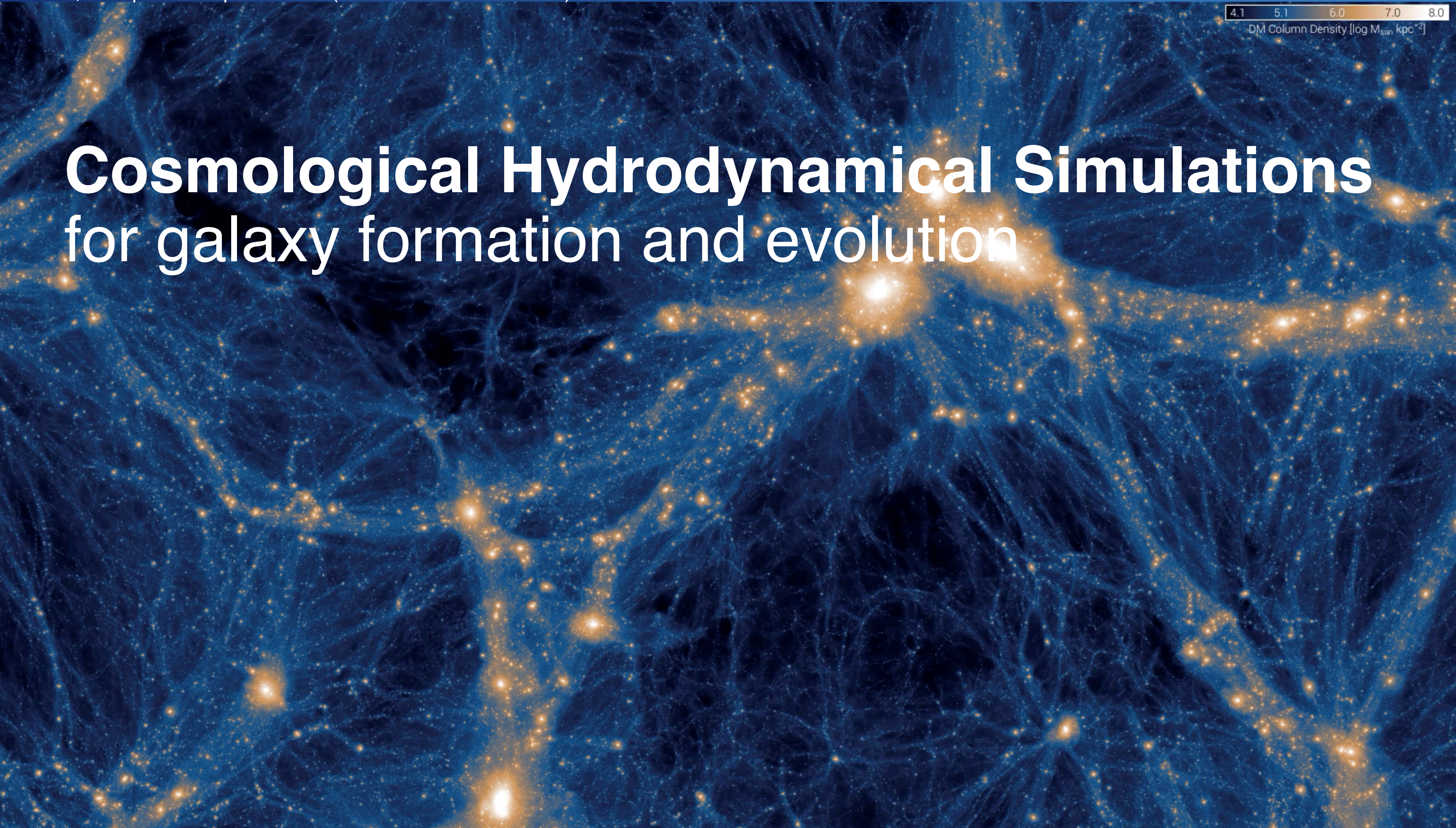
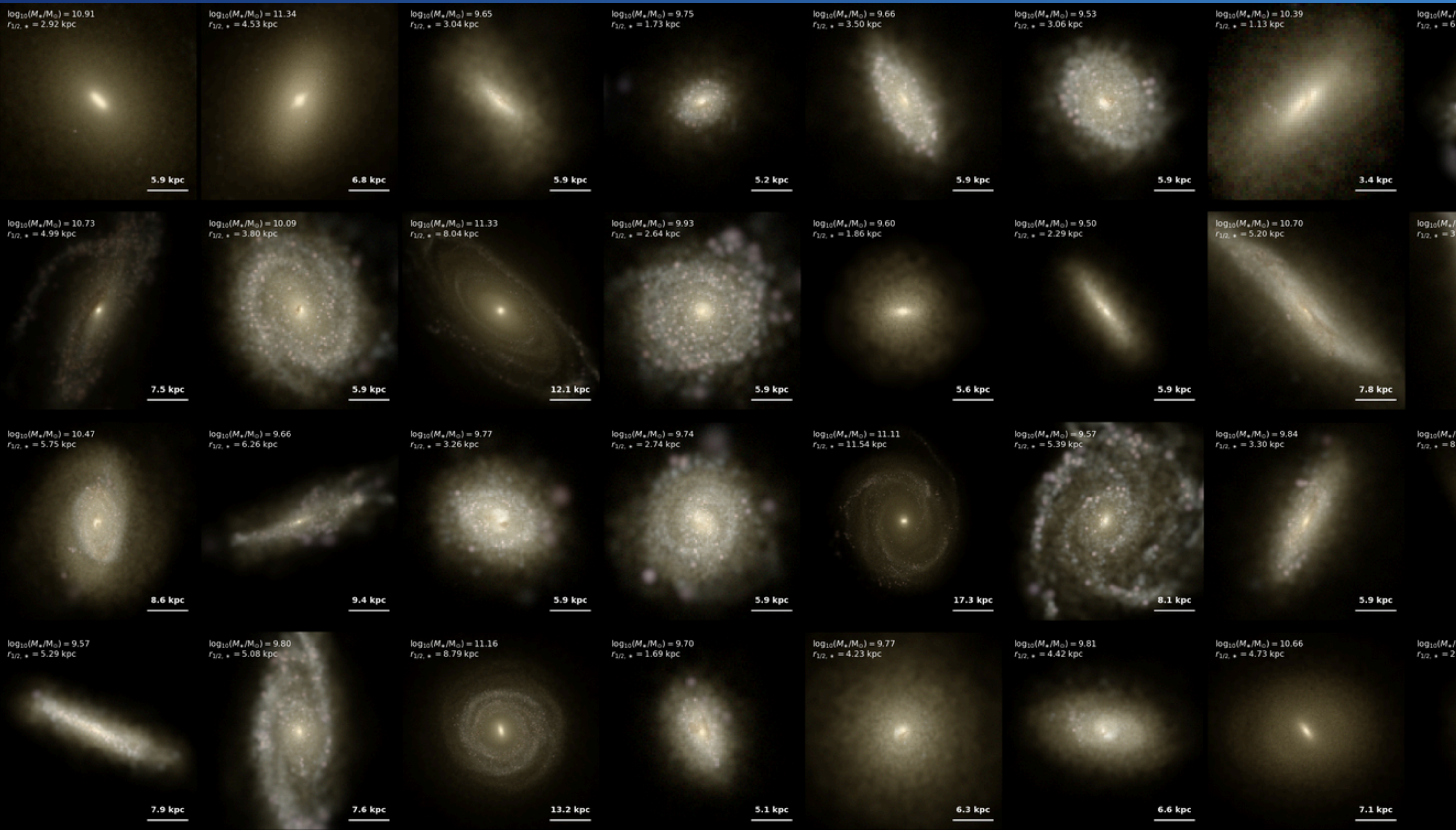
TNG50, 30 Mpc from top to bottom (Movie Credits: D. Nelson)

Cosmological Hydrodynamical Simulations
for galaxy formation and evolution



Annalisa Pillepich, ISAPP21 School, 2021/06/24


$\log_{10}(M_*/M_\odot) = 10.91$
$r_{1/2,*} = 2.92$ kpc
5.9 kpc
$\log_{10}(M_*/M_\odot) = 11.34$
$r_{1/2,*} = 4.53$ kpc
6.8 kpc
$\log_{10}(M_*/M_\odot) = 9.65$
$r_{1/2,*} = 3.04$ kpc
5.9 kpc
$\log_{10}(M_*/M_\odot) = 9.75$
$r_{1/2,*} = 1.73$ kpc
5.2 kpc
$\log_{10}(M_*/M_\odot) = 9.66$
$r_{1/2,*} = 3.50$ kpc
5.9 kpc
$\log_{10}(M_*/M_\odot) = 9.53$
$r_{1/2,*} = 3.06$ kpc
5.9 kpc
$\log_{10}(M_*/M_\odot) = 10.39$
$r_{1/2,*} = 1.13$ kpc
3.4 kpc
$\log_{10}(M_*/M_\odot) = 10.73$
$r_{1/2,*} = 4.99$ kpc
7.5 kpc
$\log_{10}(M_*/M_\odot) = 10.09$
$r_{1/2,*} = 3.80$ kpc
5.9 kpc
$\log_{10}(M_*/M_\odot) = 11.33$
$r_{1/2,*} = 8.04$ kpc
12.1 kpc
$\log_{10}(M_*/M_\odot) = 9.93$
$r_{1/2,*} = 2.64$ kpc
5.9 kpc
$\log_{10}(M_*/M_\odot) = 9.60$
$r_{1/2,*} = 1.86$ kpc
5.6 kpc
$\log_{10}(M_*/M_\odot) = 9.50$
$r_{1/2,*} = 2.29$ kpc
5.9 kpc
$\log_{10}(M_*/M_\odot) = 10.70$
$r_{1/2,*} = 5.20$ kpc
7.8 kpc
$\log_{10}(M_*/M_\odot) = 10.47$
$r_{1/2,*} = 5.75$ kpc
8.6 kpc
$\log_{10}(M_*/M_\odot) = 9.66$
$r_{1/2,*} = 6.26$ kpc
9.4 kpc
$\log_{10}(M_*/M_\odot) = 9.77$
$r_{1/2,*} = 3.26$ kpc
5.9 kpc
$\log_{10}(M_*/M_\odot) = 9.74$
$r_{1/2,*} = 2.74$ kpc
5.9 kpc
$\log_{10}(M_*/M_\odot) = 11.11$
$r_{1/2,*} = 11.54$ kpc
17.3 kpc
$\log_{10}(M_*/M_\odot) = 9.57$
$r_{1/2,*} = 5.39$ kpc
8.1 kpc
$\log_{10}(M_*/M_\odot) = 9.84$
$r_{1/2,*} = 3.30$ kpc
5.9 kpc
$\log_{10}(M_*/M_\odot) = 9.57$
$r_{1/2,*} = 5.29$ kpc
7.9 kpc
$\log_{10}(M_*/M_\odot) = 9.80$
$r_{1/2,*} = 5.08$ kpc
7.6 kpc
$\log_{10}(M_*/M_\odot) = 11.16$
$r_{1/2,*} = 8.79$ kpc
13.2 kpc
$\log_{10}(M_*/M_\odot) = 9.70$
$r_{1/2,*} = 1.69$ kpc
5.1 kpc
$\log_{10}(M_*/M_\odot) = 9.77$
$r_{1/2,*} = 4.23$ kpc
6.3 kpc
$\log_{10}(M_*/M_\odot) = 9.81$
$r_{1/2,*} = 4.42$ kpc
6.6 kpc
$\log_{10}(M_*/M_\odot) = 10.66$
$r_{1/2,*} = 4.73$ kpc
7.1 kpc

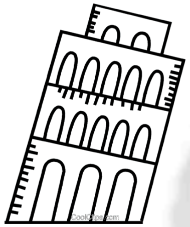
Two words about myself: Annalisa Pillepich (MPIA)

Research group leader since 2016
Max Planck Institute for Astronomy

Postdoc at
Harvard University

PhD (in Cosmology)
ETH zürich

Postdoc



Bachelor and Master
in (Theoretical) Physics at the
Universita' di Pisa

Two words about myself: Annalisa Pillepich (MPIA)

My role: **MPIA** group leader (since June 2016) in numerical astrophysics and cosmology

---

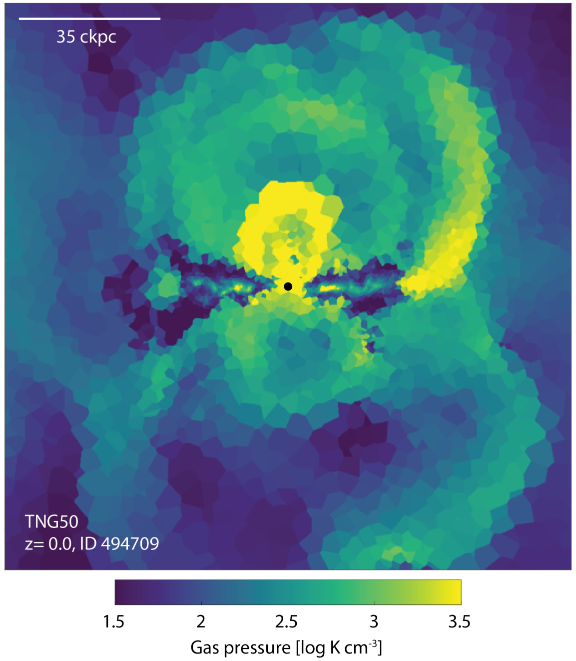
My research:

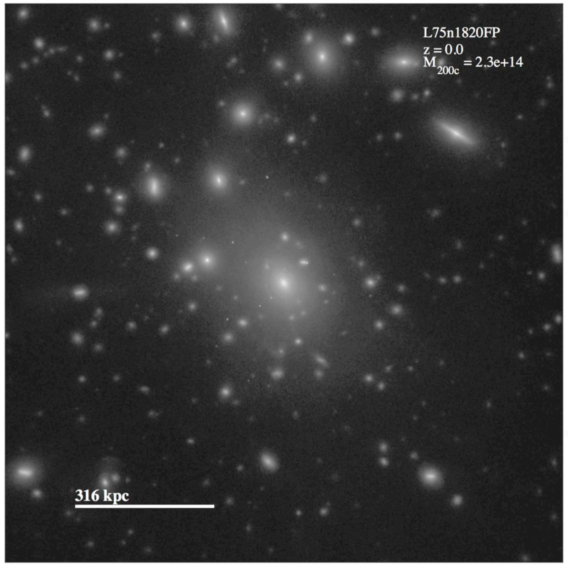
We develop, execute, analyse and contrast to observations numerical models to understand galaxies and the emergence of structures in the Universe.

Our favourite models are: cosmological, large-volume simulations including gravity+magneto-hydrodynamics + galaxy processes + star formation => population of galaxies

We run them on thousands of computing cores at National Supercomputer Facilities. Our current flagship model is: IllustrisTNG

I am also interested on Galaxy Clusters, as astrophysics labs and cosmological tools





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d. The baryonic fluid
e. Fundamentals of galaxy formation
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    a. Elements of SPH
    b. Elements of Eulerian hydrodynamics
    c. Moving-mesh and mesh-less techniques
g. The available cosmological codes
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    a. Star formation
    b. Stellar evolution
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j. Examples of state-of-the art simulations
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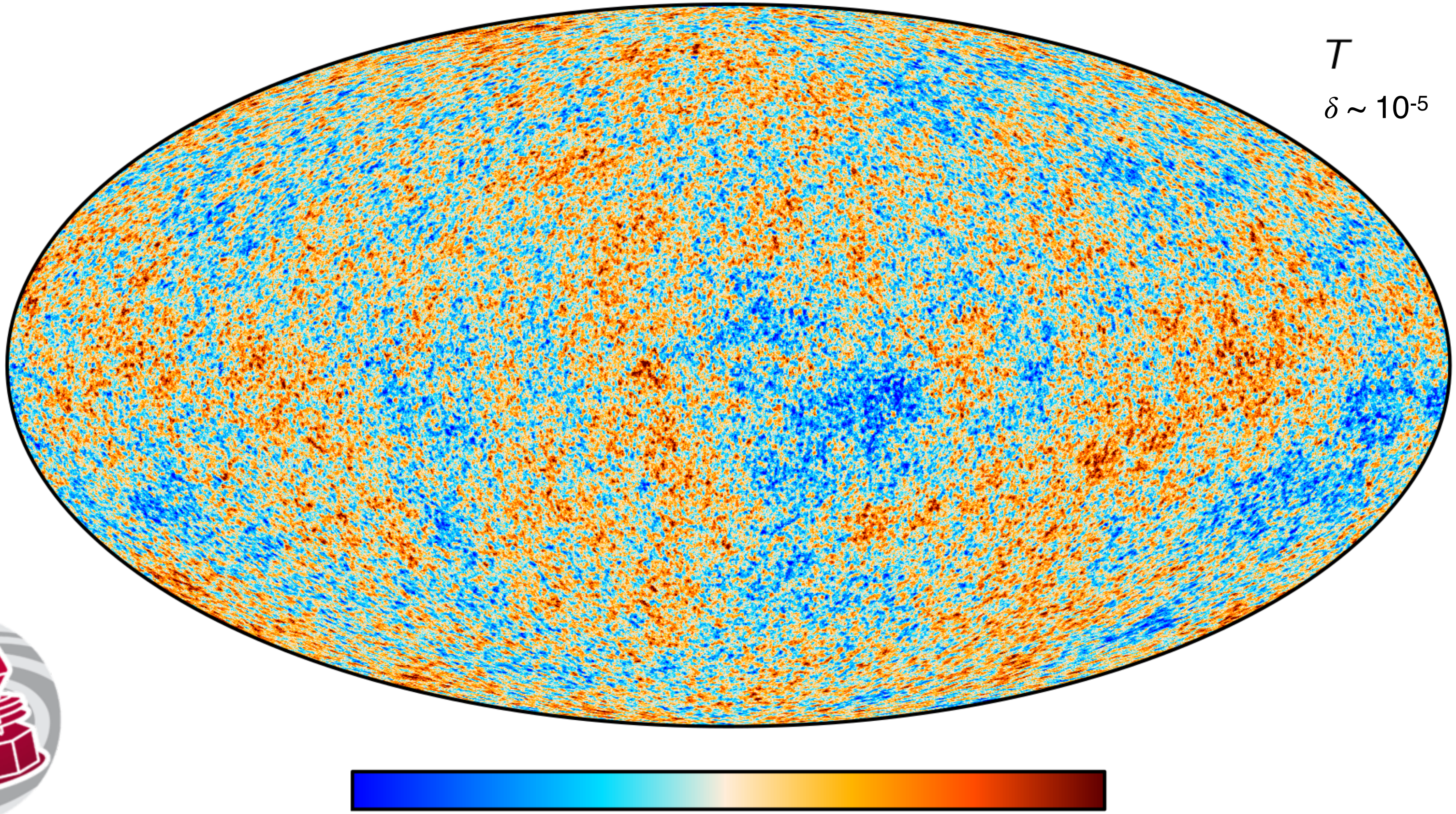
The initial conditions of the Universe

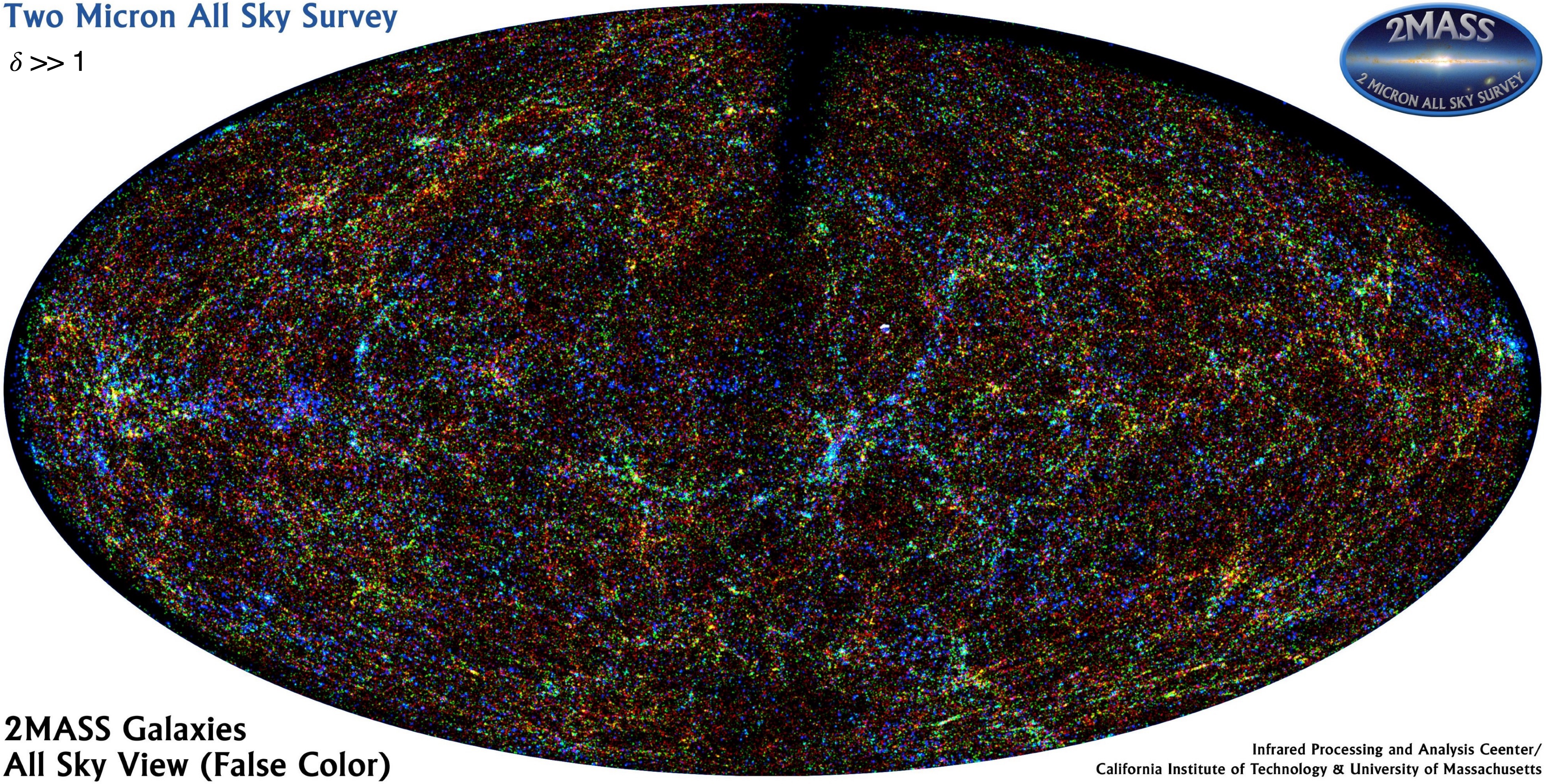


Planck Satellite (ESA)

The Universe today, after ~13.8 billion years of evolution

Two Micron All Sky Survey

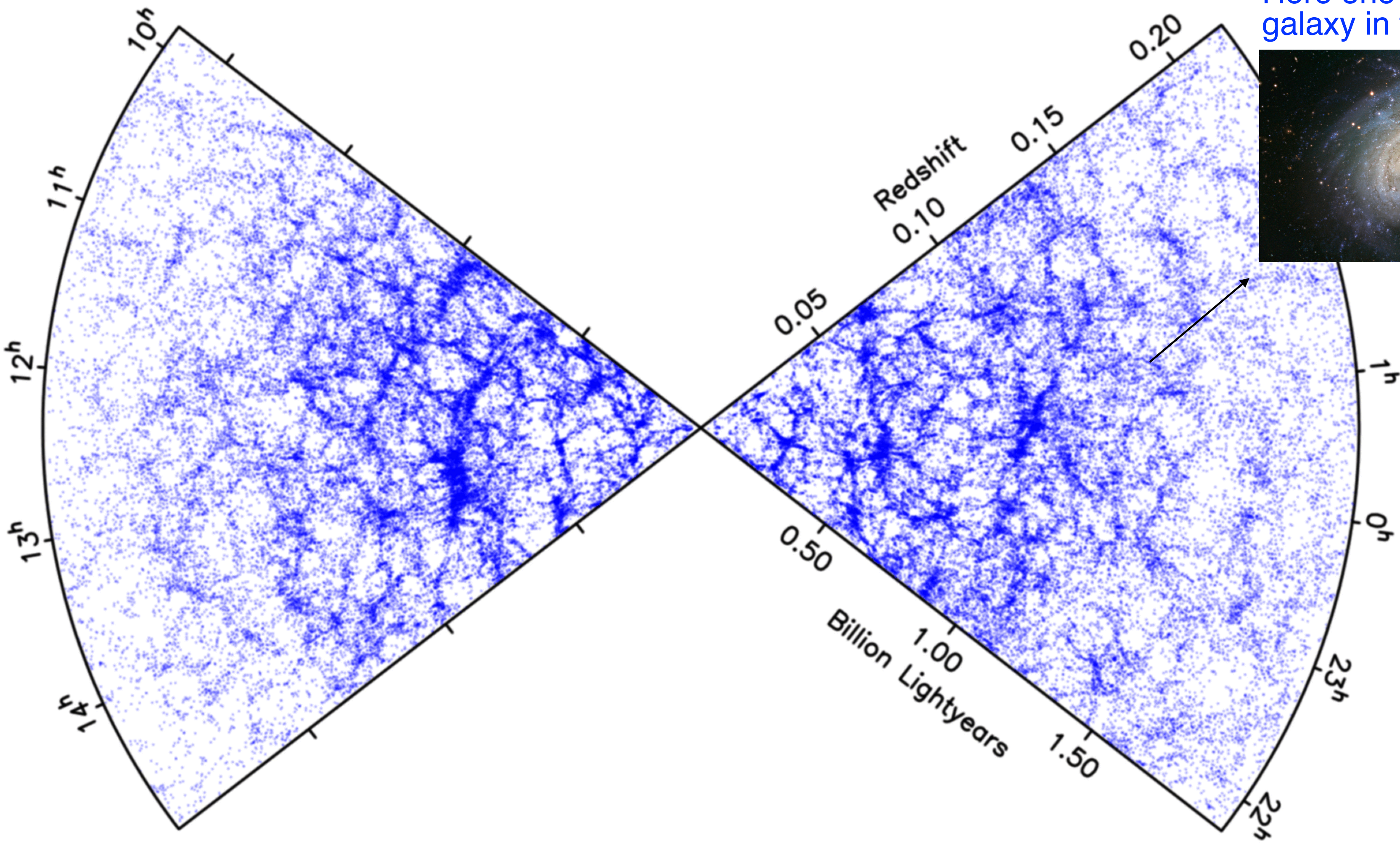
$\delta >> 1$



2MASS Galaxies
All Sky View (False Color)

Infrared Processing and Analysis Ceenter/
California Institute of Technology & University of Massachusetts

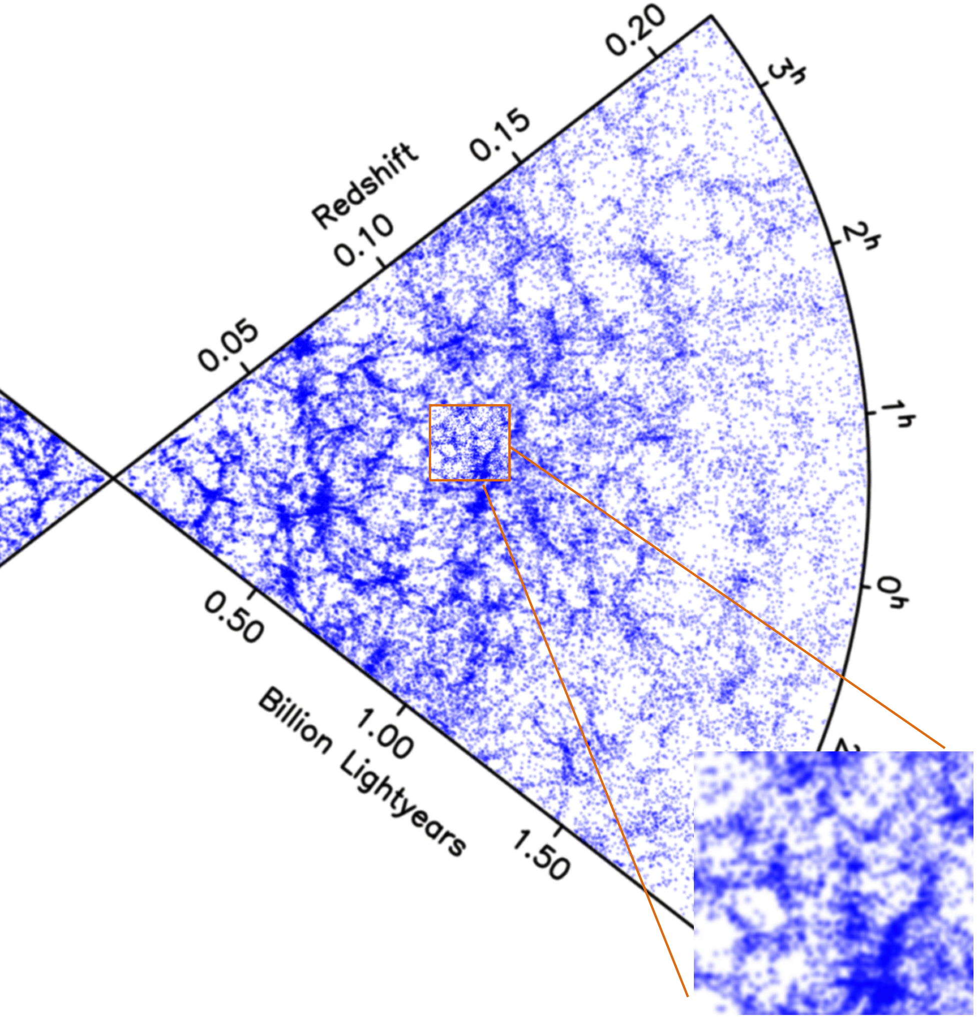
The Universe today, after ~13.8 billion years of evolution



Here one dot, one galaxy in the sky!!!

2dF Galaxy Redshift Survey (2002)

The Universe is *not* homogeneous on scales of 10-100 Mpc and below

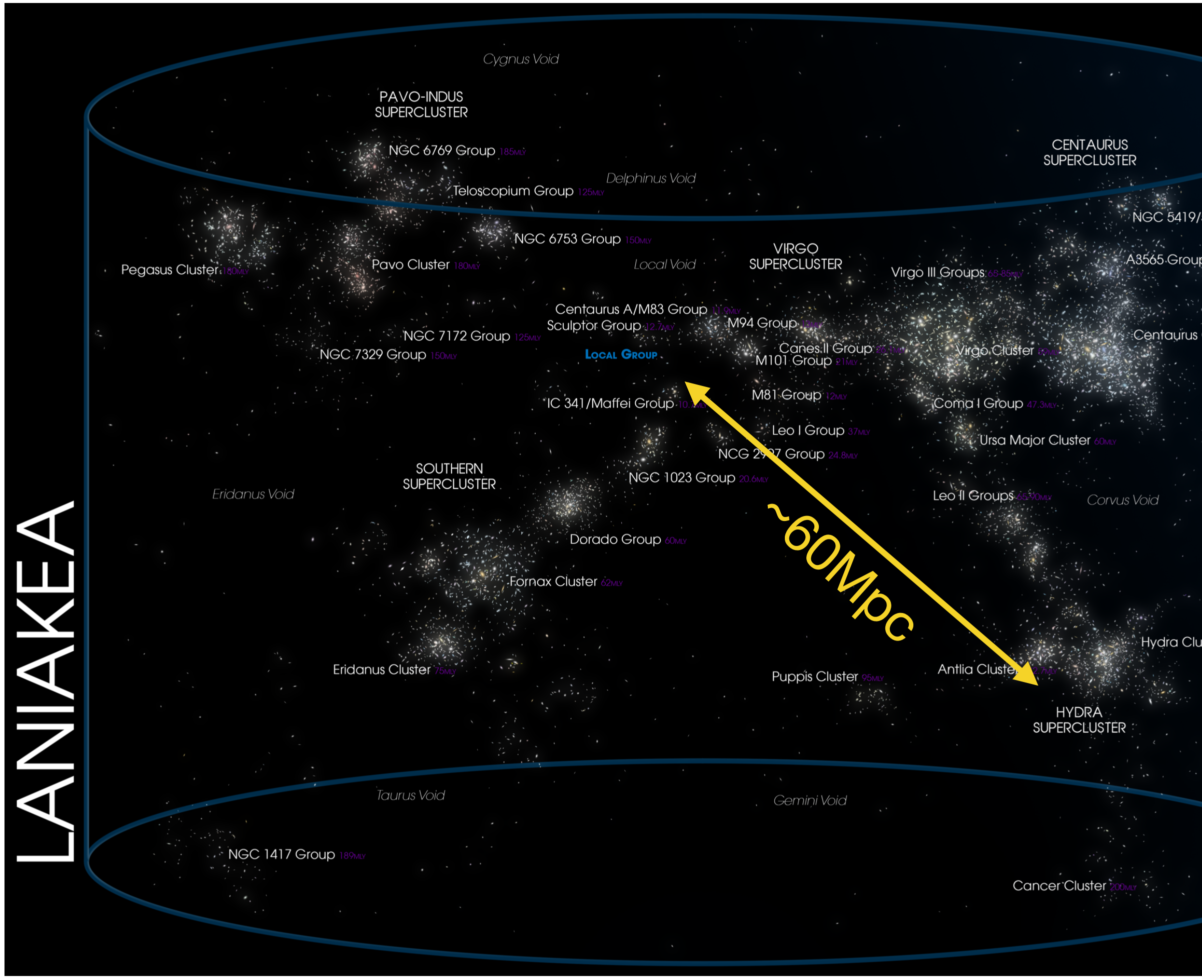


Not uniform distribution of galaxies!

2dF Galaxy Redshift Survey (2002)

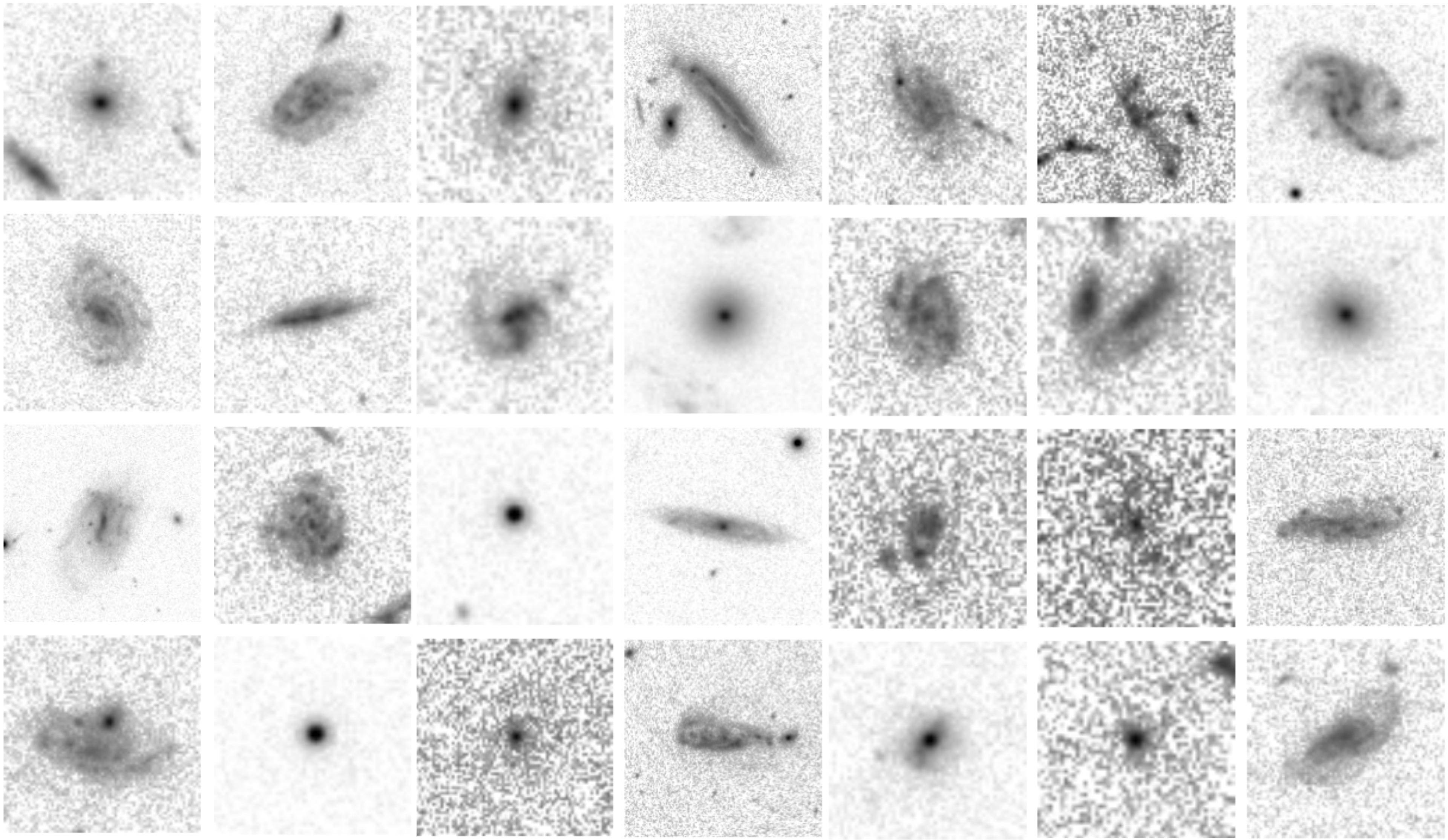
Our neighborhood:

1 Mpc = ~ 3 million light years



The Universe is certainly *not* homogeneous on the scales of galaxies, ~1-100 kpc

Observed galaxies exhibit diverse morphologies and shapes



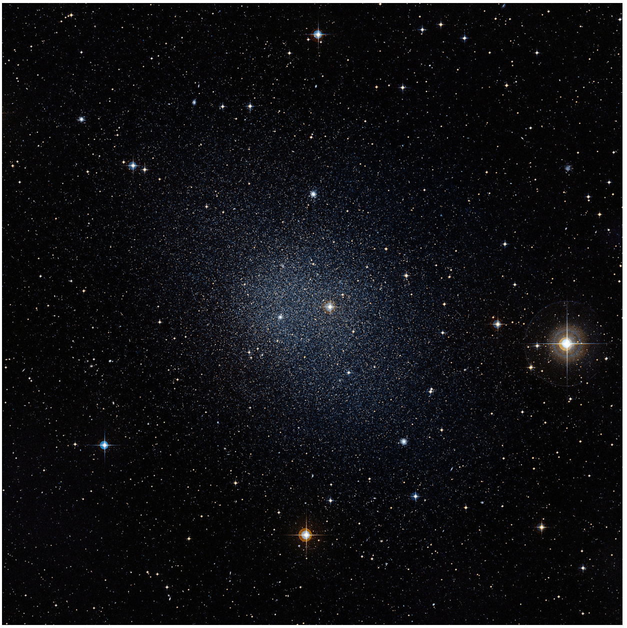
Observed galaxies exhibit diverse masses and spatial sizes

Solar Mass $M_{\odot} = 1.99 \times 10^{30}$ kg

Parsec: 1pc = 3.26 lyr = 3.086×10^{16} m

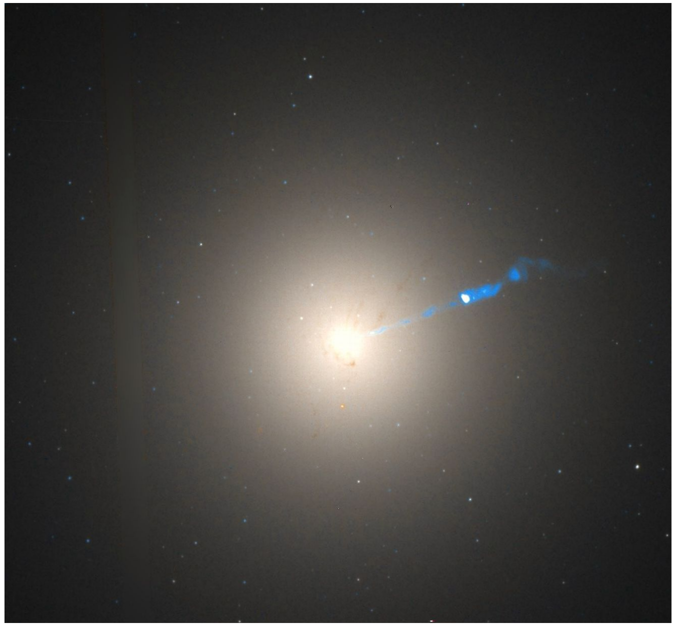
> 6 orders of magnitudes in mass

> 2 orders of magnitudes in size



10^6 solar masses
~100-500 pc

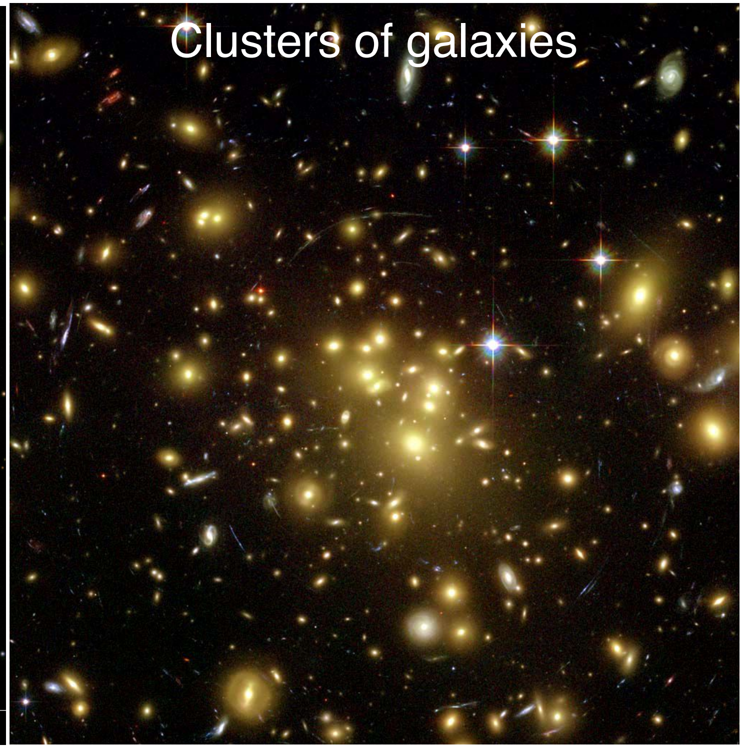
10^{10} solar masses
A few kpc



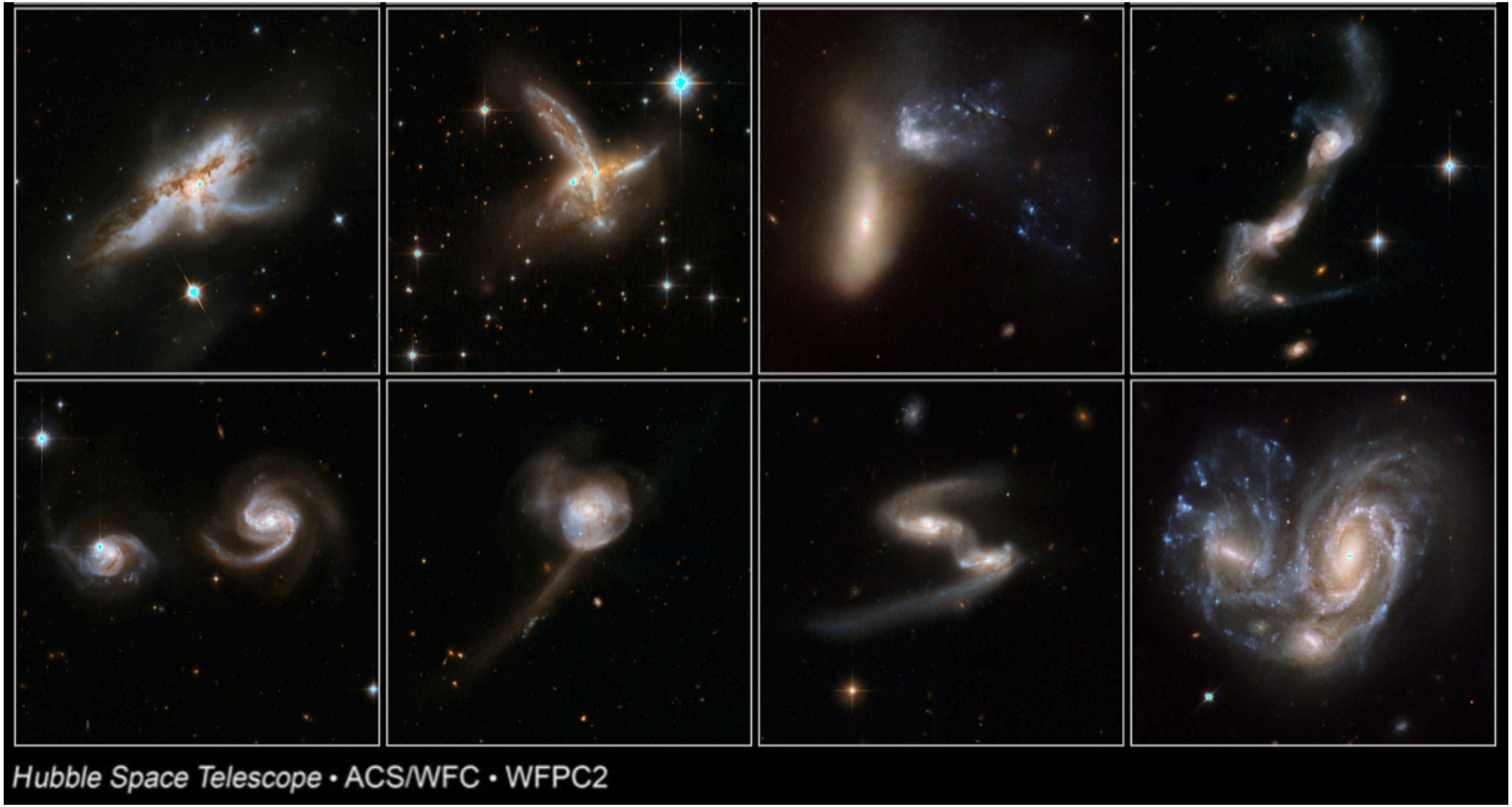
10^{12} solar masses
Tens of kpc

Observed galaxies can live in very different environments

Low densities → High densities

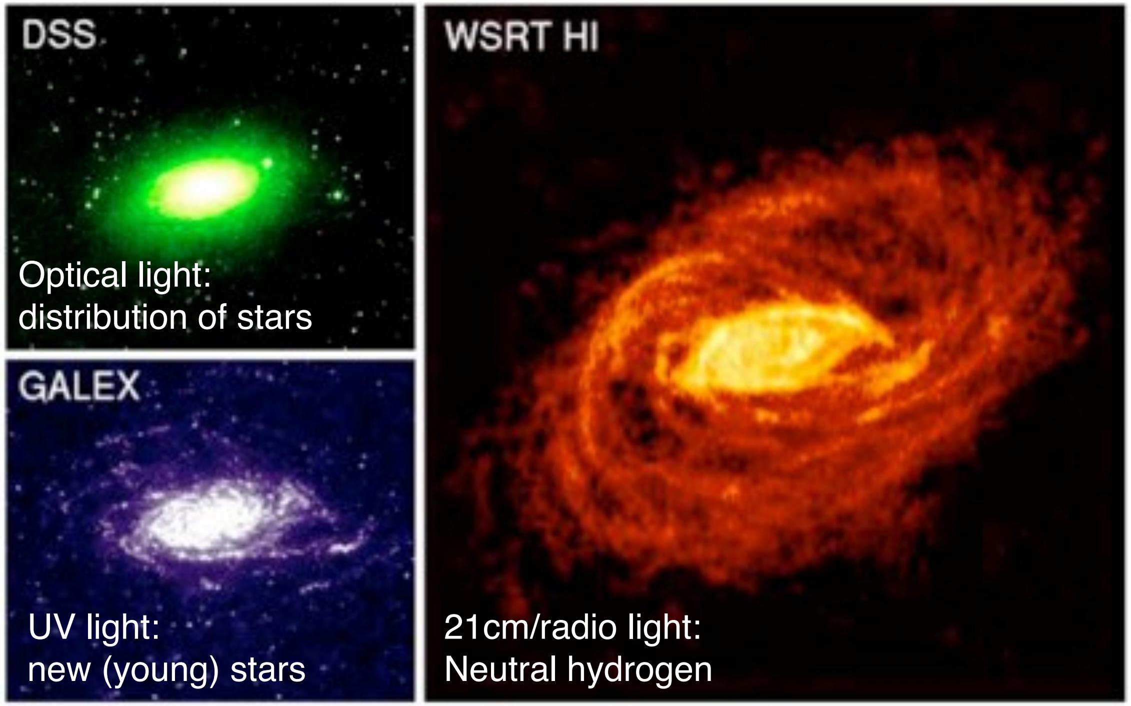


Observed galaxies can merge and interact!

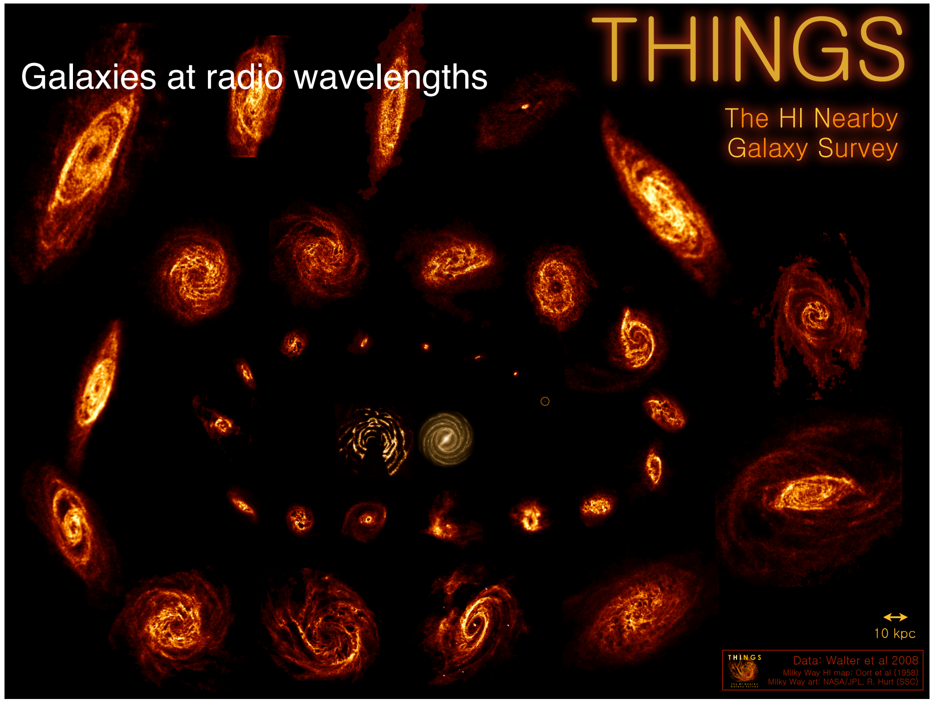


Galaxies are not just collections of stars

In (disk) galaxies, the stellar light is associated to emissions from "cold" gas, like Hydrogen (ten thousand degrees and below).



NGC 5055



The stellar bodies of galaxies are surrounded by gas!

COLD GAS
-200 to10'000 degrees C

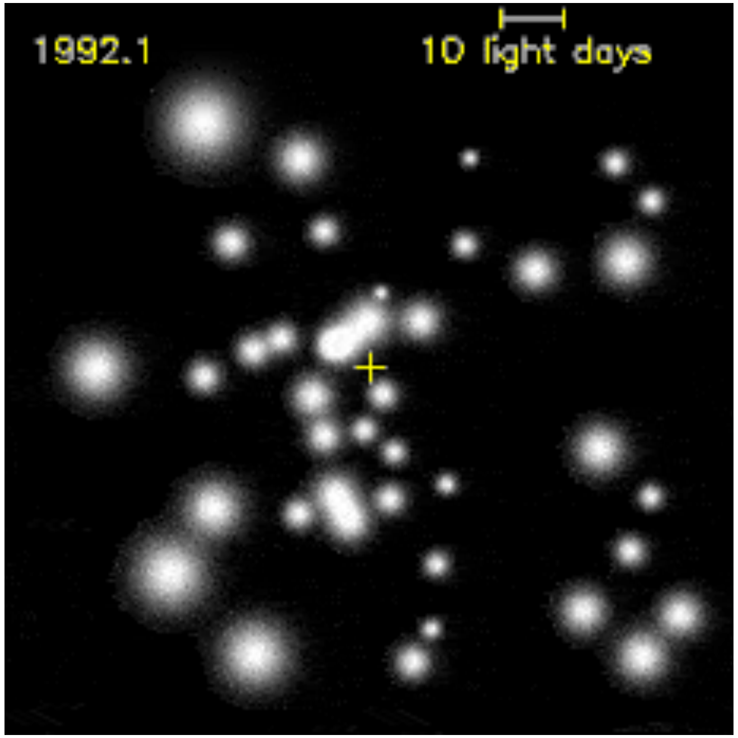
M82: stellar light in a disk + molecular gas perpendicular to the disk

HOT GAS
~million degree C

A galaxy cluster: stellar light + X-ray emitting gas (violet)

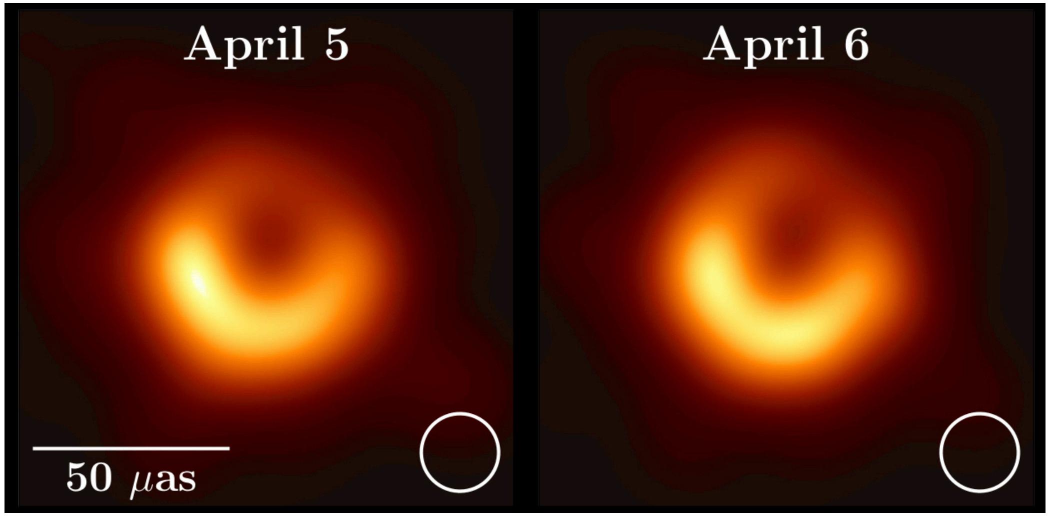
Most galaxies host even a super massive black hole at their center

The Milky Way has a SMBH at its centre: the motion of the stars is consistent with General Relativity. The mass of the MW's SMBH is relatively small: ~4x10^6 M$_\odot$



This is the first "image" of the event horizon around a black hole in a nearby galaxy.

The mass of this SMBH (>10^9 M$_\odot$) is more than a thousand times larger than the Milky Way's

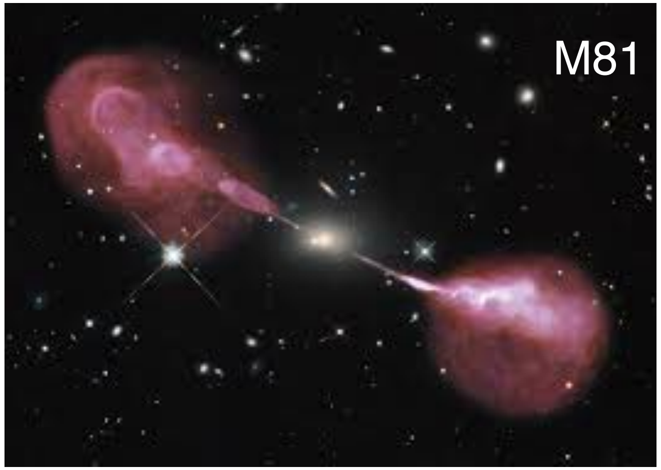


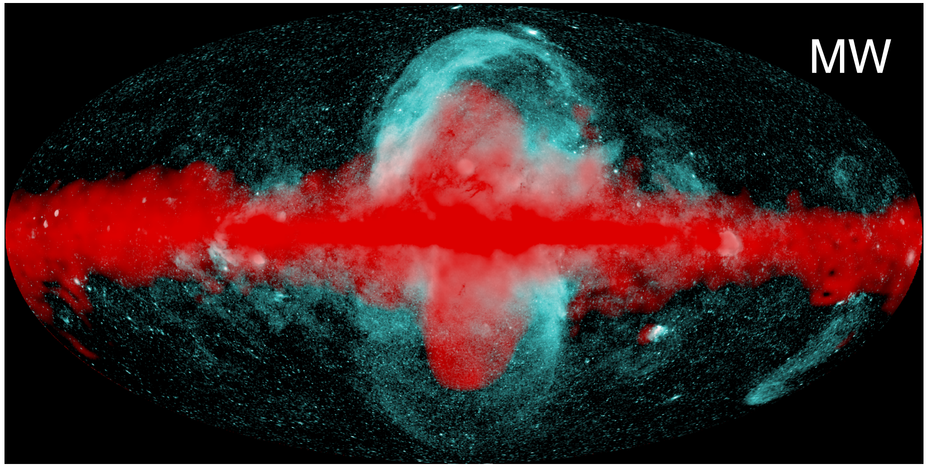
The more massive a galaxy is, the more massive its central SMBH is

But why are galaxies relevant for cosmology and gamma-ray physics?

Galaxies are the smallest “cosmological” structures

Galaxies are where things happen!





(AGN i.e. active SMBHs, supernova explosions, gas inflows and outflows, …)

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The goals of cosmological galaxy simulations

To formulate self-consistent models for
the formation+evolution of galaxies that:

a. Function across masses, spatial scales, evolutionary stages and environments

b. Return the observed statistical properties of the ***galaxy populations***

c. Reproduce the ***structural, internal properties*** of individual galaxies

The goals of cosmological galaxy simulations

1. To quantify what galaxies' properties tell us about their formation (both individually and as populations)

2. To provide interpretation to observational findings

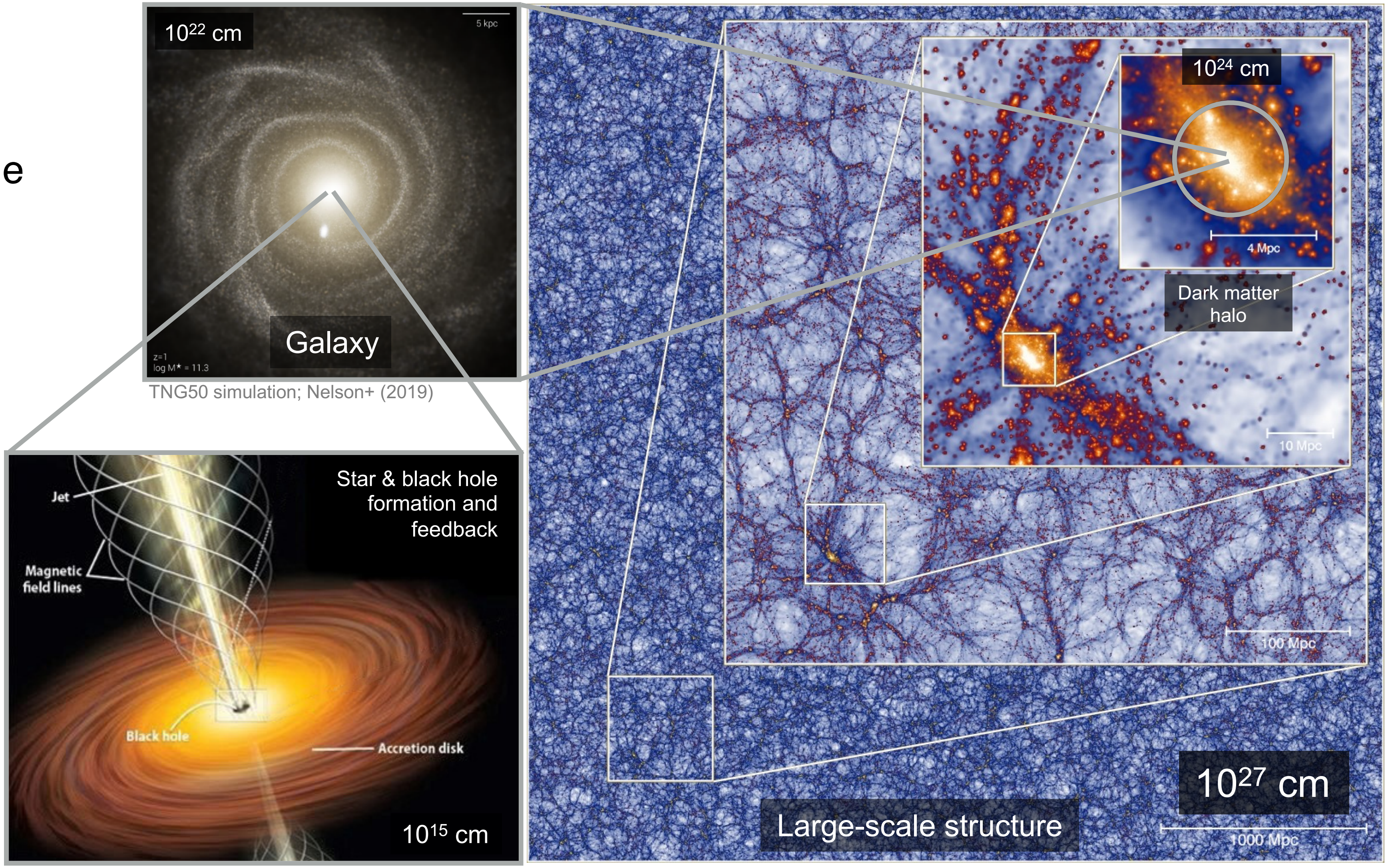
3. To ultimately infer, via comparison to observations, what galaxies can tell us about the underlying assumptions on gravity, matter and the Universe as a whole

The desiderata of cosmological galaxy simulations

1. Include the relevant physics
2. Account for the cosmological context
3. Simulate large ensembles of galaxies
   i.e. large mass ranges and diversity at a given mass
4. Match fundamental observations
5. Reach crucial physical scales

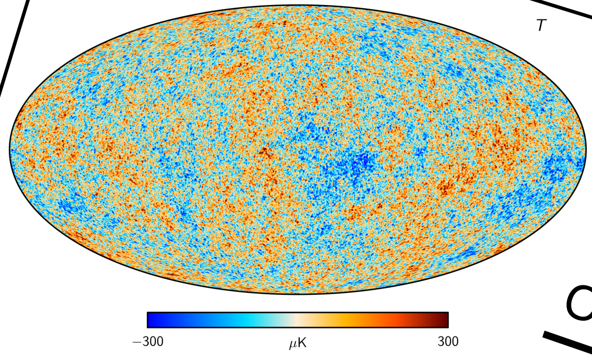
Galaxy formation is a multi-scale problem

Enormous
dynamic range
in space
(and time)



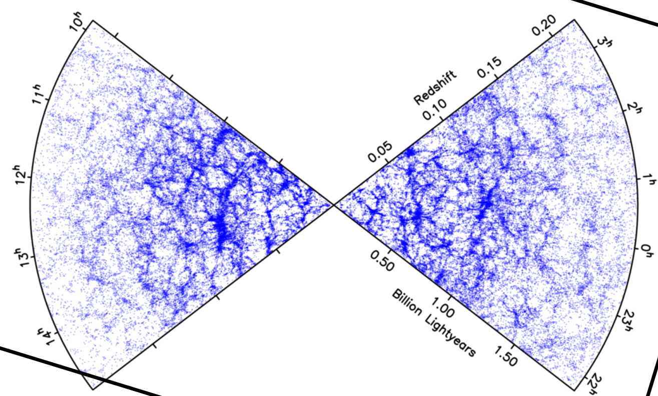
TNG50 simulation; Nelson+ (2019)

Galaxy formation is a multi-scale and multi-physics problem



Large scales

OK…



Harder…

Small scales

The Universe is highly structured

Gravity is not the only force of relevance:

Not just collapse…
Back propagation **from small to large scales**

Gravity only dominates

The Universe is ~ homogeneous and isotropic

The basic idea of Gravitational Instability and the emergence of structures

Gravity is only attractive.
Even if at the initial conditions there are tiny deviations from homogeneity, these will develop further and further as time goes by

A5.2. *GRAVITATIONAL INSTABILITY* 149

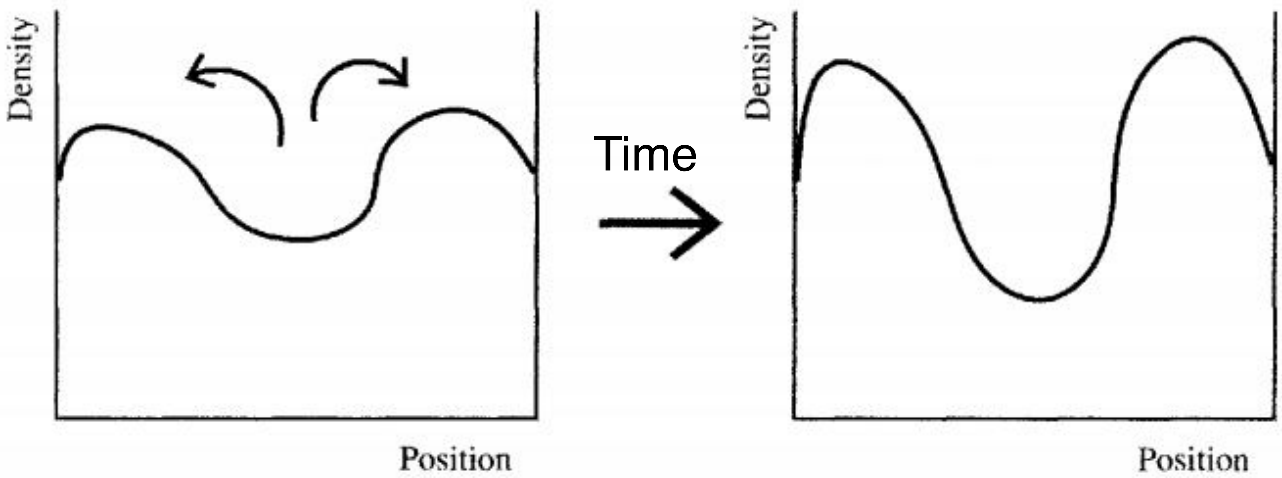


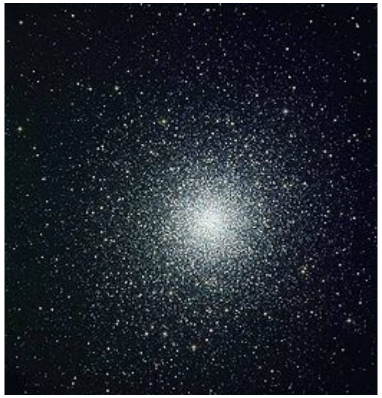
Figure A5.1 Gravity pulls material towards the denser regions, enhancing any initial irregularities.

Simulating the gravity-dominated regime

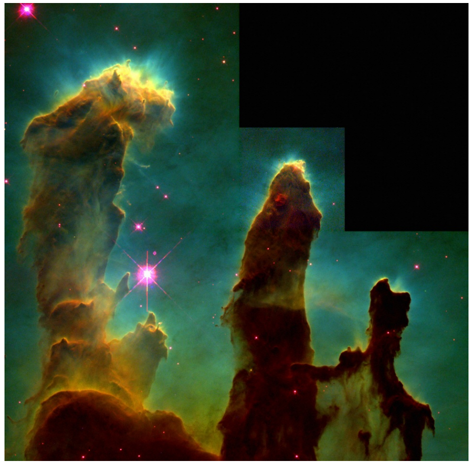
Credits: V. Springel

Simulating the small-scale regime of galaxies, from darkness to the light

Stars, also in clusters

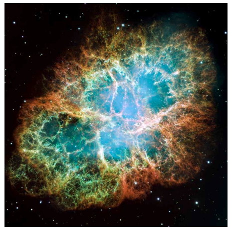


Sites of Formation of stars

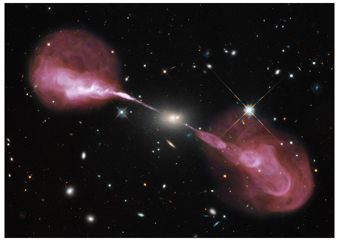


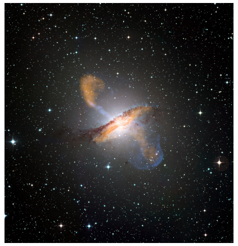
Galaxies are made of stars and cosmic gas,
i.e. “baryons”

Supernovae explosions



Gas and Dust:
Halo, Clouds, Winds





Supermassive Black Holes
at the centres

The basic idea of galaxy formation

Dark Matter

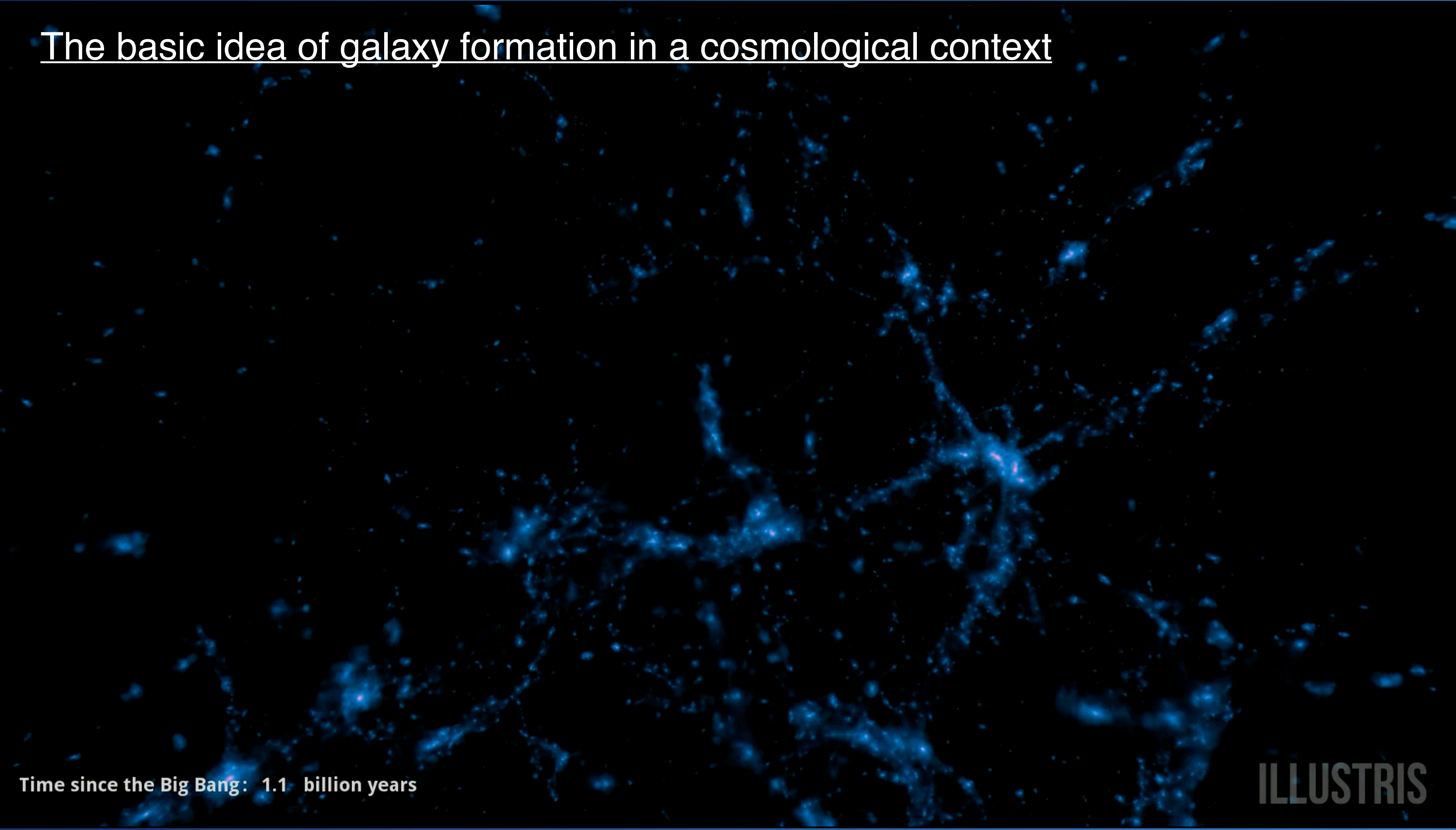
Galaxies form at the center of gravitationally-bound structures of dark matter: haloes

Gas

Dark-matter haloes are permeated of cosmic gas

Gas can cool, concentrate at the bottom of the gravitational potential well and transform into stars

The basic idea of galaxy formation in a cosmological context



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The setup and ingredients of large-scale cosmological *galaxy* simulations

Cosmological Model (e.g. LCDM)

Matter components

Dark matter

Gas

Stars

SMBHs

Magnetic fields

…

Physical processes

Gravity

Fluid dynamics

Atomic processes

…

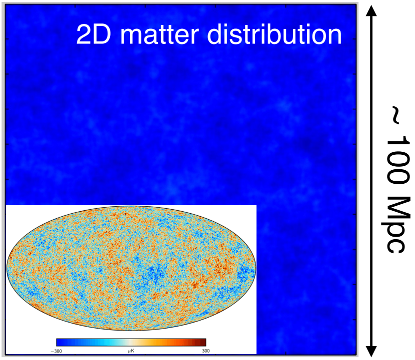
Photon propagation

Simulated domain

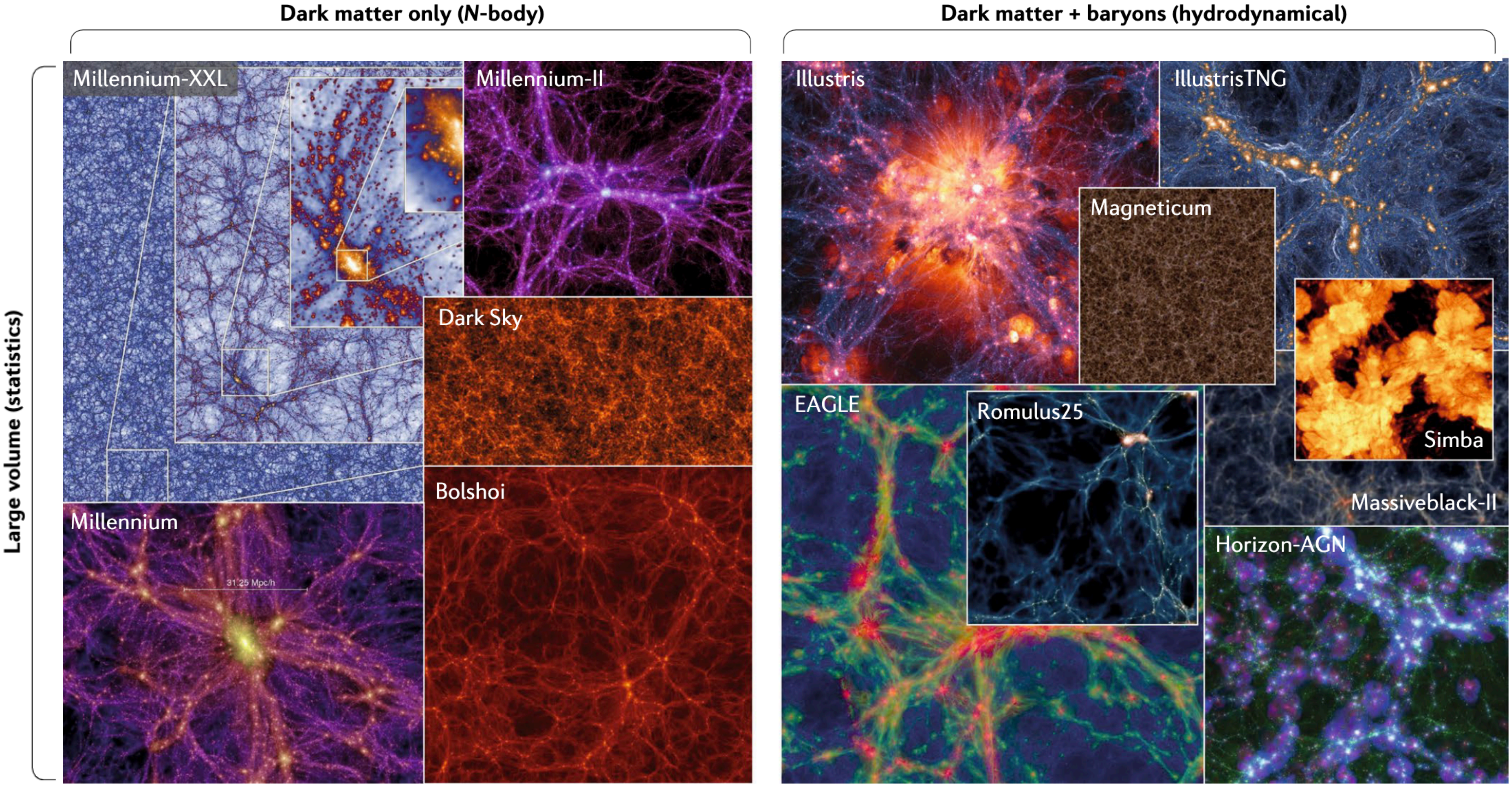
Cubes of representative portions of the Universe

(periodic boundary conditions)

From "cosmological" initial conditions

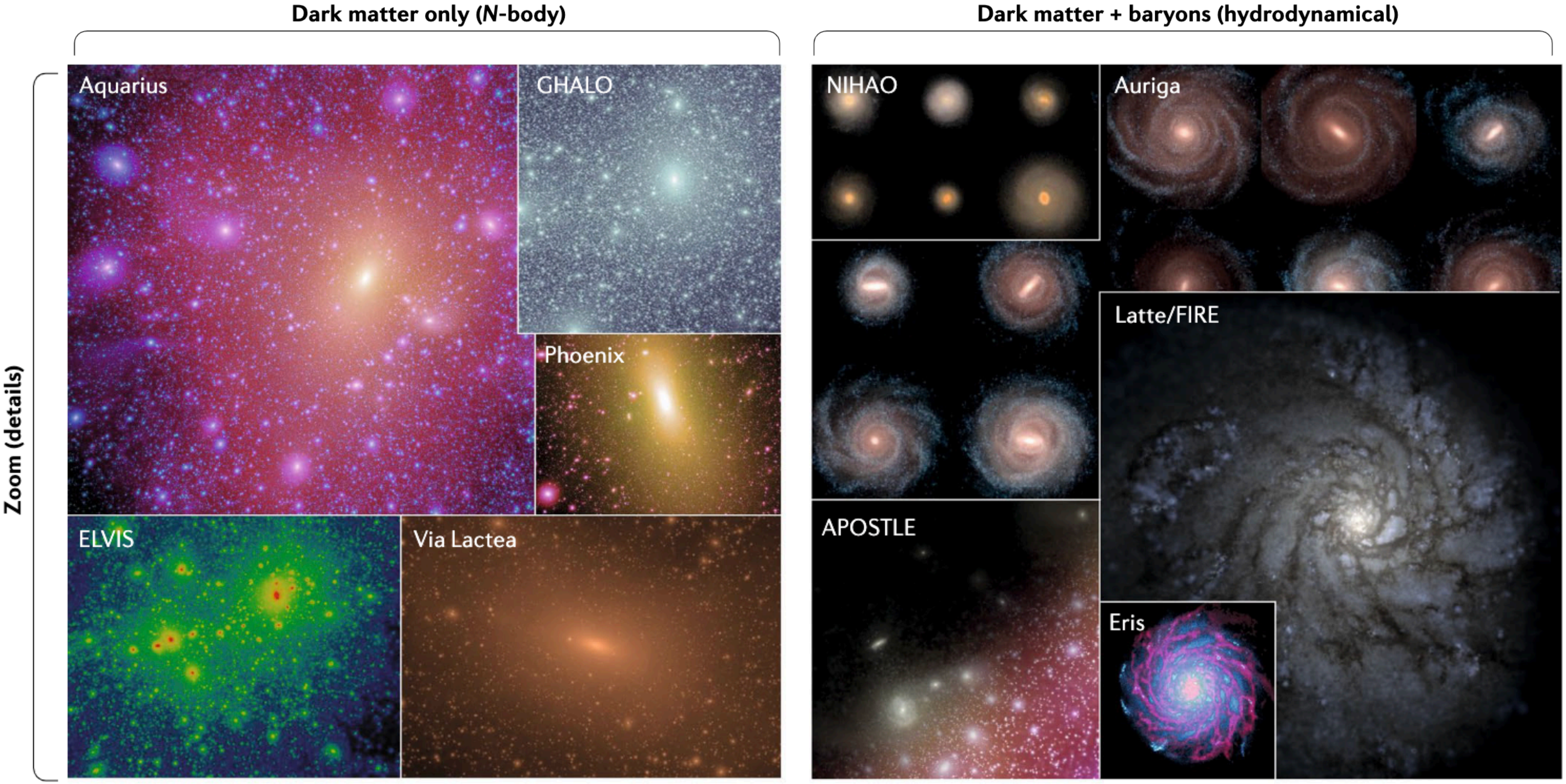


The types of large-scale cosmological *galaxy* simulations on the market



Vogelsberger+2020, Nature Technical Review

The types of large-scale cosmological *galaxy* simulations on the market



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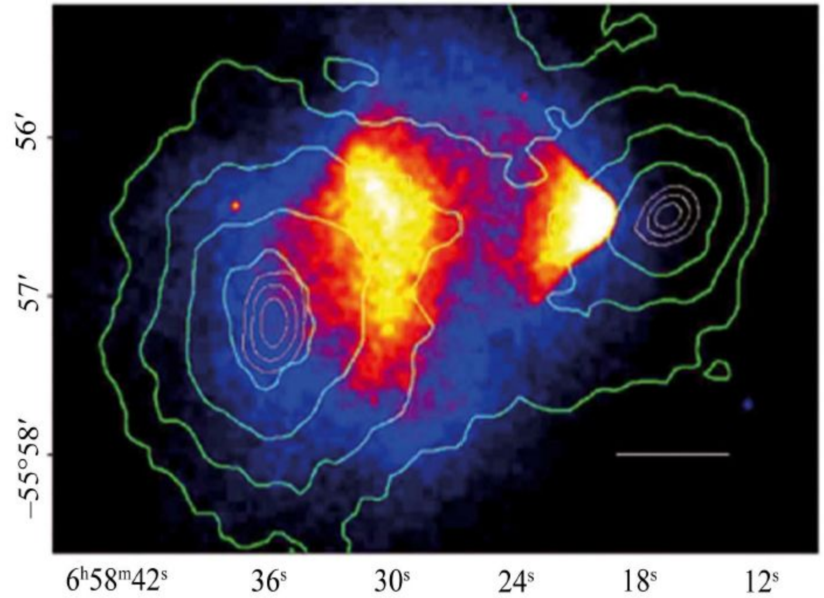
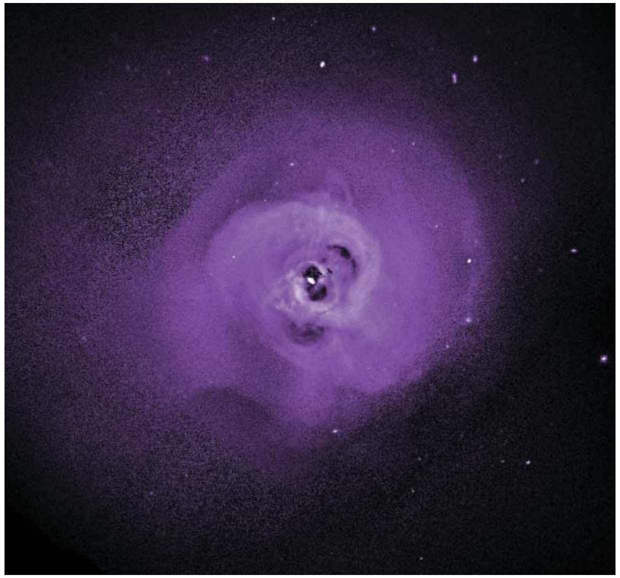
Baryons in the Universe, clearly, do not interact only via gravity

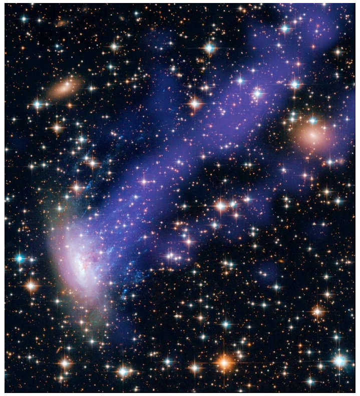
On the jargon:

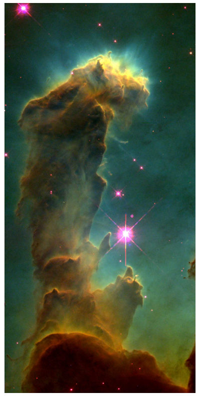
Components of the usual material objects (ions, atoms, and molecules) are called **“Baryonic matter”**

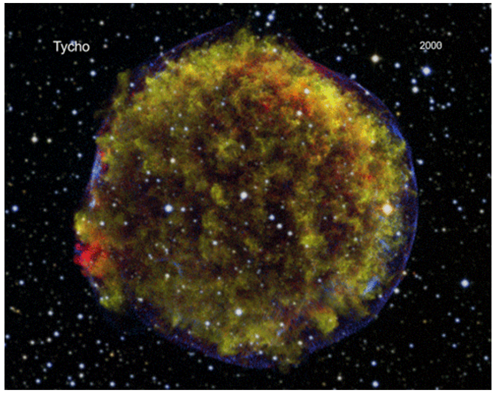
But e- are leptons!
On large scales the Universe is electrically neutral: #p = # e-
The mass density of the electrons is negligible compared to those of the baryons like p and n.











Gas dynamics is certainly more complicated than that of collisionless systems

Shocks

Hydrodynamical flows can develop shock waves, where density, velocity, temperature and specific energy jump by finite amounts

e.g. The shock connects two fluid states:
1=pre-shock
2=post-shock

$$\rho_2 > \rho_1$$
$$v_2 < v_1$$
$$T_2 > T_1$$

Mach Number

$$\mathcal{M} = \frac{v_1}{c_1}$$

Sound Speed

$$c_1^2 = \gamma P_1 / \rho_1$$

The shock itself decelerates, compresses, and heats up the fluid

In astrophysics: e.g. shocks that form when flows collide supersonically

Fluid Instabilities

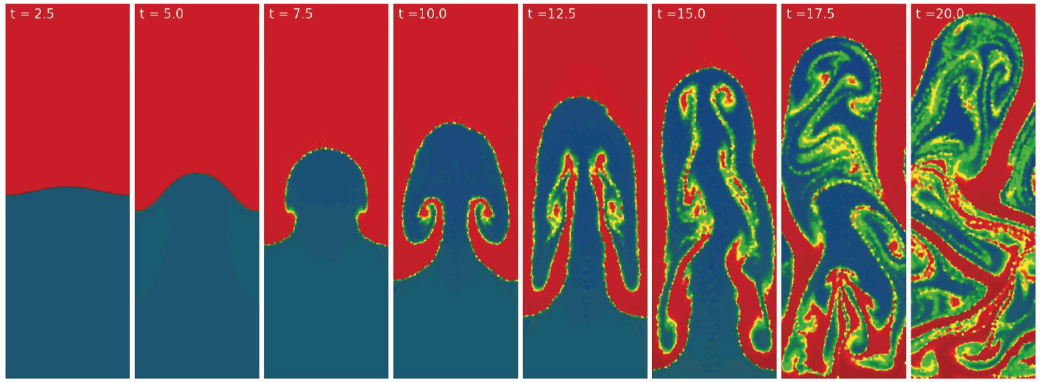
In some situations, in gaseous flows, small perturbations can rapidly grow

e.g. Rayleigh-Taylor Instability (in practice, buoyancy driven)

$\varrho > \varrho$

$\mathbf{g}$

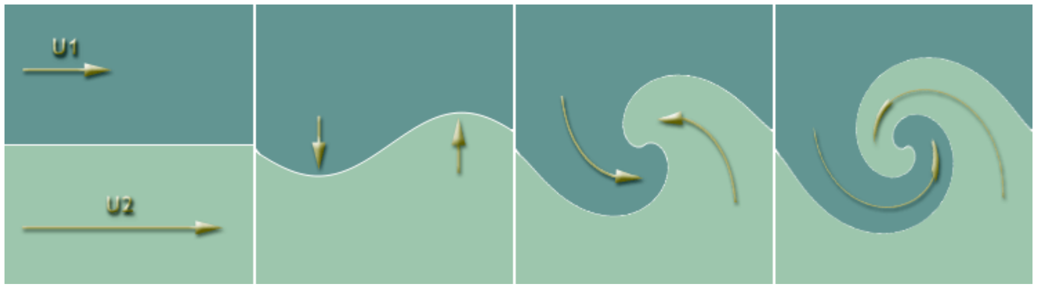
$v = v = 0$



e.g. Kelvin-Helmholtz Instability

$\varrho 2 \neq \varrho 1$

$v2 \neq v1$



Baryons (H, He, …) in the Universe can be modelled as ideal gas

Gravity is the dominant regulator of structure formation on the largest scales.
On "smaller" e.g. galactic scales, the hydrodynamics of the baryons becomes important.

Gas flows in cosmological settings are usually: "low" density and with negligible "friction".
They can be modelled by the **Euler Equations for ideal gas dynamics**:

Mass conservation (Continuity Equation)

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{v}) = 0,$$

Momentum conservation (Euler Equation)

$$\frac{\partial}{\partial t}(\rho \mathbf{v}) + \nabla(\rho \mathbf{v}\mathbf{v}^T + P) = 0,$$

Energy conservation (first law of thermodyn.)

$$\frac{\partial}{\partial t}(\rho e) + \nabla[(\rho e + P)\mathbf{v}] = 0,$$

Equation of state of ideal gas

$$P = (\gamma - 1)\rho u.$$

γ = ratio of specific heats

(ρ, **v**, P) are the **mass** density, **velocity**, and pressure.

$$e = u + \mathbf{v}^2/2$$

Is the total energy per unit mass, u is the internal **energy** per unit mass

$\gamma = 5/3$ (for monoatomic gas)

!!! Euler equations are a simplified form of the Navier-Stokes equations for inviscid fluids (viscosity = 0; Reynold number = ∞) and fluids with zero thermal conductivity

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The key ideas of modern galaxy formation rely on the cosmological setting

1. Structure formation is driven by gravitational instability (see above)

3. Stars form out of gas (very dense gas)

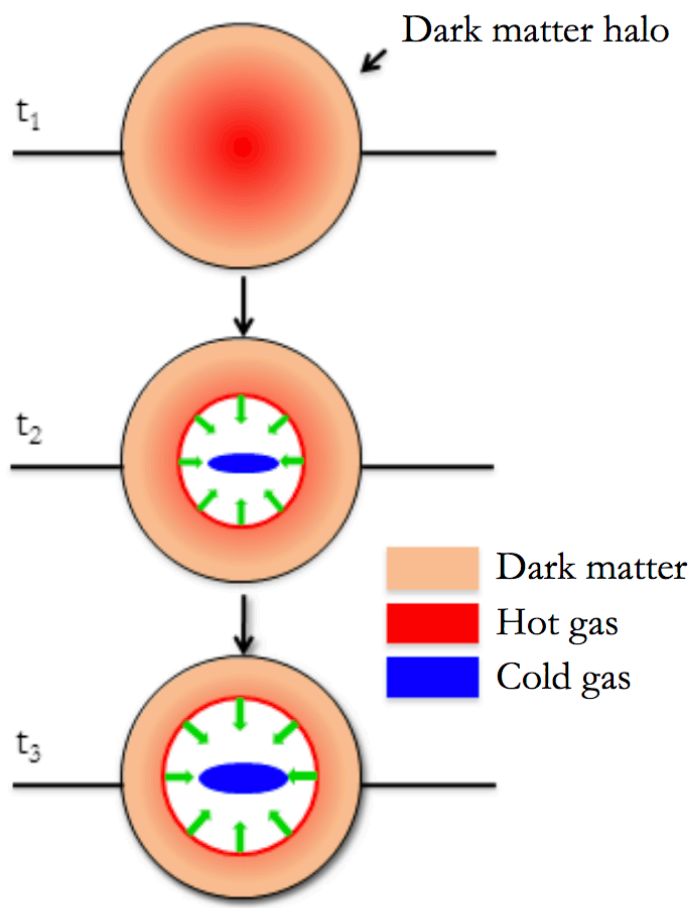
4. Dark-matter haloes get a spin due to tidal torques *Hoyle 1949*

5. Galaxies form inside dark-matter haloes, via a two-stage collapse:
   1. Dissipationless collapse of the dark-matter haloes themselves
   2. Dissipative collapse of gas: baryons collapse in the halo potential well and get shock heated *Rees & Ostriker 1977* *White & Rees 1978*

6. Gas cools mainly by radiative transitions: the typical mass of a galaxy is set by cooling arguments
   Hoyle 1953, Silk 1977, Binney 1977, Rees & Ostriker 1977

7. The formation of disk galaxies can be understood by the cooling of gas to the DM-halo centres via conservation of the angular momentum of the DM halo *Fall & Efstathiou 1980*

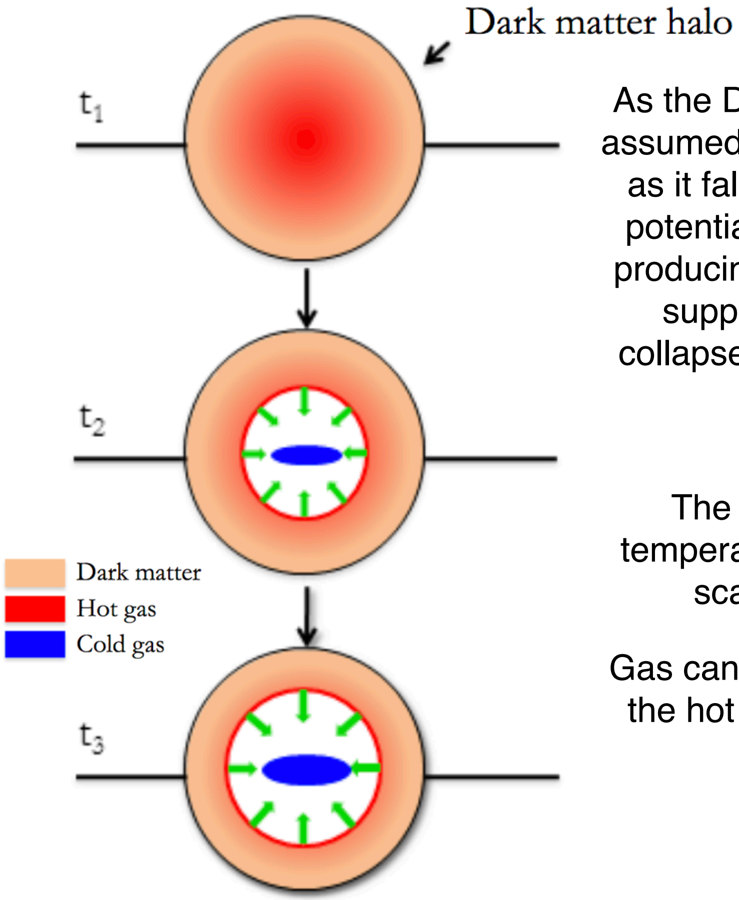
8. (Elliptical galaxies form via the merger of disk galaxies)



White & Rees 1978

The formation of galaxies rely on dissipative collapse of gas + cooling

The gas initially has the same spatial distribution as the dark matter (t < t1)



As the DM halo collapse, gas is assumed to be heated by shocks as it falls into the gravitational potential well of the dark halo, producing a hot gas halo that is supported against further collapse by the pressure of the gas (t1).

The gas attains the viral temperature of the halo (which scales as halo mass)

Gas can subsequently cool from the hot halo, through radiative processes (t2).

Baugh 2006

Virial temperature

Half of the potential energy of the infalling gas is converted into kinetic energy, which in turn is transformed into heat:

$$T_{\rm vir} \equiv T_{\rm 200c} \simeq \frac{\mu m_{\rm p} G M_{\rm 200c}}{2 k_{\rm B} R_{\rm 200c}} \simeq \frac{\mu m_{\rm p} G M_{\rm tot}}{2 k_{\rm B} R_{\rm 200c}}.$$

The role of Dissipation

Baryons (gas in particular) can radiate: this is a sign that dissipative processes are at work.

Dissipative processes = processes for which e.g. internal energy is transformed into another form.

Namely, baryons can loose energy by a number of radiative processes => reduce of thermal energy.

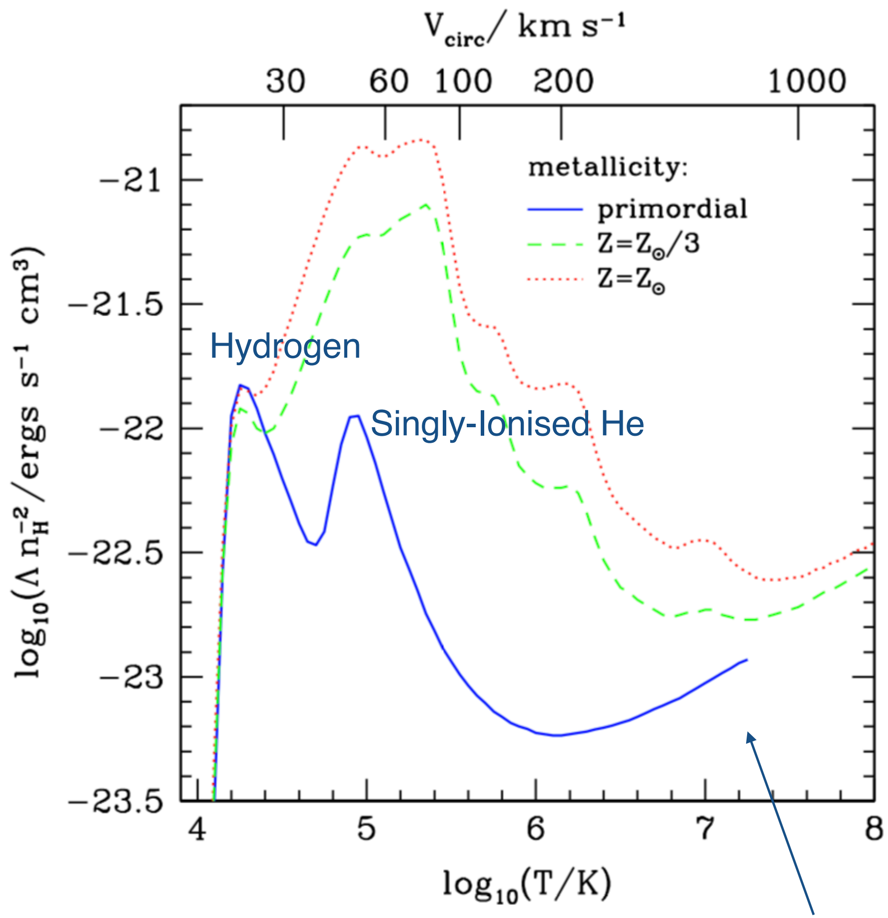
As the gas cools, the pressure of the gas drops and the removal of pressure support means that the gas sinks to the centre of the dark halo on the free-fall or dynamical timescale in the halo (step t3).

The formation of galaxies rely on dissipative collapse of gas + cooling

Mechanisms for gas cooling

- Bremsstrahlung radiation as electrons are accelerated in an ionized plasma, i.e. scattering between electrons and nuclei (10^7 K)
- Emission of photons following transitions between energy levels, due to collisional excitation, i.e. collisions between atoms and electrons (10^{4-6} K)
- inverse Compton scattering of CMB photons by electrons in the hot halo gas (only at high z)
- The excitation of rotational or vibrational energy levels in molecular hydrogen through collisions (100-1000 K)

The relative importance and efficiency of the various cooling processes depend on the **density (^2)** and **temperature** of the gas, as well as on its **chemical composition**



Bremsstrahlung (radiation by electrons experiencing acceleration in the electric field of ions): the cooling rate goes like T $^{1/2}$

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Cosmological galaxy simulations mean working between physics and numerics

The astrophysics we need to model is largely described by systems of partial differential equations:

Gravity

Poisson-Vlasov system of equations:

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \Phi}{\partial \mathbf{r}}\frac{\partial f}{\partial \mathbf{v}} = 0,$$

$$\nabla^2 \Phi = 4\pi G m \int f(\mathbf{r},\mathbf{v},t)\,\mathrm{d}\mathbf{v}.$$

Fluid dynamics

Mass conservation (Continuity Equation)

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{v}) = 0,$$

Momentum conservation (Euler Equation)

$$\frac{\partial}{\partial t}(\rho \mathbf{v}) + \nabla(\rho \mathbf{v}\mathbf{v}^T + P) = 0,$$

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$$\frac{\partial}{\partial t}(\rho e) + \nabla[(\rho e + P)\mathbf{v}] = 0,$$

Equation of state of ideal gas

$$P = (\gamma - 1)\rho u.$$

Cosmological galaxy simulations mean working between physics and numerics

The astrophysics we need to model is largely described by systems of partial differential equations….

that are coupled (for the gas), via source terms

Gravity

Poisson-Vlasov system of equations:

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \Phi}{\partial \mathbf{r}}\frac{\partial f}{\partial \mathbf{v}} = 0,$$

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Equation of state of ideal gas

$$P = (\gamma - 1)\rho u.$$

But: writing down the equations is not enough.
The ability to [numerically] solve them is required.

Cosmological galaxy simulations mean working between physics and numerics

Discretization

The fluid/matter can be "sampled" by e.g. N particles, N cells, …

↓

Mass/space resolution elements

Integration (in time)

- Explicit and implicit Euler Methods
- Runge-Kutta Methods
- The Leapfrog
- Symplectic Integrators

↓

Time steps

But: writing down the equations is not enough.
The ability to [numerically] solve them is required.

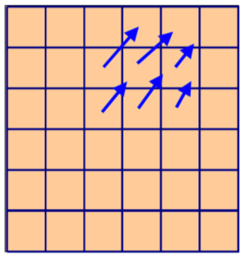
We need to solve a system of hyperbolic partial differential equations…

There exists no general solution to PDEs, as different approaches are needed for different problems.
Eulerian methods are the traditional schemes to solve the system of equations of ideal hydrodynamics.

For cosmological hydrodynamical simulations, there is a variety of fundamentally different numerical methods, of which the majority can be classified in two types:

Eulerian Hydrodynamics

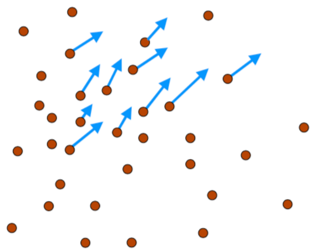
The **space** is discretised
The problem is solved on a mesh
The numerical entities are **volume elements**



(e.g. mesh methods, grid methods)

Lagrangian Hydrodynamics

The **mass** is discretised
The problem is solved on fluid elements
The numerical entities are **particles**



(e.g. smooth-particle-hydrodynamics methods, or SPH)

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The kernel interpolation is at the basis of SPH

The mass is discretised with particles.
Still, we need continuous fluid quantities to solve the equations from such discretised tracers.

This is done with a kernel summation interpolant, to e.g. estimate the density at all points in space, not just where the particles are

We want the smoothed interpolated version F_s of a field F:

$$F_s(\mathbf{r}) = \int F(\mathbf{r}')W(\mathbf{r}-\mathbf{r}', h)\,\mathrm{d}\mathbf{r}'.$$

Kernel function W(**r**, h)

If we know the field F at a set of points **r_i**, the integral can be replaced with a summation, with $V_i \sim m_i/\rho_i$

$$F_s(\mathbf{r}) \simeq \sum_j \frac{m_j}{\rho_j} F_j W(\mathbf{r}-\mathbf{r}_j, h).$$

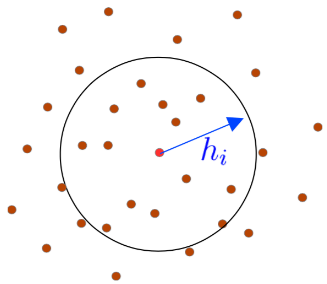
SPH density estimate: (Similarly can be done for the velocity field)

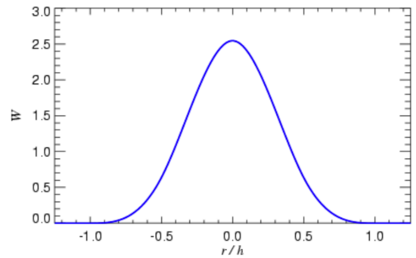
$$\rho_s(\mathbf{r}) \simeq \sum_j m_j W(\mathbf{r}-\mathbf{r}_j, h),$$

From a set of particle coordinates and their masses.

This is defined everywhere and can be differentiated!

Kernels in SPH: must be normalised to 1, simple, with spherical symmetry, best with a finite support (not Gaussians)





e.g. a cubic spline:
$W(r,h) = w(\frac{r}{2h})$

$$w_{3D}(q) = \frac{8}{\pi}\begin{cases} 1-6q^2+6q^3, & 0 \le q \le \frac{1}{2}, \\ 2(1-q)^3, & \frac{1}{2} < q \le 1, \\ 0, & q > 1, \end{cases}$$

h is the **smoothing length, and can be chosen to be adaptive.**
h_i = h(r_i, t)

Practical Idea: summation can be restricted to the ***Nngb*** *neighbors that lie within the spherical region of radius 2h around the target point ri. This corresponds to a computational cost of order O(Nngb N) for the full density estimate.* ***Nngb = 32, 64, …***

The equation of motions can be expressed and solved in Lagrangian form

Euler Equations for inviscid ideal gas in Lagrangian form

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho \nabla \cdot \mathbf{v} = 0,$$

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} + \frac{\nabla P}{\rho} = 0,$$

$$\frac{\mathrm{d}u}{\mathrm{d}t} + \frac{P}{\rho} \nabla \cdot \mathbf{v} = 0,$$

Convective derivative
$\mathrm{d}/\mathrm{d}t = \partial/\partial t + \mathbf{v} \cdot \nabla$

They are a system of partial differential equations (conservation of mass, momentum and energy) that follow from the Lagrangian:

$$L = \int \rho \left(\frac{\mathbf{v}^2}{2} - u \right) \mathrm{d}V.$$

[....] →

New discretised dynamical equations of motion that take into account for Euler equation

$$\frac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} = -\sum_{j=1}^{N} m_j \left[f_i \frac{P_i}{\rho_i^2} \nabla_i W_{ij}(h_i) + f_j \frac{P_j}{\rho_j^2} \nabla_i W_{ij}(h_j) \right],$$

$$f_i = \left[1 + \frac{h_i}{3\rho_i} \frac{\partial \rho_i}{\partial h_i} \right]^{-1},$$

A complicated system of partial differential equations has been transformed into a much simpler set of ordinary differential equations:

- Only the momentum equation needs to be solved explicitly
- The mass conservation and the total energy equation are automatically fulfilled, because the particle masses and their specific entropies stay constant

The Euler equations for cosmological applications are slightly different

Euler Equations for inviscid ideal gas in Lagrangian form

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho \nabla \cdot \mathbf{v} = 0,$$

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} + \frac{\nabla P}{\rho} = 0,$$

$$\frac{\mathrm{d}u}{\mathrm{d}t} + \frac{P}{\rho} \nabla \cdot \mathbf{v} = 0,$$

Convective derivative
$\mathrm{d}/\mathrm{d}t = \partial/\partial t + \mathbf{v} \cdot \nabla$

They are a system of partial differential equations (conservation of mass, momentum and energy) that follow from the Lagrangian:

$$L = \int \rho \left(\frac{\mathbf{v}^2}{2} - u \right) \mathrm{d}V.$$

→

Euler Equations for inviscid ideal gas in Lagrangian form, including self-gravity and gas cooling

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho \nabla \cdot \mathbf{v} = 0,$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{\nabla P}{\rho} - \nabla \Phi.$$

Gravitational Potential

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{P}{\rho} \nabla \cdot \mathbf{v} - \frac{\Lambda(u, \rho)}{\rho}.$$

Cooling Function $\Lambda(u, \rho)$

Convective derivative
$\mathrm{d}/\mathrm{d}t = \partial/\partial t + \mathbf{v} \cdot \nabla$

Equation of state
$P = (\gamma - 1)\rho u,$

With, for simple atomic ideal gas, the adiabatic exponent of the equation of state can be taken as 5/3

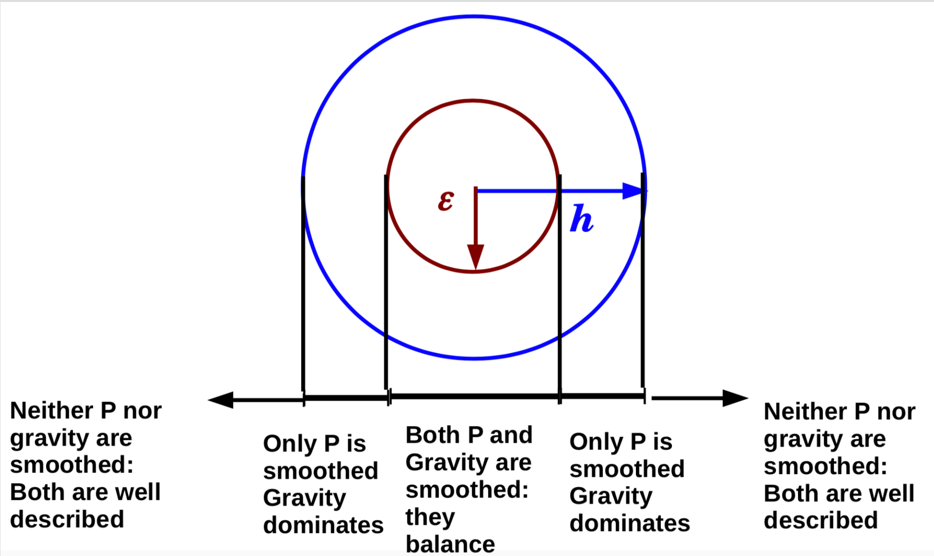
How should the kernel-smoothing length be chosen?

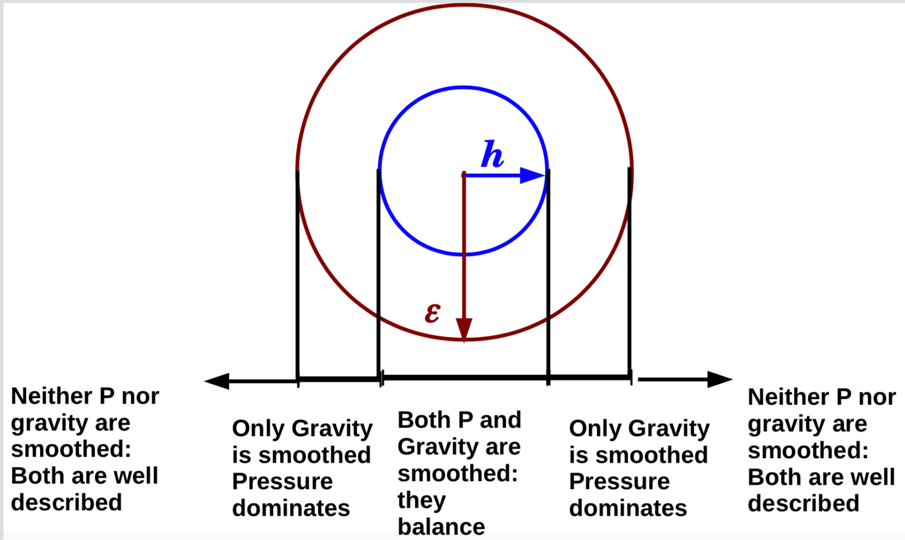
Smoothing length (hydro) must be ~ softening length (gravity)

ε *(Bate & Burkert 1995)* *h*

If ε < *h* → gravity stronger than pressure
→ spurious collapse

If *h* < ε → pressure stronger than gravity
→ spurious expansion

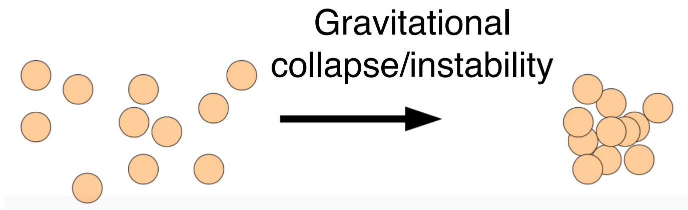




The main advantage of SPH is its natural spatial adaptivity

SPH Advantages

The resolution adjusts automatically, as the particle density increases where needed:



Galilean Invariance is automatically fulfilled

Energy, momentum, angular momentum, mass and entropy are all simultaneously conserved (if no additional viscosity)

The self-gravity of the gas naturally treated with the same accuracy as the dark matter. The numerical treatment nicely couples with the N-body treatment of the gravity

SPH Disadvantages

Mixing is completely suppressed at the particle level (e.g. metal mixing)

Shocks are broaden over a few smoothing lengths

It requires the addition of artificial viscosity to better capture shocks

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We need to solve a system of hyperbolic partial differential equations…

There exists no general solution to PDEs, as different approaches are needed for different problems.
Eulerian methods are the traditional schemes to solve the system of equations of ideal hydrodynamics.

Euler Equations for inviscid ideal gas in compact form:

$$\frac{\partial \boldsymbol{U}}{\partial t} + \nabla \cdot \boldsymbol{F} = 0$$

A conservation law :) Of mass, momentum and energy

$$\boldsymbol{U} = \begin{pmatrix} \rho \\ \rho \boldsymbol{v} \\ \rho e \end{pmatrix} = \begin{pmatrix} \rho \\ \rho \boldsymbol{v} \\ \rho u + \frac{1}{2}\rho \boldsymbol{v}^2 \end{pmatrix} = \boldsymbol{U}(\boldsymbol{x}, t),$$

State vector

$$\boldsymbol{F}(\boldsymbol{U}) = \begin{pmatrix} \rho \boldsymbol{v} \\ \rho \boldsymbol{v}\boldsymbol{v}^T + P \\ (\rho e + P)\boldsymbol{v} \end{pmatrix}$$

Flux function/vector

$$P = (\gamma - 1)\rho u$$

Equation of state

Classes of solution schemes for the PDEs:

- Finite difference methods
- Finite volume methods
- Spectral methods
- Methods of lines
- Finite element methods

In cosmological Eulerian codes, most solutions are finite volume methods (somehow a subclass of the finite difference methods):

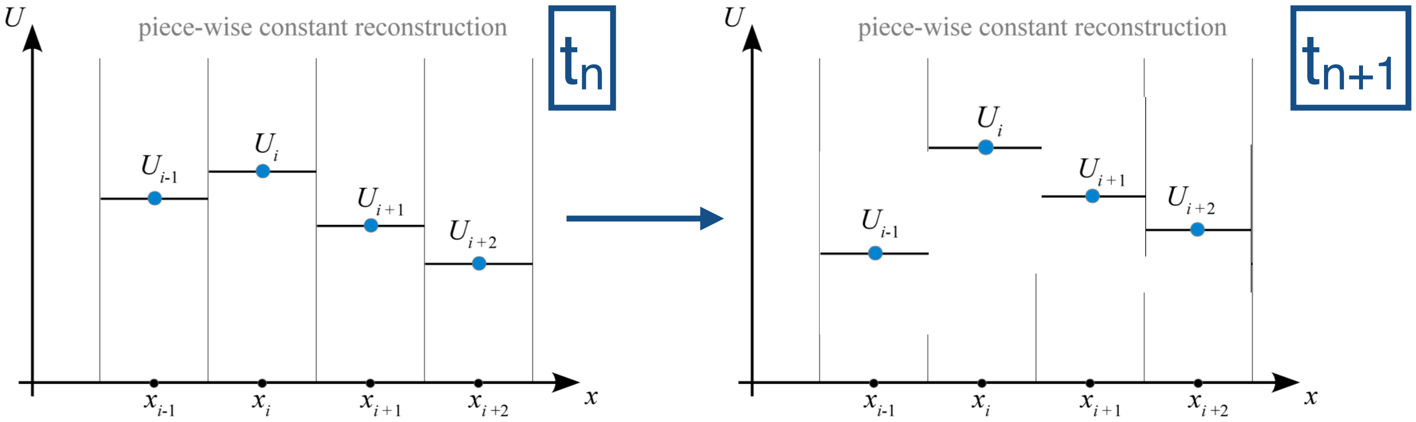
the discretization is carried out in terms of a subdivision of the system's volume into a finite number of disjoint cells.

The most practical and popular scheme is Godunov's.

The Godunov's method allows us to solve the Euler equations, …

… with the help of a Riemann Solver!

How is the problem conceived? e.g. discretisation in 1D



$$\mathbf{U} = \begin{pmatrix} \rho \\ \rho \mathbf{v} \\ \rho e \end{pmatrix}$$

$$\mathbf{F} = \begin{pmatrix} \rho \mathbf{v} \\ \rho \mathbf{v}\mathbf{v}^T + P \\ (\rho e + P)\mathbf{v} \end{pmatrix}$$

Each cell is characterised by a cell-averaged of the state vector U at a given time t_n:

$$\mathbf{U}_i = \frac{1}{V_i} \int_{\mathrm{cell}\ i} \mathbf{U}(\mathbf{x})\,\mathrm{d}V.$$

And we want the values for the state vector U at all cells at a subsequent t_n+1.

In practice, we need to know the fluxes F at the interfaces between cells

The Godunov's method allows us to solve the Euler equations, …

… with the help of a Riemann Solver!

General ideas of the Godunov's method, a Reconstruct-Evolve-Average (REA) scheme:

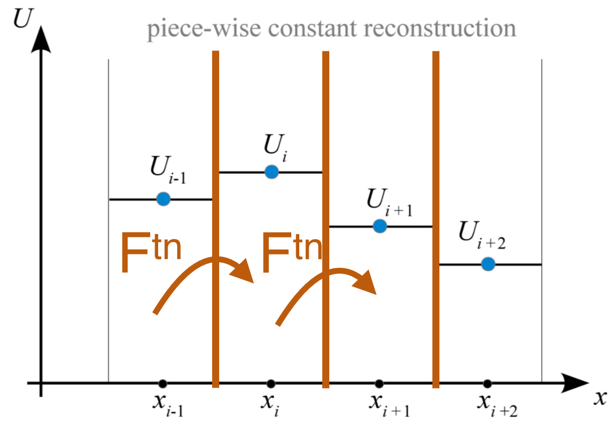
Let us discrete the volume (in 1D for simplification).

Step1. Reconstruct: the cells are characterised by cell-averaged quantities (e.g. the piece-wise constant states in the 1st order Godunov scheme)

Step2. Evolve: the reconstructed state is evolved forward in time by Δt. In the Godunov scheme at first order, this is done by treating each cell **interface** as a (piece-wise constant) initial value problem.

Step3. Average: the wave structure that results from the evolution is spatially averaged to compute new states U_n+1 for each cell.

Repeat.



Here, in practice, we need to solve a problem for the evolution of a piece-wise linear problem with given left and right states that are brought into contact at time t_n:

this is what is done by a **Riemann solver**!

$$\mathbf{F}^{\star} = \mathbf{F}_{\text{Riemann}}(\mathbf{U}_L, \mathbf{U}_R)$$

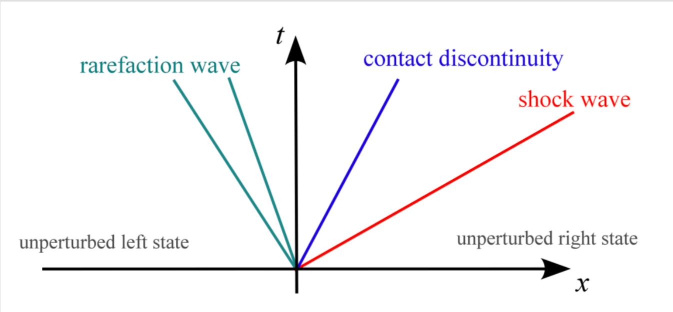
This is all OK as long as the solution (the waves emanating) from opposite side of a cell to do not interact, so Δt needs to be limited

The Riemann problem is an initial-value problem

See e.g. *Toro 1997*

The Riemann problem is an initial-value problem for hyperbolic systems (i.e. with a conservation law) that consists of two piece-wise constant states that meet at a plane at t=0. The task is to solve for the subsequent evolution at t>0.

The Riemann solution can always be formally described with characteristics for three self-similar waves:

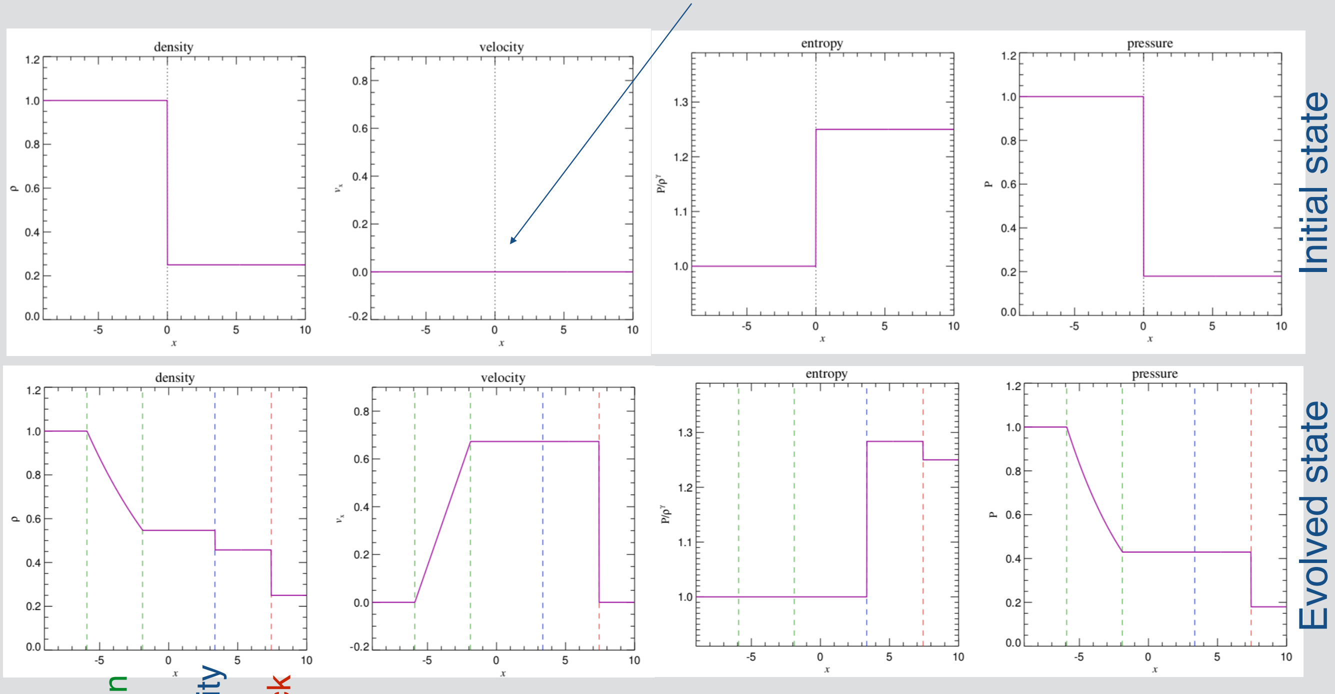


These waves propagate with constant speed.

If the solution is known at some time $t>0$, it can also be obtained at any other time.

At $x = 0$, the fluid quantities ($\rho^\star, P^\star, v^\star$) are *constant in time* for $t > 0$.

A Riemann problem example (the Sod shock-tube problem):



rarefraction

Discontinuity

shock

Shock: sudden compression of the fluid. Kinetic energy is transformed in heat: production of entropy.

Contact discontinuity: traces the original separation. Pressure and velocity are constant through this.

Rarefraction: a rarefaction wave forms when the gas suddenly expands. No actual discontinuities in the primitive variables

The Godunov scheme can be written easily for the 1D case

Each cell is characterised by a cell-averaged of the state vector U at a given time t_n:

$$\mathbf{U}_i = \frac{1}{V_i}\int_{\text{cell } i} \mathbf{U}(\mathbf{x})\,\mathrm{d}V.$$

The conservation law (see a few slides ago) can be integrated over a cell and over a finite interval of time:

$$\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} \mathrm{d}x \int_{t_n}^{t_{n+1}} \mathrm{d}t \left(\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x}\right) = 0.$$

$$\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} \mathrm{d}x\,[\mathbf{U}(x,t_{n+1}) - \mathbf{U}(x,t_n)] + \int_{t_n}^{t_{n+1}} \mathrm{d}t \left[\mathbf{F}(x_{i+\frac{1}{2}},t) - \mathbf{F}(x_{i-\frac{1}{2}},t)\right] = 0.$$

$$\Delta x \left[\mathbf{U}_i^{(n+1)} - \mathbf{U}_i^{(n)}\right] + \int_{t_n}^{t_{n+1}} \mathrm{d}t \left[\mathbf{F}(x_{i+\frac{1}{2}},t) - \mathbf{F}(x_{i-\frac{1}{2}},t)\right] = 0.$$

The fluxes F at the interfaces can be obtained via solution of a Riemann problem:

$$\mathbf{F}(x_{i+\frac{1}{2}},t) = \mathbf{F}^\star_{i+\frac{1}{2}}$$

$$\mathbf{F}^\star_{i+\frac{1}{2}} = \mathbf{F}_{\text{Riemann}}(\mathbf{U}_i^{(n)}, \mathbf{U}_{i+1}^{(n)})$$

The solution reads:

$$\mathbf{U}_i^{(n+1)} = \mathbf{U}_i^{(n)} + \frac{\Delta t}{\Delta x}\left[\mathbf{F}^\star_{i-\frac{1}{2}} - \mathbf{F}^\star_{i+\frac{1}{2}}\right].$$

Flux that flows into the cell from the left

Flux that flows out from the cell from the right

The idea of using the Riemann solution to do the time update of the states is due to Godunov, hence the name.

In general, we do not know the exact form of analytic functions U(**x**,t) and F(**x**,t).

The approximation of U and F makes it a numerical scheme. Using piece-wise constant functions makes it accurate at first order

IMPORTANT!!!
For the solution to hold, the waves emanating from one interface cannot interact with the other end of the cell:

Courant-Friedrichs-Levy or CFL condition

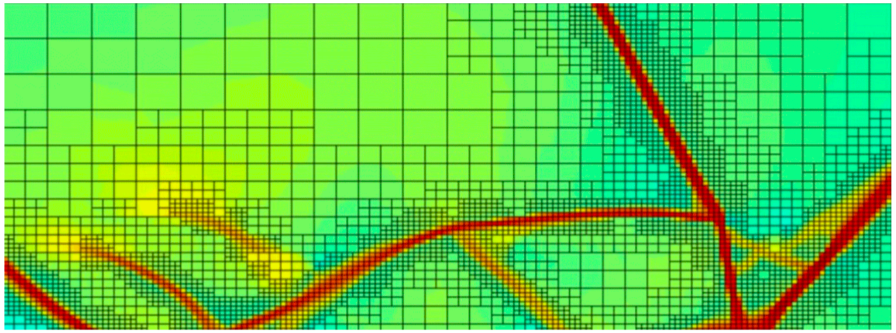
$$\Delta t \le \Delta x / c_{\max}$$

Adaptive Mesh Refinement (AMR) is typically used in cosmological simulations

In practical applications:

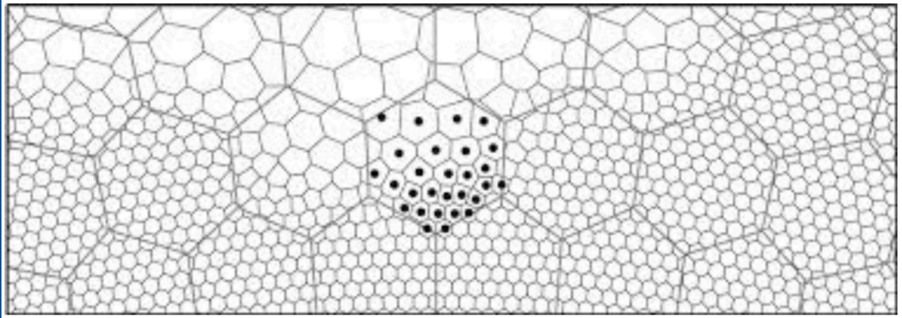
1. it is prohibitive to sample the entire space with the same (maximum) resolution (too many cells, too much memory)
2. if huge dynamical range is simulated (e.g. regions with density orders of magnitude larger → it is a waste of time to use max. resolution on very low-density regions)

Mesh refinement strategies:



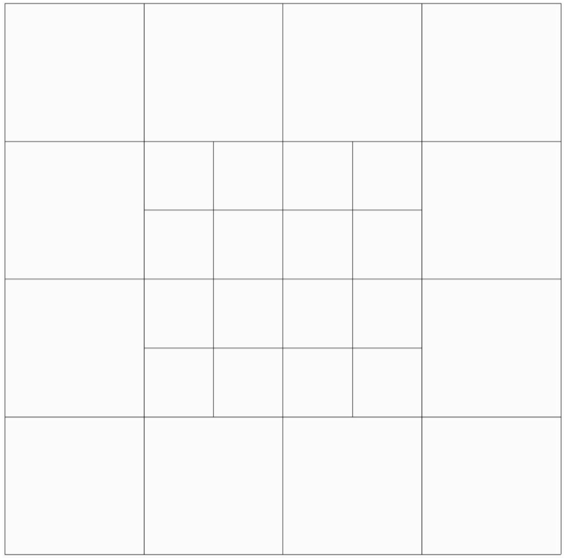
Regular:

the computational volume is divided into cubic elements (cells)



Irregular: the computational volume is divided into arbitrary-shaped cells

Regular AMR has been the common choice of grid/Eulerian codes so far, particularly e.g. Fully Threaded Trees: cartesian grids are split in 8



Refinement criteria: mass or spatial based!

The main advantage of grid/Eulerian methods is its accuracy

Grid Advantages

The scheme is a priory highly accurate

It captures shocks and contact discontinuity naturally well

It entails low numerical viscosity

Mixing occurs implicitly at the cell level

Grid Disadvantages

Galilean invariance is not necessarily fulfilled

Self-gravity on the gas needs to be done on a mesh

For AMR: refinement algorithms are really complicated

For AMR: refinement algorithms can generate spurious effects, especially at higher frequency and at boundaries

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    a. Elements of SPH
    b. Elements of Eulerian hydrodynamics
    c. Moving-mesh and mesh-less techniques
g. The available cosmological codes
h. On numerical resolution
i. Modelling the physics of galaxies
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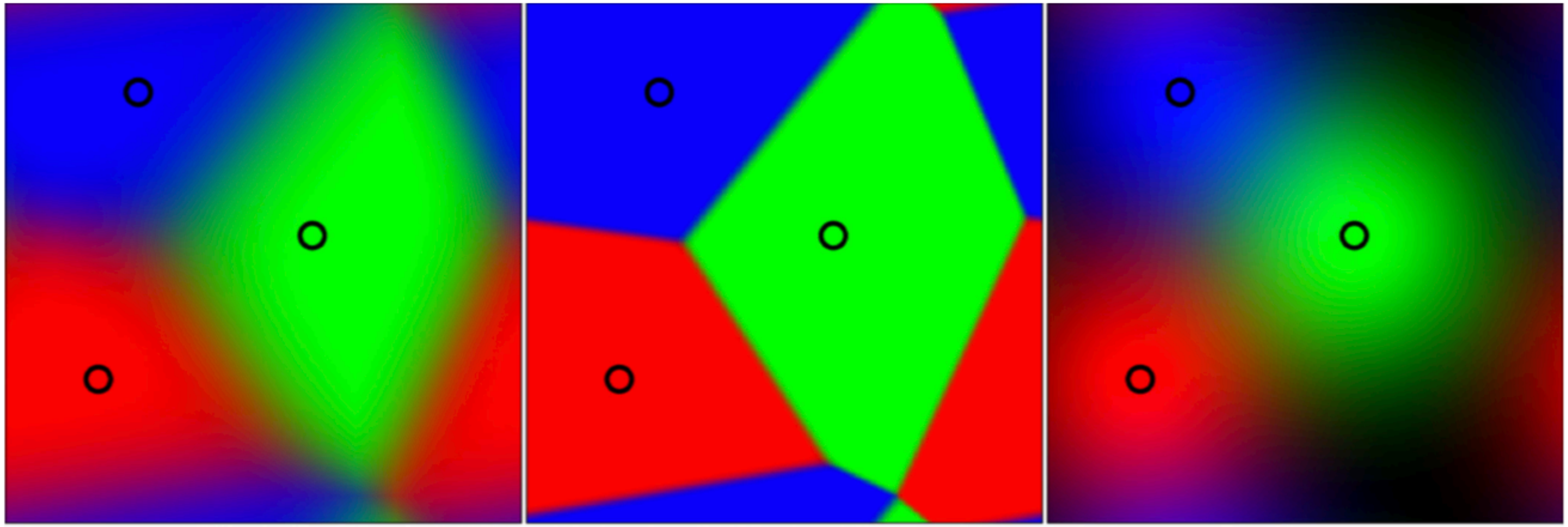
    b. Stellar evolution
    c. Feedback: needs, from stars and from SMBHs
j. Examples of state-of-the art simulations
k. Selected results

A new family of schemes brings together the advantages of SPH and AMR

There are two main examples and types of such numerical schemes used in Computational Cosmology:

Mesh-free finite volume method	**Unstructured moving mesh defined by a Voronoi tessellation**

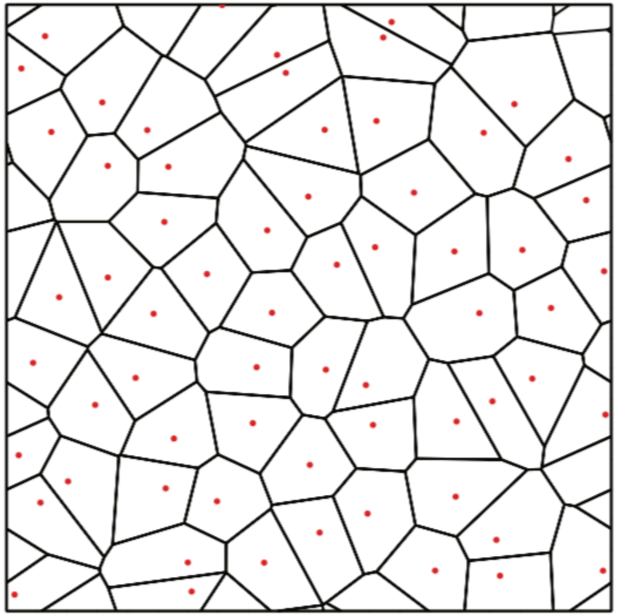


New Meshless Methods Here (MFV, MFM) | Unstructured / Moving–Mesh Methods | Smoothed–Particle Hydrodynamics

In e.g. AREPO, the Riemann problem is solved at the Voronoi-cell interfaces

Springel 2010

The space is discretised with a Voronoi tessellation



The Euler equations are solved with a finite-volume scheme where the Riemann solver is used at the interfaces of the Voronoi cells

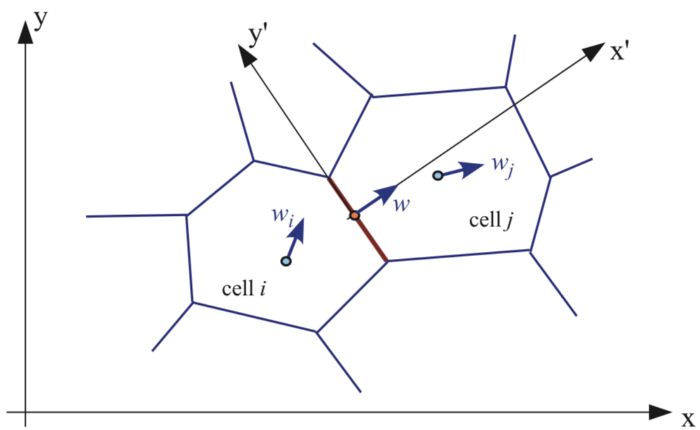
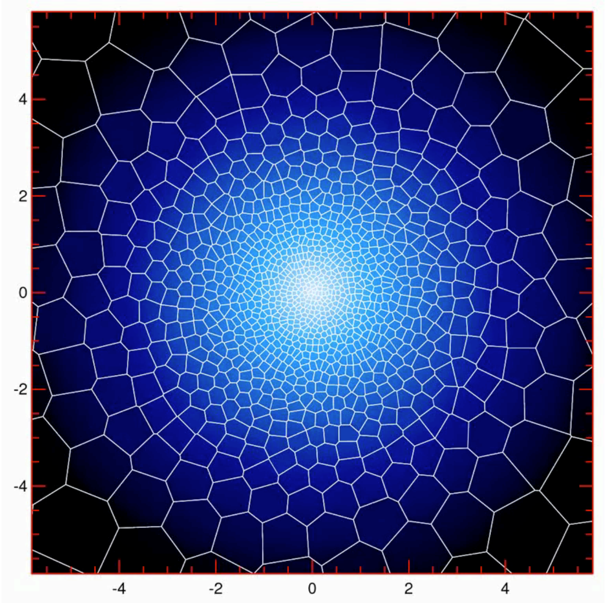


Figure 10. Geometry of the flux calculation. We use an unsplit scheme where the flux across each face is estimated based on a 1D Riemann problem. To this end, the fluid state is expressed in a frame which moves with the normal velocity $\boldsymbol{w}$ of the face, and is aligned with it. Note that the motion of the face is fully specified by the velocities of the mesh-generating points of the cells left and right of the face.

The so-called mesh-generating points move in space according to the fluid bulk velocity



Rotating Gaseous Disk + Gas Mesh

A moving-mesh code is naturally adaptive: more resolution where more mass

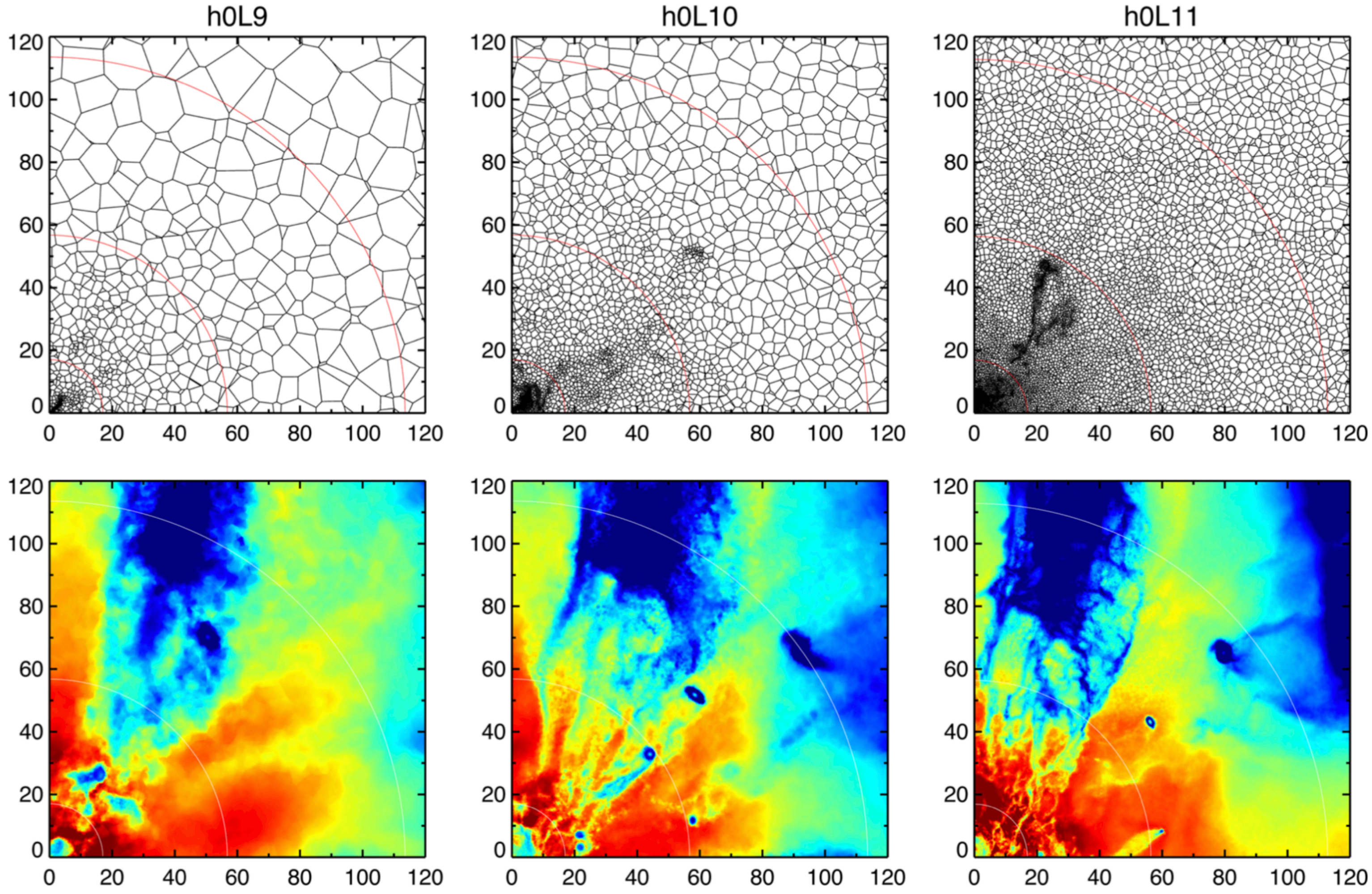
The Voronoi cells are deformable and move with the bulk motion of the gas

Refinement/derefinement of the mesh is also usually in place

(in AREPO, the mass in a cell is kept fixed, within a factor of 2: this is called **cell target mass**)

Higher densities = better resolution

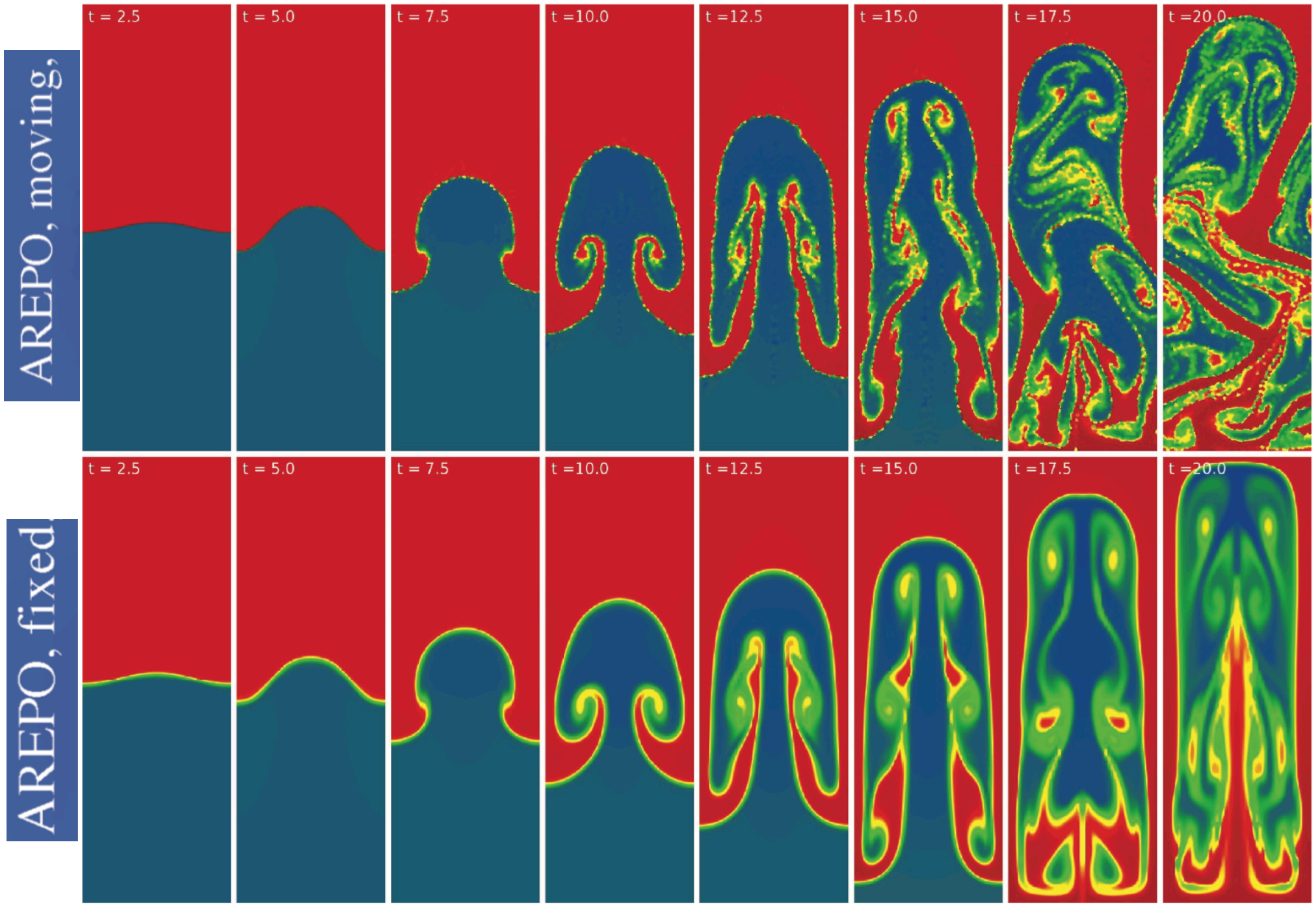
Same (quadrant of) a halo at better resolution

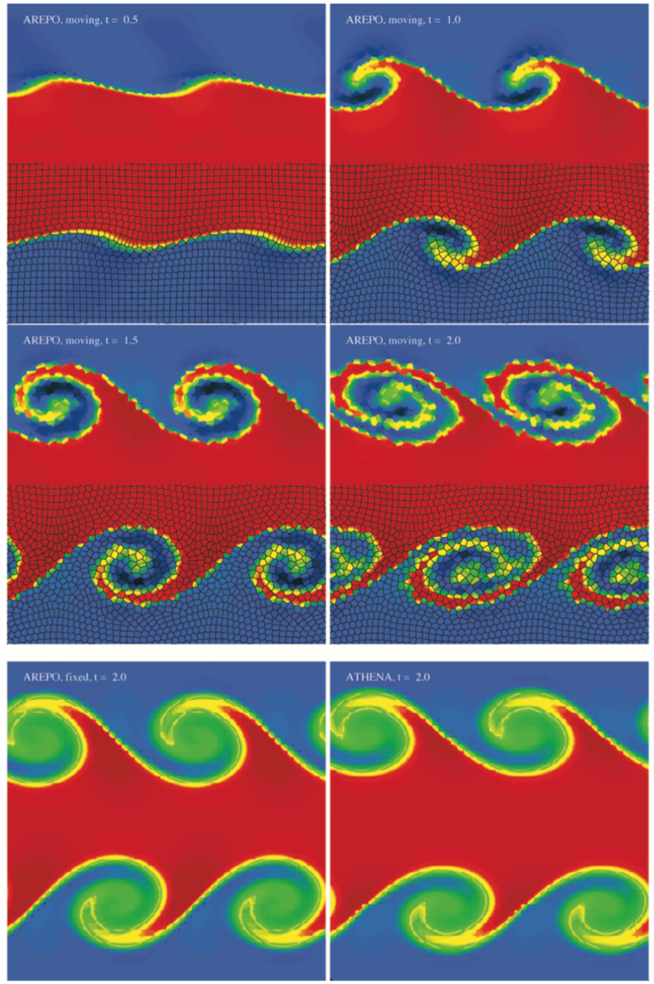


Nelson et al. 2015

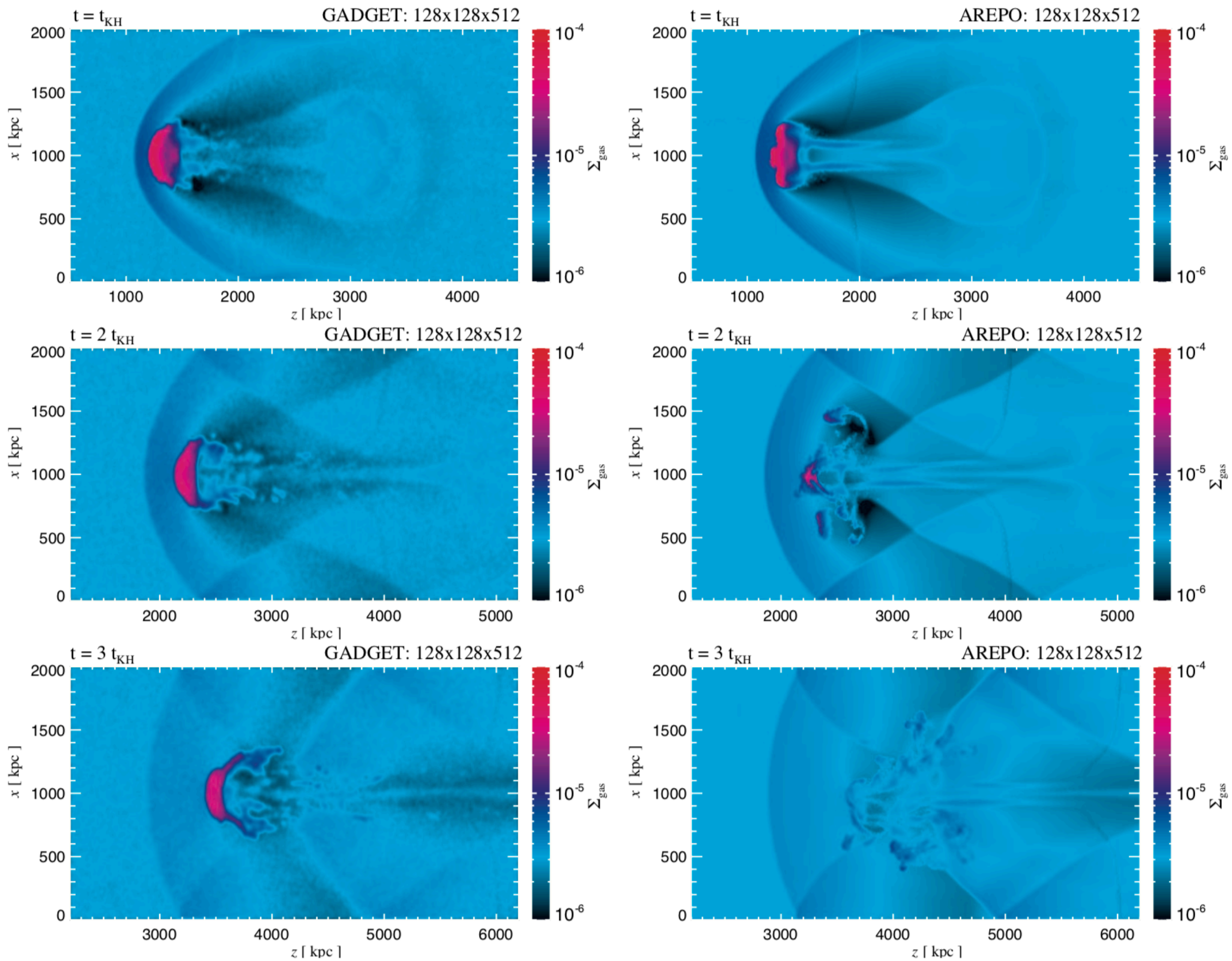
Moving-mesh codes do better jobs than fixed grids….

Springel 2010





Moving-mesh codes do better jobs than traditional SPH…



Sijacki et al. 2012

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Cosmological galaxy simulations mean working between physics and numerics

For codes to be *cosmological*:
The Universe expansion needs to be accounted for!!!
(Variable written in comoving coordinates)

Time integration i.e. time steps of the resolution elements cannot be all the same:
Individual i.e. hierarchical time stepping

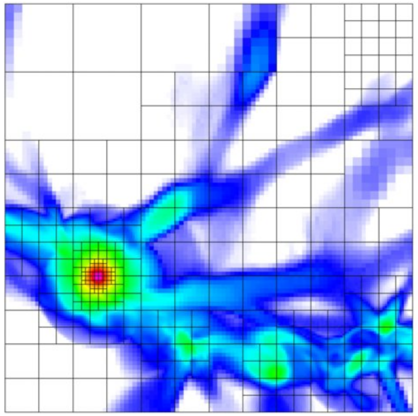
The code must be parallel, e.g. distributed-memory parallel (via MPI)

The current codes of Cosmological (Magneto)Hydrodynamical Simulations

GRID/AMR Codes

AMR:
Enzo
Ramses

AMR = adaptive mesh refinement

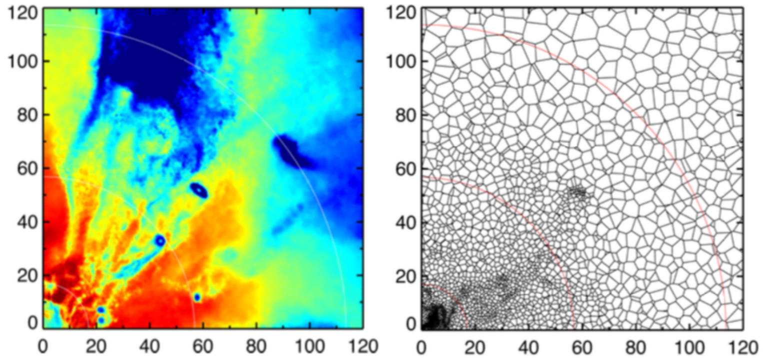


(Flash)
Static grids:
ATHENA
PLUTO

(not for cosmological sims)

Moving-mesh Codes

AREPO



Mesh-less Codes

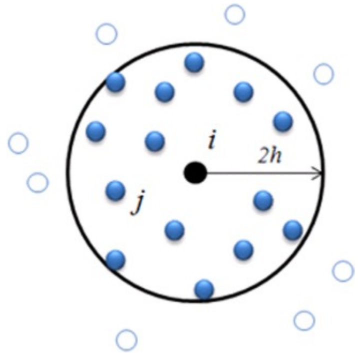
GIZMO

(Tess)
(not for cosmological sims)

SPH Codes

Traditional SPH:
GADGET
TSPH

Modern or
Corrected SPH:
P-SPH
SSPH
GASOLINE
GADGET-n
ChaNGa



Smooth Particle Hydrodynamics:

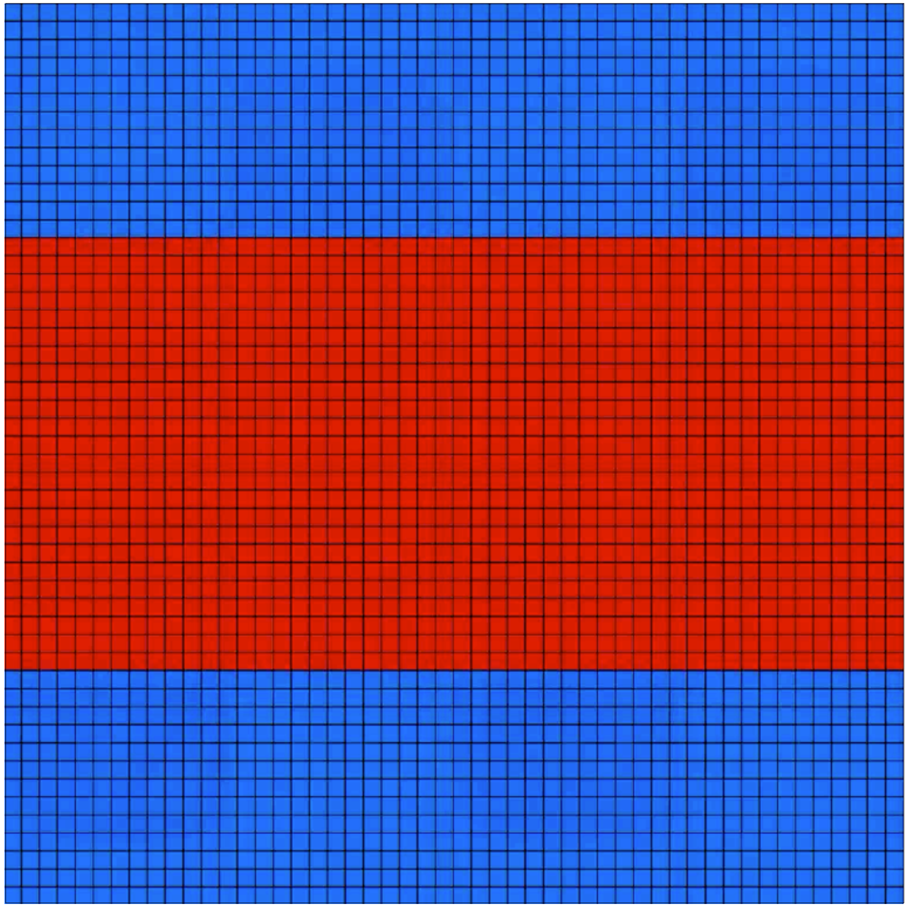
E.g. Cosmological galaxy simulations with AREPO: Illustris and IllustrisTNG

● = *particles*

resolution elements	● DARK MATTER *GAS - cells* ● STARS ● BLACK HOLES
gravity	TreePM Solver
(Magneto-) hydrodynamics	Riemann Solver on Voronoi Mesh

The Voronoi cells are deformable and move with the bulk motion of the gas

AREPO is massively parallel
(scales well up to ~30k cores, depending on the problem at hand)



Credits: V. Springel

The types of data products/outputs of typical cosmological *galaxy* simulations

SNAPSHOTS
i.e. "particle" data
∀ resolution elements, N entries

One entry one STELLAR PARTICLE

```
BirthPos                   Dataset {339778, 3}
BirthVel                   Dataset {339778, 3}
Coordinates                Dataset {339778, 3}
GFM_InitialMass            Dataset {339778}
GFM_Metallicity            Dataset {339778}
GFM_Metals                 Dataset {339778, 10}
GFM_MetalsTagged           Dataset {339778, 6}
GFM_StellarFormationTime   Dataset {339778}
GFM_StellarPhotometrics    Dataset {339778, 8}
Masses                     Dataset {339778}
ParticleIDs                Dataset {339778}
Potential                  Dataset {339778}
SubfindDMDensity           Dataset {339778}
SubfindDensity             Dataset {339778}
SubfindHsml                Dataset {339778}
SubfindVelDisp             Dataset {339778}
Velocities                 Dataset {339778, 3}
```

One entry one GAS CELL

```
CenterOfMass                 Dataset {12723085, 3}
Coordinates                  Dataset {12723085, 3}
Density                      Dataset {12723085}
ElectronAbundance            Dataset {12723085}
EnergyDissipation            Dataset {12723085}
GFM_AGNRadiation             Dataset {12723085}
GFM_CoolingRate              Dataset {12723085}
GFM_Metallicity              Dataset {12723085}
GFM_Metals                   Dataset {12723085, 10}
GFM_MetalsTagged             Dataset {12723085, 6}
GFM_WindDMVelDisp            Dataset {12723085}
GFM_WindHostHaloMass         Dataset {12723085}
InternalEnergy               Dataset {12723085}
Machnumber                   Dataset {12723085}
MagneticField                Dataset {12723085, 3}
MagneticFieldDivergence      Dataset {12723085}
Masses                       Dataset {12723085}
NeutralHydrogenAbundance     Dataset {12723085}
ParticleIDs                  Dataset {12723085}
Potential                    Dataset {12723085}
StarFormationRate            Dataset {12723085}
SubfindDMDensity             Dataset {12723085}
SubfindDensity               Dataset {12723085}
SubfindHsml                  Dataset {12723085}
SubfindVelDisp               Dataset {12723085}
Velocities                   Dataset {12723085, 3}
```

One entry one DM PARTICLE

```
Coordinates         Dataset {13481846, 3}
ParticleIDs         Dataset {13481846}
Potential           Dataset {13481846}
SubfindDMDensity    Dataset {13481846}
SubfindDensity      Dataset {13481846}
SubfindHsml         Dataset {13481846}
SubfindVelDisp      Dataset {13481846}
Velocities          Dataset {13481846, 3}
```

One entry one SMBH

```
BH_BPressure
BH_CumEgyInjection_QM
BH_CumEgyInjection_RM
BH_CumMassGrowth_QM
BH_CumMassGrowth_RM
BH_Density
BH_HostHaloMass
BH_Hsml
BH_Mass
BH_Mdot
BH_MdotBondi
BH_MdotEddington
BH_Pressure
BH_Progs
BH_U
Coordinates
Masses
ParticleIDs
Potential
SubfindDMDensity
SubfindDensity
SubfindHsml
SubfindVelDisp
Velocities
```

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Quantifying numerical resolution for the hydrodynamical schemes is complex!

Also for the gas dynamics, a scheme is characterised by mass and spatial resolution, but the meaning of those is different for different techniques: SPH vs. Grid codes, fixed cartesian grids vs. AMR grids, etc etc….

Mass Resolution

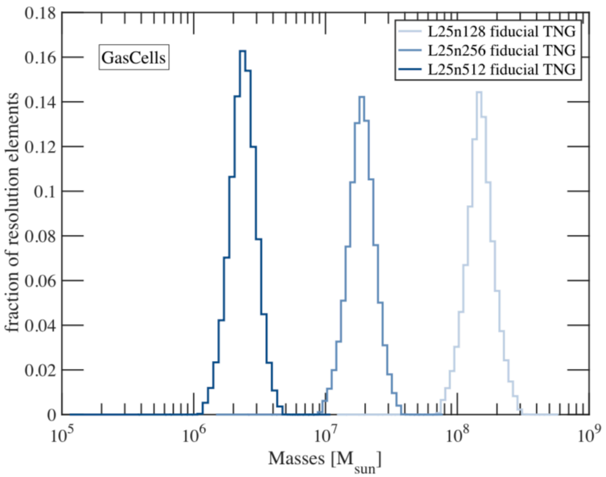
Mass of the gas element: particle/cell

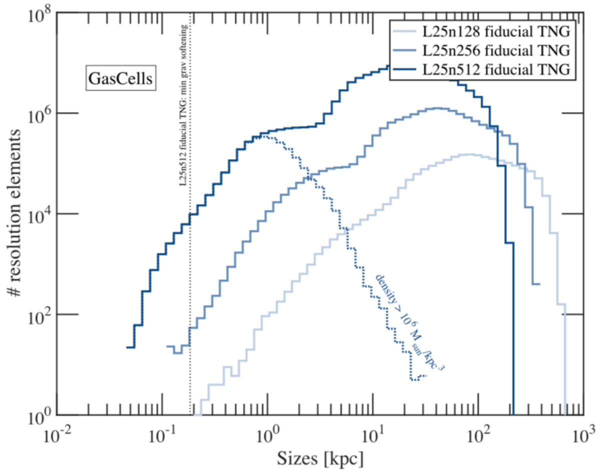
SPH: mass of the gas particle
GRID codes: target mass of the cell (if kept fixed!)

Spatial Resolution

Spatial accuracy over which the Euler equations are numerically solved

SPH: SPH kernel smoothing length (in N_ngb)
GRID codes: size of the cells





In both cases, for adaptive meshes, the physical sizes of the smoothing lengths and cells can be different across the simulated volume!

The value of the "smallest" cell is not meaningful!
The whole distribution should be provided

Notes on numerical resolution

Many numbers would be needed to quantify resolution.

Basically:

- **mass** resolution => particle/cell mass (DM, gas, stars)
- **spatial** resolution
  - gravity (softening DM, gas, stars, BHs)
  - hydrodynamics ("smoothing" length or cell size)

e.g. for a DM+GAS sim:
Size of the cubic box (L [Mpc]) +
DM+GAS particles at the ICs +
Assumption on OmegaM, Omegab =>
DM particle mass, initial gas mass

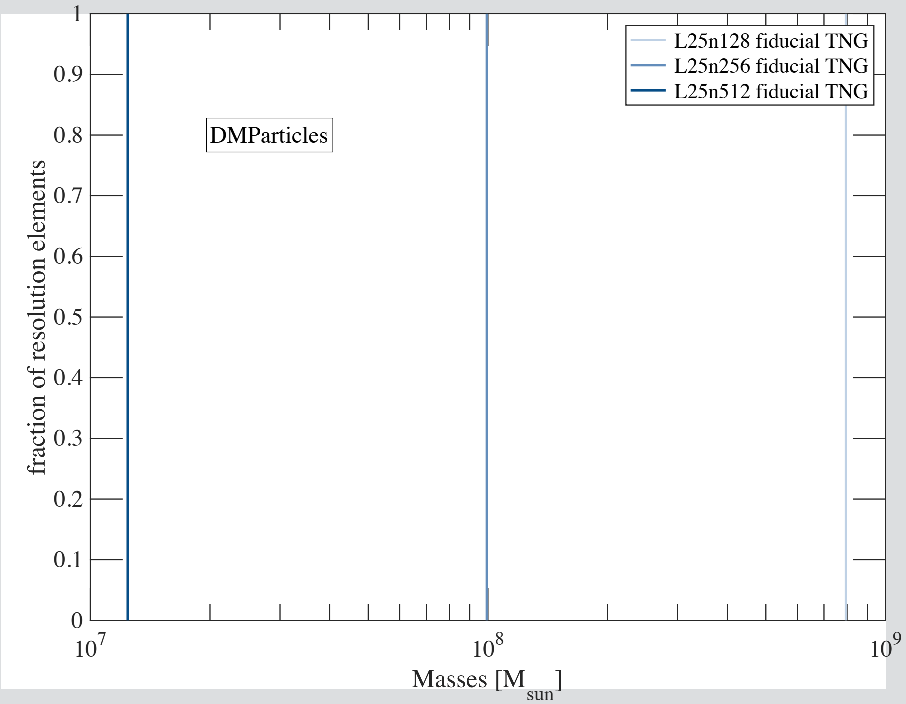
Hence e.g. L75n1820, L25n512

Run name	L_{box} (com Mpc)	N_{GAS} –	N_{DM} –	m_{baryon} ($M_\odot$)	m_{DM} ($M_\odot$)	$\epsilon^{z=0}_{DM,stars}$ (pc)	$\epsilon_{gas,min}$ (phys pc)	$\bar{r}_{cell,SF}$ (com pc)	CPU time (Mh)	N_{cores} –
TNG50(−1)	51.7	2160^3	2160^3	8.5×10^4	4.5×10^5	288	72	140	~130	16 320
TNG100(−1)	110.7	1820^3	1820^3	1.4×10^6	7.5×10^6	738	190	355	18.0	10 752
TNG300(−1)	302.6	2500^3	2500^3	1.1×10^7	5.9×10^7	1477	370	715	34.9	24 000

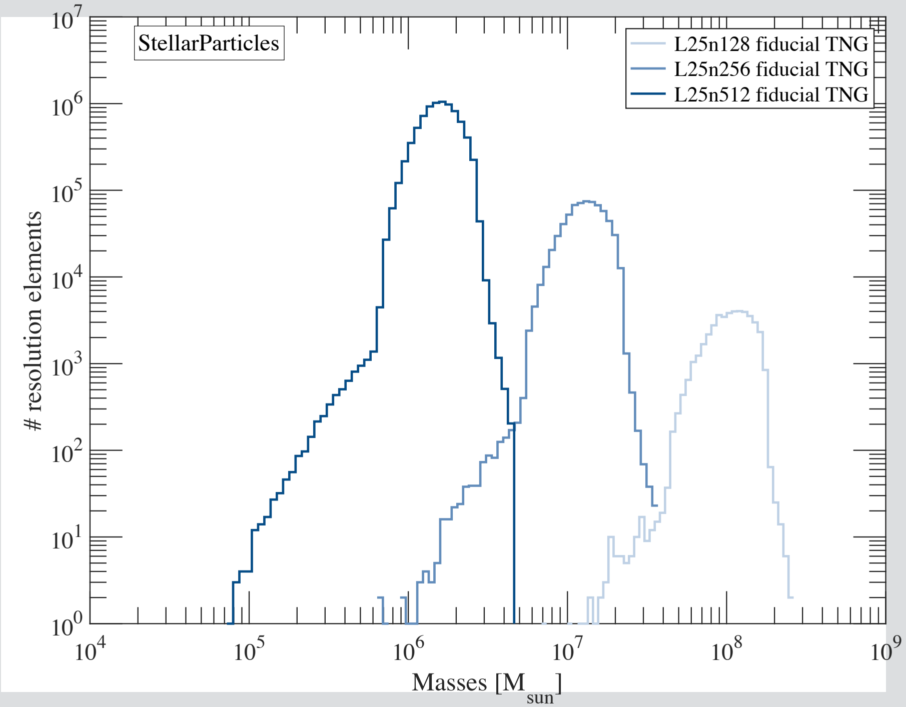
Pillepich+2019

Notes on numerical resolution

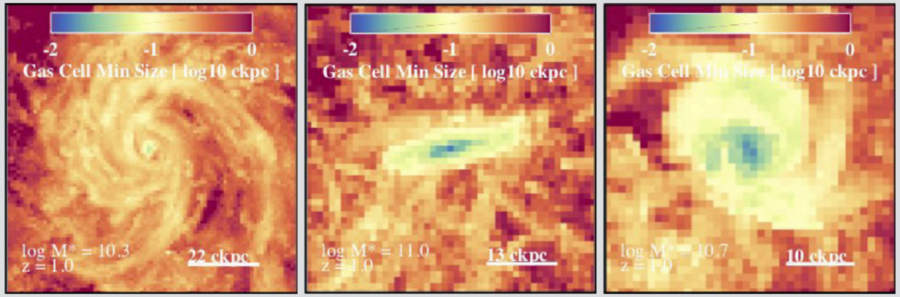
The case of DM particles, at different resolutions:



The case of star particles, at different resolutions:



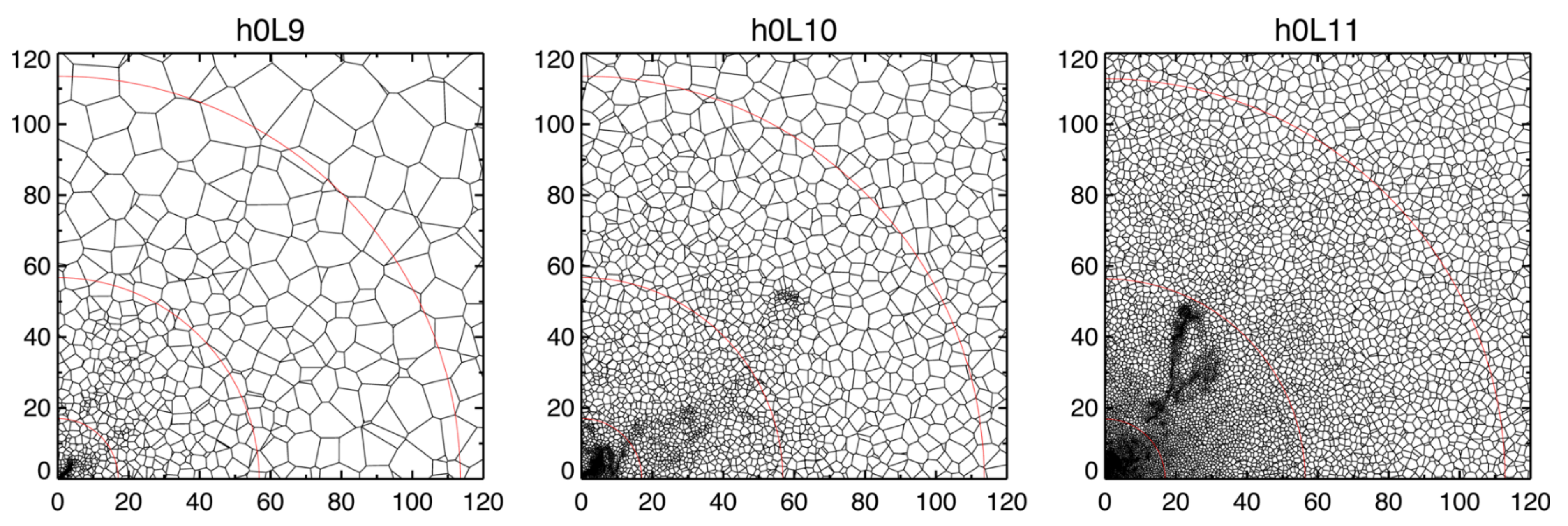
Notes on numerical resolution



TNG50: Pillepich, Nelson, Springel + 2019

The case of gas cells in AREPO:

- *Full-adaptivity of the code*
- *Gas cell target mass*
- *Refinement/de-refinement*

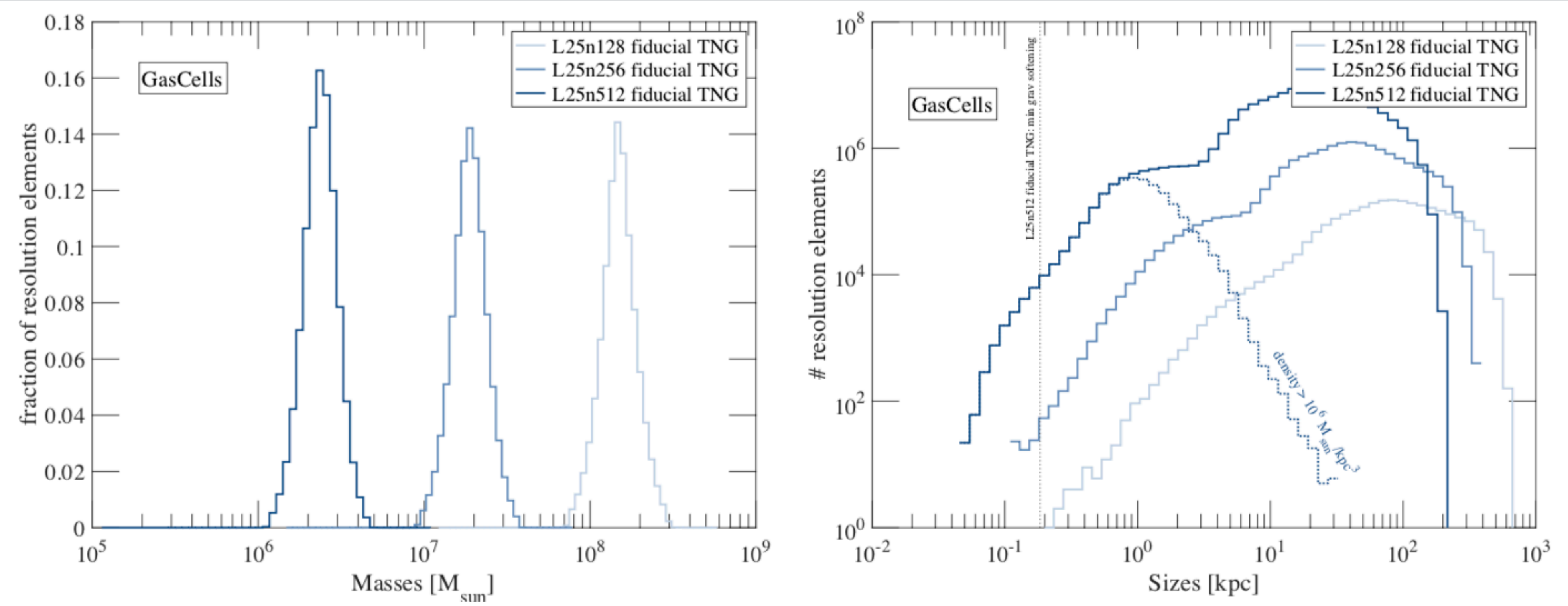


Nelson+2015

Improving resolution

Notes on numerical resolution

The case of gas cells, at different resolutions



Pillepich+2018a

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The setup and ingredients of large-scale cosmological *galaxy* simulations

Matter components

Dark matter

Gas

Stars

SMBHs

Magnetic fields

…

Physical processes

Gravity

Fluid dynamics

Atomic processes

…

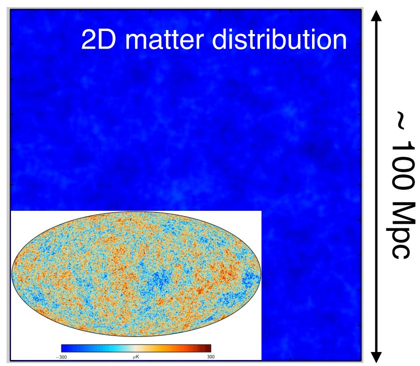
Photon propagation

Simulated domain

Cubes of representative portions of the Universe

(periodic boundary conditions)

From “cosmological” initial conditions



The setup and ingredients of large-scale cosmological *galaxy* simulations

Physical processes

Gravity

Fluid dynamics

Atomic processes

…

Photon propagation

The setup and ingredients of large-scale cosmological *galaxy* simulations

- Gravity & hydrodynamics
- Atomic processes (radiative cooling of the gas):
  - Heavy elements (metals = beyond He)
  - Molecules (H_2, CO, …)
- Star formation
  - Stellar evolution
  - Metal production and enrichment
  - Feedback: from supernovae, stellar winds, …
- (SuperMassive) Black Holes:
  - Formation
  - Growth: merging and gas accretion
  - Feedback: radiative, thermal, momentum
- Radiation (RHD):
  From stars, BHs, diffuse gas, reionization
- Magnetic Fields (MHD)
- Relativistic particle populations (cosmic rays)
- Dust (i.e. very large molecules)
- Plasma Physics (thermal conduction, …)

Essentially, all astrophysical phenomena are unresolved

(occur below the physical resolution of the sims)

They require some level of "subgrid" modelling:

$$\frac{\partial \boldsymbol{U}}{\partial t} + \nabla \cdot \boldsymbol{F} = \cancel{0}$$

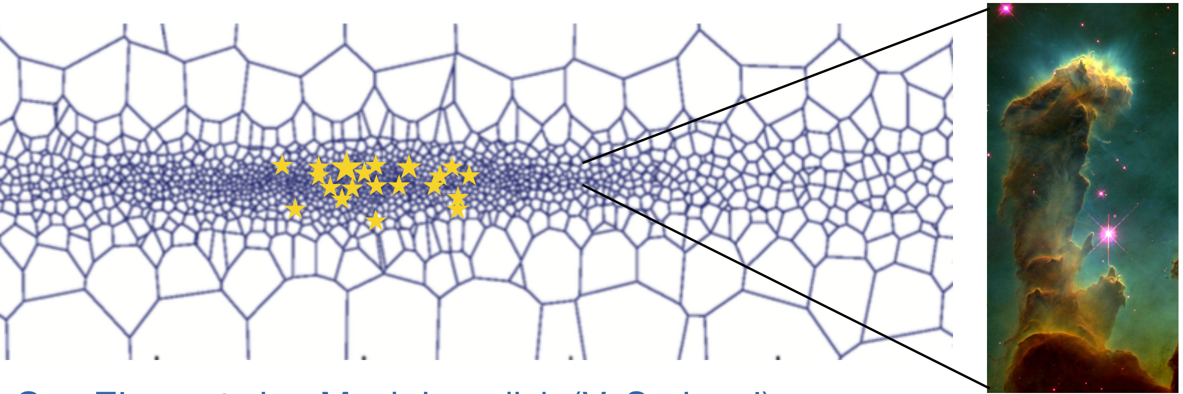
$$= S$$

Messy astrophysics adds complex, poorly understood source terms

"Laws" suggested by observations and tailored theoretical models are invoked and implemented

Via look-up "tables"

Star formation: will always be sub grid :)



Gas Elements i.e. Mesh in a disk (V. Springel)

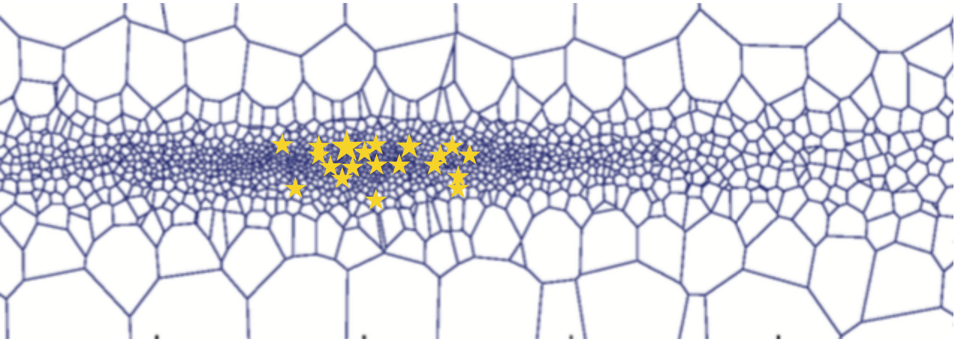
In large-volume cosmological galaxy simulations, the average gas resolution element in star-forming region is ~50-1000 pc.

We do *not* resolve (yet!) star forming regions, giant molecular clouds, supernovae explosions, HII regions, small-scales turbulence, etc etc

In galaxy simulations,
star formation = conversion of gas elements into stellar particles

Fractions of the mass of the gas elements (particles or cells) are stochastically transformed into a different type of resolution elements, usually particles, that are collision less (stellar particles interact via gravity, not hydrodynamically)

Star formation: the basic idea is to convert only dense gas

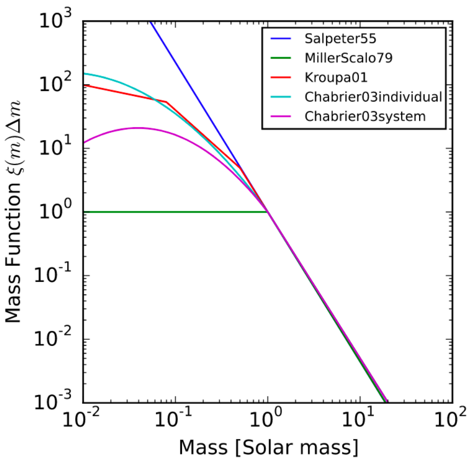


Gas Elements i.e. Mesh in a disk (V. Springel)

In the majority of the current models, gas above a certain density is converted into stars.

- Different models assume different values for such **density threshold** (e.g. in Eris, n = 5 cm^-3, Illustris, n = 0.1 cm^-3)
- Such threshold may depend of gas properties, e.g. metallicity (EAGLE), self-gravity (FIRE), …
- In some models, the star formation depends on the density of specific gas species, e.g. molecular gas (FIRE)

The newly-produced **stellar particles** represent **single age stellar populations** with a given IMF=initial mass function.



The conversion of gas mass into stellar mass is done stochastically, assuming observationally-allowed star formation time scales

SF time-scales

$$\frac{d\rho_\star}{dt} = (1-\beta)\frac{\rho_c}{t_\star}$$

SF efficiency

Parameters are de facto chosen to obtain something like a Kennicutt-Schmidt relation between Sigma_SFR and Sigma_GAS, when averaged across whole disks

Star formation: different simulations, different recipes in the detail

Eagle

Independently of resolution, the same locally-averaged KS law is recovered

$$\dot{\rho}_* = A\left(1\ \mathrm{M}_\odot\ \mathrm{pc}^{-2}\right)^{-n} \times \left(\frac{\gamma}{G} f_\mathrm{g} P_\mathrm{c}\right)^{(n-1)/2} \rho_{\mathrm{g,c}} \left(\frac{\rho_\mathrm{g}}{\rho_{\mathrm{g,c}}}\right)^{\frac{(n-1)\gamma_\mathrm{eff}}{2}+1}$$

Schaye & Dalla Vecchia (2008)

$$\rho_\star \sim 0.1\ \mathrm{cm}^{-3}, \quad \rho_\star(Z)$$

Magneticum

EoS from a multiphase model of the ISM

$$\frac{\mathrm{d}\rho_\star}{\mathrm{d}t} = \frac{\rho_c}{t_\star} - \beta\frac{\rho_c}{t_\star} = (1-\beta)\frac{\rho_c}{t_\star}$$

Springel & Hernquist (2003)

$$\rho_\star = 0.13\ \mathrm{cm}^{-3}$$
$$t_\star = 2\ \mathrm{Gyr}$$

In cosmological galaxy simulations, we do *not* resolve (yet!?!) star forming regions, giant molecular clouds, supernovae explosions, HII regions, small-scales turbulence, etc etc

Star formation = conversion of gas into stellar particles

Stellar particles represent single age stellar populations with a given IMF, which can evolve

Illustris

$$\rho_\star = 0.13\ \mathrm{cm}^{-3}$$
$$t_\star = 2.2\ \mathrm{Gyr}$$

$$\frac{\mathrm{d}\rho_\star}{\mathrm{d}t} = \frac{\rho_c}{t_\star} - \beta\frac{\rho_c}{t_\star} = (1-\beta)\frac{\rho_c}{t_\star}$$

Springel & Hernquist (2003)

IllustrisTNG

$$\rho_\star = 0.13\ \mathrm{cm}^{-3}$$
$$t_\star = 2.2\ \mathrm{Gyr}$$

$$\frac{\mathrm{d}\rho_\star}{\mathrm{d}t} = \frac{\rho_c}{t_\star} - \beta\frac{\rho_c}{t_\star} = (1-\beta)\frac{\rho_c}{t_\star}$$

Springel & Hernquist (2003)

Star particles: an example of sub-grid modelling

!!! A real star ~ 1 M☉

a star particle: 10^{4-7} M☉

Each represents an entire stellar population

- IMF at formation — What shape?
- Stars of different masses evolve along distinct tracks

At each timestep, the star ensemble ages: — Stellar evolution and metal enrichment. Yields: how much of each element?

- Synthetises heavy elements (C, O, Fe)
- Loses mass into the surrounding due to winds

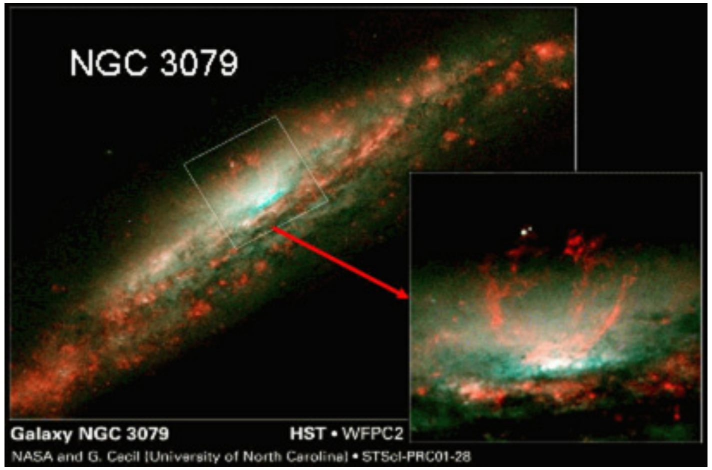
4D look up tables,
based on observations and small-scale simulations
[factor of ~few uncertatinties]

Some reach end of life:

- Undergo core-collapse supernova, with release of 10^{51} erg of energy
- (Collapse into BHs, whit dwarfs, neutron stars — "compact objects") — Feedback: how does it couple to the gas?

The resulting [mass, momentum, energy] output is coupled back to the gas

Stellar feedback: the basic idea is to reduce the amount of star-forming gas



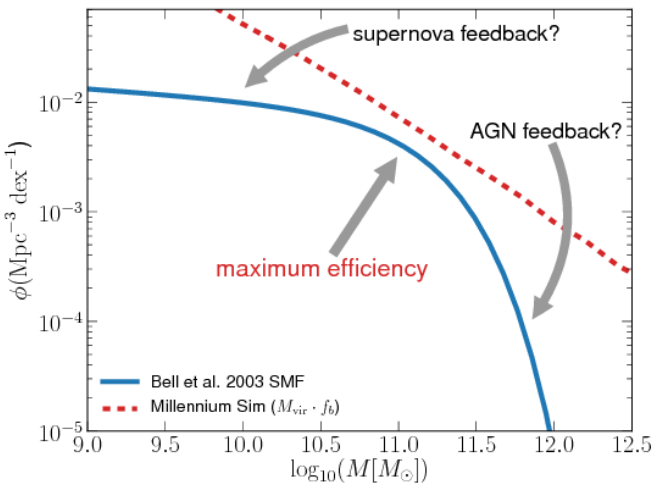
Motions of gas from the star-forming regions of galaxies are observed.

Powerful energy injections are known to be triggered by SN explosions

Feedback is needed to reproduce the amount of stellar mass of observed galaxies

Take a cosmological simulations of dark matter and gas and add recipes for star formation and gas cooling: well, all the gas would be converted quickly into stars, producing too much stellar mass.

This is know as e.g. the **overcooling problem**



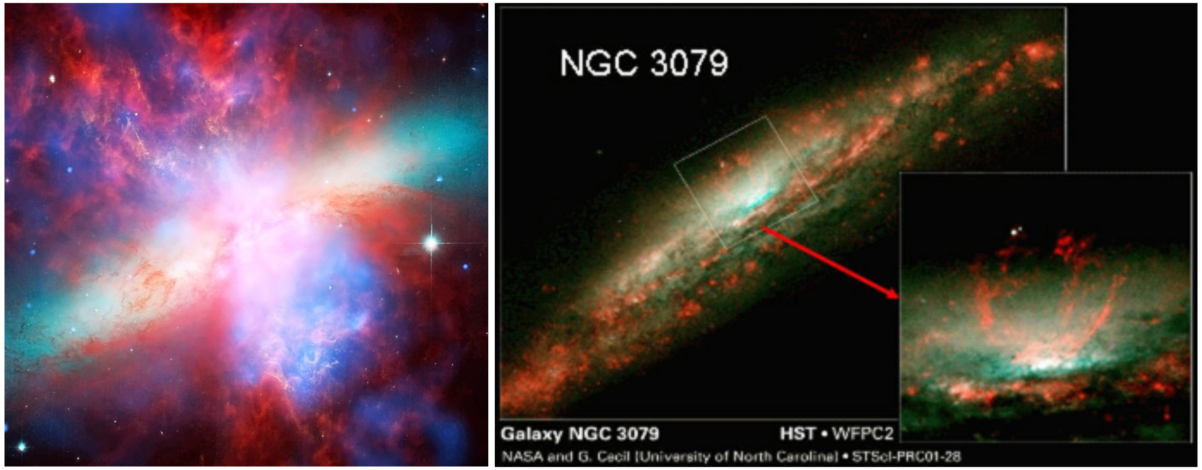
Observed galaxy stellar mass function

vs.

Predicted dark-matter halo mass function x baryonic fraction

Additional mechanisms that regulate a galaxy's star formation activity are invoked to fix the mismatch (since early 2000')

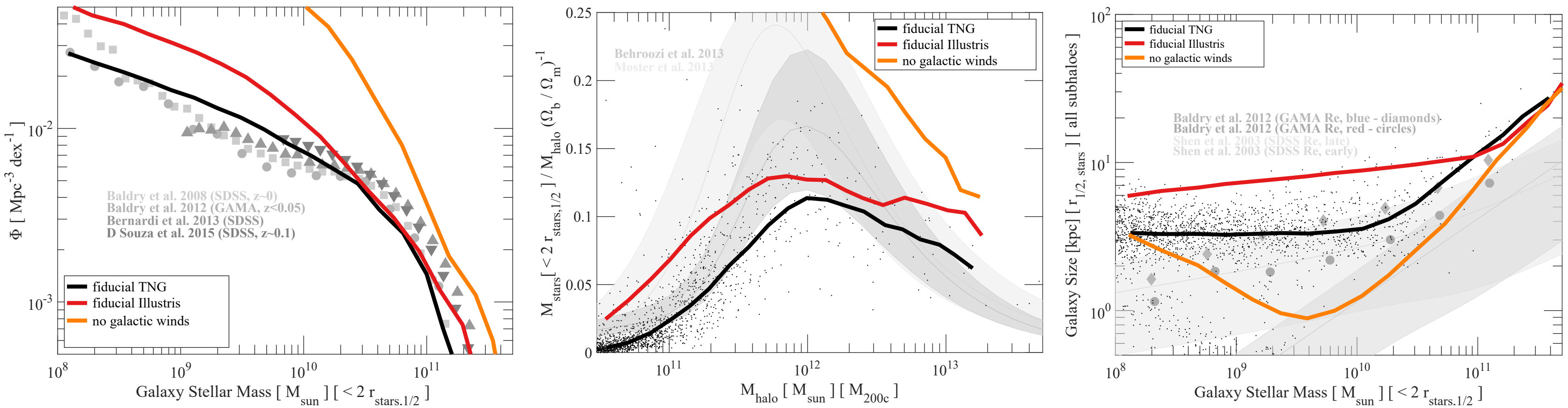
Stellar feedback: the basic idea is to reduce the amount of star-forming gas



Motions of gas from the star-forming regions of galaxies are observed.

Powerful energy injections are known to be triggered by SN explosions

Feedback is needed to reproduce the amount of stellar mass of observed galaxies



Without 'a' form of feedback acting at all masses, i.e. a mechanism that regulates conversion of gas into stars, haloes would host way too massive and compact galaxies

Pillepich, Springel, Nelson, et al. 2018a

Stellar feedback: different simulations, different recipes in the detail

Eagle

Avoid numerical overcooling with a stochastic injection of thermal energy accounting for all SNII energy in one SSP

Wind particles are stochastically selected once a SSP has evolved. **NO** decoupling from the hydro.

$$\left(t_s = \frac{h}{c_s}\right) \ll \left(t_c = \frac{u}{\Lambda}\right)$$

$$\epsilon_{\rm SNII} = 8.73 \times 10^{15}\ {\rm erg\ g^{-1}} \left(\frac{n_{\rm SNII}}{1.736 \times 10^{-2}\ {\rm M_\odot^{-1}}}\right) E_{51}$$

$$\Delta T = (\gamma - 1)\frac{\mu m_{\rm H}}{k_{\rm B}}\epsilon_{\rm SNII}\frac{m_*}{m_{\rm g,heat}} = 4.23 \times 10^7\ {\rm K} \left(\frac{n_{\rm SNII}}{1.736 \times 10^{-2}\ {\rm M_\odot^{-1}}}\right)\left(\frac{\mu}{0.6}\right) \times E_{51}\frac{m_*}{m_{\rm g,heat}}$$

Dalla Vecchia & Schaye (2012)

Magneticum

Wind particles are stochastically spawned from star forming gas particles, at first decoupled from the hydro

Idea similar to those of Illustris and TNG

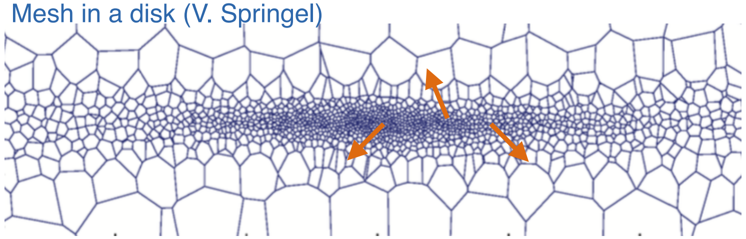
Parameters are chosen so that wind particles are launched with a fixed velocity: v_wind = 350 km/s, at all scales

Springel & Hernquist 2003
Hirschmann et al. 2014

Illustris

IllustrisTNG

Mesh in a disk (V. Springel)



Wind particles are stochastically spawned from star forming gas cells, at first decoupled from the hydro.

Choices at injection: velocity, mass loading/ energy, thermal content.

In these models:

$$\dot{E}_w = e_w\ \dot{M}_{\rm SFR},$$

\propto

$$N_{\rm SNII}\ E_{\rm SNII,51}\ 10^{51}\ {\rm erg\ M_\odot^{-1}},$$

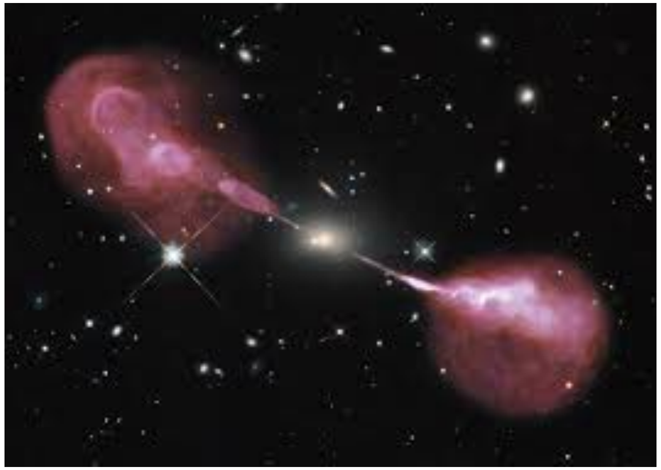
Non-local, from sf-ing gas	General approach	Non-local, from sf-ing gas
Bipolar	Directionality	Isotropic
Cold	Thermal content	Warm
$\propto$ local $\sigma_{\rm DM}$	Injection velocity	$\propto$ Local $\sigma_{\rm DM}$ with $H(z)$ scaling
–	Injection mass loading	Gas-metallicity (Z) dependent
No	Injection velocity floor	Yes: 350 km s^{-1}

TNG winds are overall faster (at all redshifts) but with lower mass loadings (at z<2) than Illustris'

Springel & Hernquist 2003
Vogelsberger, Genel, Sijacki, Torrey et al. 2013
Torrey, Vogelsberger, Genel, Sijacki et al. 2014

Springel & Hernquist 2003
Pillepich, Springel, Nelson, Genel et al. 2018

SMBH feedback: the basic idea is to mimic what observed



AGNs and SMBHs are ubiquitous

Powerful energy injections are known to be launched from the sites of SMBHs

SMBH feedback is the only mechanism that can quench entire populations of simulated massive consistently with observations

SMBH seeding, growth, and positioning: we do not know how SMBHs form

Eagle		Magneticum
$1 \times 10^5 h^{-1}$ M$_\odot$	BH Seed Mass	$M_{\rm BH}(M_*)/100$
$1 \times 10^{10} h^{-1}$ M$_\odot$	FoF Halo Mass for BH Seeding	300–500 stellar particles
un-boosted Bondy-Hoyle (w/ $C_{\rm visc}$)	BH Accretion	$\alpha = 100$ boosted Bondy-Hoyle
gas particles, Eddington limited	BH Accretion	gas particles, Eddington limited
kicked to halo potential minimum	BH Positioning	Imposed ang. mom. conservation + dynamical friction
Schaye, Crain, Bower et al. 2015		*Hirschmann, Dolag, Saro et al. 2014*

SMBHs are usually placed by hand as "sink particles":

They can grow in mass

a) by `accreting` material from the surroundings

Accretion a' la Bondi:

$$\dot{M}_{\rm BH} = \frac{4\pi\alpha G^2 M_{\rm BH}^2 \rho}{\left(c_s^2 + v^2\right)^{3/2}}$$

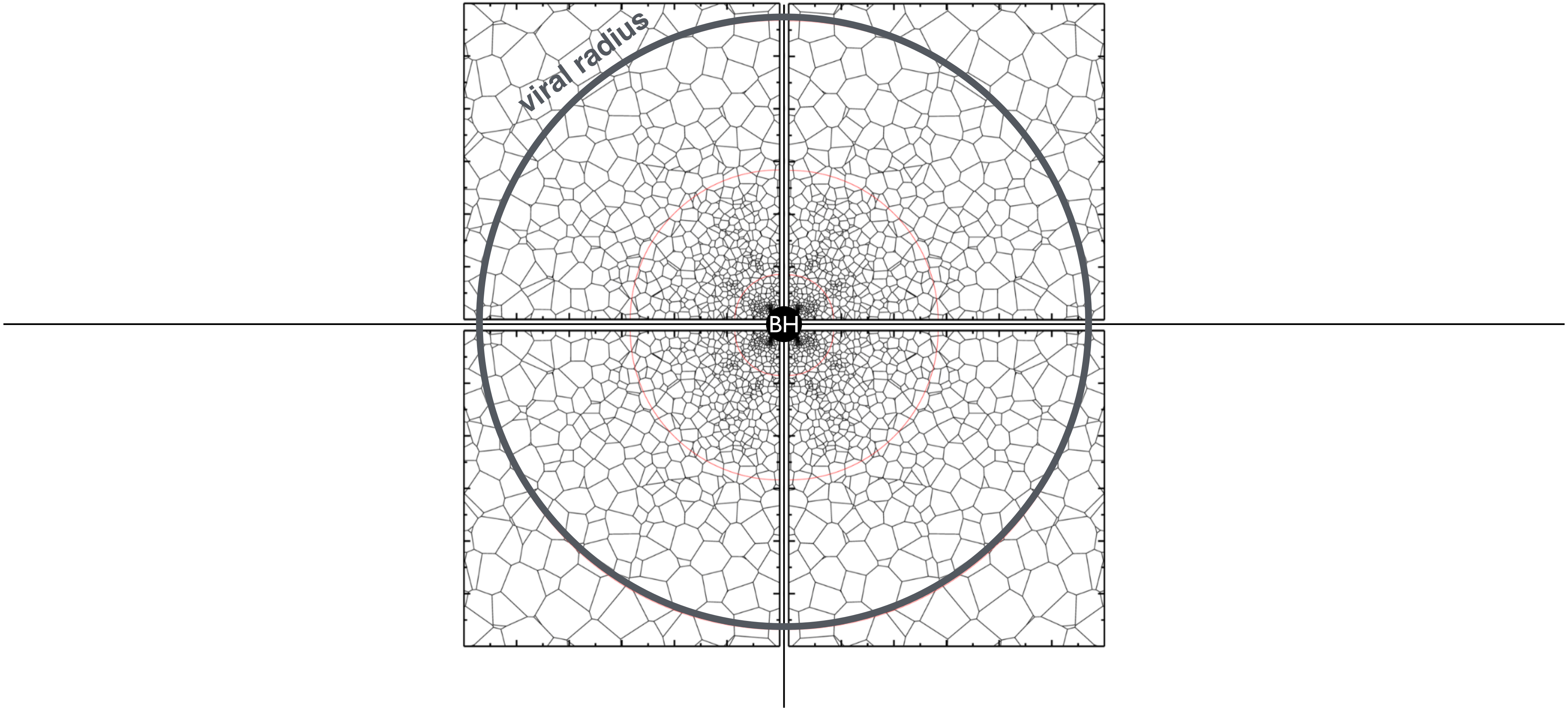
Accretion Ceiling:

$$\dot{M}_{\rm Edd} = \frac{4\pi G M_{\rm BH} m_p}{\epsilon_r \sigma_T c}$$

b) by *self-consistently* merging with other SMBHs

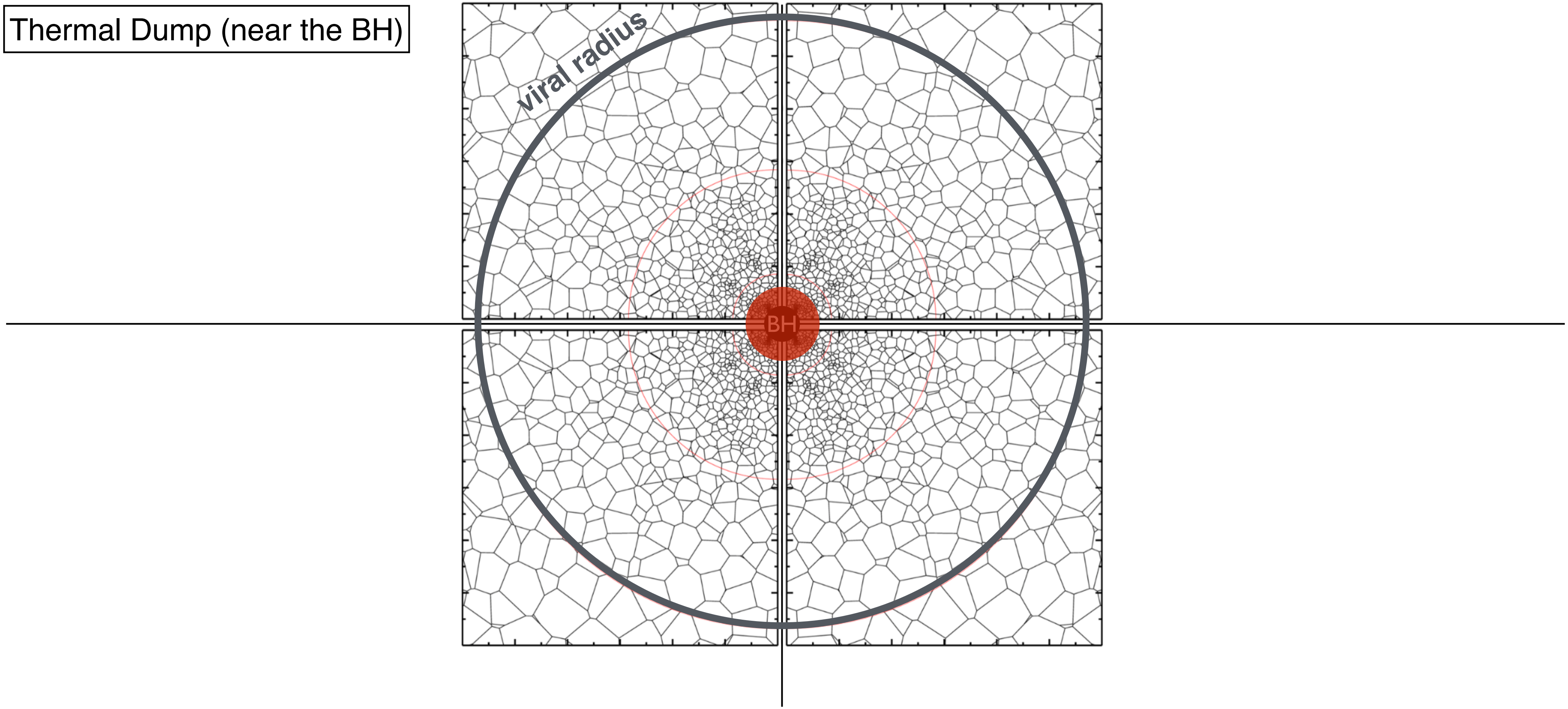
Illustris		IllustrisTNG
$1 \times 10^5 h^{-1}$M$_\odot$	BH Seed Mass	$8 \times 10^5 h^{-1}$M$_\odot$
$5 \times 10^{10} h^{-1}$M$_\odot$	FoF Halo Mass for BH seeding	$5 \times 10^{10} h^{-1}$M$_\odot$
$\alpha = 100$ Boosted Bondi-Hoyle	BH Accretion	Un-boosted Bondi-Hoyle (w/ v_A)
parent gas cell, Eddington limited	BH Accretion	nearby cells, Eddington limited
fixed to nearby potential minimum	BH Positioning	fixed to nearby potential minimum
Vogelsberger, Genel, Sijacki, Torrey et al. 2013		*Weinberger, Springel, Hernquist, Pillepich et al. 2017*

SMBH feedback: possible sub grid implementations



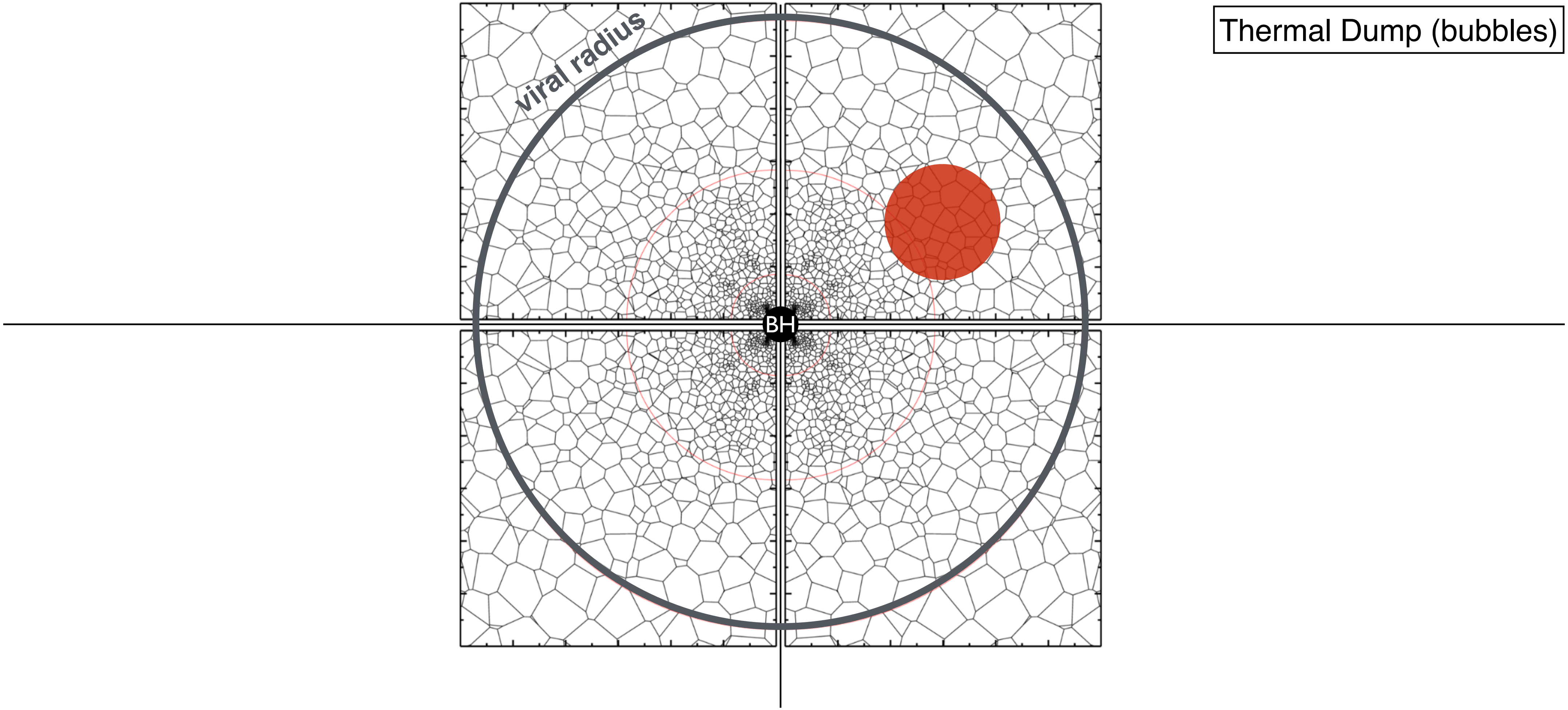
SMBH feedback: possible sub grid implementations

Thermal Dump (near the BH)

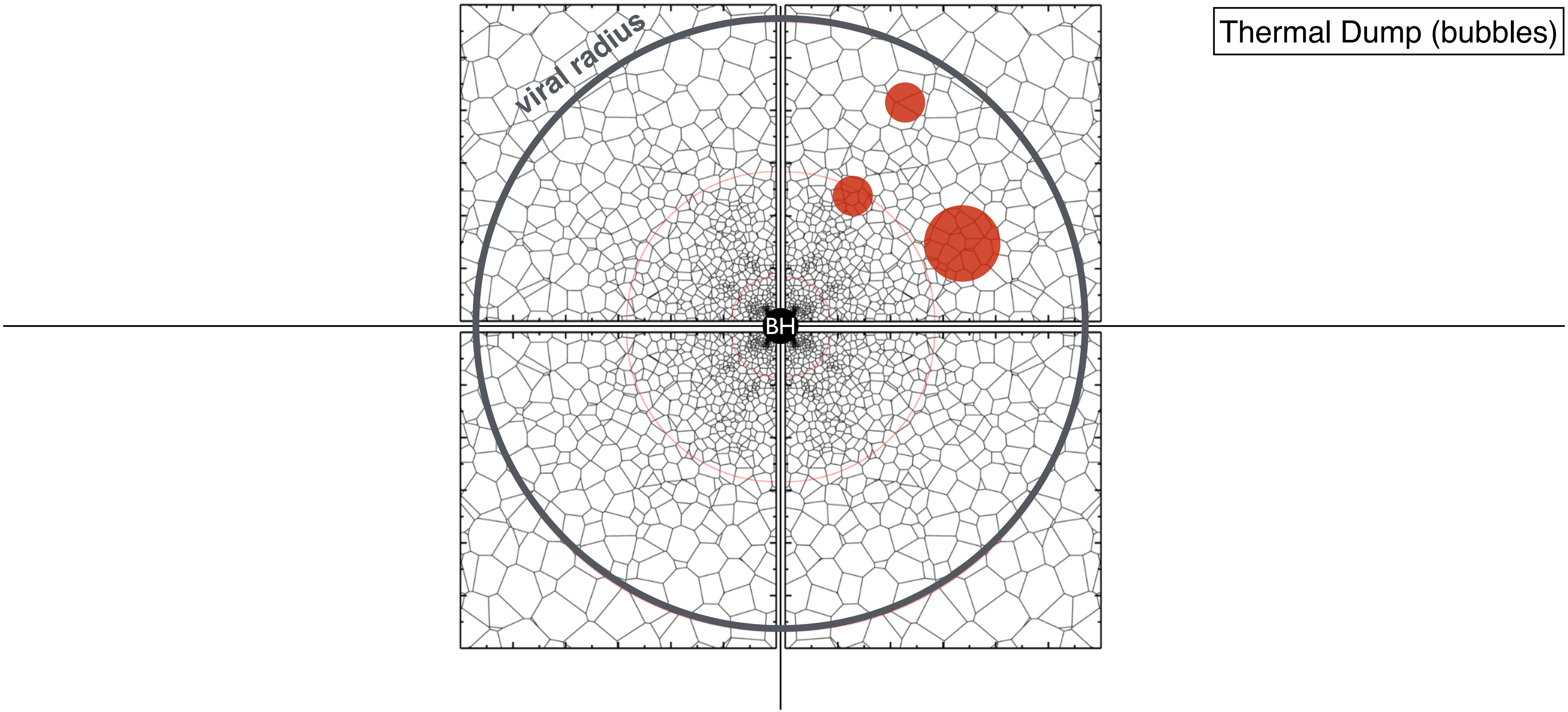


SMBH feedback: possible sub grid implementations

Thermal Dump (bubbles)

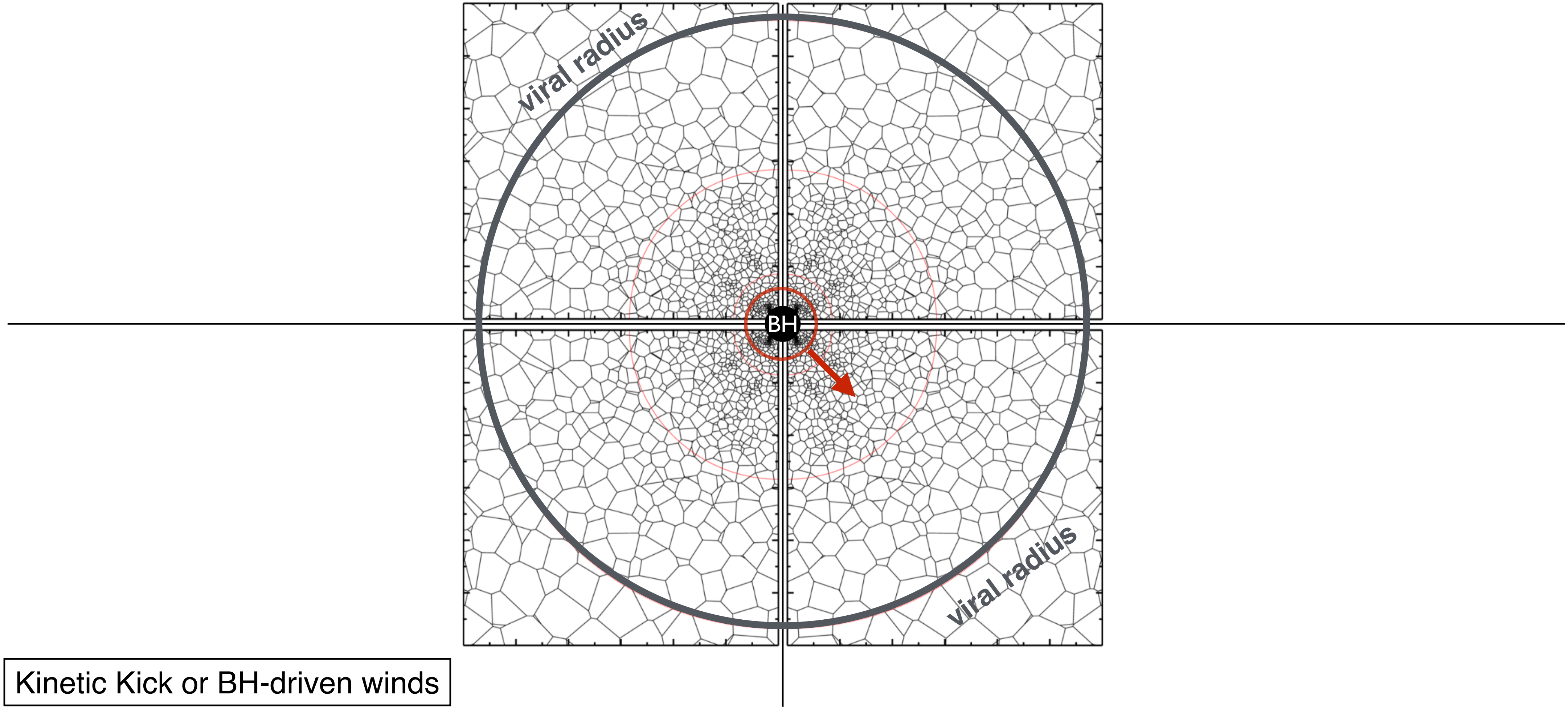


SMBH feedback: possible sub grid implementations



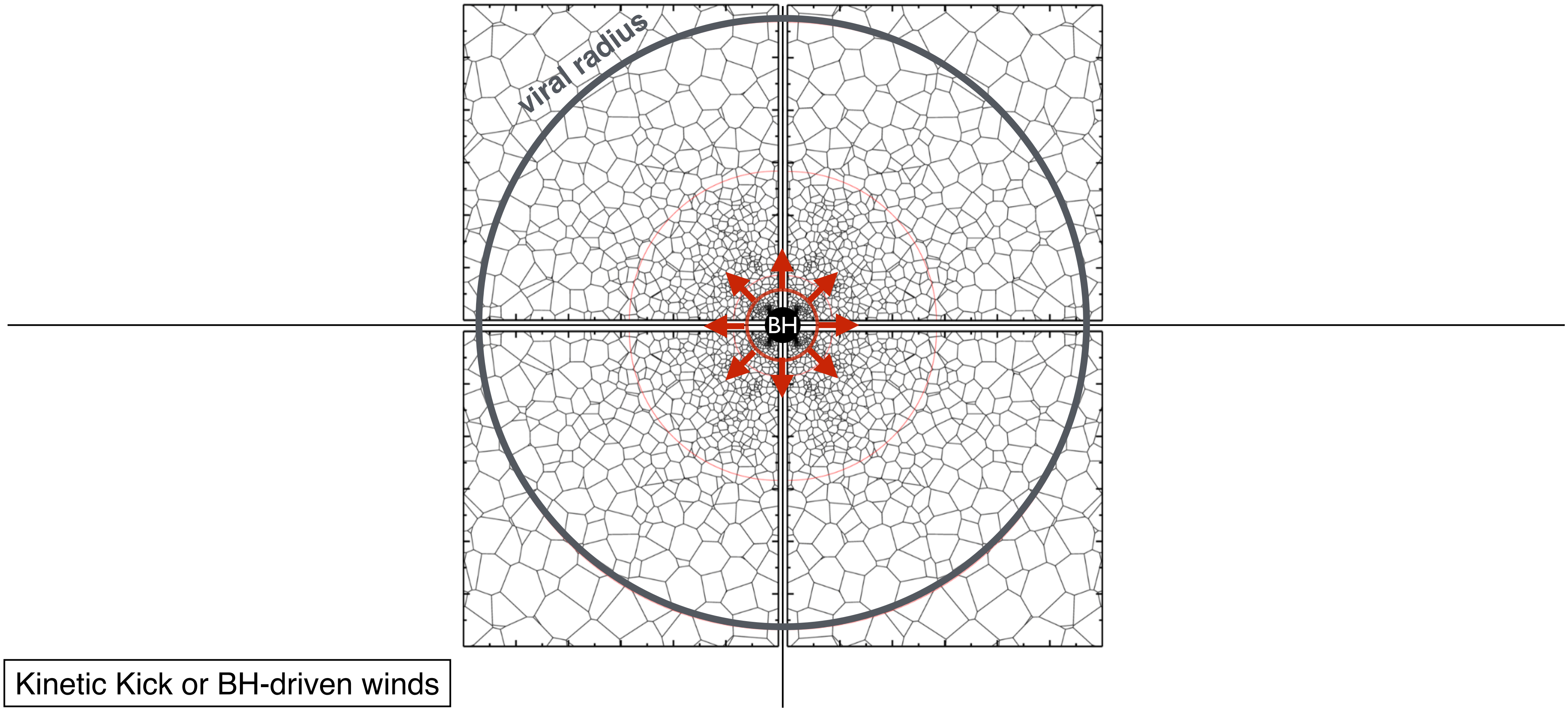
Thermal Dump (bubbles)

SMBH feedback: possible sub grid implementations



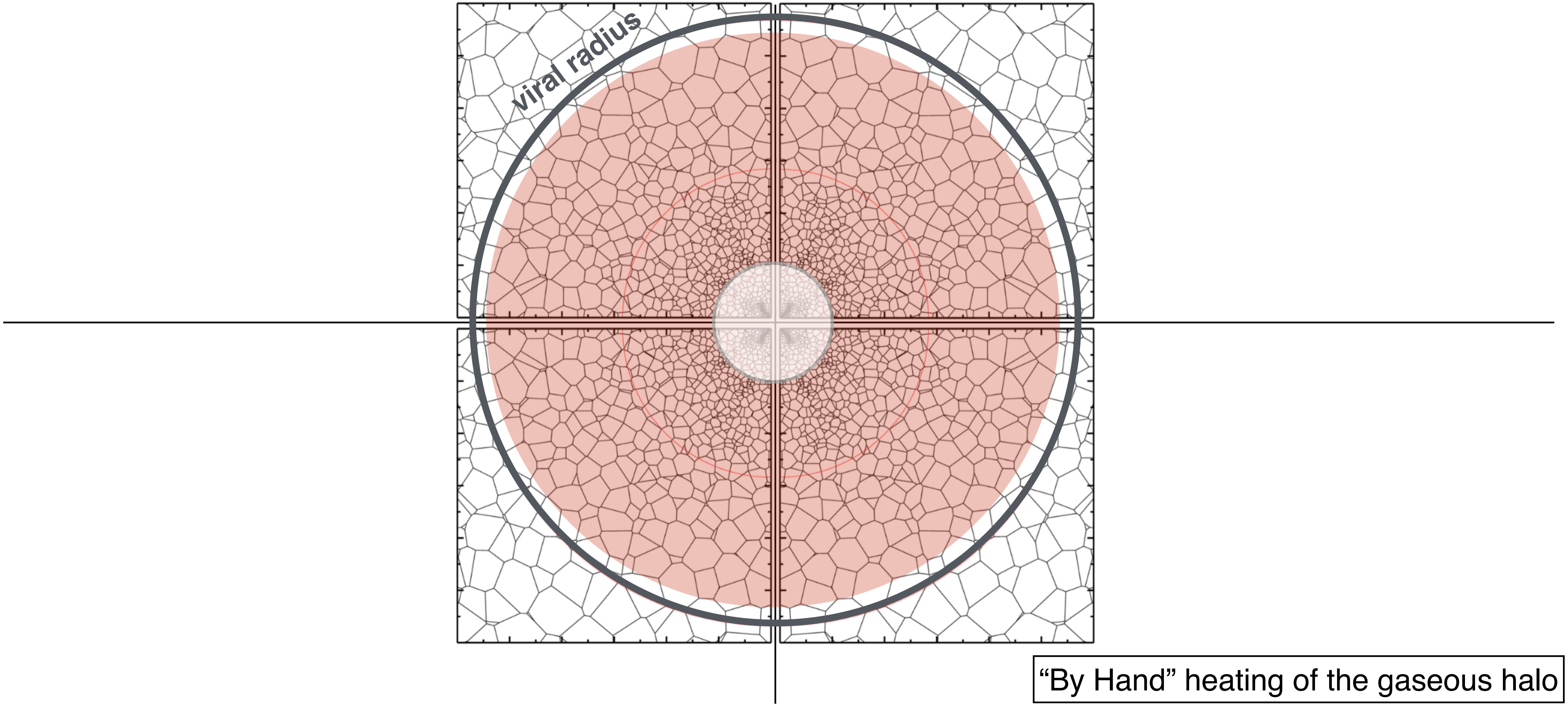
Kinetic Kick or BH-driven winds

SMBH feedback: possible sub grid implementations



Kinetic Kick or BH-driven winds

SMBH feedback: possible sub grid implementations



"By Hand" heating of the gaseous halo

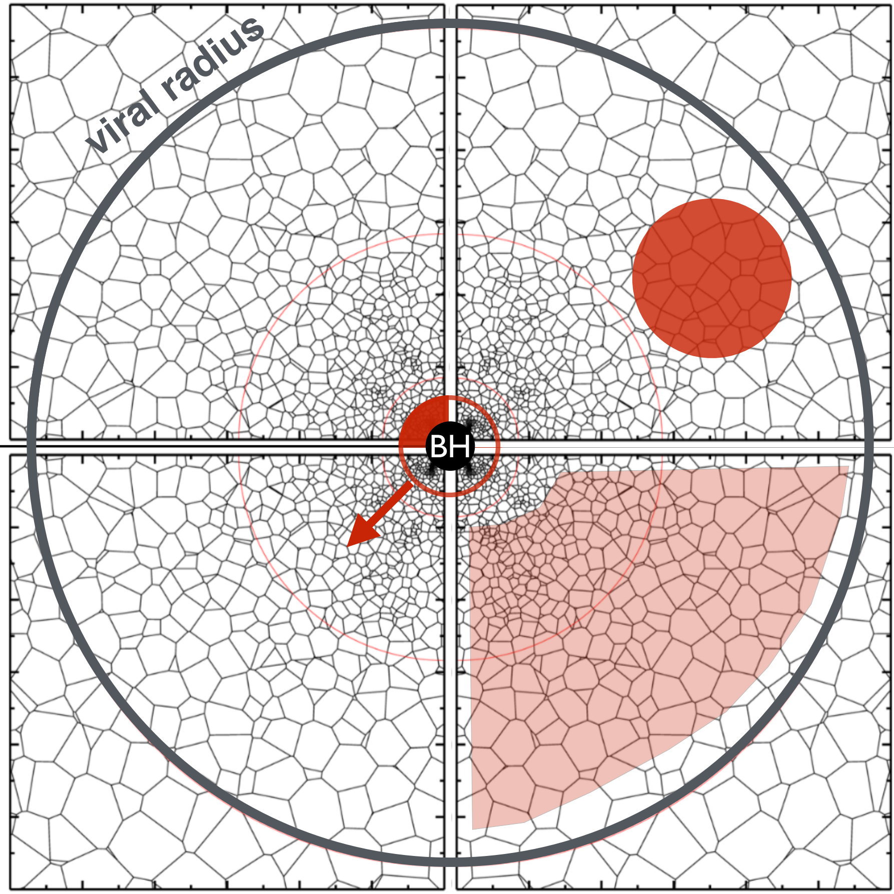
SMBH feedback: possible sub grid implementations

aka QUASAR MODE

Thermal Dump (near the BH)

Continuous?
- yes e.g. Illustris, HorizonAGN
- no e.g. Eagle

Only at high accretion rates?
- yes e.g. Illustris
- no e.g. Eagle (all the time)



aka RADIO MODE

Thermal Dump (bubbles)

Very sporadic, energetic bubbles: Illustris
More frequent, “smaller bubbles”: Auriga

Only at low accretion rates?
- yes e.g. Illustris
- no e.g. Auriga (all the time)

Kinetic Kick or BH-driven winds

Isotropic?
- no e.g. TNG, each time in different dirs
- no: bipolar e.g. HorizonAGN

Continuous?
- ~ e.g. TNG, each time in different dirs

Only at low accretion rates?
- yes e.g. TNG, HorizonAGN

See also Choi et al. 2012, 2014, 2015; Dubois et al. 2010, 2012; Weinberger et al 2017

“By Hand” heating of the gaseous halo

Affecting only non-self shielded gas e.g. Mufasa, NIHAO variations

SMBH feedback: different simulations, different recipes in the detail

Eagle

One mode:

thermal dump (“quasar”)
Intermittent

Schaye, Crain, Bower et al. 2015

Magneticum

Two “modes”:

High-accr. rates:
thermal dump (continuous)
Low-accr. rates:
x4 more energetic thermal dump

Low/High Accretion Transition: χ constant $= 0.01\,\dot{M}_{\rm Edd}$

Hirschmann, Dolag, Saro et al. 2014

Illustris

Two modes:

High-accr. rates:
thermal dump (continuous)
Low-accr. rates:
thermal ‘bubbles’ in the ICM (intermittent)

IllustrisTNG

Two modes:

High-accr. rates:
thermal dump (continuous)
Low-accr. rates:
BH-driven winds (isotropic, pulsated)

constant: $0.05\,\dot{M}_{\rm Edd}$ — Low/High Accretion Transition: χ — BH-mass dependent, $\leqslant 0.1\,\dot{M}_{\rm Edd}$

\+
A radiative feedback

Vogelsberger, Genel, Sijacki, Torrey et al. 2013

Weinberger, Springel, Hernquist, Pillepich et al. 2017

For cosmological galaxy simulations, a large and uncertain parameter space

Illustris	Model features	TNG	Technical Reference
	MHD		
No	Magnetohydrodynamics (MHD)	Yes: Powell $\nabla \cdot B$ cleaning	Pakmor et al. (2011)
–	Seed B field strength	1.6×10^{-10} phys Gauss at $z = 127$	Pakmor & Springel (2013)
–	Seed B field configuration	Uniform in random direction	Pakmor & Springel (2013)
	BHs and BH feedback		
$1 \times 10^5 h^{-1}\,\mathrm{M}_\odot$	BH seed mass	$8 \times 10^5 h^{-1}\,\mathrm{M}_\odot$	Weinberger et al. (2017)
$5 \times 10^{10} h^{-1}\,\mathrm{M}_\odot$	FoF halo mass for BH seeding	$5 \times 10^{10} h^{-1}\,\mathrm{M}_\odot$	Vogelsberger et al. (2013)
$\alpha = 100$ Boosted Bondi–Hoyle	BH accretion	Un-boosted Bondi–Hoyle (w/ v_A)	Weinberger et al. (2017)
Parent gas cell, Eddington limited	BH accretion	Nearby cells, Eddington limited	Weinberger et al. (2017)
Pixed to local potential minimum	BH positioning	Fixed to local potential minimum	Vogelsberger et al. (2013)
Two: 'Quasar/Radio'	BH feedback Modes	Two: 'high-/low-accretion state'	Weinberger et al. (2017)
Thermal Injection around BHs	High-accr-rate feedback	Thermal injection around BHs	Weinberger et al. (2017)
Thermal 'bubbles' in the ICM	Low-accr-rate feedback	BH-driven kinetic wind	Weinberger et al. (2017)
Constant: 0.05	Low/high-accretion transition: χ	BH-mass dependent, ≤ 0.1	Weinberger et al. (2017)
0.2	Radiative efficiency: ϵ_r	0.2	Weinberger et al. (2017)
$\epsilon_f \epsilon_r$, with $\epsilon_f = 0.05$	High-accretion-rate feedback factor	$\epsilon_f \epsilon_r$, with $\epsilon_f = 0.1$	Weinberger et al. (2017)
$\epsilon_m \epsilon_r$, with $\epsilon_m = 0.35$	Low-accretion-rate feedback factor	$\epsilon_{\mathrm{f,\,kin}} \leq 0.2$	Weinberger et al. (2017)
Yes	Radiative BH feedback	Yes	Vogelsberger et al. (2013)
	Galactic winds		
Non-local, from sf-ing gas	General approach	Non-local, from sf-ing gas	Vogelsberger et al. (2013)
Bipolar	Directionality	Isotropic	This paper
Cold	Thermal content	Warm	This paper
$\propto$ local σ_{DM}	Injection velocity	$\propto$ Local σ_{DM} with $H(z)$ scaling	This paper
–	Injection mass loading	Gas-metallicity (Z) dependent	This paper
No	Injection velocity floor	Yes: $350\,\mathrm{km\,s^{-1}}$	This paper
3.7	Wind velocity factor: κ_w	7.4	This paper
1.09	Wind energy factor: $\bar{e}_w$	3.6	This paper
-	Thermal Fraction: τ_w	0.1	This paper
-	Z-dependence reduction factor: $f_{w,Z}$	0.25	This paper
-	Z-dependence reference metallicity: $Z_{w,Z}$	0.002	This paper
-	Z-dependence reduction power: $\gamma_{w,Z}$	2	This paper
0.4	Metal loading of wind particles: γ_w	0.4	Vogelsberger et al. (2013)
	Stellar Evolution		
Chabrier (2003)	IMF	Chabrier (2003)	Vogelsberger et al. (2013)
$[6, 100]\,\mathrm{M}_\odot$	[min, max] SNII Mass	$[8, 100]\,\mathrm{M}_\odot$	this paper
see Table 2	Yield Tables	see Table 2	this paper
at every star timestep	ISM Chemical Enrichment	time/stellar mass discrete	this paper
	Metal Advection		
gradient extrapolation	Advection Scheme	same + renormalization	this paper
0	Initialization Metal Fractions	10^{-10} at $z = 127$	this paper
H, He, C, N, O, Ne, Mg, Si, Fe	Tracked Element Scalars	same 9 + other metals	this paper
-	Metal Tagging	from SNIa, SNII, AGB separately	Naiman et al. (2017)
-	Iron Tagging	from SNIa and SNII separately	Naiman et al. (2017)
-	r-processes	from NS-NS mergers	Naiman et al. (2017)

Pillepich et al 208a

Functional forms and inclusion of physical processes are by choice, dictated by scientific applications and computing capabilities

Parameter values are from a mixture of:
- Ab initio results.
- Smaller-scale simulations/theory.
- Observations.

A calibration/tuning step is required to constrain free parameters:
- ~10 to ~50 dimensions.
- Minimize the loss function?
- Nope. By hand, e.g. domain-based expertise.

Different simulations, different "calibration strategies"

Eagle

- z=0 galaxy stellar mass function
- z=0 galaxy sizes
- z=0 BH mass - galaxy mass

Fits to specific observational data sets

Schaye, Crain, Bower et al. 2015; Crain, Schaye, Bower et al. 2015

Magneticum

- z=0 BH mass - galaxy mass
- X-ray luminosity-mass of massive haloes
- Stellar metallicity content of L* galaxies

In comparison to some observations

Observables of reference for development of the models

Illustris

- cosmic star formation rate density
- z=0 galaxy stellar mass function
- z=0 stellar to halo mass relation
- (z=0 gas metallicity-mass relation)
- (z=0 BH mass - galaxy mass)

In comparison to some observations

Vogelsberger, Genel, Sijacki, Torrey et al. 2013
Torrey, Vogelsberger, Genel, Sijacki et al. 2014

IllustrisTNG

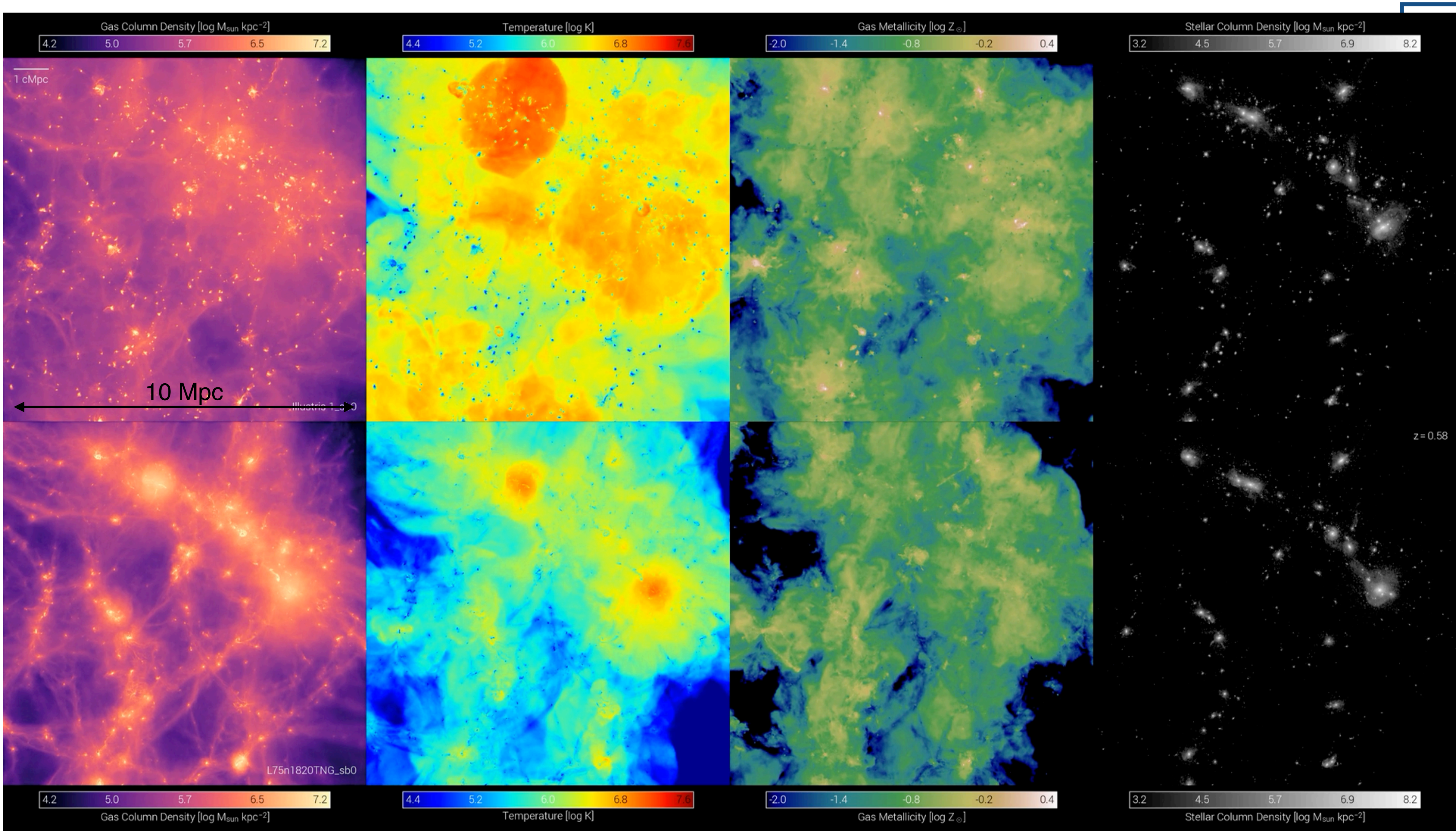
- cosmic star formation rate density
- z=0 galaxy stellar mass function
- z=0 stellar-to-halo mass relation
- z=0 gas fraction within R500c
- z=0 BH mass vs. galaxy mass relation
- z=0 stellar sizes vs. galaxy mass relation

All in comparison to Illustris model outcome

Pillepich, Springel, Nelson, Genel et al. 2018
Weinberger, Springel, Hernquist, Pillepich et al. 2017

Illustris Model

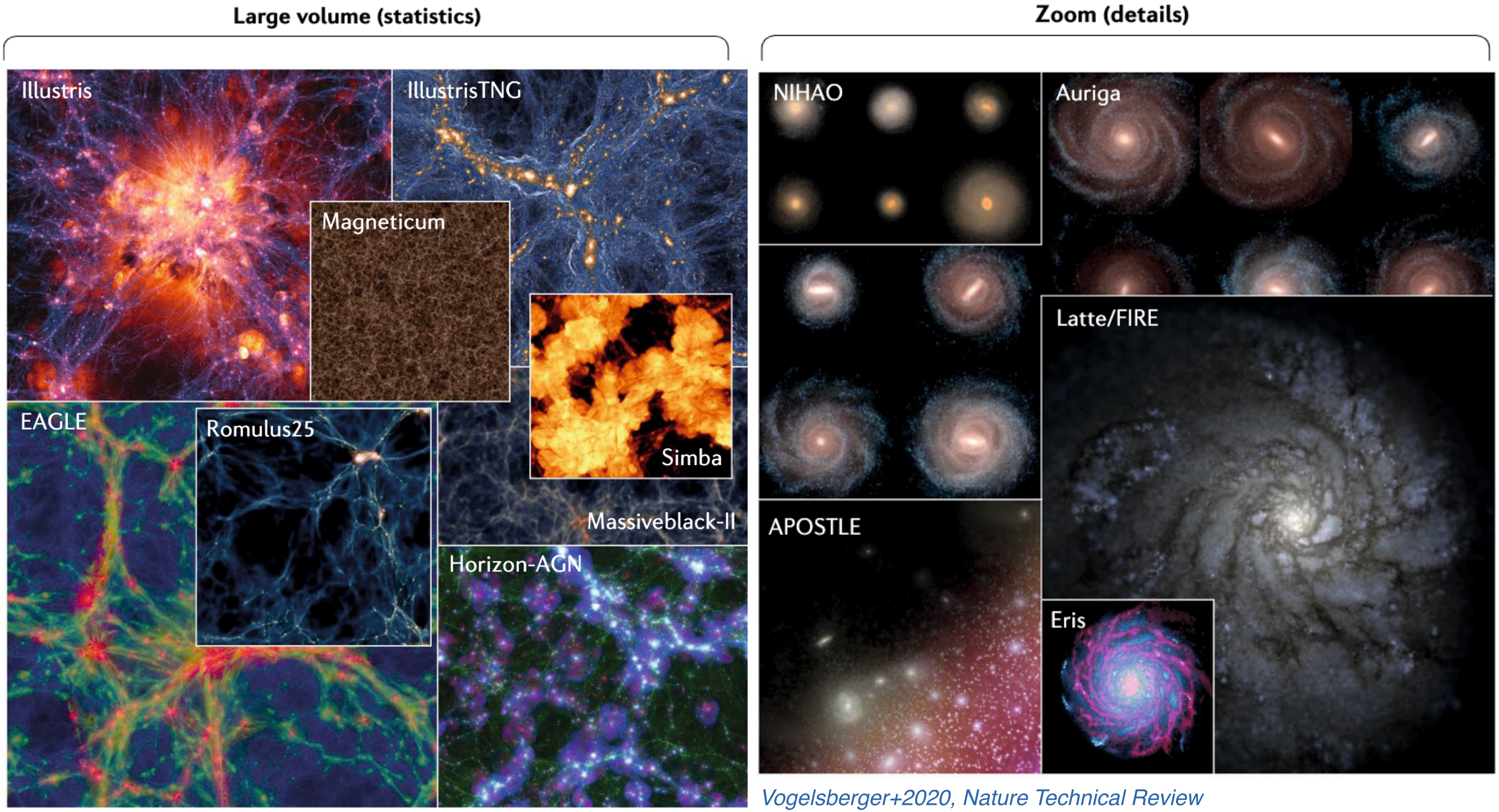
TNG Model



The goals/scope of today's lecture

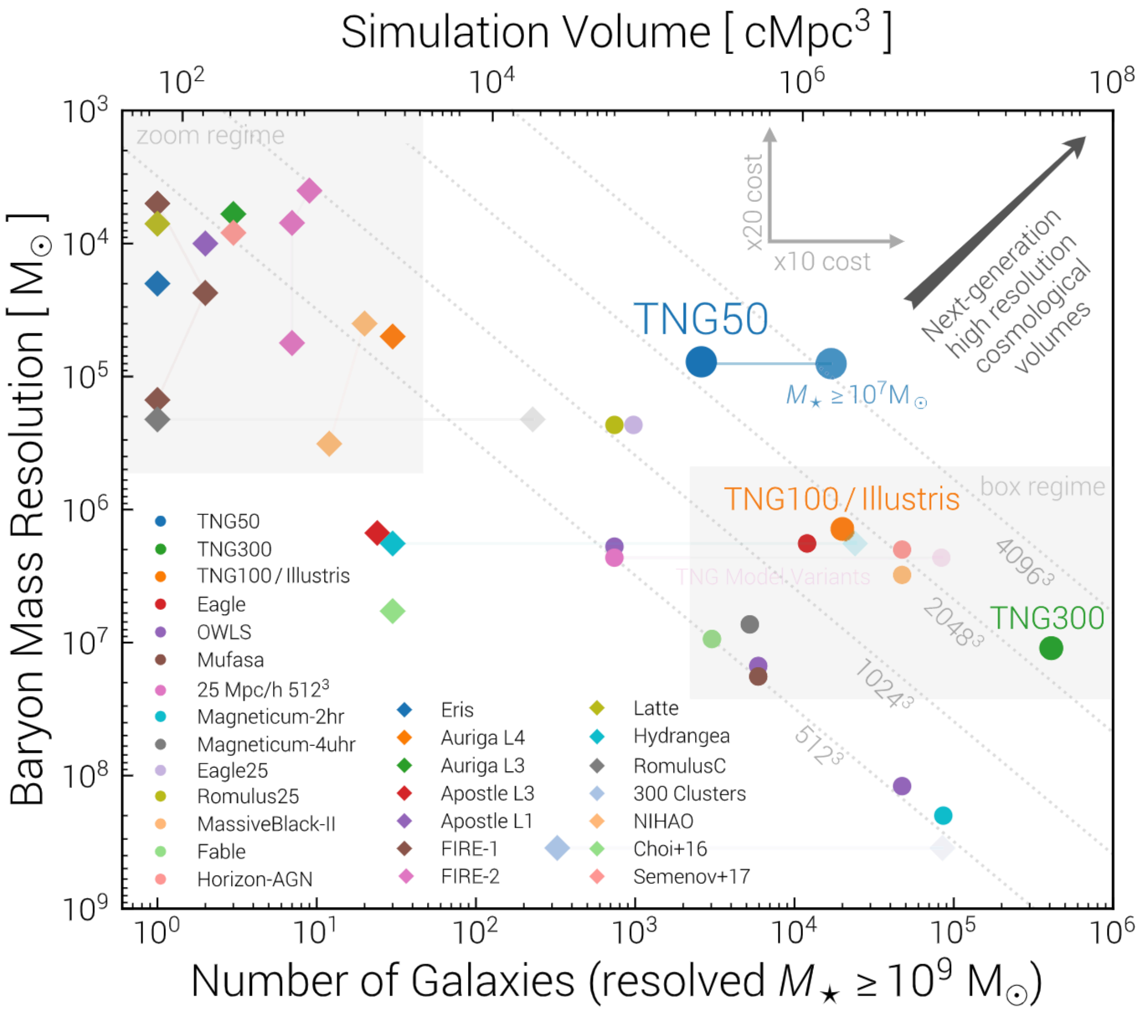
a. The cosmological context
b. The goals of cosmological galaxy simulations
c. The landscape and basic ingredients
d. The baryonic fluid
e. Fundamentals of galaxy formation
f. Euler equations and their numerical solution
    a. Elements of SPH
    b. Elements of Eulerian hydrodynamics
    c. Moving-mesh and mesh-less techniques
g. The available cosmological codes
h. On numerical resolution
i. Modelling the physics of galaxies
    a. Star formation
    b. Stellar evolution
    c. Feedback: needs, from stars and from SMBHs
j. Examples of state-of-the art simulations
k. Selected results

Examples of cosmological *galaxy* simulations of these years



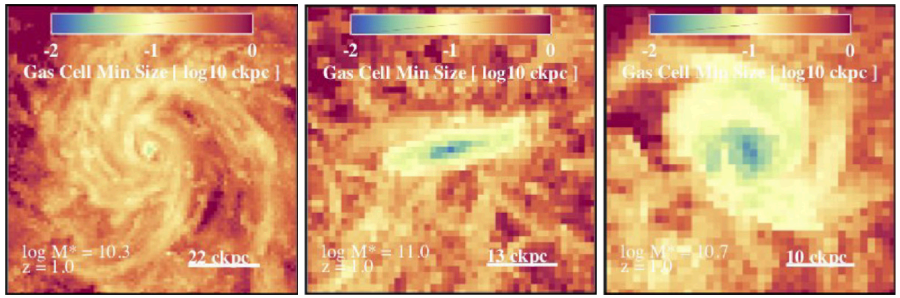
Vogelsberger+2020, Nature Technical Review

Large-scale cosmological *galaxy* simulations: a remarkable progress in ~10 yrs



Nelson, Pillepich, Springel + 2019

- Cubic volumes of synthetic universes
  - a few tens of cMpc to ~300cMpc a side
- Four main constituents:
  - Gas
  - Dark Matter
  - Stars
  - Super Massive Black Holes
- Gas/stars mass resolution: ~10^{5-7} M☉
- Spatial resolution: 100 pc - a few kpc

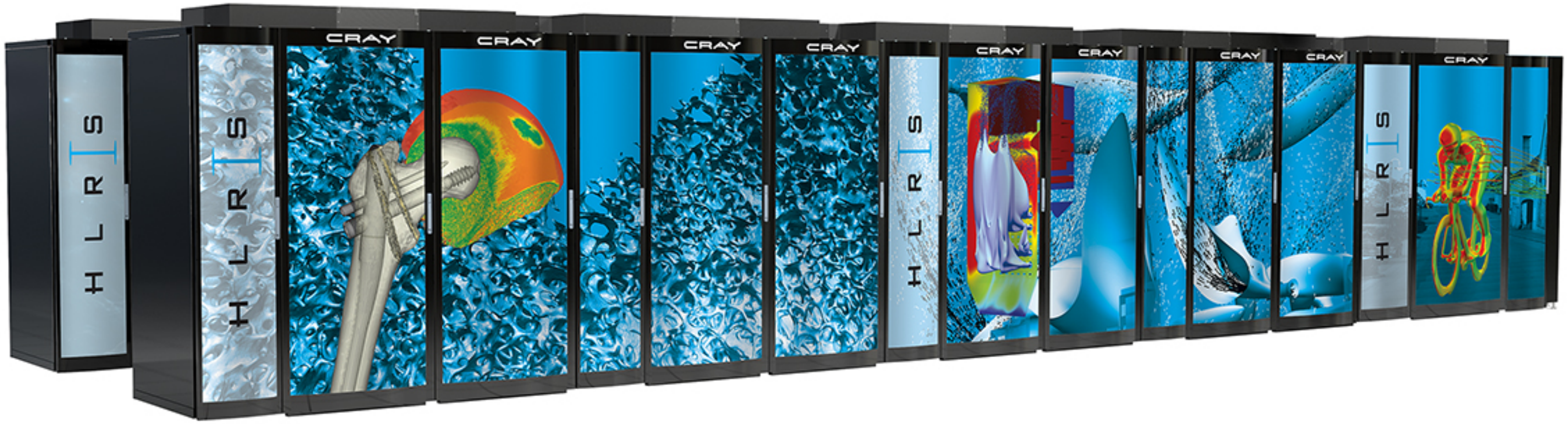


TNG50: Pillepich, Nelson, Springel + 2019

The TNG50 simulation, a field-leading computational endeavour

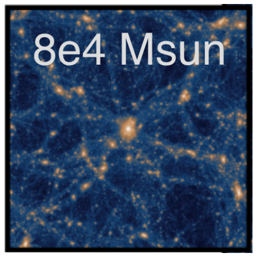
Co-PIs: D. Nelson (MPA), A. Pillepich (MPIA)

HazelHen (Stuttgart)
Cray Cluster
7712 nodes
with 24 cores/node
5.3GB of memory per core!



TNG50

Cosmological volume at zoom resolution

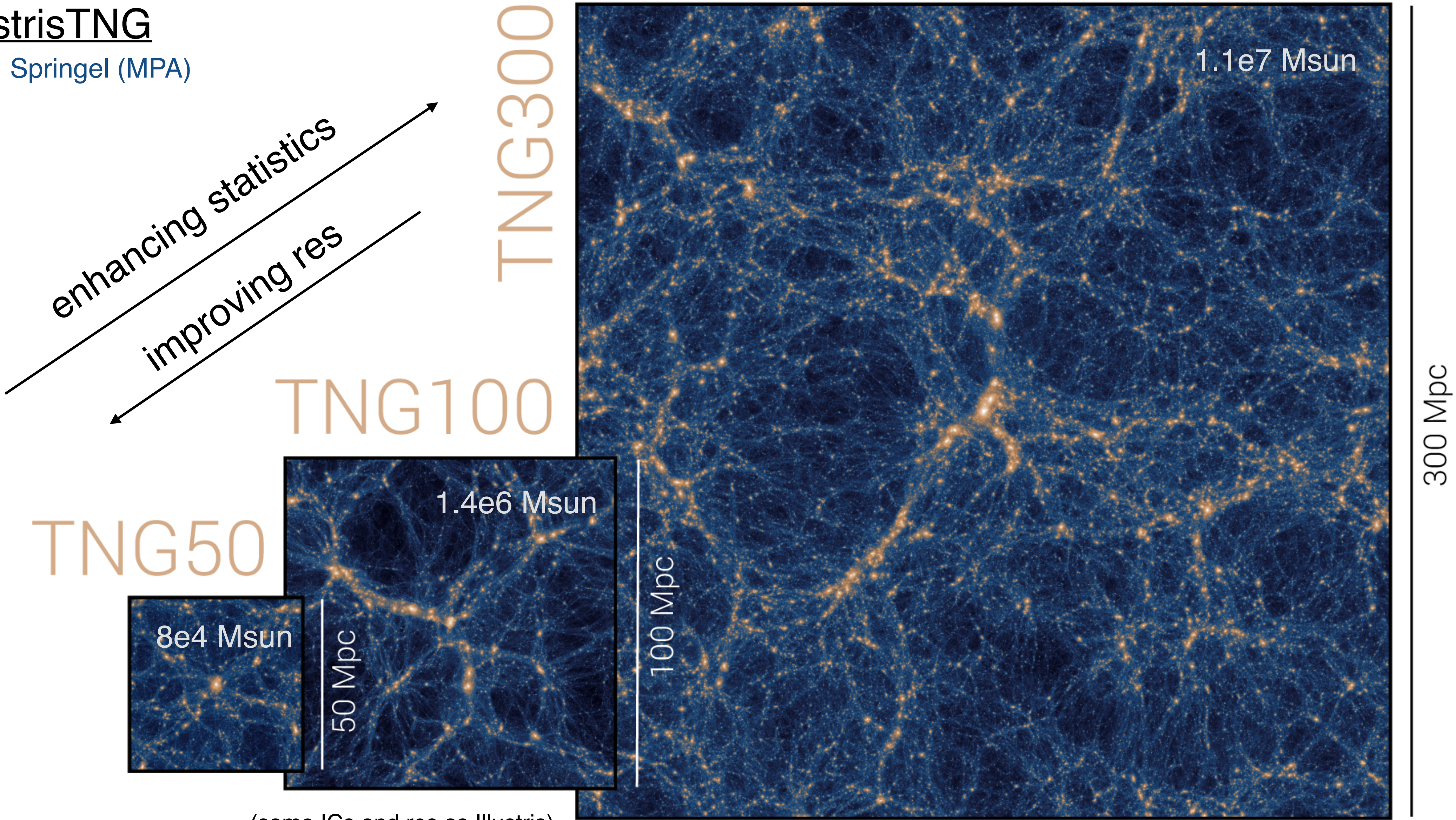


It has run for more than one year, 24/7 on 16k computing cores!

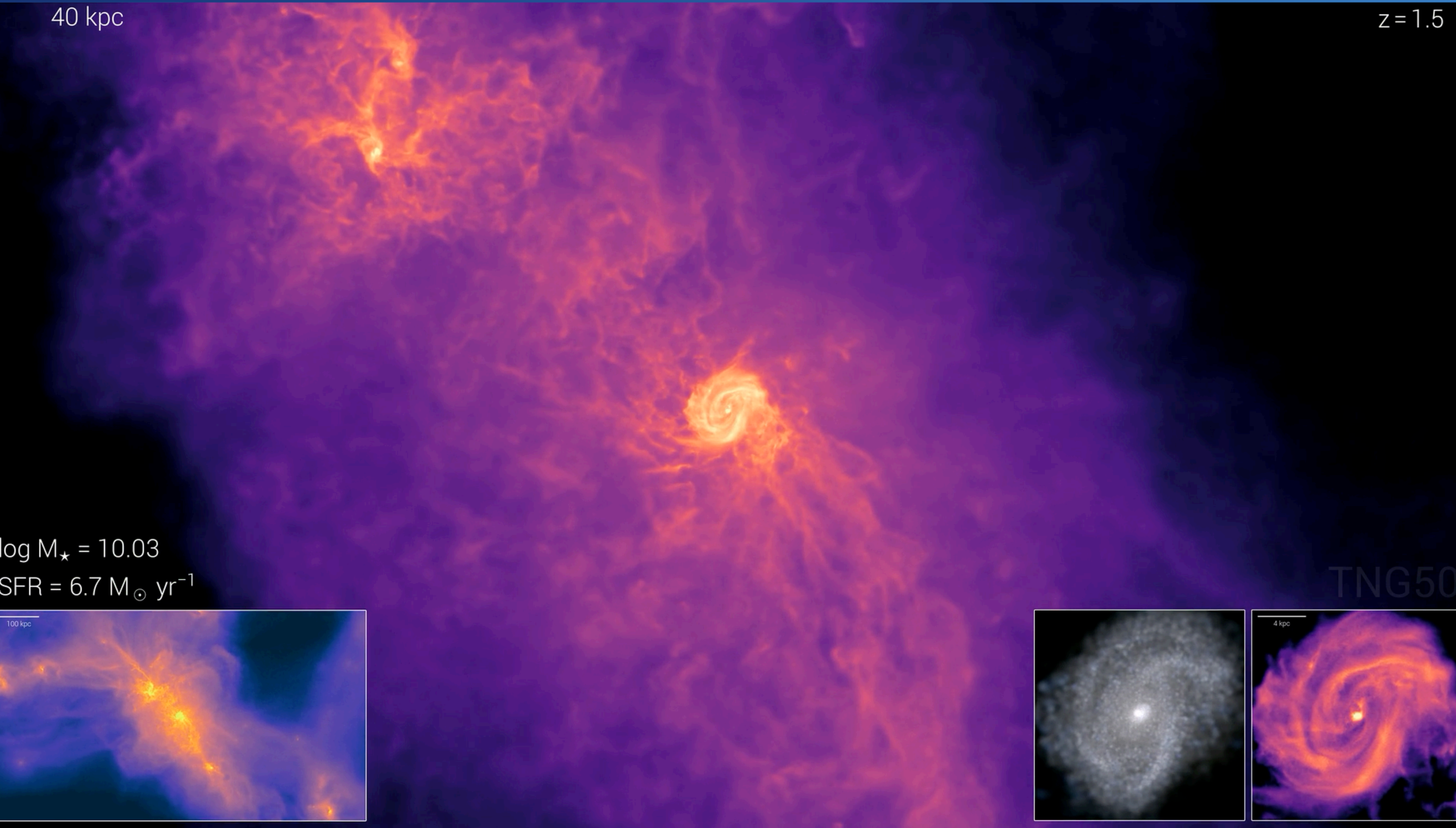
TNG50: Pillepich, Nelson et al. 2019
TNG50: Nelson, Pillepich et al. 2019

IllustrisTNG

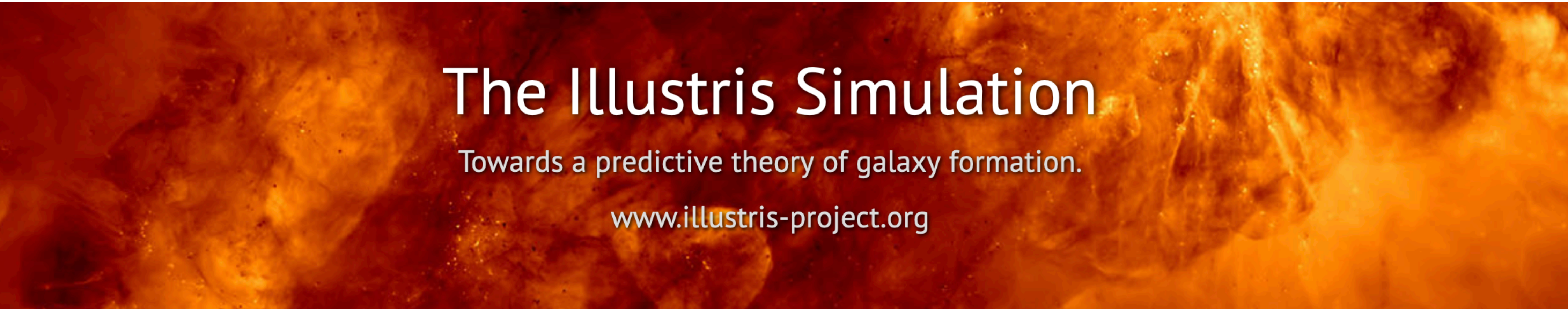
PI: V. Springel (MPA)

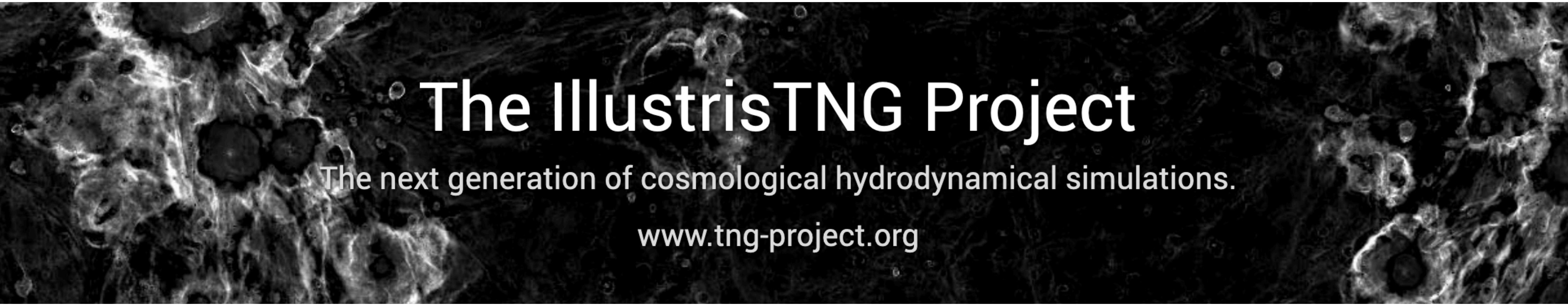


(same ICs and res as Illustris)


40 kpc
z = 1.5
log $M_\star$ = 10.03
SFR = 6.7 $M_\odot$ yr^{-1}
TNG50
100 kpc
4 kpc

IllustrisTNG: fully public output





Spelling out the Setup, i.e. a minimal glossary

III

Cosmological	as opposed to	Isolated
Gravity+Hydro		DM-only, stars-only
MHD		no B fields, gas neutral
Galaxy Formation/ Full Physics		Adiabatic (if with gas), non radiative
Uniform Volume	as opposed to	Zoom
Moving Mesh		Smooth Particle Hydro (gas particles) Cartesian Grid (yet adaptive)
Periodic Boundary Conditions		"Embedded" in a larger volume

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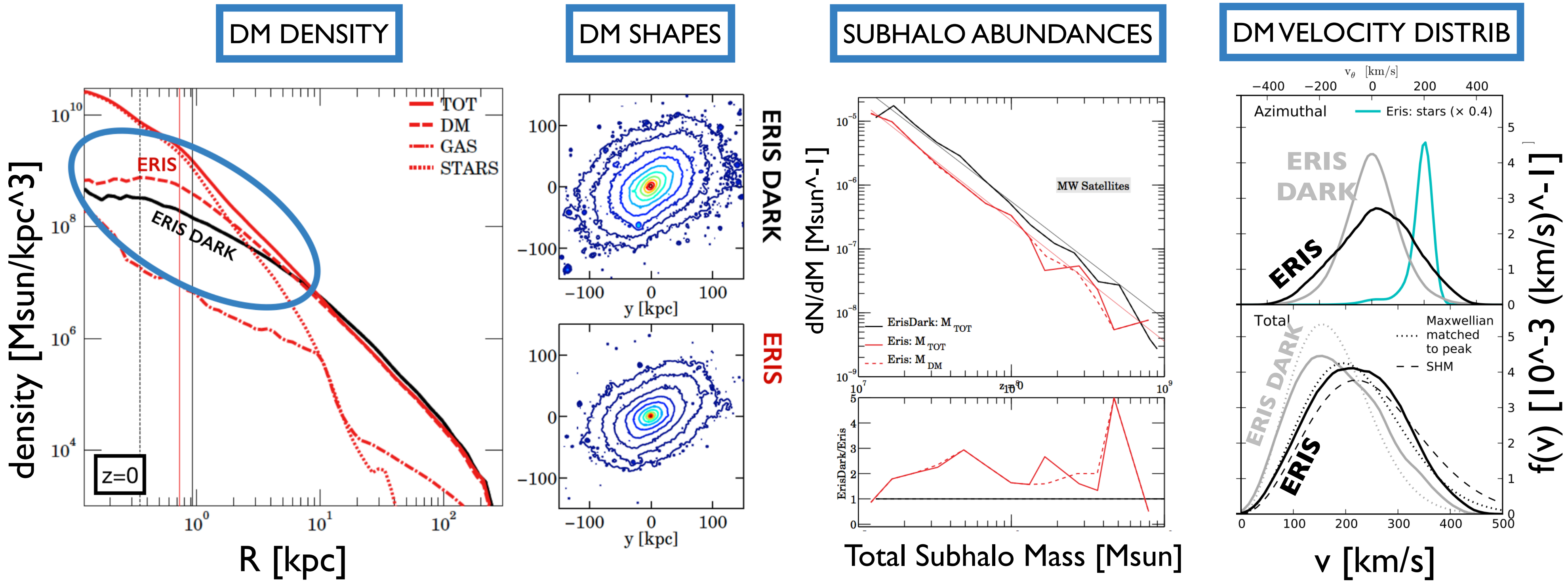
Baryonic physics alter DM expectations from N-body only calculations

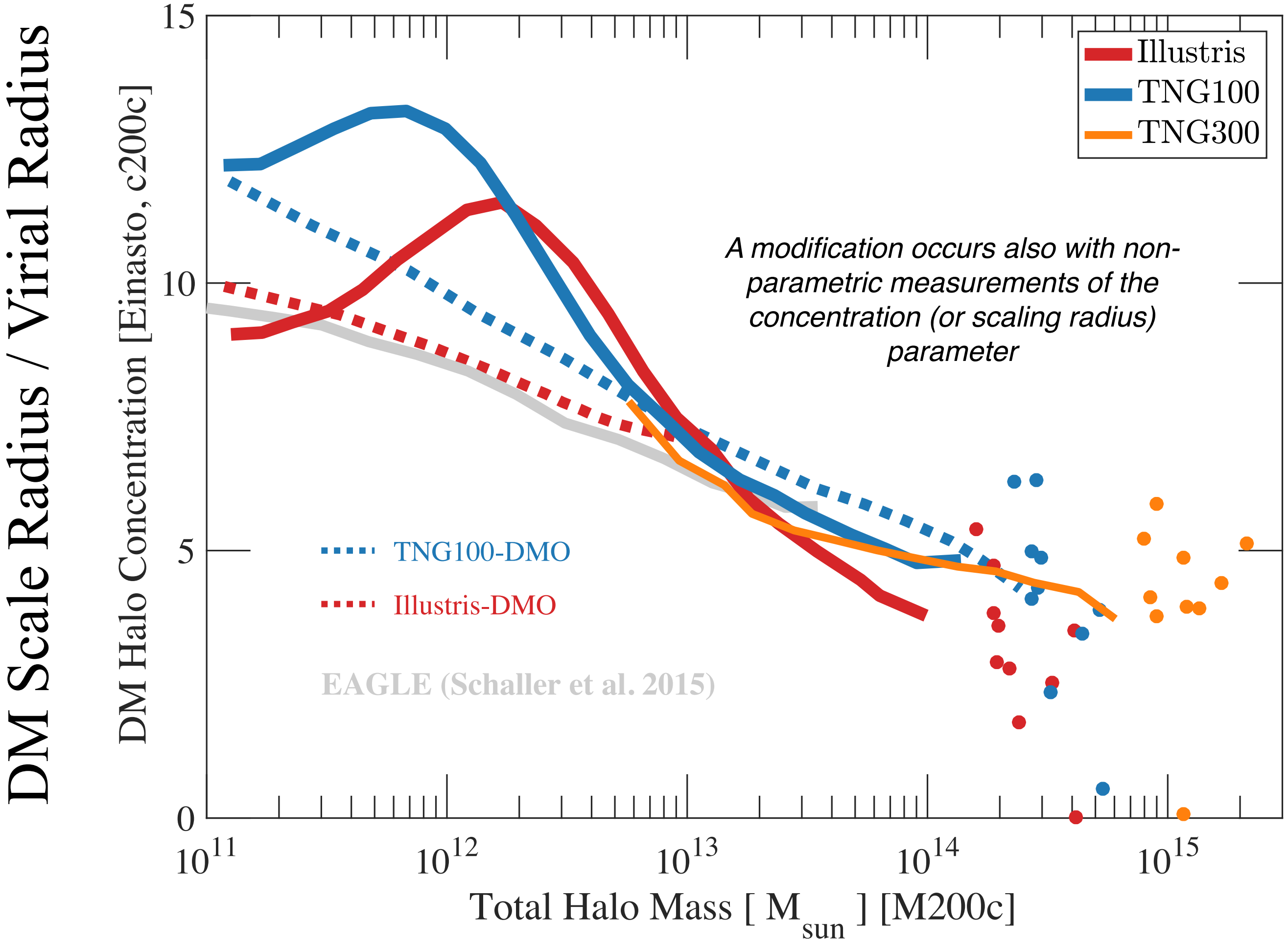
For example, in a MW-like simulated galaxy (Eris):



Pillepich [private communication]

Pillepich, Kuhlen, et al. 2014

Baryonic physics affects the distribution of dark matter within haloes



We have known since many years that more massive haloes are less concentrated than low mass ones

In both Illustris and TNG, the DM concentration-mass relation gets modified by baryonic physics, in a not-monotonic way wrt DMO predictions.

Yet, the effects of galaxy physics can be qualitatively and quantitatively different from model to model

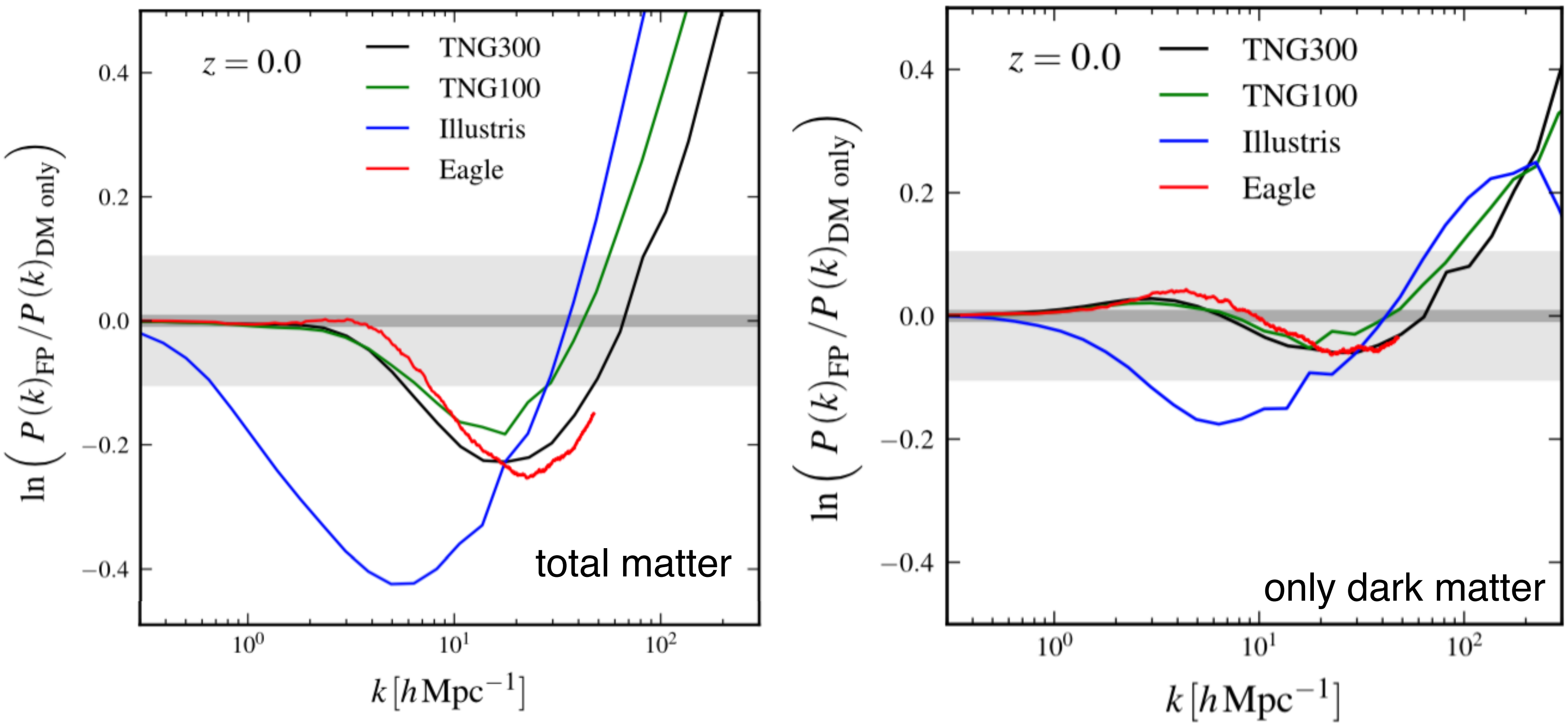
[Pillepich: preliminary]

Lovell, Pillepich et al. in 2018, Chua, Pillepich et al. 2017

Baryonic physics within galaxies affects the clustering of matter …

also the dark one! And also on scales >> haloes

Ratio between Full Physics and DM only



Springel, Pakmor, Pillepich et al. 2018

…by much more than the 1% accuracy needed to constrain dark energy.
But, different predictions from different models.


500 kpc
z = 0.55
TNG50