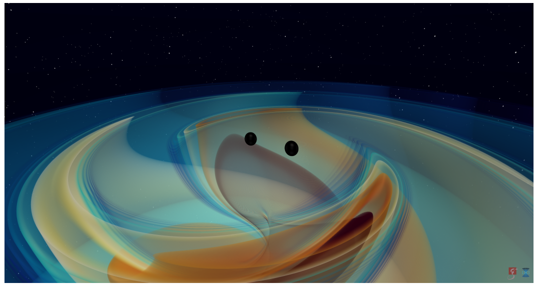
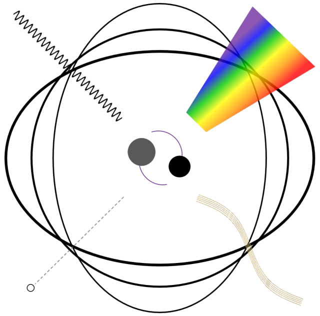
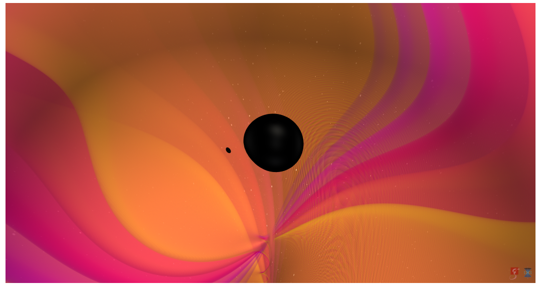
GW190521





GW190814



The Making of High-Precision Gravitational Waves to Explore the Dark Universe

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"XXVI IFT Christmas Workshop", Instituto de Física Teórica UAM/CSIC, Madrid Dec 16, 2020

Outline

- Observing gravitational waves and inferring astrophysical/physical information hinges on our ability to make precise predictions of two-body dynamics and gravitational radiation.

- How do we build the hundred-thousand accurate and efficient waveform models employed in LIGO/Virgo searches and inference studies?

- Success of interplay between analytical and numerical relativity.

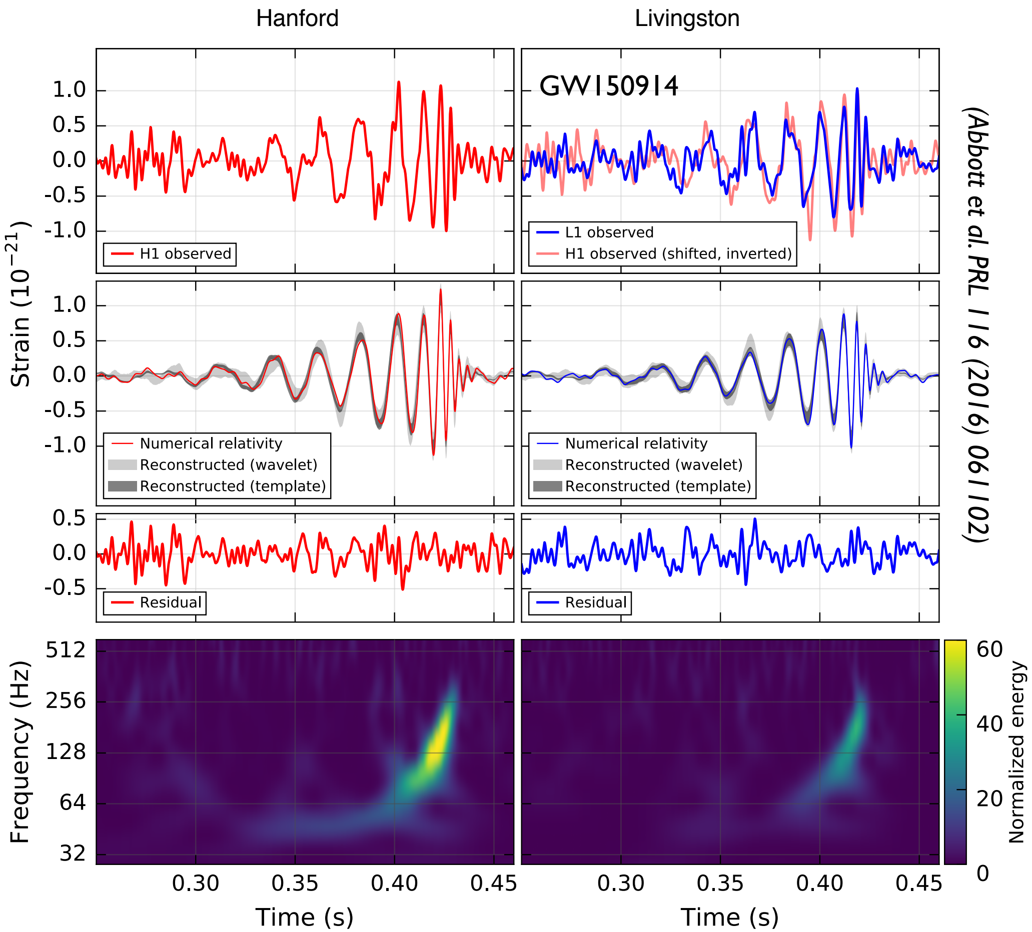
- State-of-the-art waveform models for binary black holes.

- Highlights on science (astrophysical-source properties, tests of General Relativity) from the latest observing run of LIGO and Virgo.

- What are the highest modeling priorities and accuracy requirements toward the era of high-precision GW astrophysics?

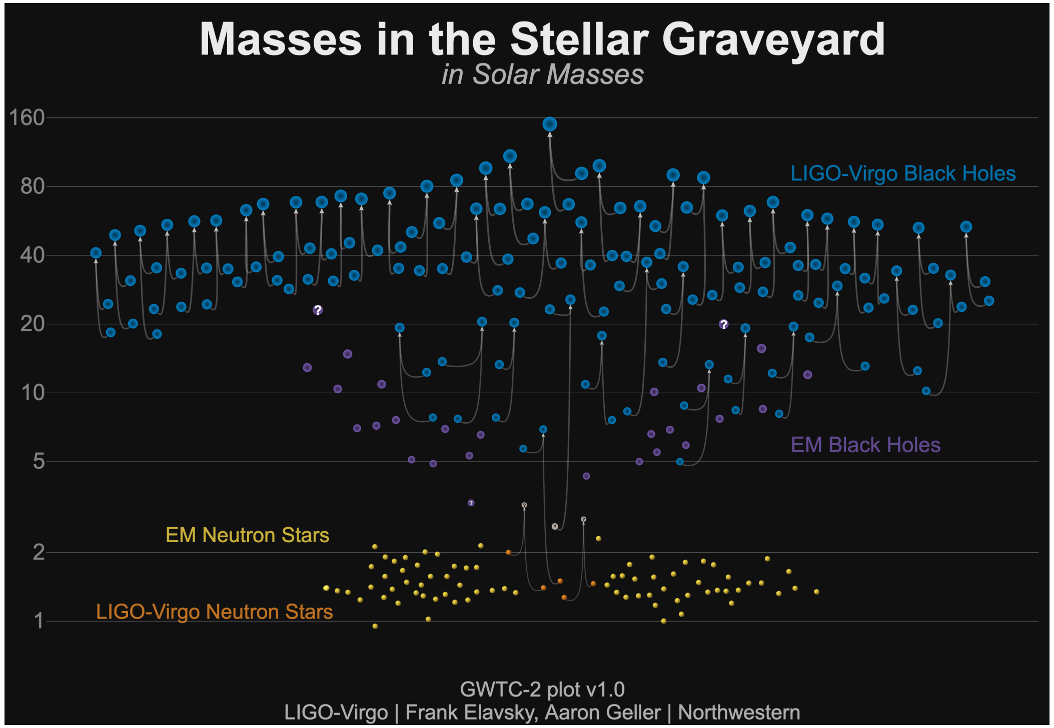
Gravitational Waves Ushered in New Era of Astrophysics

- **Discovery** of **GW** from **a binary black-hole merger** by LIGO

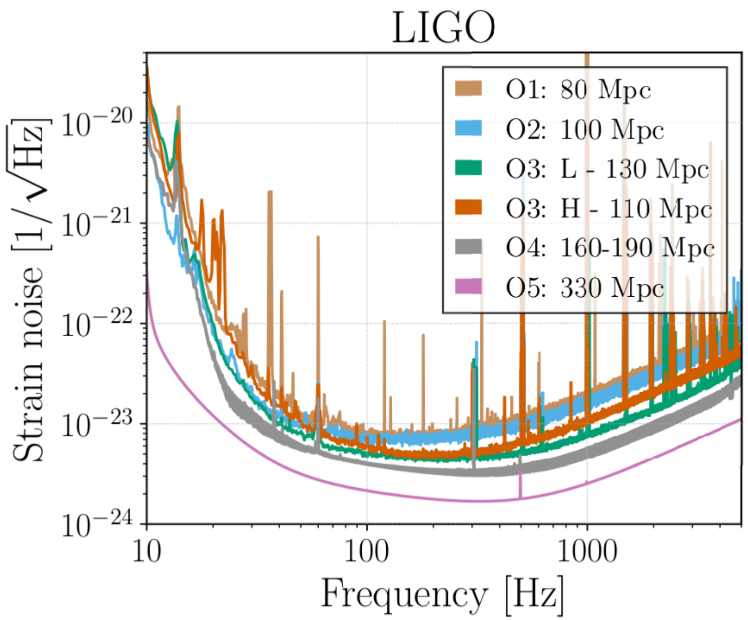


(Abbott et al. PRL 116 (2016) 061102)

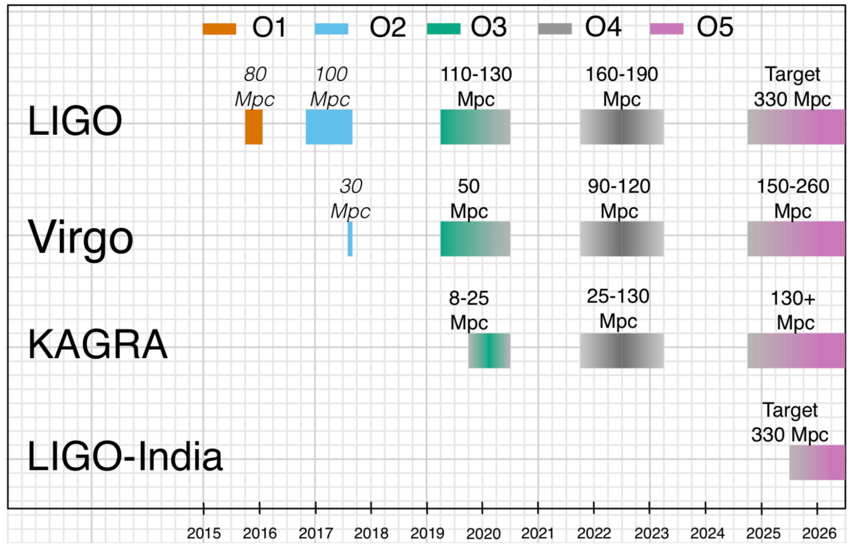
- **Since GW150914** was observed, **48 more black hole binaries (BHB)** and **two binary neutron stars (BNS) discovered** by LIGO/Virgo.



Gravitational-Wave Landscape until ~2030



(Aasi et al. Living Rev. Rel. 21, 2020)



- From **several tens to hundreds** of binary detections per year.

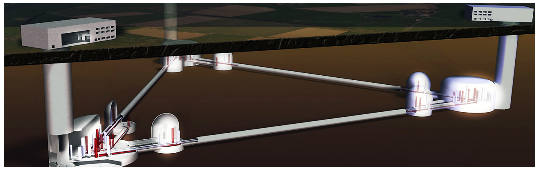
- Inference of **astrophysical properties** of BHBs, NSBHs and BNSs **in local Universe.**

Observation run	Network	Expected BNS detections	Expected NSBH detections	Expected BBH detections
O3	HLV	1^{+12}_{-1}	0^{+19}_{-0}	17^{+22}_{-11}
O4	HLVK	10^{+52}_{-10}	1^{+91}_{-1}	79^{+89}_{-44}

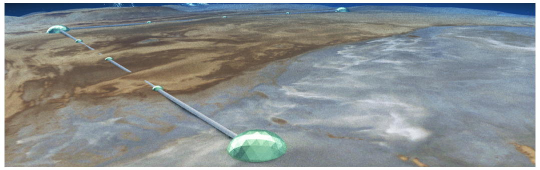
(Aasi et al. Living Rev. Rel. 21, 2020)

Gravitational-Wave Landscape after 2030 on the Ground and in Space

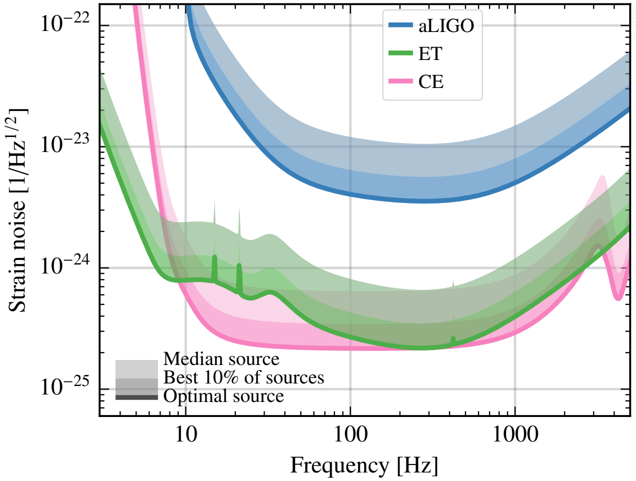
Einstein Telescope

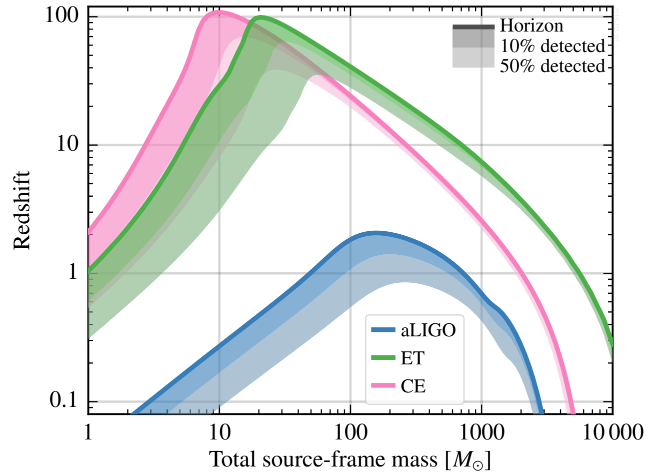


Cosmic Explorer



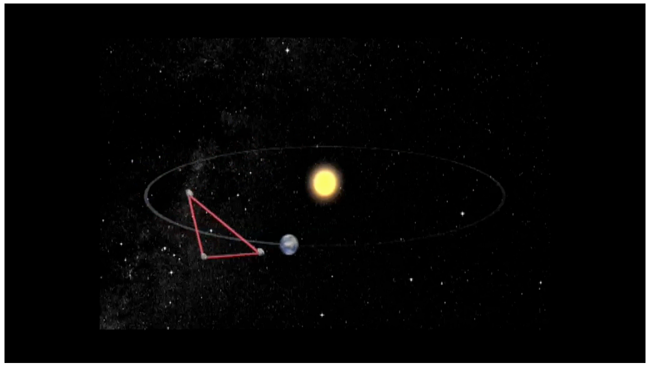
(3G Science-Case Report, in prep 20)



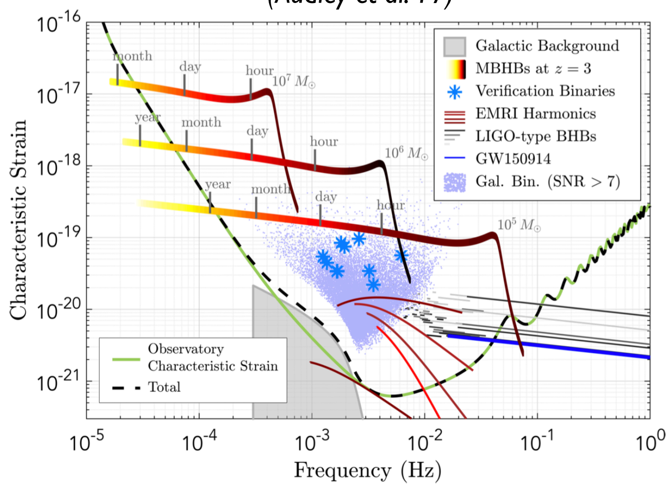


Laser Interferometer Space Antenna (LISA)

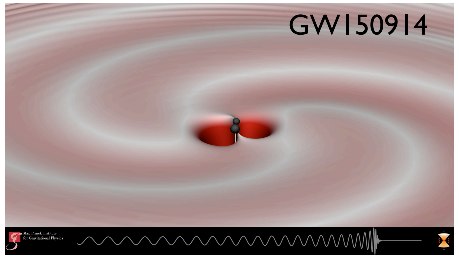
Credit: AEI/Milde Marketing



(Audley et al. 17)



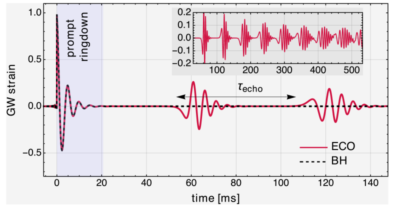
Outstanding Questions in Physics and Astrophysics



- What are the **properties** of **dynamical spacetime** (gravitational waves)?

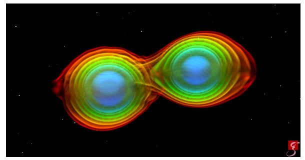
- Is **General Relativity still valid** in the highly dynamical, strong-field regime?

- Are **Nature's black holes** the black holes predicted in the **General** theory of **Relativity**?

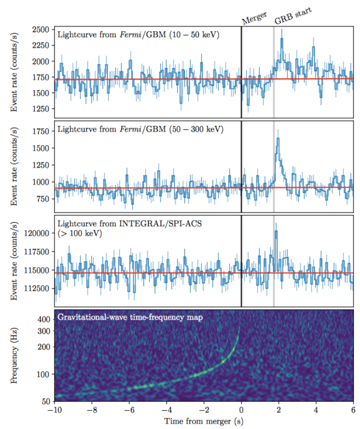


- How **black holes** and **neutron stars form**, which is their **astrophysical environment**, and how do they **form binaries**?

- How **matter behaves** under **extreme density and pressure**? Can **dark matter** make **compact objects**?

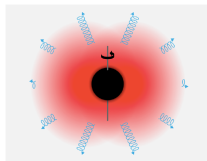


GW170817



- What's the **origin** of the **most energetic phenomena** in our Universe?

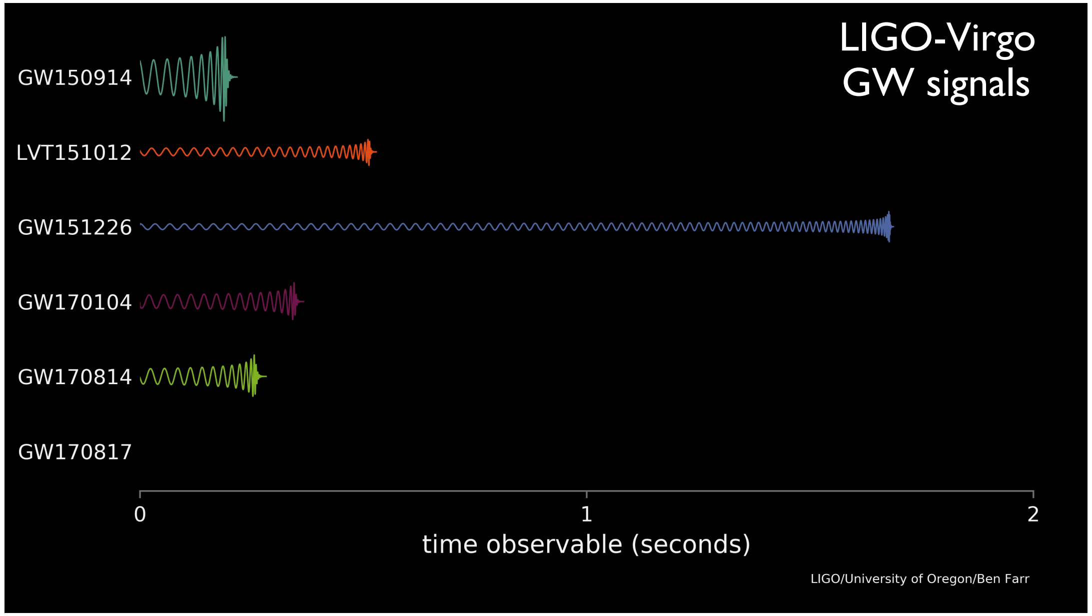
- Can we **discover new fundamental particles** (axions, ultra-light bosons)?



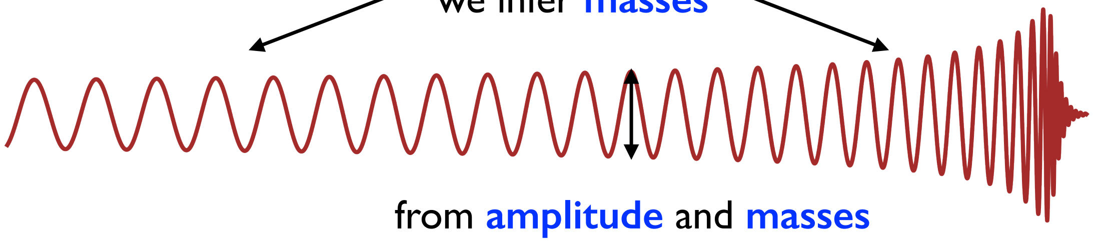
BH with boson cloud

- Can we **infer** the **cosmological model of our Universe** through gravitational-wave observations?

Gravitational Waves are Fingerprints of Sources

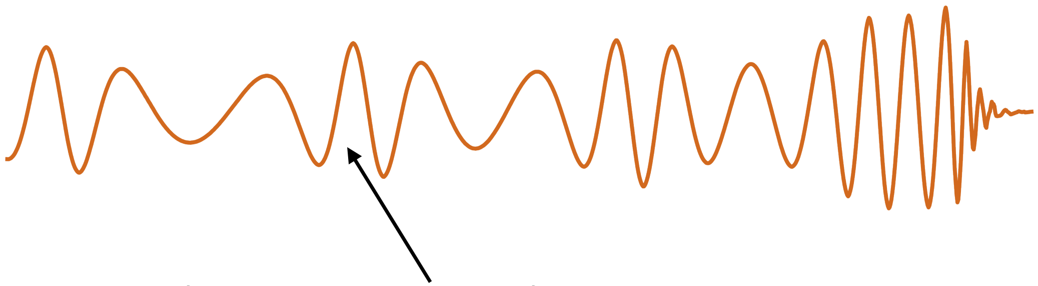


from **frequency evolution** we infer **masses**



from **amplitude** and **masses** we infer **distance**

from **time of arrival**, **amplitude and phase** at detectors we infer **sky location**



from **modulations** of **amplitude and phase** we infer **spins and eccentricity**

- At fixed binary's mass, the **higher** the **frequency,** the **smaller** the **binary's separation**, and the **later** the **inspiral stage**

$$\omega = \sqrt{\frac{GM}{r^3}} \qquad f_{\rm GW} = \frac{\omega}{\pi}$$

orbital frequency (ω)

orbital separation (r)

- Binary black holes **merge** at $f_{\rm GW} \sim \frac{4400\,\mathrm{Hz}}{M/M_{\rm Sun}}$

Solving Two-Body Problem in General Relativity

•**GR** is **non-linear theory**.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

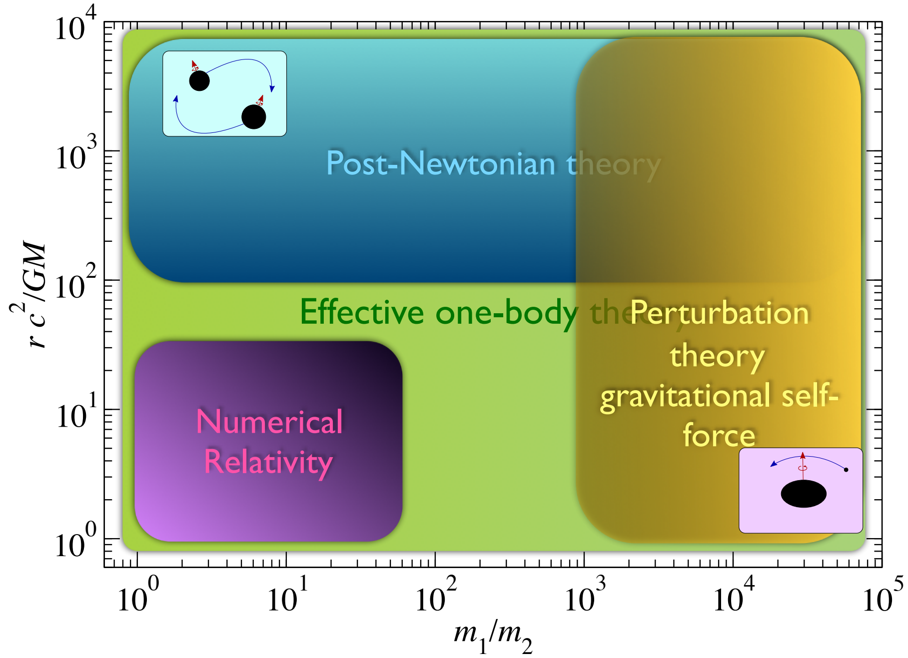
•Einstein's field equations can be solved:

-**approximately,** but **analytically** (**fast** way)

-**"exactly"**, but **numerically** on supercomputers (**slow** way)

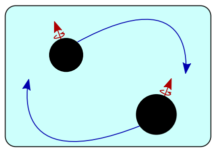
•**Synergy** between **analytical** and **numerical relativity** is **crucial** to **provide GW detectors with templates** to use for **searches** and **inference analyses**.

bound orbits: $v^2/c^2 \sim GM/rc^2$



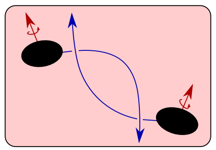
•**Post-Newtonian** (PN) (large separation, and slow motion, **bound motion**, i.e., **early inspiral**)

expansion in $v^2/c^2 \sim GM/rc^2$



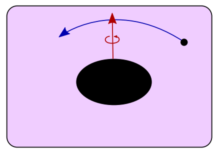
•**Post-Minkowskian** (PM) (large separation, **unbound motion**, i.e., **scattering**)

expansion in G



•**Small mass-ratio** (gravitational self-force, GSF, i.e., **early to late inspiral**)

expansion in m_2/m_1



Highly Accurate Waveform Models for GW Observations

- **GR** is **non-linear theory**.

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

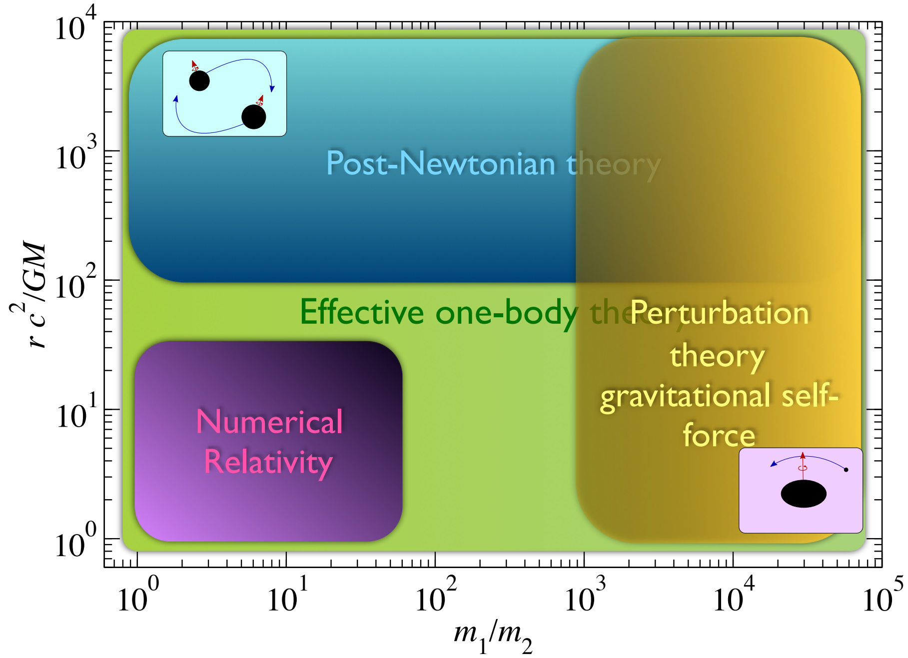
- Einstein's field equations can be solved:

-**approximately,** but **analytically** (**fast** way)

-**"exactly"**, but **numerically** on supercomputers (**slow** way)

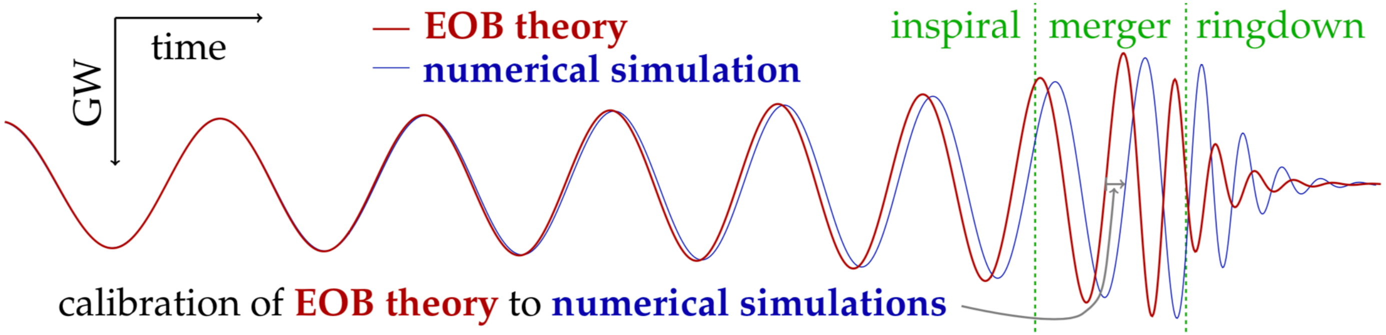
- **Synergy** between **analytical** and **numerical relativity** is **crucial** to **provide GW detectors with templates** to use for **searches** and **inference analyses**.

bound orbits: $v^2/c^2 \sim GM/rc^2$

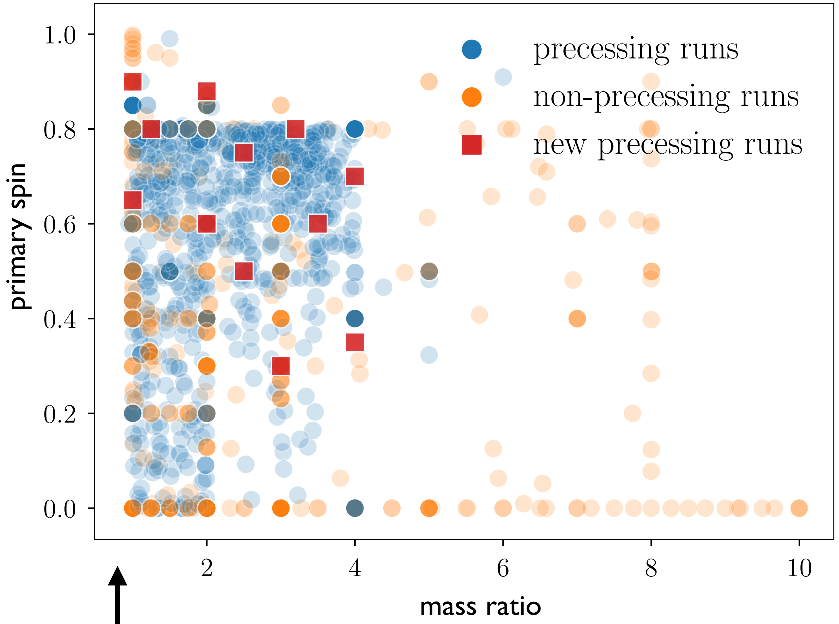


- **Effective-one-body** (EOB) (combines results from all methods, i.e., **entire coalescence**)

- **Key ideas** of EOB theory **inspired by quantum field theory.**



Numerical Relativity



- **Public Simulating eXtreme Spacetimes (SXS)** NR catalog plus **non-public SXS** NR waveforms.
  (Boyle et al. 19, Ossokine et al. 20)

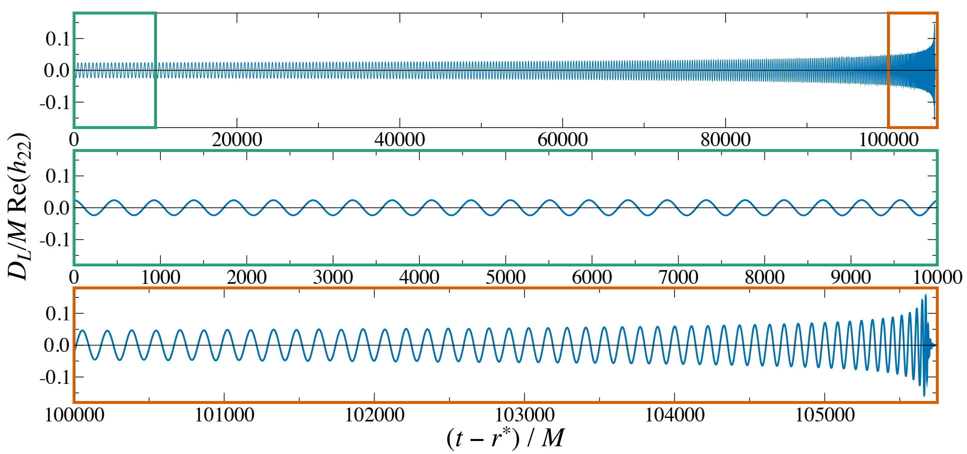
- **Other public NR catalogs.**
  (Husa et al. 15, Jani et al. 17, Healy et al. 17, 19, 20)

- Einstein's equations solved **numerically**

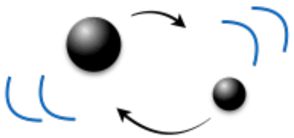
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- **376 GW cycles**, zero spins & mass-ratio 7 (8 months, few millions CPU-h)

(Szilagyi, Blackman, AB, Taracchini et al. 15)



The Effective-One-Body Approach in a Nutshell

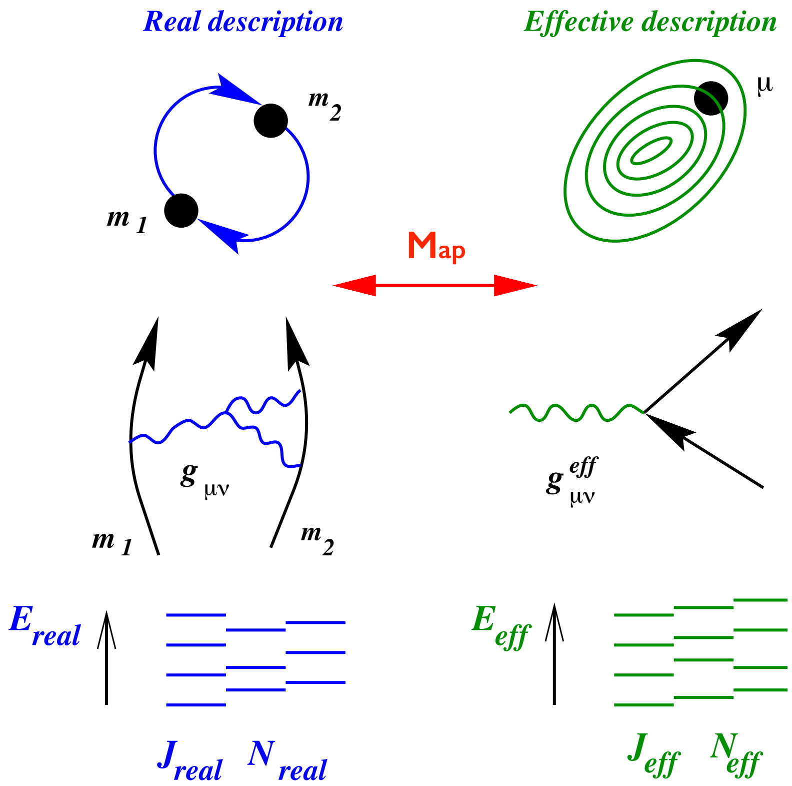


$$\nu = \frac{\mu}{M} \qquad 0 \leq \nu \leq 1/4$$

$$\mu = \frac{m_1 m_2}{M} \qquad M = m_1 + m_2$$

- **Two-body dynamics is mapped** into dynamics of **one-effective body** moving **in deformed black-hole spacetime**, deformation being the mass ratio.

- Some key **ideas** of EOB theory were **inspired by quantum** field **theory** when describing **energy** of **comparable-mass charged bodies**.



(AB & Damour 1999)

Energy for Comparable-Mass Black Holes

- Classical gravity *(AB & Damour 1999)*:

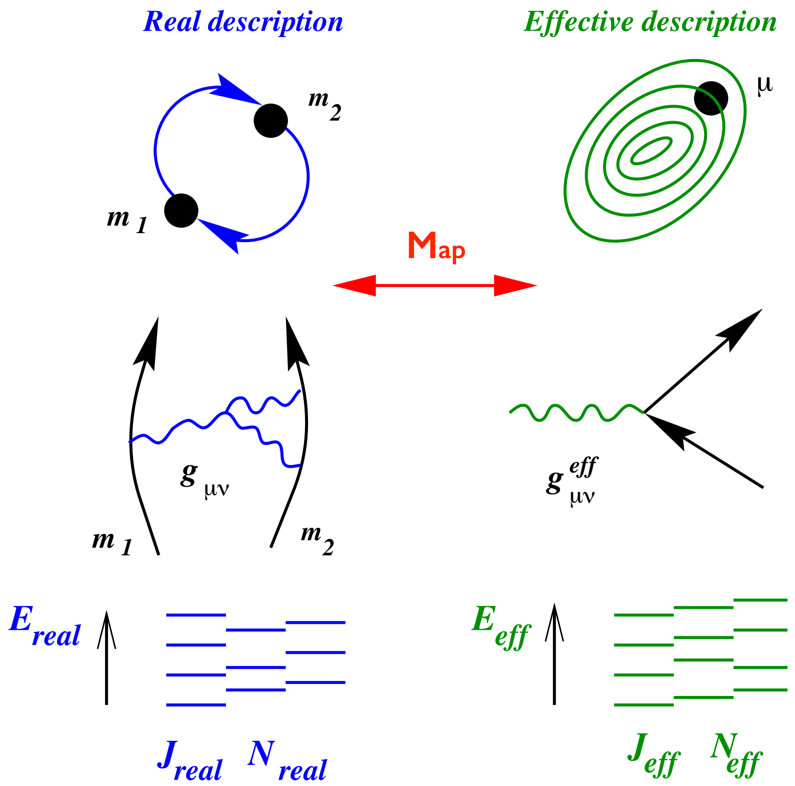
$$E_{\rm real}^2 = m_1^2 + m_2^2 + 2 m_1 m_2 \left(\frac{E_{\rm eff}}{\mu} \right)$$

- Quantum electrodynamics *(Brezin, Itzykson & Zinn-Justin 1970)*:

$$E_{\rm real}^2 = m_1^2 + m_2^2 + 2 m_1 m_2 \frac{1}{\sqrt{1 + Z^2\,\alpha^2/(n - \epsilon_j)^2}}$$

- Considering scattering states:

$$\varphi(s) \equiv \frac{s - m_1^2 - m_2^2}{2 m_1 m_2} = \frac{-(p_1 + p_2)^2 - m_1^2 - m_2^2}{2 m_1 m_2} = -\frac{p_1 \cdot p_2}{m_1 m_2}$$



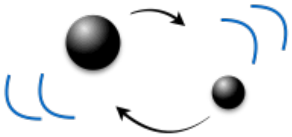
Most natural symmetric function of asymptotic momenta of two-particle system which reduces in test-mass limit $m_2 \ll m_1$ to energy of m_2 in rest-frame of m_1.

(AB & Damour 1999)

EOB Hamiltonian: Resummed Conservative Dynamics

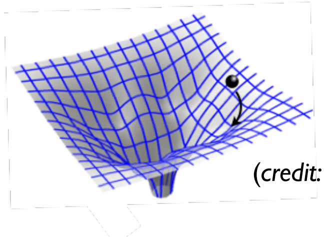
- **Real Hamiltonian**

$$H_{\rm real}^{\rm PN} = H_{\rm Newt} + H_{\rm 1PN} + H_{\rm 2PN} + \cdots$$



- **Effective Hamiltonian**

$$H_{\rm eff}^{\nu} = \mu \sqrt{A_\nu(r)\left[1 + \frac{\mathbf{p}^2}{\mu^2} + \left(\frac{1}{B_\nu(r)} - 1\right)\frac{p_r^2}{\mu^2}\right]}$$



$$ds_{\rm eff}^2 = -A_\nu(r)dt^2 + B_\nu(r)dr^2 + r^2 d\Omega^2$$

- **EOB Hamiltonian**

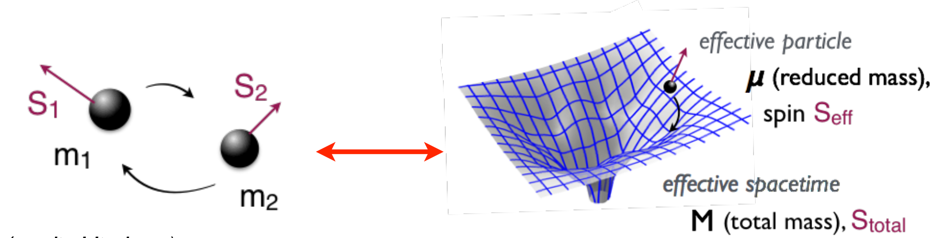
$$H_{\rm real}^{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}{}^{\nu}}{\mu} - 1\right)}$$

- Dynamics condensed $A_\nu(r)$ and $B_\nu(r)$
- $A_\nu(r)$, which encodes the energetics of circular orbits, is quite simple:

$$A_\nu(r) = 1 - \frac{2M}{r} + \frac{2M^3\nu}{r^3} + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\frac{M^4\nu}{r^4} + \frac{a_5(\nu) + a_5^{\log}(\nu)\log(r)}{r^5} + \frac{a_6(\nu)}{r^6} + \cdots$$

5PN unknown as today

EOB Conservative Spin Dynamics & Waveforms



(credit: Hinderer)

$$H_{\rm real}^{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}{}^{\nu}}{\mu} - 1\right)}$$

- **EOB equations of motion** *(AB et al. 00, 05; Damour et al. 09)*:

$$\dot{\mathbf{r}} = \frac{\partial H_{\rm real}^{\rm EOB}}{\partial \mathbf{p}} \qquad F \propto \frac{dE}{dt}, \quad \frac{dE}{dt} \propto \sum_{\ell m} |h_{\ell m}|^2$$

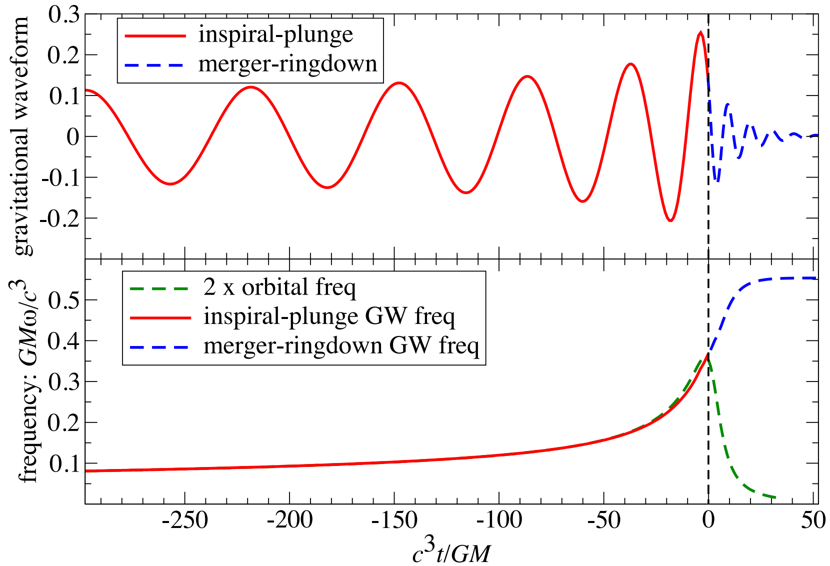
$$\dot{\mathbf{p}} = -\frac{\partial H_{\rm real}^{\rm EOB}}{\partial \mathbf{r}} + \mathbf{F} \qquad \dot{\mathbf{S}} = \{\mathbf{S}, H_{\rm real}^{\rm EOB}\}$$

- **EOB inspiral waveforms** *(AB et al. 00; Damour et al. 09, 11; Pan, AB et al. 11)*:

$$h_{\ell m}^{\rm insp-plunge} = h_{\ell m}^{\rm Newt}\, e^{-im\Phi}\, S_{\rm eff}\, T_{\ell m}\, e^{i\delta_{\ell m}}\, (\rho_{\ell m})^{\ell}\, h_{\ell m}^{\rm NQC}$$

- **EOB merger-ringdown** waveform is a **superposition of quasi-normal modes**.

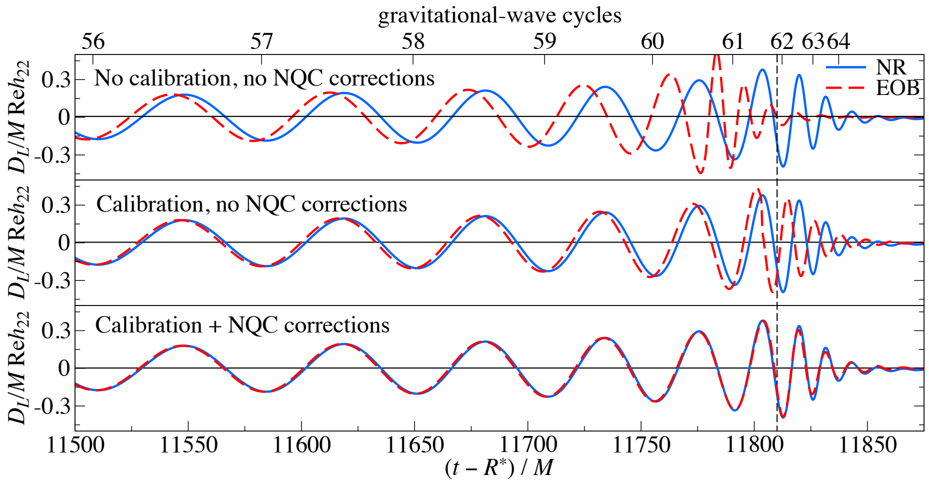
(AB & Damour 00, AB et al. 07, Damour & Nagar 07, Del Pozzo & Nagar 17, Bohé et al. 17)



(AB & Damour 00)

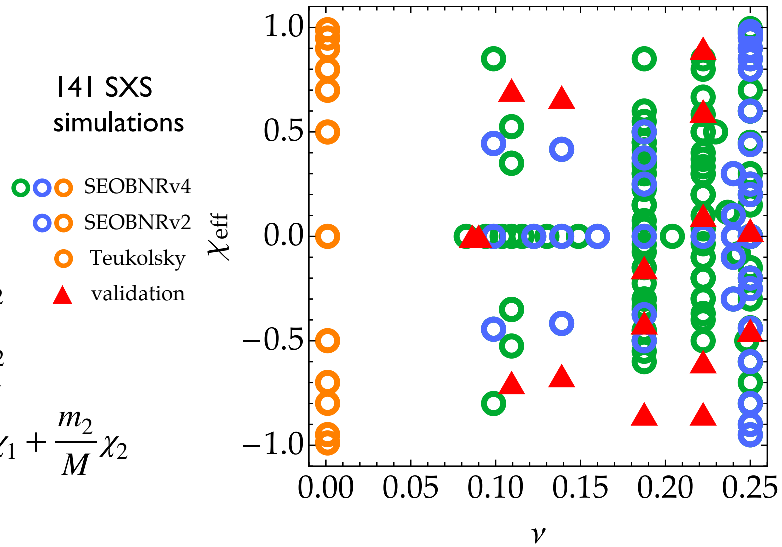
Completing EOB Waveforms with NR & Perturbation Theory Information

- We calibrate EOB to **inspiral-merger-ringdown NR** waveforms.



(credit: Taracchini)

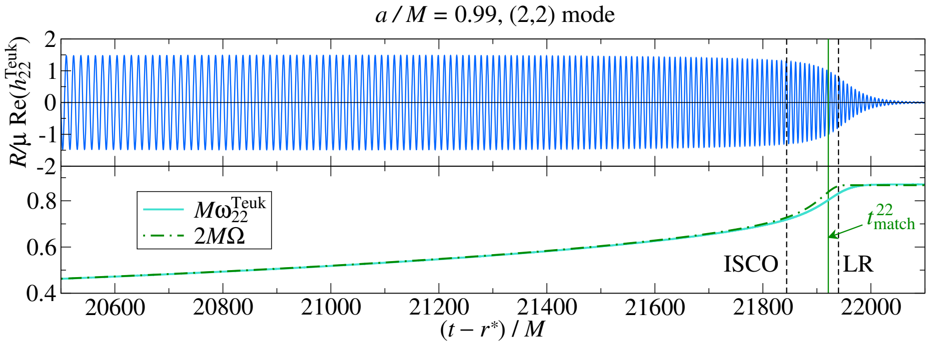
Calibration of SEOBNR for O2-O3 searches and inference studies



$$\chi_1 = S_1/m_1^2$$
$$\chi_2 = S_2/m_2^2$$
$$\chi_{\rm eff} = \frac{m_1}{M}\chi_1 + \frac{m_2}{M}\chi_2$$

(Bohé et al. 17)

- We calibrate EOB to **merger-ringdown waveforms** in **test-body limit**.



(credit: Taracchini)

(Pan, AB et al. 13, Taracchini, AB, Pan, Hinderer & SXS 14, Pürrer 15)

(Bohé, Shao, Taracchini, AB & SXS 17, Babak et al. 16; Cotesta et al. 18, 20, Ossokine et al. 20)

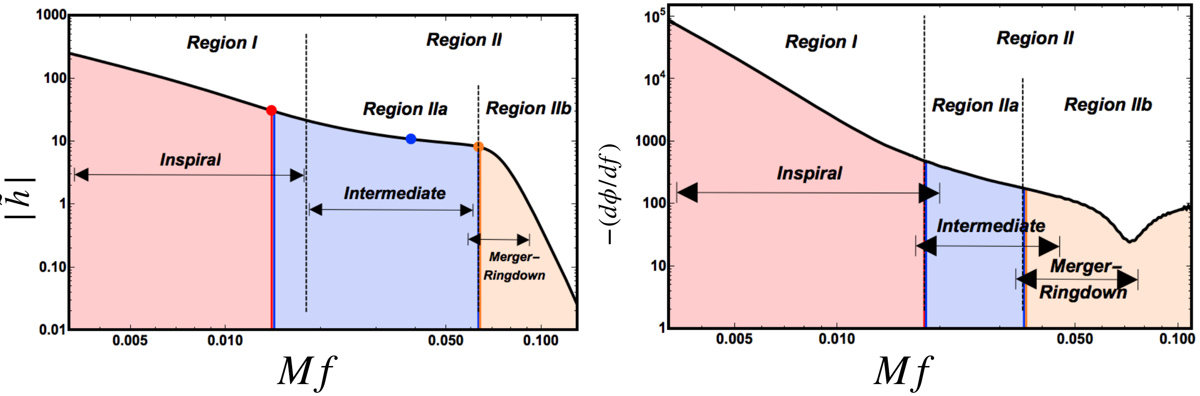
(see also Damour & Nagar 14, Nagar et al. 18, Nagar, Messina et al. 19, Nagar, Pratten et al. 20, Nagar, Riemenschneider et al. 20)

Phenomenological & NR-Surrogate Waveforms

- **Fast, frequency-domain** waveform model hybridizing EOB & NR waveforms, and then fitting.

(Schmidt et al.12; Hannam et al. 13; Khan et al. 15; Husa et al. 15; Khan et al. 18-19; García-Quíros et al. 20, Pratten et al. 20)

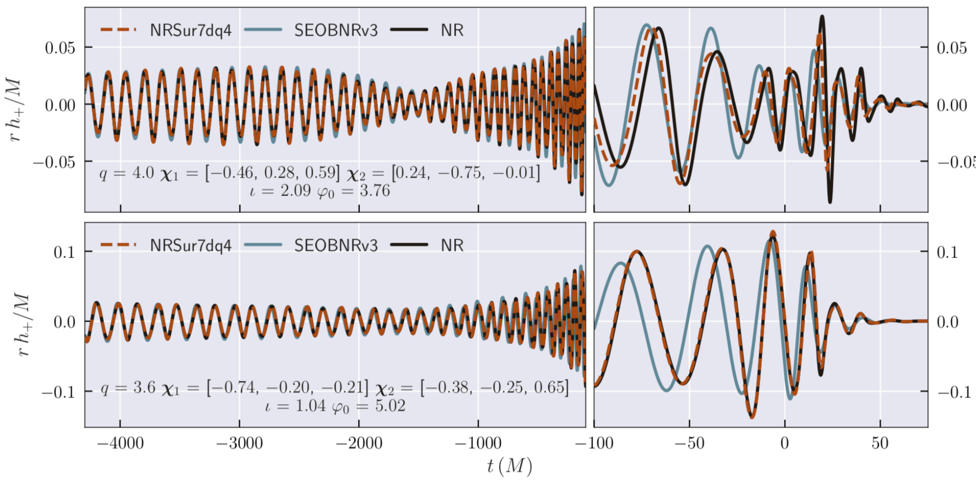
IMRPhenom $$\tilde{h}(f;\lambda_i) = \mathcal{A}(f;\lambda_i)\, e^{i\phi(f;\lambda_i)}$$



(Khan et al. 16)

- **NR surrogate models** are built **directly** by interpolating **NR simulations,** which are **selected in parameter space** using analytical waveform models.
- **Highly accurate**, but **limited in** binary's **parameter space** and **length** (~20 orbits), unless hybridized with EOBNR waveforms.

NRSur

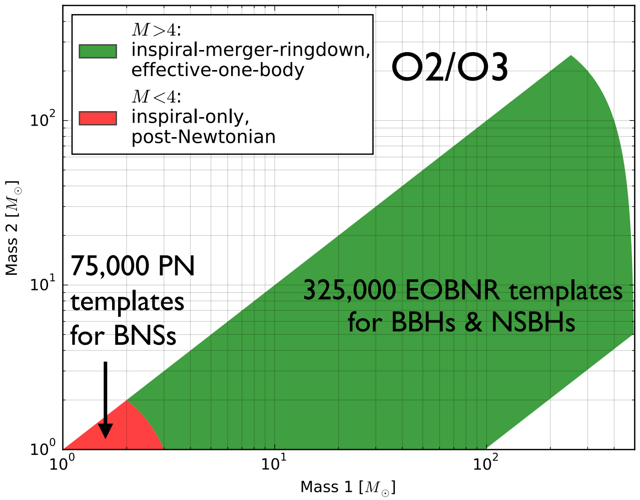


(Varma et al. 19)

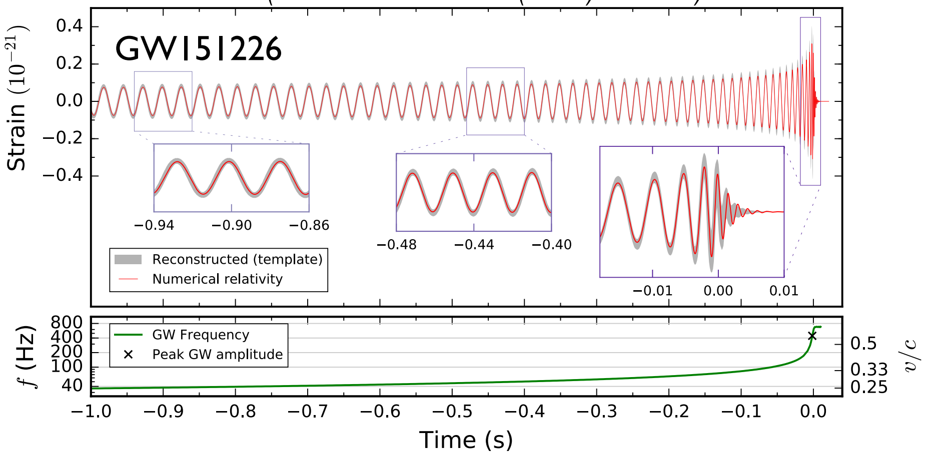
Template Bank for Modeled Searches & Possible Systematics in O1& O2

- **Matched filtering** employed

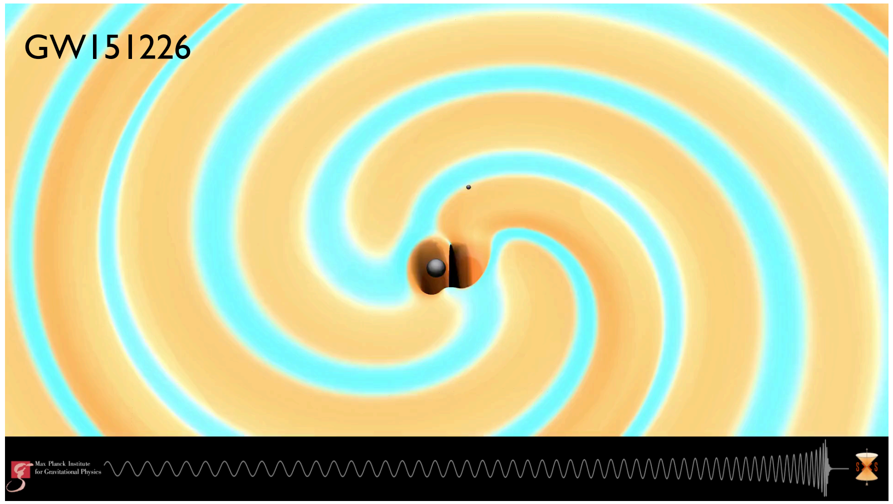
(Dal Canton & Harry 17)



(Abbott et al. PRL 116 (2016) 241103)



(Ossokine, AB & SXS project)



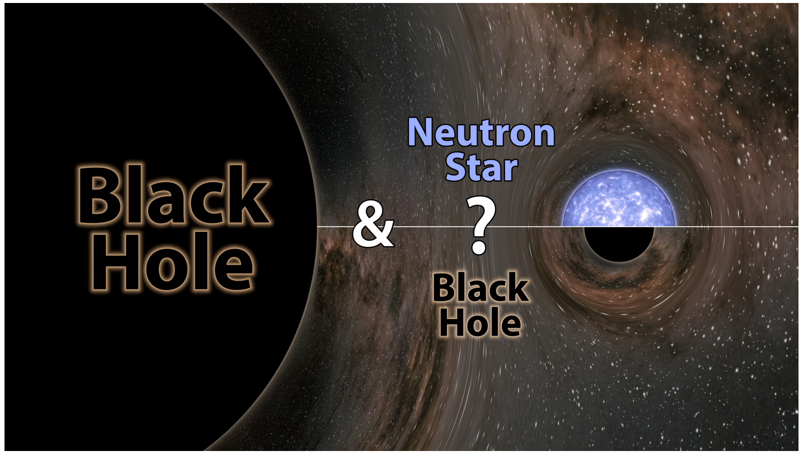
- **Systematics** due to modeling **were smaller than statistical** errors for GW events observed in **O1 & O2 runs**.

(Abbott et al. CQG 34 (2017) 104002, Abbott et al. PRX 9 (2019) 031040)

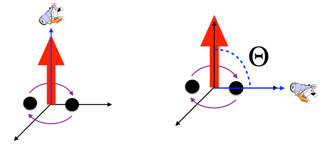
Highlights from O3a Run as we Explore the Universe

GW190814: a binary with a puzzling companion

- A black hole 23 times the mass of our Sun merging with an *object* just 2.6 times the mass of the Sun.

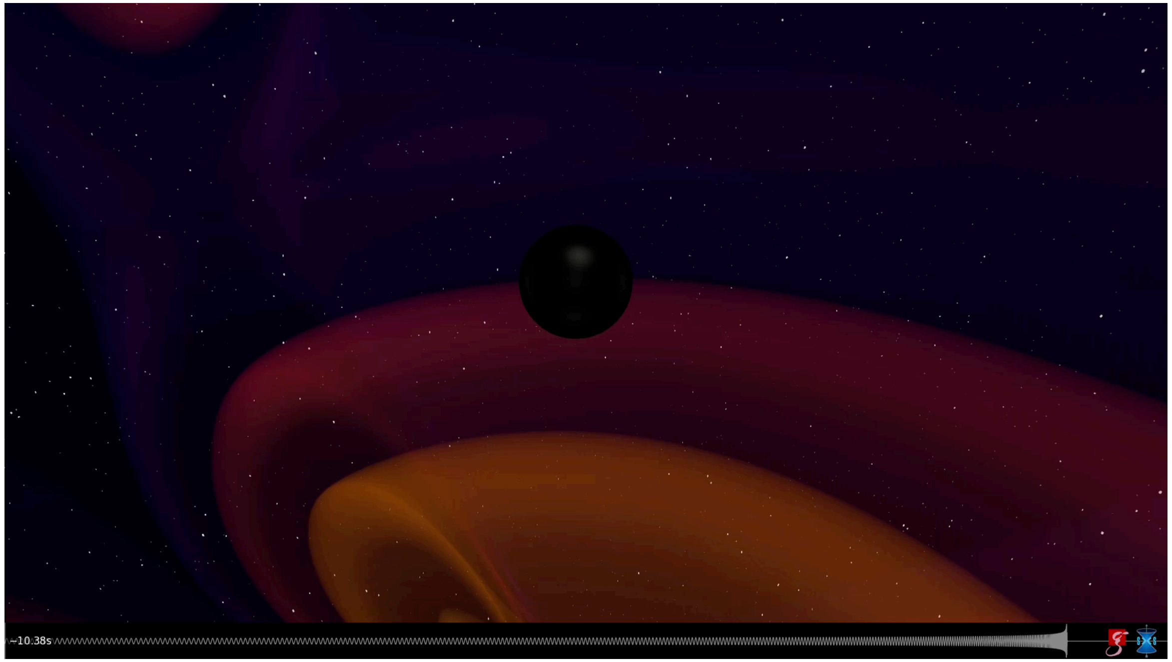


$$h_+(t;\Theta,\varphi) - i\,h_\times(t;\Theta,\varphi) = \sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell} {}_{-2}Y_{\ell m}(\Theta,\varphi)\,h_{\ell m}(t)$$



(credit: Cotesta)

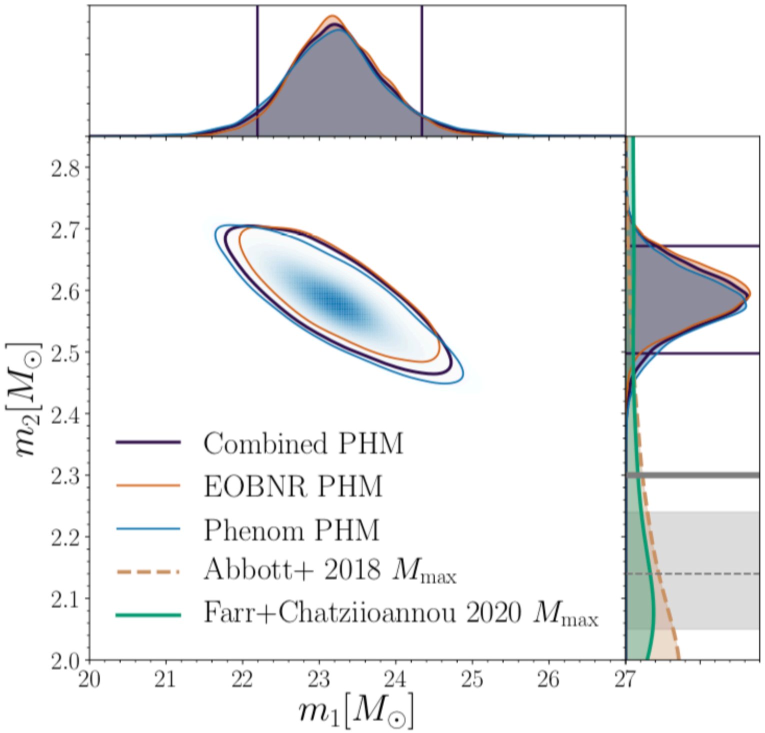
- The more substructure and complexity the binary has (e.g., masses or spins of BHs are different) the richer is the spectrum of radiation emitted.



(credit: Fischer, Pfeiffer, Ossokine & AB; SXS Collaboration)

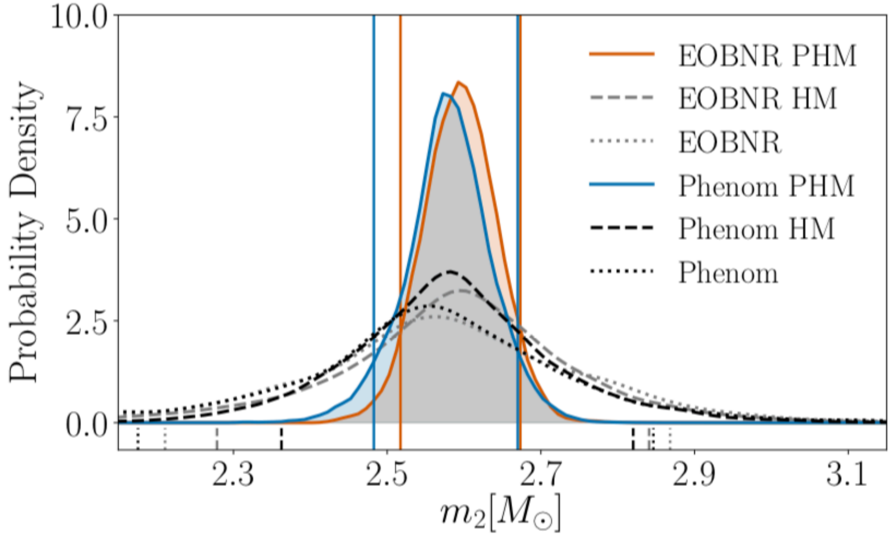
GW190814: a Binary with a Puzzling Companion

(Abbott et al. ApJ Lett 896 (2020) 2, L44)



- **More massive BH** rotated with **spin < 0.07.**
- **Systematics due to** waveform modeling **smaller than statistical** errors.

- Either the **largest neutron star** or the **smallest black hole.**



- Using waveform models with **higher-modes and spin-precession constrains more tightly** the **secondary mass.**

GW190412: a Signal Like None Before

GW190412: **a signal like none before**

(Abbott et al. PRD 102 (2020) 4)

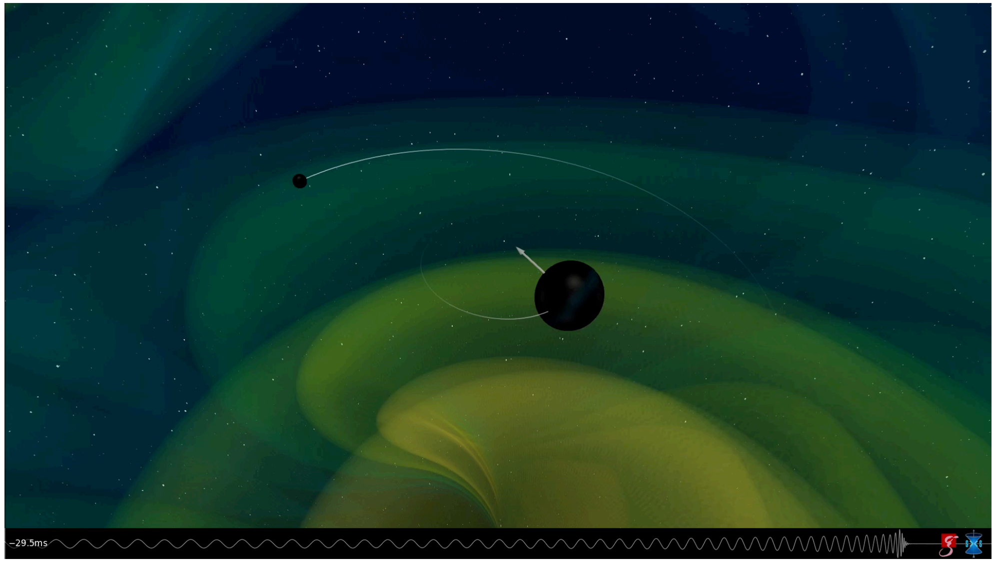
$q = m_1/m_2$

$\chi_1 = S_1/m_1^2$

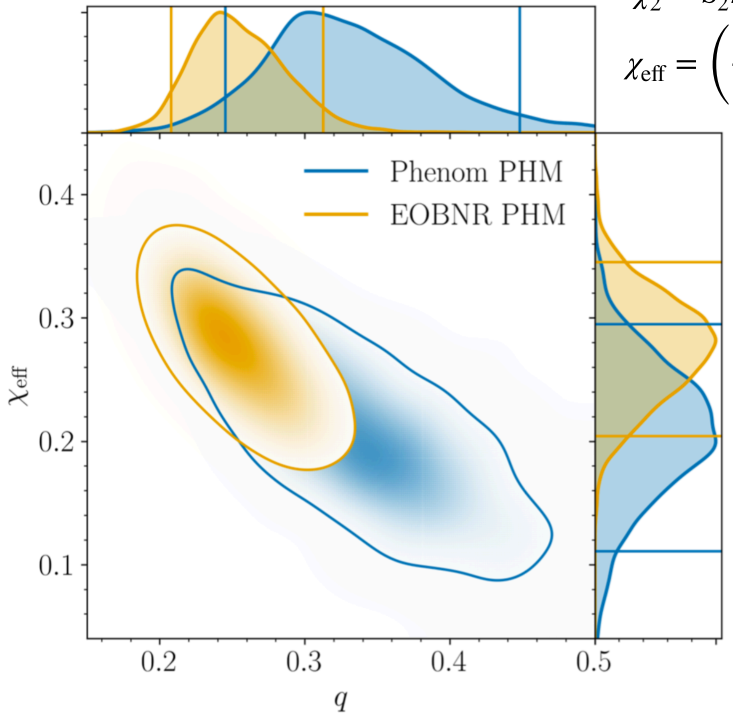
$\chi_2 = S_2/m_2^2$

$\chi_{\text{eff}} = \left(\frac{m_1}{M}\chi_1 + \frac{m_2}{M}\chi_2\right) \cdot \hat{\mathbf{L}}$

- Binary black hole with **mass asymmetry as large as 4**, and **BH spinning at about 40%** the possible maximum value allowed by General Relativity.



(credit: Fischer, Pfeiffer & AB; SXS Collaboration)



- **More massive BH** rotated with **spin** $0.17 - 0.59$ at 90 % CI

GW190412: a Signal Like None Before

GW190412: a signal like none before

(Abbott et al. PRD 102 (2020) 4)

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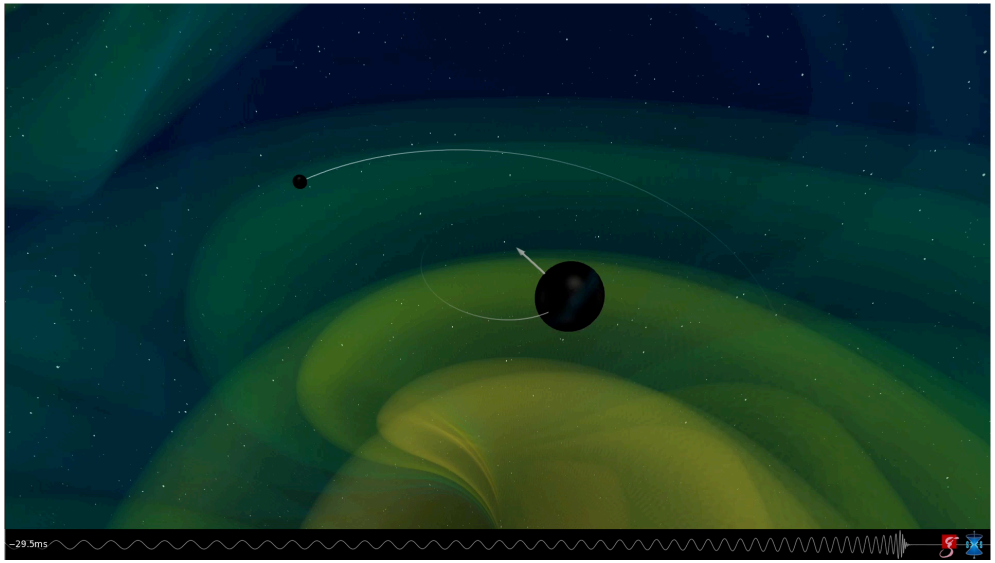
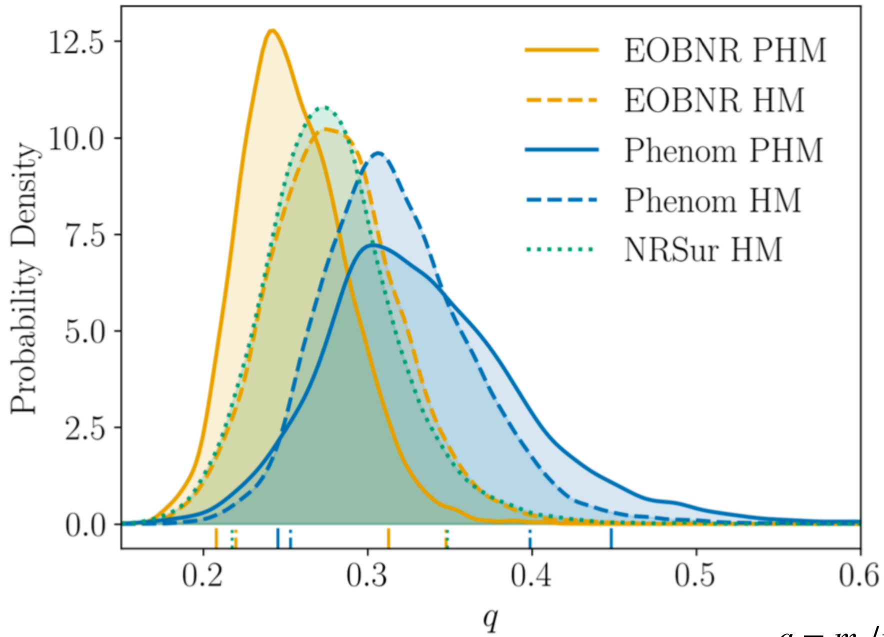


(credit: Fischer, Pfeiffer & AB; SXS Collaboration)



$q = m_1/m_2$

- Systematics due to waveform modeling are not negligible when spins and higher modes are relevant.

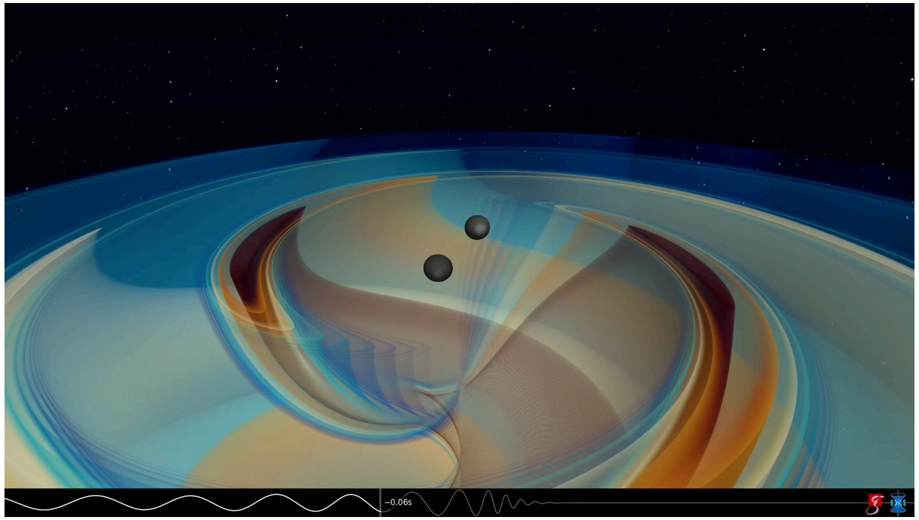
GW190521: a Signal Produced by the Largest BHs so far

GW190521: **a signal produced by the largest BHs so far**

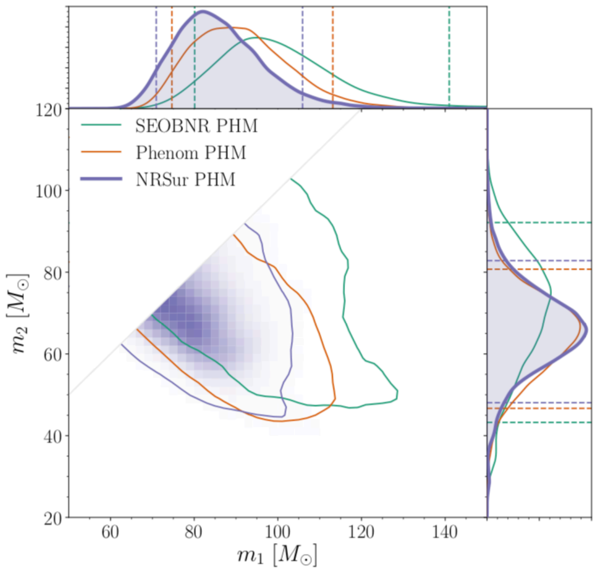
(Abbott et al. PRL 125 (2020) 10, ApJ Lett 900 (2020) L13)

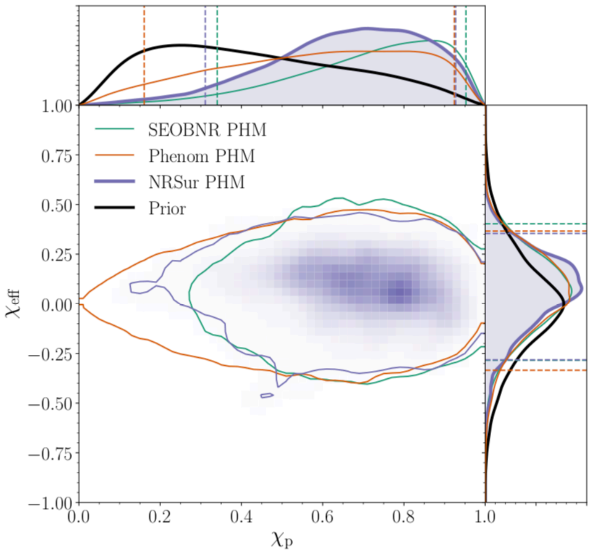
$$q = m_1/m_2$$
$$\chi_1 = S_1/m_1^2 \qquad \chi_{\rm eff} = \left(\frac{m_1}{M}\chi_1 + \frac{m_2}{M}\chi_2\right)\cdot \hat{\mathbf{L}}$$
$$\chi_2 = S_2/m_2^2$$

- A black hole **too massive** (85 times the mass of our Sun) to have been formed **from a collapsed star.**



(credit: Fischer, Pfeiffer & AB; SXS Collaboration)

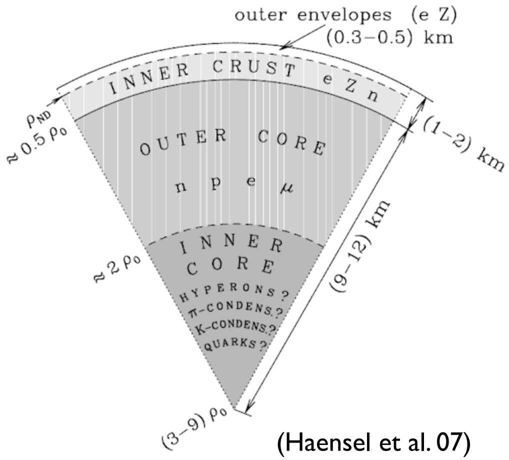




- **Systematics** due to waveform modeling **are not negligible when spin precession** and **higher modes are relevant.**

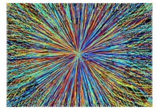
Probing Extreme-Matter with Gravitational Waves

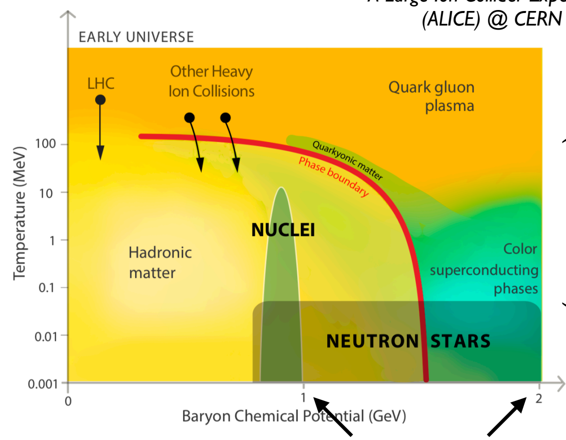
- **Neutron-star (NS)** properties:
  - mass: $1 - 3\, M_{\rm Sun}$
  - radius: $9 - 15\,{\rm km}$
  - inner core density > $2 \times (2.8 \times 10^{14})\,{\rm g/cm^3}$ (nuclear density)
  - magnetic field: $\sim 10^{15} \times$ @Earth
  - surface temperature: $\sim 10^{3} \times$ @Earth
  - pressure: $\sim 10^{27} \times$ @Earth



(Haensel et al. 07)

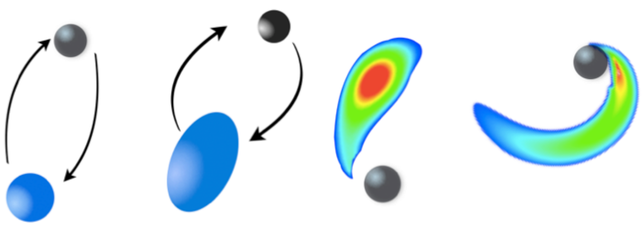
- **Conjectured** states of **matter.**


A Large Ion Collider Experiment (ALICE) @ CERN

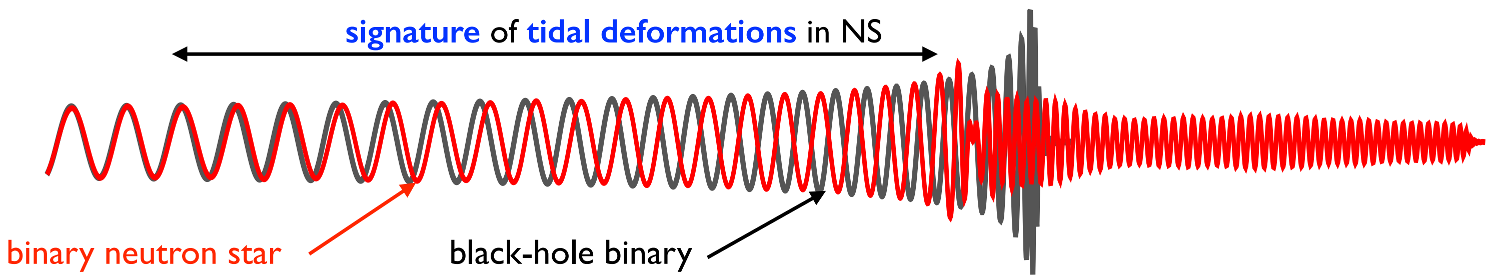


(Watts et al. 16)

- What is the **internal structure** and **composition** of **neutron stars**?


(credit: Hinderer)

- **NS equation of state** (EOS) affects gravitational **waveform** during **late inspiral, merger** and **post-merger.**



Probing Extreme-Matter with Gravitational Waves (contd.)

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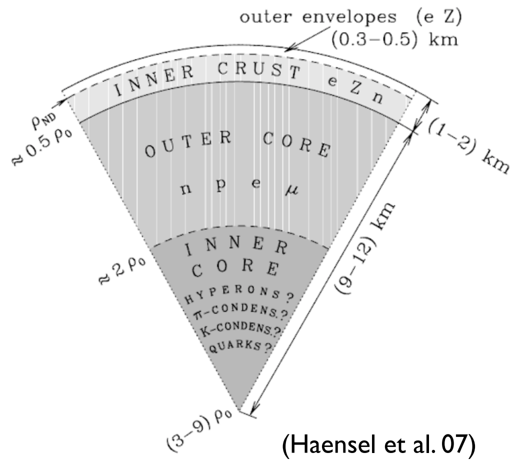
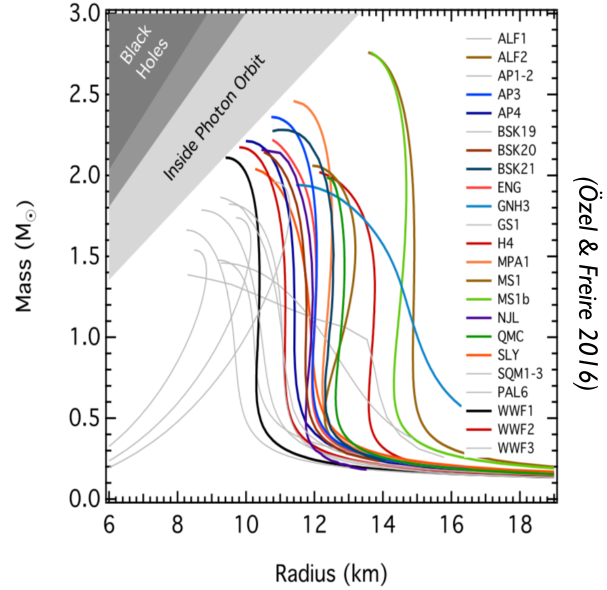


(Haensel et al. 07)



(Özel & Freire 2016)

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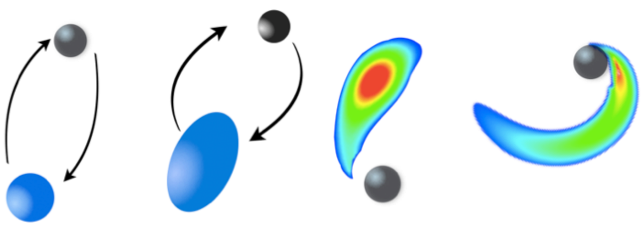

(credit: Hinderer)

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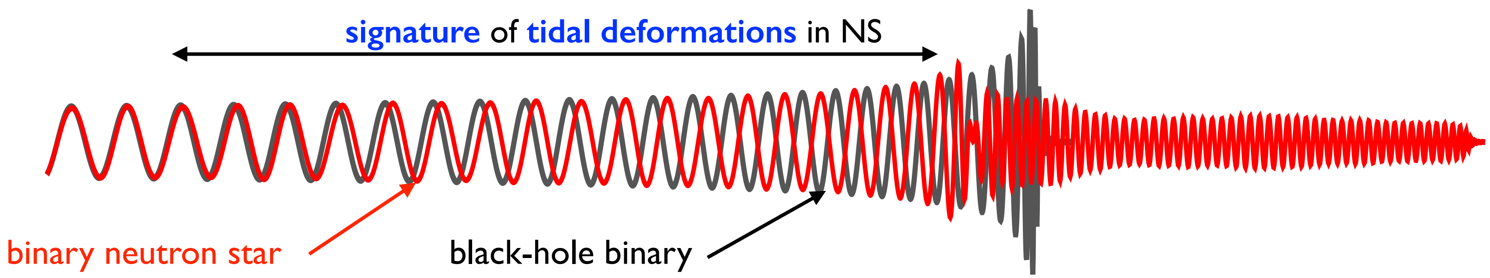
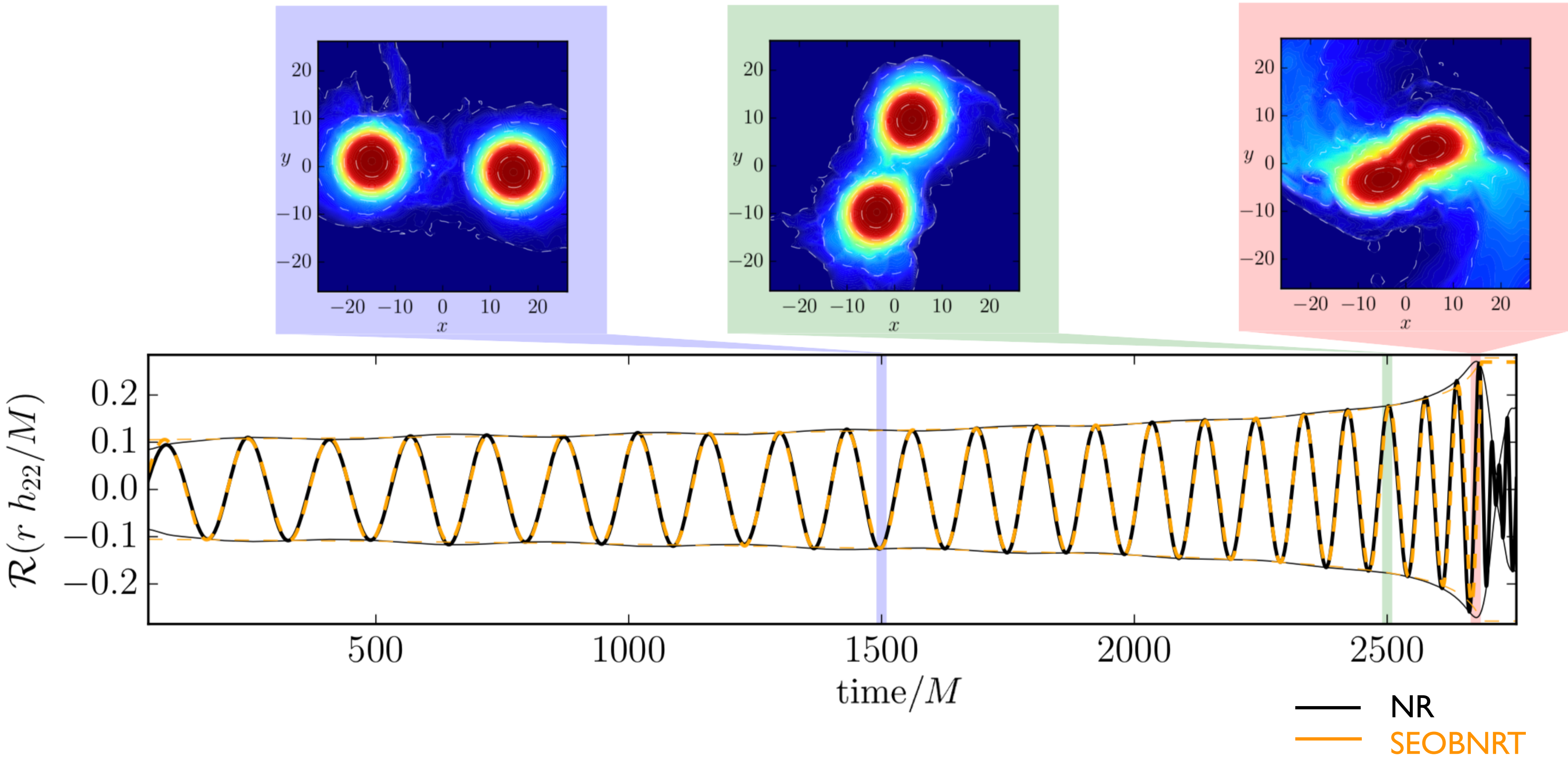


Waveforms for BNS combining analytical & numerical relativity

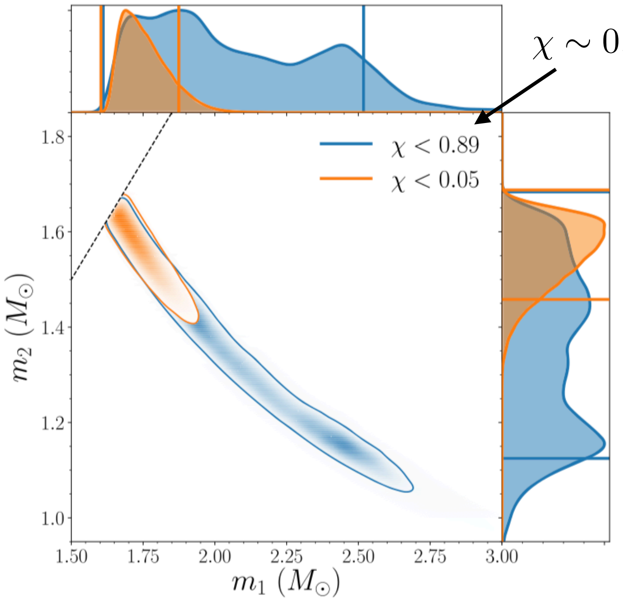
•Synergy between **analytical** and **numerical work** is **crucial.**



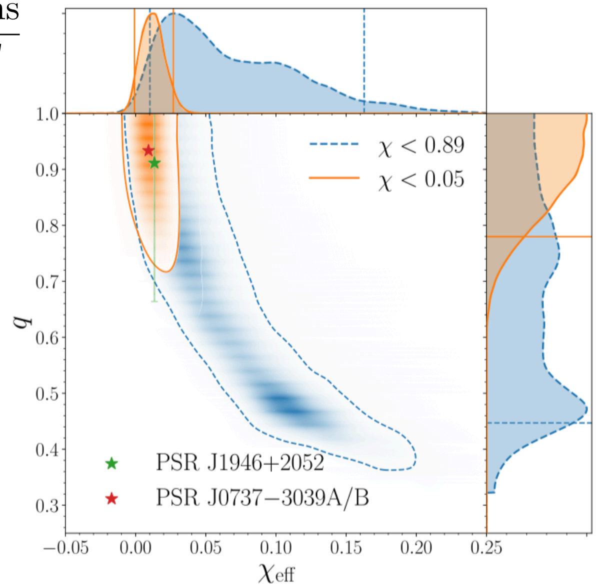
(Damour 1983, Flanagan & Hinderer 08, Binnington & Poisson 09, Vines et al. 11, Damour & Nagar 09, 12, Bernuzzi et al. 15, Hinderer, …AB … et al. 16, Steinhoff, … AB … et al. 16, Dietrich et al. 17-19, Nagar et al. 18)

GW190425: a Binary Neutron Star with Surprisingly High Mass

(*Abbott et al. ApJ Lett* 892 (2020))



$\chi \sim 0.4 \frac{1\text{ms}}{T}$



$q = m_1/m_2$

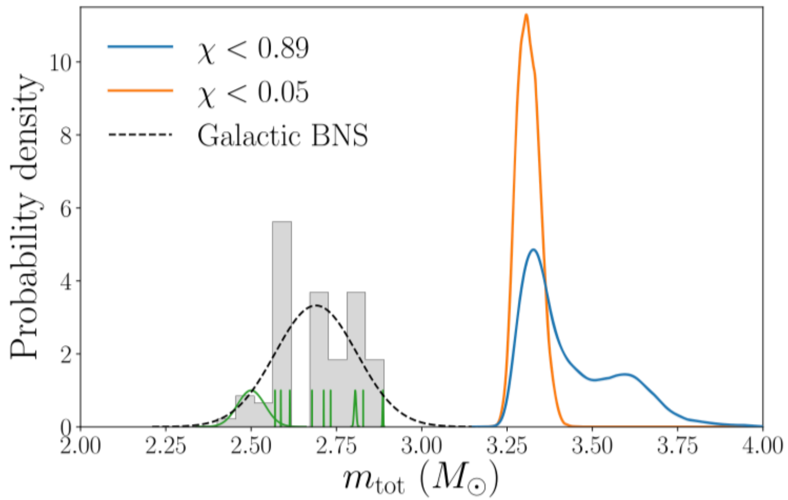
$\chi_1 = S_1/m_1^2$

$\chi_2 = S_2/m_2^2$

$\chi_{\rm eff} = \left(\frac{m_1}{M}\chi_1 + \frac{m_2}{M}\chi_2\right) \cdot \hat{\mathbf{L}}$

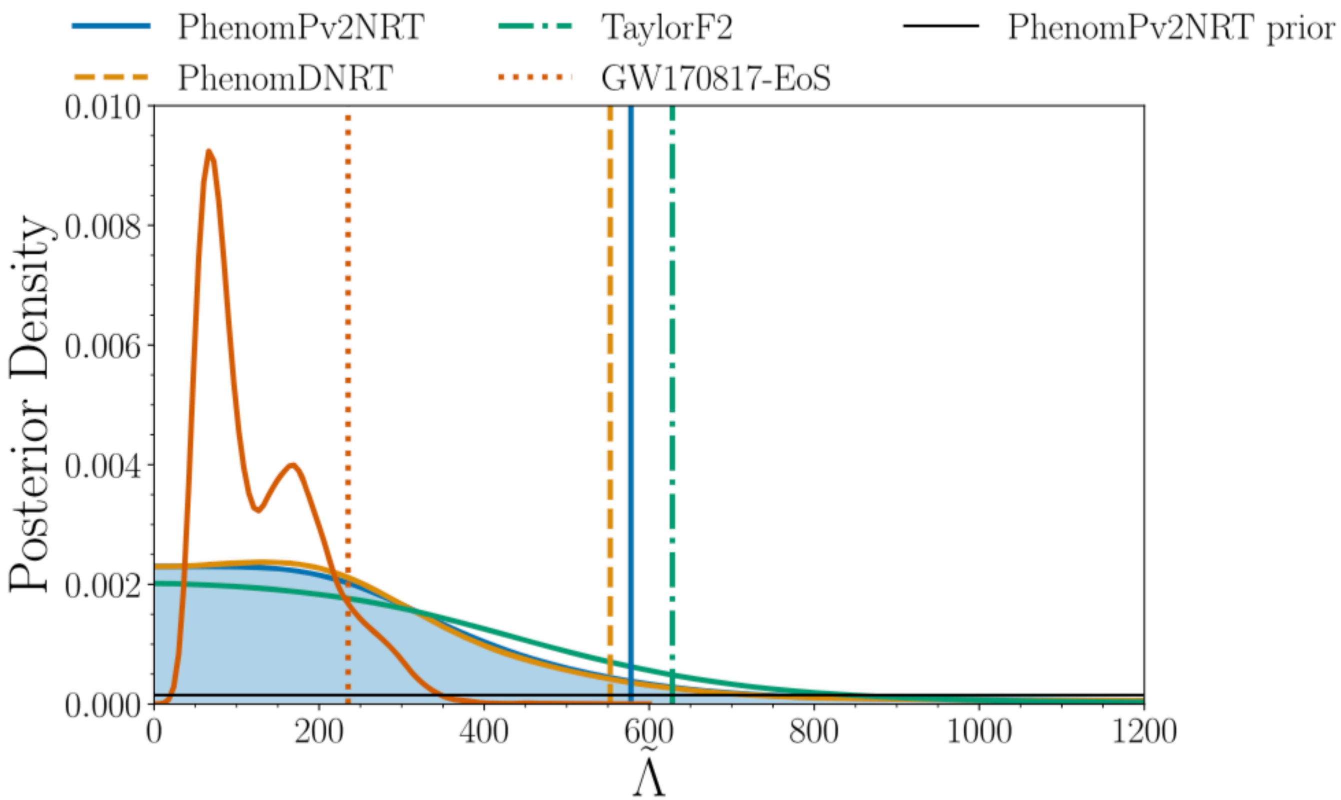
- **GW190425's masses** are **consistent with mass measurements of NSs** in binaries.

- **GW190425's total mass $3.4^{+0.3}_{-0.1} M_\odot$** is larger than BNSs in our galaxy: **new population of BNS?**



GW190425: Inference on Tidal Deformability Parameter

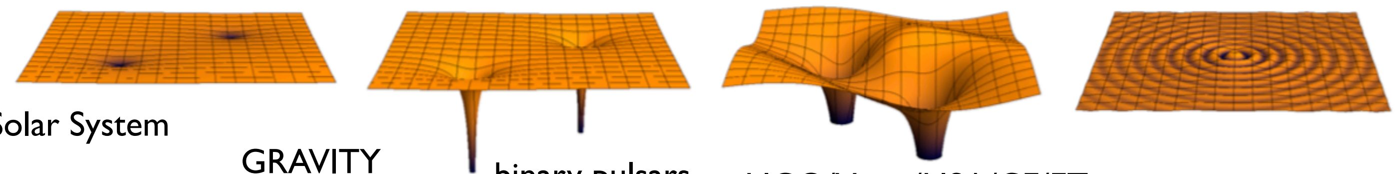
(Abbott et al. ApJ Lett 892 (2020))



• **GW190425's SNR is lower (~ 13)** than GW170817's SNR (~ 34): **looser constraint** on tidal deformability.

Tests of General Relativity: Bounding Higher-Order PN Coefficients

strong and highly dynamical spacetime



Solar System

GRAVITY

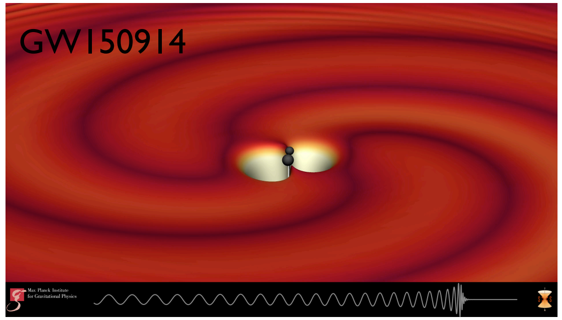
Event Horizon Telescope

binary pulsars

LIGO/Virgo/LISA/CE/ET

(Wex 2014)

(Ossokine, AB & SXS project)



- BBHs **rapidly varying orbital periods** allow us to **bound higher-order PN coefficients** in gravitational phase of GW signals.

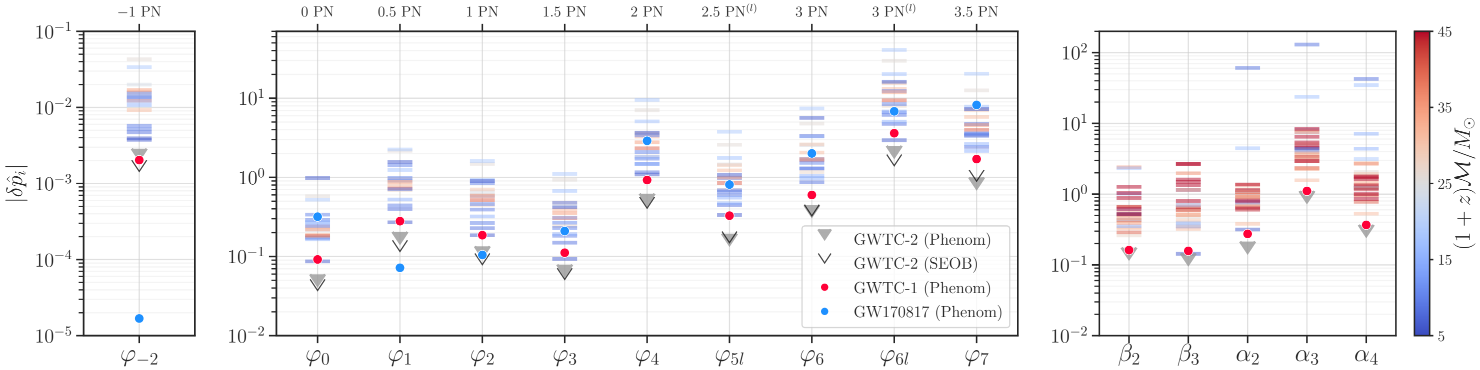
$$\tilde{h}(f) = \mathcal{A}(f) e^{i\varphi(f)}$$

$$\varphi(f) = \varphi_{\rm ref} + 2\pi f t_{\rm ref} + v^{-5} \left[\sum_{n=-2}^{7} \varphi_n^{\rm (GR)} (1 + \delta\hat{\varphi}_n) v^n \right.$$

$$v = (\pi M f)^{1/3} \qquad \left. + \sum_{n=5}^{6} \varphi_{n\,\ell}^{\rm (GR)} (1 + \delta\hat{\varphi}_{n\,\ell}) v^n \log v \right]$$

(Arun et al. 06 , Mishra et al. 10, Yunes & Pretorius 09, Li et al. 12)

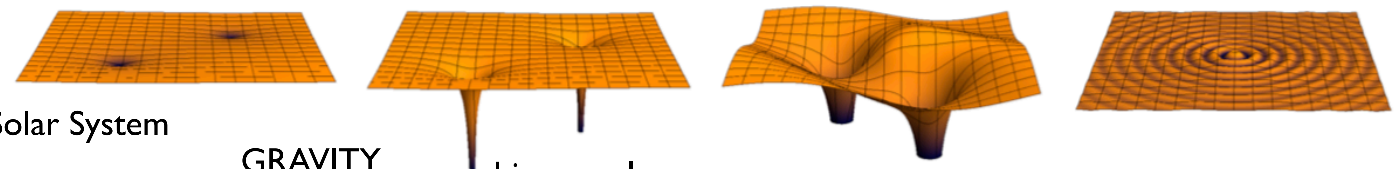
(Abbott et al. arXiv:2010.14529)



- **PN parameters** describe: **tails** of radiation due to backscattering, **spin-orbit** and **spin-spin** couplings.
- **PN parameters** take **different values** in **gravity theories** alternative to GR.

Tests of General Relativity: Bounding Higher-Order PN Coefficients

strong and highly dynamical spacetime



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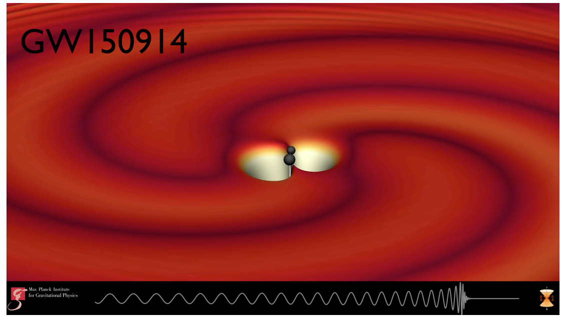

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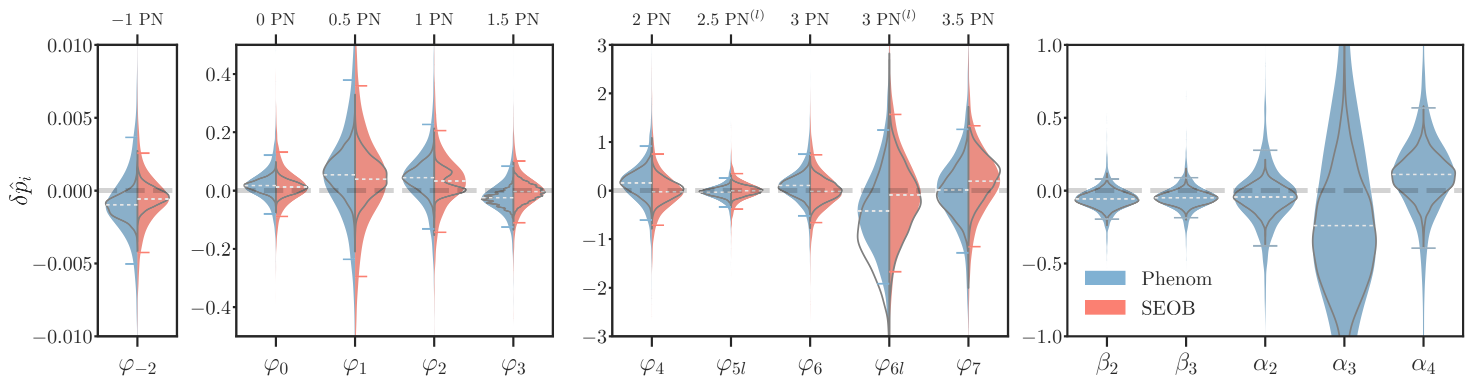
$$v = (\pi M f)^{1/3}$$

(Arun et al. 06 , Mishra et al. 10, Yunes & Pretorius 09, Li et al. 12)



(Abbott et al. arXiv:2010.14529)

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- **PN parameters** take **different values** in **gravity theories** alternative to GR.

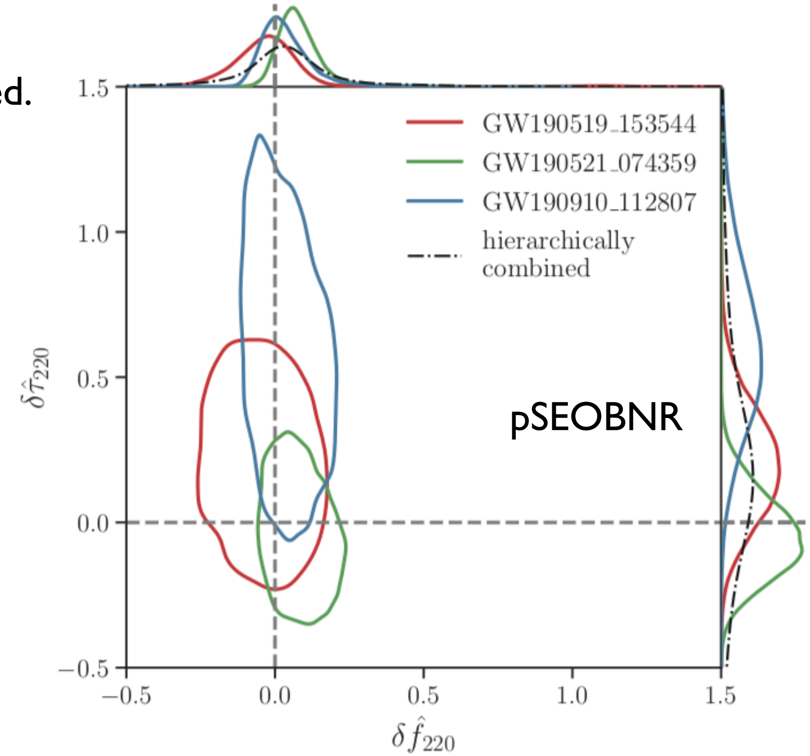
Tests of General Relativity: Remnant Properties

- In General Relativity, **remnant object** resulting from coalescence of two astrophysical BHs **is a perturbed Kerr BH**.
- The remnant **BH relaxes** to its stationary Kerr state **by emitting quasi-normal modes (QNMs).**
  (Vishveshwara 70, Press 71, Chandrasekhar et al. 75)
- The QNM's **frequencies and decay times** only depend on **BH's mass and spin** (no-hair conjecture).
  (Israel 69, Carter 71; Hawking 71, Bardeen 73)
- The **no-hair conjecture can be disproved** if more than one QNM is observed.
  (Dreyer et al. 2004, Berti et al. 2006, Gossan et al. 2012, Meidam et al. 2014)
- **Inspiral-merger-ringdown** waveform model **with parameterized QNM**'s frequency and decay time (pSEOBNR):
  (Brito, AB & Raymond 18, Ghosh, Brito & AB in prep 21)

$$f_{220} = f_{220}^{\rm GR}\,(1 + \delta\hat{f}_{220}) \qquad \delta\hat{f}_{220} = 0.03^{+0.38}_{-0.35}$$

$$\tau_{220} = \tau_{220}^{\rm GR}\,(1 + \delta\hat{\tau}_{220}) \qquad \delta\hat{\tau}_{220} = 0.16^{+0.98}_{-0.98}$$

- **Results obtained also with** model that includes only **a superposition of damped sinusoids**.
  (Abbott et al. PRL 116 (2016) 221101, Carullo et al. 19, Isi et al. 19, Abbott et al. arXiv:2010.14529)

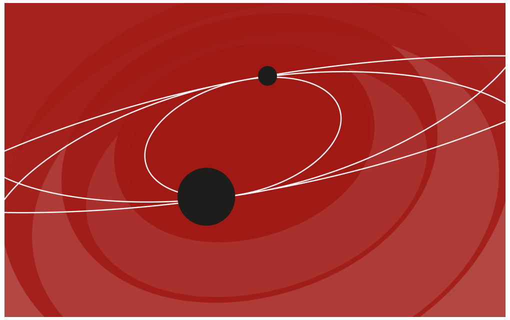


(Abbott et al. arXiv:2010.14529)

Scattering Amplitude: A New Way to Study 2-body Problem

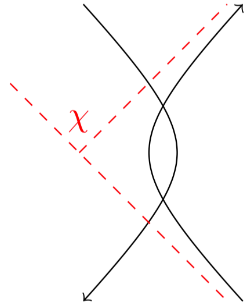
- Relativistic 2-body dynamics

(credit: Carvalho)

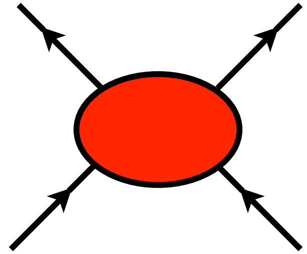


- Classical scattering: scattering angle χ

(credit: Steinhoff)



- Quantum scattering amplitude



e.g., in Born approximation: Fourier transform of potential is related to scattering amplitude

- 2-body **Hamiltonian at 3PM (2 loops)** for nonspinning BHs. *(Cheung et al. 19, 20, Bern et al. 19, Blümlein et al. 20, Kälin et al. 20)*

Small parameter is **$GM/rc^2 \ll 1$, $v^2/c^2 \sim 1$,** large separation, natural **for unbound motion/scattering**

$$H(\boldsymbol{p},\boldsymbol{r}) = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V(\boldsymbol{p},\boldsymbol{r})$$

$$V(\boldsymbol{p},\boldsymbol{r}) = \sum_{i=1}^{\infty} c_i(\boldsymbol{p}^2)\left(\frac{G}{|\boldsymbol{r}|}\right)^i$$

$$E = E_1 + E_2 \qquad \gamma = E/m$$

$$\xi = E_1 E_2/E^2 \qquad \sigma = \frac{p_1 \cdot p_2}{m_1 m_2}$$

$$V^{(1)}(\mathbf{p},\mathbf{q}) = \int \frac{\mathbf{d^3 r}}{(\mathbf{2}\pi)^\mathbf{3}}\, \mathcal{M}^{\rm tree}(\mathbf{p},\mathbf{q})\, \mathbf{e}^{-\mathbf{ir}\cdot\mathbf{q}}$$

↑ amplitude

$$c_1 = \frac{\nu^2 m^2}{\gamma^2 \xi}\left(1 - 2\sigma^2\right)$$

Results from Interplay with Scattering Amplitude Methods & EFT

(Bern et al. 19)

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN		
1PM	$(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8$	$+ v^{10}$	$+ v^{12}$	$+ v^{14}$	$+ \dots)$	G^1
2PM		$(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8$	$+ v^{10}$	$+ v^{12}$	$+ \dots)$	G^2
3PM			$(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8$	$+ v^{10}$	$+ \dots)$	G^3
4PM				$(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8$	$+ \dots)$	G^4
5PM					$(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ \dots)$	G^5
6PM						$(1$	$+ v^2$	$+ v^4$	$+ \dots)$	G^6
⋮										

• 2-body **Hamiltonian at 3PM (2 loops)** for nonspinning BHs. *(Cheung et al. 19, 20, Bern et al. 19, Blümlein et al. 20, Kälin et al. 20)*

Small parameter is $GM/rc^2 \ll 1$, $v^2/c^2 \sim 1$, large separation, natural **for unbound motion/scattering**

$$H(\boldsymbol{p},\boldsymbol{r}) = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V(\boldsymbol{p},\boldsymbol{r})$$

$$V(\boldsymbol{p},\boldsymbol{r}) = \sum_{i=1}^{\infty} c_i(\boldsymbol{p}^2)\left(\frac{G}{|\boldsymbol{r}|}\right)^i$$

$$E = E_1 + E_2 \qquad \gamma = E/m$$

$$\xi = E_1 E_2/E^2 \qquad \sigma = \frac{p_1 \cdot p_2}{m_1 m_2}$$

$$V^{(1)}(\mathbf{p},\mathbf{q}) = \int \frac{\mathrm{d}^3\mathbf{r}}{(2\pi)^3}\, \mathcal{M}^{\text{tree}}(\mathbf{p},\mathbf{q})\, \mathbf{e}^{-\mathbf{i}\mathbf{r}\cdot\mathbf{q}}$$

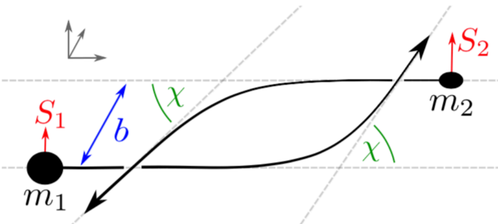
↑ amplitude

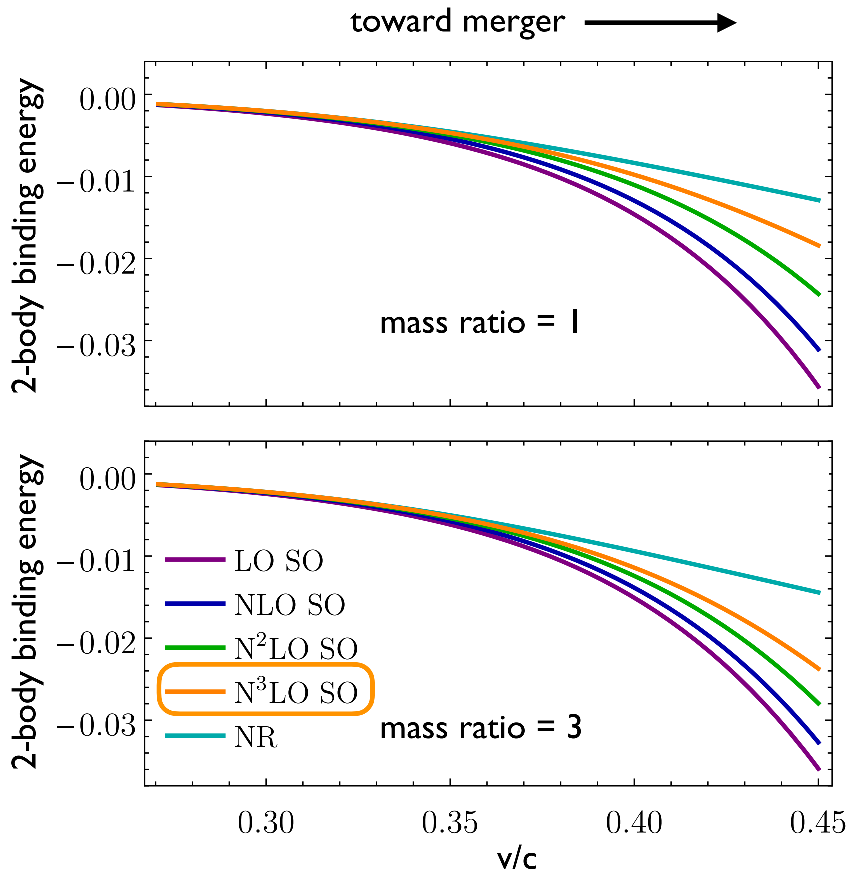
$$c_1 = \frac{\nu^2 m^2}{\gamma^2 \xi}(1 - 2\sigma^2)$$

Results from Interplay with Scattering Amplitude Methods & EFT (contd.)

- 2-body **spin-orbit** (SO) **Hamiltonian at 4.5PN** computed using **EFT** or **interplay** between **bound and unbound orbits**, and gravitational **self-force** results.

(Levi et al. 20, Antonelli et al. 20)





(Antonelli et al. 20)

- 2-body non-spinning **Hamiltonian at 5PN & 6PN** *partially* computed using **EFT** or **interplay** between **bound and unbound orbits**, and gravitational **self-force** results.

(Foffa et al. 19, Blümlein et al. 20, Damour 20, Bini, Damour & Geralico 20)

- 2-body **Hamiltonian at 2PM (1 loop)** for spinning, precessing BHs.

(Bini et al. 17, 18, Vines 18, Bern et al. 20)

- **Results** can be easily **included** into **EOB formalism**.

(Damour 19, Antonelli et al. 19)

Toward High-Precision Gravitational-Wave Astrophysics

- **Observing** gravitational waves and **inferring astrophysical/physical information** hinges on our **ability** to make highly **precise predictions** of **two-body dynamics** and **gravitational radiation.**

- Crucial to **improve waveform models** for BBHs and binaries with matter for LIGO and Virgo **upcoming runs** and for **future detectors** (Cosmic Explorer, Einstein Telescope & LISA). **Waveform accuracy** would need to be **improved by one or two orders of magnitude** depending on the parameter space.

- Unique opportunity for **theoretical particle physicists to contribute**.

- **Conservative dynamics** →

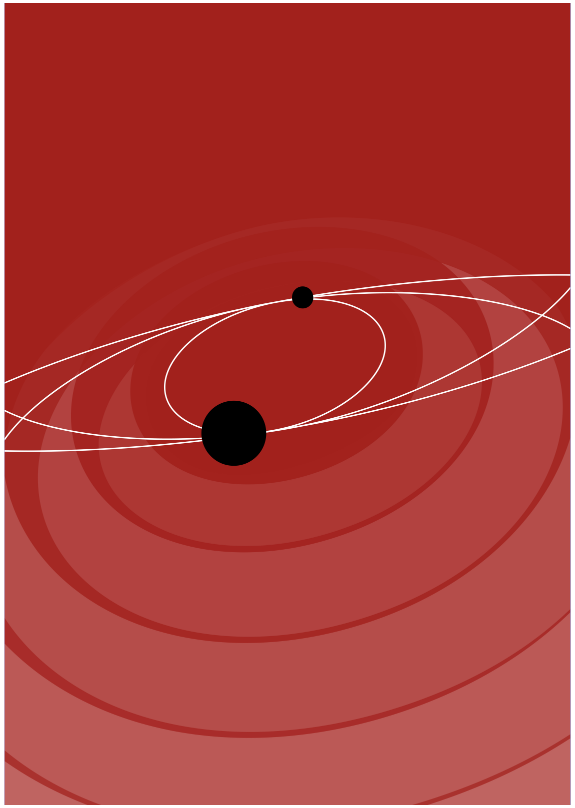
PN order	0	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5
no spin	N	1PN		2PN		3PN		4PN		5PN		6PN	
spin-orbit			LO SO		NLO SO		N2LO SO		N3LO SO				
spin^2				LO S2		NLO S2		N2LO S2		N3LO S2			
spin^3							LO S3		NLO S3				
spin^4								LO S4		NLO S4			
spin^5											LO S5		NLO S5
spin^6												LO S6	

need up to
1PM / tree
2PM / 1-loop
3PM / 2-loop
4PM / 3-loop
5PM / 4-loop
6PM / 5-loop
7PM / 6-loop

(credit: Vines)

N.B. **Resummation** methods (e.g., EOB) can **accelerate accuracy**.

- **Plus radiation!**



(credit: Carvalho)

Thanks!