Option pricing on a chip

AI goes MAD
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June 2022

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OUTLINE

Present Small Scale Quantum Algorithms

One photon classifier

One photon universal approximant

One photon photonic chip for option pricing

Present Small Scale Quantum algorithms

Quantum Algorithms

Gate circuits

Search - Grover
QFT - Shor

Annealing

Adiabatic Evolution

Variational

Autoencoders
Eigensolvers
Classifiers

Realistic Quantum Algorithms for the NISQ Era

Gate circuits

Search - Grover
QFT - Shor

Annealing

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Variational Quantum Circuits

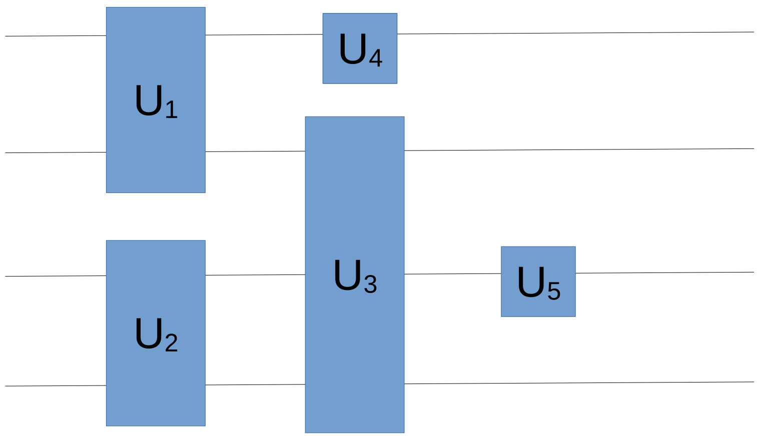
Learn from AI

Supervised learning
Unsupervised learning
Reinforcement learning

Q learning
Convolutional NN
Adversarial NN

Universal Approximant
Relation to Gaussian process
...

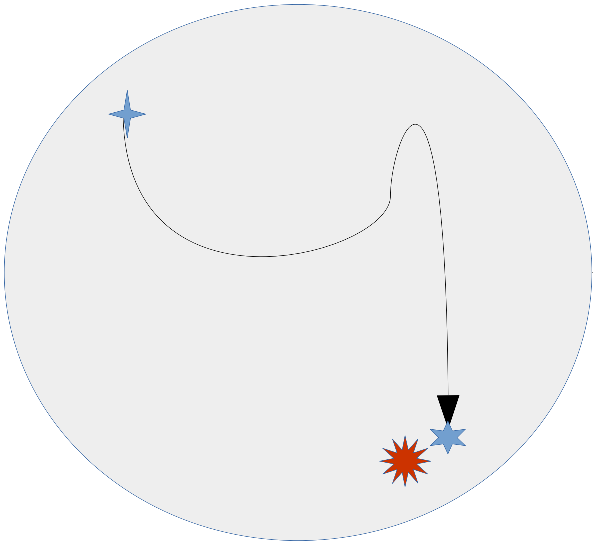
Rational for Variational Quantum Circuits



$$U(\vec{\alpha}) = U_n \dots U_2 U_1$$

Variational quantum states

= Explore a large (Hilbert) space



Near optimal solution

Classical characterization of a global unitary

Q Computer is a machine that can generate variational states

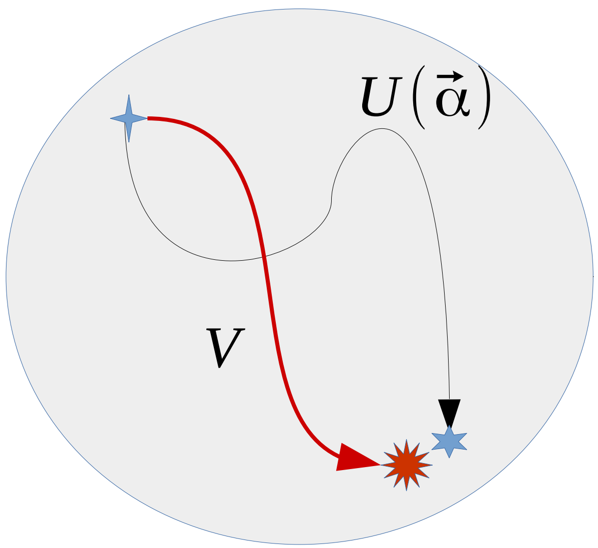
Variational Quantum Computer!!

Solovay-Kitaev Theorem

Circuit approximation to *V*

Let $\{U_i\}$ be a dense set of unitaries

$$|U_k \ldots U_2 U_1 - V| < \delta$$



optimal solution

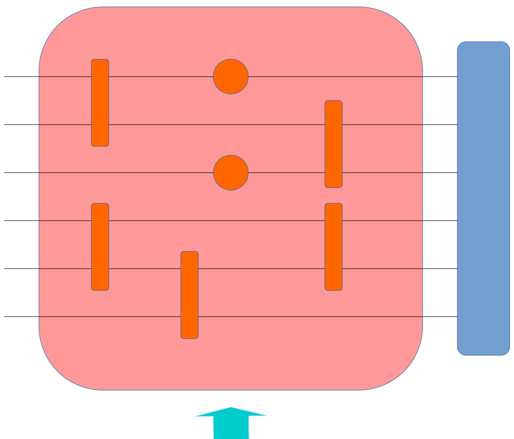
$$k \sim O\left(\log^c \frac{1}{\delta}\right) \qquad c < 4$$

To learn the e.g. ground state, only a column is needed

Variational Quantum Eigensolvers

Aspuru-Guzik et al.
IBM
Zapata
Blatt

any optimization problem



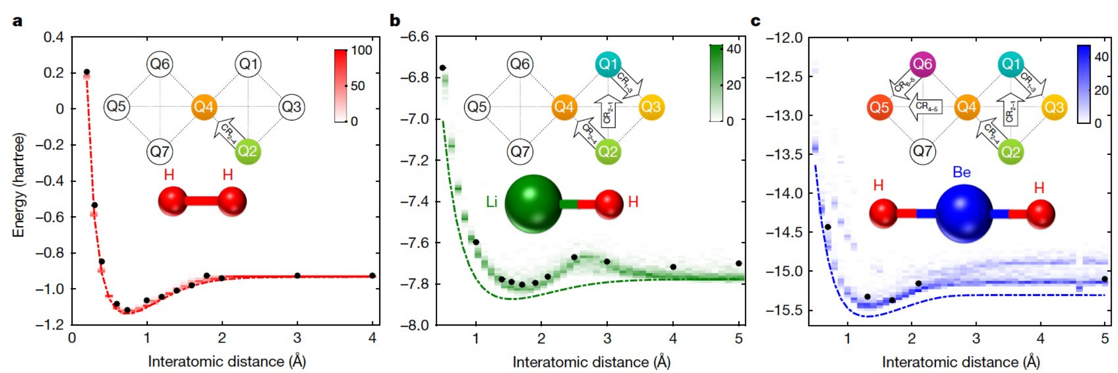
Read out

Energy
Correlations

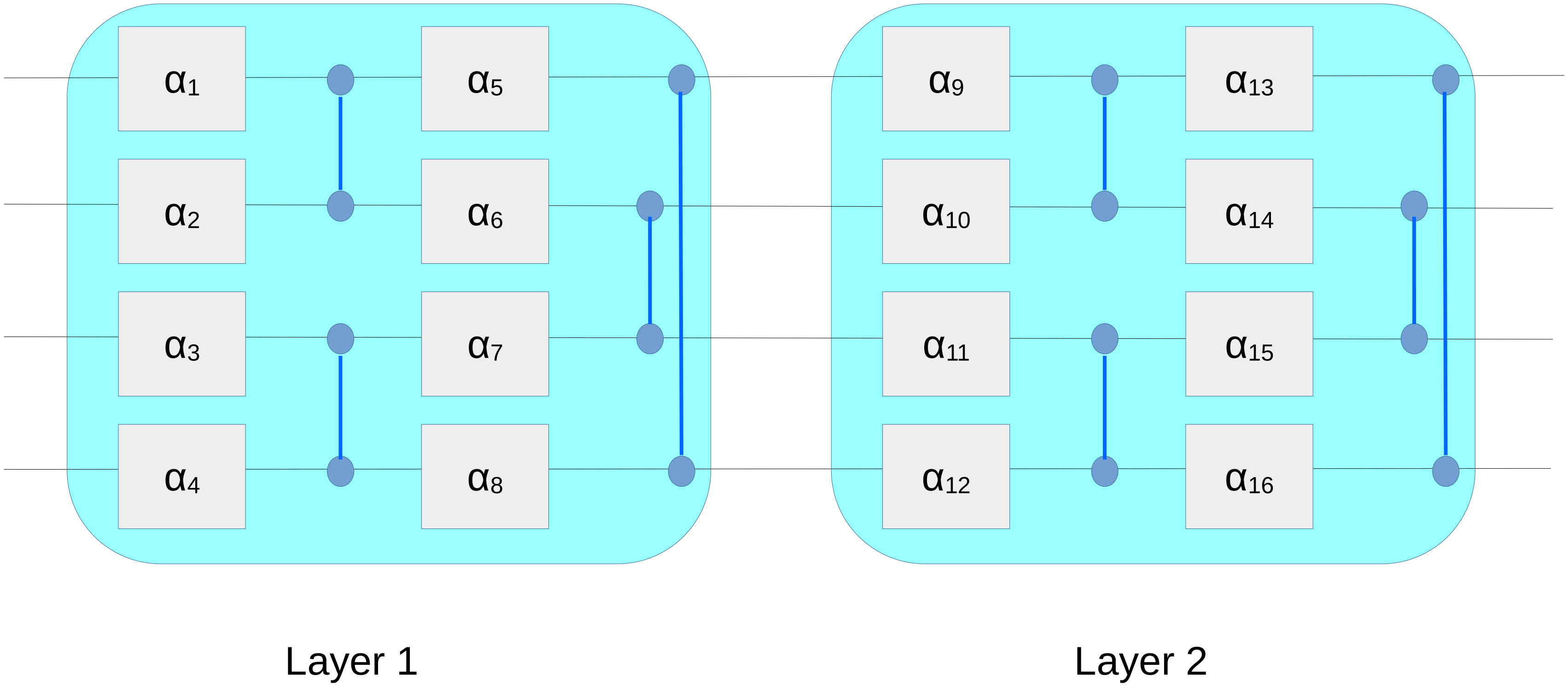
Machine learning
on circuit design

Hybrid classical/quantum algorithms

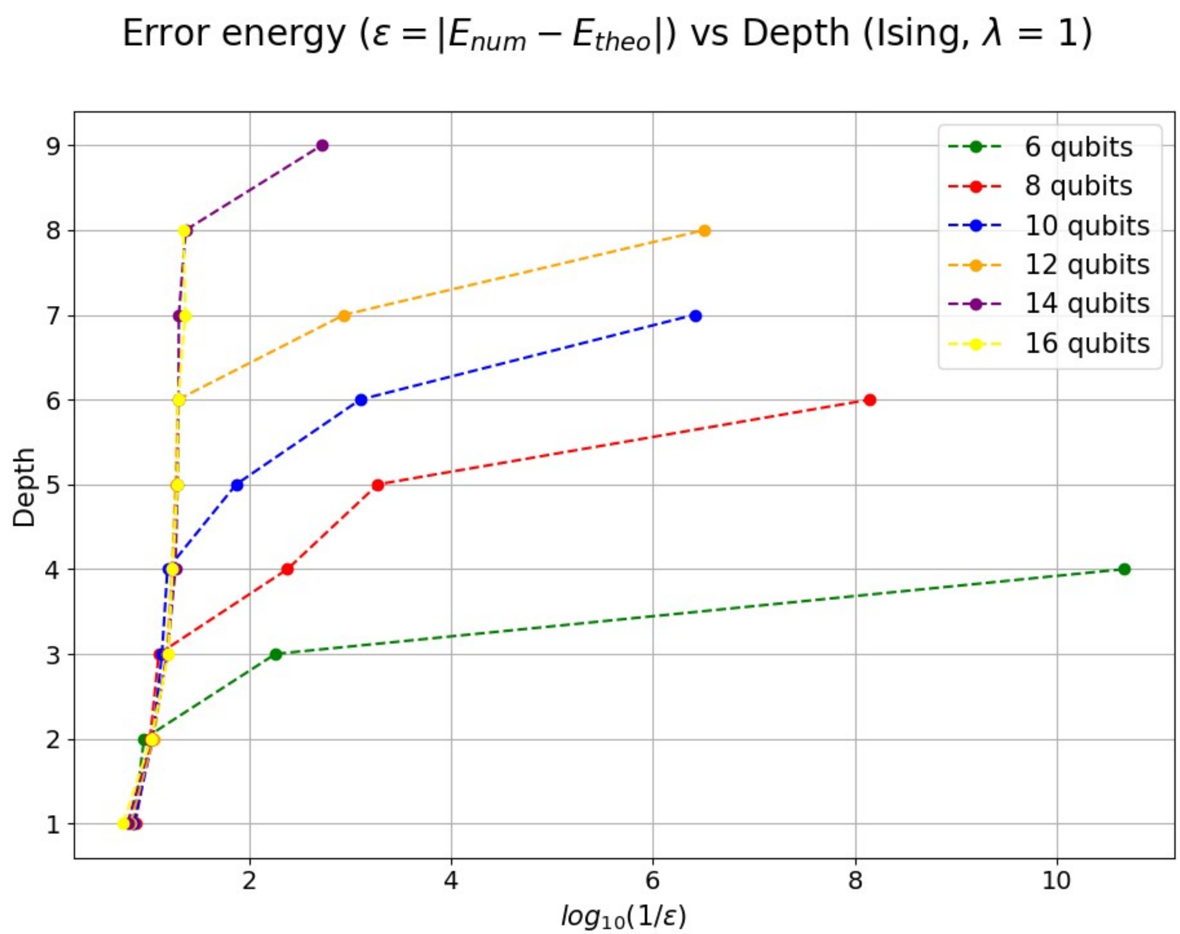
Quantum Chemistry



Example of variational processing



Note that qubits only talk to each other after some layers are operated!
Cave at: Barren plateaus



Entropy generation

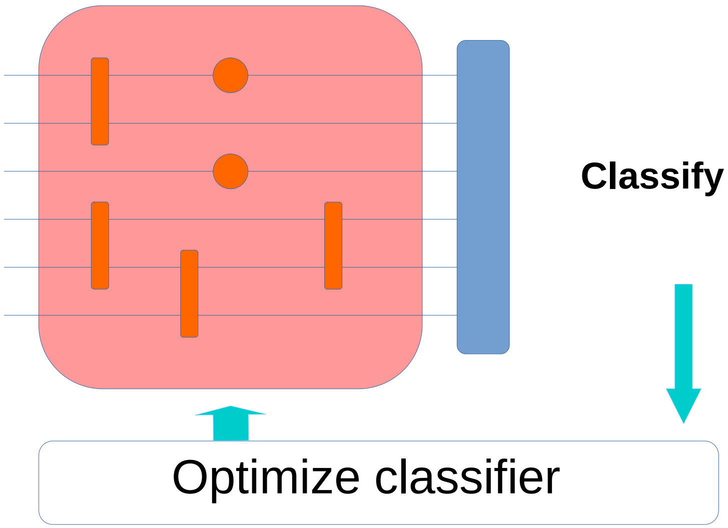
Entropy refinement

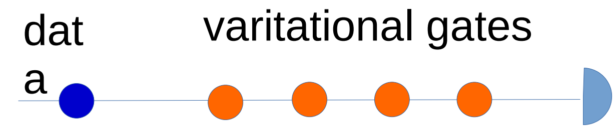
Simulation of CFT much better than bounds from Solovay-Kitaev

C. Bravo-Prieto, J. Lumbreras, L. Tagliacozzo, JIL (20

One Photon Quantum Classifier

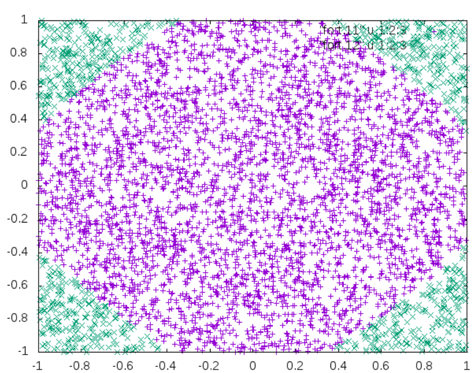
Variational quantum classifier

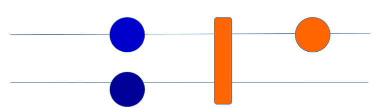




1 qubit naive classifier is useless

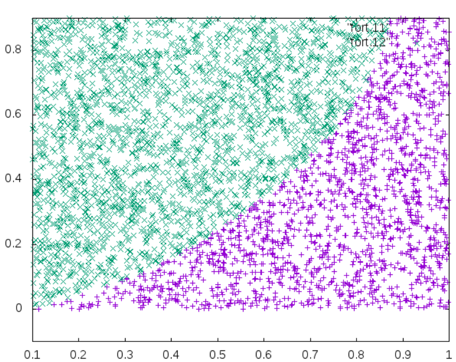
Classify points for a circle



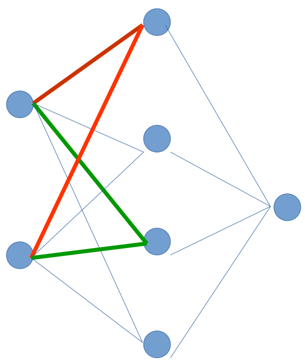


96.6% success
better if output 00 vs 11

Classify points for a parabola



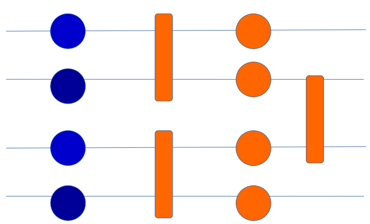
What is the minimal useful circuit?



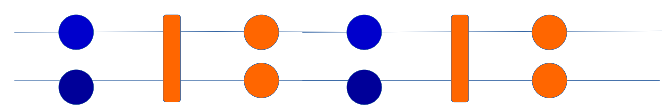
Parallel processing
= data are entered several times

Th: the larger the first hidden layer,
the more processing capabilities

Possible improvements



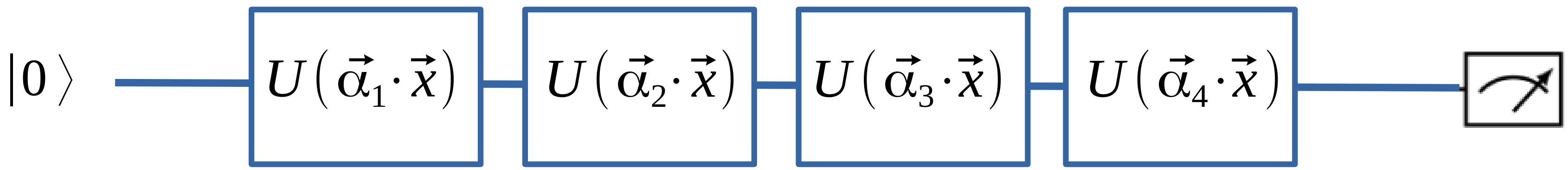
Enter data
twice



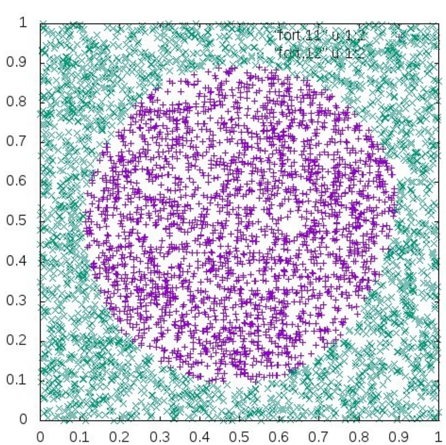
Re-enter data

A. Pérez-Salinas, A Cervera-Lierta, E. Gil, JIL (2019)

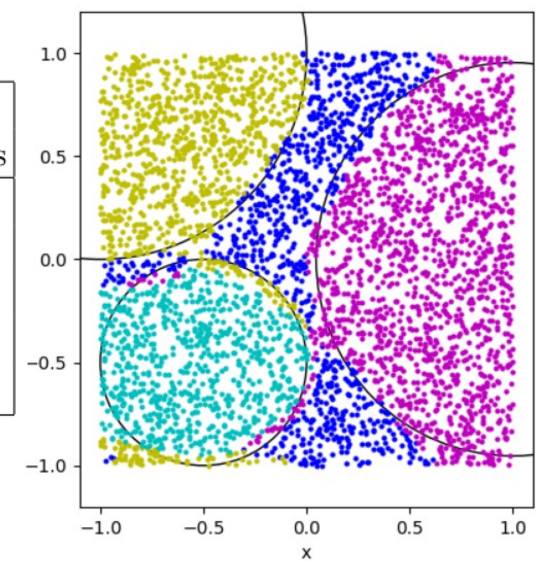
Re-uploading for a universal classifier with a single qubit



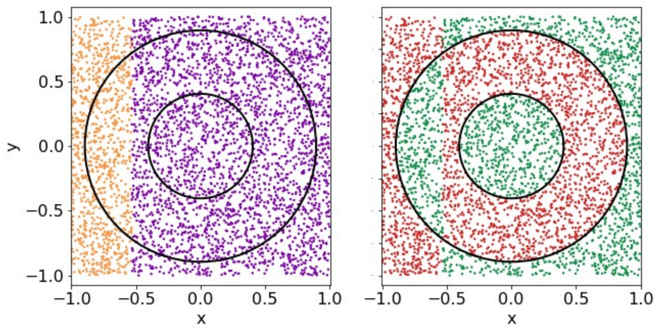
D dimensional via re-uploading
K categories via final measurement



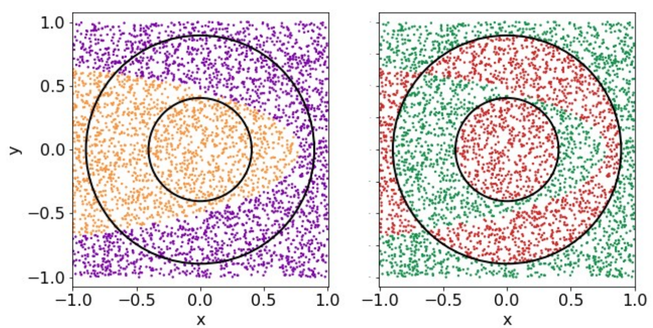
Layers	2 classes			4 classes	
	Circle	Sphere	Hypersphere	Wavy-lines	3-circles
1	75.2%	70.2%	68.0%	70.4%	74.5%
2	89.7%	75.0%	72.6%	88.2%	83.0%
6	92.8%	86.5%	93.2%	89.8%	83.8%
10	96.1%	91.7%	85.5%	90.0%	91.6%
20	96.9%	93.0%	89.2%	89.4%	92.3%



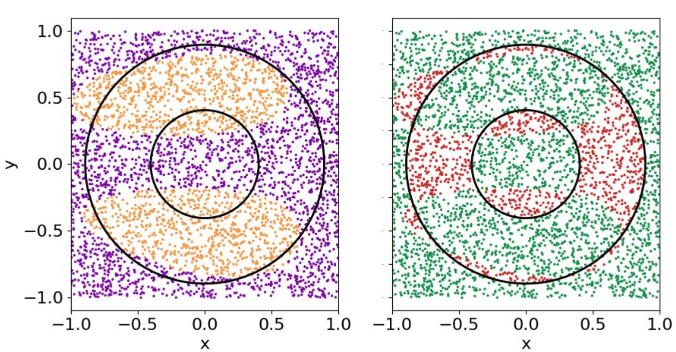
L 1



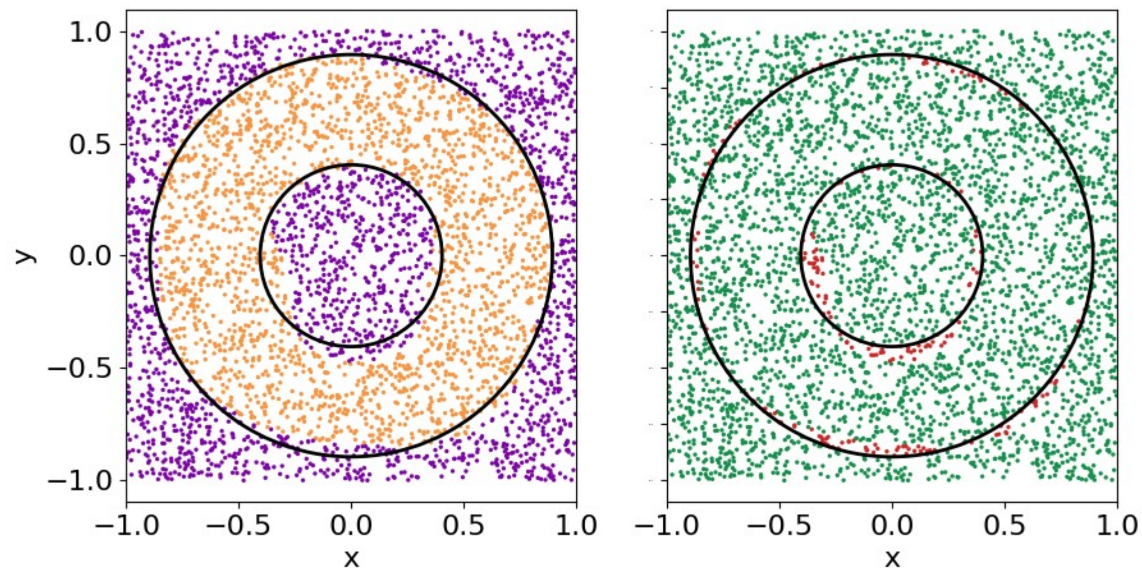
L 2



L 3

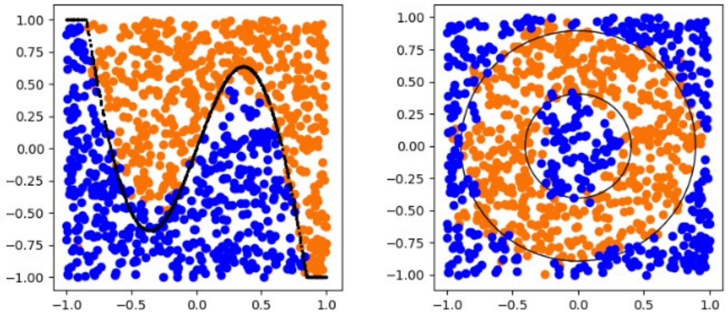


L 6



6 QC layers = 100 hidden NN neurons

And now, the experiment...



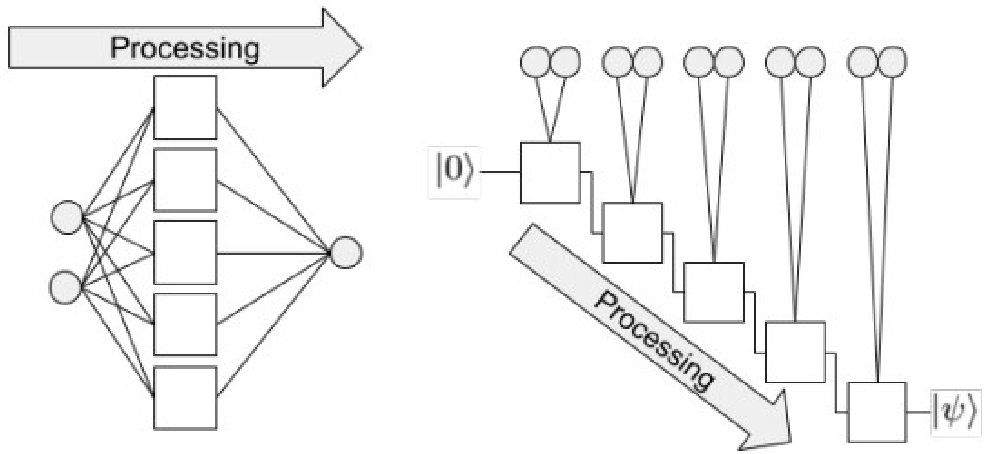
4 layers

Problem	Classical classifiers NN	SVC	Quantum classifier χ^2_{wf}	QPU χ^2_{wf}
Circle	0.96	0.97	0.97	*0.96(0.93) ± 0.01
3 circles	0.88	0.66	0.90	0.85 ± 0.04
Hypersphere	0.98	0.95	0.78	*0.76(0.64)±0.01
Sphere	0.97	0.95	0.72	0.59±0.06
Squares	0.98	0.96	0.97	0.92±0.04
Non-Convex	0.99	0.77	0.95	0.91±0.03
Wavy Lines	0.95	0.82	0.94	0.90±0.04
Binary annulus	0.94	0.79	0.92	0.84 ±0.03
Annulus	0.96	0.77	0.94	0.89 ± 0.05

An Ion based single qubit quantum classifier in NISQ era (2021)

T. Dutta, A. Perez-Salinas, J. Phua Sing Cheng, JIL, Manas Mukherjee

data re-uploading = query complexity



(a) Neural network (b) Quantum classifier

One Photon Universal Approximant

Let's face it: How powerful is 1 qubit?

What is the representation capability of 1 qubit?

Can we get any complex function as a result of the read-out?

$$z(x)=\sum_{n=-N}^{N} c_n e^{i 2\pi n x}$$

Harmonic analysis: Fourier series is a universal approximant

e.g. point-by-point convergence to any bounded function

Define a fundamental gate, and a circuit

$$U(x;\omega,\alpha,\beta,\varphi,\lambda)=R_z\left(\frac{\alpha+\beta}{2}\right)R_y(\lambda)R_z\left(\frac{\alpha-\beta}{2}\right)R_z(\omega x)R_y(\varphi)$$

$$U=\prod_i^k U_i(x;\omega_i,\alpha_i,\beta_i,\varphi_i,\lambda_i)$$

Notation

$$a_+=\cos\lambda\cos\varphi\, e^{i\alpha} \qquad b_+=-\cos\lambda\sin\varphi\, e^{i\alpha} \qquad c_+=\sin\lambda\cos\varphi\, e^{-i\beta} \qquad d_+=-\sin\lambda\sin\varphi\, e^{-i\beta}$$
$$a_-=-\sin\lambda\sin\varphi\, e^{i\beta} \qquad b_-=-\sin\lambda\cos\varphi\, e^{i\beta} \qquad c_-=\cos\lambda\sin\varphi\, e^{-i\alpha} \qquad d_-=\cos\lambda\cos\varphi\, e^{-i\alpha}$$

$$U=\begin{pmatrix} a_+e^{i\omega x}+a_-e^{-i\omega x} & b_+e^{i\omega x}+b_-e^{-i\omega x} \\ c_+e^{i\omega x}+c_-e^{-i\omega x} & d_+e^{i\omega x}+d_-e^{-i\omega x} \end{pmatrix}$$

Proceed by induction

Trivially, an initial gate with zero frequency delivers the first fourier term

Then, if N is true, prove N+1

$$\prod_{i=1}^{N} U_i = \begin{pmatrix} \sum_{n=N}^{N} A_n e^{i2\pi nx} & \sum_{n=N}^{N} B_n e^{i2\pi nx} \\ \sum_{n=N}^{N} C_n e^{i2\pi nx} & \sum_{n=N}^{N} D_n e^{i2\pi nx} \end{pmatrix}$$

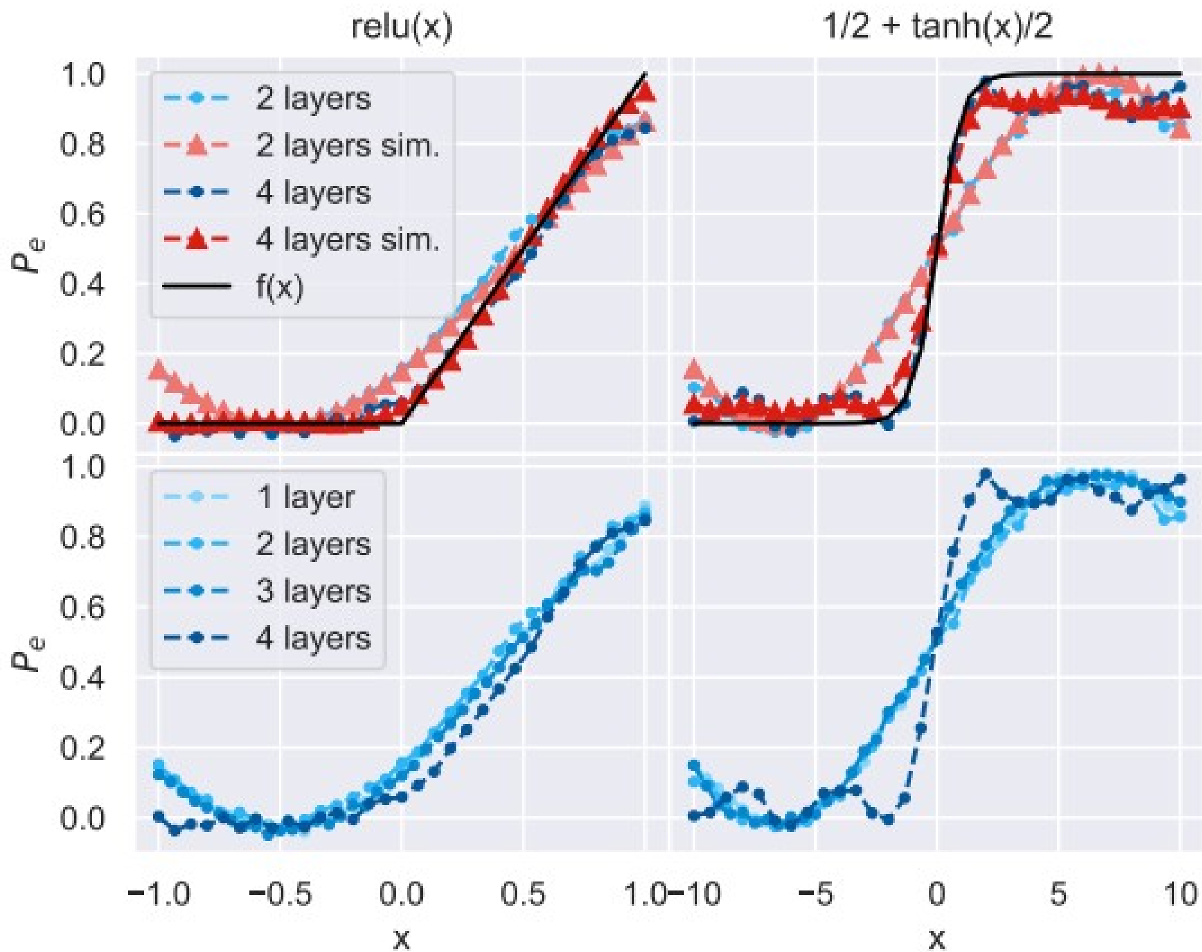
Indeed, e.g.

$$\widetilde{A}_0 = A_0 a_- + C_0 b_-$$
$$\widetilde{A}_{\pm n} = A_{\pm n} a_- + C_{\pm n} b_- + A_{\pm(n-1)} a_+ + C_{\pm(n-1)} b_+$$
$$\widetilde{A}_{\pm(N+1)} = A_{\pm N} a_+ + C_{\pm N} b_+$$

And similar expressions for the rest of terms

A. Pérez-Salinas, D. López, P. Forn-Díaz, JIL (2020)

Experimental proof!



N.B.

Both NN and Quantum Circuits are much better than a Fourier series because of the freedom of choosing-training frequencies

Adaptive Fourier expansion

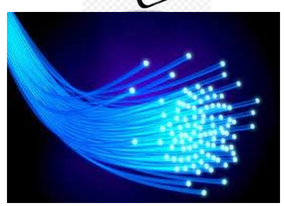
Role of many qubits?

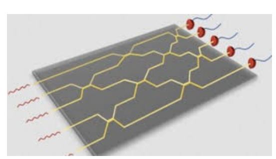
One photon quantum chip for Option Pricing

S. Ramos-Calderer, A. Pérez-Salinas, D. García-Martín, C. Bravo-Prieto, J. Cortada, J. Planaguma, and J. I. Latorre, Quantum unary approach to option pricing, PRA 103, 032414 (2021).

Focus

on sustainability
on demo/PoC





Integrated photonics

(Toy) System solutions

Use cases:
- Monte Carlo
- Edge computing
- Linear algebra
- ...

Need not be quantum

The (classical) problem

Asset evolves with stochastic equation
Monte Carlo simulation is necessary to price options

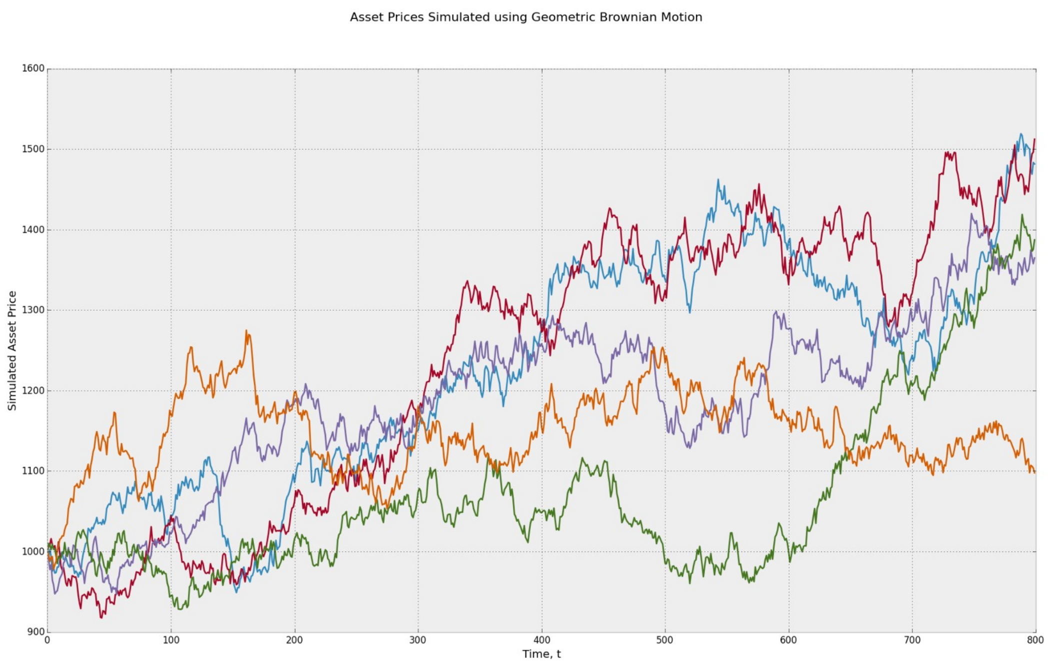
Asset value

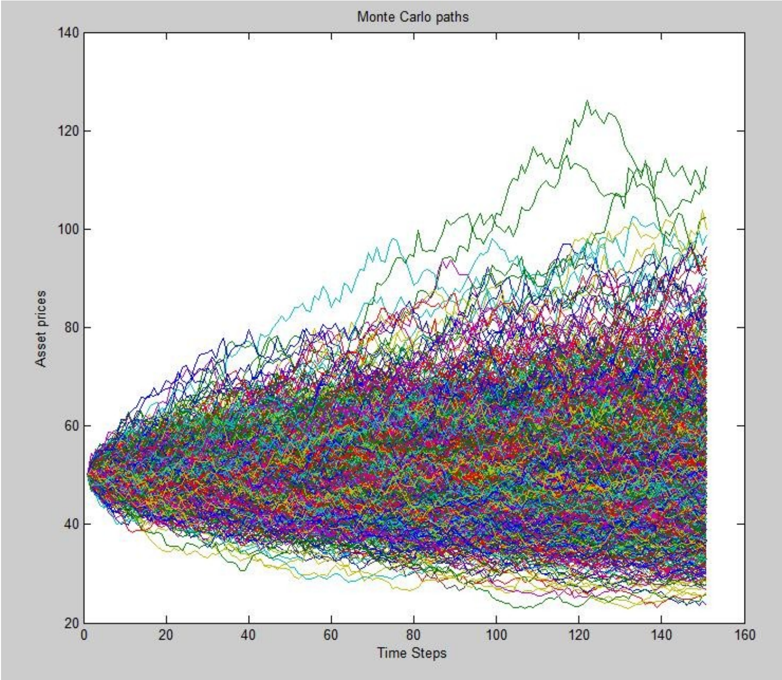
$$dS_t = S_t r\, dt + S_t \sigma\, dW_t,$$

Interest rate   Volatility   Stochastic variable

$$payoff = \sum_{S_T > k} Prob(S_T) S_T$$

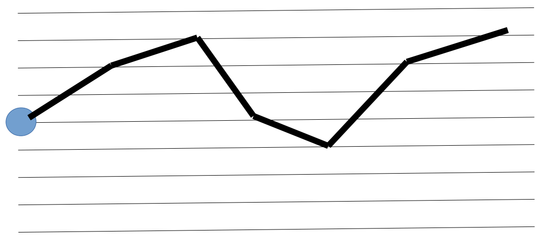
Strike   Maturity time



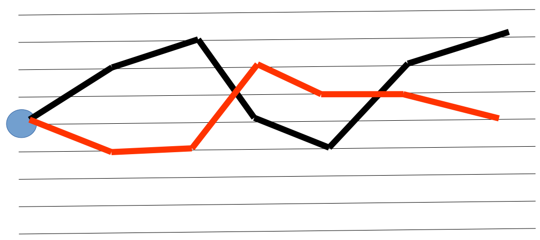


Computationally expensive Monte Carlo simulation!

Unary representation for asset value: path entanglement



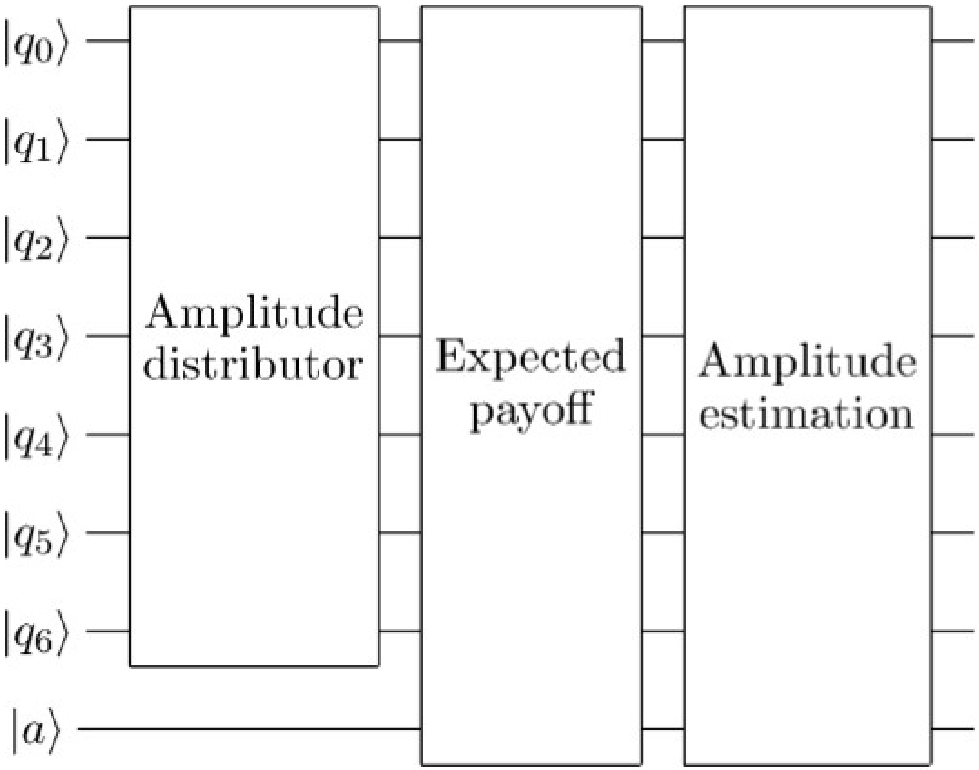
$$|\psi(T)\rangle = |0100000000\rangle$$

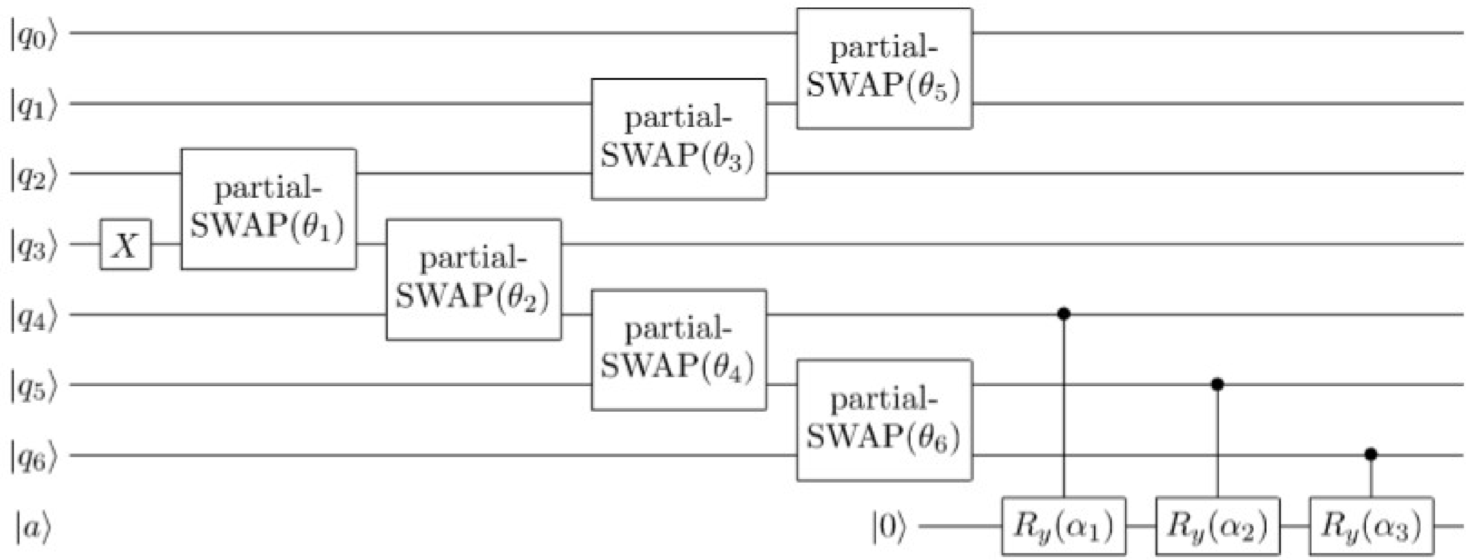


$$|\psi(T)\rangle = \frac{1}{\sqrt{2}}\left(|0100000000\rangle + |0000100000\rangle\right)$$

$$|\psi(T)\rangle = \sum_i \sqrt{p_i}|i\rangle$$

Quantum circuit structure for option pricing

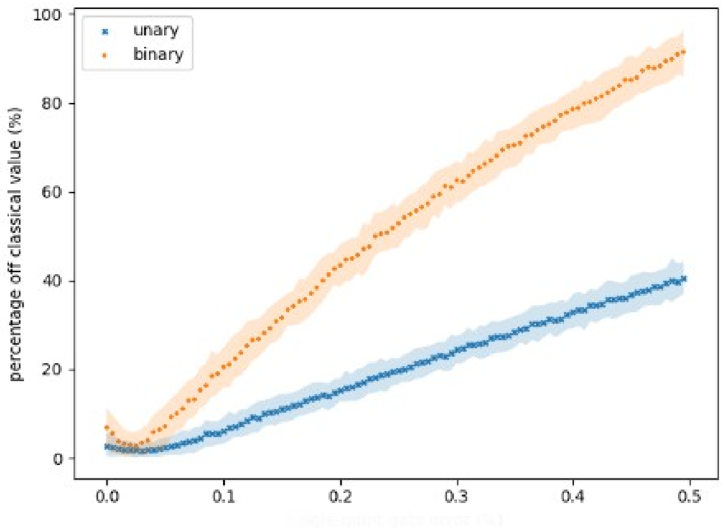




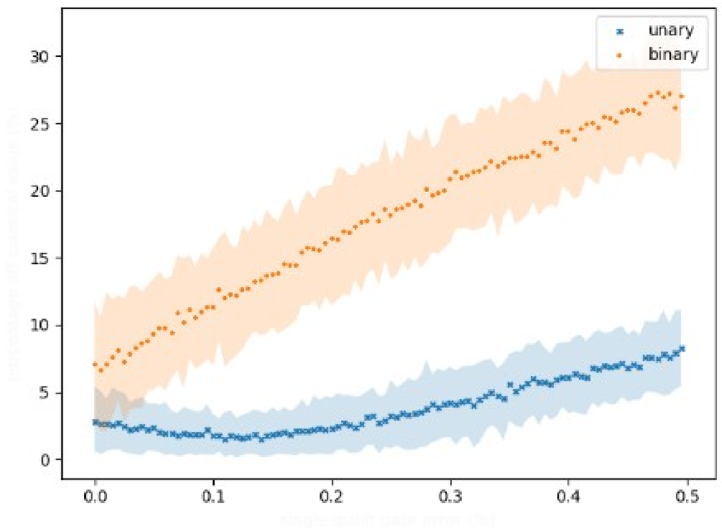
$$|\Psi\rangle = \sum_{S_i<K}^{n-1} \sqrt{p_i}\,|u_i\rangle\,|0\rangle + \sum_{S_i>K}^{n-1} \sqrt{p_i}\cos(\alpha_i/2)\,|u_i\rangle\,|0\rangle + \sum_{S_i>K}^{n-1} \sqrt{p_i}\sqrt{\frac{S_i-K}{S_{max}-K}}\,|u_i\rangle\,|1\rangle$$

$$P(|1\rangle) = \sum_{S_i>K} p_i \frac{S_i-K}{S_{max}-K}.$$

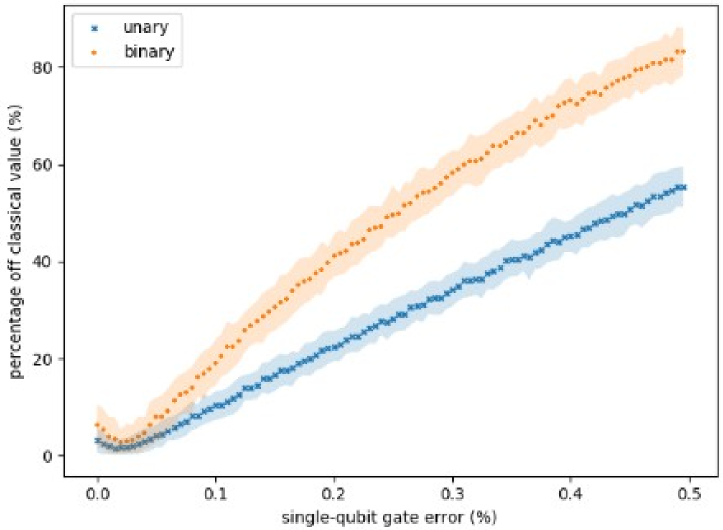
Errors and postselection



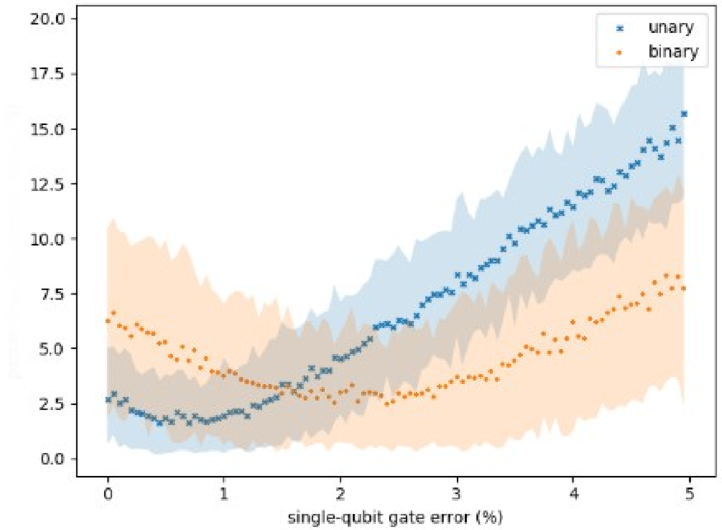
(a) bitflip error



(b) phaseflip error



(c) bitphase error

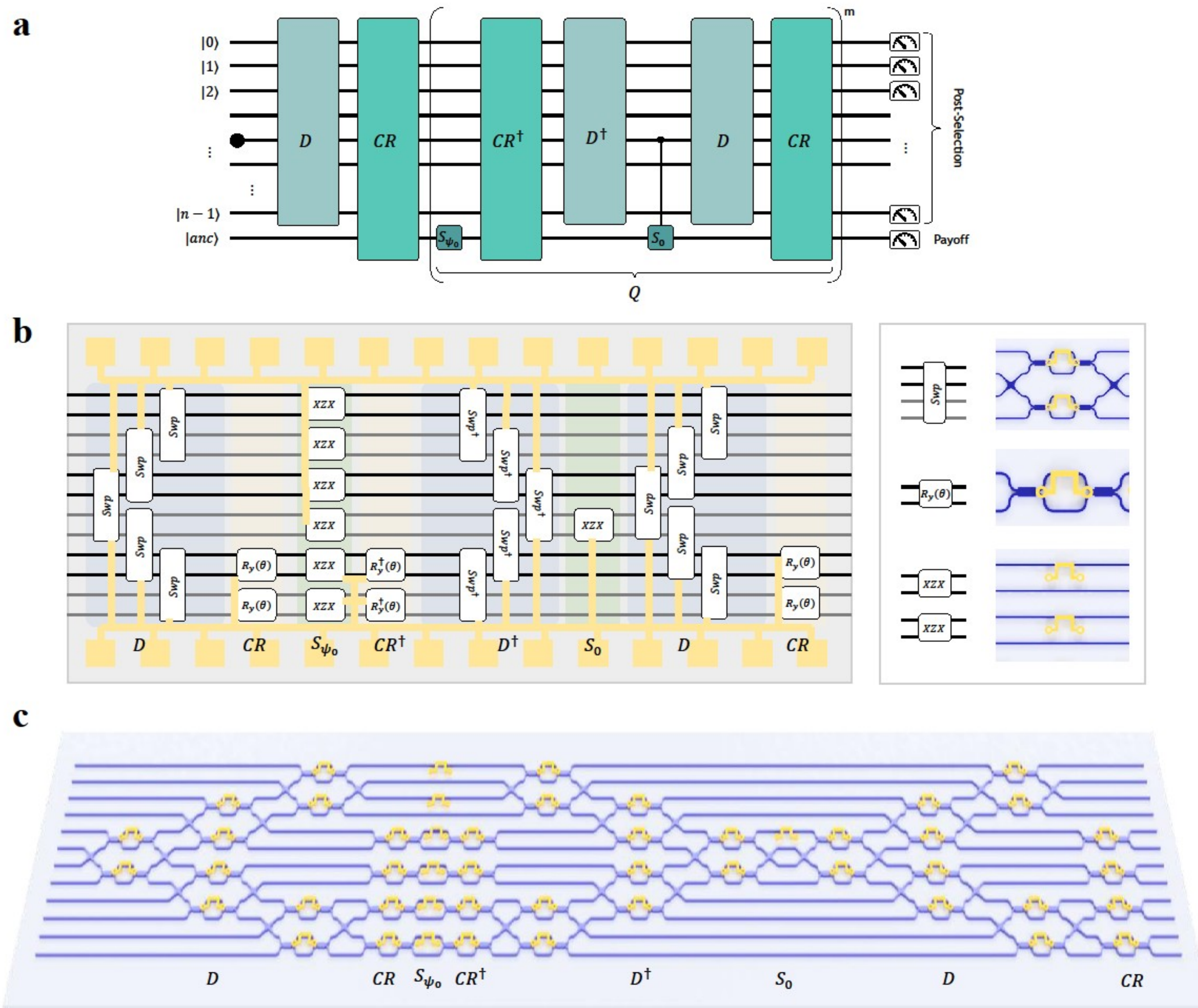


(d) measurement error

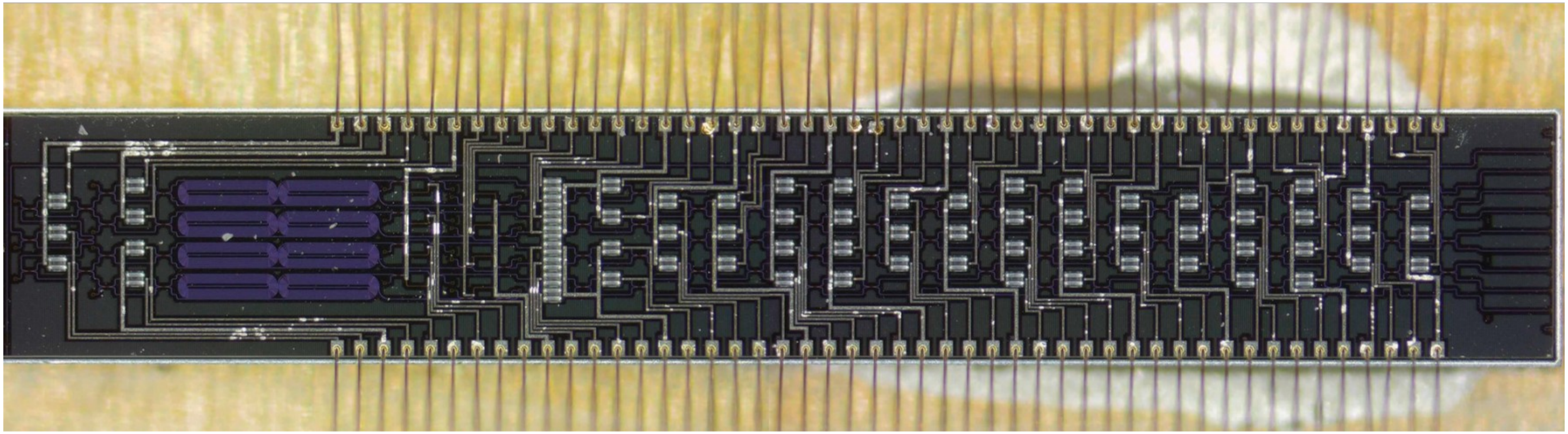
Quantum unary circuit for Option Pricing on a photonic chip

H. Zhang, L. X. Wan, S. Ramos-Calderer, Y. C. Zhan, W. K. Mok,,H. X. Lin, H. Cai, G. Q. Lo, X. S. Luo, L. C. Kwek, J. I. Latorre, and A. Q. Liu
Quantum unary approach to option pricing: GANs in a photonic chip

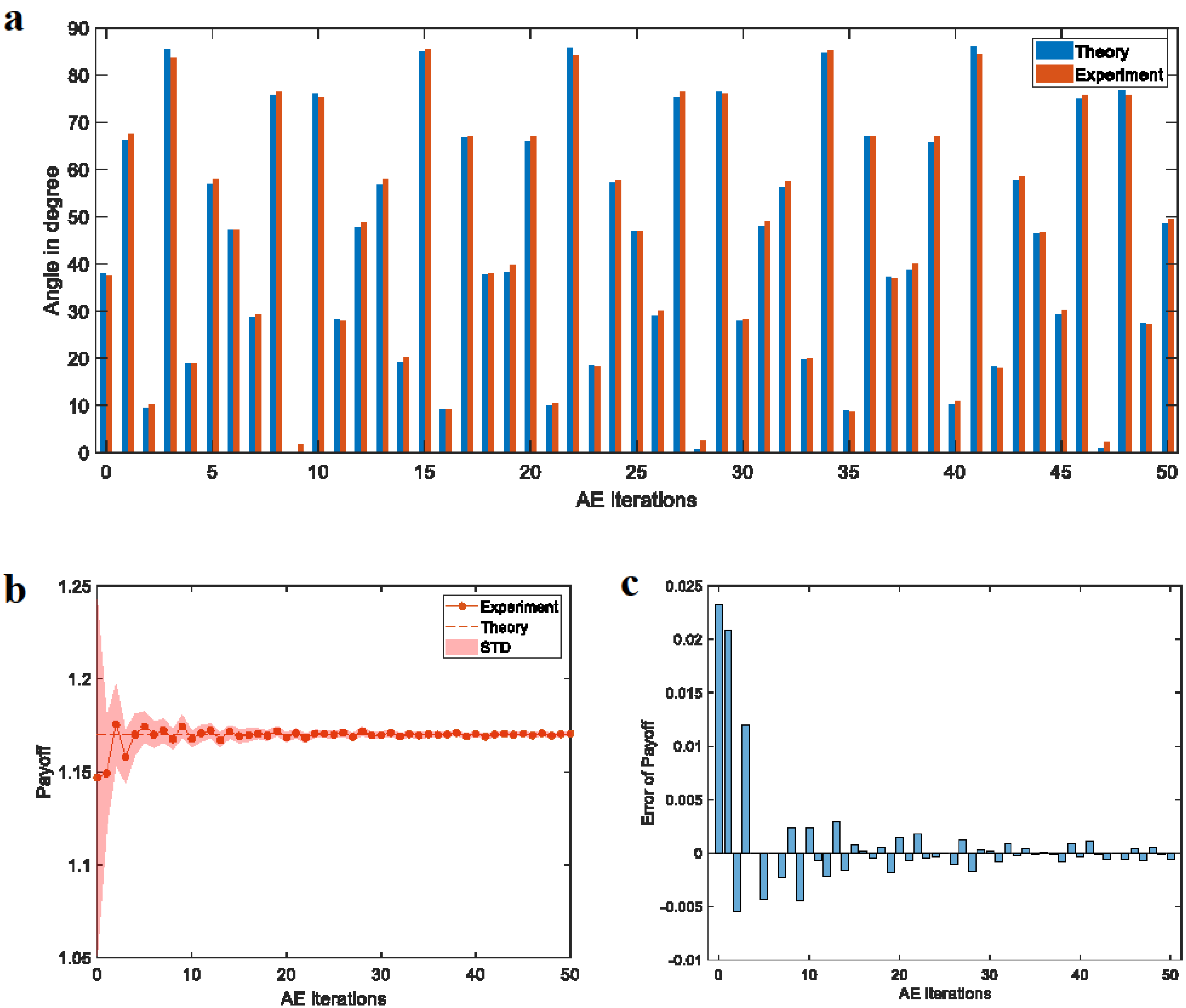
Quantum unary approach to option pricing: GANs in a photonic chip



Demo chip



Quantum Science and Engineering Centre (QSec@NTU)
Institute of Microelectronics, A*STAR
Advanced Micro Foundry
National Institute of Education
NTU
CQT
TII, Abu Dhabi

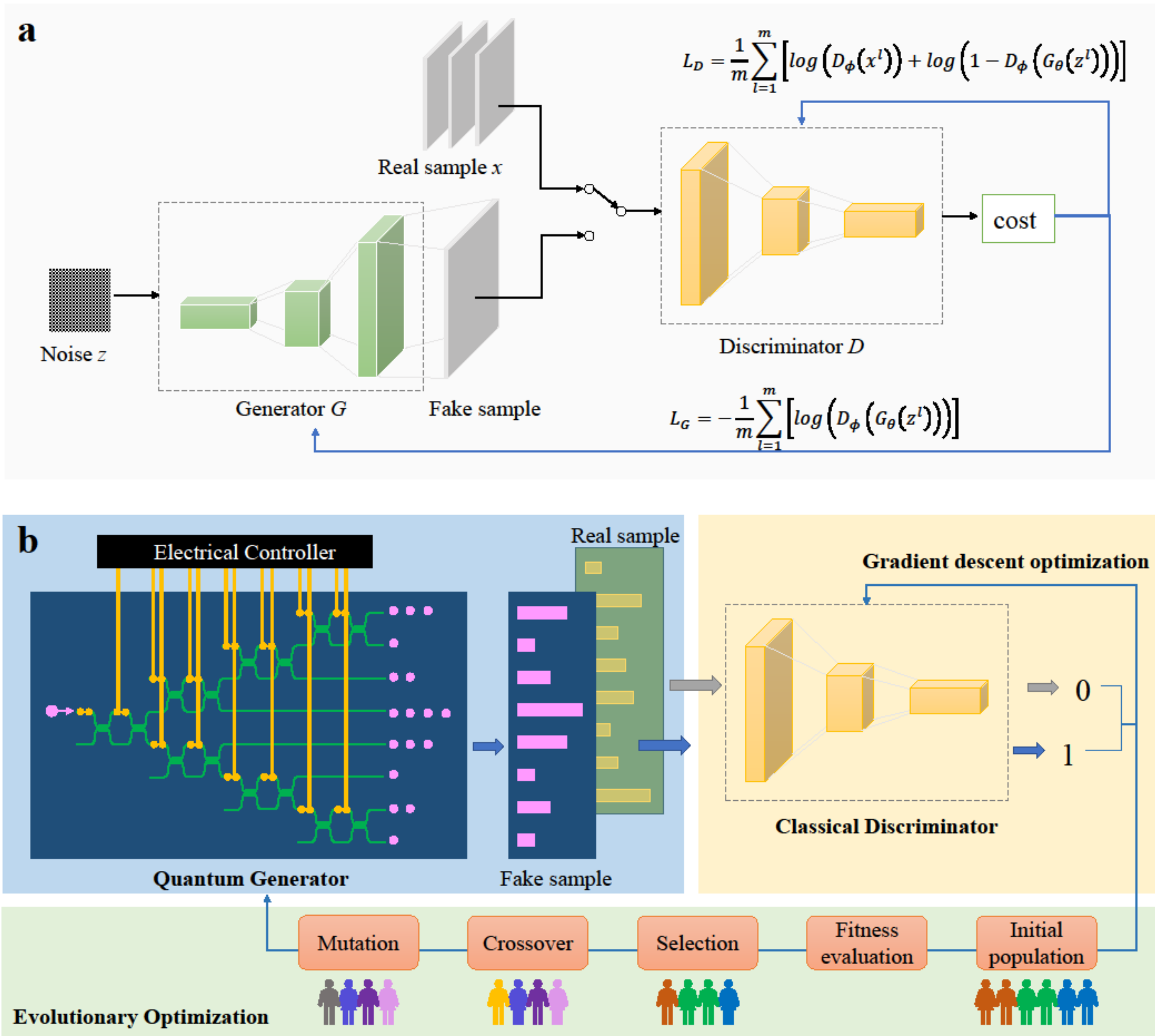

a
Angle in degree
AE iterations
Theory
Experiment
b
Payoff
AE iterations
Experiment
Theory
STD
c
Error of Payoff
AE iterations

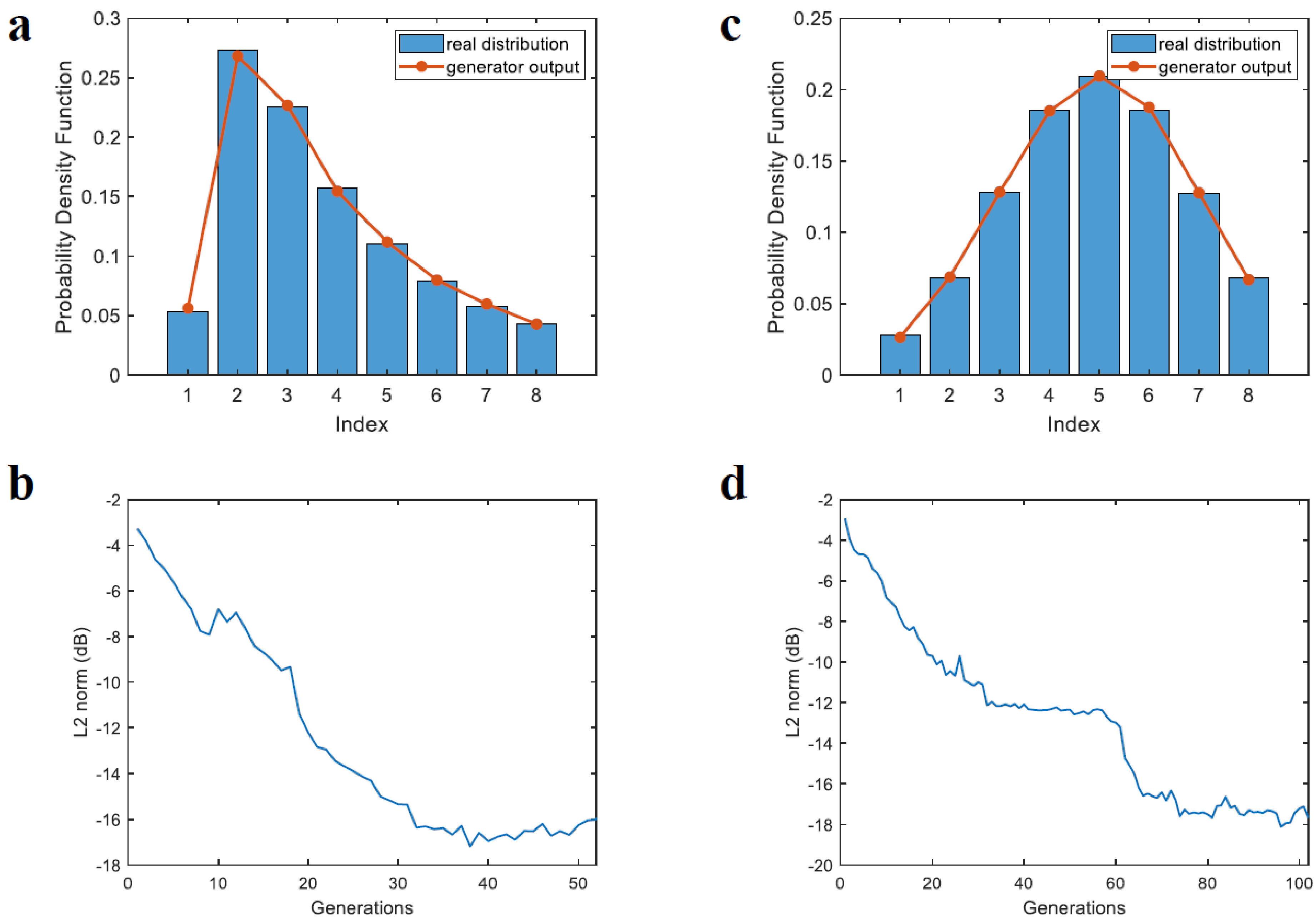
Quantum unary circuit on a photonic chip

trained with GANs

Centre for Quantum Technologies

NUS National University of Singapore




a
real distribution
generator output
Probability Density Function
0.3
0.25
0.2
0.15
0.1
0.05
0
1 2 3 4 5 6 7 8
Index
c
real distribution
generator output
Probability Density Function
0.25
0.2
0.15
0.1
0.05
0
1 2 3 4 5 6 7 8
Index
b
L2 norm (dB)
-2
-4
-6
-8
-10
-12
-14
-16
-18
0 10 20 30 40 50
Generations
d
L2 norm (dB)
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
0 20 40 60 80 100
Generations

Very likely, no need for single photon

Conclusion

1 qubit is a lot

Reuploading as a resource: query compexity

Low Entanglement compatible with quantum advantage
(negative signs in the Q operator)

Photonic chips are natural platforms for some simple tasks

Need to rethink 5-50 qubits

ThanQs!