A consistency check for the standard cosmological model with Gaussian Processes

Matteo Martinelli

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Madrid, June 15th, 2022

based on :

L. Perenon, MM, S. Ilic, R. Maartens, M. Lochner Phys.Dark Univ. 34 (2021)

L. Perenon, MM, R. Maartens, S. Camera, C. Clarkson *coming soon*

Outline

1. The cosmological standard model
2. A consistency check for ΛCDM
3. Gaussian Process reconstruction
4. Results and conclusions **(PRELIMINARY!!)**

Overview

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The GR-ΛCDM model

The cosmological principle (homogeneous and isotropic Universe), together with the framework of General Relativity allows to describe the expansion of the Universe

$$ds^2 = -dt^2 + a^2(t)dx^2\,, \qquad G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$H^2(a) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\sum_i \rho_i\,, \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\sum_i \rho_i(1+3w_i)$$

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- "baryons": how cosmologists call standard matter
- Dark Matter: unknown component responsible for most of the mass of the Universe
- Dark Energy: accounts for late time accelerated expansion

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GR+FLRW+baryons+DM+Λ = ΛCDM

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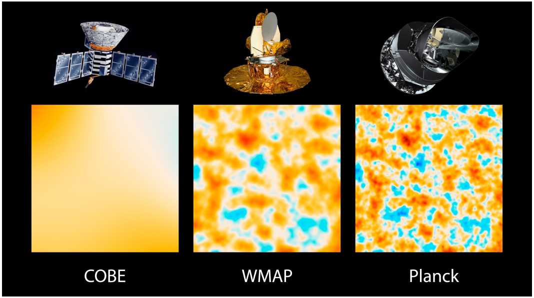

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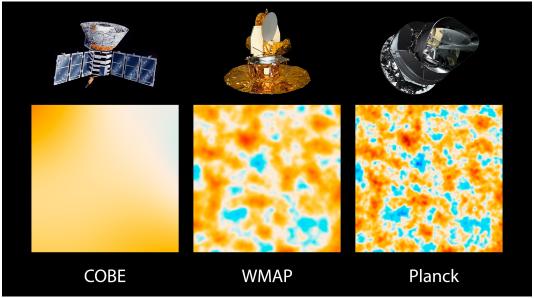

We describe the Universe with **small** perturbations of the FLRW metric

$$ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1-2\Phi)d\vec{x}^2$$

$$k^2\Psi = -4\pi G a^2 \rho\Delta \qquad k^2\left[\Phi+\Psi\right] = -8\pi G a^2 \rho\Delta$$

A good match for observations

Using these assumptions we can obtain prediction for observables and compare with data.

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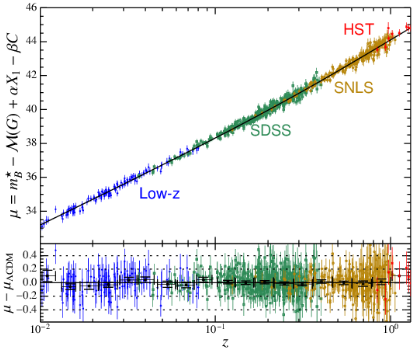

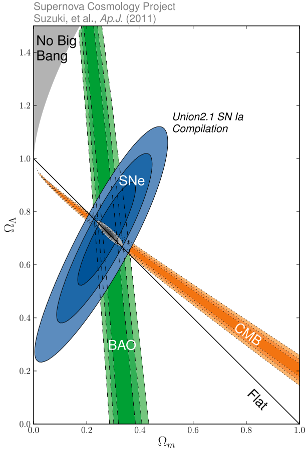

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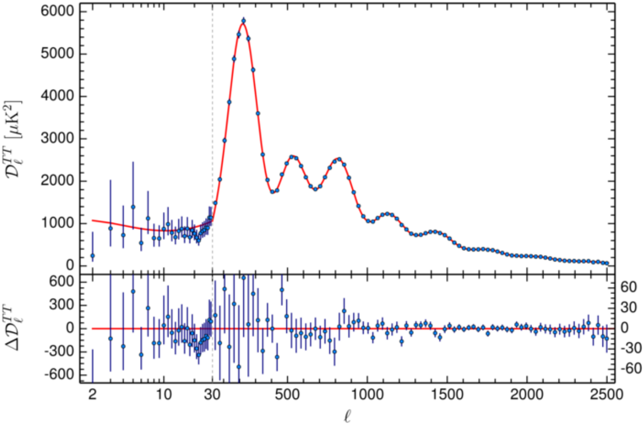

Planck 2015 results I. Overview of products and scientific results

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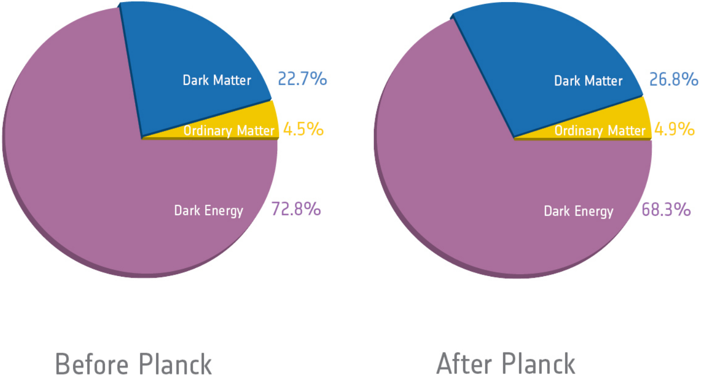

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Despite its success, ΛCDM still has some open theoretical questions

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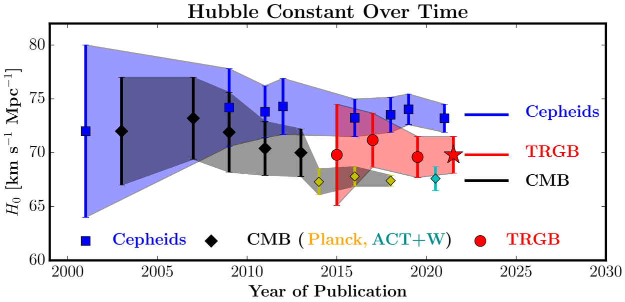

W. Freedman arXiv: 2106.15656

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How can we solve these tensions?

- Blame systematics!
- New physics!

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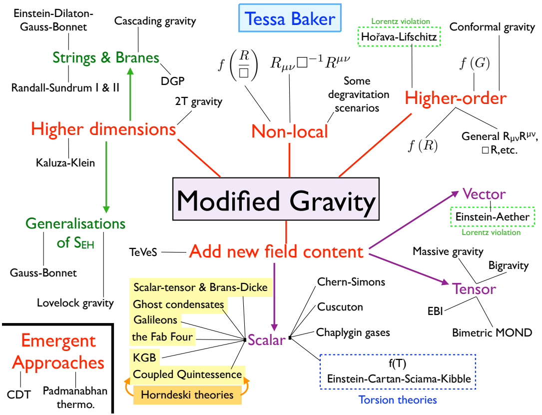

Bull, Akrami et al. Physics of the Dark Universe 12 (2016)

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We can identify the key features of ΛCDM and parameterize deviations from them

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$$w_{\rm de} = w(a)$$
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$$k^2\left[\Phi+\Psi\right] = -8\pi G\Sigma(a,k) a^2 \rho\Delta$$
$$\frac{\Phi}{\Psi} = \eta(a,k)$$

Observables for cosmological evolution

In order to test the ΛCDM model, we want to constrain the function $w_{\rm de}(z)$.

This is not directly observable, but enters theoretical predictions for quantities that are, e.g. $H(z)$ and $f(z)$.

At the background level, one can obtain measurements of $H(z)$ through Cosmic Chronometers (CC), i.e. measurements of the rate of expansion through redshift and age measurements of galaxies (based on star formation models).

Galaxy surveys can instead provide information on the growth rate by observing the clustering of cosmological structures

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Here we use a compilation of $H(z)$ and $f(z)$ data points that is extracted from current observations

R. Bernardo, D. Grandón, J. Levi Said, V. Cardenas Phys. Dark Univ. 36 (2022)
F. Avila, A. Bernui, A. Bonilla, R. Nunes arXiv:2201.07829

Connecting $w_{\rm de}$ to observables: $w_{\rm bg}$

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Exploiting the continuity equation for DE in a flat ($\Omega_k = 0$) FLRW Universe

$$\frac{\mathrm{d}\ln H(z)}{\mathrm{d}\ln(1+z)} \equiv -\frac{\dot{H}(z)}{H^2(z)} = \frac{3}{2} + \frac{3}{2}\left[1-\Omega_{\rm m}(z)\right] w_{\rm de}(z)\,,$$

we can express the EoS parameter of dark energy as a function of only background quantities

$$\boxed{w_{\rm bg}(z) = \frac{1}{1-\Omega_{\rm m}(z)}\left[\frac{2}{3}\frac{\mathrm{d}\ln H(z)}{\mathrm{d}\ln(1+z)} - 1\right]}$$

We saw that $H(z)$ can be obtained from CC observations, while

$$\Omega_{\rm m}(z) = \Omega_{\rm m,0}\frac{H_0^2}{H^2(z)}\,.$$

Thus, $w_{\rm bg}$ can be completely specified by observations.

Connecting $w_{\rm de}$ to observables: $w_{\rm gr}$

The same approach can be used starting for the linear evolution equations of matter perturbations

$$\ddot{\delta}_{\rm m}(z) + 2\,H(z)\,\dot{\delta}_{\rm m}(z) - \frac{3}{2}\,\Omega_{\rm m}(z)\,H^2(z)\,\delta_{\rm m}(z) = 0\ ,$$

which can be rewritten as an evolution equation for the growth rate

$$\frac{{\rm d}\ln f(z)}{{\rm d}\ln(1+z)} = f(z) + \frac{1}{2} + \frac{3}{2}\,\left[\Omega_{\rm m}(z) - 1\right]\,w_{\rm de}(z) - 3\,\frac{\Omega_{\rm m}(z)}{2\,f(z)}\ ,$$

from which we can express $w_{\rm de}$ as a function of observable background and perturbation quantities

$$\boxed{w_{\rm gr}(z) = \frac{1}{1-\Omega_{\rm m}(z)}\,\left[\frac{2}{3}\,\left(f(z) - \frac{{\rm d}\ln f(z)}{{\rm d}\ln(1+z)}\right) - \frac{\Omega_{\rm m}(z)}{f(z)} + \frac{1}{3}\right]\ .}$$

Consistency checklist

Our aim is to constrain both $w_{\rm bg}$ and $w_{\rm gr}$ from observational data and compare the results we find.
Depending on this comparison we can obtain information on possible departures from ΛCDM:

- $w_{\rm bg} \neq -1$ departure from flat-ΛCDM caused by either curvature, DE, or MG, or even a combination of them.

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  - $w_{\rm bg} \neq -1 \neq w_{\rm gr}$, competing effects at play in the two sectors (e.g. massive neutrinos vs MG);
  - If $w_{\rm bg} = -1$, only perturbations affected (MG?), it can also hint for problems in how we obtain the data on $f(z)$.

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Reconstruction instead of fitting

Investigations of $w_{\rm de}$ are usually performed parameterizing such a function and constraining the free parameters, e.g. using CPL

$$w_{\rm de}(z) = w_0 + w_a \frac{z}{1+z}\,.$$

However, the limited freedom of parameterization could hide features present in the data that could be crucial to identify departures from ΛCDM and of extended models.

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The development of machine learning techniques provided tools that can be used to reconstuct the functions of interest without the need to assume their trends.

Here we decided to use Gaussian Processes to reconstruct cosmological functions from data.

Gaussian Process

Reconstructing a function with GP corresponds to assuming that the the values f^* at points X^* are Gaussian random variables with mean μ^* and variance K.

Observational data (y) are instead Gaussian variables, with noise entering the covariance matrix (C).

Assuming f^* and y come from the same distribution

$$\begin{bmatrix} y \\ f^* \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu \\ \mu^* \end{bmatrix}, \begin{bmatrix} K(X,X)+C & K(X,X^*) \\ K(X^*,X) & K(X^*,X^*) \end{bmatrix}\right).$$

K is the so-called kernel, which is an "arbitrary" function: this will contain hyper-parameters that need to be optimized by minimizing the likelihood

$$\ln \mathcal{L} = -\frac{1}{2}(y-\mu)^T \left[K(X,X)+C\right]^{-1}(y-\mu) - \frac{1}{2}\ln|K(X,X)+C| - \frac{n}{2}\ln 2\pi$$

Reconstruction a function with GP

Once the kernel is chosen, and we obtain the optimal hyper parameters, we can reconstruct the mean and covariance of the function we are interested in

$$\mathrm{mean}(f^*) = \mu^* + K(X^*, X)[K(X, X) + C]^{-1}(y - \mu),$$
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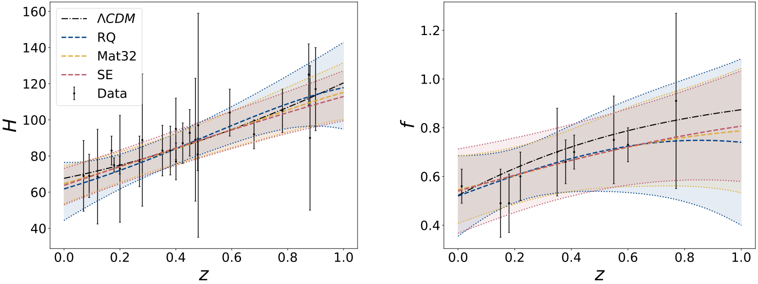

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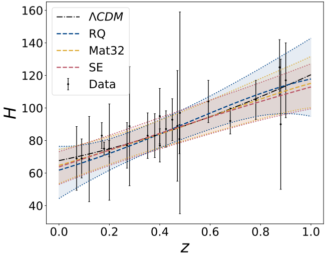
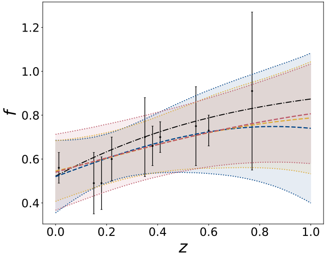
- mean priors μ have to be chosen and we need to check that the reconstruction is not dominated by this choice;
- the hyperparameters contained in the kernel need to be optimized and special attention is needed for their prior range as it needs to be wider than the constraints obtained from data;
- results should be reasonably stable changing the kernel choice.

Stability checks

We perform our reconstruction with 3 different kernels to test its stability:

- Squared Exponential (SE), simple and smooth reconstruction;
- Matern 3/2 (Mat32), captures sharp variations and performs well with noisy data;
- Rational Quadratic (RQ), combination of many SE kernels, one extra parameter.





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We also check that

- prior ranges for hyper parameters are wide enough, i.e. they are completely constrained by data and not limited by the prior;
- changing the mean prior (we use $\mu = 0$ and the ΛCDM prediction) does not change the results.

Pipeline for ΛCDM consistency test

In order to apply our consistency test, we follow the procedure below:

1. we choose kernel, hyper parameters range and mean prior for the GP;
2. we apply the GP reconstruction to our CC and galaxy data and obtain the reconstructed $H(z)$ and $f(z)$ functions;
3. from the former, we derive the reconstruction of $\Omega_{\rm m}(z) = \Omega_{\rm m,0} H_0^2/H^2(z)$, where $\Omega_{\rm m,0}$ is given by an external prior (independent measurements);
4. using these reconstructed functions and their derivatives, we obtain $w_{\rm bg}(z) = w_{\rm bg}(H, H', \Omega_{\rm m})$ and $w_{\rm gr}(z) = w_{\rm gr}(f, f', \Omega_{\rm m})$;
5. we compare the two functions and draw conclusions on the validity of ΛCDM.

Are we really model independent?

While GP allows to reconstruct functions without assumptions on their trend, but we are still affected by assumptions done in obtaining the equations for $w_{\rm de}$ and $w_{\rm gr}$:

- scale independent growth: this is an assumption that breaks down in MG (or with massive neutrinos). Our approach needs to be generalized to include this possibility;

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- scale independent growth: this is an assumption that breaks down in MG (or with massive neutrinos). Our approach needs to be generalized to include this possibility;
- flat Universe: we assume $\Omega_k = 0$. While the effects of this possible extra component would show up as deviations from $w_i = -1$, this assumption leads to reconstruction issue as this requires $\Omega_{\rm m}(z) < 1$ and

$$w_i \propto \frac{1}{1 - \Omega_{\rm m}(z)}$$

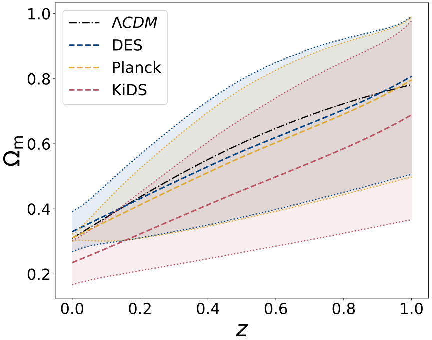
The $\Omega_{\mathrm{m}}(z)$ issue

The last point is quite relevant as for realizations crossing $\Omega_{\mathrm{m}} = 1$, the w_i functions will diverge!

To account for this we introduce a hard cut, rejecting all realizations that cross this boundary.

The amount of rejected changes depending on the choice of prior for $\Omega_{\mathrm{m},0}$, with KiDS being the least affected.

Having data on $\Omega_{\mathrm{m}}(z)$ could help with this issue.



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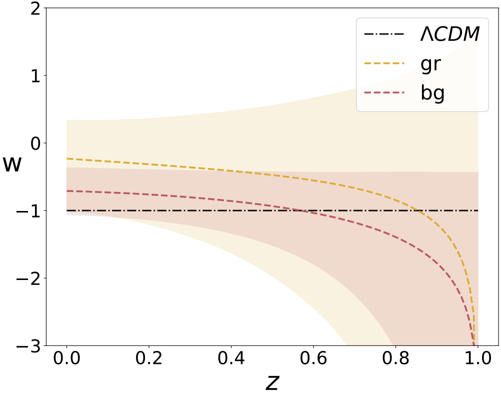
Current data results **(PRELIMINARY)**

When we reconstruct our $w(z)$ function using the current CC and galaxy data we find that the two functions are compatible with each other.

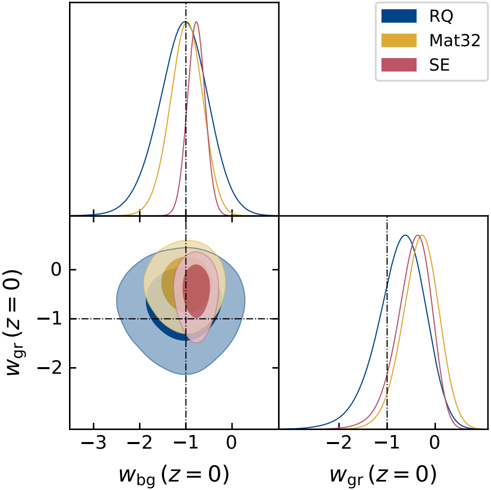
At "high" redshifts we see the effect of $\Omega_{\rm m}(z)$ getting close to one, when matter dominates over dark energy.



Current data results ($z = 0$) **(PRELIMINARY)**

If we project at current time ($z = 0$) we see an interesting hint of $w_{\rm gr} > -1$, although not statistically significant ($< 2\sigma$).

This trend is stable changing the specifics of GP, and it is most likely due to the preference of galaxy data for a suppressed growth (the S_8 tension).



Expectation for future experiments

We also attempted to understand how much future data can improve this test, and if this can detect a breakdown of ΛCDM should the Universe not follow this model.

In order to do so, we produced synthetic data for $H(z)$ and $f(z)$ in 10 redshift bins with a 1% error, so that we mimic what is expected from Stage IV experiments.

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We produce the data assuming two fiducial cosmologies:

- a simple DE model that affects both the reconstruction of $H(z)$ and $f(z)$

$$w(z) = w_0 + w_a \frac{z}{1+z}$$

- a modified gravity model where $H(z)$ is unchanged but perturbations growth is modified

$$f(z) = \Omega_{\rm m}^{\gamma}(z)$$

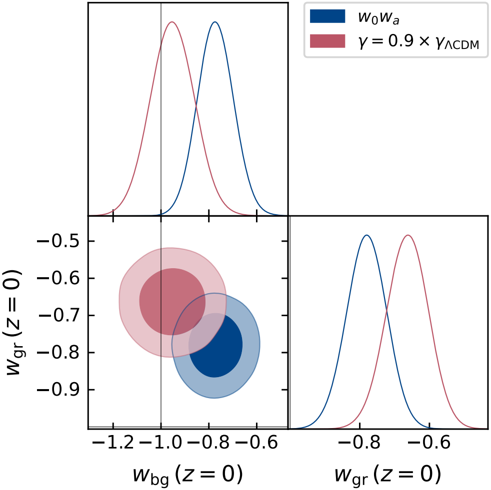
with $\gamma = 0.545$ being the GR limit, while we take a 10% variation from it.

E. Linder Phys.Rev.D 72 (2005)

Forecast results **(PRELIMINARY)**

The more precise data and prior on $\Omega_{\rm m,0}$ reduce the dependency on the choice of kernel and priors, while the realization rejected by the $\Omega_{\rm m}(z) < 1$ are reduced.

The method is able to detect both departures from ΛCDM that we assume and to distinguish between them.

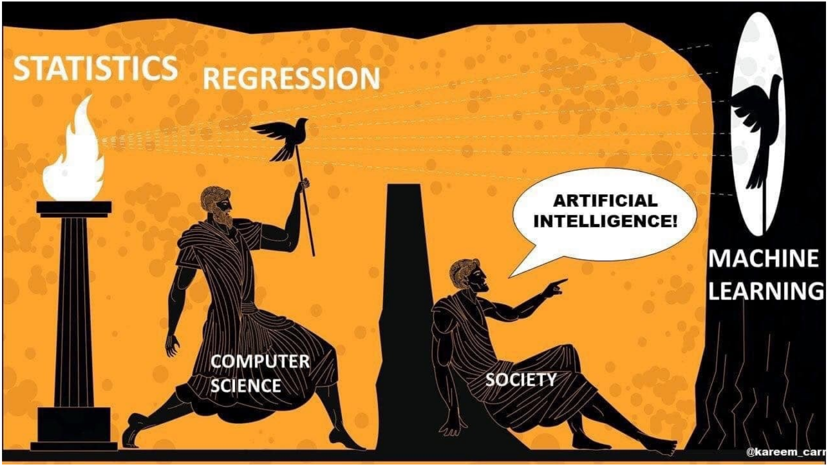


Conclusions

- Starting from the predictions of the ΛCDM model, we can derive equations for the DE EoS starting from background and perturbations;
- we compare $w_{\rm bg}$ and $w_{\rm gr}$ to detect failures of ΛCDM and the reason for it (DE, MG, ...);
- we use GP, rather than parameterizations, to reconstruct the quantities on which w_i depend;
- applying this method to current data (CC and galaxy surveys) shows agreement with ΛCDM... as usual :(
- A hint for $w_{\rm gr} > -1$ is present in the results ($< 2\sigma$) most likely connected to lower growth preferred by galaxy data.

Take home messages

- Exploring deviations from standard model with consistency checks allows to test the current model without assuming another;
- using GP or other ML reconstruction methods (e.g. GA) allows to avoid parameterizing these functions to fit the data, thus ensuring that we can catch all features;
- assumptions are still present in how the w_i functions are obtained, and they affect our reconstruction. A more general derivation is needed;
- future data will allow to lower the dependency of the results on the fine details of the GP (and on assumptions like the $\Omega_{\rm m}(z)$ cut). Model independent approaches require very good data to work!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SOCIETY
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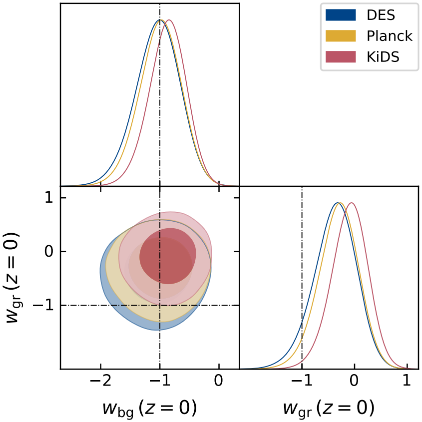
Impact of $\Omega_{\mathrm{m},0}$ prior

We cannot obtain estimates for w_{bg} and w_{gr} by only reconstructing $H(z)$ and $f(z)$; we need an external prior on $\Omega_{\mathrm{m},0}$ to obtain $\Omega_{\mathrm{m}}(z) = \Omega_{\mathrm{m},0}(H_0/H(z))^2$.

This introduces an arbitrary choice of the prior, but we tested different possibility finding results compatible with each other.

The KiDS prior gives the most significant deviation from ΛCDM, as this survey is the one with the highest S_8 tension.



Derivatives of GPs

In the expressions for $w_{\rm bg}$ and $w_{\rm gr}$ we do not have only $H(z)$ and $f(z)$ but also their derivatives.

The use of GP allows to obtain such derivatives without the need of directly obtaining data on these, which would imply to "take derivatives of the data" (very noisy procedure)

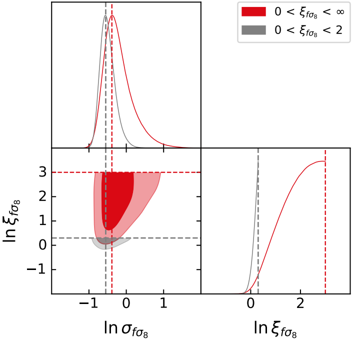
Indeed, once a function $g(x)$ is reconstructed via GP, it is possible to obtain its derivative as the derivative of a GP is still a GP

$$y' \sim \mathcal{N}(\frac{\mathrm{d}}{\mathrm{d}x}\mu, \frac{\mathrm{d}}{\mathrm{d}x}\Sigma)$$

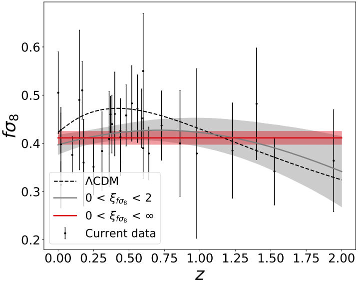
with the derivative of the covariance matrix defined by the derivative of the original kernel

$$k'(x, x^*) = \frac{\partial^2}{\partial x \partial x^*} k(x, x^*)$$

Issues with hyperparameters



If data are scattered, sparse, with large errors, it can be extremely difficult to find a suitable prior range for hyper-parameters.



Different ranges can produce completely different results. Important to have data that can produce constraints.

L. Perenon, MM, S. Ilic, R. Maartens, M. Lochner Phys.Dark Univ. 34 (2021)

Multi-task GP

Current and upcoming galaxy data do not only provide information on $f(z)$, but also on $\sigma_8(z)$ and $f\sigma_8(z)$. We might want to exploit all information we have available, but these functions are obviously correlated, we should not reconstruct them separately

The solution for this a "Multi-task" GP, where we consider the correlation both in data and kernels

$$C = \begin{bmatrix} \mathrm{cov}(f,f) & \mathrm{cov}(f,\sigma_8) & \mathrm{cov}(f,f\sigma_8) \\ \mathrm{cov}(f,\sigma_8) & \mathrm{cov}(\sigma_8,\sigma_8) & \mathrm{cov}(\sigma_8,f\sigma_8) \\ \mathrm{cov}(f,f\sigma_8) & \mathrm{cov}(\sigma_8,f\sigma_8) & \mathrm{cov}(f\sigma_8,f\sigma_8) \end{bmatrix},$$

$$\tilde{K} = \begin{bmatrix} K_{f,f} & K_{f,\sigma_8} & K_{f,f\sigma_8} \\ K_{f,\sigma_8} & K_{\sigma_8,\sigma_8} & K_{\sigma_8,f\sigma_8} \\ K_{f,f\sigma_8} & K_{\sigma_8,f\sigma_8} & K_{f\sigma_8,f\sigma_8} \end{bmatrix}$$

with the off-diagonal terms in $\tilde{K}$ being the convolution of the two single kernels.

B. Haridasu, V. Lukovic, M. Moresco, N. Vittorio JCAP 10 (2018)