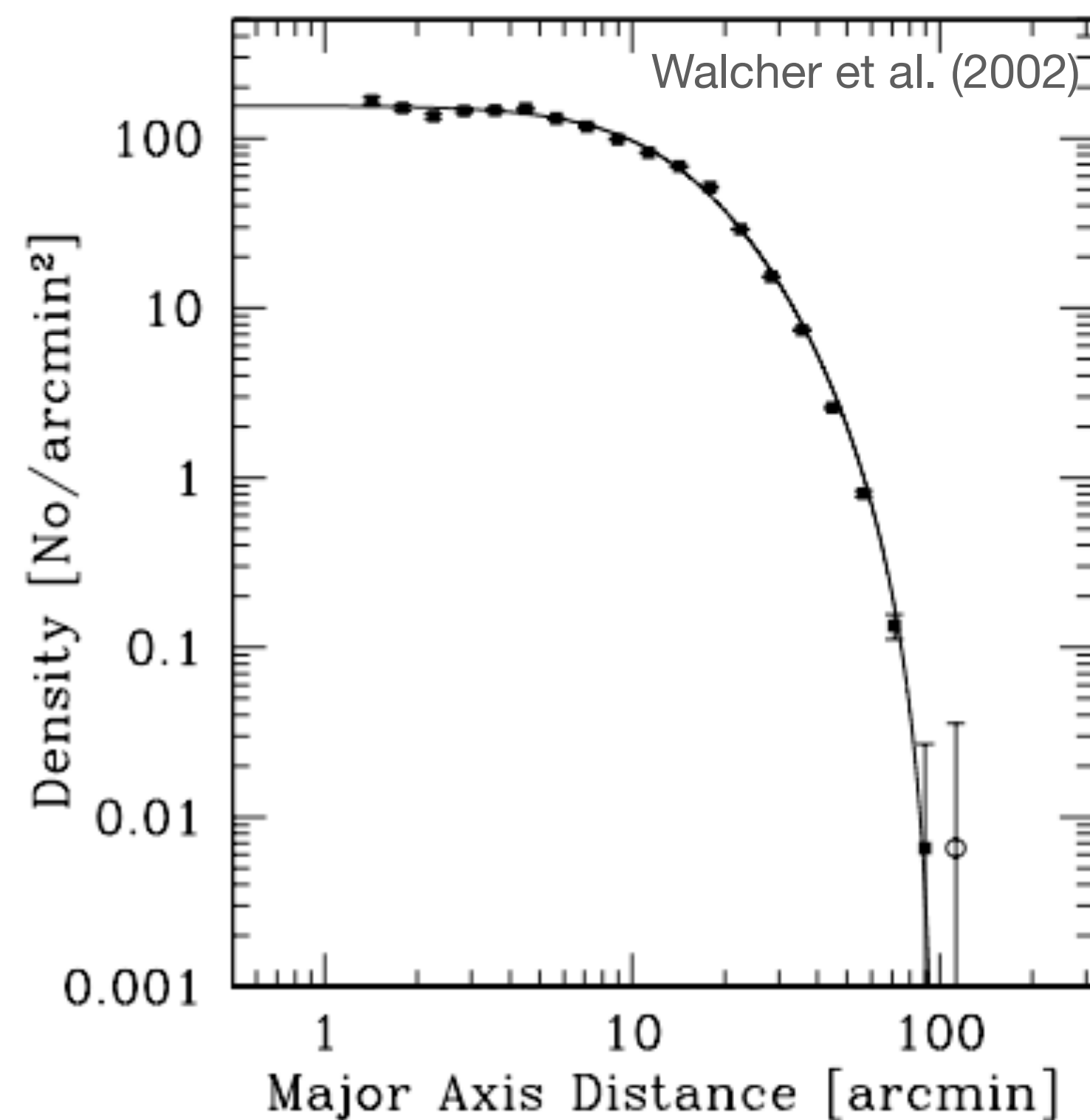


Dynamics - velocity dispersion $\sigma_p^2(R)$

Dwarf spheroidal galaxies

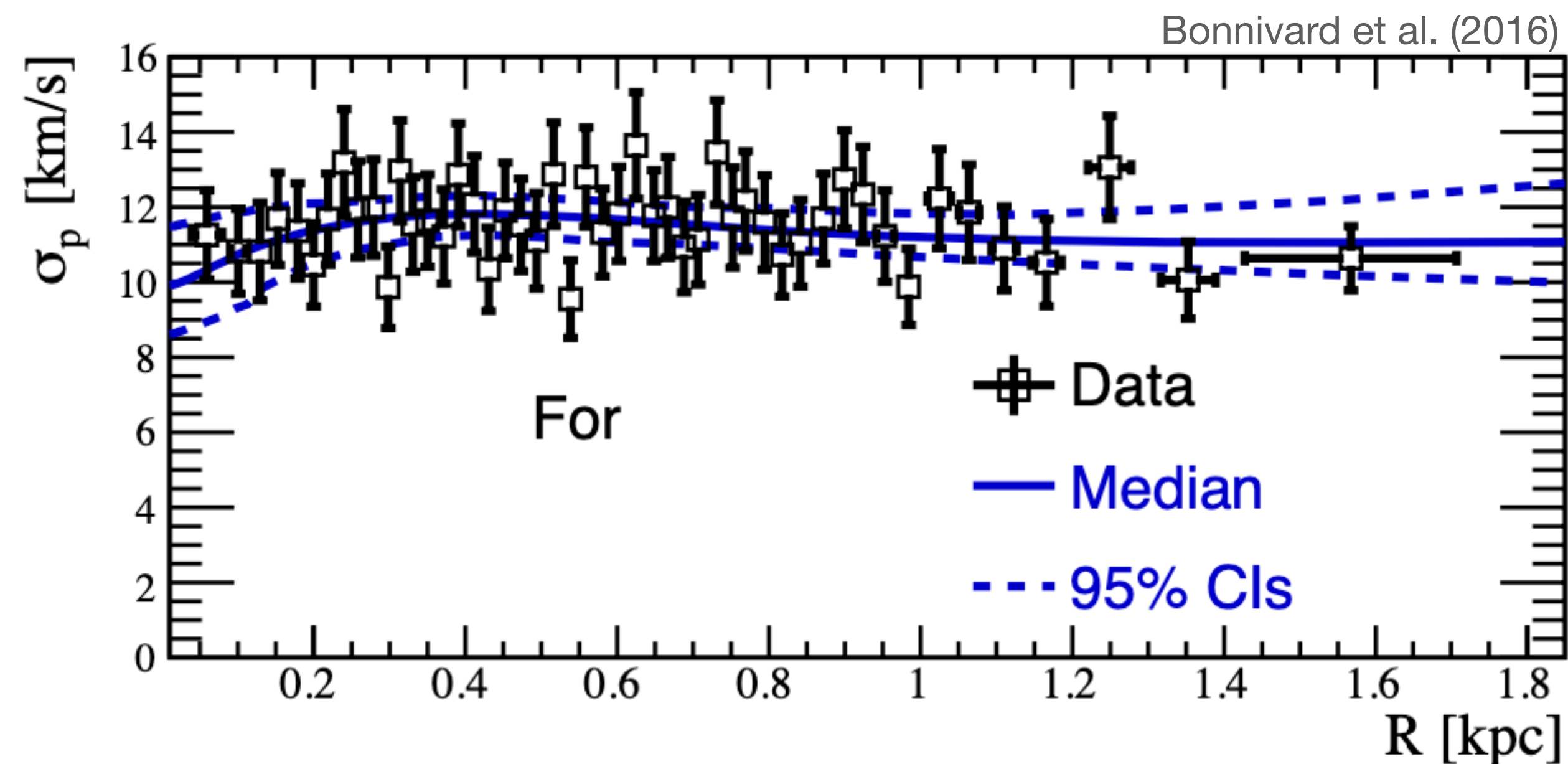
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Spectroscopy of individual stars in the object



Dynamics - velocity dispersion $\sigma_p^2(R)$

Dwarf spheroidal galaxies

Jeans modeling

Light and velocity dispersion profiles

$$\begin{array}{ccc}
 \downarrow & I(R) & \sigma_p^2(R) \uparrow \\
 \text{de-project} & & \text{project} \\
 \downarrow & \nu(r) & \overline{v_r^2}(r) \downarrow
 \end{array}$$

Jeans equation: solve for $\nu \overline{v_r^2}$

$$\frac{1}{\nu(r)} \frac{d}{dr} (\nu(r) \overline{v_r^2}(r)) + 2 \frac{\beta_{\text{ani}}(r) \overline{v_r^2}(r)}{r} = - \frac{GM(r)}{r^2}$$

$$\beta_{\text{ani}} = 1 - \overline{v_\theta^2} / \overline{v_r^2} \quad M(r) = \int_0^r 4\pi s^2 \rho(s) ds$$

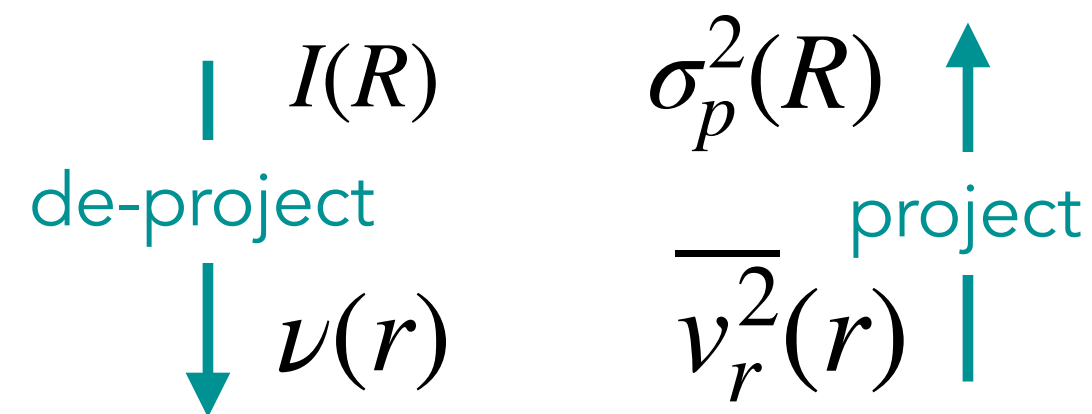
given a DM profile parameterisation
(NFW, Einasto, or more general form)

$$\rho(r) = f(\rho_s, r_s, \alpha_i)$$

Dwarf spheroidal galaxies

Jeans modeling

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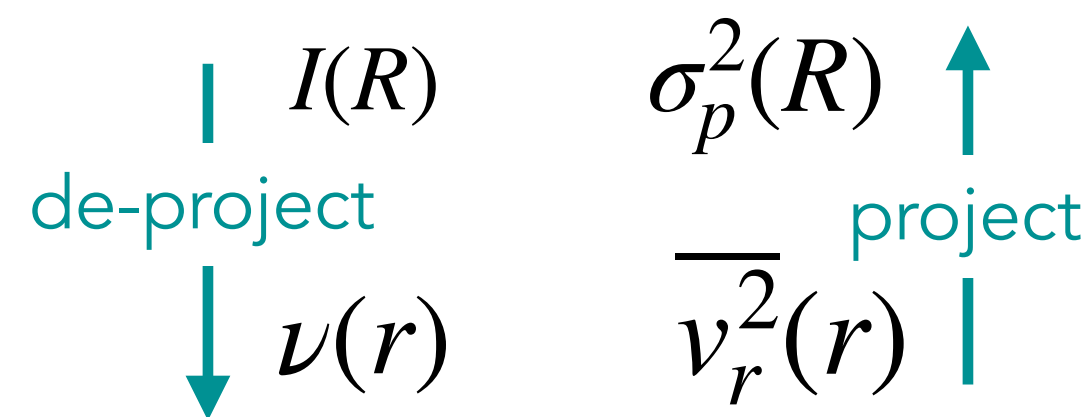
$$\rho(r) = f(\rho_s, r_s, \alpha_i)$$

1. start from collisionless Boltzmann equation
 2. integrate moments 0 and 1 over velocities
 3. combine them to get the Jeans equation
- See *Binney and Tremaine (2008)*

Dwarf spheroidal galaxies

Jeans modeling

Light and velocity dispersion profiles



Jeans equation: solve for $\nu \overline{v_r^2}$

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given a DM profile parameterisation
(NFW, Einasto, or more general form)

$$\rho(r) = f(\rho_s, r_s, \alpha_i)$$

Given the observables $I(R)$ and $\sigma_p^2(R)$,
fit for the DM profile parameters

$$(\rho_s, r_s, \alpha_i)$$

Jeans equation assumes

- Spherical symmetry
- Dynamical equilibrium
- No rotation

Not necessarily true

Parametric approach

- Light profile (Plummer, King,...)
- Anisotropy (zero, constant, $\beta(r)$)
- DM profile (NFW, core, Einasto)

different choices
=
different results

Bayesian inference needs

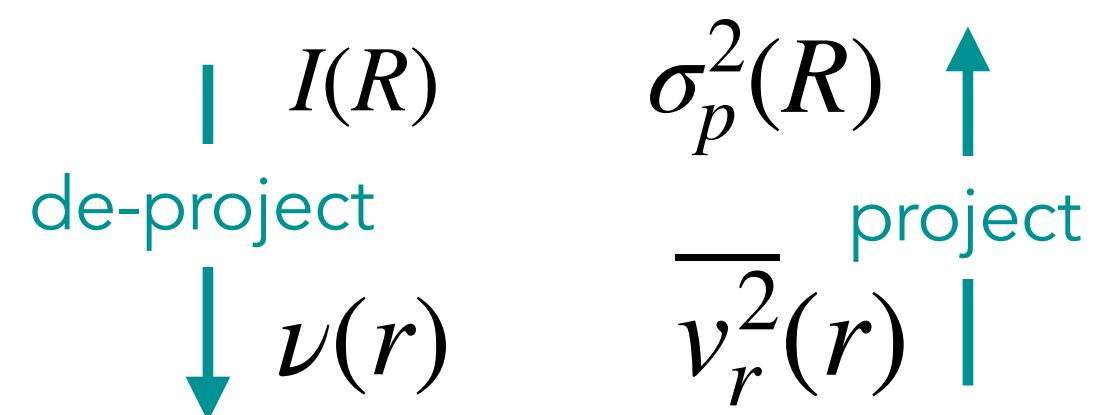
- Likelihood (binned or unbinned)
- Priors ("informative" or not)



Dwarf spheroidal galaxies

Jeans modeling

Light and velocity dispersion profiles



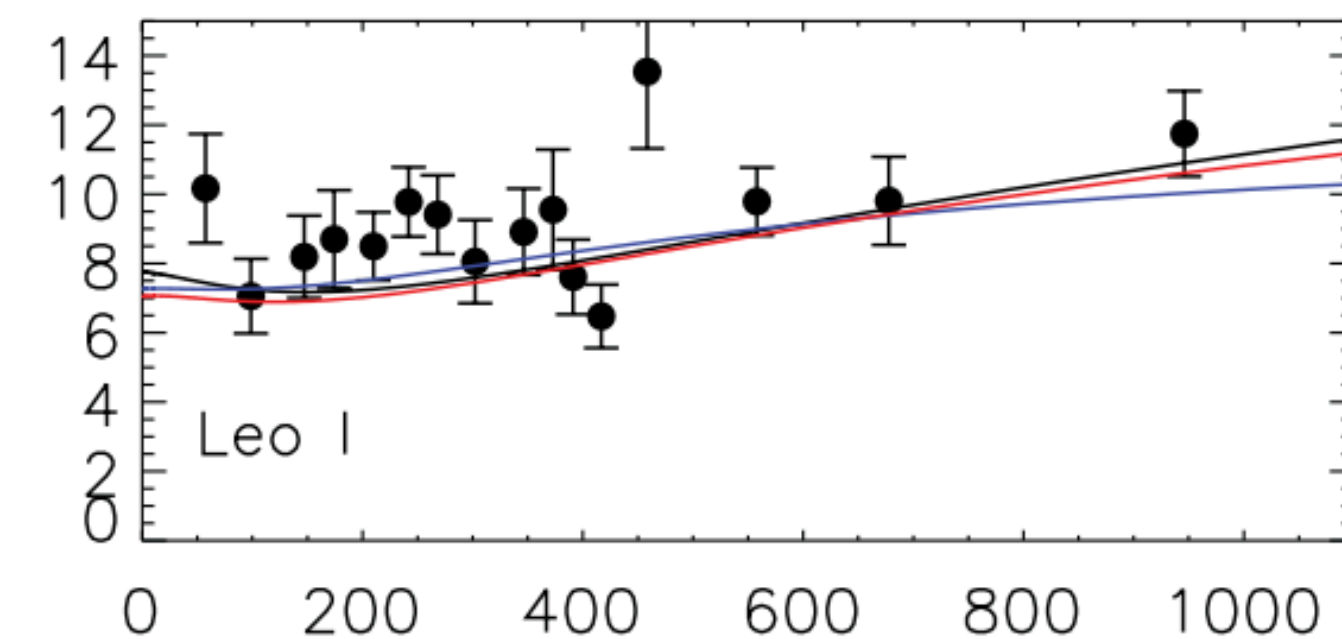
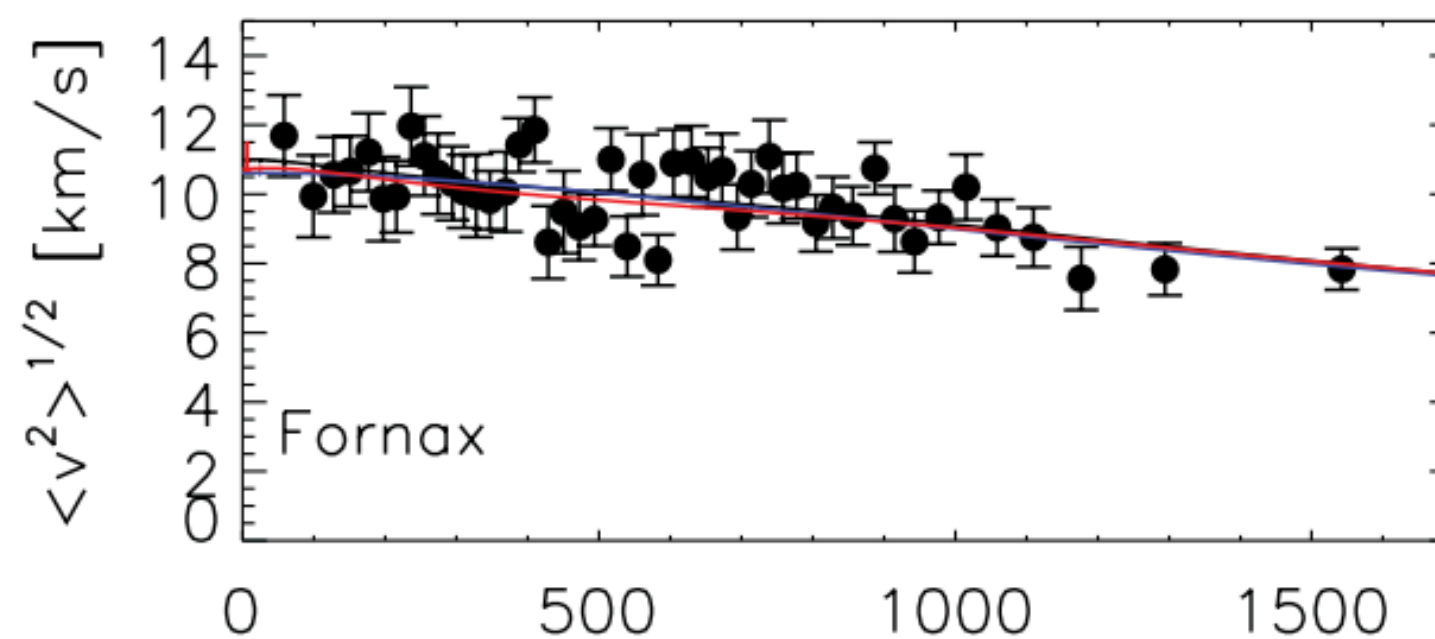
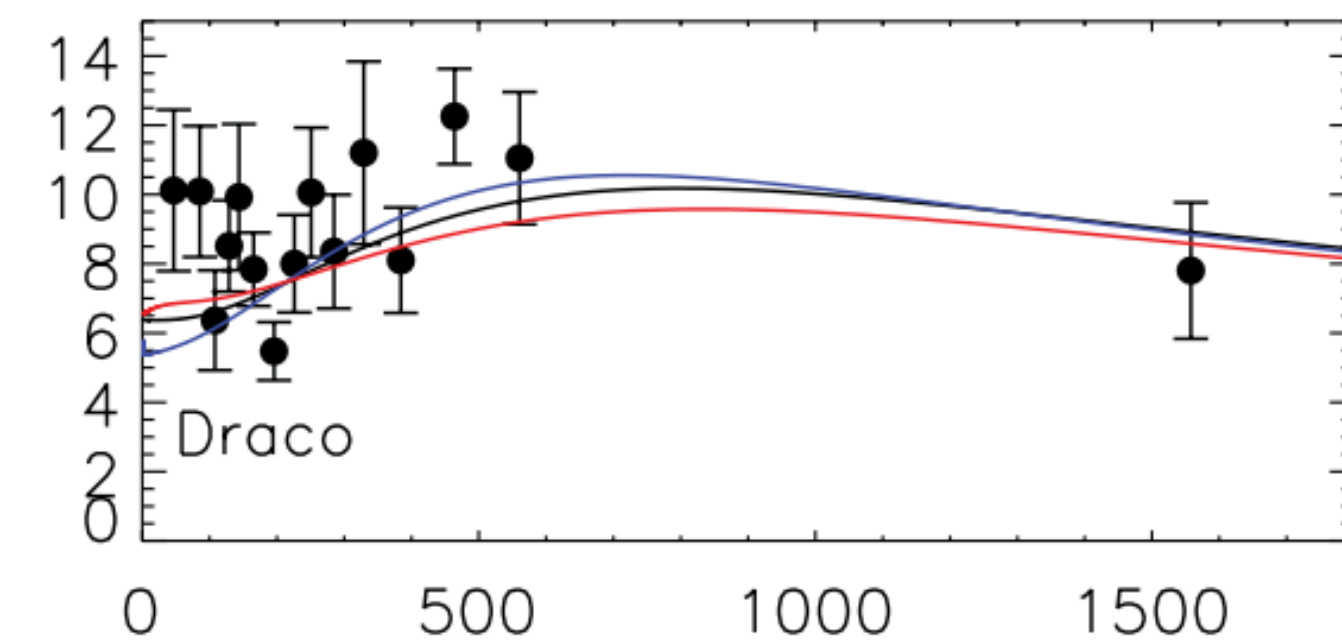
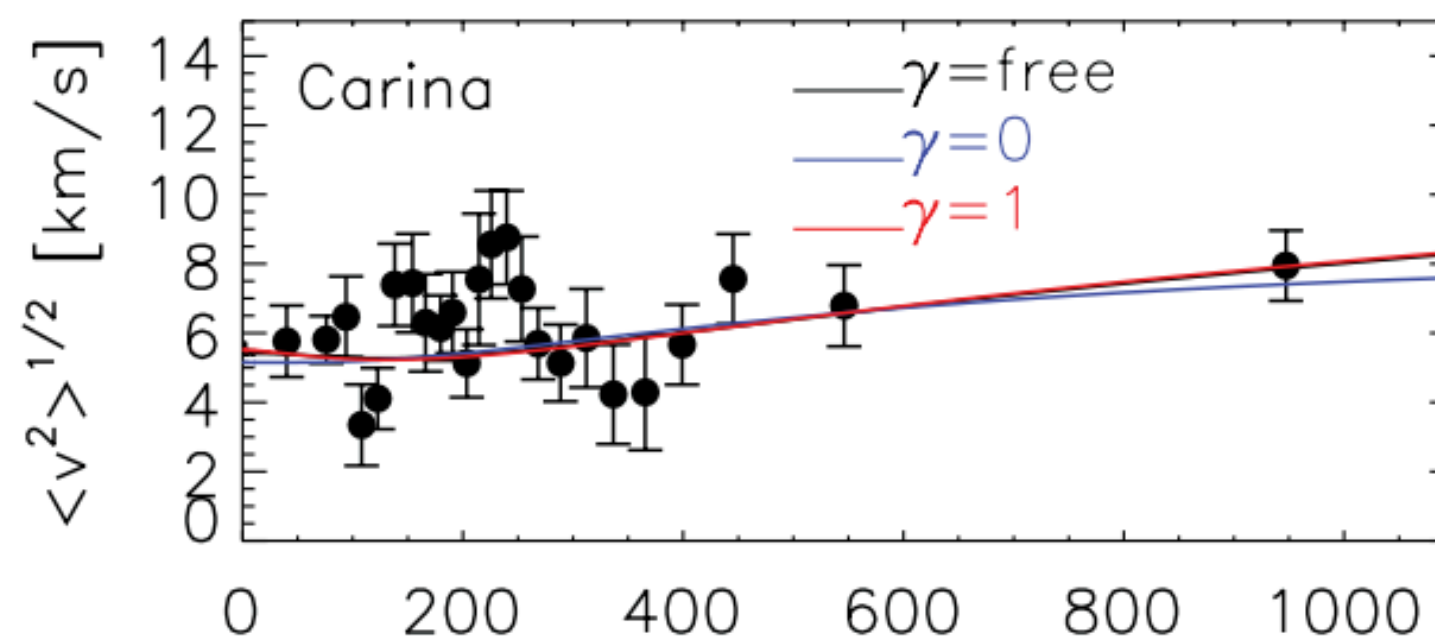
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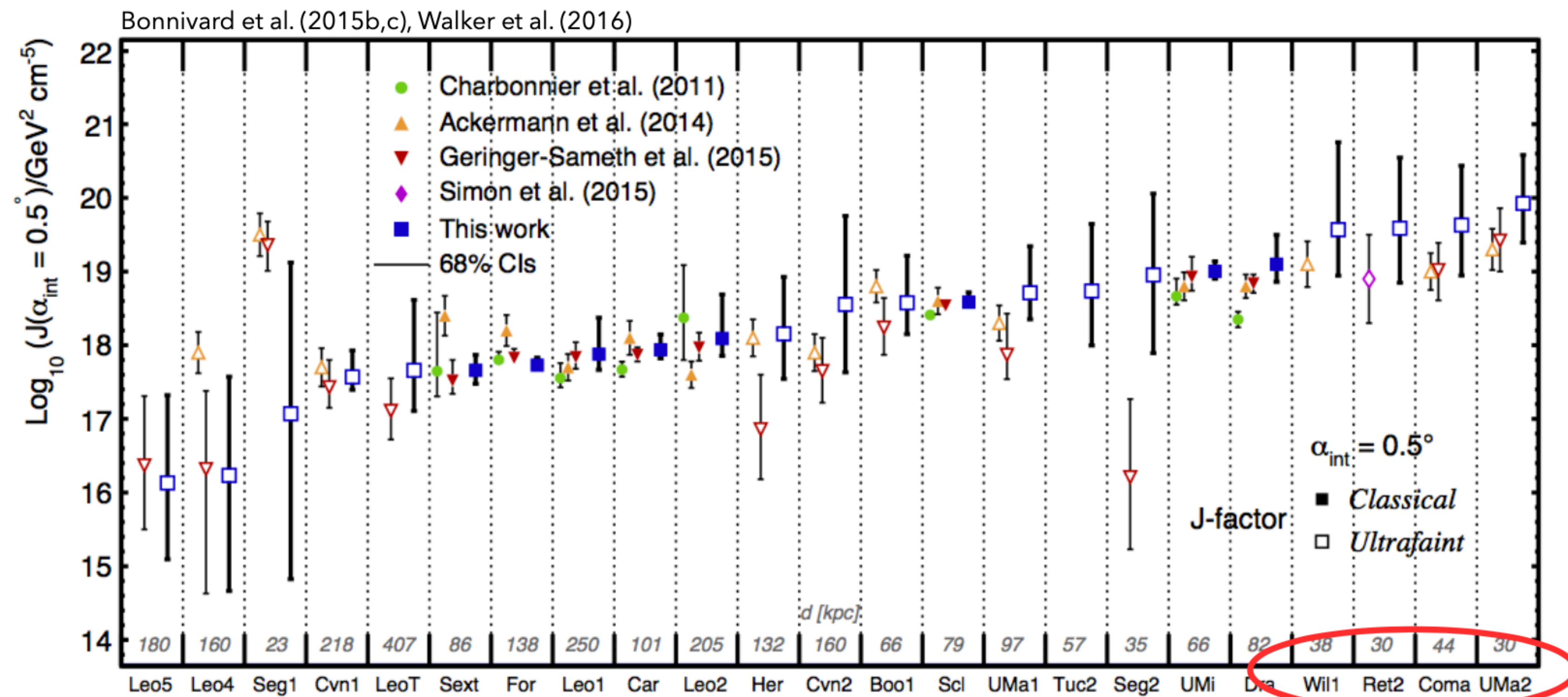
$$\rho(r) = f(\rho_s, r_s, \alpha_i)$$



NB: The inner slope is not well constrained

Dwarf spheroidal galaxies

Ranking fo best targets?

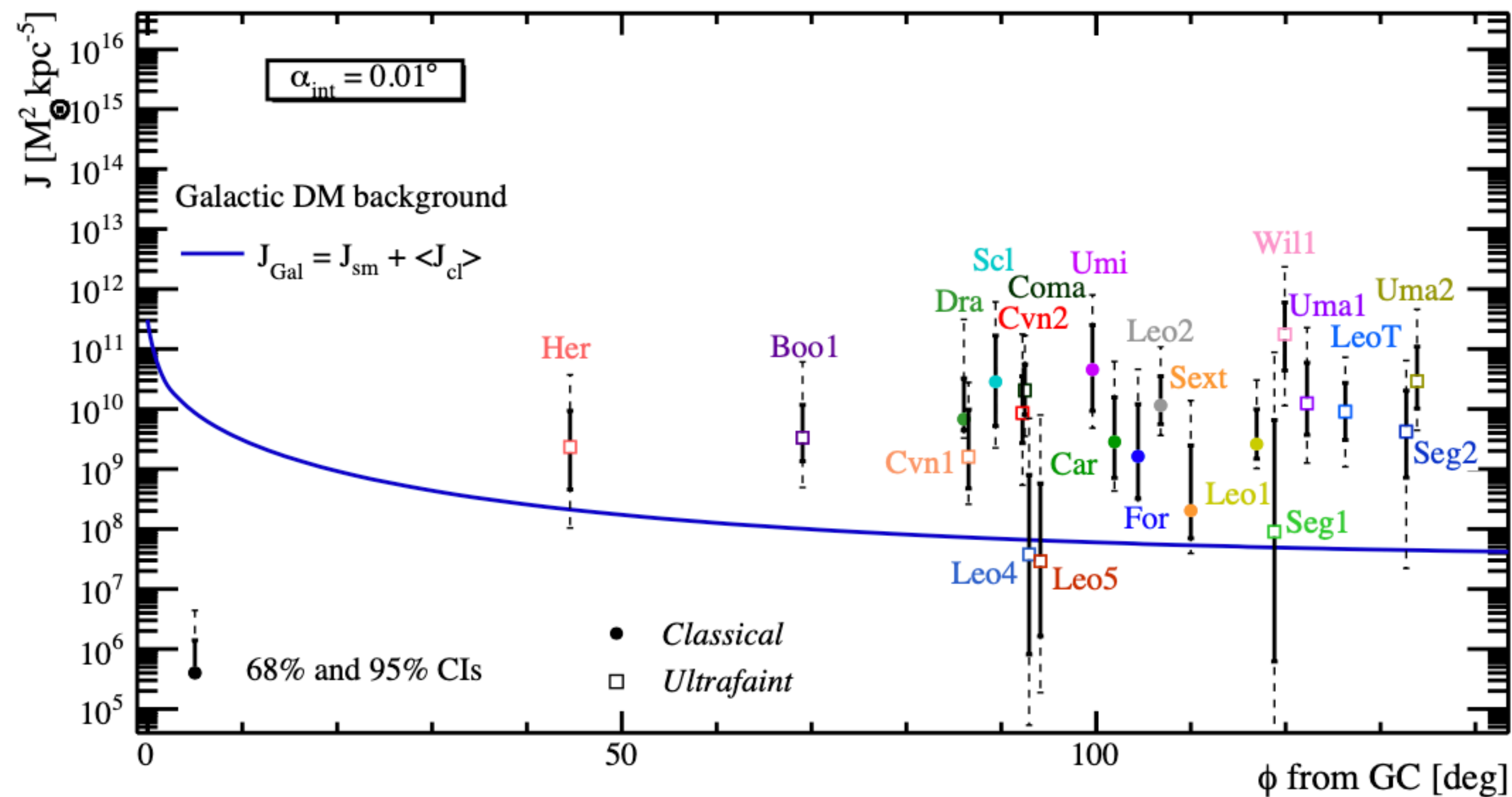


Distance is the main driver for the J-factor

Error bars depends on the size of the data sample and on the modeling choices (number of free parameters, priors, etc.)

Dwarf spheroidal galaxies

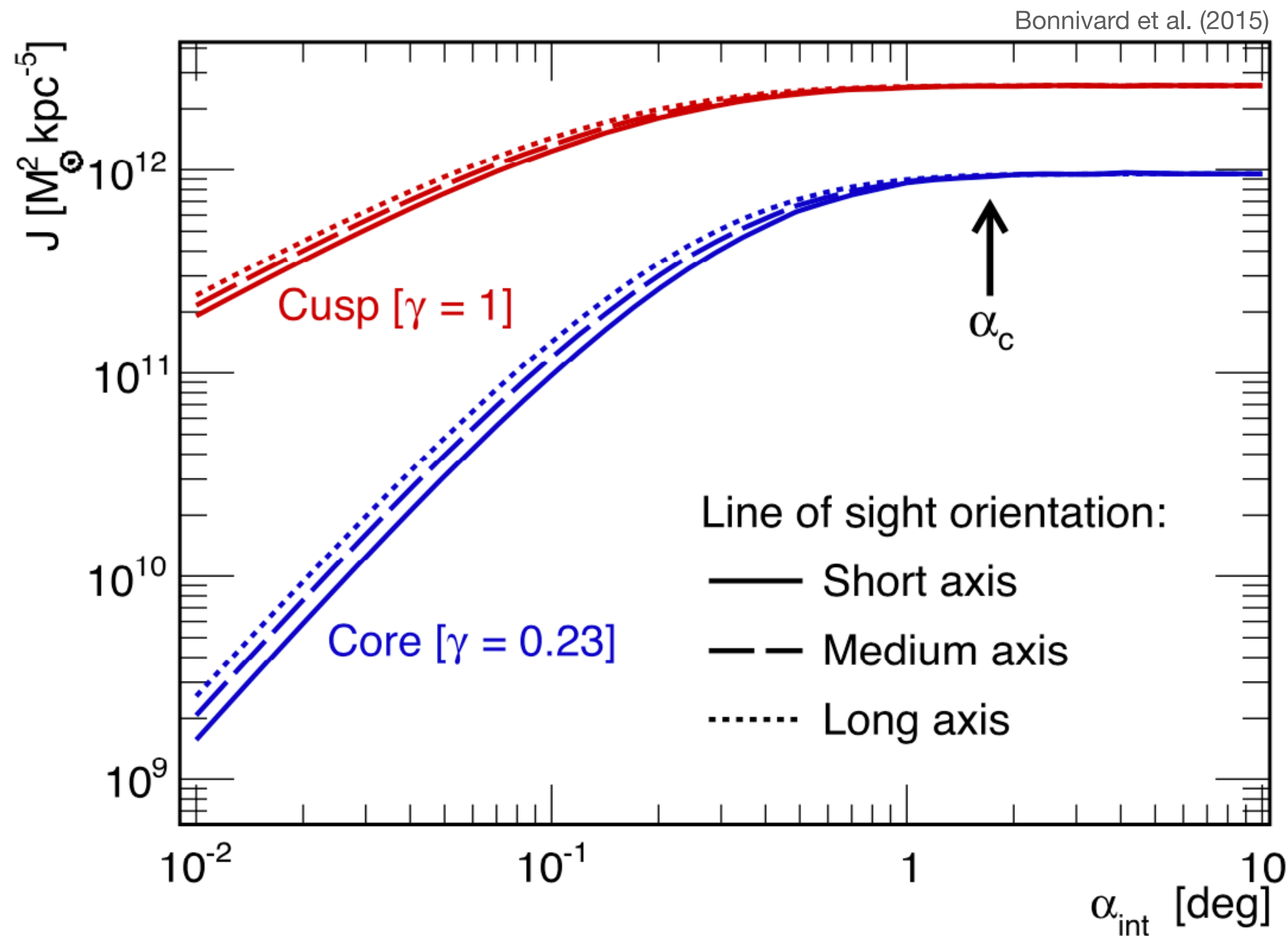
Contribution of the MW halo ?



For small integration angles the MW halo contribution may be neglected

Dwarf spheroidal galaxies

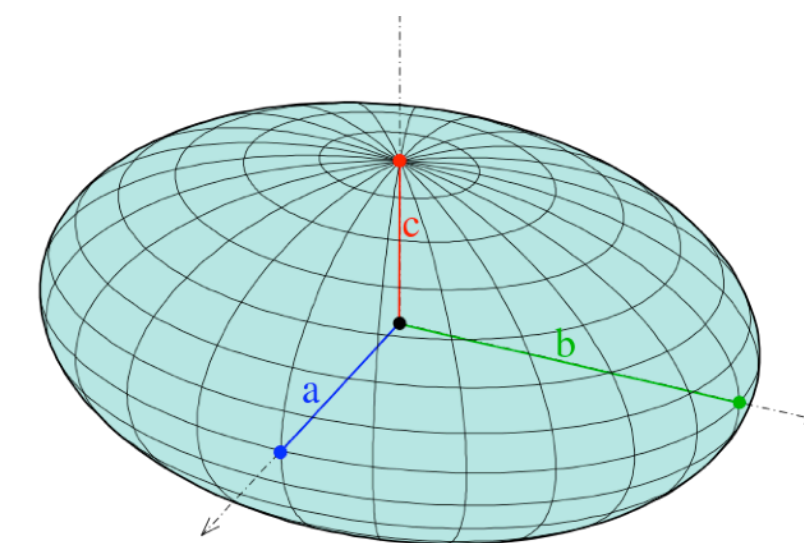
Triaxiality and projection effects (mock data)



Triaxial Zhao profile

$$\rho(r) = \frac{\rho_s}{(r_e/r_s)^\gamma [1 + (r_e/r_s)^\alpha]^{(\beta-\gamma)/\alpha}}$$

$$r_e = \sqrt{\frac{X^2}{a^2} + \frac{Y^2}{b^2} + \frac{Z^2}{c^2}}$$

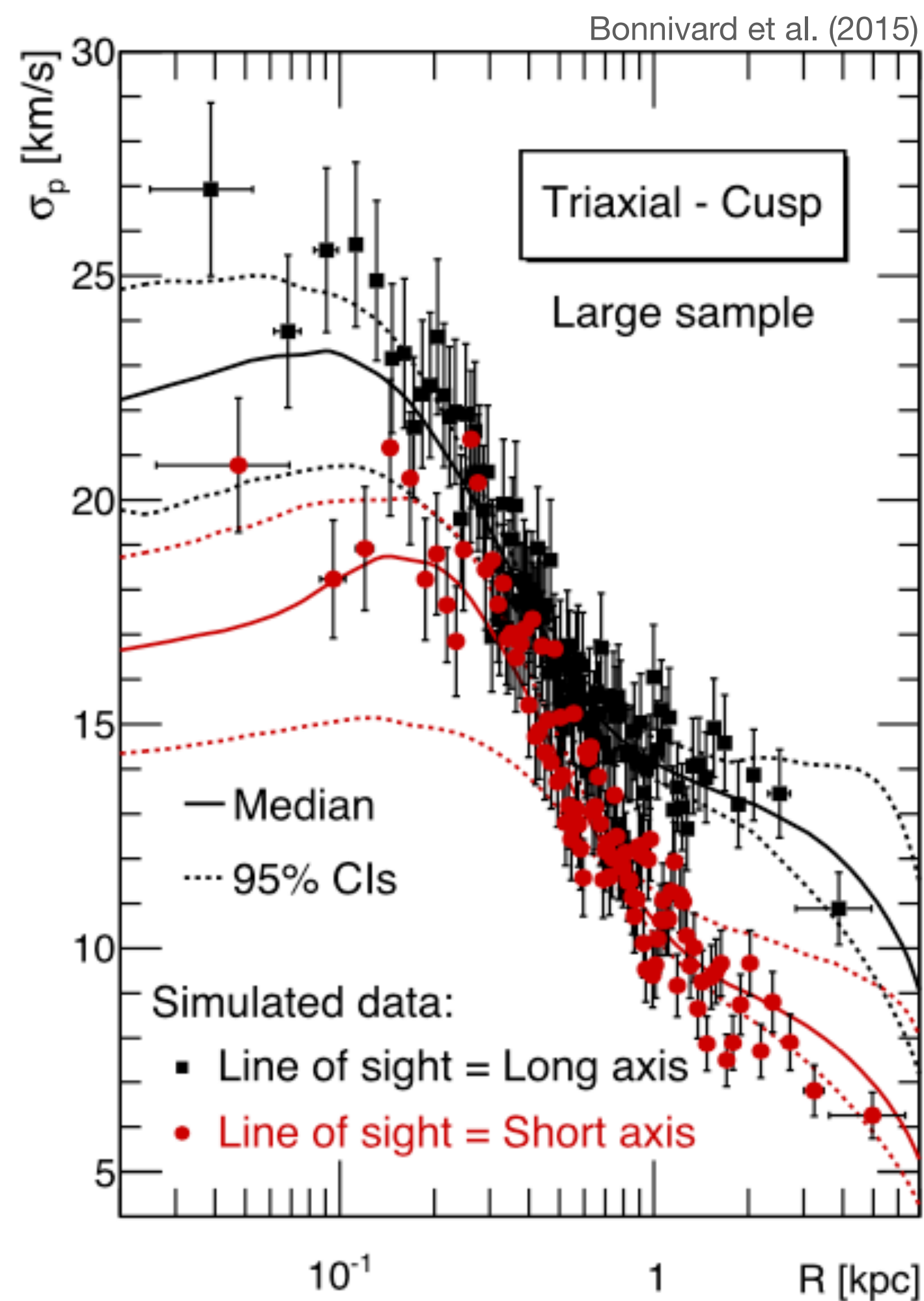


Typically, dSph-like halos in simulations have $a/b \sim 0.8$, $a/c \sim 0.6$

Compute the J-factor for the 3 l.o.s
 → little impact on the value of the J-factor
 → good news, but...

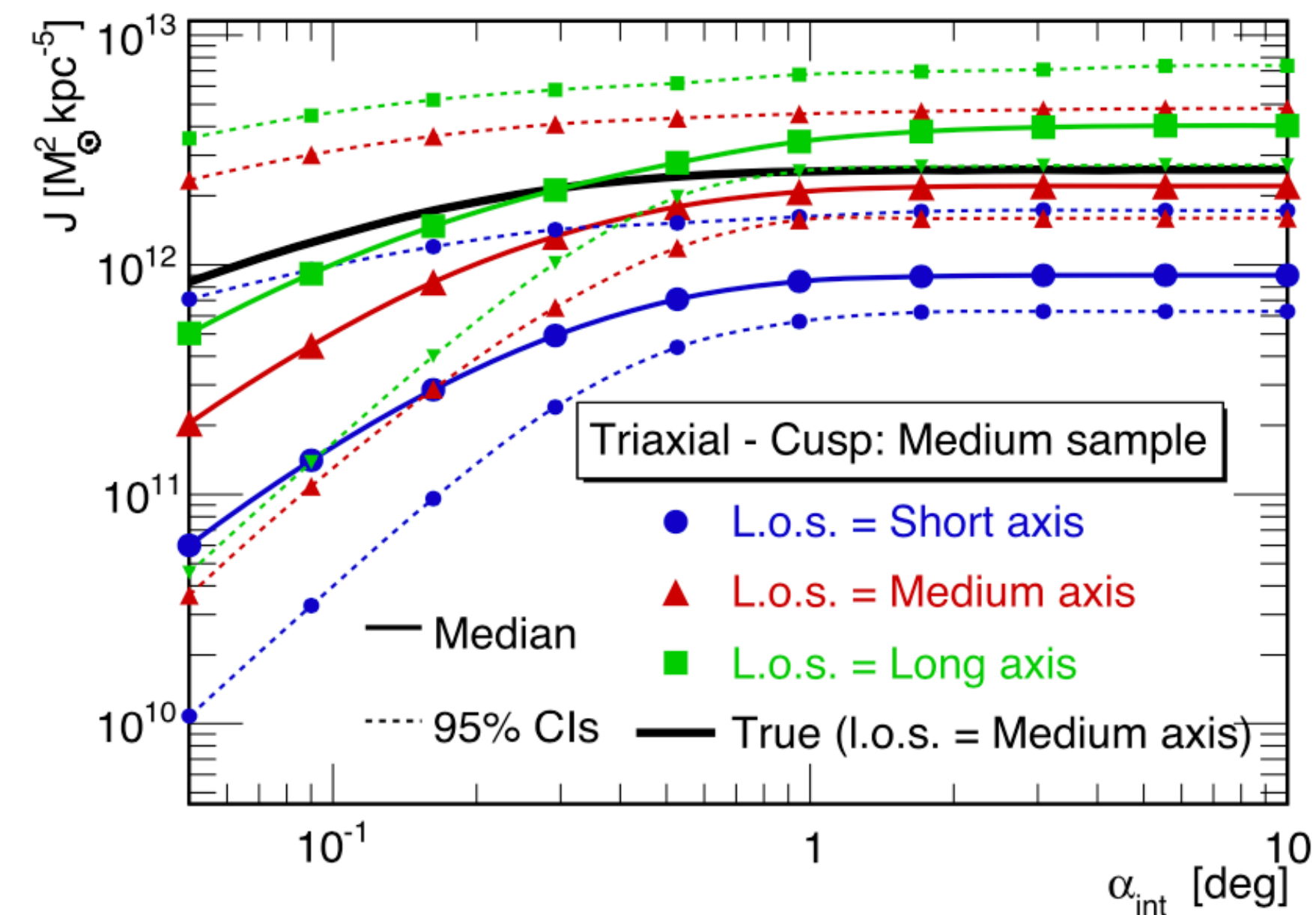
Dwarf spheroidal galaxies

Triaxiality and projection effects (mock data)



Triaxiality strongly impacts the projected velocity dispersion profile

Spherical Jeans analysis to constrain $\rho(r) \rightarrow J$ will yield bias values



Solutions exist (but more expensive computationally)

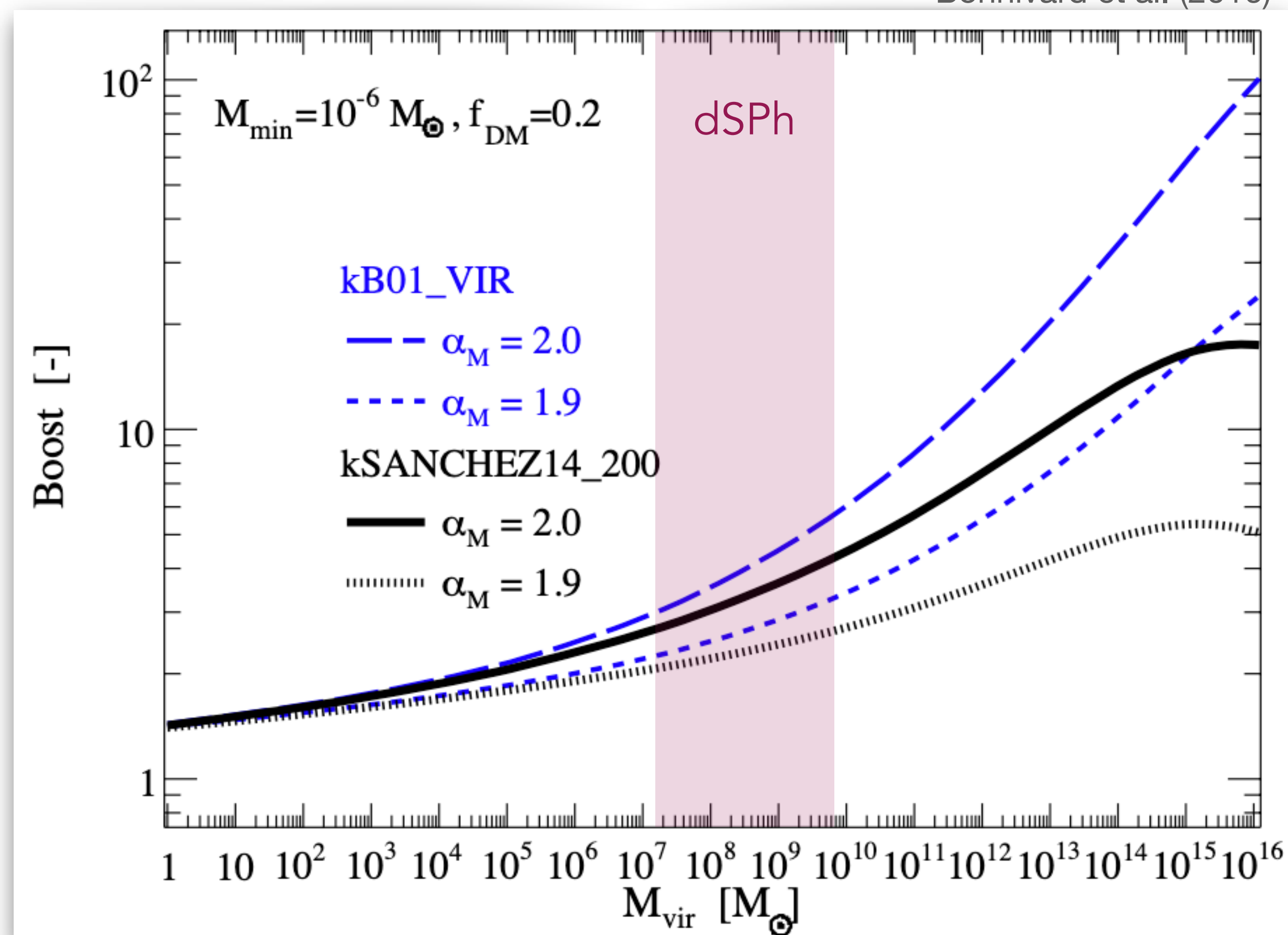
- axisymmetric Jeans analysis (e.g. Hayashi et al. 2016)
- made-to-measure models

Dwarf spheroidal galaxies

And what about substructure boost?

Negligible boost from substructures,
so generally not considered at the
scale of dSph galaxies

Bonnivard et al. (2016)



Dwarf spheroidal galaxies

Summary

- dSph are arguably the best targets to place stringent constraints on DM: High J-factors thanks to a favorable combination of distance and density
- Stellar kinematics trace the underlying gravitational potential
 - Standard approach: spherical Jeans analysis in Bayesian framework to constrain the DM density
 - Need to be careful of possible biases introduced by modeling choices: extensive checking on mock data!
 - Continuous development:
 - ▶ axisymmetric Jeans analysis (e.g. Hayashi et al. 2016)
 - ▶ informative priors (e.g. Ando et al. (2020))
 - ▶ non-parametric approach + higher orders of the velocity distribution (e.g. Read & Steger 2017)



DM modeling of galactic targets

1. Galactic center region
2. Dwarf spheroidal galaxies
3. Dark galactic clumps

Dark galactic haloes

- Low mass dark haloes pertain the Galactic DM halo
- Some may be quite close to us
- DM annihilation in a dark clump would be seen as a point-like emission in γ -rays, with no counterpart
- If no-detection, place limits *provided a model of **the subhalo population***
- Conversely to Galactic halo or dSph galaxies, all we can rely on are results from numerical simulations or semi-analytical modeling to use as ingredients

Dark galactic haloes

The ingredients

From Part 1, recall the 7 ingredients to describe the average contribution of substructures to J-factor

$$\langle J_{\text{subs}} \rangle = N_{\text{tot}} \int_0^{\Delta\Omega} \int_{l_{\text{min}}}^{l_{\text{max}}} \frac{d\mathcal{P}_V}{dV}(r(l, \Omega)) dl d\Omega \int_{M_{\text{min}}}^{M_{\text{max}}} \frac{d\mathcal{P}_M}{dM}(M) \times \int_{c_{\text{min}}(M)}^{c_{\text{max}}(M)} \frac{d\mathcal{P}_c}{dc}(c, M) \mathcal{L}(M, c) dc dM$$

$\mathcal{L}(M, c) = \int_{V_{\text{halo}}} \rho^2(M, c) dV$

Those distributions give a full statistical description of the subhalo population

- can generate realisations
- early studies: stick to one configuration
- to bracket modeling uncertainties, need to explore various options for each of these

Dark galactic haloes

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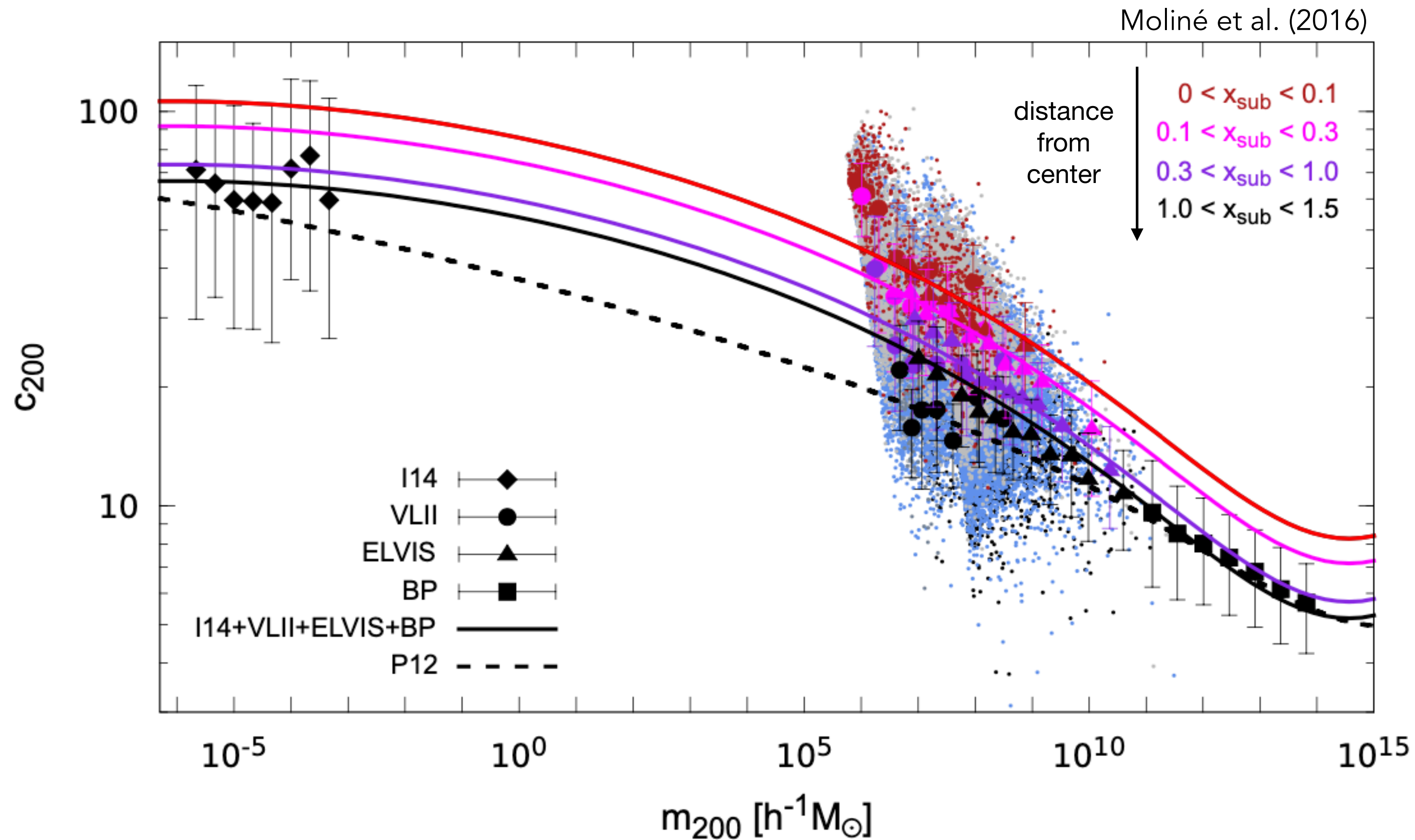
Hütten et al (2016)

	Model	VAR0	LOW	VAR1	VAR2	VAR3	VAR4	VAR5	VAR6a	VAR6b	HIGH
Varied parameters	inner profile	NFW	E	E	E	E	E	E	E	E	E
	α_m	1.9	1.9	2.0	1.9	1.9	1.9	1.9	1.9	1.9	1.9
	σ_c	0.14	0.14	0.14	0.24	0.14	0.14	0.14	0.14	0.14	0.14
	$\bar{\varrho}_{\text{subs}}$	E-AQ	E-AQ	E-AQ	E-AQ	M-VLII	E-AQ	E-AQ	E-AQ	E-AQ	M-VLII
	N_{calib}	150	150	150	150	150	300	150	150	150	300
	sub-subhalos?	no	no	no	no	no	no	yes	no	no	no
	$c(m)$	SP	SP	SP	SP	SP	SP	SP	Moliné	P-VLII	P-VLII

Dark galactic haloes

Subhalo specificities: radial-dependent concentration

$$\langle J_{\text{subs}} \rangle = N_{\text{tot}} \int_0^{\Delta\Omega} \int_{l_{\text{min}}}^{l_{\text{max}}} \frac{d\mathcal{P}_V}{dV}(r(l, \Omega)) dl d\Omega \int_{M_{\text{min}}}^{M_{\text{max}}} \frac{d\mathcal{P}_M}{dM}(M) \times \int_{c_{\text{min}}(M)}^{c_{\text{max}}(M)} \frac{d\mathcal{P}_c}{dc}(c, M) \mathcal{L}(M, c) dc dM$$



Compared to field haloes, subhaloes are subject to tidal stripping, making them more compact.

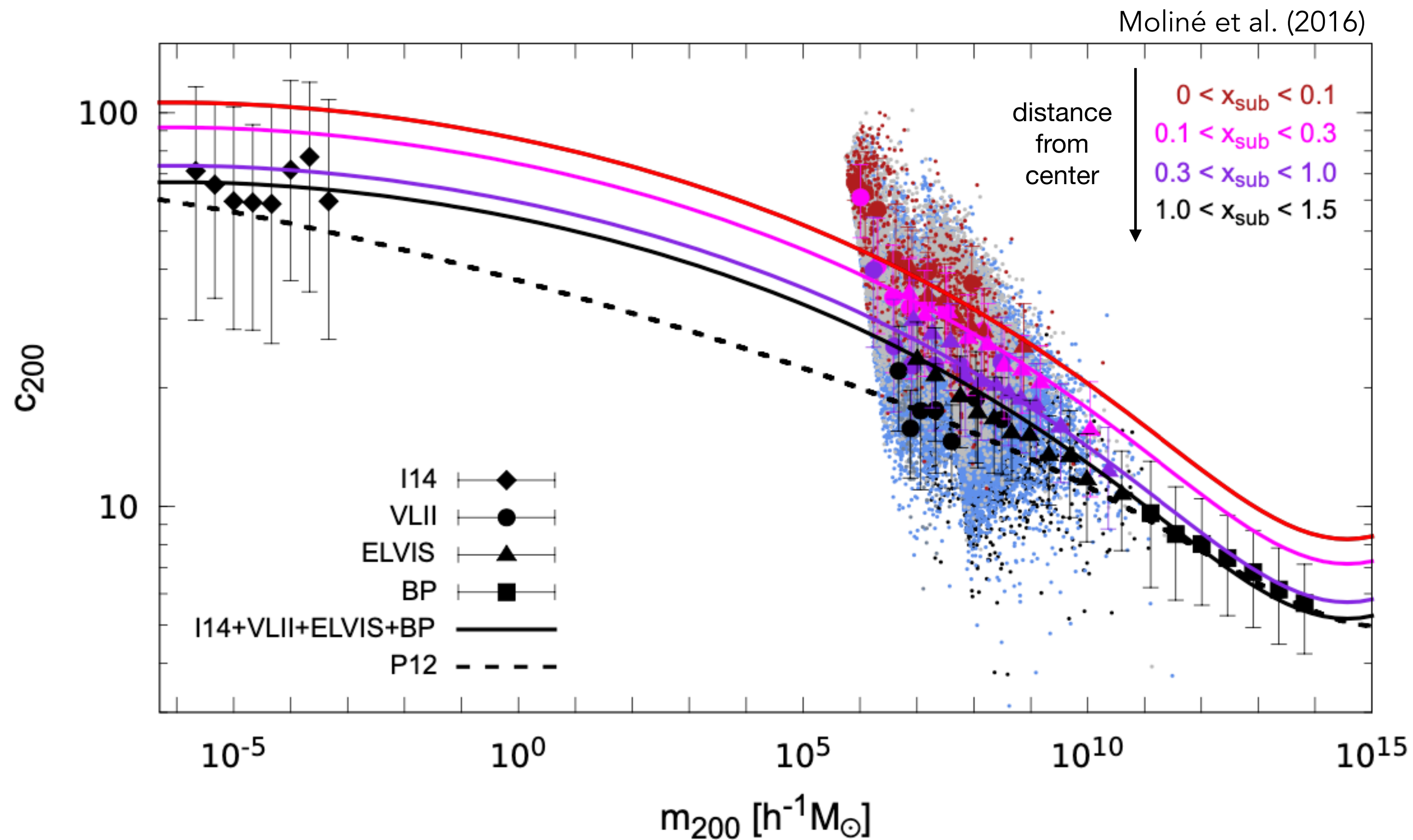
The closer to the center of the host halo, the more concentrated the subhalos.

$$c(M) \rightarrow c(M, r)$$

Dark galactic haloes

Subhalo specificities: radial-dependent concentration

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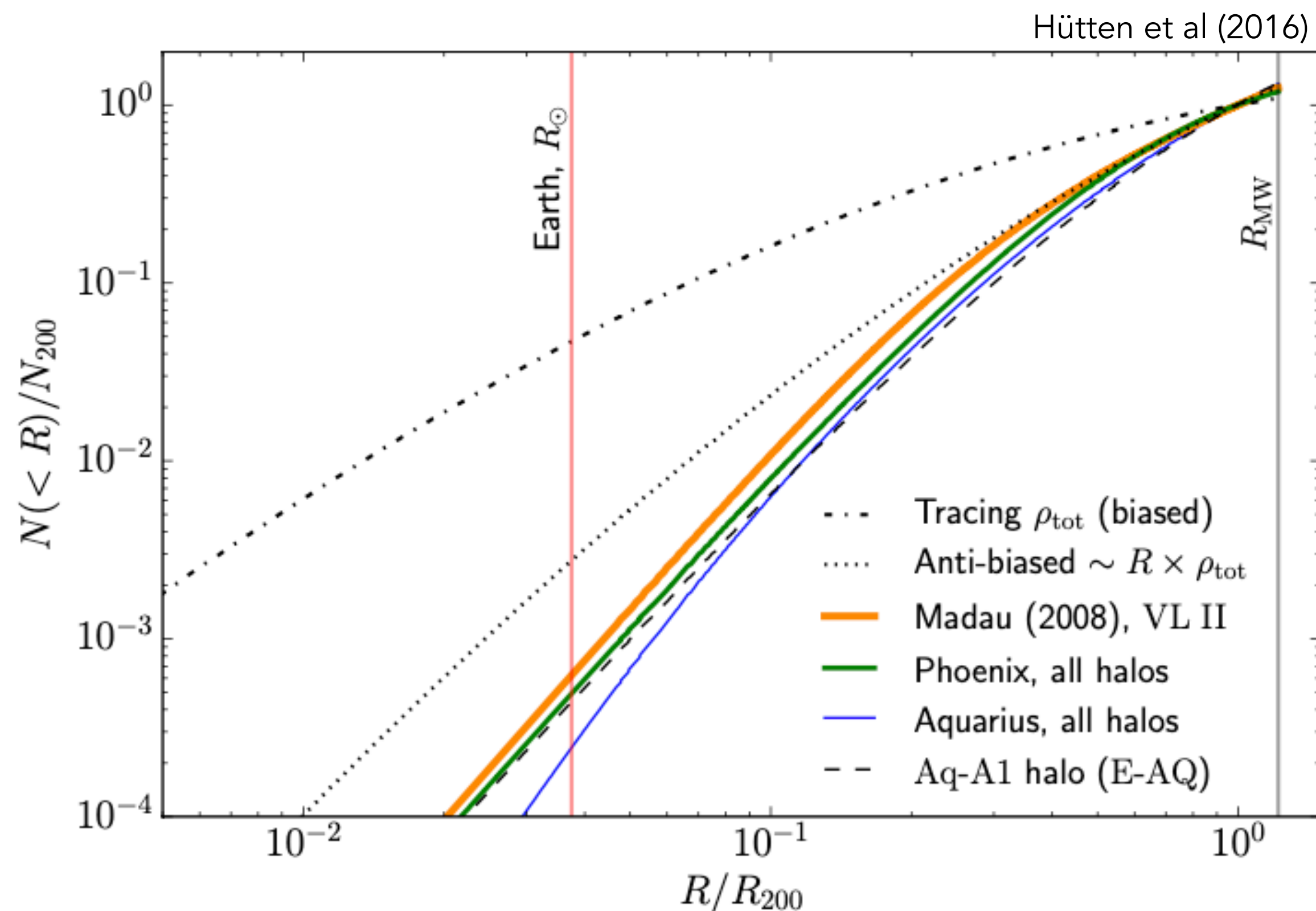
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Dark galactic haloes

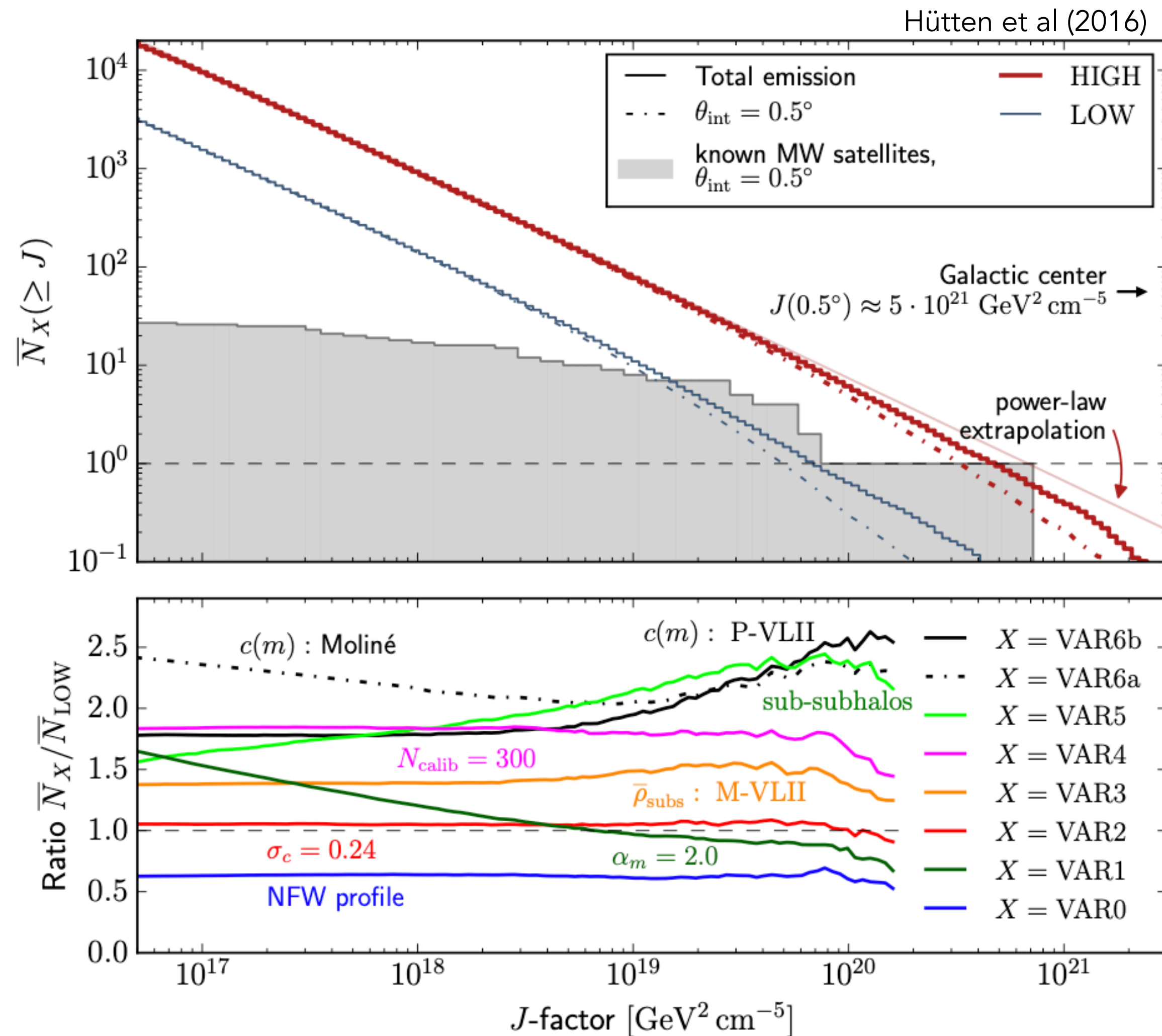
Subhalo specificities: tidal disruption from the host DM halo



- unevolved population: spatial distribution follows the total/smooth distribution
- tidal stripping/disruption due to the strong gravitational gradient in the inner region
→ reduces the number of haloes in the inner region of the host halo
- effect captured naturally captured by simulations, but can also be model from (semi-)analytical considerations (Han et al. 2016)

Dark galactic haloes

Cumulative source count distribution

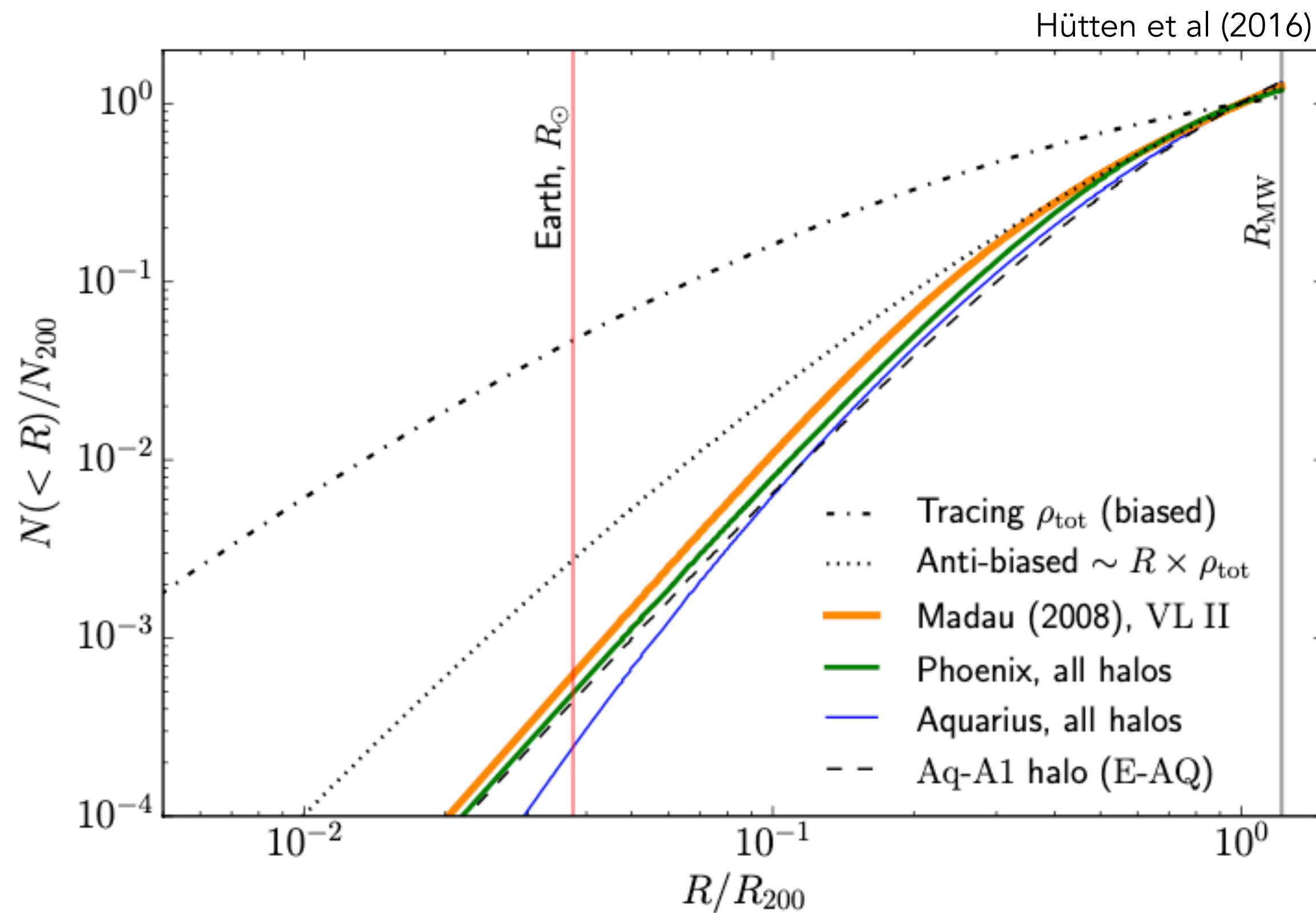


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	$\bar{\rho}_{\text{subs}}$	E-AQ	E-AQ	E-AQ	E-AQ	M-VLII	E-AQ	E-AQ	E-AQ	E-AQ	M-VLII
	N_{calib}	150	150	150	150	150	300	150	150	150	300
	sub-subhalos?	no	no	no	no	no	no	yes	no	no	no
	$c(m)$	SP	SP	SP	SP	SP	SP	SP	Moliné	P-VLII	P-VLII

Modeling choices matter and the cumulative source count distribution can vary by ~ 1 order of magnitude

Dark galactic haloes

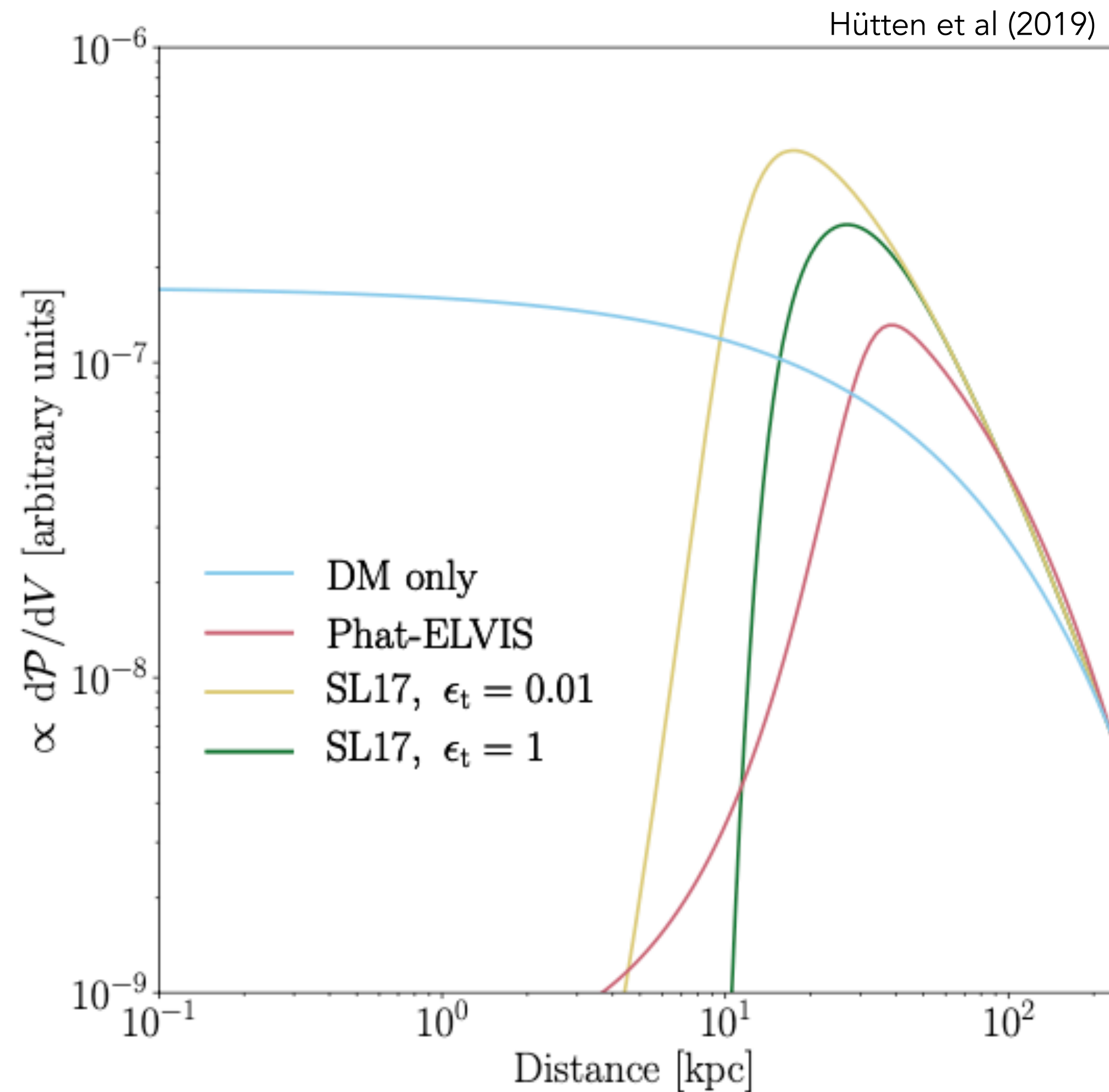
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Dark galactic haloes

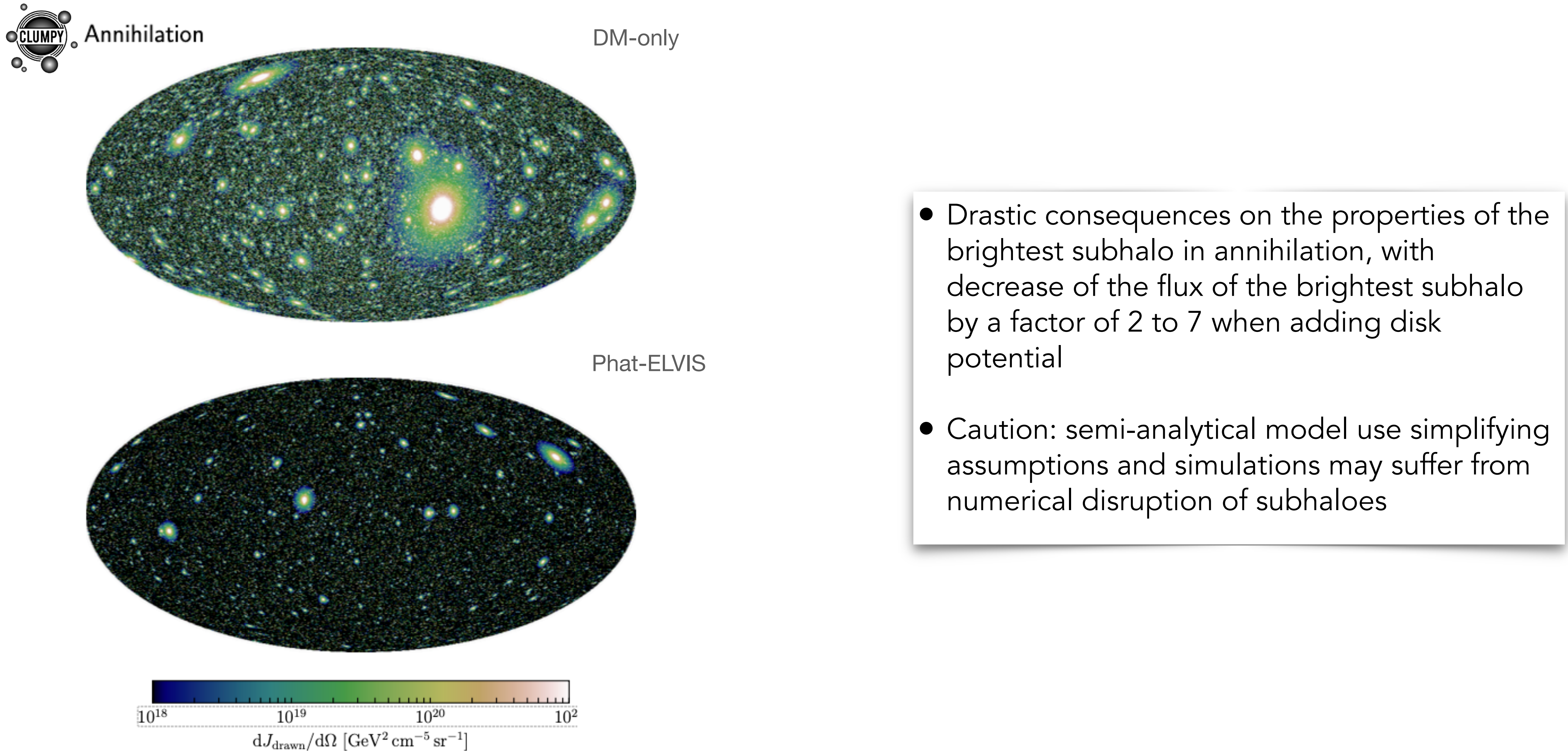
Subhalo specificities: tidal disruption from the host DM halo + baryonic disk



- adding potential from a baryonic disk has an even stronger impact, with total depletion of subhaloes in the innermost regions
- effect captured captured by simulations with added disk potential (Kelley et al. 2019), but can also be modeled from (semi-)analytical considerations (Stref & Lavalle 2017)

Dark galactic haloes

Subhalo specificities: tidal disruption from the host DM halo + baryonic disk



Dark galactic haloes

Summary

- Dark subhaloes in the MW Galactic halo can be constraining “targets”
→ Fermi-LAT all sky survey, CTA planned extragalactic survey
- Modeling of the subhalo population requires 7 ingredients. Constraints may only come from numerical simulations or semi-analytical modeling
- Need to better pin down the effect of tidal stripping in the full MW potential to get a better picture

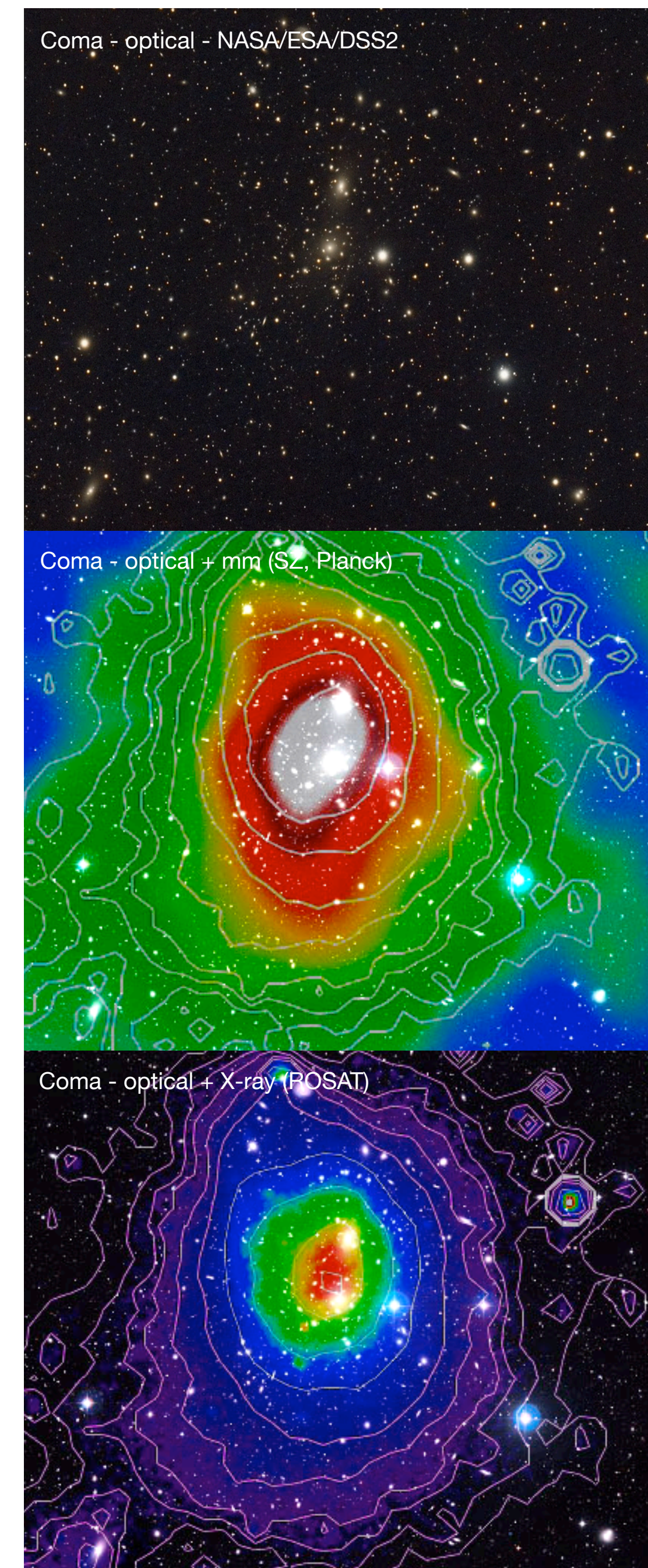
DM modeling of extragalactic targets

1. Galaxy clusters
2. The extragalactic diffuse exotic signal

Galaxy clusters

Overview

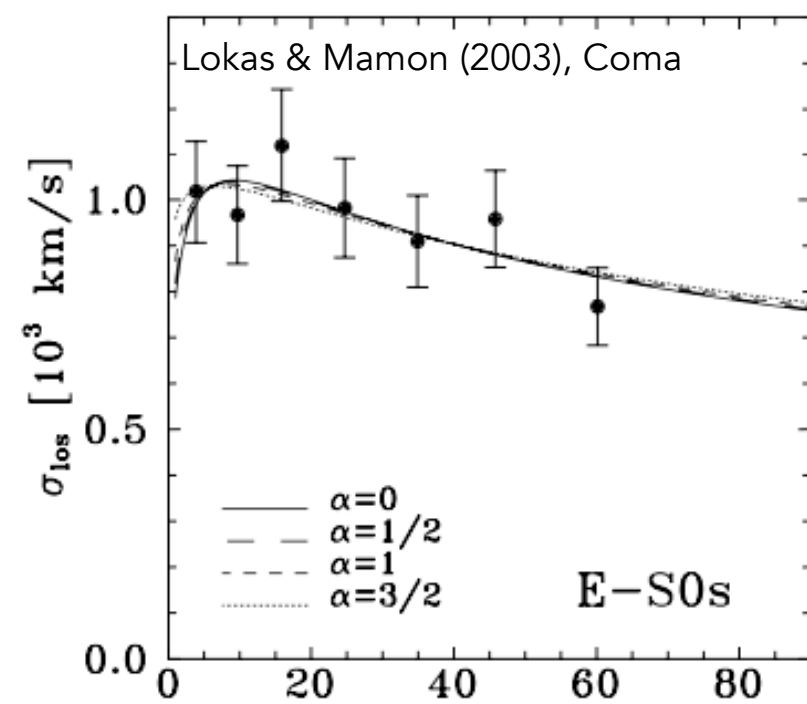
- Last stage of structure formation; largest gravitationally-bound objects in the universe
- $\sim 10^{14} - 10^{15} M_{\text{sun}}$, densest regions in the universe
 - $\sim 80\%$ dark matter
 - $\sim 15\%$ hot gas
 - \sim a few% galaxies
- "Close by" clusters: Virgo (~ 16 Mpc), Coma (~ 102 Mpc)
 - large $1/d^2$ dilution the exotic signal
- Observationally
 - X-rays: hot gas emission (free-free from ICM e-)
 - mm-wavelength: hot gas - CMB interaction (SZ effect)
 - Visible, infrared: galaxies
 - γ -rays? expected emission from interaction between gas and cosmic rays (background for DM indirect detection)



Galaxy clusters

Mass, DM profile determination

Dynamical estimation (visible, spectroscopy)

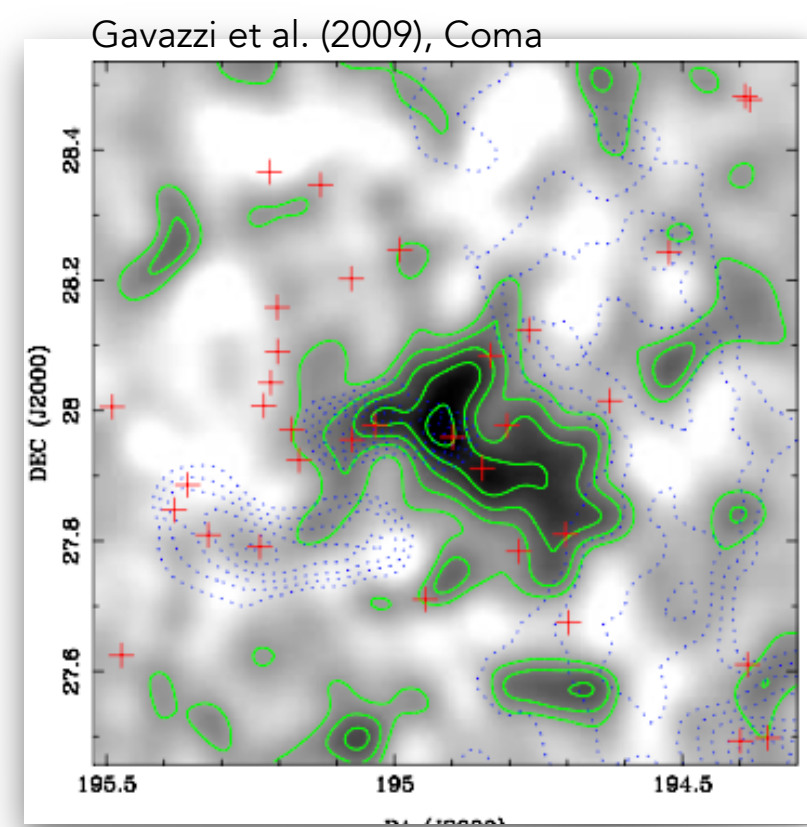


Measure velocity dispersion of galaxies in the cluster
 → use virial theorem to get the mass
 → perform Jeans analysis and fit for the density profile



In the Jeans equation (\neq dSph)
 $M_{\text{tot}}(r) = M_{\text{DM}}(r) + M_{\text{gas}}(r) + M_{\text{stars}}(r)$

Weak lensing mass estimate (visible)

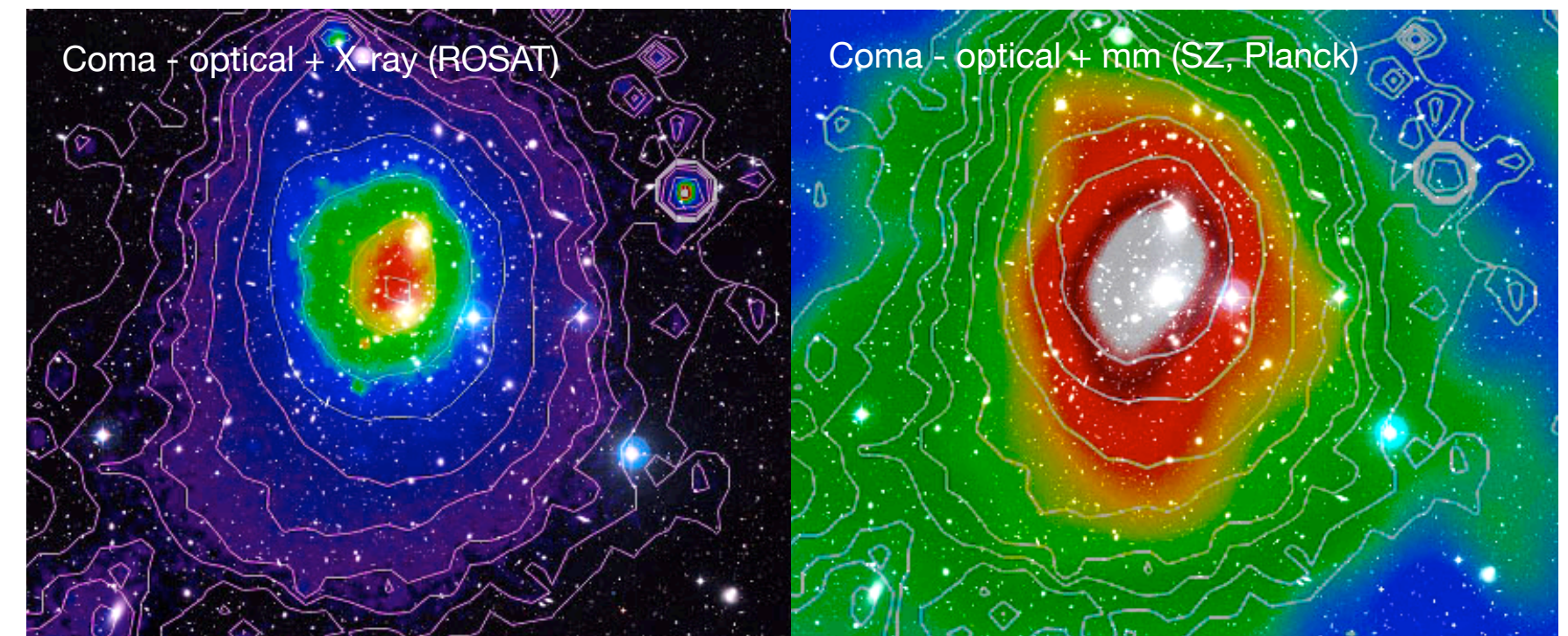


Weak gravitational lensing:

- the shape of background galaxies are coherently distorted in the presence of a foreground cluster (lens).
- the amount of distortion depends on the project mass density of the lens

No assumption on dynamical state of the system

Mass estimation from baryonic proxies (X-rays, SZ)



$$n_e(r), T_e(r)$$

$$P(r) = n_e(r)kT_e(r)$$



Assume hydrostatic equilibrium

$$\frac{dP}{dr} = -\rho g = -\rho \frac{G M_{\text{HSE}}(r)}{r^2}$$

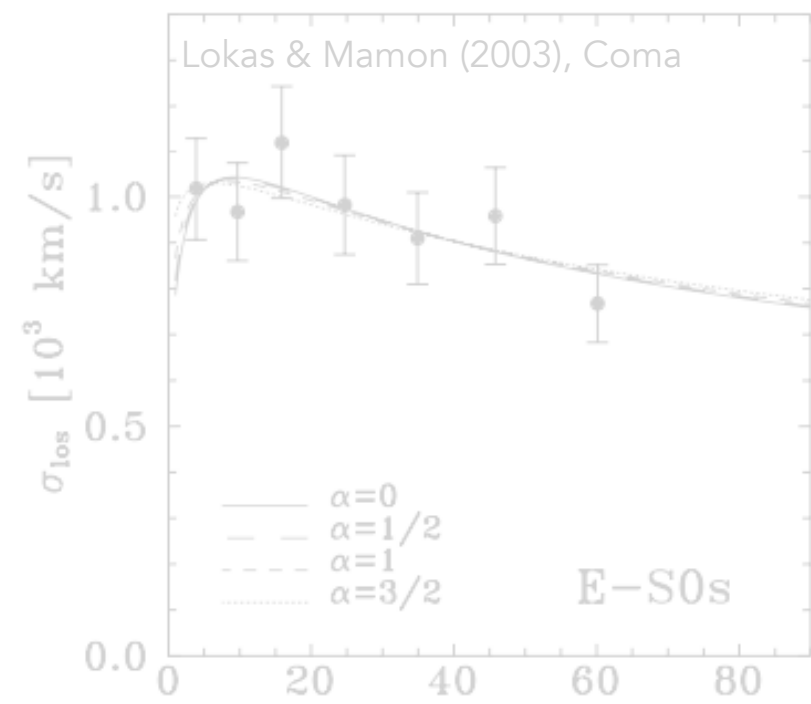
$$M_{\text{HSE}}(r) = -\frac{r^2}{G\mu m_p n_e(r)} \times \frac{dP(r)}{dr}$$

- If HSE is wrong, reconstructed mass may be biased
- Large catalogs

Galaxy clusters

Mass, DM profile determination

Dynamical estimation (visible, spectroscopy)



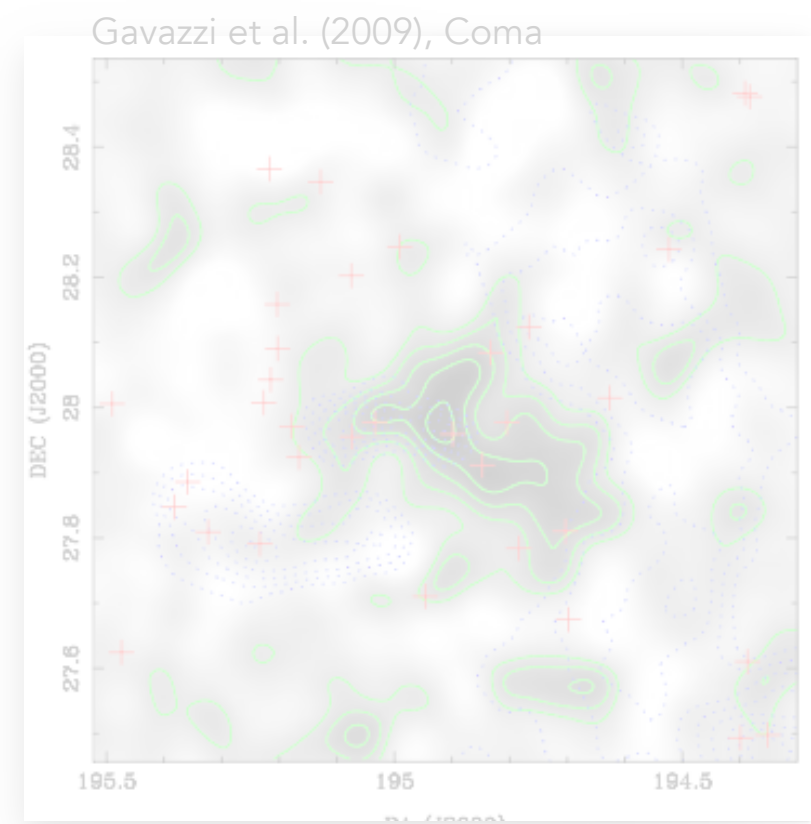
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In the Jeans equation (\neq dSph)
 $M_{\text{tot}}(r) = M_{\text{DM}}(r) + M_{\text{gas}}(r) + M_{\text{stars}}(r)$

At the cluster scale, NFW profiles are generally a good fit the data

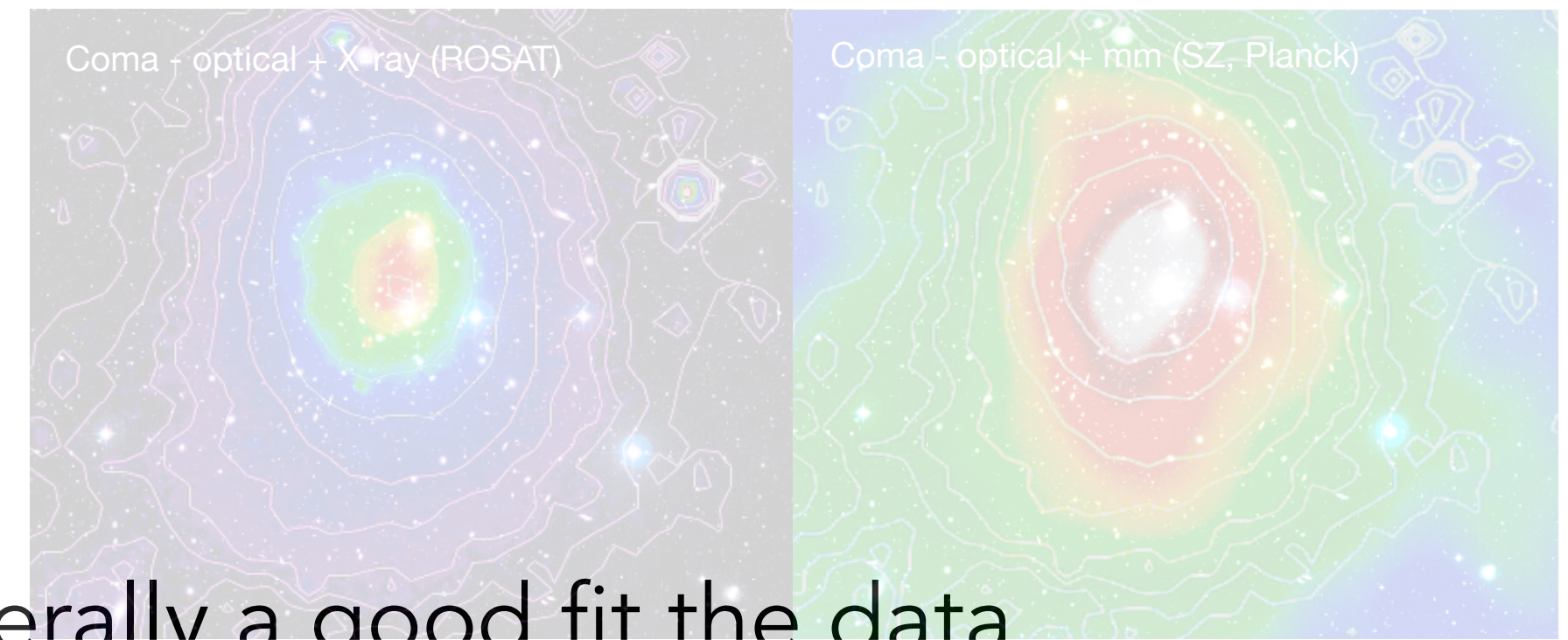
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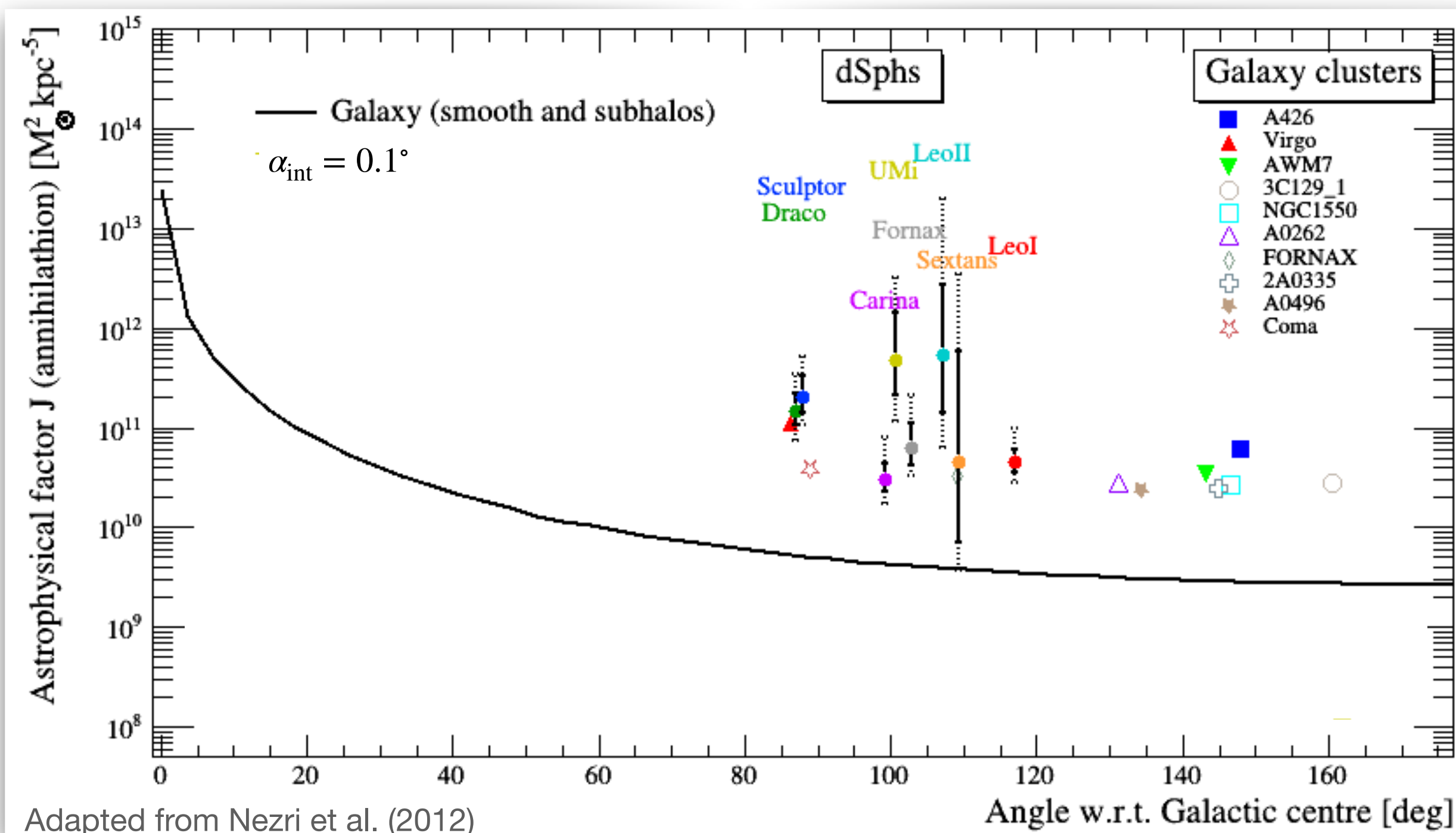
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Galaxy clusters

J-factors, boost from substructures?

Varied type of information available from the literature

- directly get ρ_s, r_s
- get the mass with a given radius, e.g. $M_{500,c}$
 - in that case, need to use a M-c relation
 - be careful with the mass definition!



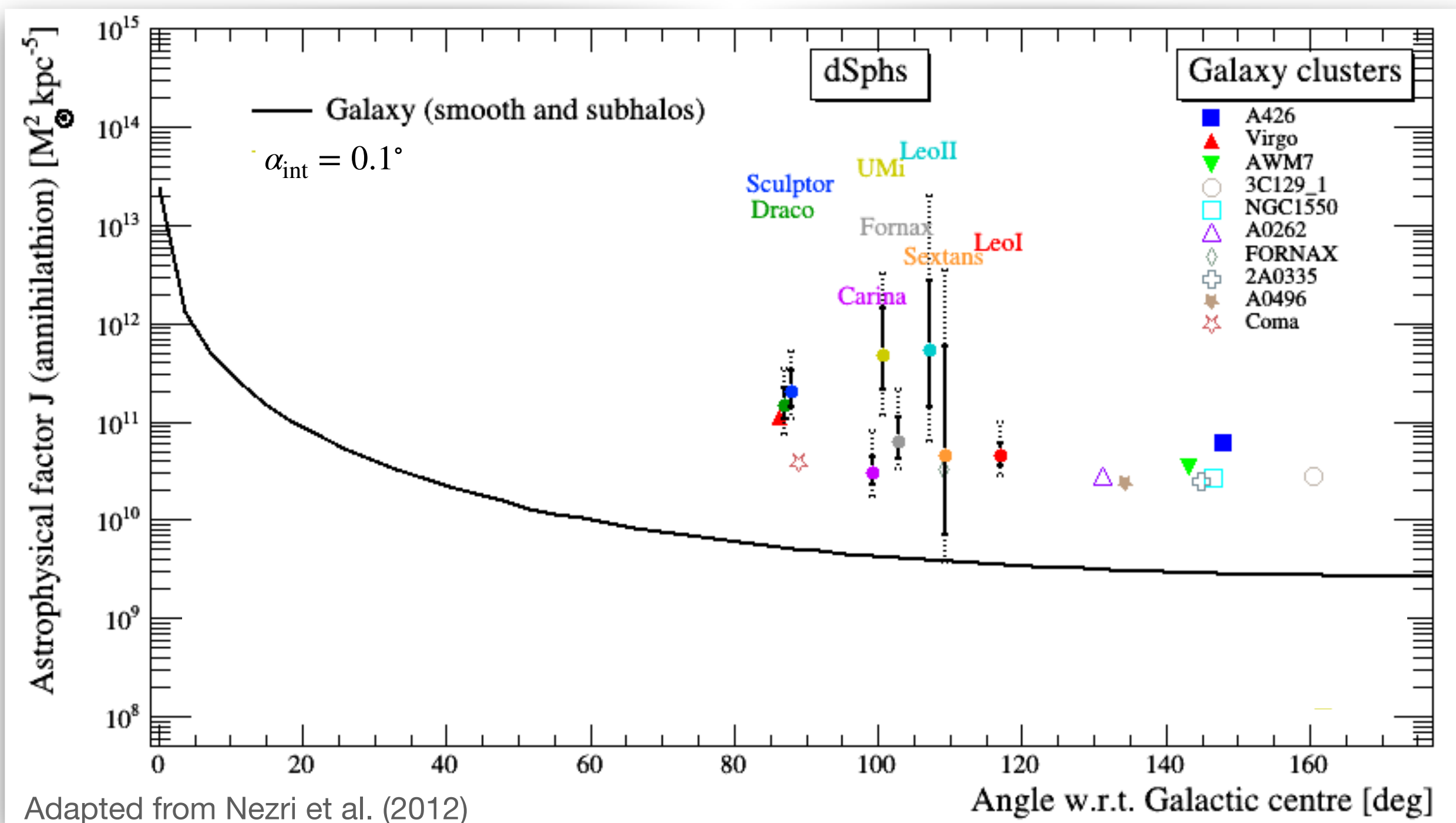
Best cluster J-factors \lesssim than that of dSphs

Galaxy clusters

J-factors, boost from substructures?

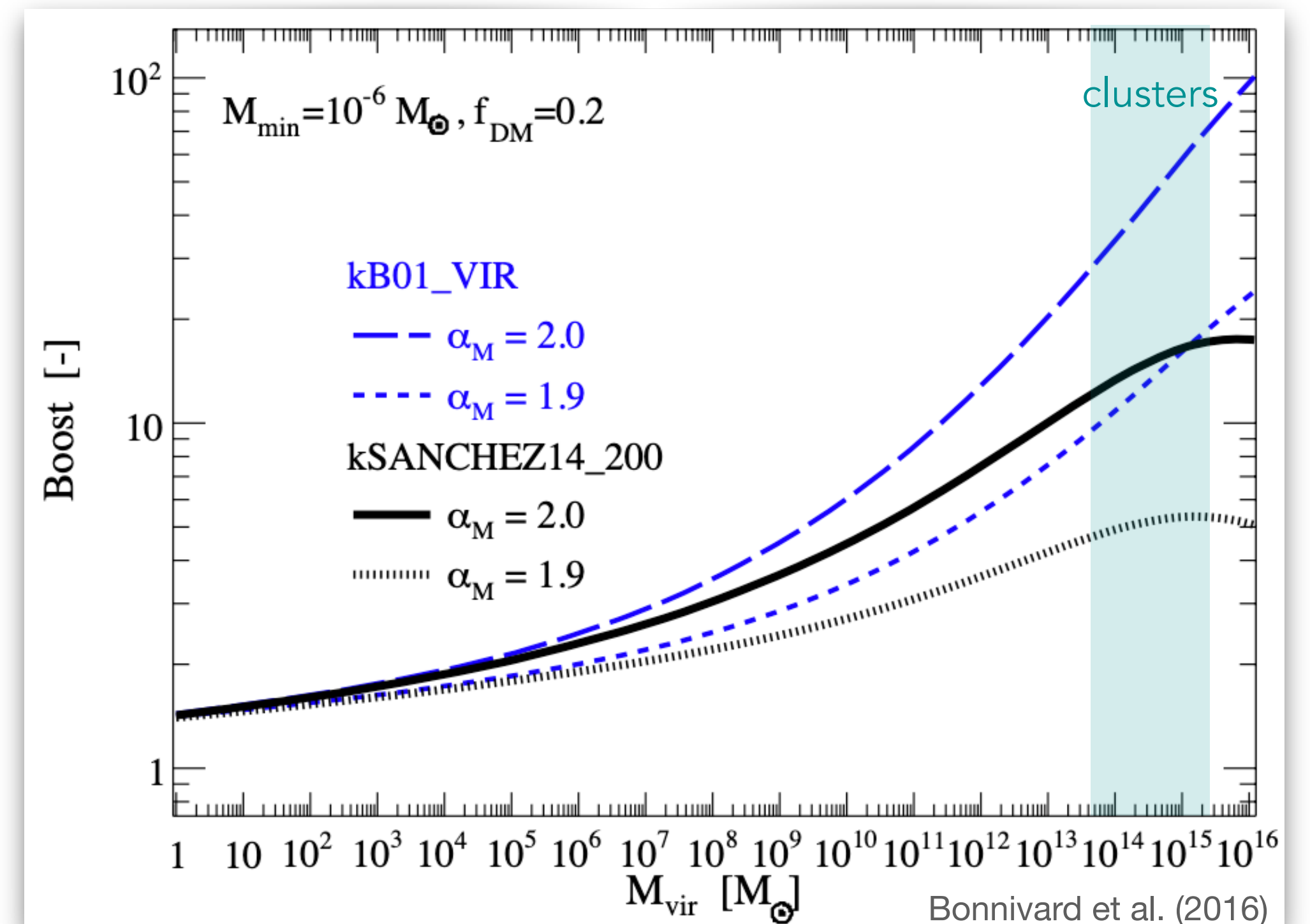
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Boost in galaxy clusters?

- early 2010s, boost ~ 1000 ! Power-law extrapolation down to $10^{-6} M_{\text{sun}}$ of M-c relations obtained from simulations ($M > 10^{10} M_{\text{sun}}$)
- Now, using flattened M-c relation, boost ~ 10 -50



Galaxy clusters

Summary

- Galaxy clusters are the densest part of the universe
- Large X-ray catalogs of “close-by” clusters. Option to stack the signal in survey data
- Their masses/profiles can be determined in multiple wavelength
 - allow to cross-check results
 - cuspy profiles, NFW
- Substructures may boost the annihilation signal by $\sim 10-50$
- J-factors \lesssim than that of dSphs

DM modeling of extragalactic targets

1. Galaxy clusters
2. The extragalactic diffuse exotic signal

The diffuse extragalactic signal

Getting started

- Dark matter in the entire universe annihilates
→ gives rise to an isotropic exotic gamma-ray signal
- Recall: at the Galactic scale, we had

$$\frac{d\Phi}{dE}(E, \vec{k}, \Delta\Omega(\alpha_{\text{int}})) = \underbrace{\frac{\langle\sigma v\rangle}{4\pi \delta m_{\text{DM}}^2} \sum_f \frac{dN^f}{dE} B_f}_{\text{particle physics}} \times \underbrace{\int_0^{2\pi} d\beta \int_0^{\alpha_{\text{int}}} \sin\alpha d\alpha \int_0^{l_{\text{max}}} \rho^2(\vec{k} l, \alpha, \beta) dl}_{\text{Astrophysical "J-factor": } [M_{\odot}^2 \text{ kpc}^{-5}] \text{ or } [\text{GeV}^2 \text{cm}^{-5}]}$$

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particle physics
Astrophysical "J-factor": [$M_{\odot}^2 \text{kpc}^{-5}$] or [$\text{GeV}^2 \text{cm}^{-5}$]

Can we separate the spectral and astrophysical part when considering the extragalactic emission? Why?

What new ingredients do we need?

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Can we separate the spectral and astrophysical part when considering the extragalactic emission? Why?

No. Integration along l_{os} = integration over redshift range and spectrum depends on redshift

What new ingredients do we need?

- Cosmology
- Halo mass function
- EBL absorption model

The diffuse extragalactic signal

$$I(E_\gamma) = \left\langle \frac{d\Phi}{dE_\gamma d\Omega} \right\rangle_{\text{sky}} = \frac{\bar{\rho}_{\text{DM},0}^2 \langle \sigma v \rangle}{8\pi m_\chi^2} \int_0^{z_{\text{max}}} c dz \frac{(1+z)^3}{H(z)} \langle \delta^2(z) \rangle \frac{dN_{\text{source}}^\gamma}{dE_e} \Big|_{E_e=(1+z)E_\gamma} \times e^{-\tau(z, E_\gamma)}$$

Structure formation Source Spectrum and EBL absorption

"Intensity multiplier"

$$\langle \delta^2(z) \rangle = \frac{1}{\bar{\rho}_{\text{m},0}^2} \int dM \frac{dn}{dM}(M, z) \times \mathcal{L}(M, z)$$

single halo luminosity

Halo mass function

$$\frac{dn}{dM}(M, z) = f(\sigma, z) \frac{\bar{\rho}_{\text{m},0}}{M} \frac{d \ln \sigma^{-1}}{dM}$$

Variance of the density field on scale defined by R(M)

$$\sigma^2(M, z) = \frac{D(z)^2}{2\pi^2} \int P_{\text{lin}}(k, z=0) \widehat{W}^2(kR) k^2 dk$$

Linear matter power spectrum
= f(cosmo)

The diffuse extragalactic signal

Capturing the modeling uncertainties

Hütten et al (2018)

Reference intensity: I_0 ($M \geq 10^{10} M_\odot$, no subhalos)			
<i>Physics properties</i>	Reference I_0	Variations $I_{0, \text{var}}$	$ I_0 - I_{0, \text{var}} /I_0$
Halo mass function [†]	R16 [28]	T08 [32], B16 [55]	$\lesssim 40\%$
Density profile ρ_{halo}	$\alpha_E = 0.17$	$\alpha_E = 0.15, \alpha_E = 0.22$, NFW	$\lesssim 20\%$
$c_\Delta(M_\Delta)$ relation [‡]	C15 [29]	L16 [30], C15- $\sigma_c=0.2$, (S14)	$\lesssim 10\%$
Cosmology (h, Ω_i, P_k) [§]	<i>Planck</i> -R16 [28]	WMAP7 [56], (WMAP-T08)	$\lesssim 10\%$
Overdensity definition	Δ_{vir} (3.3)	Δ_c (3.1) or Δ_m (3.2)=200	$\lesssim 5\%$
EBL model*	I13 [57]	F08 [58], D11 [59], G12 [60]	$\lesssim 5 - 40\%$
Total CDM contribution: I_l (extrapolation to low masses) ($M \geq M_{\text{min}}$, no subhalos)			
<i>Field halo properties</i>	Values (default in bold)		I_l/I_0 ($\simeq 5$)
Slope of dn/dM , α_M	1.85, 1.9 , 1.95		$\sim 4 - 14$
Minimal mass M_{min}	10^{-12} , 10^{-6} , $10^{-3} M_\odot$		$\sim 4 - 8$
Density profile ρ_{halo}	$\alpha_E = 0.15$, 0.17 , 0.22, NFW, Ishiyama [61]		$\sim 4 - 8$
$c_\Delta(M_\Delta)$ relation [‡]	C15 [29], L16 [30], (S14 [33])		$\sim 3 - 8$
...including boost from subhalos: I_b ($m \geq m_{\text{min}}$ with $m_{\text{min}} \equiv M_{\text{min}}$)			
<i>(Sub-)halo properties</i>	Values (default in bold)		I_b/I_l ($\simeq 1.5$)
Mass fraction f_{subs}	10%, 20% , 40%		$\sim 1.2 - 2.2$
Minimal mass m_{min}	10^{-12} , 10^{-6} , $10^{-3} M_\odot$		$\sim 1.3 - 1.8$
$c_\Delta(M_\Delta)$ relation [‡]	C15 [29], L16 [30], (S14 [33])		$\sim 1.3 - 1.7$
Density profile ρ_{subhalo}	$\alpha_E = 0.15$, 0.17 , 0.22, NFW, Ishiyama [61]		$\sim 1.3 - 1.7$
Slope of dP/dm , α_m	1.85, 1.9 , 1.95		$\sim 1.4 - 1.7$
dP/dV profile	Aquarius [62], Phoenix [63], $\propto \rho_{\text{host}}$		$\sim 1.49 - 1.51$

[†] T08 (Tinker et al., 2008), B16 (Bocquet et al., 2016), R16 (Rodríguez-Puebla et al., 2016)

[‡] S14 (Sánchez-Conde & Prada, 2014, [33]), C15 (Correa et al., 2015), L16 (Ludlow et al., 2016)

[§] *Planck*-R16 (MultiDark-*Planck* simulations used in Rodríguez-Puebla et al., 2016), WMAP-T08 (Cosmology used in T08, [32])

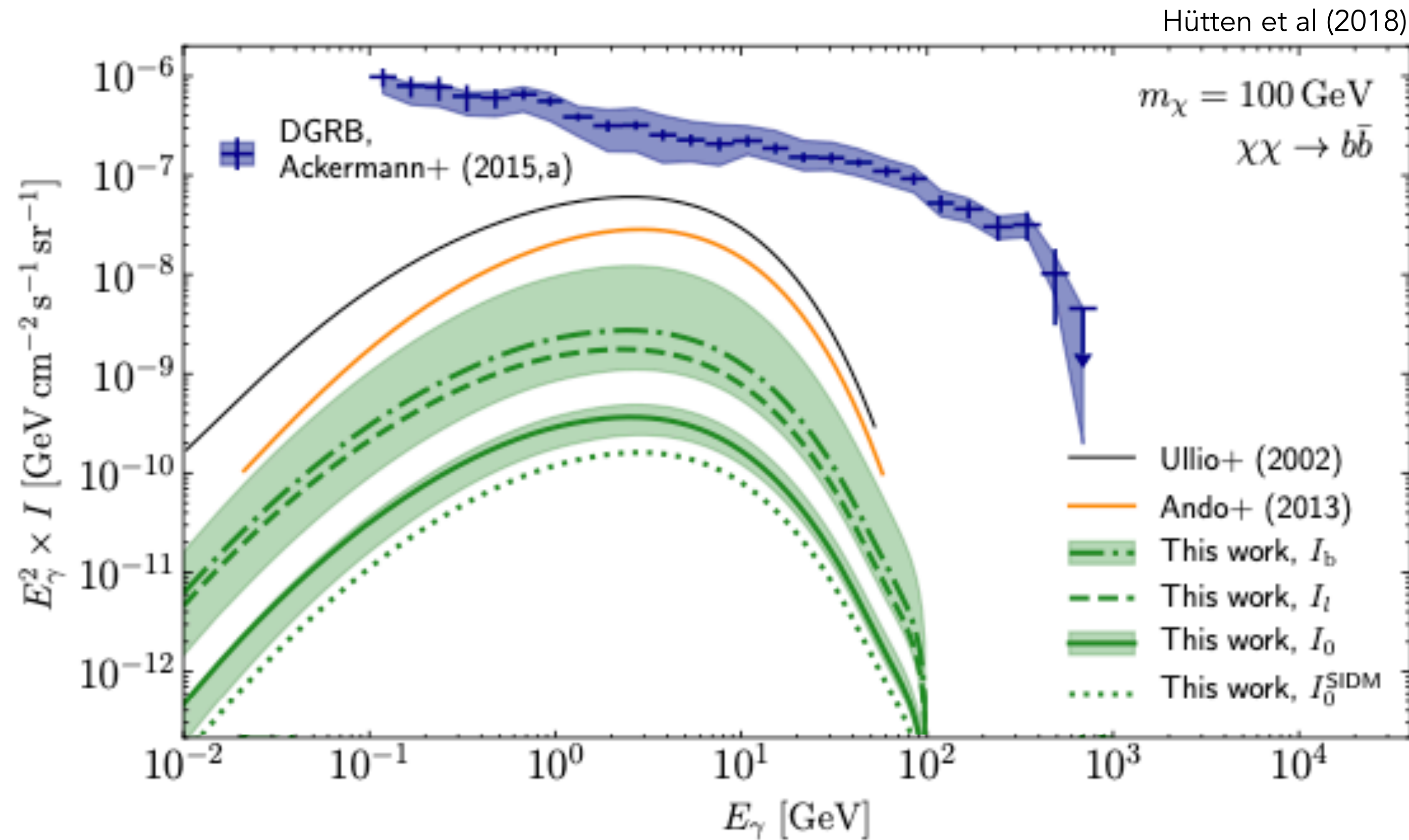
* F08 (Franceschini et al., 2008), D11 (Domínguez et al., 2011), Gilmore et al. (2012), and I13 (Inoue et al., 2013)

A lot of possible options, all available in



The diffuse extragalactic signal

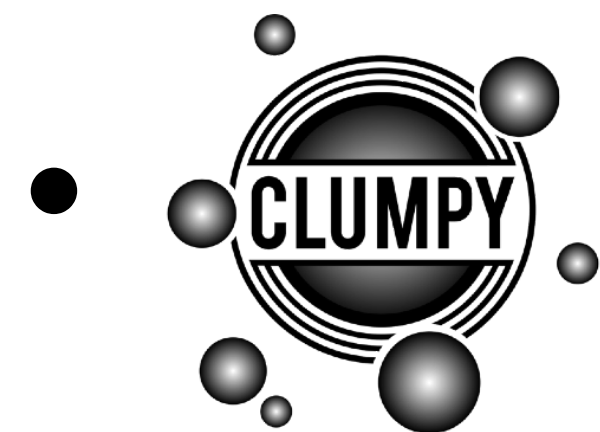
Estimation of the signal



Modeling uncertainties: ~ 1 order of magnitude

So, to conclude

- Robust constraints on DM properties require robust J- (or D-) factors determination
- Numerical simulations provide a lot of insight into modeling of DM distribution
- Apart for dark clumps, observational information (dynamics, etc.) specific to the targets under scrutiny can help up infer the DM profile
- Doing so, there are a lot of user-defined choices that may impact the results so remember to check the dependence of the results with respect to a range of these choices.



v3.1 is out! Update for this afternoon's hands on if you wish/can.