Everybody's going NuTs 2022

Institute for Theoretical Physics, UAM – CSIC

16 May – 17 June, Madrid

Dark Matter from sterile-sterile neutrino mixing

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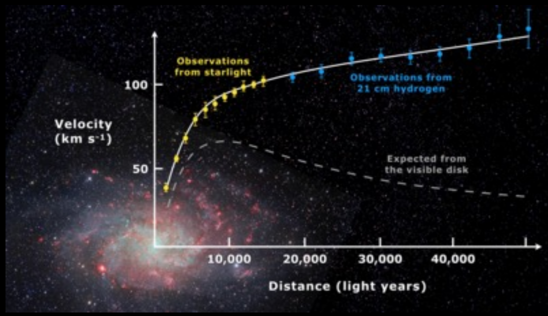
Dark Matter

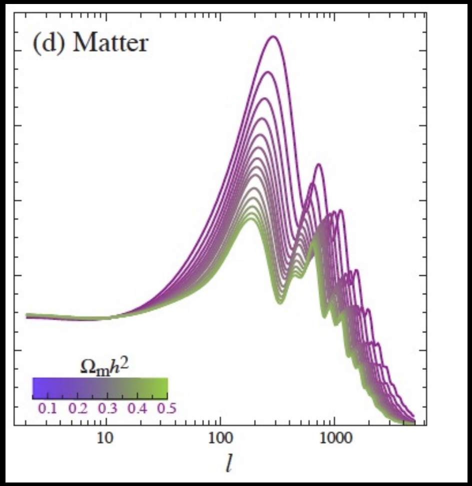
At the present time dark matter acts as a cosmic glue keeping together

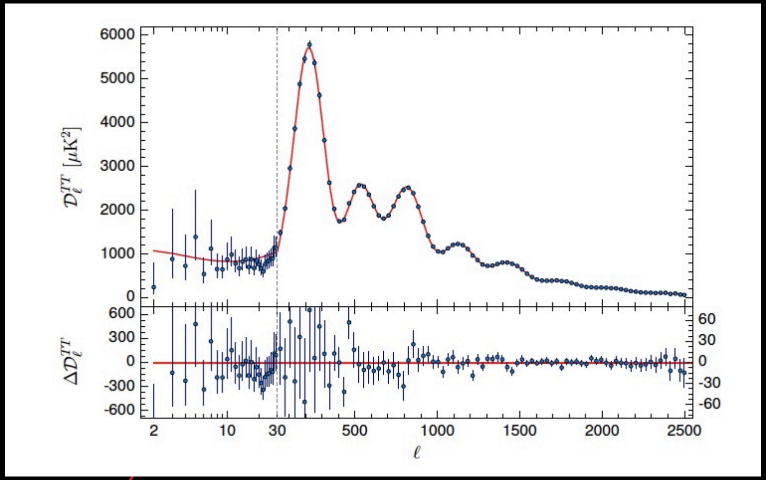
Stars in galaxies.... ... and galaxies in clusters of galaxies (such as in Coma cluster)



...but it also has to be primordial and non-baryonic to understand structure formation and CMB anisotropies

(Hu, Dodelson, astro-ph/0110414) (Planck 2018, 1807.06209)



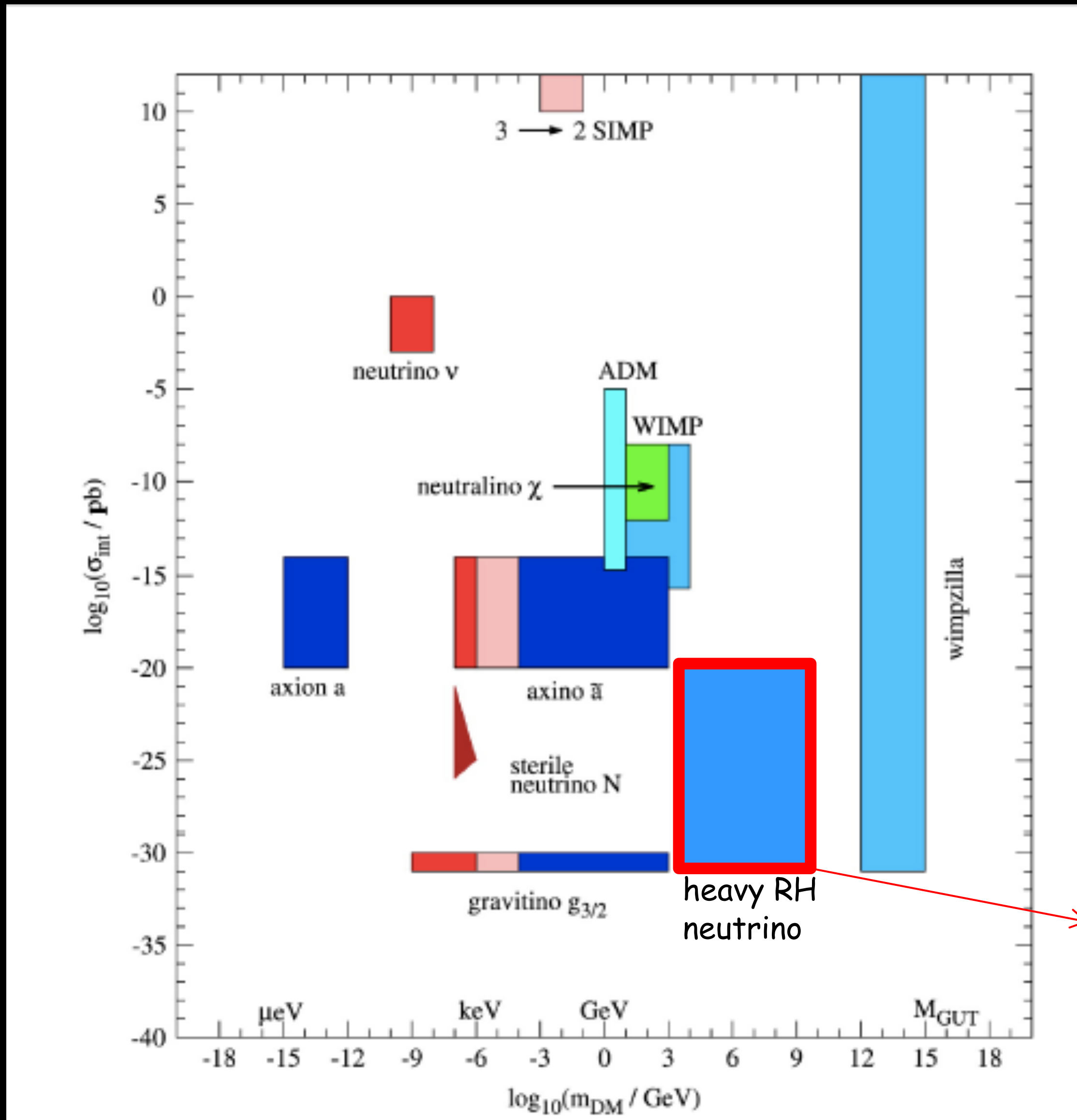


(CMB + BAO)

$$\boxed{\Omega_{CDM,0}h^2 = 0.11933 \pm 0.0009 \sim 5\Omega_{B,0}h^2}$$

Beyond the WIMP paradigm

(from Baer et al.1407.0017)

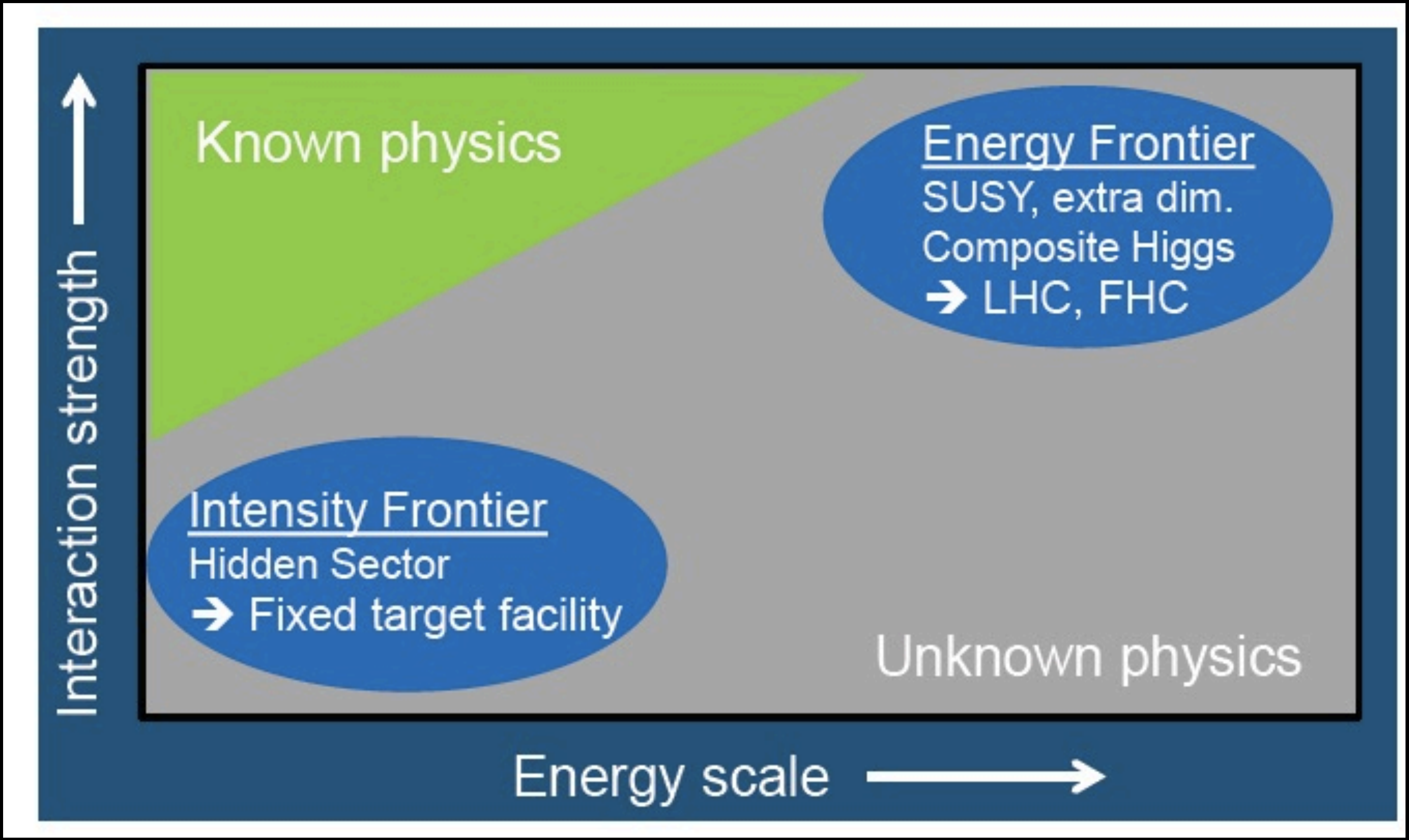


(PDB, Anisimov '08)

The more we know the less we understand?

Right-handed neutrino laboratory searches

(SHIP proposal, 1504.04855)



Dark matter from active-sterile neutrino mixing

(Dodelson Widrow '94; Shi, Fuller '99; Dolgov and Hansen '00; Asaka, Blanchet,Shaposhnikov '05)

- Type-I seesaw Lagrangian $-\mathcal{L}^{\nu}_{mass} = \overline{\nu}_L m_D \nu_R + \frac{1}{2}\overline{\nu^c_R} M \nu_R + h.c. = -\frac{1}{2}(\overline{\nu_L}\ \overline{\nu^c_R})\begin{pmatrix} 0 & m_D^T \\ m_D & M \end{pmatrix}\begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + h.c.$

- LH-RH neutrino mixing

$$\nu_{1L} \simeq U^{\dagger}_{1\alpha}\left(\nu_{L\alpha} - \frac{m_{D\alpha 1}}{M_1}\nu^c_{R1}\right)$$

$$N_{1R} \simeq \nu_{1R} + \frac{m_{D\alpha 1}}{M_1}\nu^c_{L\alpha}$$

- For $M_1 \ll m_e$ $\Rightarrow$ $\tau_1 = 5\times 10^{26}\,s\left(\frac{M_1}{keV}\right)^{-5}\left(\frac{10^{-8}}{\theta^2}\right) \gg t_0$ $\quad \theta^2 \equiv \frac{\sum_\alpha |m_{D\alpha 1}|^2}{M_1^2}$

- Solving Boltzmann equations an abundance is produced at T~100 MeV:

$$\Omega_{N_1}h^2 \sim 0.1\,\frac{\theta^2}{10^{-8}}\left(\frac{M_1}{\text{keV}}\right)^2 \sim \Omega_{DM,0}h^2$$

- The lightest neutrino mass $m_1 \lesssim 10^{-5}$ eV $\Rightarrow$ hierarchical limit
- The N_1's also radiatively decay and this produces constraints from X-rays (or opportunities to observe it).
- Considering also structure formation constraints, one is forced to consider a resonant production induced by a large lepton asymmetry
- L ~10^{-4} (3.5 keV line?). (Horiuchi et al. '14; Bulbul at al. '14; Abazajian '14)

Heavy RH neutrino as dark matter ?

(Anisimov,PDB '08)

What production mechanism? For high masses just a tiny abundance is needed:

$$N_{DM} \simeq 10^{-9} (\Omega_{DM,0} h^2) N_\gamma (t_{prod}) \frac{\mathrm{TeV}}{M_{DM}}$$

Suppose a RH neutrino has tiny Yukawa couplings (e.g., proportional to a small symmetry breaking parameter):

$$m_D \simeq \begin{pmatrix} \varepsilon_{e1} & m_{De2} & m_{De3} \\ \varepsilon_{\mu 1} & m_{D\mu 2} & m_{D\mu 3} \\ \varepsilon_{\tau 1} & m_{D\tau 2} & m_{D\tau 3} \end{pmatrix} \text{ or } m_D \simeq \begin{pmatrix} m_{De1} & \varepsilon_{e2} & m_{De3} \\ m_{D\mu 1} & \varepsilon_{\mu 2} & m_{D\mu 3} \\ m_{D\tau 1} & \varepsilon_{\tau 2} & m_{D\tau 3} \end{pmatrix} \text{ or } m_D \simeq \begin{pmatrix} m_{De1} & m_{De2} & \varepsilon_{e3} \\ m_{D\mu 1} & m_{D\mu 2} & \varepsilon_{\mu 3} \\ m_{D\tau 1} & m_{D\tau 2} & \varepsilon_{\tau 3} \end{pmatrix}$$

$$m_D = V_L^\dagger D_{m_D} U_R \qquad D_{m_D} \equiv v\, diag(h_A, h_B, h_C) \text{ with } h_A \le h_B \le h_C$$

$$\boxed{\tau_{DM} = \frac{4\pi}{h_A^2\, M_{DM}} = 0.87\, h_A^2\, 10^{-26} \frac{\mathrm{TeV}}{M_{DM}} s} \quad \Rightarrow \quad \boxed{\tau_{DM} > \tau_{DM}^{\min} \simeq 10^{28} s \Rightarrow h_A < 10^{-27} \sqrt{\frac{\mathrm{TeV}}{M_{DM}} \times \frac{10^{28}\,\mathrm{s}}{\tau_{DM}^{\min}}}}$$

Too small to reproduce the correct abundance with any production mechanism within a minimal type-I seesaw extension

Many proposed production mechanisms

Recently many production mechanisms have been proposed especially to address IceCube initially seemingly anomalous PeV neutrino events:

- from $SU(2)_R$ extra-gauge interactions (LRSM) (Fornengo,Niro, Fiorentin);
- from inflaton decays (Anisimov,PDB'08; Higaki, Kitano, Sato '14);
- from resonant annihilations through SU(2)' extra-gauge interactions (Dev, Kazanas,Mohapatra,Teplitz, Zhang '16);
- From new $U(1)_Y$ interactions connecting DM to SM (Dev, Mohapatra,Zhang '16);
- From $U(1)_{B-L}$ interactions (Okada, Orikasa '12);
- …………………

In all these models IceCube data are fitted through fine tuning of parameters responsible for decays (they are post-dictive)

DM from Higgs induced neutrino mixing

(Anisimov '06, Anisimov,PDB '08)

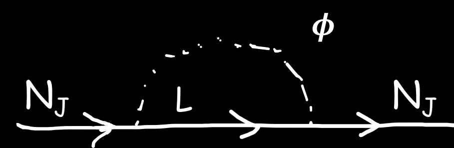
Assume new (5-dim) interactions with the standard Higgs:

$$\mathcal{L}_A = \frac{\lambda_{IJ}}{\Lambda}\phi^\dagger\phi\,\overline{N_I^c}N_J$$

In general they are non-diagonal in the Yukawa basis: this generates a RH neutrino mixing.
Consider a 2 RH neutrino mixing for simplicity. Interactions generate effective potentials:

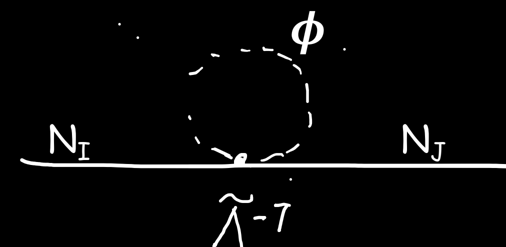
From the Yukawa interactions:

$$V_J^Y = \frac{T^2}{8E_J}h_J^2$$



From the new interactions:

$$V_{IJ}^\Lambda = \frac{T^2}{12\Lambda}\lambda_{IJ}$$



$$\widetilde{\Lambda} = \Lambda / \lambda_{DM\text{-}S}$$

Effective mixing Hamiltonian (monocromatic approximation p~3T):

$$\Delta H \simeq \begin{pmatrix} -\dfrac{\Delta M^2}{4p} - \dfrac{T^2}{16p}h_S^2 & \dfrac{T^2}{12\widetilde{\Lambda}} \\ \dfrac{T^2}{12\widetilde{\Lambda}} & \dfrac{\Delta M^2}{4p} + \dfrac{T^2}{16p}h_S^2 \end{pmatrix}$$

$$\Delta M^2 \equiv M_S^2 - M_{DM}^2$$

If $\Delta M^2 < 0$ ($M_{DM} > M_S$) there is a resonance at: $z_{res} \equiv \dfrac{M_{DM}}{T_{res}} = \dfrac{h_S M_{DM}}{2\sqrt{M_{DM}^2 - M_S^2}}$

Resonant production described by Landau-Zener?

Density matrix calculation of the relic abundance

(P.Di Bari, K. Farrag, R. Samanta, Y. Zhou, 1908.00521)

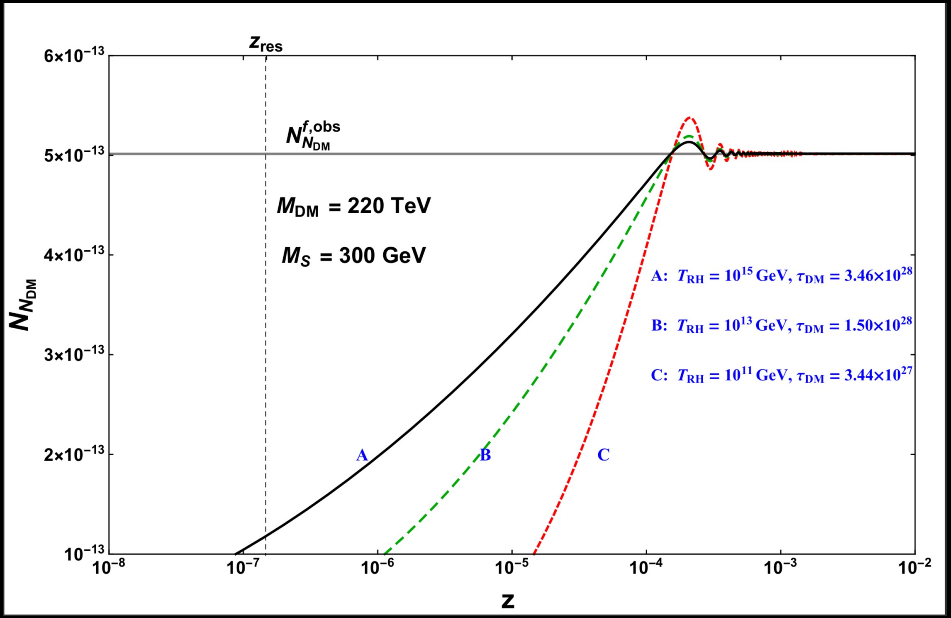
Density matrix equation for the DM-source RH neutrino system

$$\frac{dN_{IJ}}{dt} = -i\,[\Delta H, N]_{IJ} - \begin{pmatrix} 0 & \frac{1}{2}(\Gamma_D + \Gamma_S) N_{DM-S} \\ \frac{1}{2}(\Gamma_D + \Gamma_S) N_{S-DM} & (\Gamma_D + \Gamma_S)(N_{N_S} - N_{N_S}^{eq}) \end{pmatrix}$$

A numerical solution shows that a simple calculation based on the Landau-Zener approximation overestimates the relic abundance by a few orders of magnitude (especially in the hierarchical case)

The resonance occurs before oscillations develop ⇒ the production is non-resonant

initial
N_S thermal
abundance

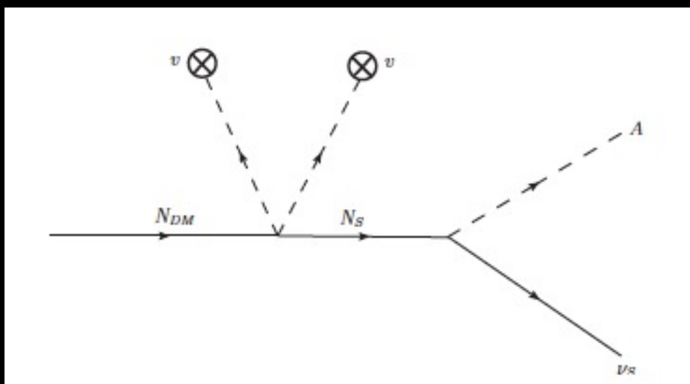


Constraints from decays

(Anisimov,PDB '08; Anisimov,PDB'10; P.Ludl.PDB,S.Palomarez-Ruiz'16)

2 body decays ($M_S > M_W$)

DM neutrinos unavoidably decay today into A+leptons (A=H,Z,W) through the same mixing that produced them in the very early Universe



$$\theta_{\Lambda 0} = \frac{2\, v^2/\widetilde{\Lambda}}{M_{\rm DM}\,(1 - M_{\rm S}/M_{\rm DM})}\,.$$

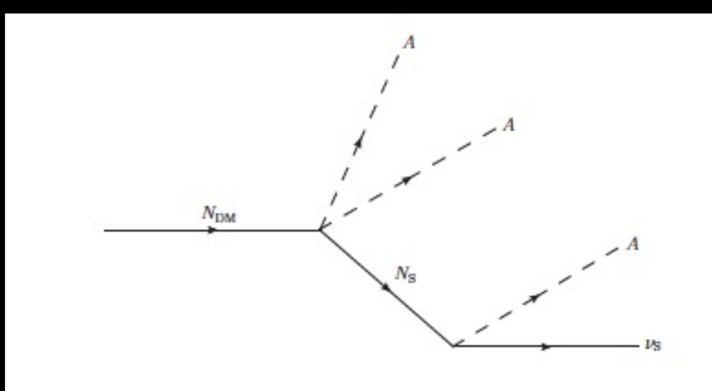
mixing angle today

Lower bound on M_{DM} ($\tau_{28} \equiv \tau_{DM}^{min}/10^{28}$s)

$$M_{\rm DM} \geq M_{\rm DM}^{\rm min} \simeq 54\,{\rm TeV}\,\alpha_{\rm S}\,\tau_{28}\,\left(\frac{M_{\rm S}}{M_{\rm DM}}\right)$$

4 body decays

$$N_{\rm DM} \to 2\,A + N_{\rm S} \to 3\,A + \nu_{\rm S}\ (A = W^{\pm}, Z, H).$$



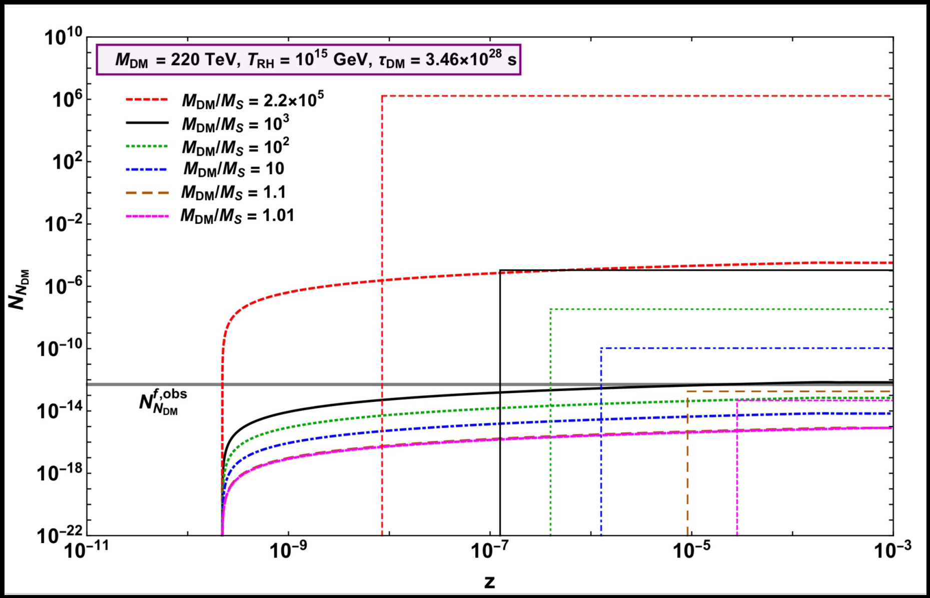
Upper bound on M_{DM} ($\tau_{28} \equiv \tau_{DM}^{min}/10^{28}$s)

$$M_{\rm DM} \lesssim 5.3\,{\rm TeV}\,\alpha_{\rm S}^{-\frac{2}{3}}\,z_{\rm res}^{-\frac{1}{3}}\,\tau_{28}^{-\frac{1}{3}}\,\left(\frac{N_{N_{\rm S}}}{N_\gamma}\right)_{\rm res}^{\frac{1}{3}}\,\left(\frac{M_{\rm DM}}{M_{\rm S}}\right)^{\frac{2}{3}}$$

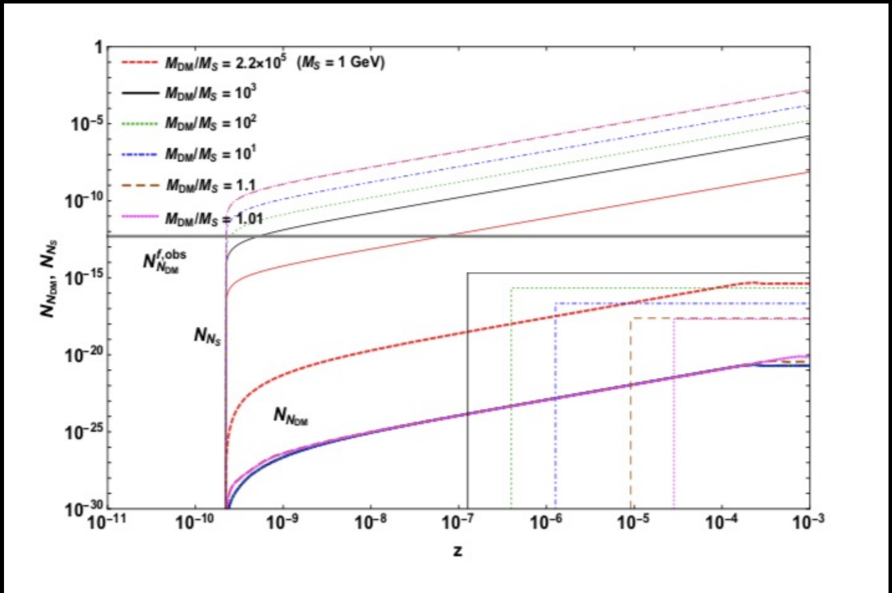
3 body decays and annihilations also can occur but yield weaker constraints

Failure of Landau-Zener approximation in the hierarchical case

initial thermal N_S-abundance

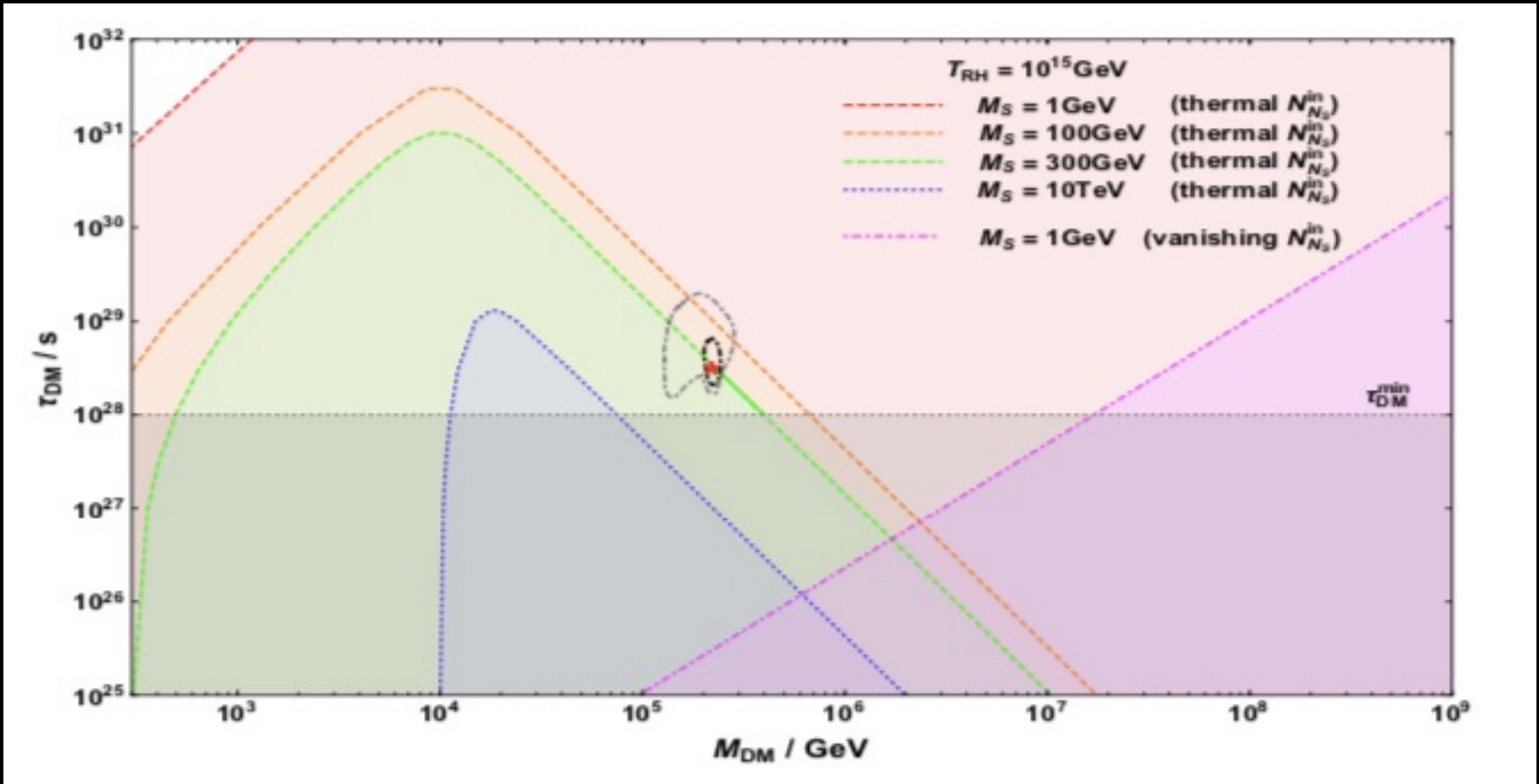


initial vanishing N_S-abundance



DM lifetime vs. mass plane: allowed regions

(P.Di Bari, K. Farrag, R. Samanta, Y. Zhou, 1908.00521)

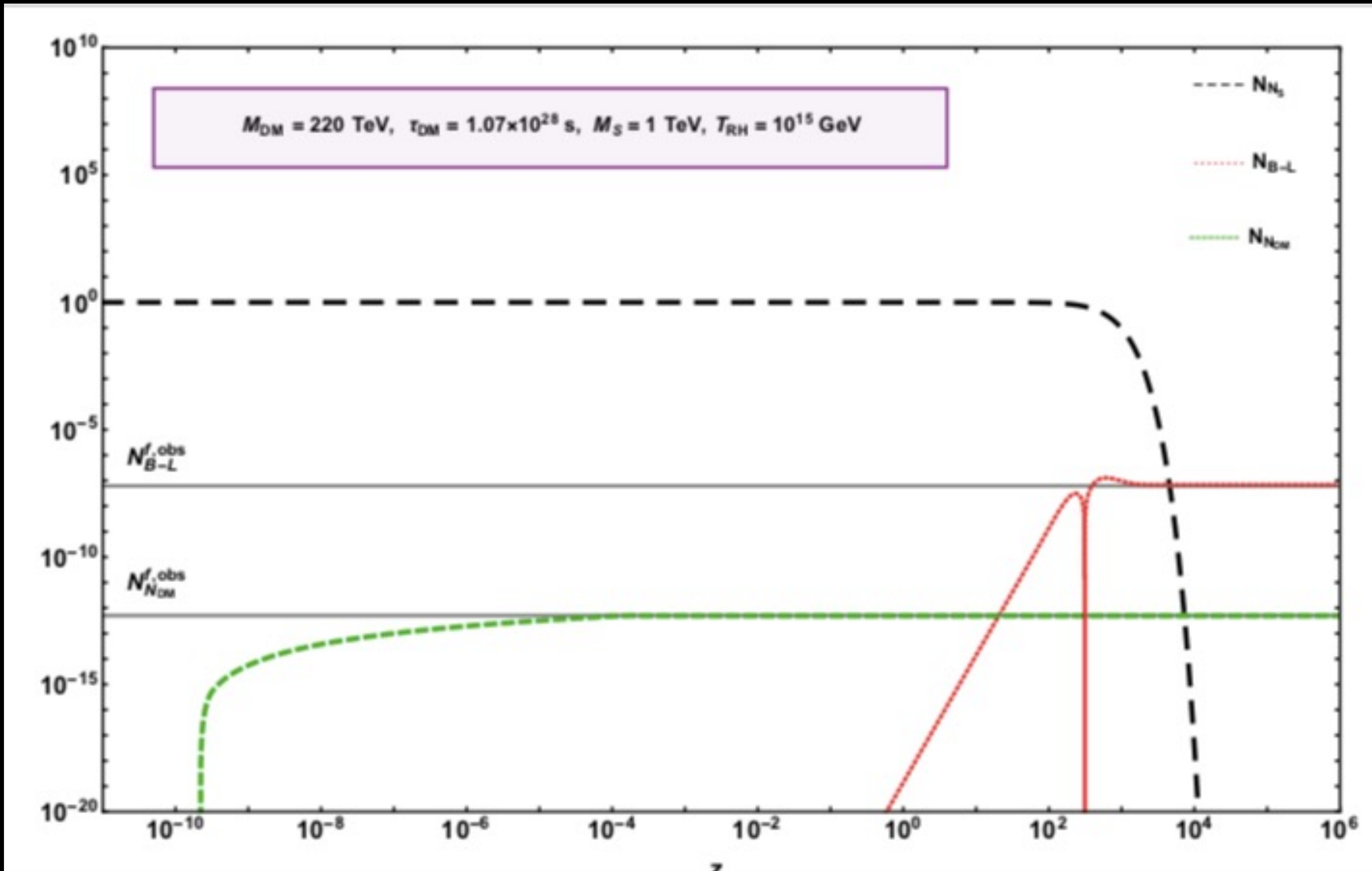


Solutions only for initial thermal N_S abundance, unless $M_S \sim 1$ GeV

Unifying Leptogenesis and Dark Matter

(PDB, K. Farrag, R. Samanta, Y. Zhou, 1908.00521)

A solution for initial thermal N_S abundance:

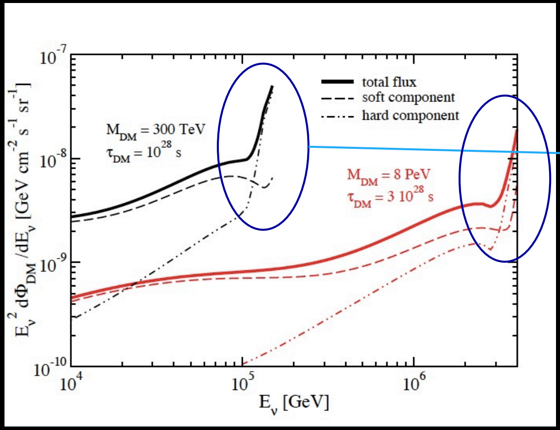


Very high energy neutrinos from decays

(Anisimov,PDB,0812.5085;PDB, P.Ludl,S. Palomarez-Ruiz 1606.06238)

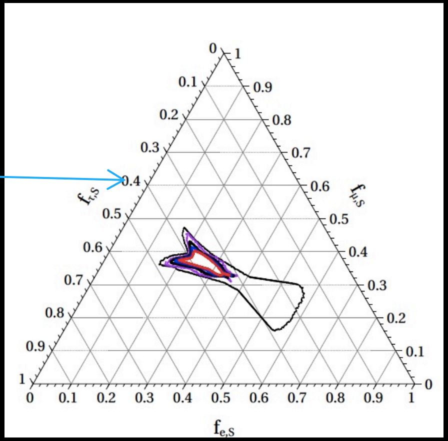
- DM neutrinos unavoidably decay today into A+leptons (A=H,Z,W) through the same mixing that produced them in the very early Universe
- Potentially testable high energy neutrino contribution

Energy neutrino flux



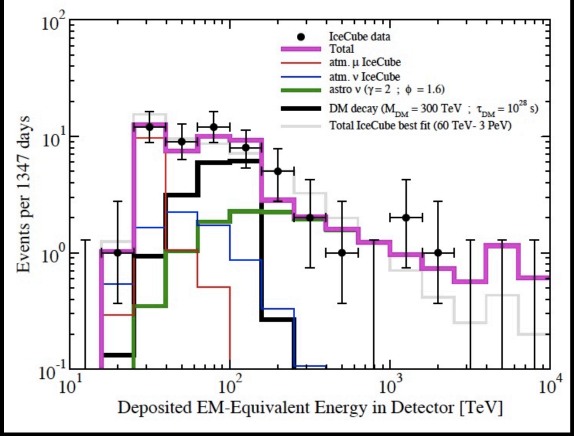
Flavour composition at the detector

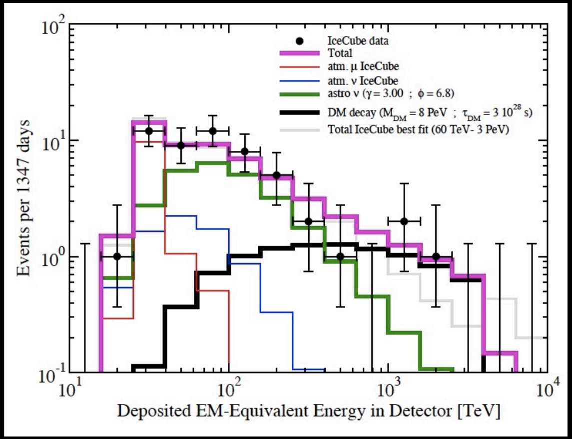
Hard component



Neutrino events at IceCube: 2 examples

M_{DM}=300TeV

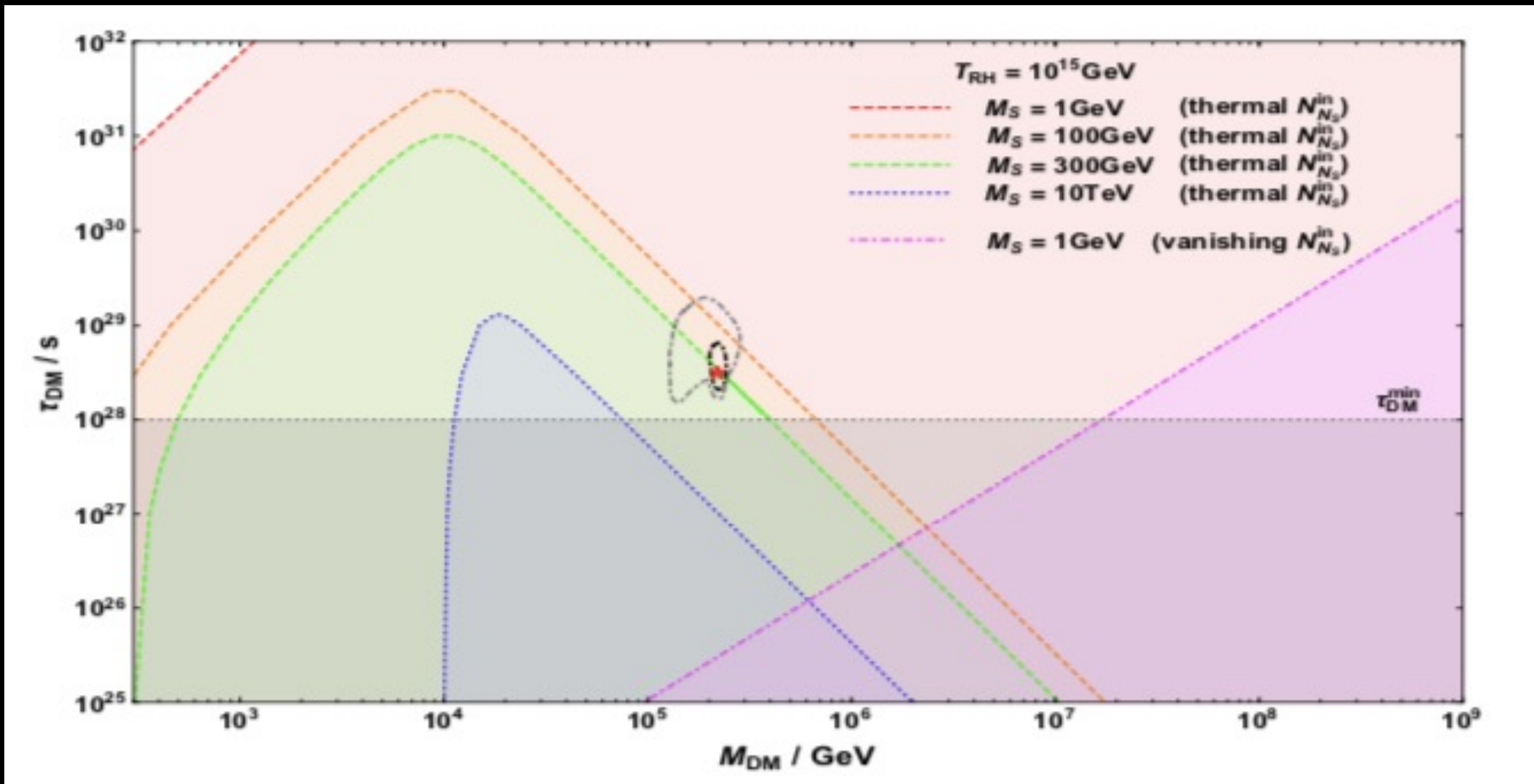




M_{DM}=8 PeV

DM lifetime vs. mass plane: allowed regions

(P.Di Bari, K. Farrag, R. Samanta, Y. Zhou, 1908.00521)



Solutions only for initial thermal N_S abundance, unless M_S~ 1 GeV

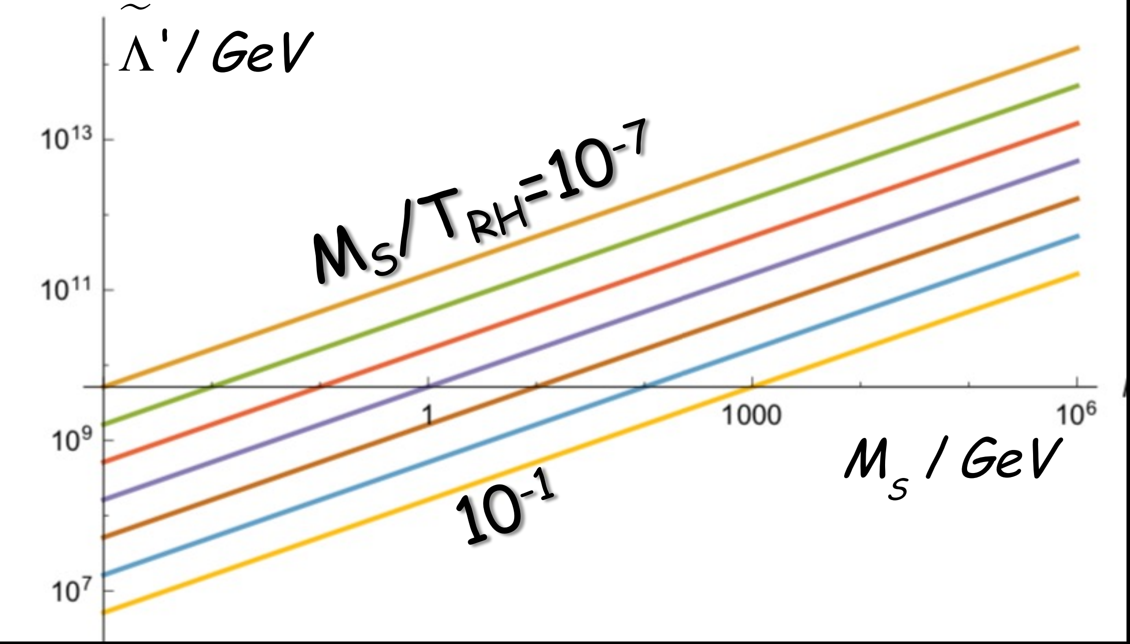
Higgs portal interactions of the source RH neutrinos

(PDB, A. Murphy, in preparation)

$$\mathcal{L}_A = \frac{\lambda_{DM-S}}{\Lambda}\phi^\dagger\phi\,\overline{N^c_{DM}}N_S + \frac{\lambda_{S-S}}{\Lambda}\phi^\dagger\phi\,\overline{N^c_S}N_S$$

Can these interactions thermalise the source RH neutrinos?

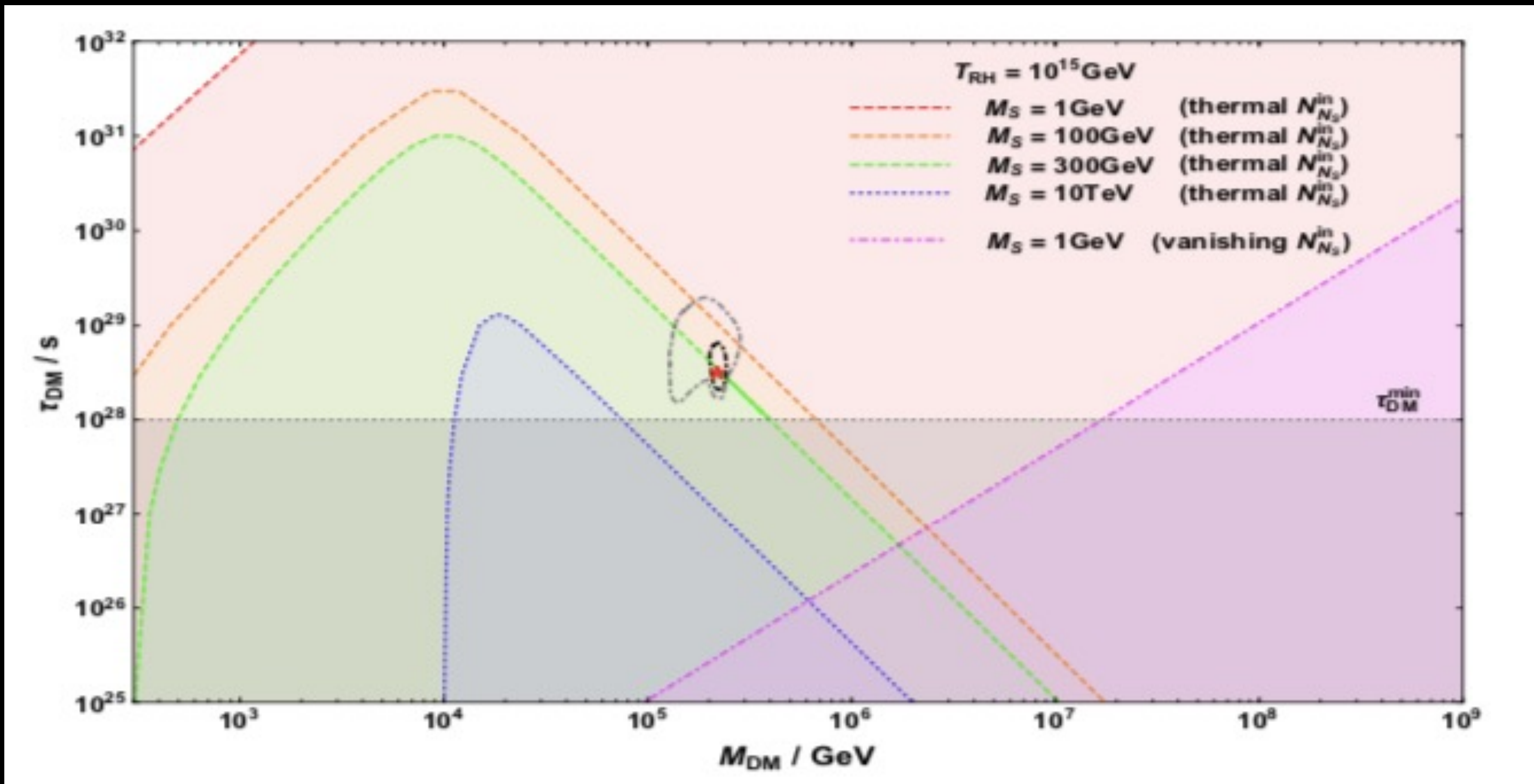
$\tilde{\Lambda}' = \Lambda / \lambda_{S-S}$



Below the lines there is a fast thermalization

DM lifetime vs. mass plane: allowed regions

(P.Di Bari, K. Farrag, R. Samanta, Y. Zhou, 1908.00521)



Solutions only for initial thermal N_S abundance, unless M_S~ 1 GeV

Mass varying source right-handed neutrino

(PDB, D. Marfatia, YL. Zhou 2001.07637)

$$-\mathcal{L}_\lambda = \frac{1}{2} M_{\rm DM} \overline{N^c_{\rm DM}} N_{\rm DM} + \frac{1}{2} M_{\rm D} \overline{N^c_{\rm D}} N_{\rm D} + \frac{\lambda_{\rm S}}{2} \eta \overline{N^c_{\rm S}} N_{\rm S} + \frac{1}{\tilde{\Lambda}} \Phi^\dagger \Phi \overline{N^c_{\rm D}} N_{\rm S} + \frac{1}{\tilde{\Lambda}} \Phi^\dagger \Phi \overline{N^c_{\rm DM}} N_{\rm D} + \text{h.c.}\,. \quad (1)$$

The scalar field η acquires a vev v_η during a first order phase transition and accordingly N_S acquires a space-time dependent mass:

$$M_S(x,t) = \lambda_S \, v_\eta(x,t)$$

The bubble wall profile is well described by a kink solution found in the thin wall approximation:

$$v_\eta(r,t) = \frac{1}{2} \bar{v}_\eta \left[1 - \tanh\left(\frac{r - v_{\rm w}(t - t_\star)}{\Delta_{\rm w}}\right)\right] ,$$

Thermal effects and density matrix equation

(PDB, D. Marfatia, YL. Zhou 2001.07637)

Also this time we need to account for thermal masses from both interactions:

$$\widetilde{M}^2_{\rm S}(x,t) = M^2_{\rm S}(x,t) + \frac{T^2}{4}\, h^2_{\rm S} + \frac{T^2}{8}\, \lambda^2_{\rm S}\, N_{N_{\rm S}}\, N_\eta$$

We get then the following effective Hamiltonian:

$$\Delta\mathcal{H}_{IJ} \simeq \begin{pmatrix} -\frac{\Delta\widetilde{M}^2}{4\,p} & \Delta H_{\rm mix} \\ \Delta H_{\rm mix} & \frac{\Delta\widetilde{M}^2}{4\,p} \end{pmatrix}$$

$$\Delta H_{\rm mix} \equiv T^2/(12\,\widetilde{\Lambda})$$

$$\widetilde{\Delta M^2}(x,t) = M^2_S(x,t) - M^2_D$$

And again we can write a density matrix equation (I,J = D,S):

$$\frac{dN_{IJ}}{dt} = -i\,[\Delta\mathcal{H}, N]_{IJ} - \begin{pmatrix} 0 & \Gamma_{\rm dec} \\ \Gamma_{\rm dec} & \Gamma_{\rm prod} \end{pmatrix}$$

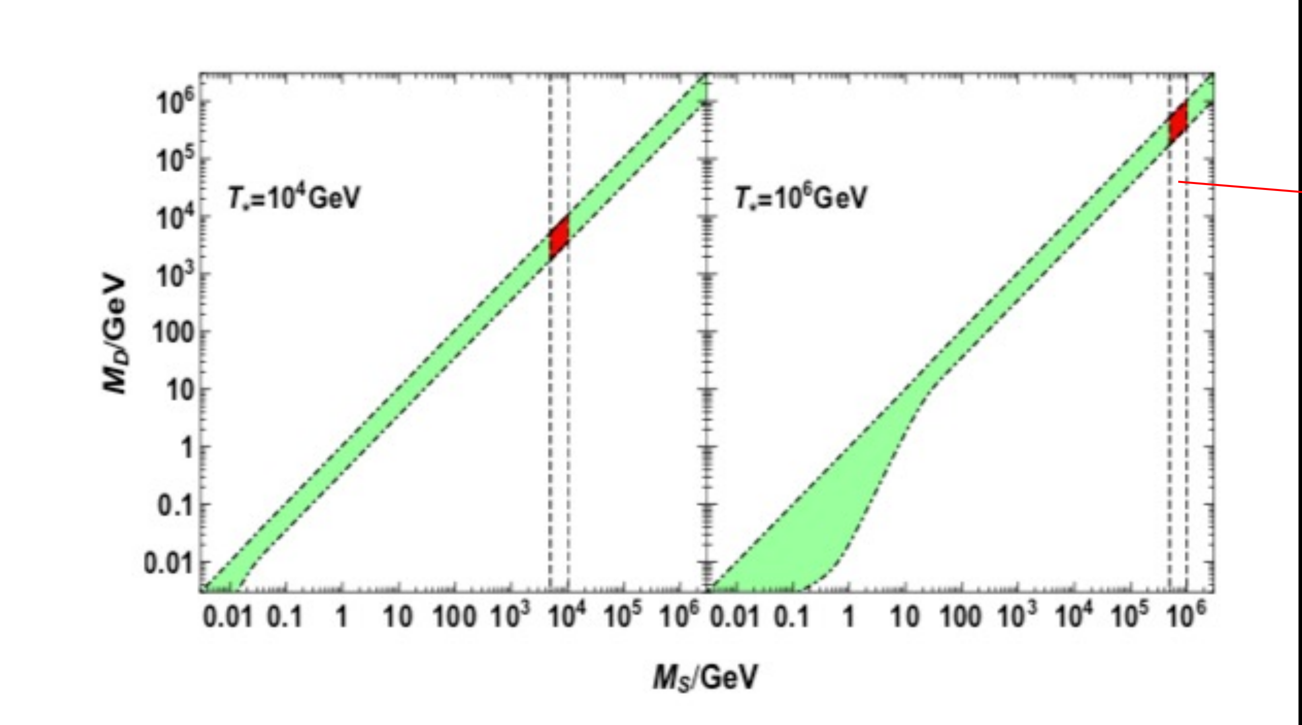
$$\frac{dN_{N_S}}{dt} = -\Gamma_{prod}(N_{N_S} - N^{eq}_{N_S}) \quad with \;\; \Gamma_{prod} \simeq 2\,\Gamma_{\eta\to N_S N_S} \Rightarrow N_{N_S} \simeq N^{eq}_{N_S}$$

Resonance condition

(PDB, D. Marfatia, YL. Zhou 2001.07637)

$$\Delta\widetilde{M}^2(r, t_{\rm res}) = 0, \iff \widetilde{M}_{\rm S}^2(r, t_{\rm res}) = M_{\rm D}^2$$

$$\iff \frac{M_{\rm D}^2}{M_{\rm S}^2} \simeq \left[\frac{1}{2} - \frac{1}{2}\tanh\left(\frac{r - v_{\rm w}\,(t_{\rm res} - t_\star)}{\Delta_{\rm w}}\right)\right]^2 + \frac{T_\star^2}{6\bar{v}_\eta^2},$$



Landau-Zener holds in this vertical band

Constraints from dark matter decays

(PDB, D. Marfatia, YL. Zhou 2001.07637)

Since the DM mass is below the Higgs and gauge boson mass the dominant decaying mode is DM → $\nu e^+ e^-$

$$\Gamma_{N_{\rm DM}\to\nu\,\ell_\alpha^+\ell_\alpha^-} = \frac{(\theta_{\Lambda 0}^{\rm D-S}\,\theta_{\Lambda 0}^{\rm DM-D})^2}{96\,\pi^3}\,\frac{\overbrace{\overline{m}_\alpha}^{\gtrsim 0.1\, m_{sol} \sim 1\,\mathrm{meV}}}{M_{\rm S}}\,G_{\rm F}^2\,M_{\rm DM}^5\,,$$

$$\theta_{\Lambda 0}^{\rm D-S(DM-D)} = 2v^2/(\widetilde{\Lambda}\,(M_{\rm S(D)} - M_{\rm D(DM)}))$$

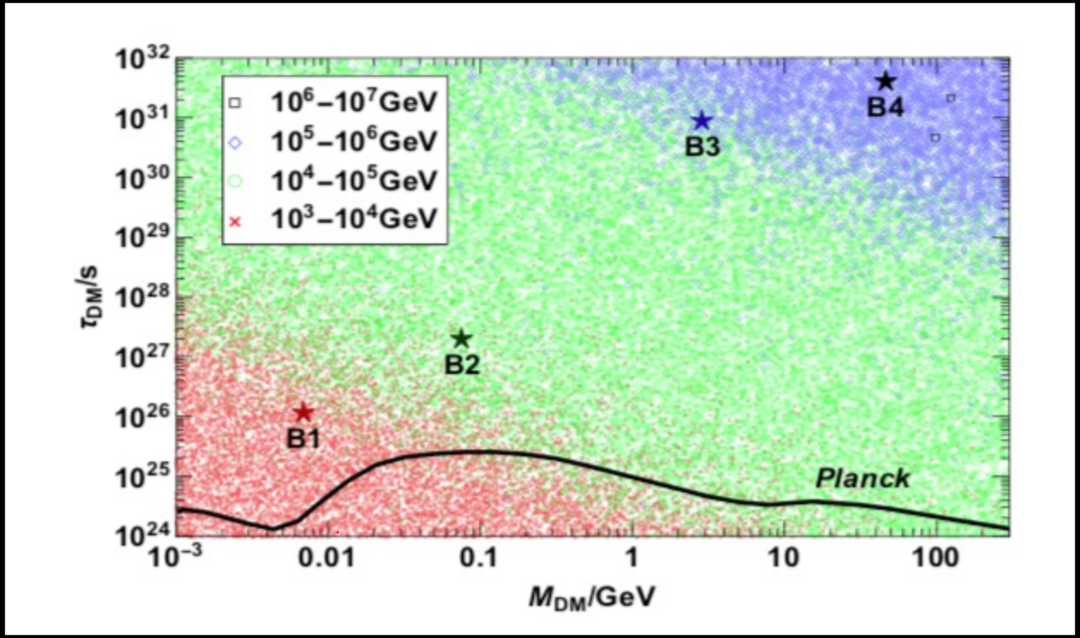
One has constraints both from CMB anisotropies (decays affect reionization history) and from X and γ-ray diffuse backgrounds (EGRET, FERMI, INTEGRAL) placing a lower bound $\tau_{\rm DM} \gtrsim 10^{25}$ s

Searching for solutions

(PDB, D. Marfatia, YL. Zhou 2001.07637)

The scan is done over $M_{DM}, M_S, T^*, \tau_{DM}$

All points reproduce $\Omega_{DM}h^2 \sim 0.12$



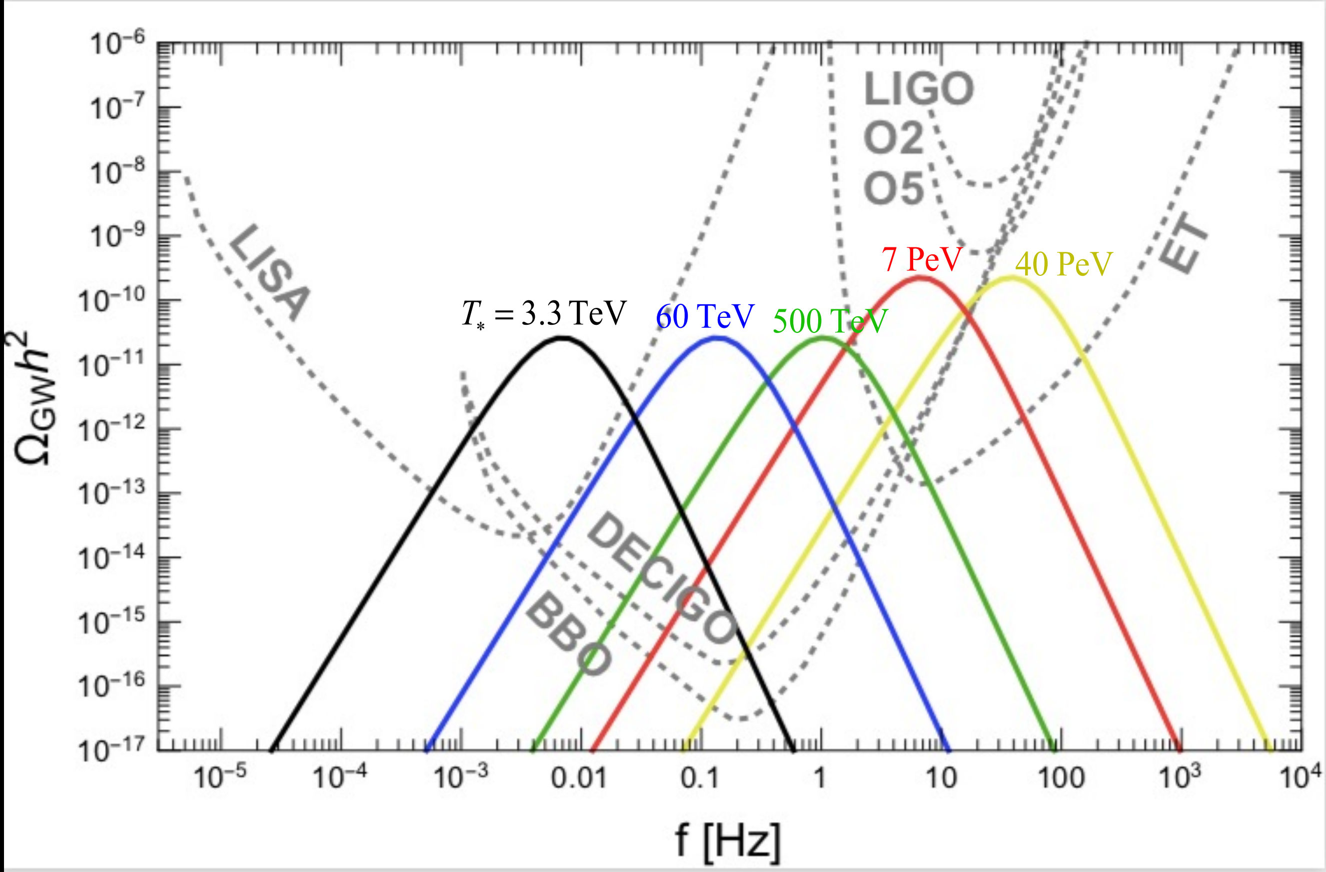
	$\frac{T_\star}{\mathrm{PeV}}$	$\frac{\tau_{\mathrm{DM}}}{10^{26}\mathrm{s}}$	$\frac{M_{\mathrm{S}}}{\mathrm{TeV}}$	$\frac{M_{\mathrm{D}}}{\mathrm{TeV}}$	$\frac{M_{\mathrm{DM}}}{\mathrm{GeV}}$	v_{w}	α	$\frac{\beta}{H_\star}$
B1	$3\cdot10^{-3}$	1.219	1.57	0.567	$7\cdot10^{-3}$	0.90	0.10	10
B2	0.016	21.26	12.9	7.72	0.077	0.90	0.10	10
B3	0.106	$9.25\cdot10^{4}$	93.3	72.6	2.92	0.90	0.10	10
B4	1.052	$4.24\cdot10^{5}$	666	666	46.69	0.95	0.15	5
B5	10.75	$4.69\cdot10^{17}$	$8.7\cdot10^{3}$	$5.3\cdot10^{3}$	175.8	0.95	0.15	5

TABLE I: Benchmark points obtained for $v_{\mathrm{w}}/\Delta_{\mathrm{w}} = T_\star/50$.

GWs from SFOPTs

$$\Omega_{GW}^{PT} h^2(f) \simeq \omega(f; f_{peak})$$

(from PDB, D. Marfatia, YL. Zhou 2001.07637)



How to calculate Ω_{GW}? Which scale and what new physics ?

First order phase transition associated to Majorana mass generation in the Majoron model

(PDB, D. Marfatia, YL. Zhou 2106.00025)

$$-\mathcal{L}_{N_I+\sigma} = \overline{L_\alpha}\, h_{\alpha I}\, N_I\, \widetilde{\Phi} + \frac{\lambda_I}{2}\sigma\, \overline{N_I^c}\, N_I + V_0(\sigma) + h.c. \quad (respecting\ U_L(1)\ symmetry)$$

$$\sigma = \frac{1}{\sqrt{2}}(\sigma_1 + i\sigma_2)\,, \qquad <\sigma> = \frac{v_T}{\sqrt{2}}$$

At the end of the σ-phase transition, after SB, L is violated and

$$\sigma = \frac{e^{i\theta}}{\sqrt{2}}(v_0 + S + iJ) \qquad M_I = \lambda_I \frac{v_0}{\sqrt{2}} \sim M \ (seesaw\ scale)$$

Dirac neutrino mass matrix $m_D = v_{ew}\, h/\sqrt{2}$ generated after EWSB

At the moment let us assume $T_* > v_{ew}$ (high scale scenarios)

After both symmetry breakings: $m_\nu = -\frac{v_{ew}^2}{2}\frac{h_{\alpha I} h_{\beta I}}{M_I}$

Given the measured values of the neutrino oscillation mass scales, RH neutrinos thermalise prior to the phase transition and contribute to the thermal potential

DARK SECTOR ≡ N_I 's + J + S VISIBLE SECTOR ≡ SM particles

The minimal model

$$V_0(\sigma) = -\mu^2 |\sigma|^2 + \lambda |\sigma|^4 \;\Rightarrow\; v_0 = \sqrt{\mu^2/\lambda} \qquad (\lambda, \mu^2 > 0)$$

In the broken phase: $\sigma = \frac{e^{i\theta}}{\sqrt{2}}(v_0 + S + iJ)$

J is a massless Majoron and S has a mass $m_S=(2\lambda)^{1/2}v_0$

For the one-loop finite temperature effective potential one finds a polynomial

$$V_{\rm eff}^T(\sigma_1) \simeq D\,(T^2 - T_0^2)\sigma_1^2 - A\,T\,\sigma_1^3 + \frac{1}{4}\lambda_T\,\sigma_1^4\,,$$

The minimal model

$$2DT_0^2 = \lambda v_0^2 + \frac{N}{8\pi^2}\frac{M^4}{v_0^2} - \frac{3}{8\pi^2}\lambda^2 v_0^2$$

↳ destabilization temperature ("end" of phase transition)

$$D = \frac{\lambda}{8} + \frac{N}{24}\frac{M^2}{v_0^2}\,; \quad A = \frac{(3\lambda)^{3/2}}{12\pi}$$

Finally:

$$\lambda_T = \lambda - \frac{NM^4}{8\pi^2 v_0^2}\log\frac{a_F T^2}{e^{3/2}M^2} + \frac{9\lambda^2}{16\pi^2}\log\frac{a_B T^2}{e^{3/2}m_s^2}$$

→ You can't increase N arbitrarily!

(Espinosa, Quiros '07)
(Kehayias, Profumo '09)

Adding an auxiliary scalar

very heavy real scalar

$$V_0(\sigma,\eta) = V_0(\sigma) + \underbrace{V_{\eta\sigma}(\sigma,\eta) + V_\eta(\eta)}_{\text{new terms}}$$

the most important term is contained in $V_{\eta\sigma}$:

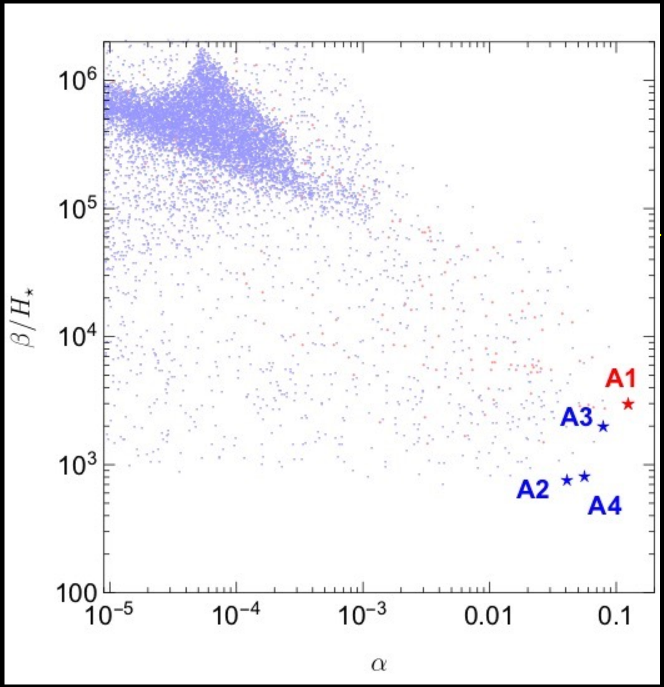
$$V_{\eta\sigma}(\eta,\sigma) = \frac{\delta_1}{2}|\sigma|^2\eta + \frac{\delta_2}{2}|\sigma|^2\eta^2$$

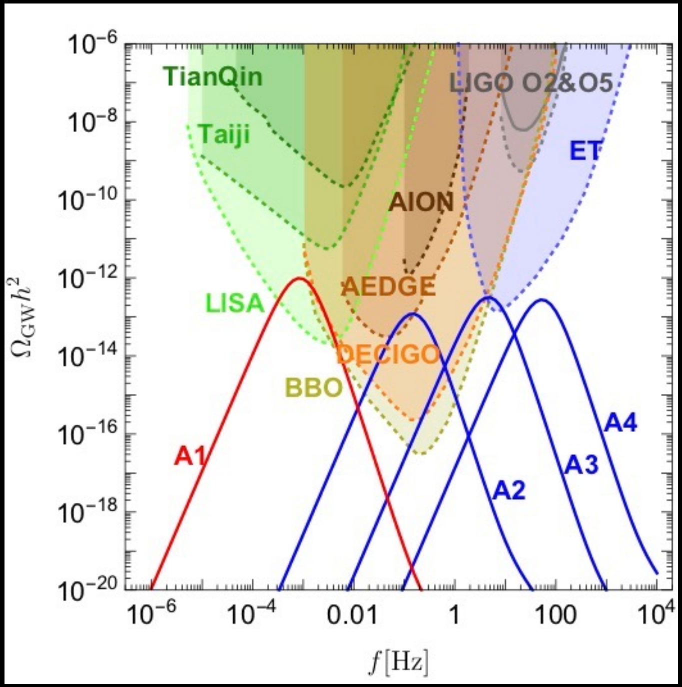
η undergoes a PT settling to its true vacuum prior to the σ-PT

$$\Rightarrow V^T_{eff}(\sigma_1,\tilde{\mu}) = \frac{1}{2}\tilde{M}_T^2\sigma_1^2 - (AT+\tilde{\mu})\sigma_1^3 + \frac{1}{4}\lambda_T\sigma_1^4$$

$\tilde{\mu} \propto \delta_2$

Adding an auxiliary scalar: GW spectrum





	Inputs				Predictions			
	m_S/GeV	$\tilde{\mu}$/GeV	M/GeV	v_0/GeV	$T_\star$/GeV	α	$\beta/H_\star$	a_0
A1	0.06190	0.0005857	0.5361	3.5873	0.6504	0.1248	2966	0.05951
A2	156.2	13.15	465.6	1014	721	0.04139	754.8	0.3886
A3	1036	13.72	7977	44424	9180	0.08012	1975	0.06268
A4	43874	1856	181099	567378	247807	0.05611	809.7	0.1944

SUMMARY

- Within the current phenomenological results DM puzzle might have a solution at higher scales than those usually explored

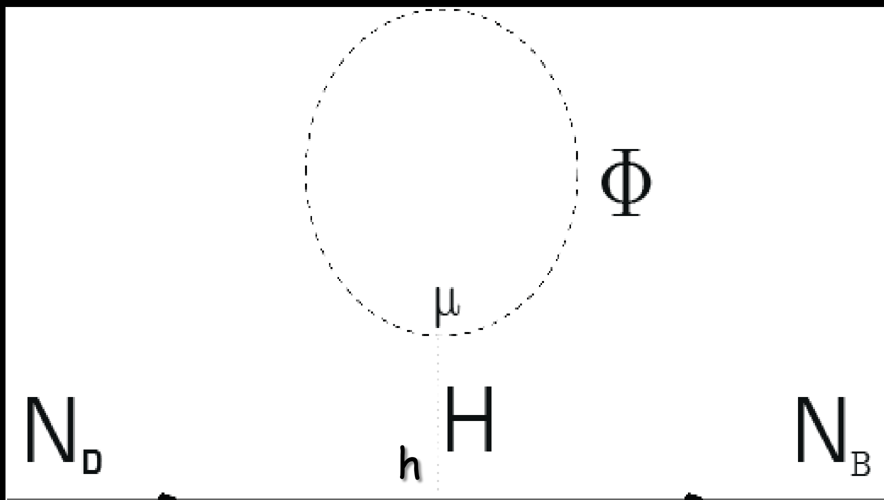
- Neutrino physics is a good place where to look for such a solution. A high scale DM requires to extend the usual type-I seesaw Lagrangian (able to explain neutrino masses and mixing and the matter-antmatter asymmetry via leptogenesis).

- Higgs induced RH-RH neutrino mixing provides a way to produce dark neutrinos with the right abundance and….also to make them shining.

- Density matrix calculations are crucial and seem to suggest new possibilities such as production during a phase transition, interplay with Higgs portal interactions, …..

A possible GUT origin ?

(Anisimov,PDB, 2010, unpublished)



$$\frac{1}{\Lambda_{\rm eff}} = \frac{h\mu}{M_{\rm GUT}^2}$$

$$\Lambda_{\rm eff} >> M_{\rm GUT} \,!$$

or also:

$\Rightarrow \Lambda \sim y^{-2} M_F$

$\rightsquigarrow$ some heavy fermion ($M_F \sim M_{GUT}$)