How warm are non-thermal relics?
Lyman-α bounds on out-of-equilibrium dark matter

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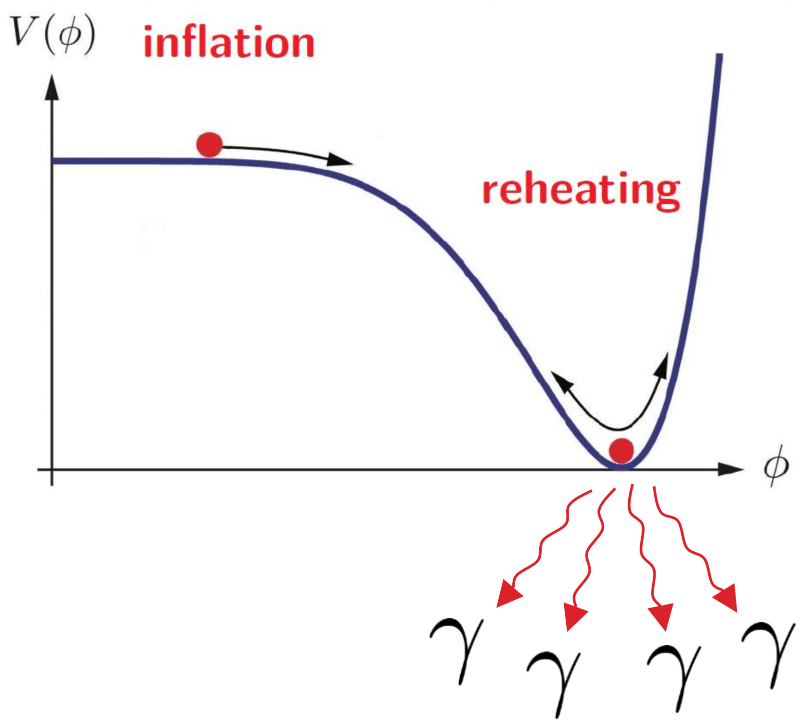
Based on [**arXiv:2011.13458**]
in collaboration with **G. Ballesteros** & **M. A. G. Garcia**

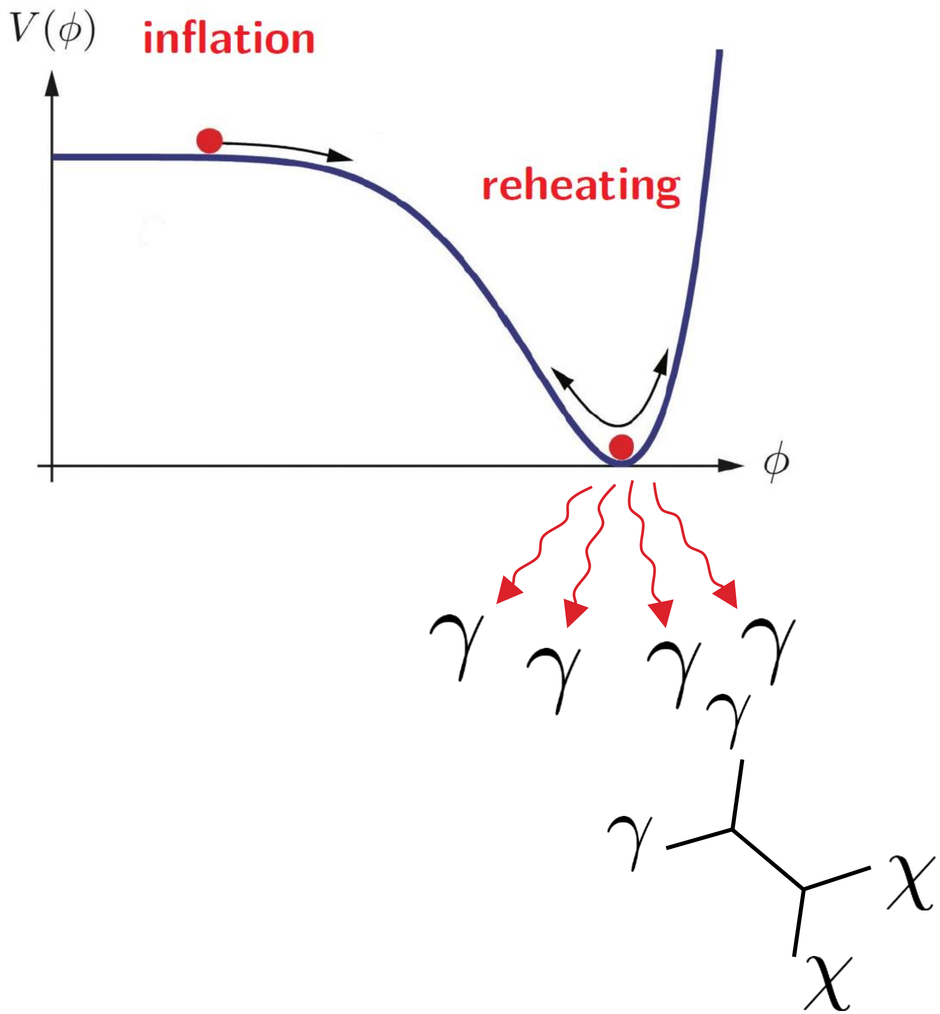
DM production during reheating



ϕ : **inflaton**

γ : **generic SM particle**

DM production during reheating



ϕ : **inflaton**

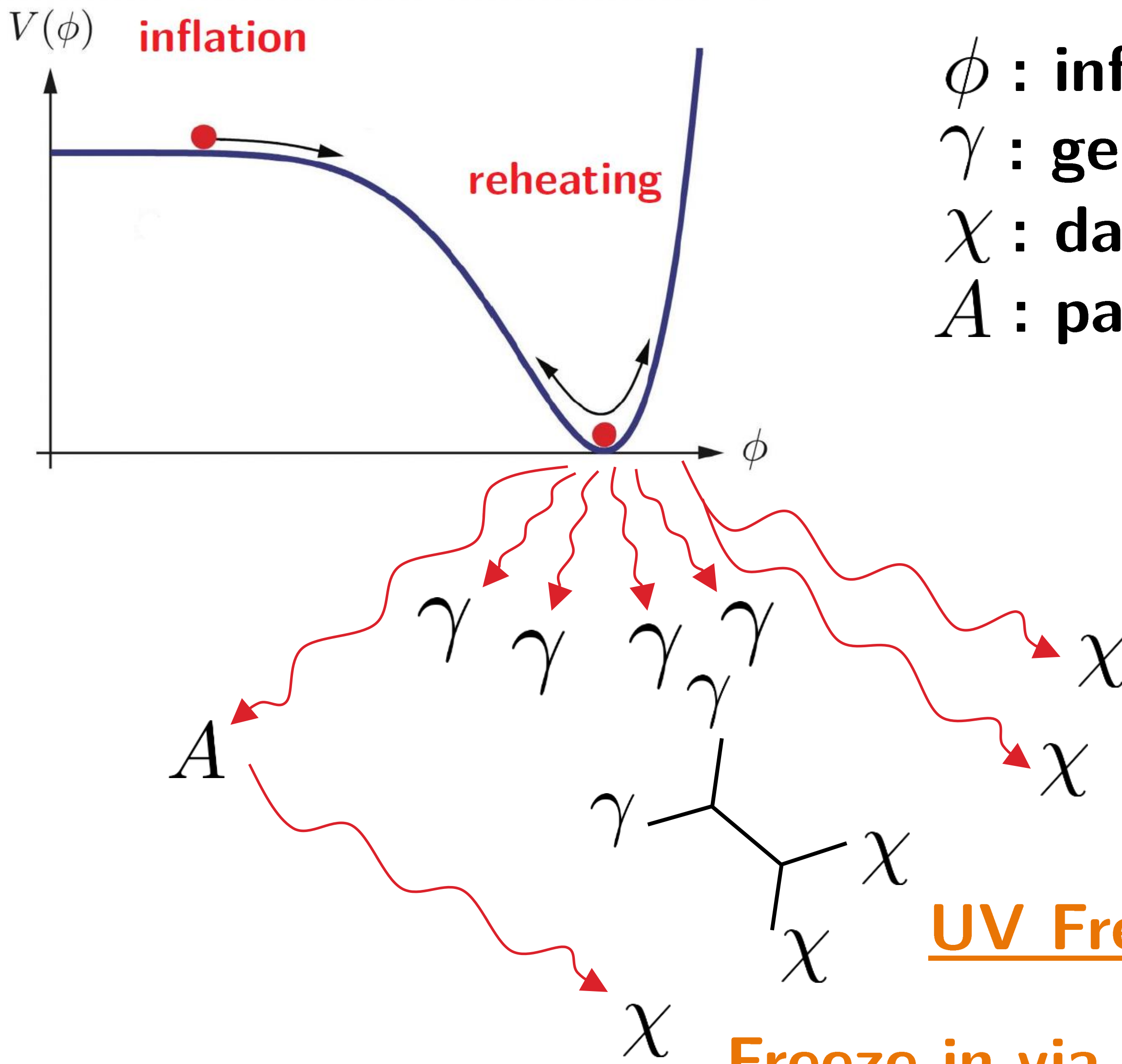
γ : **generic SM particle**

χ : **dark matter**

$$\sigma(s) = \frac{s^{\frac{n}{2}}}{\Lambda^{n+2}}$$

UV Freeze-in via scattering

DM production during reheating



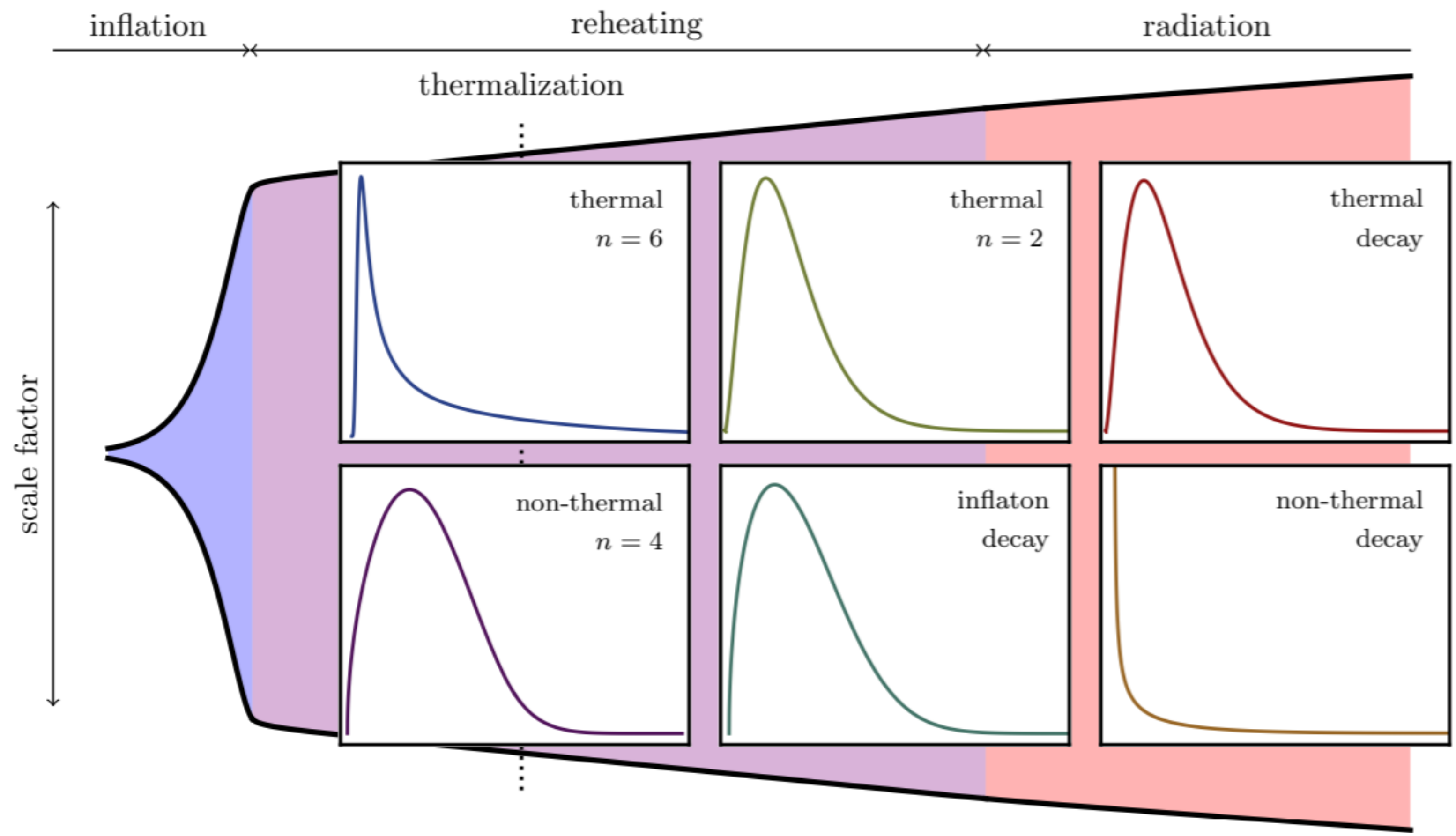
ϕ : inflaton
γ : generic SM particle
χ : dark matter
A : particle (SM or not)

Decay of inflaton

UV Freeze-in via scattering

Freeze-in via decay

DM phase space distribution



- Most of the distributions fitted by $f(q) \propto q^{\alpha} \exp\left(-\beta\, q^{\gamma}\right)$

What is the cosmological imprint of out-of-equilibrium dark matter?

Cosmological imprint

- **Cosmological role** of out-of-equilibrium dark matter via

$$\bar{\rho} = 4\pi \left(\frac{T_\star}{a}\right)^4 \int q^2 \epsilon \bar{f}(q)\, \mathrm{d}q$$

energy-density

$$\bar{P} = \frac{4\pi}{3} \left(\frac{T_\star}{a}\right)^4 \int q^2 \frac{q^2}{\epsilon} \bar{f}(q)\, \mathrm{d}q$$

pressure

$$q \equiv \frac{p\, a(t)}{T_\star}$$ **: comoving momentum**

$$\epsilon = \sqrt{q^2 + \left(\frac{m_{\rm DM}\, a}{T_\star}\right)^2}$$

- Define $w \equiv \bar{P}/\bar{\rho}$ **: equation-of-state parameter**

- In pure $\Lambda\mathrm{CDM}$: $w = 0$ precisely (**Cold = pressureless**)

- But $w \neq 0$! ➡ **Non-Cold Dark Matter** cosmology

Non-Cold Dark Matter $w \neq 0$

- Expanding quantities around **homogenous background**

$$f(\boldsymbol{x}, \boldsymbol{p}, \tau) = \bar{f}(|\boldsymbol{p}|, \tau)[1 + \Psi(\boldsymbol{x}, \boldsymbol{p}, \tau)]$$

- In matter domination, matter **overdensities** δ follow

$$\ddot{\delta} + \mathcal{H}\dot{\delta} - \frac{3}{2}\mathcal{H}^2 \left(1 - \frac{k^2}{k_{\mathrm{FS}}^2}\right)\delta = 0 \qquad w \ll 1$$

where $k_{\mathrm{FS}}^2(a) = \frac{9}{10}\frac{\mathcal{H}^2}{w(a)}$ is the **Free-Streaming wavenumber**

$\mathrm{d}\tau \equiv a\,\mathrm{d}t$: **Conformal time** τ

$\mathcal{H} \equiv a\,H$: **Conformal Hubble rate**

$w \equiv \bar{P}/\bar{\rho}$: **Equation-of-state parameter**

[C. Ma & E. Bertschinger. ApJ 455 (1995) 7-25]

[J. Lesgourgues & T. Tram, JCAP 09 (2011) 032]

[M. Kunz, S. Nesseris, I. Sawicki, PRD 94, 023510 (2016)]

[G. Ballesteros, M. A. G. Garcia & **MP**, 2011.13458]

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- If $w=0$ all modes grow "**democratically**" : **CDM** limit
  $w \neq 0$ **cutoff** in the power spectrum at $k_{\mathrm{H}}(a) \equiv \left[\int_0^a \frac{1}{k_{\mathrm{FS}}(\tilde{a})}\frac{\mathrm{d}\tilde{a}}{\tilde{a}}\right]^{-1}$

- **Only w controls the cutoff scale!**

$\mathrm{d}\tau \equiv a\,\mathrm{d}t$: **Conformal time τ**
$\mathcal{H} \equiv a\,H$: **Conformal Hubble rate**
$w \equiv \bar{P}/\bar{\rho}$: **Equation-of-state parameter**

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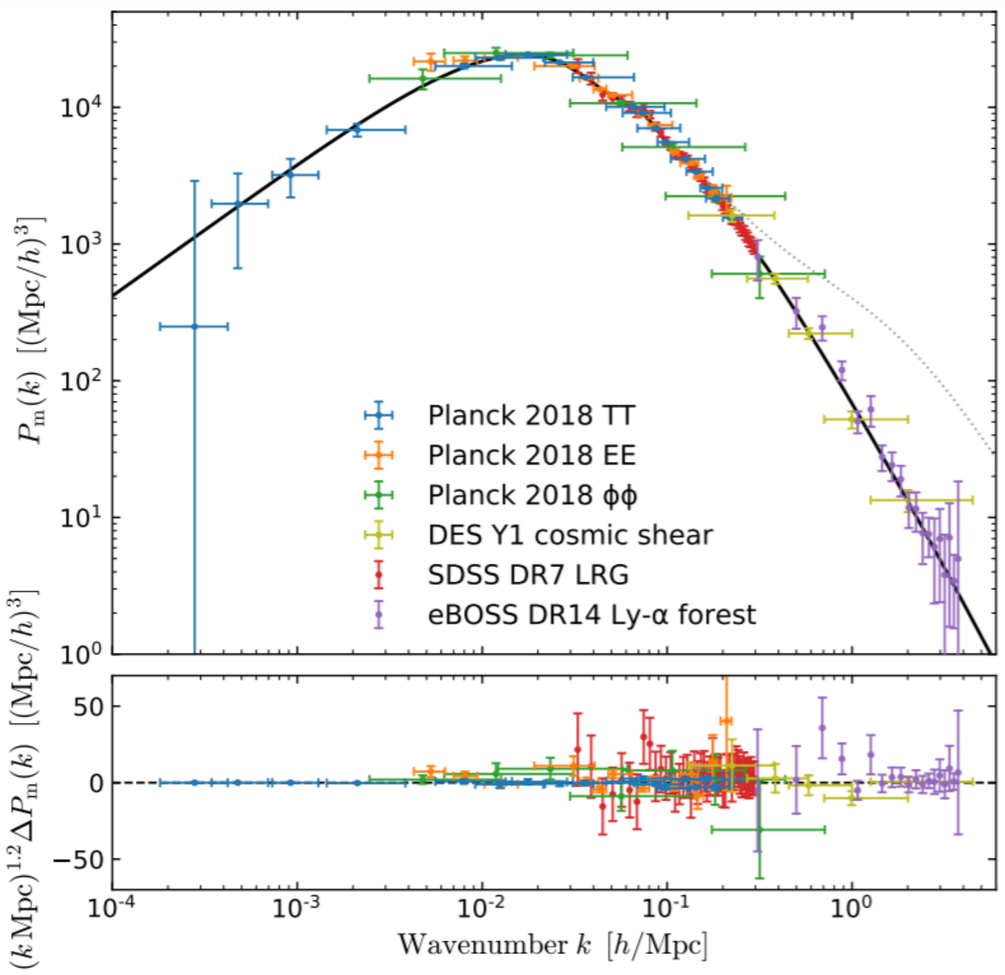
[G. Ballesteros, M. A. G. Garcia & **MP**, 2011.13458]

Non-Cold Dark Matter $w \neq 0$

- Small scales of power spectrum probed by **Lyman-alpha forest**

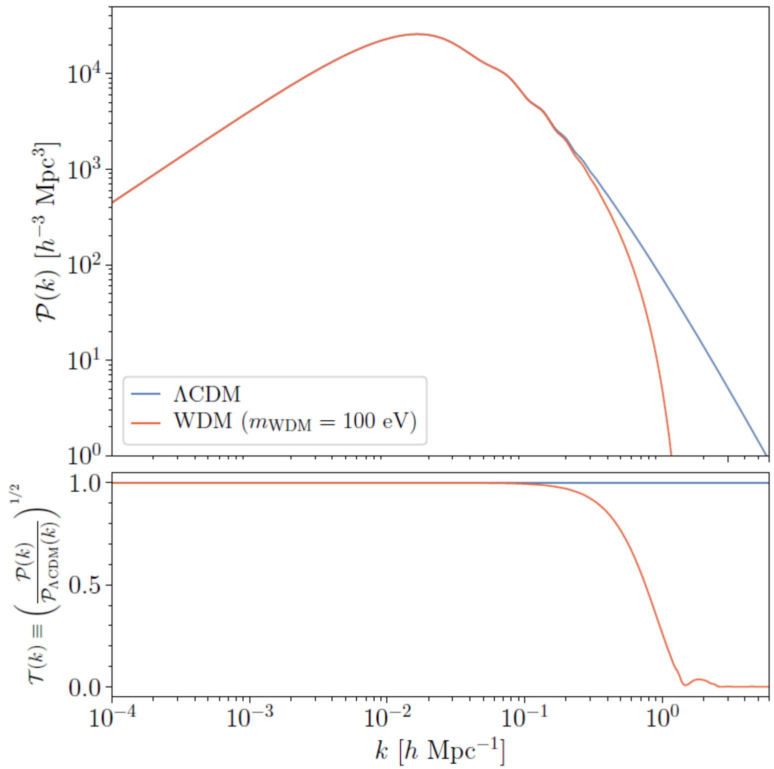


[S. Chabanier, M. Millea, N. Palanque-Delabrouille, MNRAS 489 (2019) 2, 2247-2253]

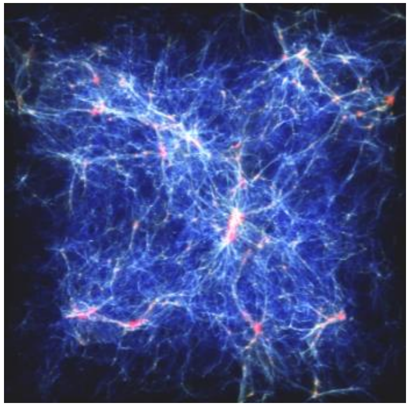
Non-Cold Dark Matter $w \neq 0$

- Lyman-alpha forest constraints Warm Dark Matter (**WDM**)

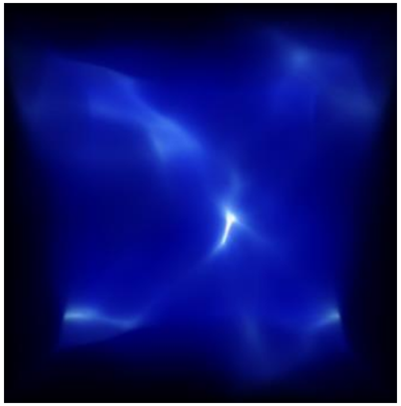
$$\bar{f}_{\rm WDM}(q) = \frac{1}{1 + e^{q/T_{\rm WDM}}} \quad \Rightarrow \quad \Omega_{\rm WDM} h^2 \simeq \left(\frac{m_{\rm WDM}}{94\ \rm eV}\right)\left(\frac{T_{\rm WDM}}{T_\nu}\right)^3$$



ΛCDM



WDM



$m_{\rm WDM} = 100\,\rm eV$

[J. Baur, N. Palanque-Delabrouille, C. Yèche, C. Magneville, M. Viel, JCAP 08 (2016) 012]

How warm is Non-Cold Dark Matter?

- From **Lyman-alpha** forest $m_{\rm WDM}^{\rm Ly-\alpha} > (1.9 - 5.3)$ keV at 95% C.L.

[Braur et al. JCAP 08 (2016) 012 – Iršič et al. PRD 96 (2017) 2, 023522
Palanque Delabrouille et al. JCAP 04 (2020) 038 – Viel et al. PRD 88 (2013) 043502
Viel et al. PRD 71 (2005) 063534 – Narayanan et al. ApJ 543 (2000) L103-L106]

$$w_{\rm WDM}(a) \simeq 6 \times 10^{-15}\, a^{-2} \left(\frac{\rm keV}{m_{\rm WDM}}\right)^{8/3} \Rightarrow w_{\rm WDM}(a=1) < 10^{-15}$$

- Constraints much **stronger** than **CMB**! $w_{\rm WDM}(a=1) < 10^{-10}$

[M. Kunz, S. Nesseris, I. Sawicki, PRD 94, 023510 (2016)]

- How **cold** are **WIMPs** ?

$$w(a) \simeq 10^{-29} \left(\frac{1}{a^2}\right) \left(\frac{20\,T_F}{m_{\rm DM}}\right) \left(\frac{100\ {\rm GeV}}{m_{\rm DM}}\right)^2 \left(\frac{100}{g_*^F}\right)^{2/3} \Rightarrow$$

- **How to translate Lyman-alpha WDM bounds on any DM ?**

$$w(m_{\rm DM}) = w_{\rm WDM}(m_{\rm WDM}^{\rm Ly-\alpha})$$

[S. Colombi, S. Dodelson, L. M. Widrow ApJ. 458 (1996) 1 - Kamada, N. Yoshida, K. Kohri, T. Takahashi JCAP 03 (2013) 008
K. J. Bae, R. Jinno, A. Kamada, K. Yanagi JCAP 03 (2020) 042 - A. Kamada & K. Yanagi JCAP 1911 (2019) 029]

How warm is Non-Cold Dark Matter?

$w - \text{matching}$

$$w \simeq \frac{\delta P}{\delta \rho} = \frac{T_\star^2}{3 m_{\rm DM}^2} \frac{\langle q^2 \rangle}{a^2}$$

$$m_{\rm DM} = m_{\rm WDM}^{\rm Ly-\alpha} \left(\frac{T_\star}{T_{\rm WDM}} \right) \sqrt{\frac{\langle q^2 \rangle}{\langle q^2 \rangle_{\rm WDM}}}$$

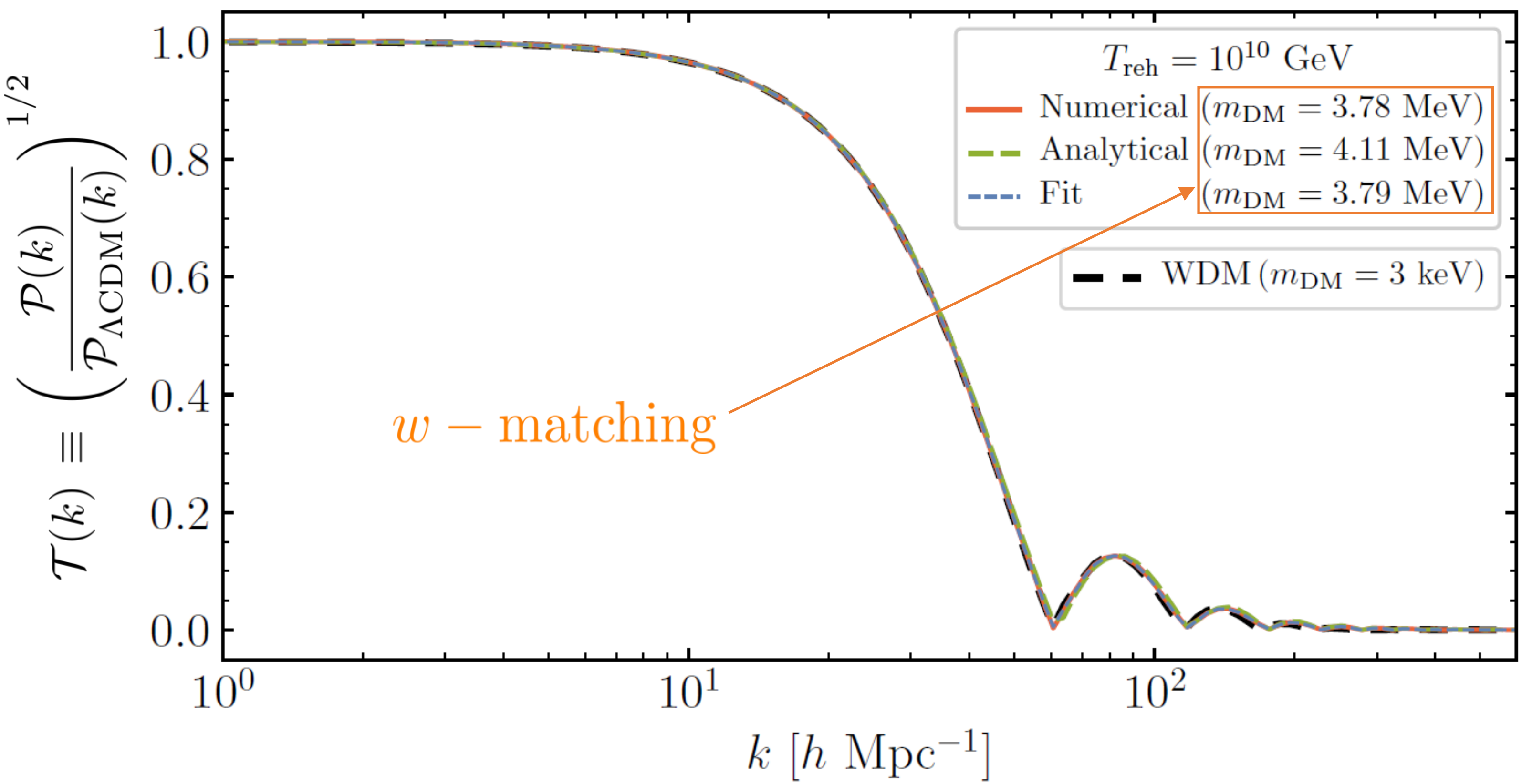
- Compute **2nd moment of distribution** + **determine** $T_\star$
- If **distribution** can be fitted by $f(q) \propto q^\alpha \exp\left(-\beta\, q^\gamma\right)$

$w - \text{matching}$ ➡

$$m_{\rm DM} \simeq 7.56 \text{ keV} \left(\frac{m_{\rm WDM}^{\rm Ly-\alpha}}{3 \text{ keV}} \right)^{4/3} \left(\frac{\langle p \rangle_0}{T_0} \right) \sqrt{\frac{\Gamma\left(\frac{3+\alpha}{\gamma}\right) \Gamma\left(\frac{5+\alpha}{\gamma}\right)}{\Gamma^2\left(\frac{4+\alpha}{\gamma}\right)}}$$

How warm is Non-Cold Dark Matter?

- Example: **inflaton decay** case computed using **CLASS**
  [D. Blas, J. Lesgourgues & T. Tram JCAP 07 (2011) 034 - J. Lesgourgues & T. Tram, JCAP 09 (2011) 032]



- **Excellent agreement** with $w-$matching for **all distributions**!

Inflaton decay

- **Lyman-alpha** bounds translate into

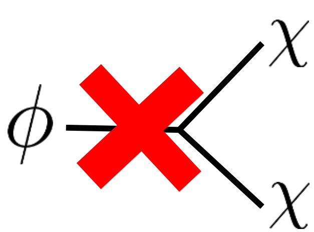
$$m_{\rm DM} \gtrsim \left(\frac{m_{\rm WDM}^{\rm Ly-\alpha}}{3\ {\rm keV}}\right)^{4/3} \left(\frac{106.75}{g_{*s}^{\rm reh}}\right)^{1/3} \left(\frac{m_\phi}{3\times 10^{13}\ {\rm GeV}}\right) \left(\frac{10^{10}\ {\rm GeV}}{T_{\rm reh}}\right) \begin{cases} 3.78\ {\rm MeV}\,, & {\rm Numerical}\,, \\ 4.11\ {\rm MeV}\,, & {\rm Analytical}\,, \\ 3.79\ {\rm MeV}\,, & {\rm Fit}\,. \end{cases}$$

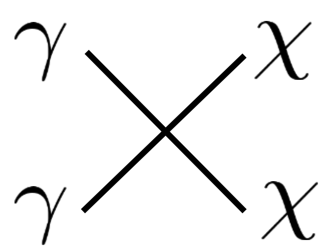
- For **low reheating temperature** $T_{\rm reh} \ll m_\phi$

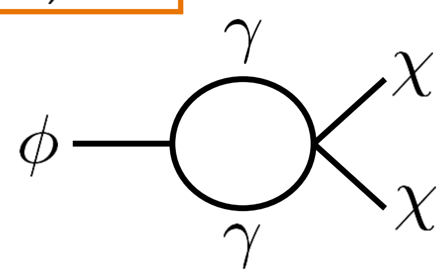
$$m_{\rm DM} \gtrsim {\rm EeV}$$

- **Combining** with **relic density** condition

$${\rm Br}_\chi < 1.5\times 10^{-4} \left(\frac{g_{*s}^{\rm reh}}{106.5}\right)^{1/3} \left(\frac{3\,{\rm keV}}{m_{\rm WDM}^{\rm Ly-\alpha}}\right)^{4/3}$$

- Even if 

, since 

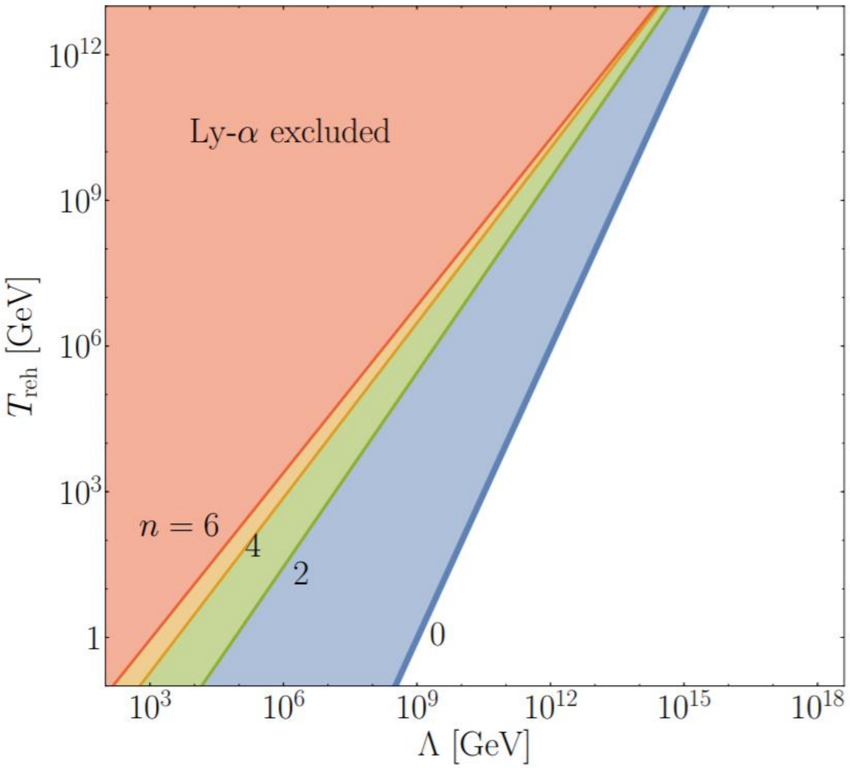
then 

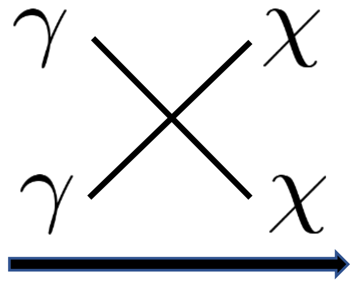
[K. Kaneta, Y. Mambrini & Keith A. Olive Phys.Rev.D 99 (2019) 6, 063508]

UV freeze-in via scattering

$$m_{\rm DM} \gtrsim \left(\frac{m_{\rm WDM}^{\rm Ly-\alpha}}{3\ \rm keV}\right)^{4/3} \left(\frac{106.75}{g_{*s}^{\rm reh}}\right)^{1/3} \begin{cases} 7.27\ (7.17)\ \rm keV\,, & \rm FF \quad Numerical\ (Fit)\,, \quad n=0 \\ 8.48\ (8.73)\ \rm keV\,, & \rm FF \quad Numerical\ (Fit)\,, \quad n=2 \\ 8.52\ (8.05)\ \rm keV\,, & \rm FF \quad Numerical\ (Fit)\,, \quad n=4 \end{cases}$$

- **Combine** with **relic density** condition



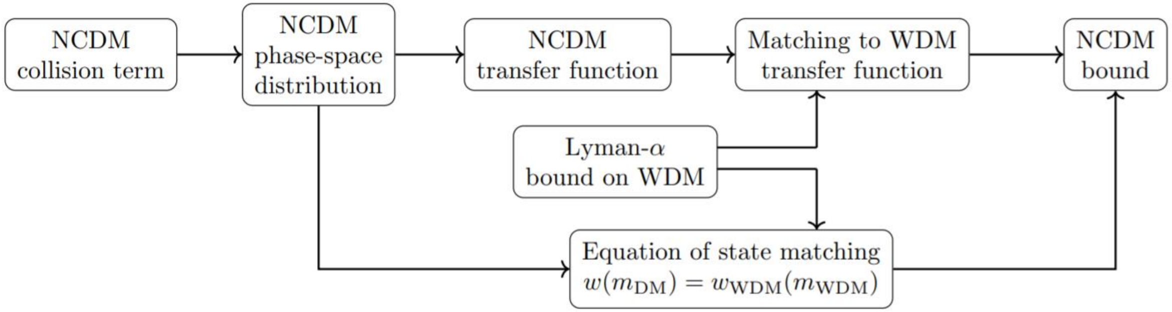


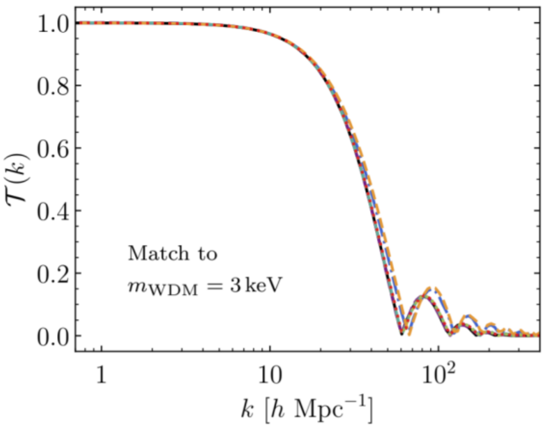
DM production

$$\sigma(s) = \frac{s^{\frac{n}{2}}}{\Lambda^{n+2}}$$

- **Apply to any UV freeze-in model!**

Summary





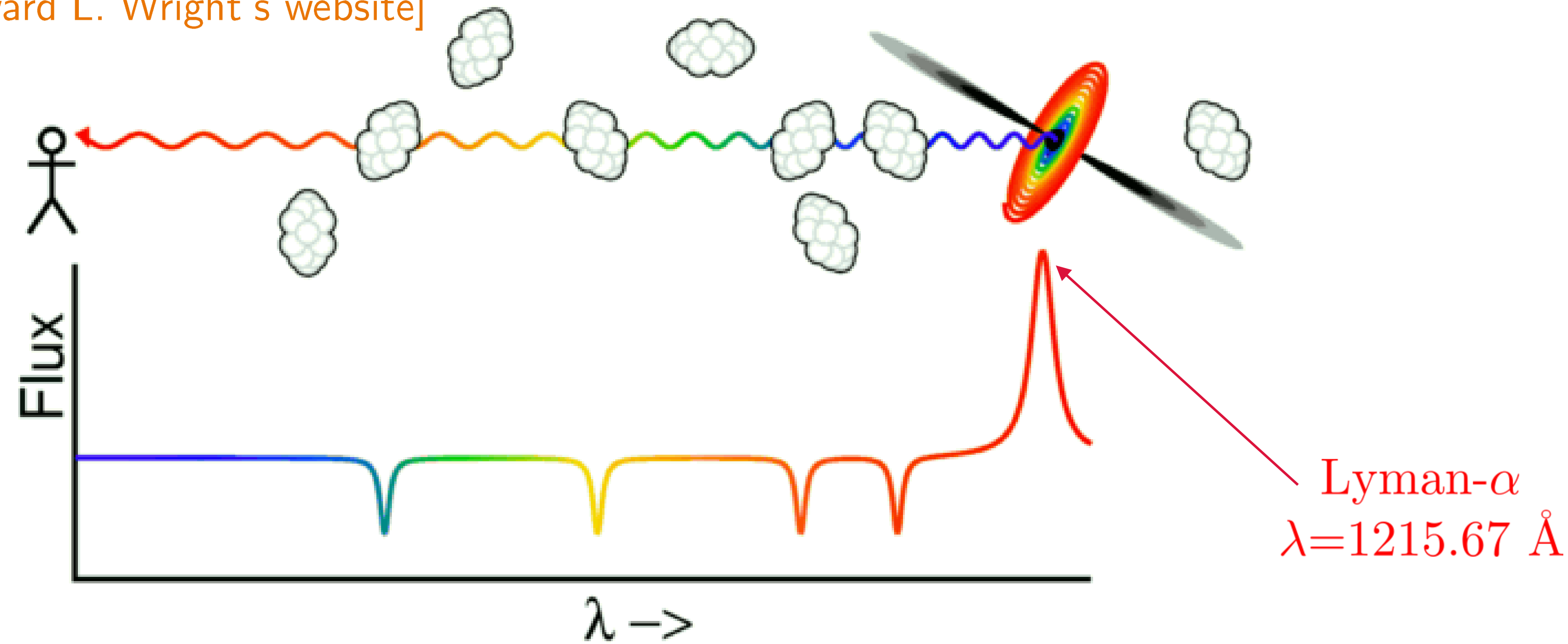
- **Out-of-equilibrium DM** produced **after inflation** can be **probed** by **Lyman-alpha**
- Much more **in the paper!** [arXiv:2011.13458]
- **Dark matter is cold.**

Thank you for your attention

Back-up Slides

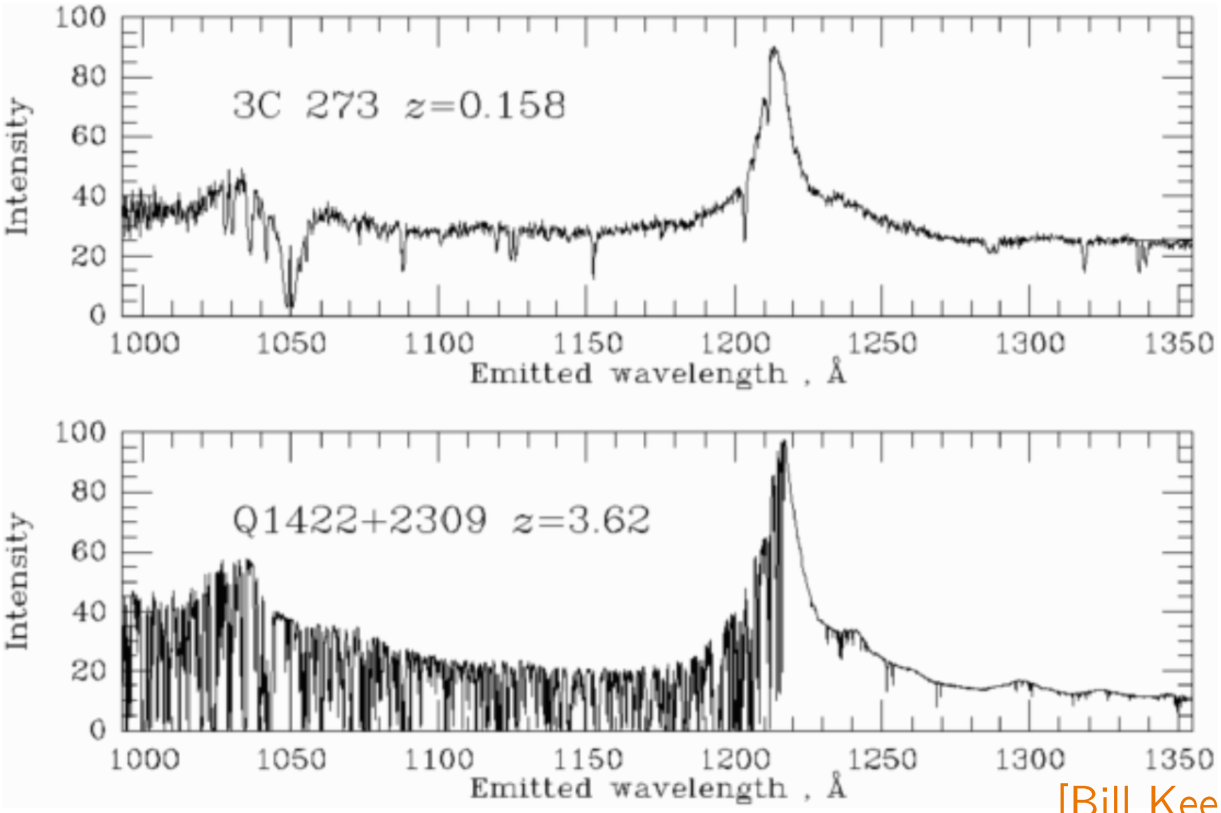
Lyman-alpha forest

[Edward L. Wright's website]



- Quasi-Stellar Objects (QSO) are luminous astrophysical objects powered by gas spiraling at high velocity into an massive black hole
- Light emitted by distant QSO is absorbed in foreground structures
- Allows for a 1D measure of overdensities along line of sight

Lyman-alpha forest



[Bill Keel's website]

- Comparison of QSO spectra at low and high redshift in QSO rest frame

DM Phase space distribution

- Obtain **phase space distribution** by solving **Boltzmann** equation

$$\frac{\partial f_\chi}{\partial t} - H|\boldsymbol{p}|\frac{\partial f_\chi}{\partial|\boldsymbol{p}|} = \mathcal{C}[f_\chi(|\boldsymbol{p}|, t)]$$

$$n_\chi(t) = \frac{g_\chi}{(2\pi)^3}\int \mathrm{d}^3\boldsymbol{p}\, f_\chi(p_0, t)$$

number-density

$$\rho_\chi(t) = \frac{g_\chi}{(2\pi)^3}\int \mathrm{d}^3\boldsymbol{p}\, p_0\, f_\chi(p_0, t)$$

energy-density

- **Collision term** for processes $\chi + a + b + \cdots \longleftrightarrow i + j + \cdots$

$$\begin{aligned}\mathcal{C}\left[f_\chi\right] = -\frac{1}{2p_0}\int \frac{g_a \mathrm{d}^3\boldsymbol{p}_a}{(2\pi)^3 2p_{a0}}\frac{g_b \mathrm{d}^3\boldsymbol{p}_b}{(2\pi)^3 2p_{b0}}\cdots\frac{g_i \mathrm{d}^3\boldsymbol{p}_i}{(2\pi)^3 2p_{i0}}\frac{g_j \mathrm{d}^3\boldsymbol{p}_j}{(2\pi)^3 2p_{j0}}\cdots \\ \times (2\pi)^4\,\delta^{(4)}(p_\chi + p_a + p_b + \cdots - p_i - p_j - \cdots) \\ \times \Big[|\mathcal{M}|^2_{\chi+a+b+\cdots\longrightarrow i+j+\cdots}\, f_a f_b \cdots f_\chi (1\pm f_i)(1\pm f_j)\cdots \\ - |\mathcal{M}|^2_{i+j+\cdots\longrightarrow \chi+a+b+\cdots}\, f_i f_j \cdots (1\pm f_a)(1\pm f_b)\cdots(1\pm f_\chi)\Big]\end{aligned}$$

NCDM Cosmology

- Expand around (homogenous) **background quantities**

$$f(\boldsymbol{x},\boldsymbol{p},\tau)=\bar{f}(|\boldsymbol{p}|,\tau)[1+\Psi(\boldsymbol{x},\boldsymbol{p},\tau)]$$

- Expand **fluctuations** in term of **Legendre polynomials**

$$\Psi(\boldsymbol{k},\hat{\boldsymbol{n}},q,\tau)=\sum_{\ell=0}^{\infty}(-i)^{\ell}(2\ell+1)\Psi_{\ell}(\boldsymbol{k},q,\tau)P_{\ell}(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{n}})$$

- Express **fluctuations** in terms of **Legendre coefficients**

$$\delta\bar{\rho}=4\pi\left(\frac{T_\star}{a}\right)^4\int q^2\epsilon\bar{f}(q)\Psi_0\,\mathrm{d}q, \qquad \text{energy density fluctuation}$$

$$\delta\bar{P}=\frac{4\pi}{3}\left(\frac{T_\star}{a}\right)^4\int q^2\frac{q^2}{\epsilon}\bar{f}(q)\Psi_0\,\mathrm{d}q, \qquad \text{pressure (density) fluctuation}$$

$$(\bar{\rho}+\bar{P})\theta=4\pi k\left(\frac{T_\star}{a}\right)^4\int q^3\bar{f}(q)\Psi_1\,\mathrm{d}q, \qquad \text{velocity divergence}$$

$$(\bar{\rho}+\bar{P})\sigma=\frac{8\pi k}{3}\left(\frac{T_\star}{a}\right)^4\int q^2\frac{q^2}{\epsilon}\bar{f}(q)\Psi_2\,\mathrm{d}q, \qquad \text{anisotropic stress.}$$

NCDM Cosmology

- The phase space distribution satisfies **collisionless Boltzmann equation**

$$\frac{\partial f}{\partial \tau} + \frac{\mathrm{d}x^i}{\mathrm{d}\tau}\frac{\partial f}{\partial x^i} + \frac{\mathrm{d}q}{\mathrm{d}\tau}\frac{\partial f}{\partial q} + \frac{\mathrm{d}n_i}{\mathrm{d}\tau}\frac{\partial f}{\partial n_i} = 0\,,$$

- **Plugging** distribution expansion in **Legendre polynomials** give

$$\mathrm{d}s^2 = a(\tau)\big(-\mathrm{d}\tau^2 + (\delta_{ij} + h_{ij})\,\mathrm{d}x^i\,\mathrm{d}x^j\big)$$

in **synchronous** gauge ($h \equiv h_{ii}$)

$$\dot{\Psi}_0 = -\frac{qk}{\epsilon}\Psi_1 + \frac{1}{6}\dot{h}\frac{\mathrm{d}\ln\bar{f}}{\mathrm{d}\ln q}\,,$$

$$\dot{\Psi}_1 = \frac{qk}{3\epsilon}\Big(\Psi_0 - 2\Psi_2\Big)\,,$$

$$\dot{\Psi}_2 = \frac{qk}{5\epsilon}\Big(2\Psi_1 - 3\Psi_3\Big) - \Big(\frac{1}{15}\dot{h} + \frac{2}{5}\dot{\eta}\Big)\frac{\mathrm{d}\ln\bar{f}}{\mathrm{d}\ln q}\,,$$

$$\dot{\Psi}_\ell = \frac{qk}{(2\ell+1)\epsilon}\Big(\ell\Psi_{\ell-1} - (\ell+1)\Psi_{\ell+1}\Big)\,, \quad [\ell \geq 3]$$

For a non-relativistic species, higher multipoles are typically suppressed by (positive) powers of $q/\epsilon \sim p/m_{\mathrm{DM}}$, making any Ψ_ℓ with $\ell \geq 2$ much smaller than Ψ_0 and Ψ_1. In this case, the Boltzmann hierarchy can be truncated imposing $\Psi_\ell = 0$ for $\ell > 1$. In this (non-relativistic) case Ψ_0 depends only mildly on the variable q, and the integrals are dominated by the low $q \ll \epsilon$ regime so that we can identify $\delta P/\delta\rho \simeq \bar{P}/\bar{\rho} = w$.

NCDM Cosmology

- **Neglecting higher multipoles**, for very **non-relativistic** DM, integrating over momenta gives

$$\dot{\delta} = -(1+w)\left(\theta + \frac{\dot{h}}{2}\right) - 3\mathcal{H}\left(\hat{c}_s^2 - w\right)\delta + 9\mathcal{H}^2(1+w)\left(\hat{c}_s^2 - c_a^2\right)\frac{\theta}{k^2}\,,$$

$$\dot{\theta} = -\mathcal{H}\left(1 - 3\hat{c}_s^2\right)\theta + \frac{\hat{c}_s^2}{1+w}k^2\delta\,,$$

- In **matter domination**, from **Einstein equations**, metric perturbation follow

$$\ddot{h} + \mathcal{H}\dot{h} + 3(1+3w)\mathcal{H}^2\delta = 0\,,$$

- Which can be translated to evolution of **matter density fluctuations**

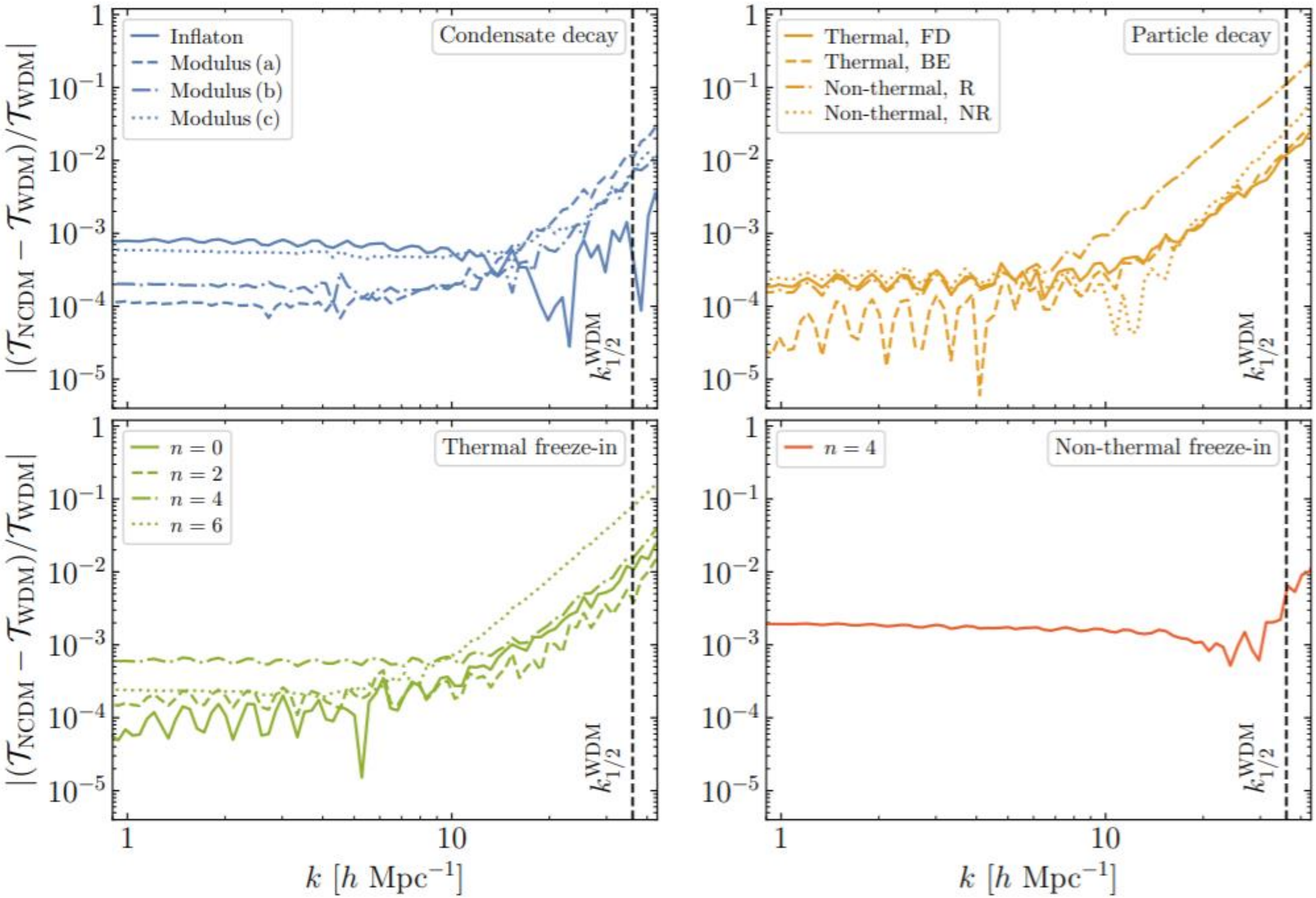
$$\ddot{\delta} + \mathcal{H}\dot{\delta} - \frac{3}{2}\mathcal{H}^2\left(1 - w\frac{10}{9}\frac{k^2}{\mathcal{H}^2}\right)\delta = 0\,.$$

General phase space distribution

$$f(q) \propto q^{\alpha} \exp\left(-\beta\, q^{\gamma}\right)$$

Scenario		α	β	γ
Inflaton decay		-3/2	0.74	1.00
Moduli decay	during reheating	-3/2	1.00	3/2
	after reheating	-1.00	1.00	2.00
Thermal decay		-1/2	1.00	1.00
Non-thermal decay	non-relativistic	-	-	-
	relativistic	-5/2	0.74	2.00
UV Freeze-in ($n = 0$)	BB	0.70	1.13	1.00
	FB	0.51	1.10	1.00
	FF	0.29	1.11	1.00
UV Freeze-in ($n = 2$)	BB	0.51	0.91	1.00
	FB	0.42	0.90	1.00
	FF	0.33	0.90	1.00
UV Freeze-in ($n = 4$)	BB	0.21	0.06	1.98
	FB	0.21	0.06	2.04
	FF	0.21	0.05	2.10
UV Freeze-in ($n = 6$)	BB	-	-	-
	FB	-	-	-
	FF	-	-	-
Non-thermal UV Freeze-in		-3/2	2.5	2.6

Precision on transfer functions



Contribution to $N_{\rm eff}$?

$$\begin{aligned}\Delta N_{\rm eff} &= \frac{8}{7}\left(\frac{T}{T_\nu}\right)^4 \frac{\rho_\chi - m_{\rm DM} n_\chi}{\rho_\gamma} \\ &= \frac{8\pi\Omega_\chi}{7\Omega_\gamma}\left(\frac{g_{*s}(T)}{g_{*s}^0}\right)^{4/3}\left(\frac{T}{T_\nu}\right)^4\left(\frac{T_\star}{m_{\rm DM}}\right) \\ &\times\left[\left\langle\sqrt{q^2+\left(\frac{g_{*s}^0}{g_{*s}(T)}\right)^{2/3}\left(\frac{m_{\rm DM}}{T_\star}\right)^2\left(\frac{T_0}{T}\right)^2}\right\rangle - \left(\frac{g_{*s}^0}{g_{*s}(T)}\right)^{1/3}\left(\frac{m_{\rm DM}}{T_\star}\right)\left(\frac{T_0}{T}\right)\right].\end{aligned}$$

- **Saturating** the Lyman-alpha bound gives

$$\begin{aligned}\Delta N_{\rm eff,max} &\simeq \frac{1.4\times10^{-4}}{\sqrt{\langle q^2\rangle}}\left(\frac{g_{*s}(T)}{g_{*s}^0}\right)^{4/3}\left(\frac{\Omega_\chi h^2}{0.1}\right)\left(\frac{3\,\rm keV}{m_{\rm WDM}}\right)^{4/3}\left(\frac{T}{T_\nu}\right)^4 \\ &\times\left[\left\langle\sqrt{q^2+\mu_*(T)^2}\right\rangle - \mu_*(T)\right],\end{aligned}$$

$$\mu_*(T) \equiv \sqrt{\langle q^2\rangle}\left(\frac{g_{*s}^0}{g_{*s}(T)}\right)^{1/3}\left(\frac{3\,\rm keV}{m_{\rm WDM}}\right)^{4/3}\left(\frac{7.56\,\rm keV}{T}\right).$$

$$\Delta N_{\rm eff}(T_{\rm BBN}) \lesssim 5.4\times10^{-4}\left(\frac{\langle q\rangle}{\sqrt{\langle q^2\rangle}}\right)\left(\frac{\Omega_\chi h^2}{0.1}\right)\left(\frac{3\,\rm keV}{m_{\rm WDM}}\right)^{4/3},$$