Phenomenological aspects of neutrino mass models with flavour and CP symmetries

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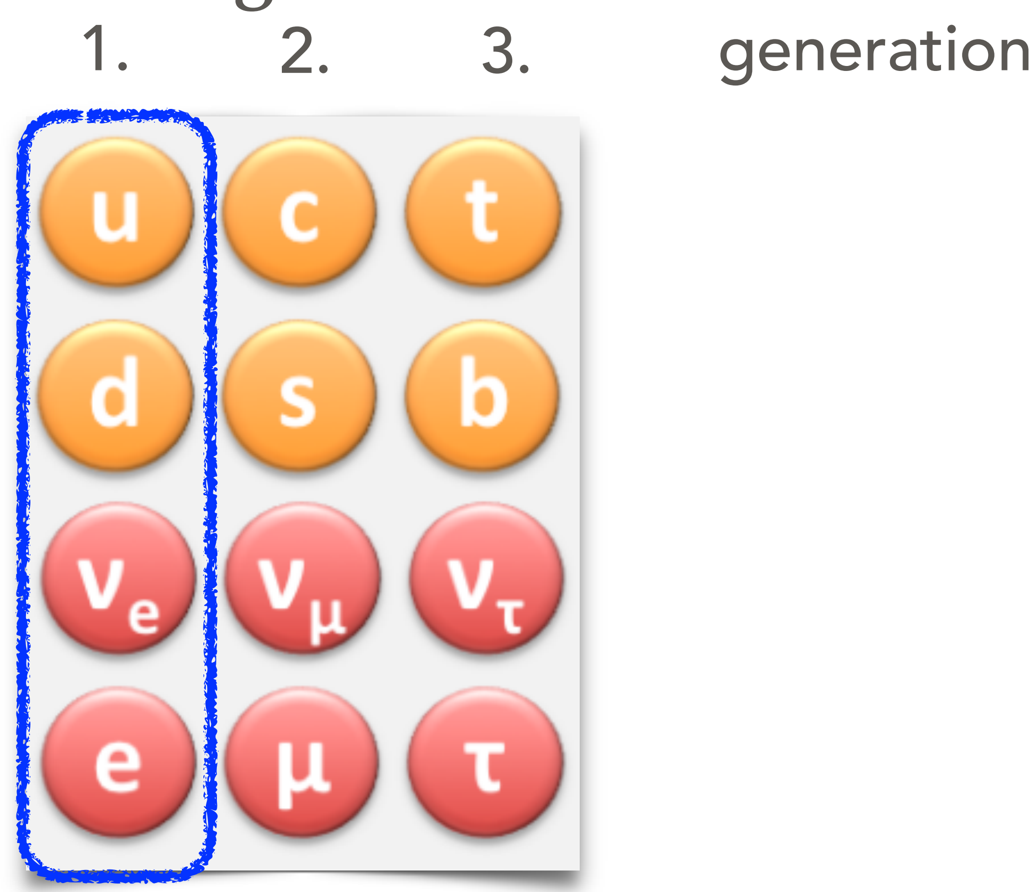
Overview

- Introduction — Flavour and CP symmetries
- Example of flavour and CP symmetries
- Scenario with inverse seesaw mechanism
- Scenario with type I seesaw mechanism
- Summary and Outlook

Introduction — Flavour and CP symmetries

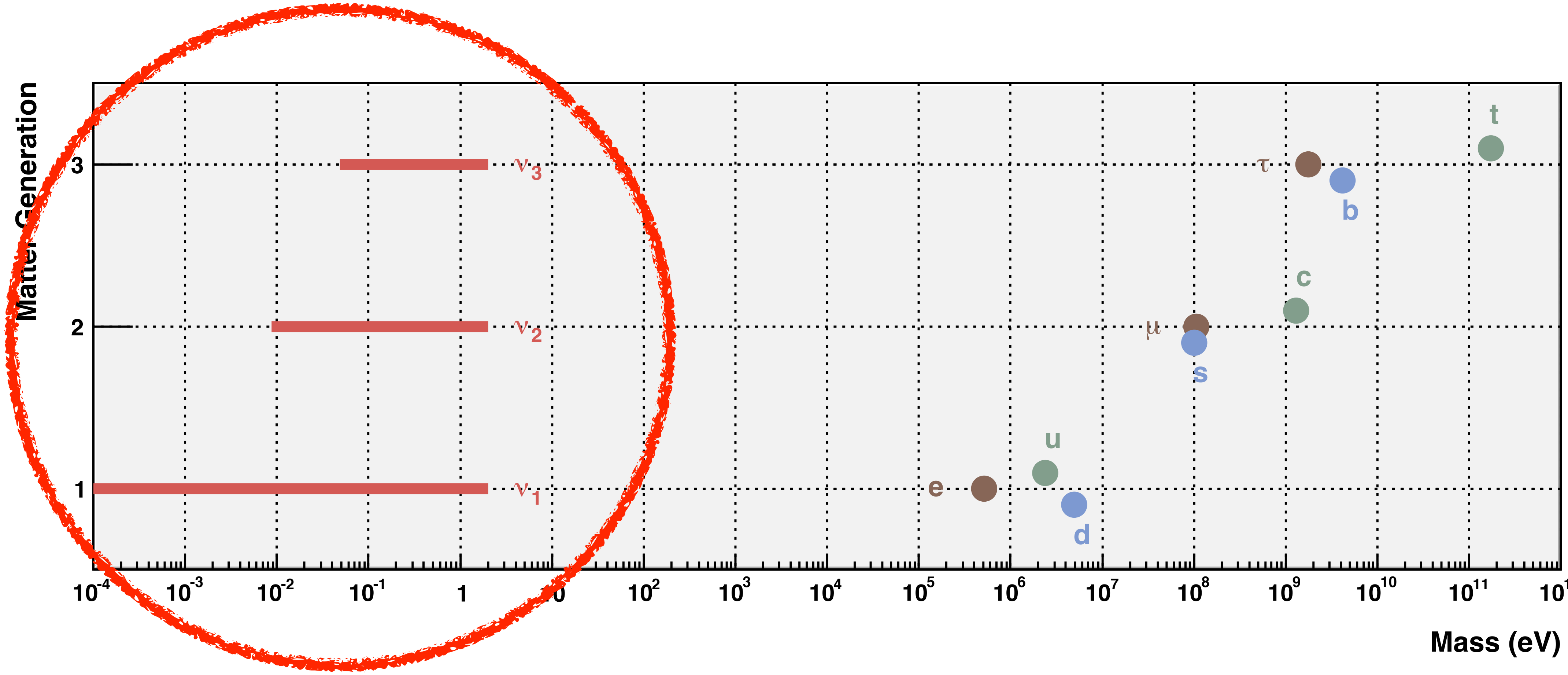
- **Replication of fermion generations**



- No explanation of three generations in the Standard Model (SM)
- Anomaly freedom of SM gauge group does not depend on number of generations
- Only first generation needed for `our world'
- Hints for more generations? … maybe a sterile neutrino

Introduction — Flavour and CP symmetries

- Replication of fermion generations
- **Fermion masses**

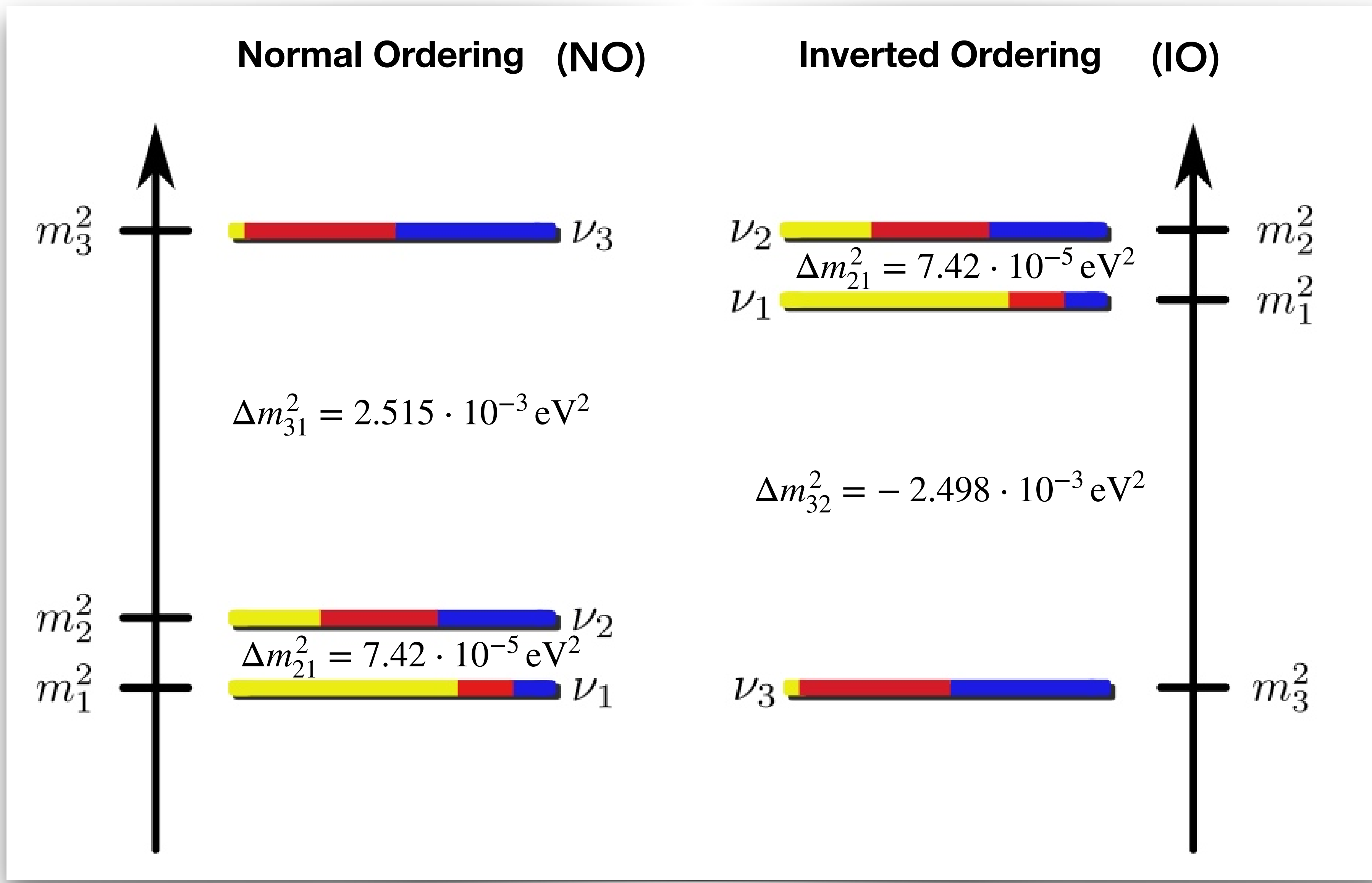


- Strong hierarchy among charged fermion masses, especially up-type quarks
- Neutrinos are much lighter and may have different hierarchy

Introduction — Flavour and CP symmetries

- Replication of fermion generations
- **Fermion masses**

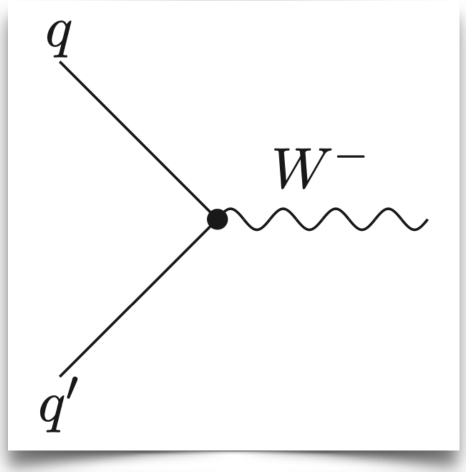
- Neutrinos are much lighter and may have different hierarchy



NuFIT 5.1 ('21)

Introduction — Flavour and CP symmetries

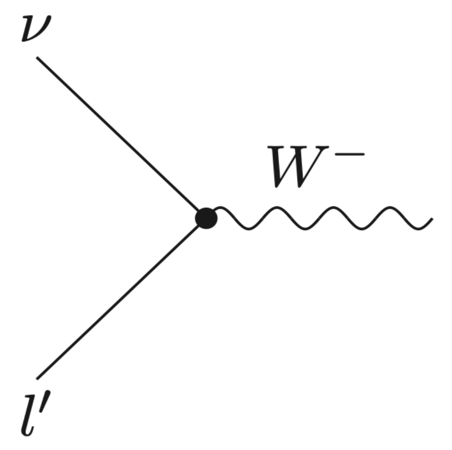
- Replication of fermion generations
- Fermion masses
- **Quark and lepton mixing**



$$\begin{pmatrix} 0.97 & 0.22 & 3.7 \cdot 10^{-3} \\ 0.22 & 0.97 & 0.042 \\ 9.0 \cdot 10^{-3} & 0.041 & 0.999 \end{pmatrix}$$

PDG ('20)

Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix



$$\begin{pmatrix} 0.82 & 0.55 & 0.15 \\ 0.29 & 0.59 & 0.75 \\ 0.49 & 0.59 & 0.64 \end{pmatrix}$$

NuFIT 5.1 ('21)

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix

Introduction — Flavour and CP symmetries

- Replication of fermion generations
- Fermion masses
- Quark and lepton mixing

- Features can be described in the SM,
  but
  - Number of free parameters in flavour sector is by far the largest in the SM
  - Yukawa couplings span several orders of magnitude
  - Origin of neutrino masses is unclear
  - Striking differences among quarks and leptons are not understood

Introduction — Flavour and CP symmetries

- Many ideas have been put forward in order to explain fermion masses and mixing and also (some of) the flavour anomalies.
- One very interesting approach is to assume a **new symmetry, acting on flavour space**, e.g.

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \to \begin{pmatrix} q_2 \\ q_3 \\ q_1 \end{pmatrix}$$

  with q_i being the ith quark generation.
  This constrains the couplings in the flavour sector.
- This approach is inspired by the success of gauge symmetries.

Properties of this new symmetry G_f ?

Introduction — Flavour and CP symmetries

Properties of this new symmetry G_f ?

G_f could be …

- … abelian or non-abelian
- … continuous or discrete
- … local or global
- … spontaneously broken or explicitly
- … broken arbitrarily or to non-trivial subgroups
- … broken at low or high energies

Its maximal possible size depends on the chosen gauge group.

Introduction — Flavour and CP symmetries

Properties of this new symmetry G_f ?

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Its maximal possible size depends on the chosen gauge group.

Introduction — Flavour and CP symmetries

Properties of this new symmetry G_f?

G_f could be …

- … abelian or **non-abelian** (**three generations**)
- … continuous or **discrete** (**preferred directions**)
- … local or global
- … spontaneously broken or explicitly
- … broken arbitrarily or to **non-trivial subgroups** (**predictive**)
- … broken at low or high energies

Its maximal possible size depends on the chosen gauge group.

Introduction — Flavour and CP symmetries

There are many options …

- Dihedral symmetries D_n as well as D_n'
- Symmetric and alternating groups, S_n and A_n
- Discrete subgroups of modular group
- Groups $\Sigma(n\,\varphi)$
- Adding CP symmetries
- Series of groups $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$ — also with CP
- …

Introduction — Flavour and CP symmetries

Much research has been performed in this direction,
see e.g. works by

Altarelli, Antusch, Branco, Calibbi, Centelles Chulia, Chen, Chu, Dasgupta, de Medeiros Varzielas, Ding, Everett, Feruglio, Gavela, Gehrlein, Girardi, Gonzalez Felipe, Grimus, CH, He, Hirsch, Joaquim, King, Lavoura, Luhn, Mahanthappa, Machado, Medina, Melis, Meloni, Merlo, Meroni, Mohapatra, Neder, Nilles, Nishi, Pas, Pascoli, Petcov, Rodejohann, Schumacher, Serodio, Shimizu, Smirnov, Spinrath, Srivastava, Stuart, Tanimoto, Titov, Valle, Vicente, Vien, Vives, Xu, Yamamoto, Ziegler, …

as well as the following reviews

Ishimori et al. ('10), King/Luhn ('13), Feruglio/Romanino ('19); Grimus/Ludl ('11)

Introduction — Flavour and CP symmetries

There are many options …

- Dihedral symmetries D_n as well as D'_n
- Symmetric and alternating groups, S_n and A_n
- Discrete subgroups of modular group
- Groups $\Sigma(n\,\varphi)$
- Adding CP symmetries
- **Series of groups $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$ — also with CP**
- …

Example of flavour and CP symmetries

Series of groups $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$

- Have 3-dim irrep(s)
- Can also offer 1-dim irreps and 2-dim irreps
- Are subgroups of SU(3)

$\Delta(3\,n^2)$

Luhn/Nasri/Ramond ('07)

$$a^3 = e \;,\quad c^n = e \;,\quad d^n = e \;,$$
$$c\,d = d\,c \;,\quad a\,c\,a^{-1} = c^{-1} d^{-1} \;,\quad a\,d\,a^{-1} = c$$

$$g = a^\alpha c^\gamma d^\delta \quad \text{with} \quad \alpha = 0, 1, 2 \;,\; 0 \leq \gamma, \delta \leq n-1$$

A well-known member is the permutation group A_4

Example of flavour and CP symmetries

Series of groups $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$

- Have 3-dim irrep(s)
- Can also offer 1-dim irreps and 2-dim irreps
- Are subgroups of SU(3)

$\Delta(6\,n^2)$ Add to relations of $\Delta(3\,n^2)$ Escobar/Luhn ('08)

$$b^2 = e \;\; , \;\; (a\,b)^2 = e \;\; , \;\; b\,c\,b^{-1} = d^{-1} \;\; , \;\; b\,d\,b^{-1} = c^{-1}$$

$$g = a^\alpha b^\beta c^\gamma d^\delta \quad \text{with} \quad \alpha = 0,1,2\,, \;\; \beta = 0,1\,, \;\; 0 \leq \gamma, \delta \leq n-1$$

A well-known member is the permutation group S_4

Example of flavour and CP symmetries

Add CP as further symmetry

Grimus/Rebelo ('95),

Ecker/Grimus/Neufeld ('84,'87,'88)

- Motivation:
  For more than one generation of certain particle species, define CP that also acts on generations of particles, e.g.

  $$\Phi_i(x) \quad \rightarrow \quad X_{ij}\,\Phi_j^\dagger(x_P) \;\; \text{with} \;\; (x_P)_\mu = x^\mu$$

  with

  $$X X^\dagger = X X^\star = 1$$

- Phenomenological viewpoint: Feruglio/CH/Ziegler ('12)
  adding CP and exploring the interplay between flavour and CP symmetries opens new avenues for description of mixing (new patterns and constraints on Majorana phases)

Harrison/Scott ('02), Grimus/Lavoura ('03)

- CP is involution and corresponds to automorphism of flavour symmetry Holthausen/Lindner/Schmidt ('12), Chen et al. ('14)

Example of flavour and CP symmetries

Breaking of symmetries

Feruglio/CH/Ziegler ('12)

Idea: Keep some residual symmetry among charged leptons and neutrinos, G_e and G_ν, with $G_e \neq G_\nu$

Mismatch of symmetries corresponds to lepton mixing

G_f and CP

$\swarrow$ $\searrow$

charged leptons	neutrinos
G_e	$G_\nu = Z_2 \times$ CP
U_e	$U_\nu = \Omega_\nu R(\theta) K_\nu$

$\searrow$ $\swarrow$

$$U_{PMNS} = U_e^\dagger \Omega_\nu R(\theta) K_\nu$$

Example of flavour and CP symmetries

Breaking of symmetries

Feruglio/CH/Ziegler ('12)

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G_f and CP

$\swarrow$ $\searrow$

Minimal choice Z_3	charged leptons	neutrinos	Choice Z_2 and CP
	G_e	$G_\nu = Z_2 \times$ CP	
3 different masses	U_e	$U_\nu = \Omega_\nu R(\theta) K_\nu$	Majorana masses

$\searrow$ $\swarrow$

$U_{PMNS} = U_e^\dagger \Omega_\nu R(\theta) K_\nu$ Free parameter, Majorana phases

Size of masses not explained

Example of flavour and CP symmetries

CH/Meroni/Molinaro ('14)

$$G_f \text{ and } \mathrm{CP}$$

$\swarrow$ $\searrow$

charged leptons	neutrinos
G_e	$G_\nu = Z_2 \times \mathrm{CP}$
U_e	$U_\nu = \Omega_\nu R(\theta) K_\nu$

$\searrow$ $\swarrow$

$$U_{PMNS} = U_e^\dagger \Omega_\nu R(\theta) K_\nu$$

Generator Q

Generator Z

CP X

$$X = \Omega_\nu \, \Omega_\nu^T$$

[for concreteness $K_\nu = 1$]

n from group

$$(Q = a, Z = c^{n/2}, X = a\, b\, c^s d^{2s} P_{23}) \; ,$$
$$(Q = a, Z = c^{n/2}, X = c^s d^t P_{23}) \; ,$$
$$(Q = a, Z = b\, c^m d^m, X = b\, c^s d^{n-s} P_{23}) \, ,$$

$$X = g\, X_0$$
$$X_0 = P_{23}$$

m $\qquad$ s and t $\qquad$ free

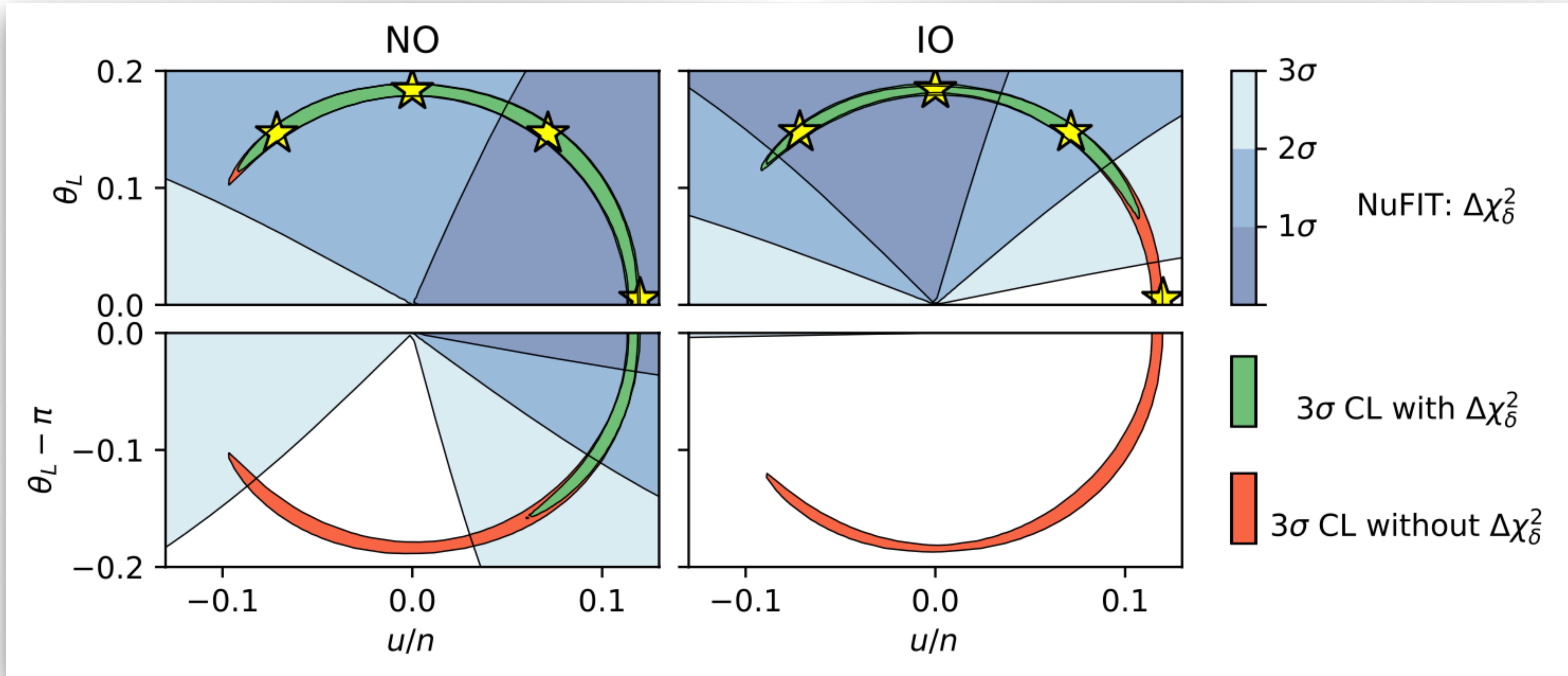
Four different types of mixing patterns with different properties

Case 1) **Case 2)** **Case 3 a)** **Case 3 b.1)**

Example of flavour and CP symmetries

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 2)



$u = 2\,s - t$

$v = 3\,t$ relevant mainly for Majorana phase

Example of flavour and CP symmetries

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 2)

$n = 14$

u	$u=-1$	$u=0$	$u=+1$
θ_L	0.146 (0.148)	0.184	0.146 (0.148)
$\sin^2\theta_{12}$	0.341	0.341	0.341
$\sin^2\theta_{13}$	0.0222 (0.0224)	0.0222 (0.0224)	0.0222 (0.0224)
$\sin^2\theta_{23}$	0.437	0.5	0.563
$\Delta\chi^2$	9.25 (11.2)	10.8 (12.5)	8.27 (8.62)

$\sin\delta = -1$ for $u = 0$

$\sin\delta \approx -0.811\,(-0.813)$ for $u = \pm 1$

several choices for v admitted

Scenario with inverse seesaw mechanism

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

- Consider a scenario of **(3,3) ISS**, i.e. 3 generations of LH doublets, 3 generations of N_i and S_j, all of them gauge singlets

Mohapatra / Valle ('86), Mohapatra (´86), Bernabeu et al. (´87), Gonzalez-Garcia / Valle (´89)

$$-(y_D)_{\alpha i}\,\overline{L^c_\alpha}\,H\,N^c_i - (M_{NS})_{ij}\,\overline{N}_i\,S_j - \frac{1}{2}\,(\mu_S)_{kl}\,\overline{S^c_k}\,S_l + \text{h.c.}$$

Mass matrix of neutral states

$$\mathcal{M}_{\text{Maj}} = \begin{pmatrix} \mathbb{0} & m_D & \mathbb{0} \\ m_D^T & \mathbb{0} & M_{NS} \\ \mathbb{0} & M_{NS}^T & \mu_S \end{pmatrix} \quad \text{with} \;\; m_D = y_D\,\frac{v}{\sqrt{2}}$$

- Light neutrino masses

$|\mu_S| \ll |m_D| \ll |M_{NS}|$:

$$m_\nu = m_D \left(M_{NS}^{-1}\right)^T \mu_S\, M_{NS}^{-1}\, m_D^T$$

Scenario with inverse seesaw mechanism

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

- We take

$$\alpha_R \sim 1 \qquad L_\alpha \sim 3 \,, N_i \sim 3 \,, S_j \sim 3$$

[detail: use additional Z_3 to distinguish e, μ, τ]

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[detail: use additional Z_3 to distinguish e, μ, τ]

$$L_\alpha \sim 3 \;, N_i \sim 3 \;, S_j \sim 3$$

Charged lepton mass matrix

Generator
Q=a

[detail: diagonal subgroup of Z_3 from flavour symmetry and additional Z_3]

$$\begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

[basis choice: Q=a diagonal]

Scenario with inverse seesaw mechanism

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[detail: use additional Z_3 to distinguish e, μ, τ]

$L_\alpha \sim 3 \, , N_i \sim 3 \, , S_j \sim 3$

Neutrino mass matrix

Generator Z

CP X

$$\mathcal{M}_{\text{Maj}} = \begin{pmatrix} \mathbb{0} & m_D & \mathbb{0} \\ m_D^T & \mathbb{0} & M_{NS} \\ \mathbb{0} & M_{NS}^T & \mu_S \end{pmatrix} \quad \text{with} \quad m_D = y_D \, \frac{v}{\sqrt{2}}$$

Scenario with inverse seesaw mechanism

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

Option 1

- We take

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[detail: use additional Z_3 to distinguish e, μ, τ]

Neutrino mass matrix

$$-(y_D)_{\alpha i}\, \overline{L}^c_\alpha\, H\, N^c_i - (M_{NS})_{ij}\, \overline{N}_i\, S_j - \frac{1}{2}\, (\mu_S)_{kl}\, \overline{S}^c_k\, S_l + \text{h.c.}$$

No symmetry breaking

$$m_D = y_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \frac{v}{\sqrt{2}} \quad \text{with} \;\; y_0 > 0$$

$$M_{NS} = M_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{with} \;\; M_0 > 0$$

Scenario with inverse seesaw mechanism

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

Option 1

- We take

$$\alpha_R \sim 1$$

[detail: use additional Z_3 to distinguish e, μ, τ]

$$L_\alpha \sim 3 \;, N_i \sim 3 \;, S_j \sim 3$$

Neutrino mass matrix

$$-(y_D)_{\alpha i}\, \overline{L}^c_\alpha\, H\, N^c_i - (M_{NS})_{ij}\, \overline{N}_i\, S_j - \frac{1}{2}\, (\mu_S)_{kl}\, \overline{S}^c_k\, S_l + \text{h.c.}$$

Symmetry breaking

$$U_S^T\, \mu_S\, U_S = \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}$$

$$U_S = \Omega(\mathbf{3})\, R_{fh}(\theta_S)$$

Scenario with inverse seesaw mechanism

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

Option 1

- We take

$$\alpha_R \sim 1 \qquad L_\alpha \sim 3 \,, N_i \sim 3 \,, S_j \sim 3$$

[detail: use additional Z_3 to distinguish e, μ, τ]

Light neutrino mass matrix

$$m_\nu = \frac{y_0^2\, v^2}{2\, M_0^2}\, \mu_S = \frac{y_0^2\, v^2}{2\, M_0^2}\, U_S^\star \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix} U_S^\dagger$$

Neutrino masses

$$m_i = \frac{y_0^2\, v^2}{2\, M_0^2}\, \mu_i \quad \text{for} \quad i = 1, 2, 3$$

Lepton mixing

$$\widetilde{U}_{\rm PMNS} = \Omega(\mathbf{3})\, R_{fh}(\theta_S)$$

at leading order

Scenario with inverse seesaw mechanism

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

Option 1

- We take

$$\alpha_R \sim 1 \qquad L_\alpha \sim 3\ , N_i \sim 3\ , S_j \sim 3$$

[detail: use additional Z_3 to distinguish e, μ, τ]

Heavy states

$$M_{h,i} = M_0 - \frac{\mu_i}{2} \quad \text{and} \quad M_{h,i+3} = M_0 + \frac{\mu_i}{2} \quad \text{with} \quad i = 1, 2, 3 .$$

Back to light neutrinos

… go beyond leading order Hettmansperger/Lindner/Rodejohann ('11)

- potentially new contributions to m_ν
- effects of non-unitarity

Scenario with inverse seesaw mechanism

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

Option 1

Back to light neutrinos

… go beyond leading order

- potentially new contributions to m_ν
- effects of non-unitarity

$$m_\nu^1 = -\frac{1}{2} m_D \left(M_{NS}^{-1}\right)^T \left[\mu_S\, M_{NS}^{-1}\, m_D^T\, m_D^\star \left(M_{NS}^{-1}\right)^\dagger + \left(M_{NS}^{-1}\right)^\star m_D^\dagger\, m_D \left(M_{NS}^{-1}\right)^T \mu_S\right] M_{NS}^{-1}\, m_D^T$$

$$m_\nu^1 = -\frac{y_0^4\, v^4}{4\, M_0^4}\, \mu_S = -\frac{y_0^4\, v^4}{4\, M_0^4}\, U_S^\star \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix} U_S^\dagger$$

Compare to

$$m_\nu = \frac{y_0^2\, v^2}{2\, M_0^2}\, \mu_S = \frac{y_0^2\, v^2}{2\, M_0^2}\, U_S^\star \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix} U_S^\dagger$$

Scenario with inverse seesaw mechanism

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

Option 1

Back to light neutrinos

... go beyond leading order

- potentially new contributions to m_ν
- effects of non-unitarity

$$\widetilde{U}_{\rm PMNS} = \left(\mathbb{1} - \eta\right) U_0$$

$$\eta = \frac{1}{2}\, m_D^\star \left(M_{NS}^{-1}\right)^\dagger M_{NS}^{-1}\, m_D^T$$

$$\eta = \frac{y_0^2\, v^2}{4\, M_0^2}\, \mathbb{1} \equiv \eta_0\, \mathbb{1}$$

Universal effect in flavour α and for different patterns Case 1) Case 2) Case 3 a) Case 3 b.1)

Compare to

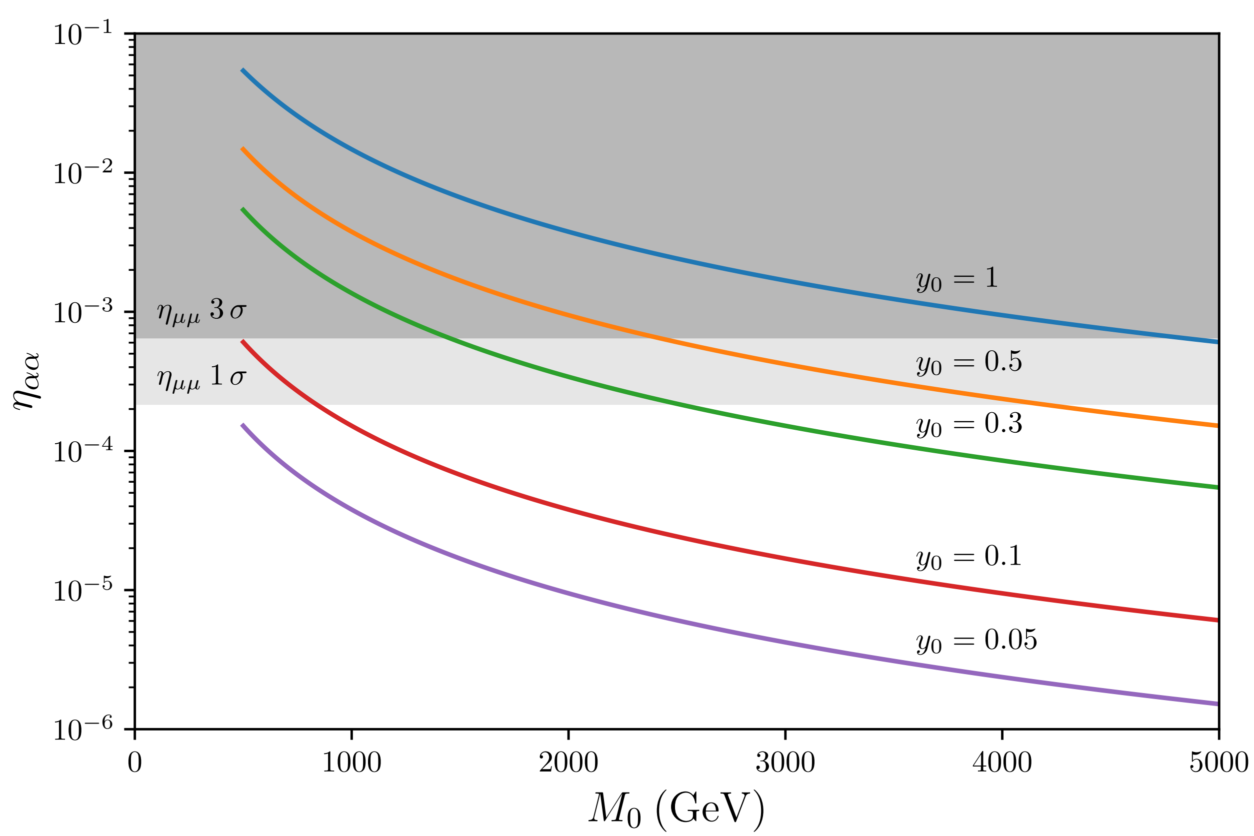
$$\widetilde{U}_{\rm PMNS} = \Omega(\mathbf{3})\, R_{fh}(\theta_S)$$

Scenario with inverse seesaw mechanism

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

Option 1

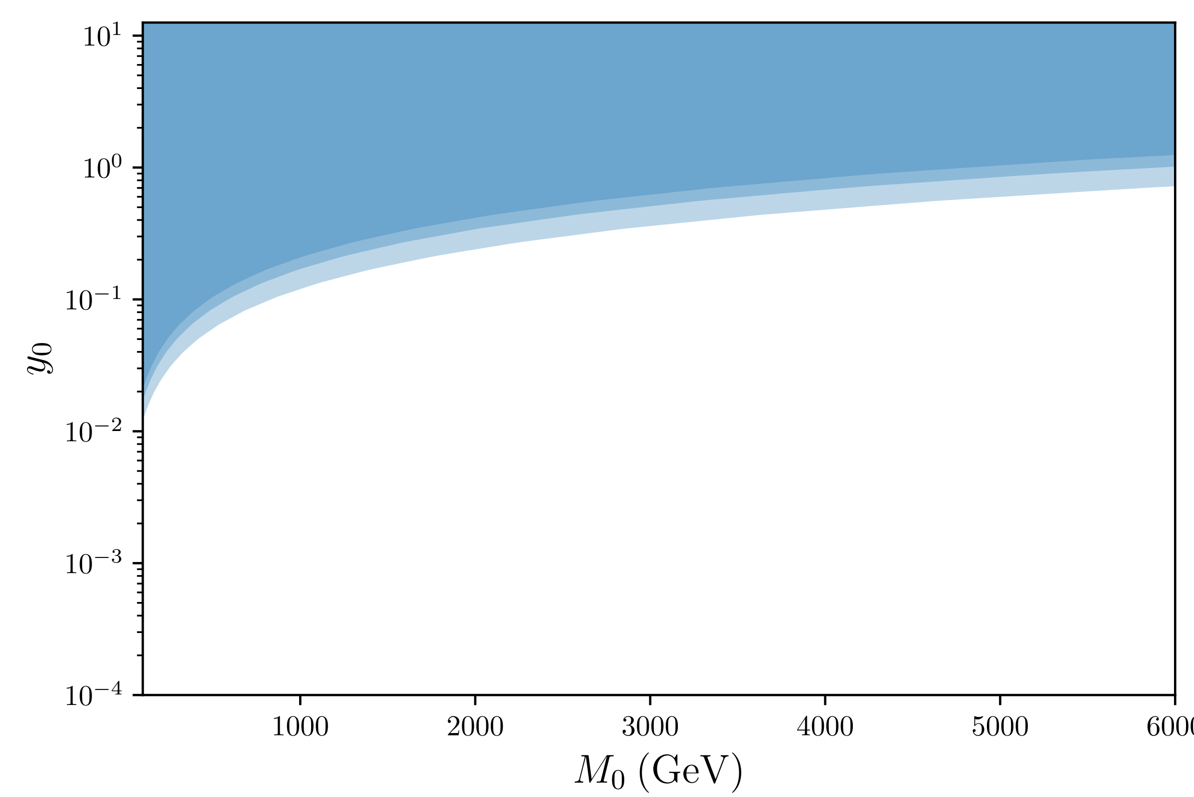
Constraints from non-unitarity



Strongest bound comes from $\eta_{\mu\mu}$

$$|\eta_{\alpha\beta}| \leq \begin{pmatrix} 1.3 \cdot 10^{-3} & 1.2 \cdot 10^{-5} & 1.4 \cdot 10^{-3} \\ 1.2 \cdot 10^{-5} & 2.2 \cdot 10^{-4} & 6.0 \cdot 10^{-4} \\ 1.4 \cdot 10^{-3} & 6.0 \cdot 10^{-4} & 2.8 \cdot 10^{-3} \end{pmatrix}$$

Fernandez-Martinez et al. ('16)



Scenario with inverse seesaw mechanism

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

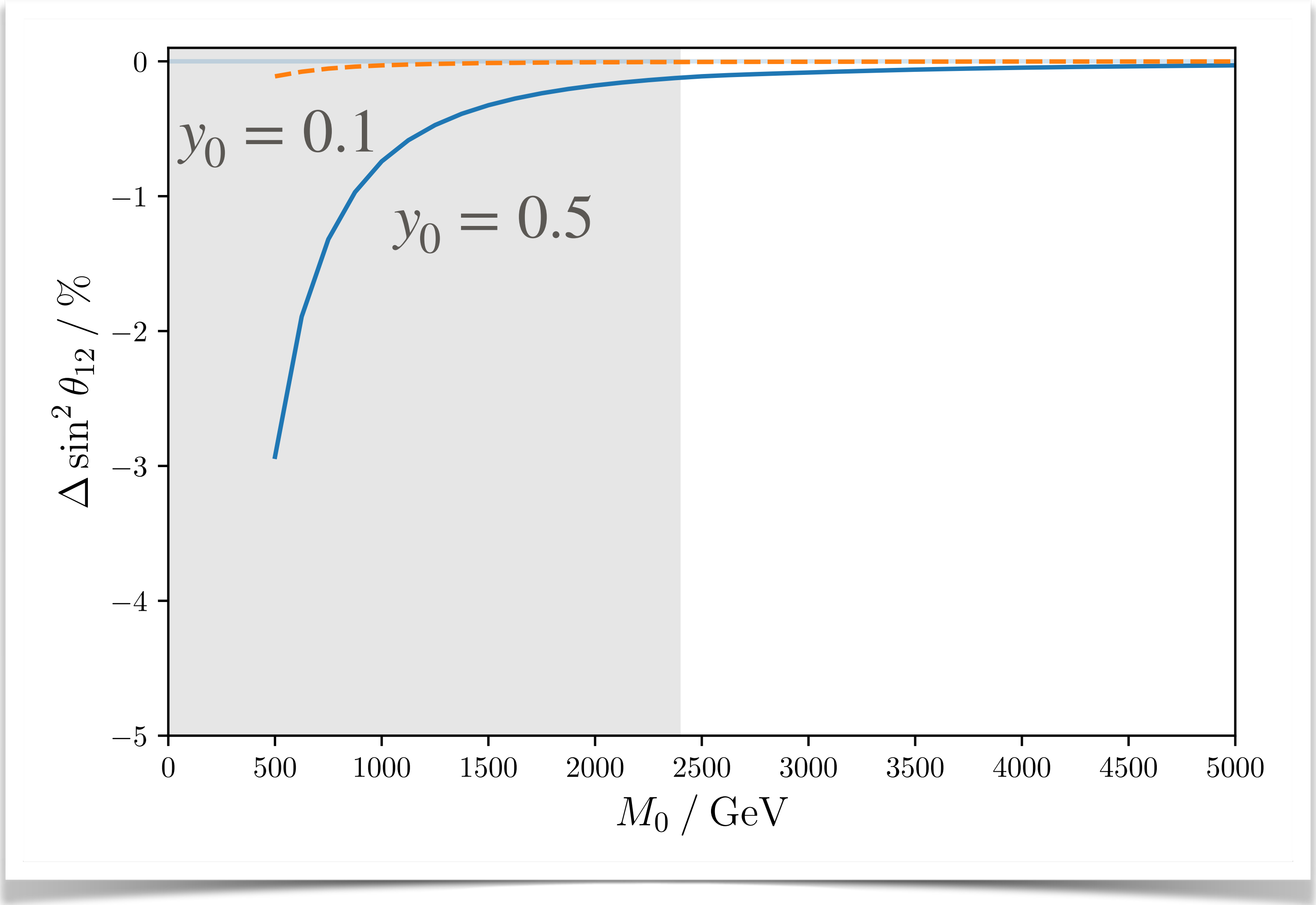
Option 1

Effect on lepton mixing

Case 1)

$$\sin^2\theta_{12} = \frac{1}{2+\cos 2\theta}$$

$$\theta \approx 0.18$$



Scenario with inverse seesaw mechanism

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

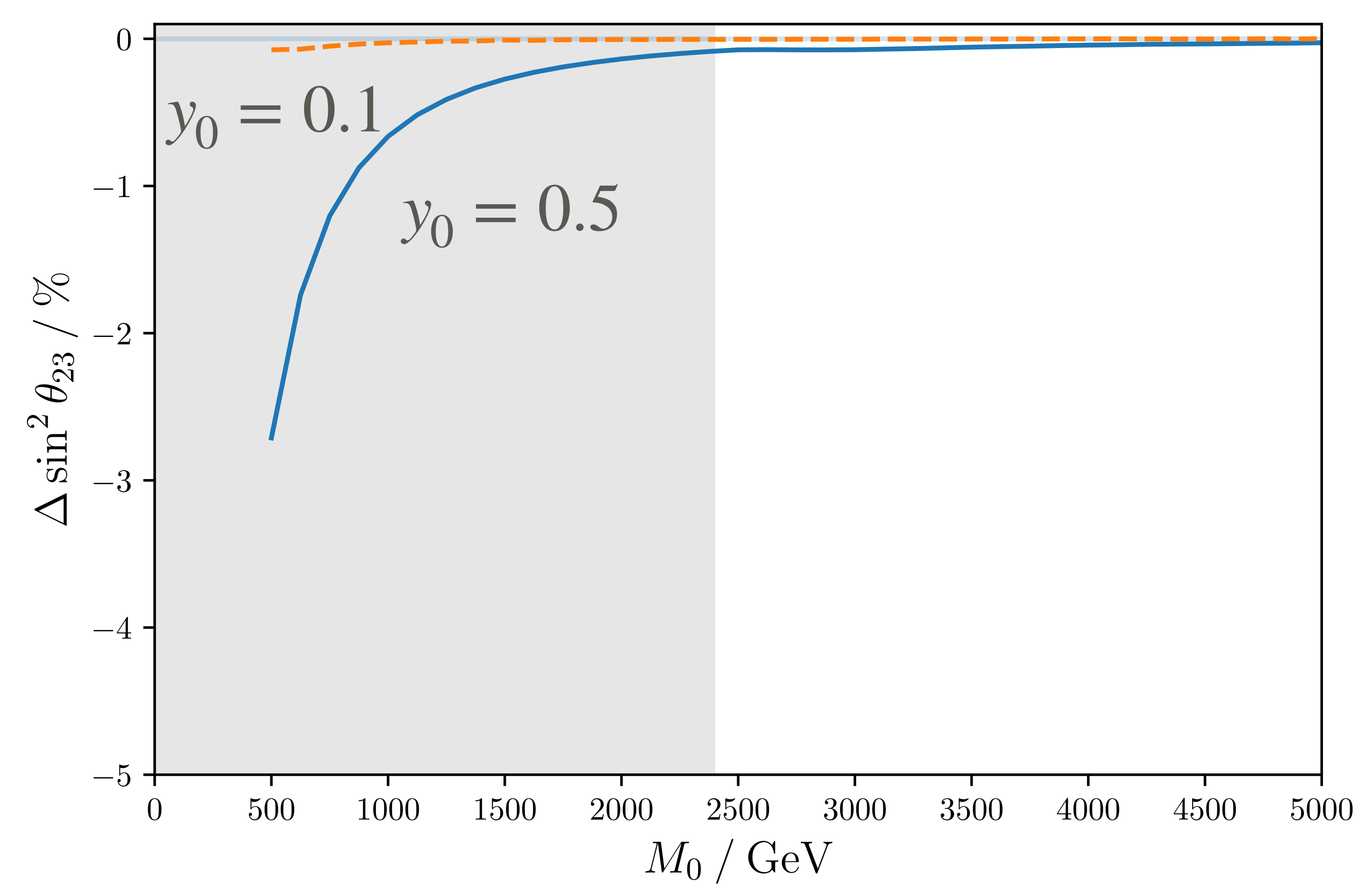
Option 1

Effect on lepton mixing

Case 1)

$$\sin^2\theta_{23} = \frac{1}{2}\left(1 + \frac{\sqrt{3}\,\sin 2\theta}{2+\cos 2\theta}\right)$$

$$\theta \approx 0.18$$



Scenario with inverse seesaw mechanism

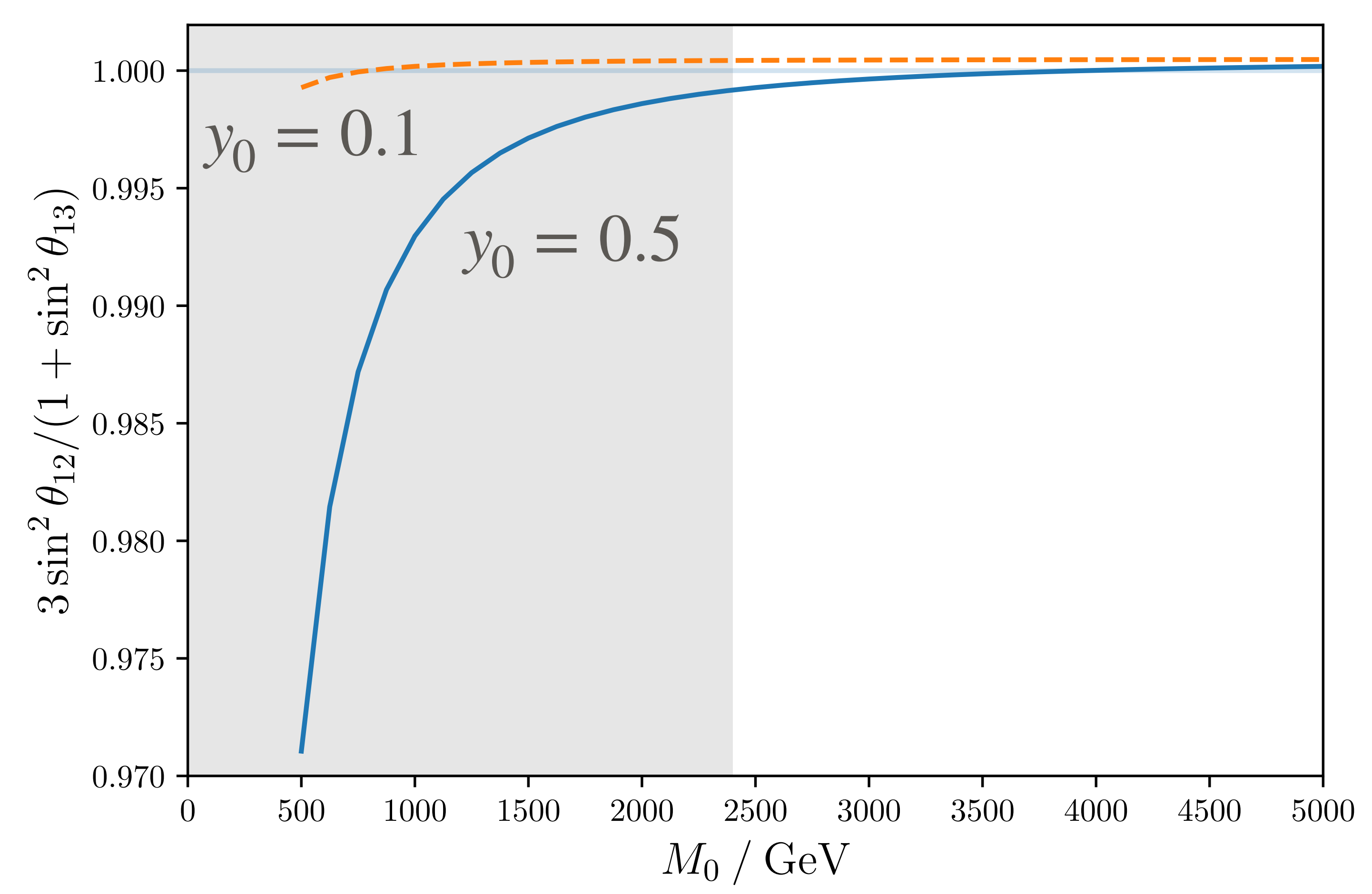
[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

Option 1

Effect on lepton mixing

Case 1)

$$\sin^2\theta_{12} \approx \frac{1}{3}\left(1+\sin^2\theta_{13}\right)$$



Scenario with inverse seesaw mechanism

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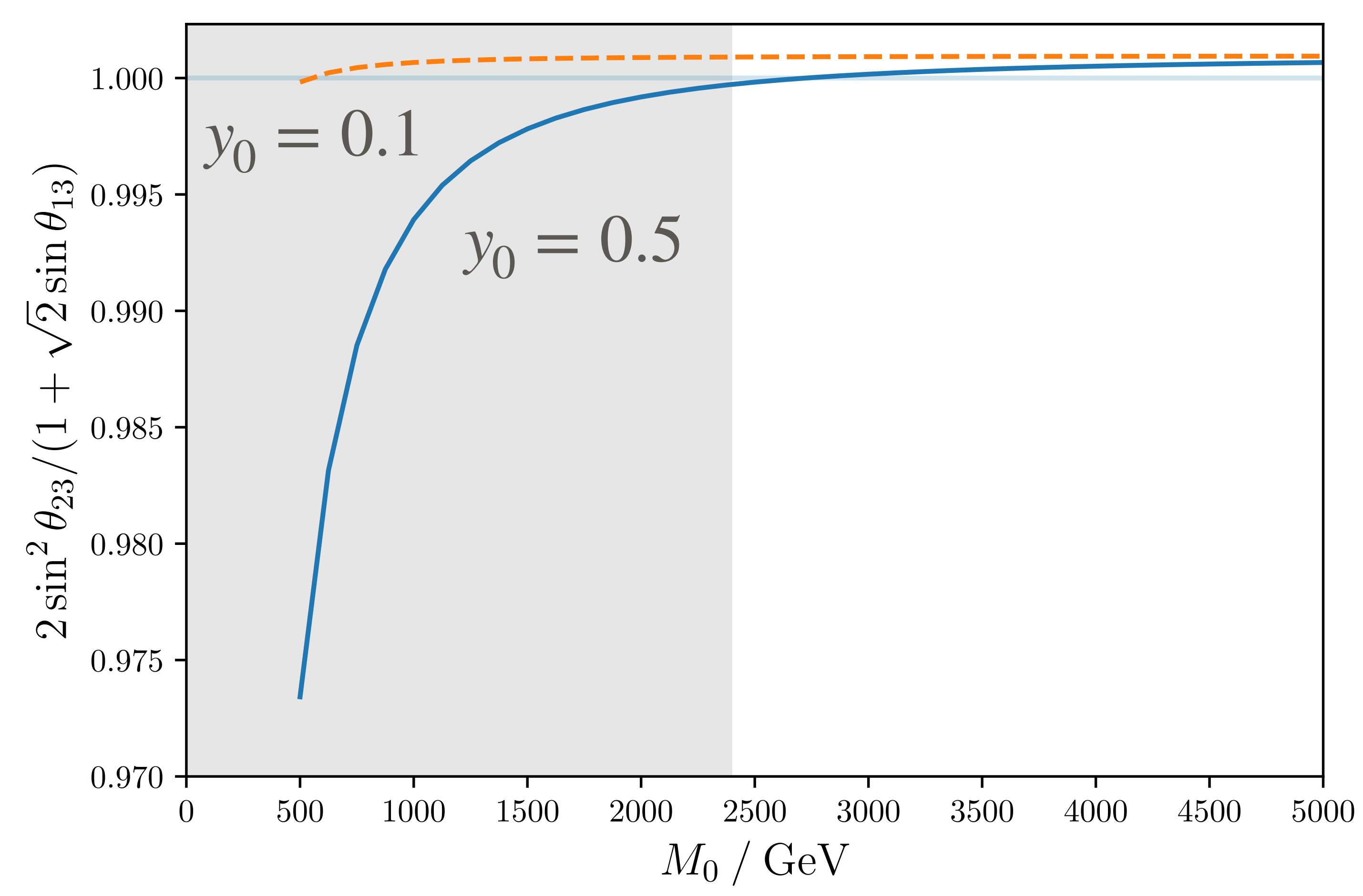
Option 1

Effect on lepton mixing

Case 1)

$$\sin^2\theta_{23} \approx \frac{1}{2}\left(1 \pm \sqrt{2}\,\sin\theta_{13}\right)$$



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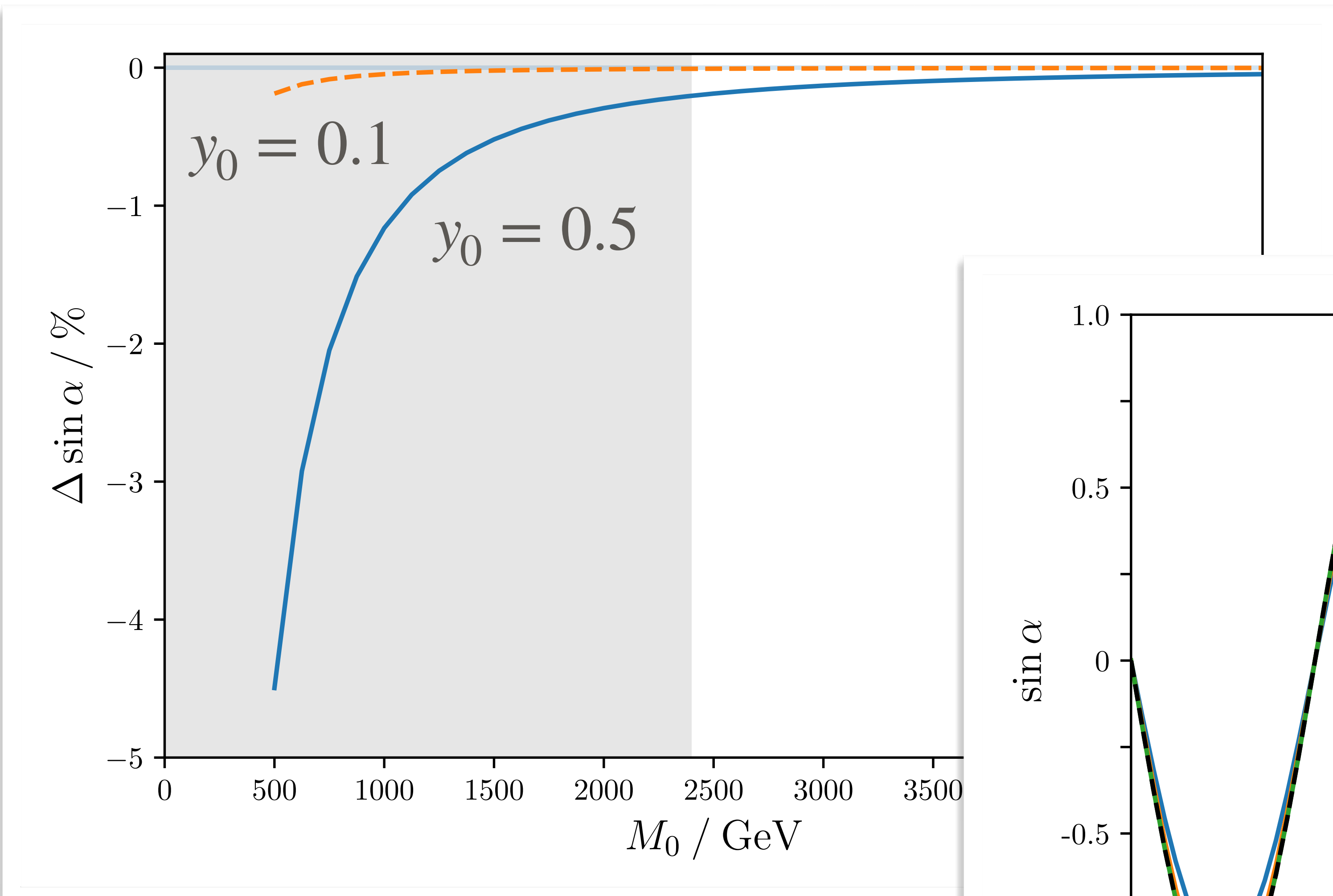
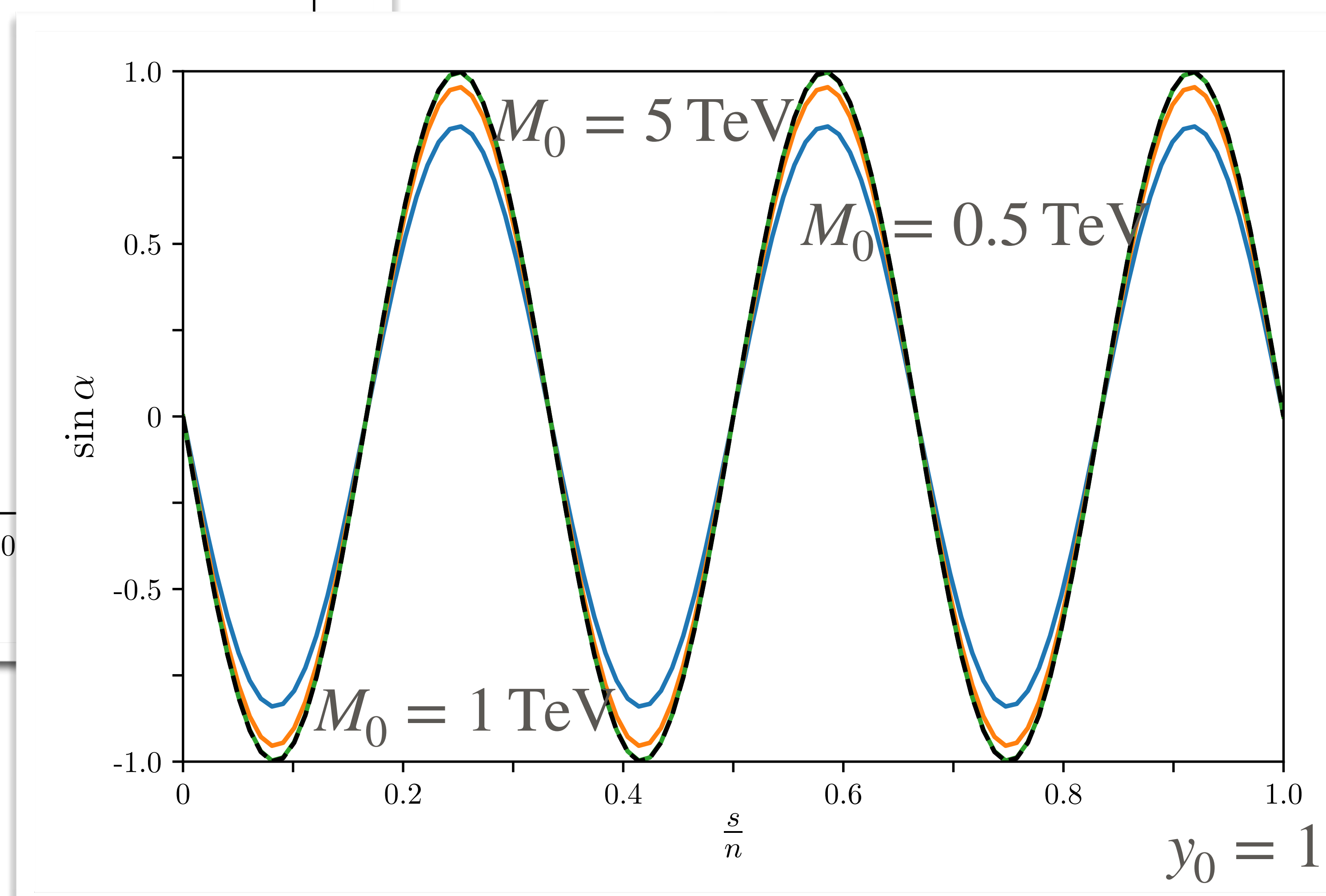


$$\sin\alpha = -\sin 6\,\phi_s$$

$$\phi_s = \frac{\pi\, s}{n}$$



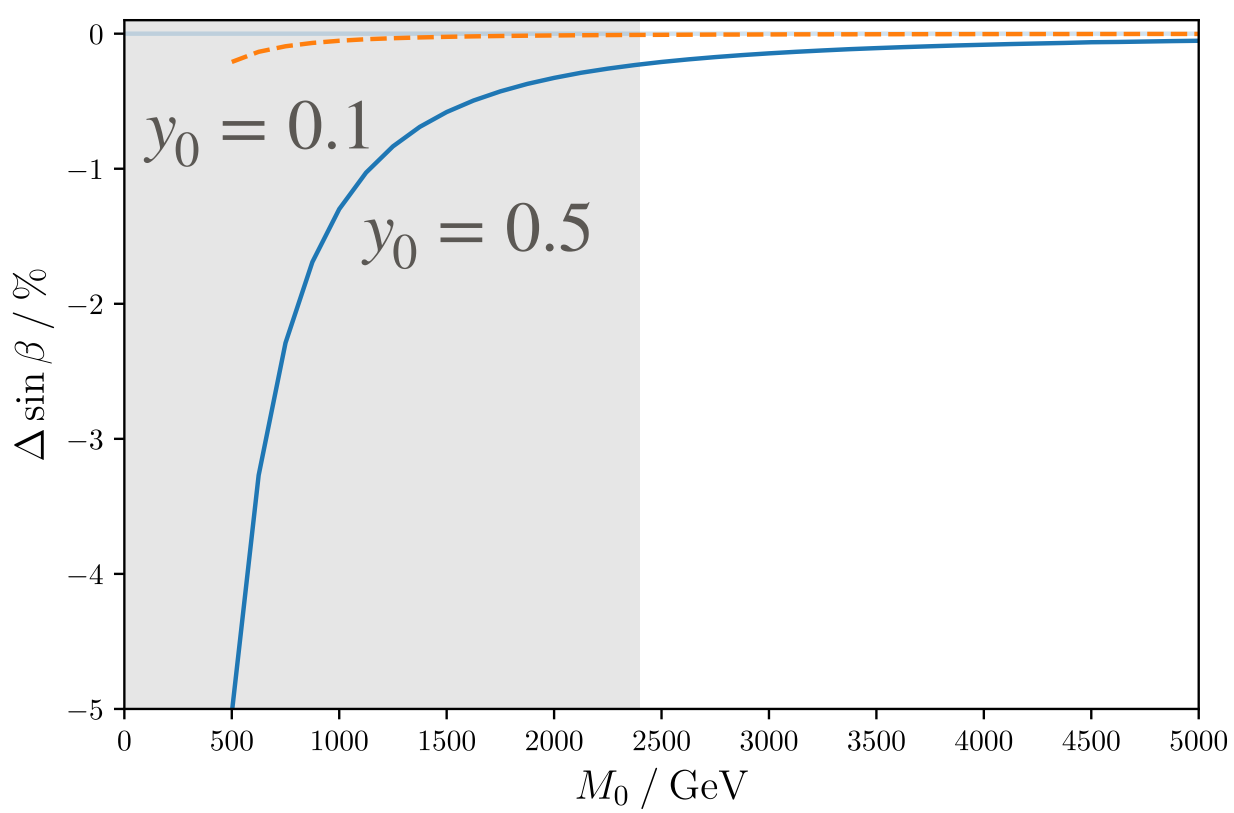
Scenario with inverse seesaw mechanism

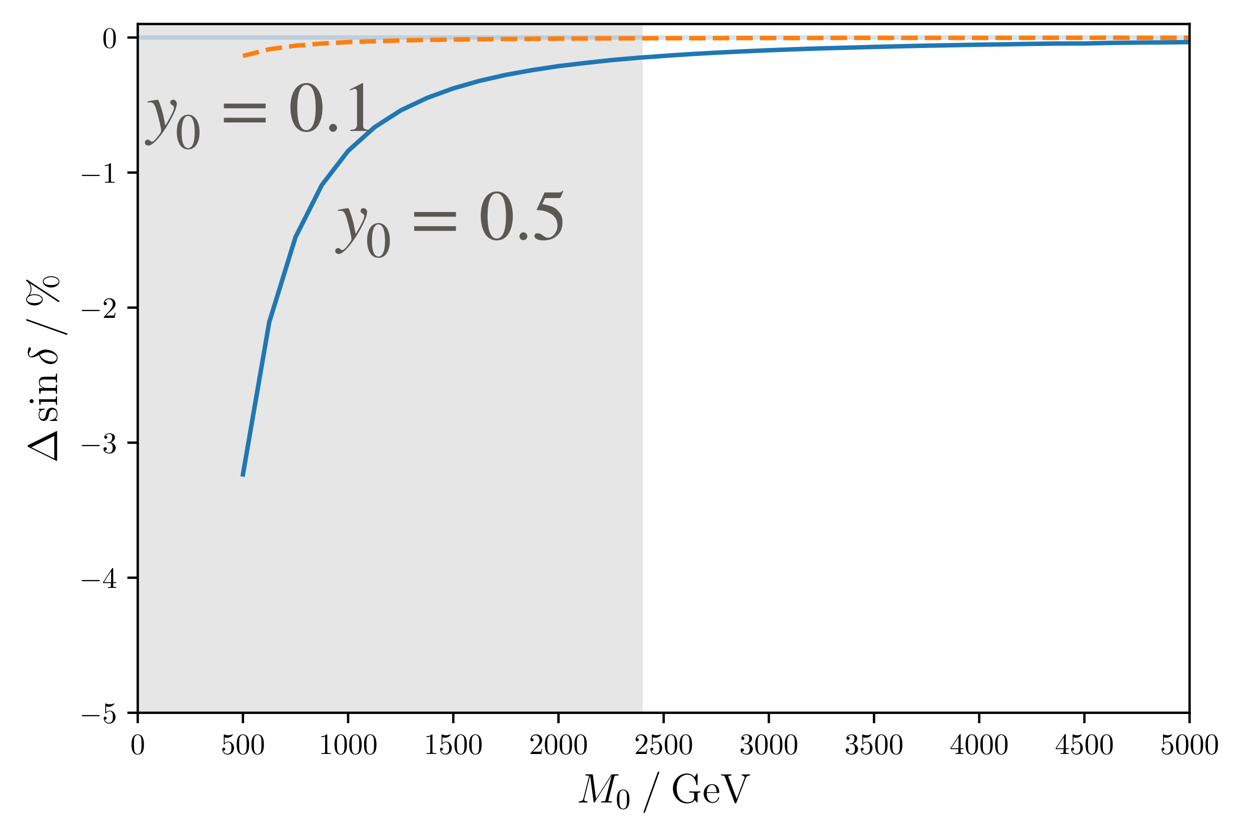
[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

Option 1

Effect on lepton mixing

Case 2)





Analysis also performed for **Case 3 a)** **Case 3 b.1)**

Scenario with inverse seesaw mechanism

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

Option 1

Charged lepton flavour violation

Relevant points

- Lepton number and flavour breaking are both encoded in the matrix

$$U_S^T \, \mu_S \, U_S = \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}$$

$$U_S = \Omega(\mathbf{3}) \, R_{fh}(\theta_S)$$

- Non-unitarity effects are flavour-diagonal and flavour-universal

$$\eta = \frac{y_0^2 \, v^2}{4 \, M_0^2} \, \mathbb{1} \equiv \eta_0 \, \mathbb{1}$$

- Mass spectrum of heavy states is peculiar:
  they form pseudo-Dirac pairs with very small mass splitting and all three such pairs have a common mass scale

$$M_{h,i} = M_0 - \frac{\mu_i}{2} \quad \text{and} \quad M_{h,i+3} = M_0 + \frac{\mu_i}{2} \quad \text{with} \quad i = 1, 2, 3 \, .$$

Scenario with inverse seesaw mechanism

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

Option 1

Charged lepton flavour violation

Relevant points

- Lepton number and flavour breaking are both encoded in the matrix μ_S
- Non-unitarity effects are flavour-diagonal and flavour-universal
- Mass spectrum of heavy states is peculiar:
  they form pseudo-Dirac pairs with very small mass splitting
  and all three such pairs have a common mass scale

Conclusion

Rates of charged lepton flavour violating processes

$\ell_\beta \to \ell_\alpha \gamma$ $\quad$ $\ell_\beta \to 3\,\ell_\alpha$ $\quad$ $\mu - e$ conversion

are very suppressed!

for general formulae see Alonso et al. ('12), Ilakovac/Pilaftsis ('95)

Scenario with inverse seesaw mechanism

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

Option 1

Some more comments

- Neutrinoless double beta decay   Blennow et al. ('10)
- Effect of perturbations of symmetry breaking scenario
- Study other phenomenology, e.g. leptogenesis
- Scrutinise correlations between different observables
- Other interesting options
- Explore cases with different number of sterile states
- Building concrete model

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

- Consider a scenario of **type I seesaw with 3 RH neutrinos**, i.e. 3 generations of LH doublets and 3 generations of gauge singlets $\nu_{R\,i}$

Minkowski ('77), Glashow ('80), Gell-Mann/Ramond/Slansky ('79), Mohapatra/Senjanovic ('80), Yanagida ('80), Schechter/Valle ('80)

$$\mathcal{L} \supset \mathrm{i}\,\overline{\nu_R}\,\partial\!\!\!/\,\nu_R - \frac{1}{2}\overline{\nu_R^c}\,M_R\,\nu_R - \overline{l_L}\,Y_D\,\varepsilon H^*\,\nu_R + \mathrm{h.c.}$$

- Light neutrino masses

$$m_\nu = -\,m_D\,M_R^{-1}\,m_D^T$$ with $$m_D = Y_D\,\langle H\rangle$$

- Heavy neutrino masses are (nearly) degenerate. This is achieved with the help of a symmetry and a suitable playground for low-scale leptogenesis

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

- We take

$$\alpha_R \sim 1 \qquad\qquad l_{L\,\alpha} \sim 3\ ,\nu_{R\,i} \sim 3'$$

[detail: use additional Z_3 to distinguish e, μ, τ]

see also Dev/CH/Molinaro ('18); Chauhan/Dev ('22)

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

- We take

$\alpha_R \sim 1$

[detail: use additional Z_3 to distinguish e, μ, τ]

$l_{L\alpha} \sim 3 \quad \nu_{Ri} \sim 3'$

irreducible, faithful, complex

Reason:
Fully explore the predictive power of flavour and CP symmetry

CH/Meroni/Molinaro ('14)

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

- We take

$$\alpha_R \sim 1$$

[detail: use additional Z_3 to distinguish e, μ, τ]

$$l_{L\alpha} \sim 3 \,, \nu_{R\,i} \sim 3'$$

irreducible, in general unfaithful, real

Reason: (flavour-universal) mass term for $\nu_{R\,i}$ w/o breaking flavour and CP symmetry

Requires n to be even [important detail: in some cases n not divisible by 4]

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

- We take

$$\alpha_R \sim 1 \qquad l_{L\alpha} \sim 3\,, \nu_{R\,i} \sim 3'$$

[detail: use additional Z_3 to distinguish e, μ, τ]

Charged lepton mass matrix

Generator Q=a

[detail: diagonal subgroup of Z_3 from flavour symmetry and additional Z_3]

$$\begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

[basis choice: Q=a diagonal]

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

- We take

$\alpha_R \sim 1$ $\qquad$ $l_{L\alpha} \sim 3 \,, \nu_{R\,i} \sim 3'$

[detail: use additional Z_3 to distinguish e, μ, τ]

Neutral lepton sector

Generator Z

CP X

$$\mathcal{L} \supset \mathrm{i}\, \overline{\nu_R} \partial\!\!\!/ \nu_R - \frac{1}{2} \overline{\nu_R^c} M_R \nu_R - \overline{l_L} Y_D \, \varepsilon H^* \nu_R + \mathrm{h.c.}$$

No symmetry breaking $\qquad$ **Symmetry breaking**

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

- We take

$$\alpha_R \sim 1 \qquad l_{L\alpha} \sim 3\,, \nu_{R\,i} \sim 3'$$

[detail: use additional Z_3 to distinguish e, μ, τ]

Neutral lepton sector

Generator Z

CP X

No symmetry breaking

$$M_R = M_R^0 = M \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Symmetry breaking

$$Z(\mathbf{3})^\dagger \, Y_D \, Z(\mathbf{3'}) = Y_D$$

$$X(\mathbf{3})^* \, Y_D \, X(\mathbf{3'}) = Y_D^*$$

$$Y_D = \Omega(\mathbf{3}) \, R_{ij}(\theta_L) \, \mathrm{diag}(y_1, y_2, y_3) \, P_{kl}^{ij} \, R_{kl}(-\theta_R) \, \Omega(\mathbf{3'})^\dagger$$

CH/Molinaro ('16)

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Neutral lepton sector

$$Y_D = \Omega(\mathbf{3})\, R_{ij}(\theta_L)\, \mathrm{diag}(y_1, y_2, y_3)\, P^{ij}_{kl}\, R_{kl}(-\theta_R)\, \Omega(\mathbf{3}')^\dagger$$

$$X(\mathbf{3}) = \Omega(\mathbf{3})\, \Omega(\mathbf{3})^T$$

$$X(\mathbf{3}') = \Omega(\mathbf{3}')\, \Omega(\mathbf{3}')^T$$

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Neutral lepton sector

$$Y_D = \Omega(\mathbf{3}) R_{ij}(\theta_L) \,\mathrm{diag}(y_1, y_2, y_3)\, P^{ij}_{kl} R_{kl}(-\theta_R) \Omega(\mathbf{3}')^\dagger$$

$$X(\mathbf{3}) = \Omega(\mathbf{3})\,\Omega(\mathbf{3})^T \qquad X(\mathbf{3}') = \Omega(\mathbf{3}')\,\Omega(\mathbf{3}')^T$$

rotations in the (ij)- and (kl)-plane, $i, j, k, l = 1, 2, 3$ with $i < j$ and $k < l$,

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Neutral lepton sector

$$Y_D = \Omega(\mathbf{3})\, R_{ij}(\theta_L)\, \mathrm{diag}(y_1, y_2, y_3)\, P_{kl}^{ij}\, R_{kl}(-\theta_R)\, \Omega(\mathbf{3}')^\dagger$$

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rotations in the (ij)- and (kl)-plane, $i, j, k, l = 1, 2, 3$ with $i < j$ and $k < l$,

three real parameters, namely the couplings y_f, $f = 1, 2, 3$.

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Neutral lepton sector

$$Y_D = \Omega(\mathbf{3})\, R_{ij}(\theta_L)\, \mathrm{diag}(y_1, y_2, y_3)\, P_{kl}^{ij}\, R_{kl}(-\theta_R)\, \Omega(\mathbf{3}')^\dagger$$

$$X(\mathbf{3}) = \Omega(\mathbf{3})\, \Omega(\mathbf{3})^T$$

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rotations in the (ij)- and (kl)-plane, $i, j, k, l = 1, 2, 3$ with $i < j$ and $k < l$,

three real parameters, namely the couplings y_f, $f = 1, 2, 3$.

permutation matrix P_{kl}^{ij}

Only needed in certain instances

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Neutral lepton sector

$$Y_D = \Omega(\mathbf{3})\, R_{ij}(\theta_L)\, \mathrm{diag}(y_1, y_2, y_3)\, P_{kl}^{ij}\, R_{kl}(-\theta_R)\, \Omega(\mathbf{3}')^\dagger$$

$$X(\mathbf{3}) = \Omega(\mathbf{3})\, \Omega(\mathbf{3})^T$$

$$X(\mathbf{3}') = \Omega(\mathbf{3}')\, \Omega(\mathbf{3}')^T$$

rotations in the (ij)- and (kl)-plane, $i, j, k, l = 1, 2, 3$ with $i < j$ and $k < l$,

three real parameters, namely the couplings y_f, $f = 1, 2, 3$.

permutation matrix P_{kl}^{ij}

In total five free real parameters **corresponding to three light neutrino masses, one free parameter for lepton mixing and one free parameter related to RH neutrinos**

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Briefly on lepton mixing

Charged leptons do not contribute to lepton mixing.

If $\text{diag}\,(y_1, y_2, y_3)\, P_{kl}^{ij}\, R_{kl}(-\theta_R)\, \Omega(\mathbf{3}')^\dagger\, M_R^{-1}\, \Omega(\mathbf{3}')^*\, R_{kl}(\theta_R)\, \left(P_{kl}^{ij}\right)^T \text{diag}\,(y_1, y_2, y_3)$ is diagonal,

then

$$U_{\text{PMNS}} = \Omega(\mathbf{3})\, R_{ij}(\theta_L)\, K_\nu$$

If not, replace θ_L by $\widetilde{\theta}_L$.

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Neutral lepton sector

$$M_R = M_R^0 = M \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Possible small symmetry breaking **in order to generate small mass splitting between RH neutrinos at Lagrangian level**

$$\delta M_R = \kappa\, M \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

due to residual symmetry among charged leptons

$$a(\mathbf{3}')^T\, \delta M_R\, a(\mathbf{3}') = \delta M_R$$

Then

$$M_1 = M\,(1 + 2\,\kappa) \;\; \text{and} \;\; M_2 = M_3 = M\,(1 - \kappa)$$

Note: in most of the analysis we use this mass spectrum of RH neutrinos, if splittings are induced at all.

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Neutral lepton sector

$$M_R = M_R^0 = M \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Possible small symmetry breaking **in order to generate small mass splitting between RH neutrinos at Lagrangian level**

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due to residual symmetry among charged leptons

$$a(\mathbf{3}')^T\, \delta M_R\, a(\mathbf{3}') = \delta M_R$$

Further splitting (example)

$$\Delta M_R = \lambda\, M \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

splits all RH neutrino masses

$$M_1 = M\,(1 + 2\,\kappa)\;,\;\; M_2 = M\,(1 - \kappa + \lambda)\;\text{ and }\; M_3 = M\,(1 - \kappa - \lambda)$$

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Solve quantum kinetic equations numerically.

$$i\frac{dn_{\Delta_\alpha}}{dt} = -2i\frac{\mu_\alpha}{T}\int\frac{d^3k}{(2\pi)^3}\,\mathrm{Tr}\left[\Gamma_\alpha\right] f_N\left(1-f_N\right) \quad + i\int\frac{d^3k}{(2\pi)^3}\,\mathrm{Tr}\left[\tilde{\Gamma}_\alpha\left(\bar{\rho}_N-\rho_N\right)\right],$$

$$i\,\frac{d\rho_N}{dt} = \left[H_N,\rho_N\right] - \frac{i}{2}\left\{\Gamma,\rho_N-\rho_N^{eq}\right\} \quad - \frac{i}{2}\sum_\alpha \tilde{\Gamma}_\alpha\left[2\frac{\mu_\alpha}{T} f_N\left(1-f_N\right)\right],$$

$$i\,\frac{d\bar{\rho}_N}{dt} = -\left[H_N,\bar{\rho}_N\right] - \frac{i}{2}\left\{\Gamma,\bar{\rho}_N-\rho_N^{eq}\right\} \quad + \frac{i}{2}\sum_\alpha \tilde{\Gamma}_\alpha\left[2\frac{\mu_\alpha}{T} f_N\left(1-f_N\right)\right].$$

- $\rho_N, \bar{\rho}_N$ — momentum averaged density matrices for two helicities of N_i
- H_N — effective Hamiltonian
- f_N — Fermi-Dirac distribution for N_i
- μ_α — flavoured lepton chemical potentials
- $\Gamma, \Gamma_\alpha, \tilde{\Gamma}_\alpha$ — different thermal interaction rates
- n_{Δ_α} — comoving lepton number densities

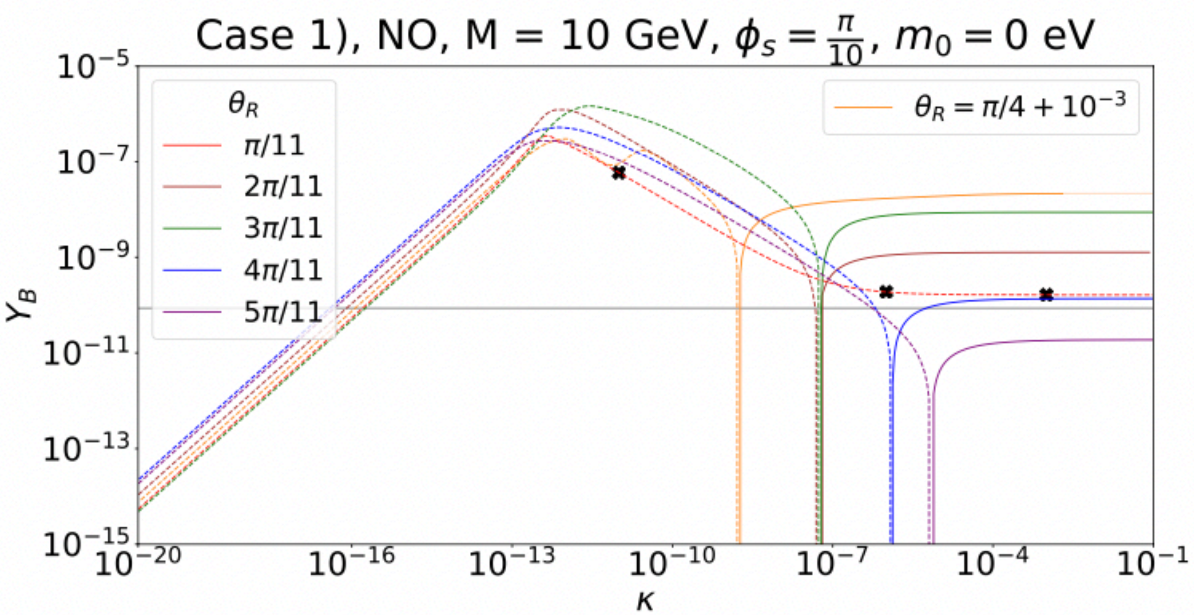
for review see Garbrecht ('18)

see also Ghiglieri/Laine ('17), Klaric/Shaposhnikov/Timiryasov ('21)

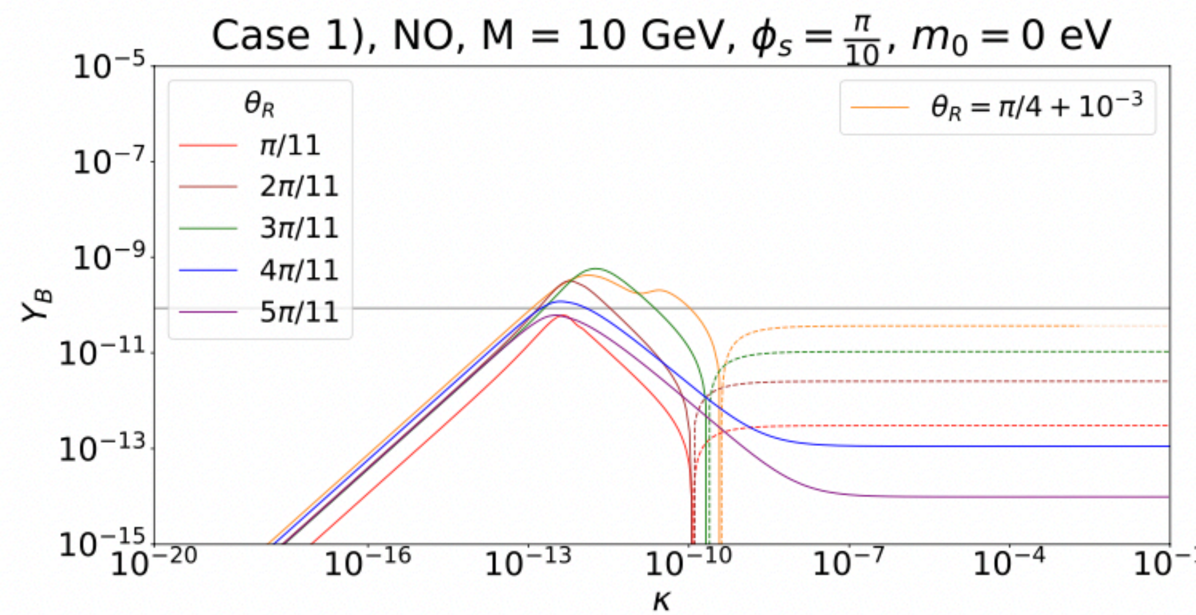
Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1)



(c) Vanishing initial conditions.

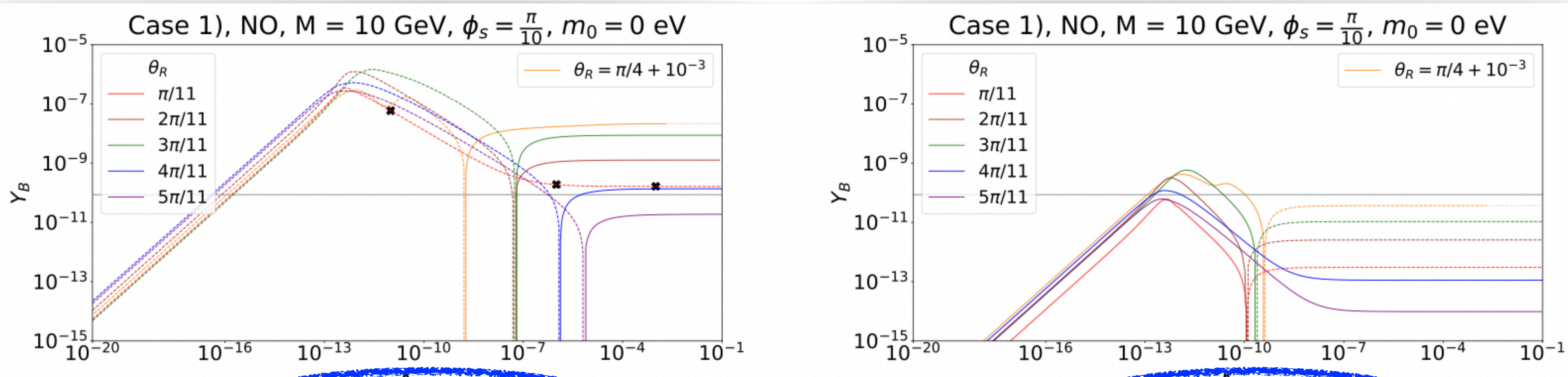


(d) Thermal initial conditions.

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1)



(c) Vanishing initial conditions. (d) Thermal initial conditions.

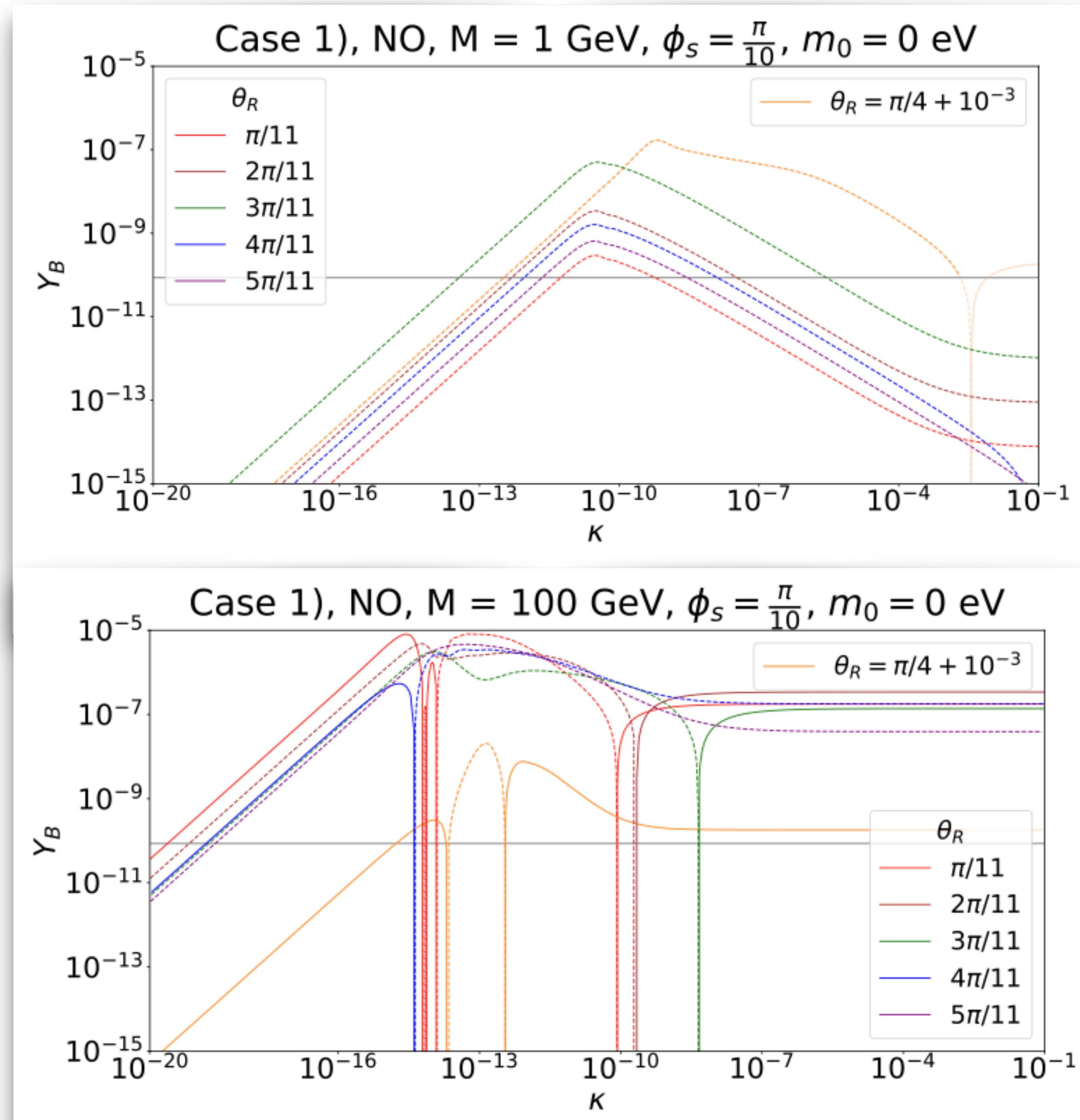
We consider two types of initial abundances of N_i

- Vanishing initial conditions are realised in e.g. νMSM
- Thermal initial conditions are realised in framework with additional interactions of N_i below reheating temperature

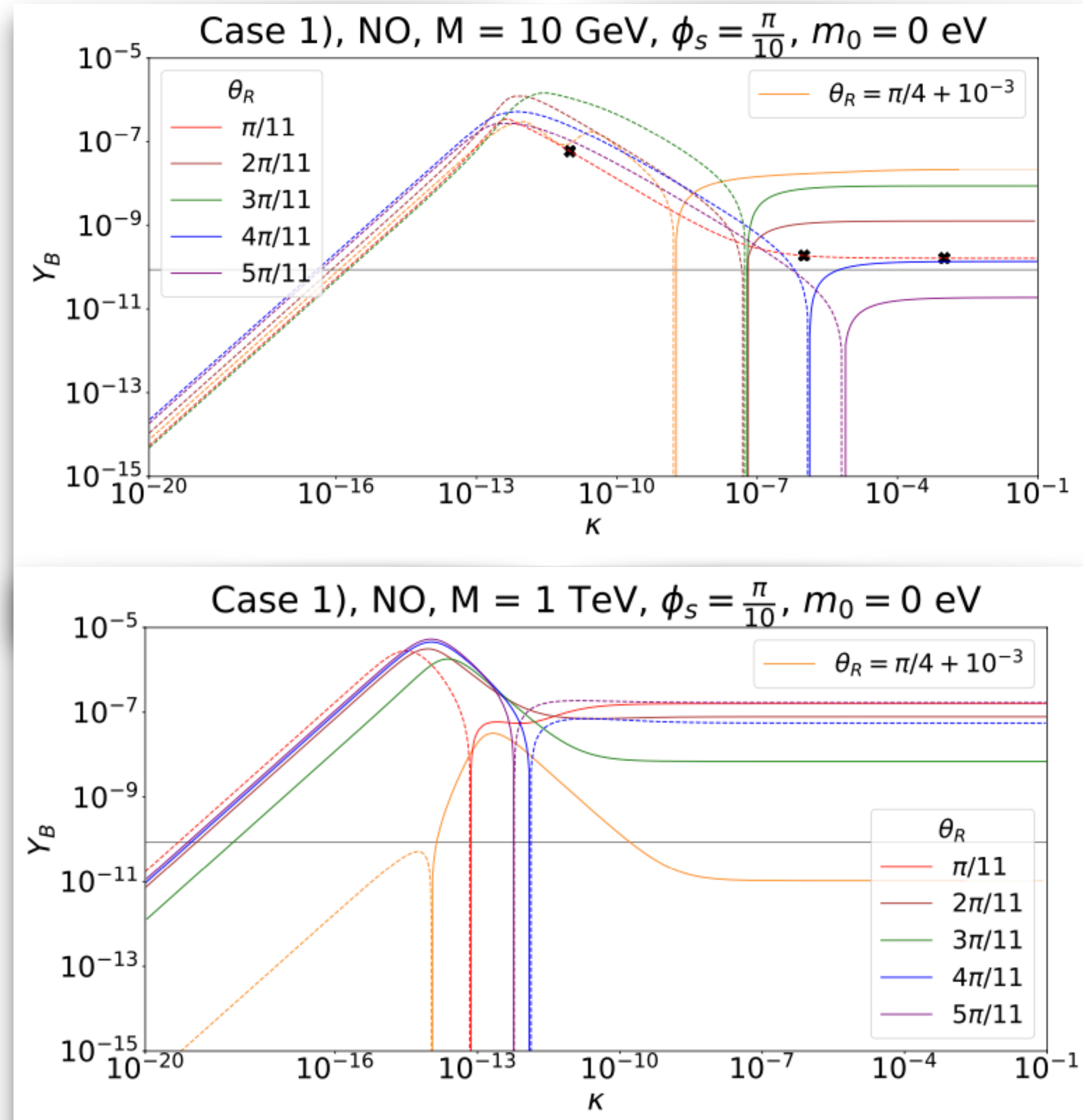
Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1)



(e) Vanishing initial conditions.

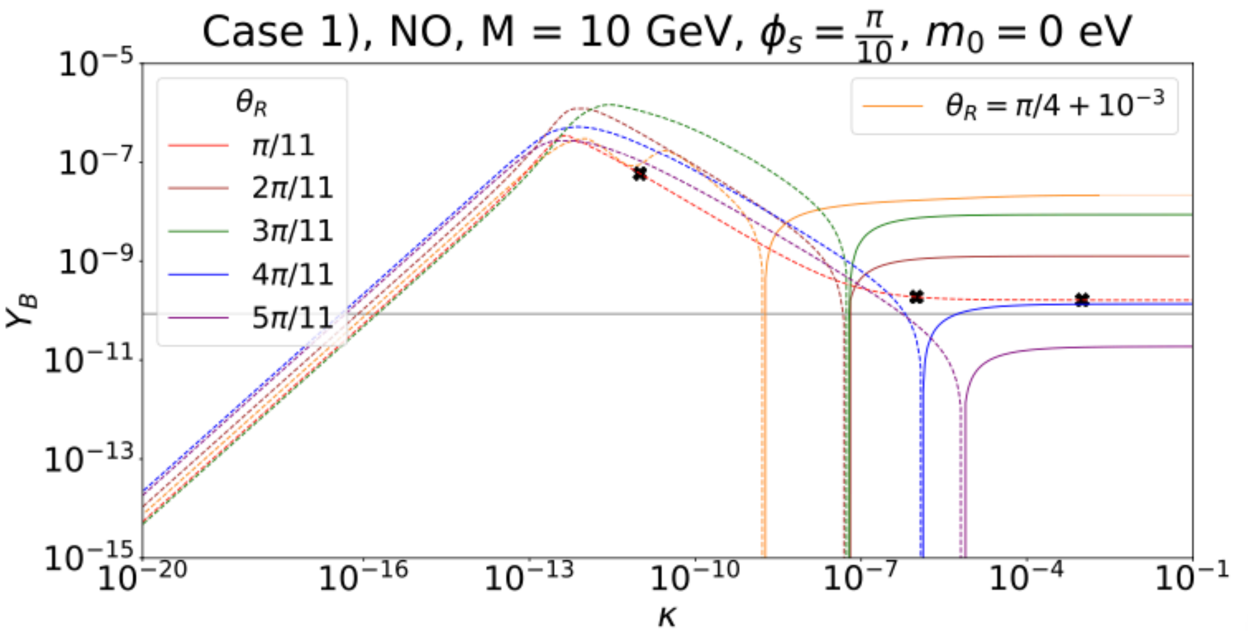


(g) Vanishing initial conditions.

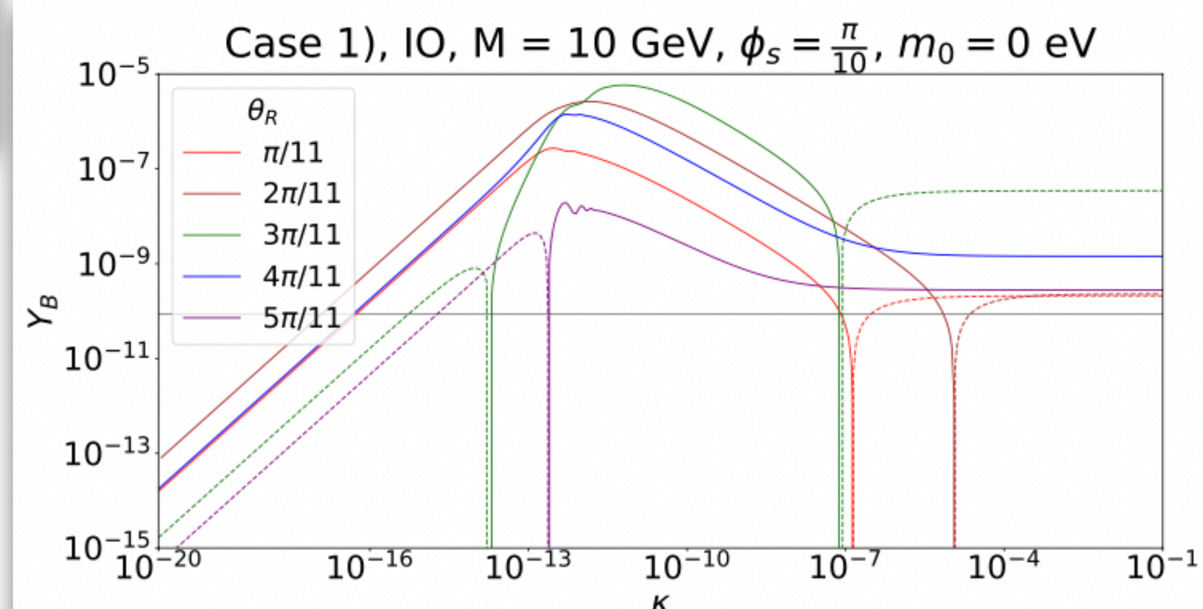
Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1)



(c) Vanishing initial conditions.

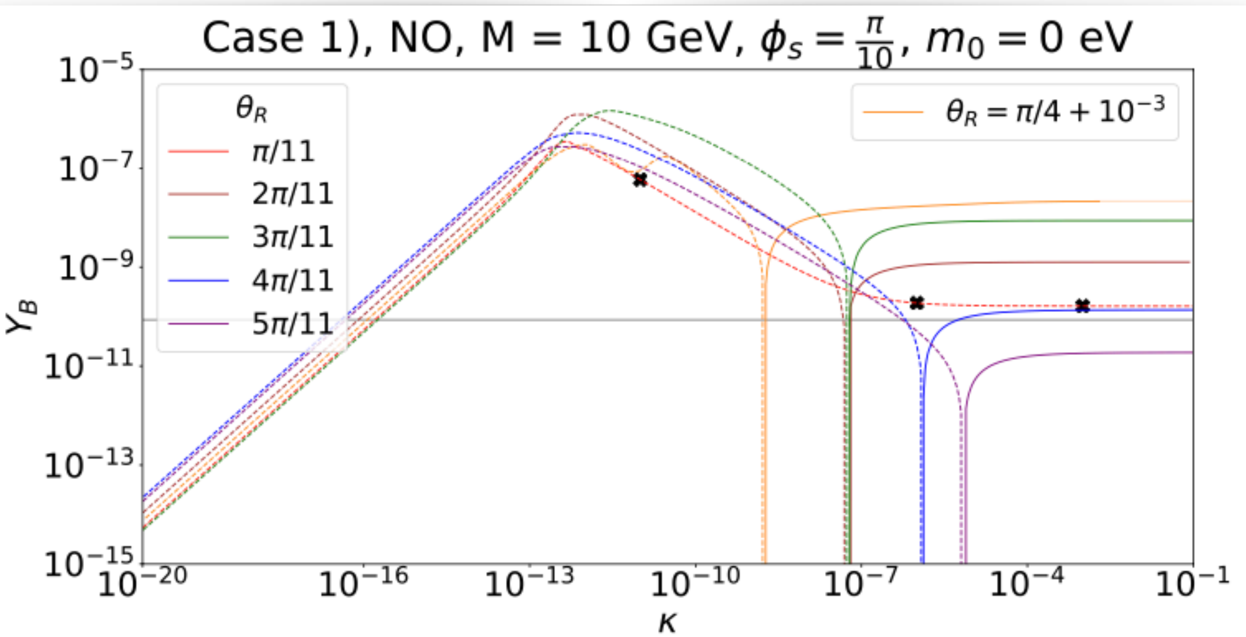


(a) Vanishing initial conditions.

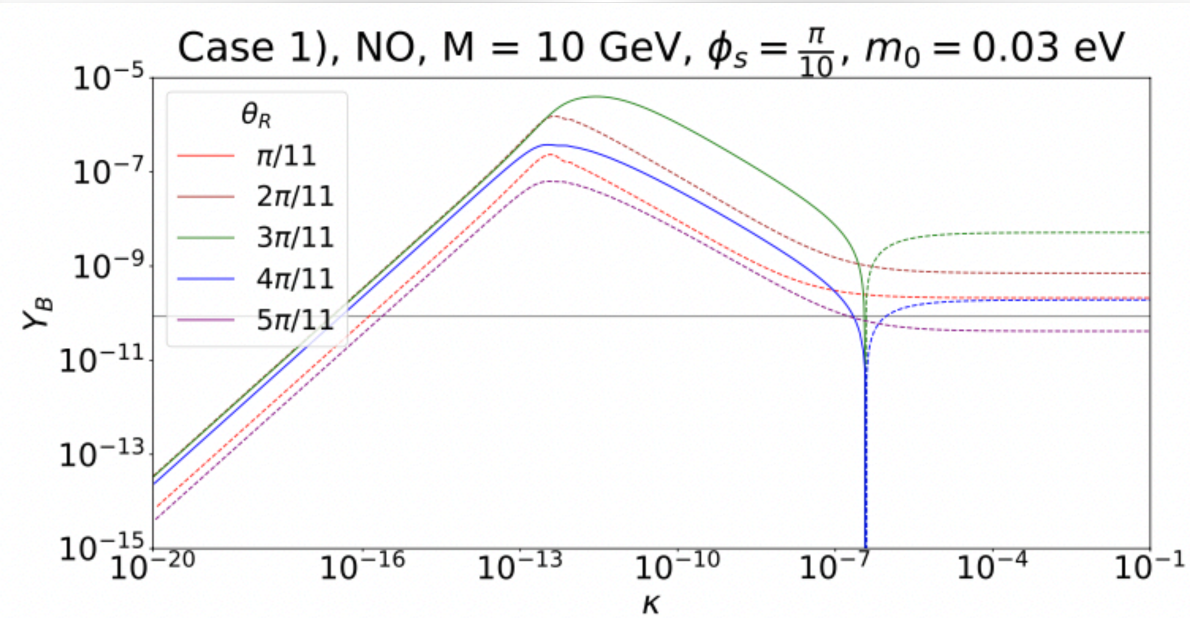
Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1)



(c) Vanishing initial conditions.



(b) Vanishing initial conditions.

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1)

Impact of free angle θ_R
For illustration consider light neutrino masses with strong NO, i.e. $m_0 = 0$ and normally ordered light neutrino masses.
This means

$$\mathrm{diag}\,(y_1, y_2, y_3)\, P_{kl}^{ij}\, R_{kl}(-\theta_R)\, \Omega(\mathbf{3}')^\dagger\, M_R^{-1}\, \Omega(\mathbf{3}')^*\, R_{kl}(\theta_R)\, \left(P_{kl}^{ij}\right)^T \mathrm{diag}\,(y_1, y_2, y_3)$$

becomes diagonal
and

$$m_2 = \frac{y_2^2\, \langle H \rangle^2}{M}$$

$$m_1 = 0 \;\; \text{and} \;\; m_3 = \frac{y_3^2\, \langle H \rangle^2}{M}\, |\cos 2\,\theta_R|$$

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1)

Impact of free angle θ_R

For illustration consider light neutrino masses with strong NO, i.e. $m_0 = 0$ and normally ordered light neutrino masses.

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becomes diagonal

and

$$m_2 = \frac{y_2^2\, \langle H \rangle^2}{M}$$

$$m_1 = 0 \;\; \text{and} \;\; m_3 = \frac{y_3^2\, \langle H \rangle^2}{M} |\cos 2\,\theta_R|$$

For $\theta_R \approx \frac{\pi}{4}$ or odd multiples of it, $\cos 2\,\theta_R \approx 0$ and y_3 large for m_3 fixed.

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1)

Impact of free angle θ_R

For illustration consider light neutrino masses with strong NO, i.e. $m_0 = 0$ and normally ordered light neutrino masses.

This means

$$\mathrm{diag}\,(y_1, y_2, y_3)\, P^{ij}_{kl}\, R_{kl}(-\theta_R)\, \Omega(\mathbf{3}')^\dagger\, M_R^{-1}\, \Omega(\mathbf{3}')^*\, R_{kl}(\theta_R) \left(P^{ij}_{kl}\right)^T \mathrm{diag}\,(y_1, y_2, y_3)$$

becomes diagonal

and

$$m_2 = \frac{y_2^2\,\langle H \rangle^2}{M}$$

$$m_1 = 0 \;\; \text{and} \;\; m_3 = \frac{y_3^2\,\langle H \rangle^2}{M}\,|\cos 2\,\theta_R|$$

but then also check

$$m_3 \approx \frac{y_3^2\,\langle H \rangle^2}{M} |\kappa + \cos 2\theta_R|$$

i.e. assume

$$\kappa \ll |\cos 2\theta_R|$$

Works also for some combinations in

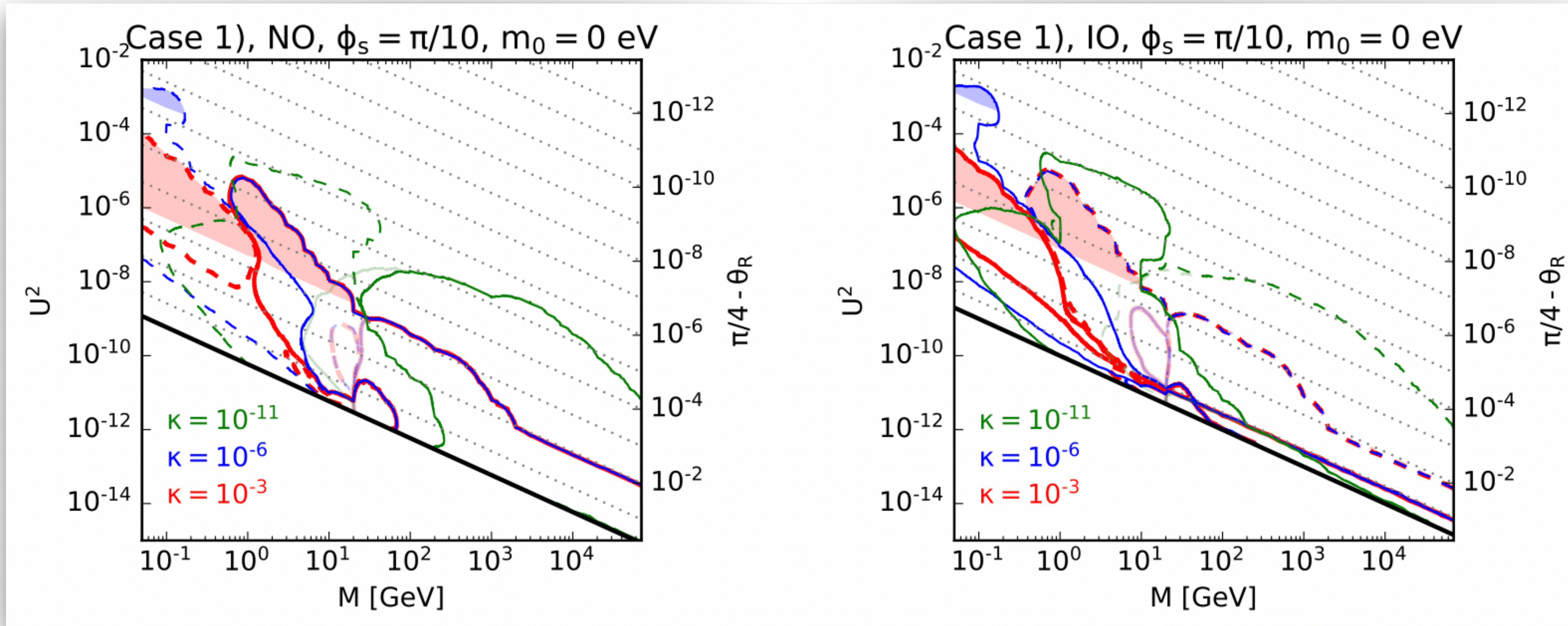
Case 2) Case 3 a)

Case 3 b.1)

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

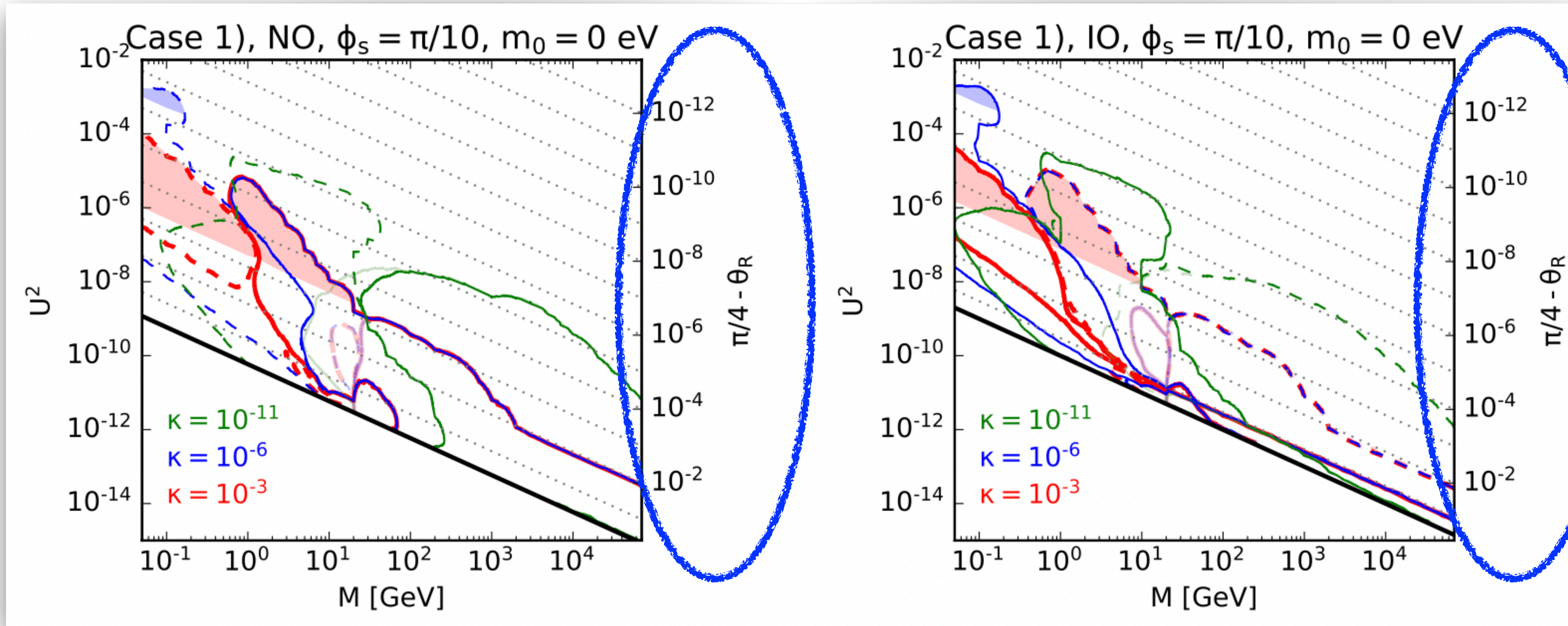
Case 1)



Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1)

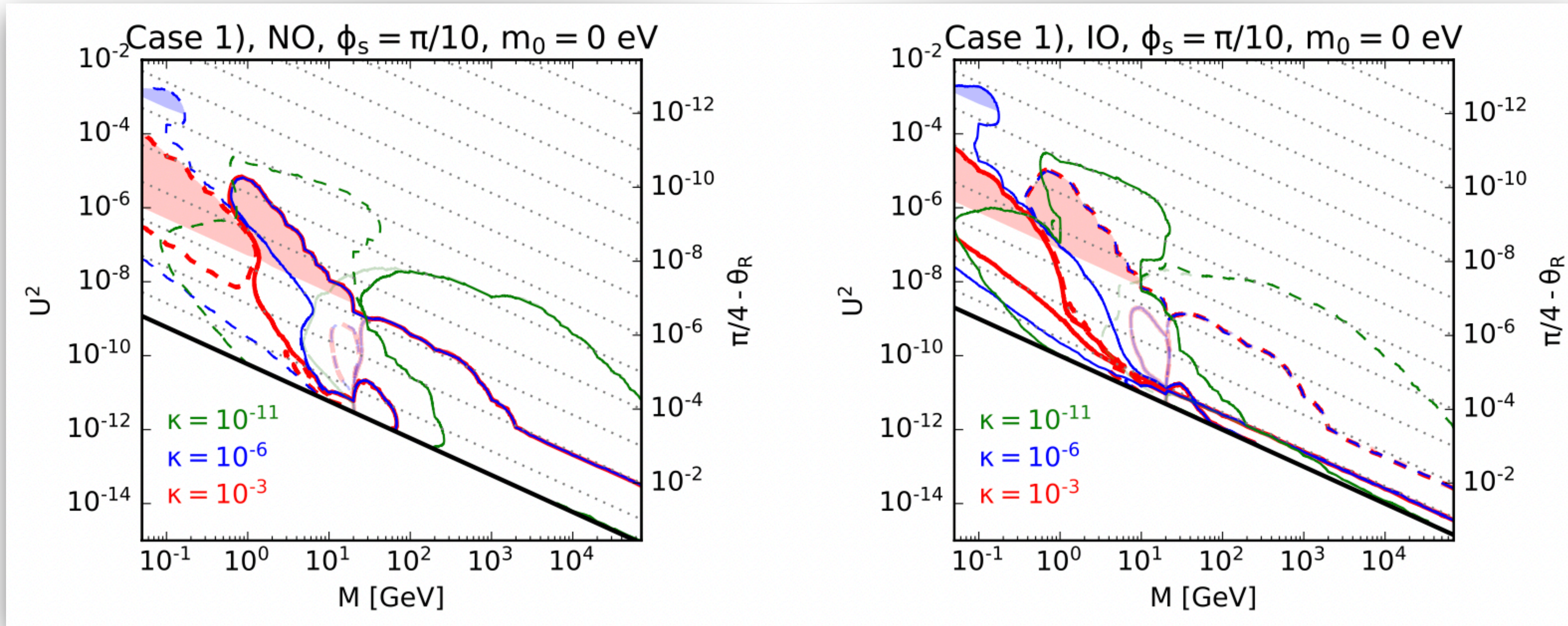


Values of θ_R so close to $\frac{\pi}{4}$ are not (always) a tuning, but related to enhanced residual symmetry, i.e. check $Y_D^\dagger Y_D$

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1)

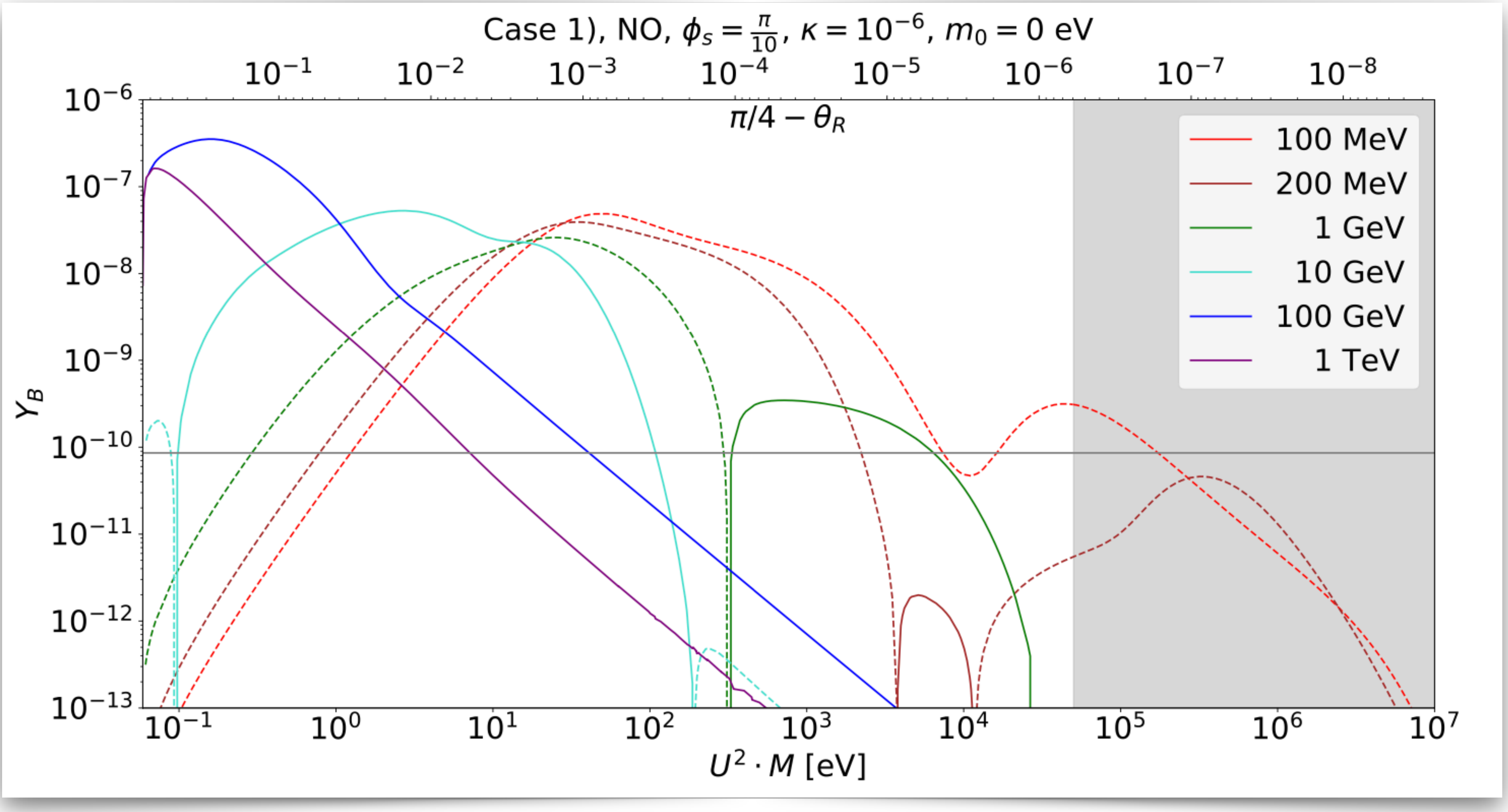


Shaded areas are due to condition for κ and θ_R.
Vanishing and thermal initial conditions are displayed.
Line shapes reflect sign of BAU.

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1)



Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Way towards capturing main dependencies analytically

- CP-violating combinations
- washout parameter

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Way towards capturing main dependencies analytically
- CP-violating combinations
- washout parameter

CP-violating combinations: see for related work Hernandez et al. ('15)

Perturbatively solve quantum kinetic equations in H_N and Γ

Leading term for lepton asymmetries

$$\mathrm{Tr}\left[\tilde{\Gamma}_\alpha(\bar{\rho}_N - \rho_N)\right] \propto \mathrm{Tr}\left(\tilde{\Gamma}_\alpha\, [H_N, \Gamma]\right) \quad \text{with} \quad \alpha = e, \mu, \tau.$$

Three types of CP-violating combinations are found

$$\begin{aligned}
C_{\mathrm{LFV},\alpha} &= i\,\mathrm{Tr}\Big(\left[\hat{M}_R^2, \hat{Y}_D^\dagger \hat{Y}_D\right] \hat{Y}_D^\dagger P_\alpha \hat{Y}_D\Big)\,, \\
C_{\mathrm{LNV},\alpha} &= i\,\mathrm{Tr}\Big(\left[\hat{M}_R^2, \hat{Y}_D^\dagger \hat{Y}_D\right] \hat{Y}_D^T P_\alpha \hat{Y}_D^*\Big)\,, \\
C_{\mathrm{DEG},\alpha} &= i\,\mathrm{Tr}\Big(\left[\hat{Y}_D^T \hat{Y}_D^*, \hat{Y}_D^\dagger \hat{Y}_D\right] \hat{Y}_D^T P_\alpha \hat{Y}_D^*\Big)
\end{aligned}$$

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

$$\begin{aligned} C_{\mathrm{LFV},\alpha} &= i\,\mathrm{Tr}\Big(\Big[\hat{M}_R^2, \hat{Y}_D^\dagger \hat{Y}_D\Big]\, \hat{Y}_D^\dagger P_\alpha \hat{Y}_D\Big)\,, \\ C_{\mathrm{LNV},\alpha} &= i\,\mathrm{Tr}\Big(\Big[\hat{M}_R^2, \hat{Y}_D^\dagger \hat{Y}_D\Big]\, \hat{Y}_D^T P_\alpha \hat{Y}_D^*\Big)\,, \\ C_{\mathrm{DEG},\alpha} &= i\,\mathrm{Tr}\Big(\Big[\hat{Y}_D^T \hat{Y}_D^*, \hat{Y}_D^\dagger \hat{Y}_D\Big]\, \hat{Y}_D^T P_\alpha \hat{Y}_D^*\Big) \end{aligned}$$

with

$$P_e = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\,,\quad P_\mu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\,,\quad P_\tau = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and in mass basis of heavy states, i.e.

$$\hat{Y}_D = Y_D\, U_R$$

$$U_R = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & i \\ 0 & 1 & -i \end{pmatrix}$$

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

$$C_{\mathrm{LFV},\alpha} = i\,\mathrm{Tr}\Big(\big[\hat{M}_R^2, \hat{Y}_D^\dagger \hat{Y}_D\big] \hat{Y}_D^\dagger P_\alpha \hat{Y}_D \Big)$$

Note the following

- Dominant combination when N_i are in relativistic regime
- Only leads to lepton flavour asymmetry, since

$$\sum_\alpha C_{\mathrm{LFV},\alpha} = 0.$$

- Crucially depends on a flavoured washout

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

$$C_{\mathrm{LFV},\alpha} = i\,\mathrm{Tr}\left(\left[\hat{M}_R^2, \hat{Y}_D^\dagger \hat{Y}_D\right] \hat{Y}_D^\dagger P_\alpha \hat{Y}_D\right)$$

Note the following

- Dominant combination when N_i are in relativistic regime
- Only leads to lepton flavour asymmetry, since

$$\sum_\alpha C_{\mathrm{LFV},\alpha} = 0.$$

- Crucially depends on a flavoured washout

$$C_{\mathrm{LNV},\alpha} = i\,\mathrm{Tr}\left(\left[\hat{M}_R^2, \hat{Y}_D^\dagger \hat{Y}_D\right] \hat{Y}_D^T P_\alpha \hat{Y}_D^*\right)$$

Note the following

- Sizeable for intermediate/larger masses of N_i
- Directly violates lepton number with

$$C_{\mathrm{LNV}} = \sum_\alpha C_{\mathrm{LNV},\alpha} \neq 0$$

compare to flavoured decay asymmetries $\epsilon_{i\alpha}$ see Dev et al. ('17)

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

$$C_{\mathrm{DEG},\alpha} \;=\; i\,\mathrm{Tr}\Big(\Big[\hat{Y}_D^T\,\hat{Y}_D^*,\hat{Y}_D^\dagger\,\hat{Y}_D\Big]\,\hat{Y}_D^T\,P_\alpha\,\hat{Y}_D^*\Big)$$

Note the following

- Only this CP-violating combination could be non-zero for zero κ and λ
- Only possible at intermediate temperatures $M/T \sim 1$
- Only leads to lepton flavour asymmetry, since $\sum_\alpha C_{\mathrm{DEG},\alpha} = 0.$

Furthermore, for the limit $\lambda \ll \kappa \lesssim 1$ consider subset of two mass-degerate states. Define $(\hat{Y}_{(23)})_{\alpha i} = (\hat{Y}_D)_{\alpha i} \quad \text{for} \quad i \in \{2,3\}$

For $\lambda = 0$ we only need

$$C^{(23)}_{\mathrm{DEG},\alpha} = i\,\mathrm{Tr}\Big(\Big[\hat{Y}_{(23)}^T\,\hat{Y}_{(23)}^*,\hat{Y}_{(23)}^\dagger\,\hat{Y}_{(23)}\Big]\,\hat{Y}_{(23)}^T\,P_\alpha\,\hat{Y}_{(23)}^*\Big)$$

Clearly, $\sum_\alpha C^{(23)}_{\mathrm{DEG},\alpha} = 0.$

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Way towards capturing main dependencies analytically

- CP-violating combinations
- washout parameter

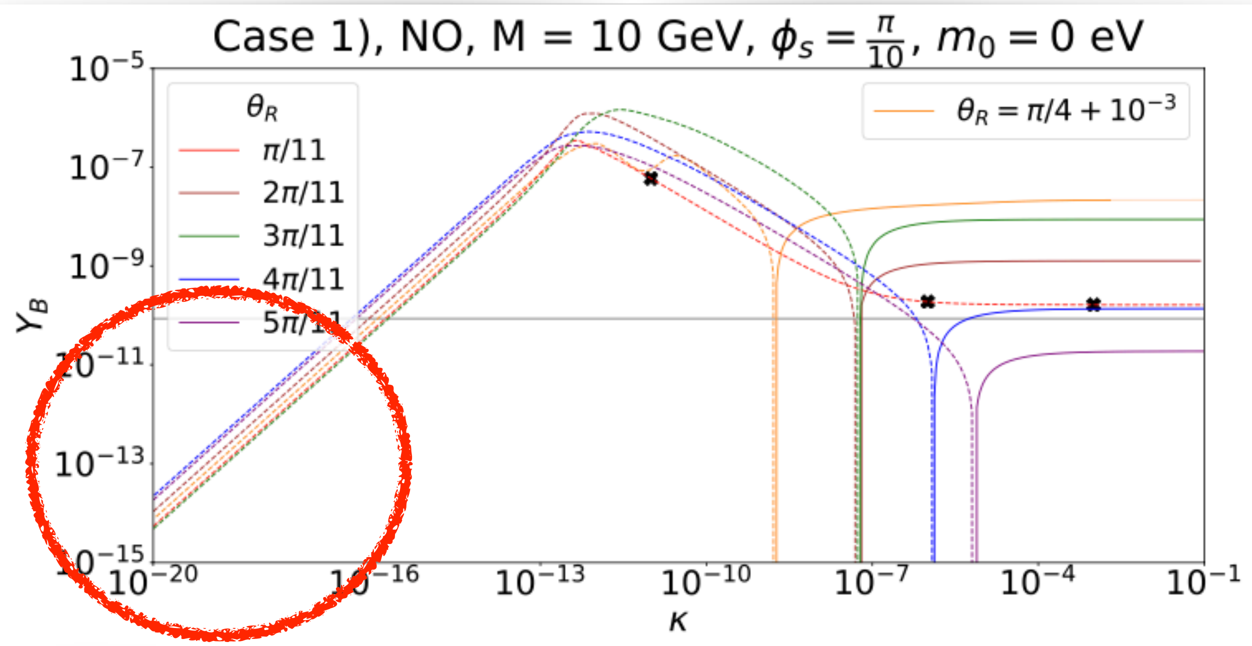
Flavoured washout parameter:

$$f_\alpha = \frac{(\hat{Y}_D \hat{Y}_D^\dagger)_{\alpha\alpha}}{\mathrm{Tr}\left(\hat{Y}_D \hat{Y}_D^\dagger\right)} .$$

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1)



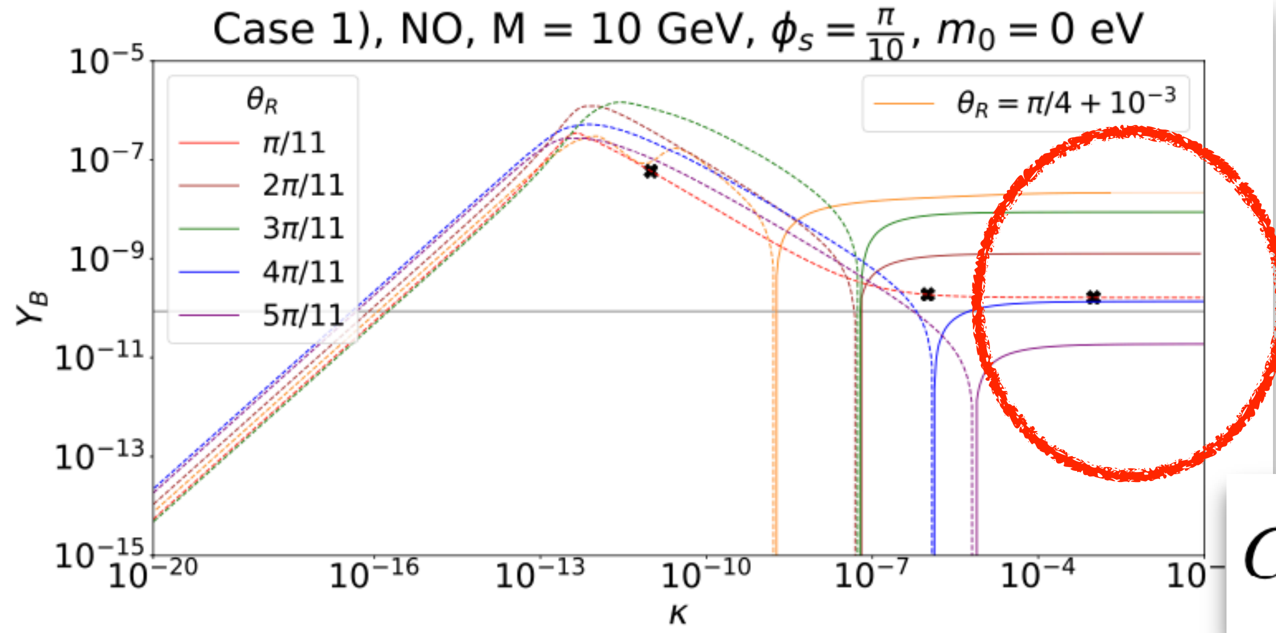
(c) Vanishing initial conditions.

$C_{\mathrm{DEG},\alpha}$ is zero.

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1)



(c) Vanishing initial conditions.

$C^{(23)}_{\mathrm{DEG},\alpha}$ is non-zero

$$C^{(23)}_{\mathrm{DEG},\alpha} = -\frac{4}{27}\, y_2\, \Delta y^2_{13}\, \left(2\,\Delta y^2_{12} - \Delta y^2_{13}\,(1 + 2\cos 2\theta_R)\right)\, (y_1 \cos\theta_{L,\alpha} \sin\theta_R - y_3 \sin\theta_{L,\alpha} \cos\theta_R)\, \sin 2\,\theta_R\, \cos 3\,\phi_s$$

$$\Delta y^2_{ij} = y_i^2 - y_j^2$$

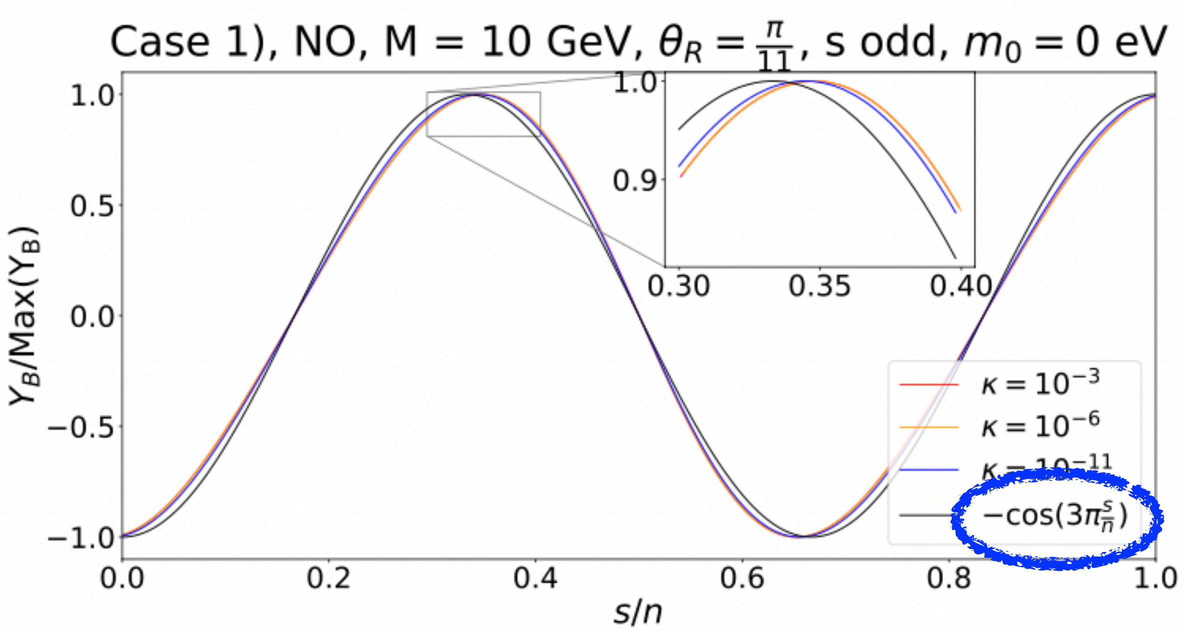
$$\theta_{L,\alpha} = \theta_L + \rho_\alpha \frac{4\pi}{3} \quad \text{with} \quad \rho_e = 0,\ \rho_\mu = +1,\ \rho_\tau = -1$$

$$\phi_s = \frac{\pi\, s}{n}$$

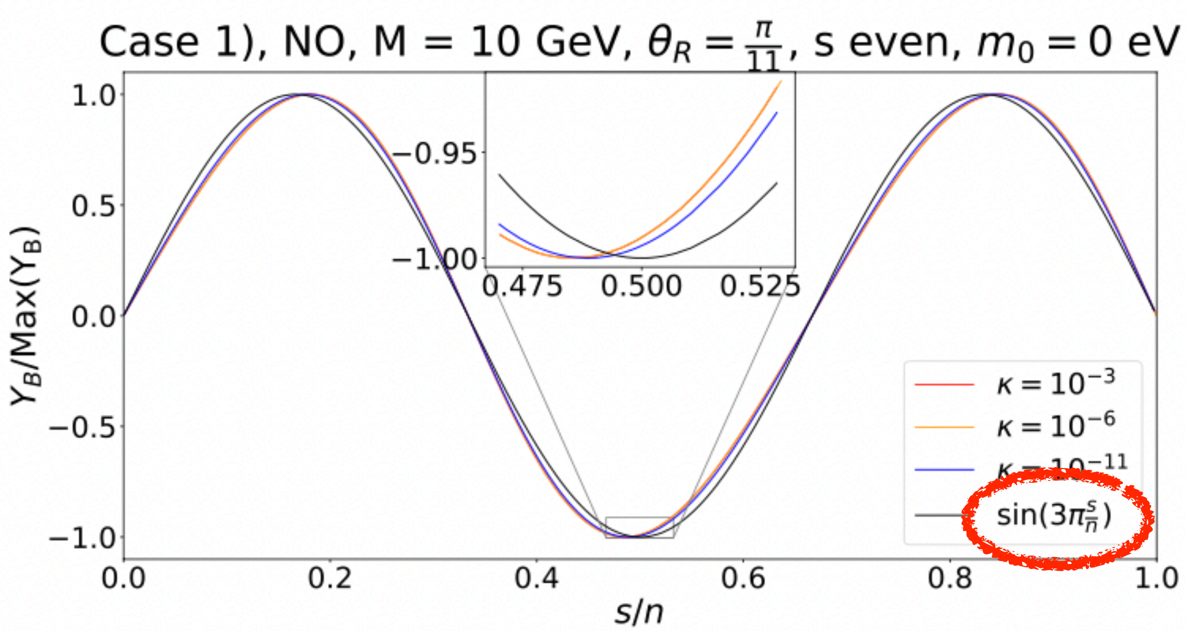
Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1)



(a) Vanishing initial conditions.



(b) Vanishing initial conditions.

$$C_{\mathrm{LFV},\alpha} = \frac{4}{9} M^2 y_2 \Delta\sigma_{12} \left(y_1 \Delta y_{12}^2 \cos\theta_{L,\alpha} \cos\theta_R - y_3 \Delta y_{23}^2 \sin\theta_{L,\alpha} \sin\theta_R \right) \sin 3\phi_s$$

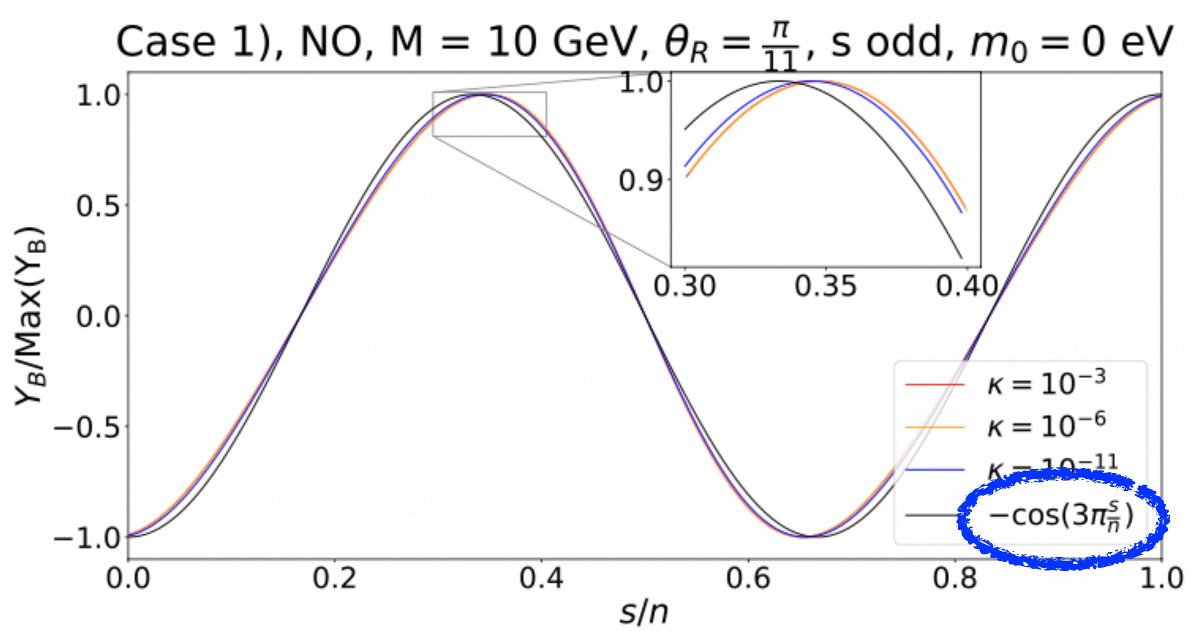
$$C_{\mathrm{LNV},\alpha} = \frac{4}{9} M^2 y_2 \Delta\sigma_{12} \Big(y_1 (\Delta y_{23}^2 - \Delta y_{13}^2 \cos 2\theta_R) \cos\theta_{L,\alpha} \cos\theta_R - y_3 (\Delta y_{12}^2 + \Delta y_{13}^2 \cos 2\theta_R) \sin\theta_{L,\alpha} \sin\theta_R \Big) \sin 3\phi_s$$

$$\Delta\sigma_{12} = (3\kappa - \lambda)(2 + \kappa + \lambda)$$

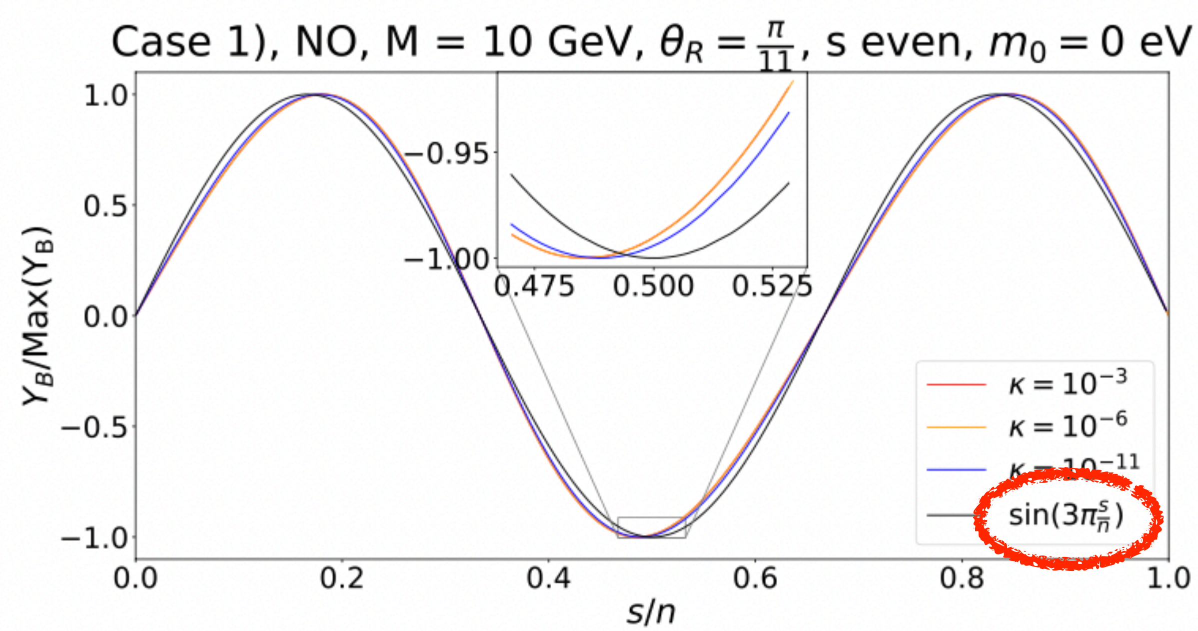
Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1)



(a) Vanishing initial conditions.



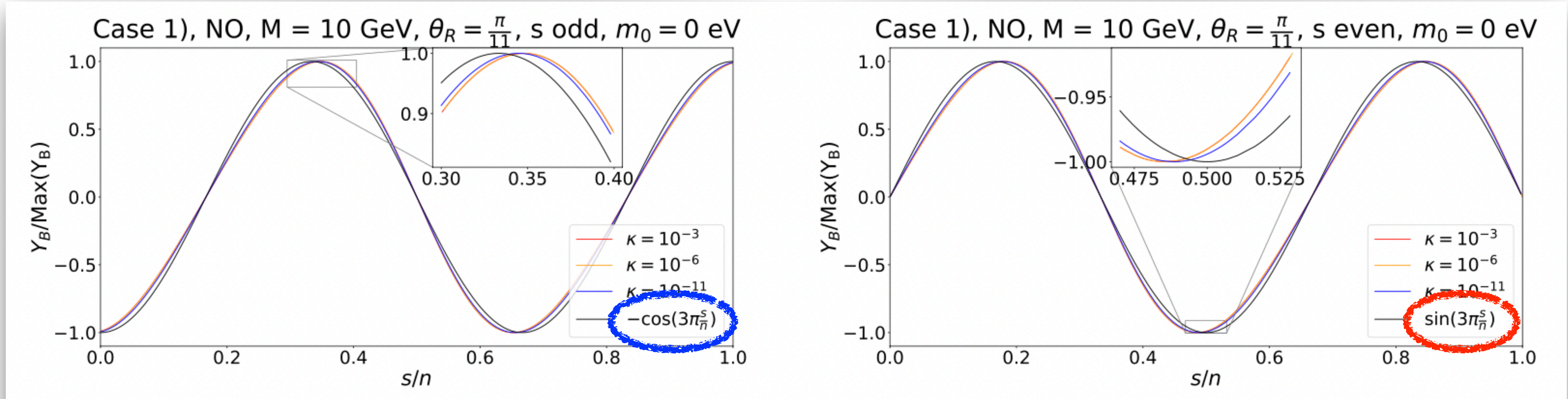
(b) Vanishing initial conditions.

$$f_\alpha = \frac{1}{3}\left(1 + \frac{\Delta y_{13}^2}{\Sigma y^2}\cos 2\,\theta_{L,\alpha}\right)$$ with $$\Sigma y^2 = y_1^2 + y_2^2 + y_3^2 \,.$$

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1)



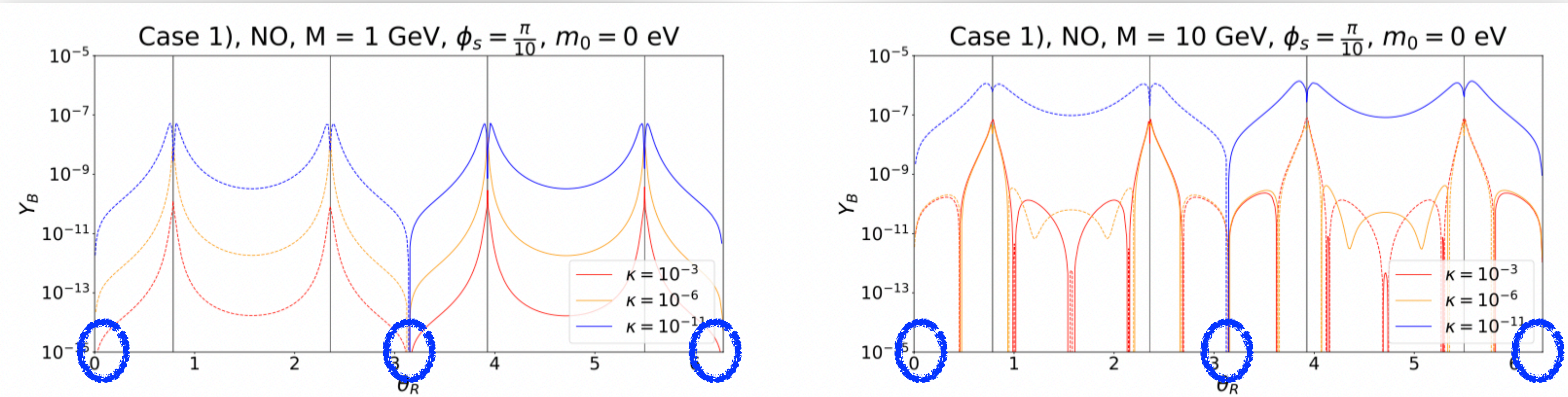
(a) Vanishing initial conditions. (b) Vanishing initial conditions.

Majorana phase α fulfils $|\sin\alpha| = |\sin\left(\frac{6\pi s}{n}\right)|$

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1)



(a) Vanishing initial conditions. (b) Vanishing initial conditions.

$$C_{\mathrm{LFV},\alpha} = \frac{4}{9} M^2 y_2 \Delta\sigma_{12} \left(y_1 \Delta y_{12}^2 \cos\theta_{L,\alpha} \cos\theta_R - y_3 \Delta y_{23}^2 \sin\theta_{L,\alpha} \sin\theta_R \right) \sin 3\phi_s$$

$$C_{\mathrm{LNV},\alpha} = \frac{4}{9} M^2 y_2 \Delta\sigma_{12} \left(y_1 (\Delta y_{23}^2 - \Delta y_{13}^2 \cos 2\theta_R) \cos\theta_{L,\alpha} \cos\theta_R - y_3 (\Delta y_{12}^2 + \Delta y_{13}^2 \cos 2\theta_R) \sin\theta_{L,\alpha} \sin\theta_R \right) \sin 3\phi_s$$

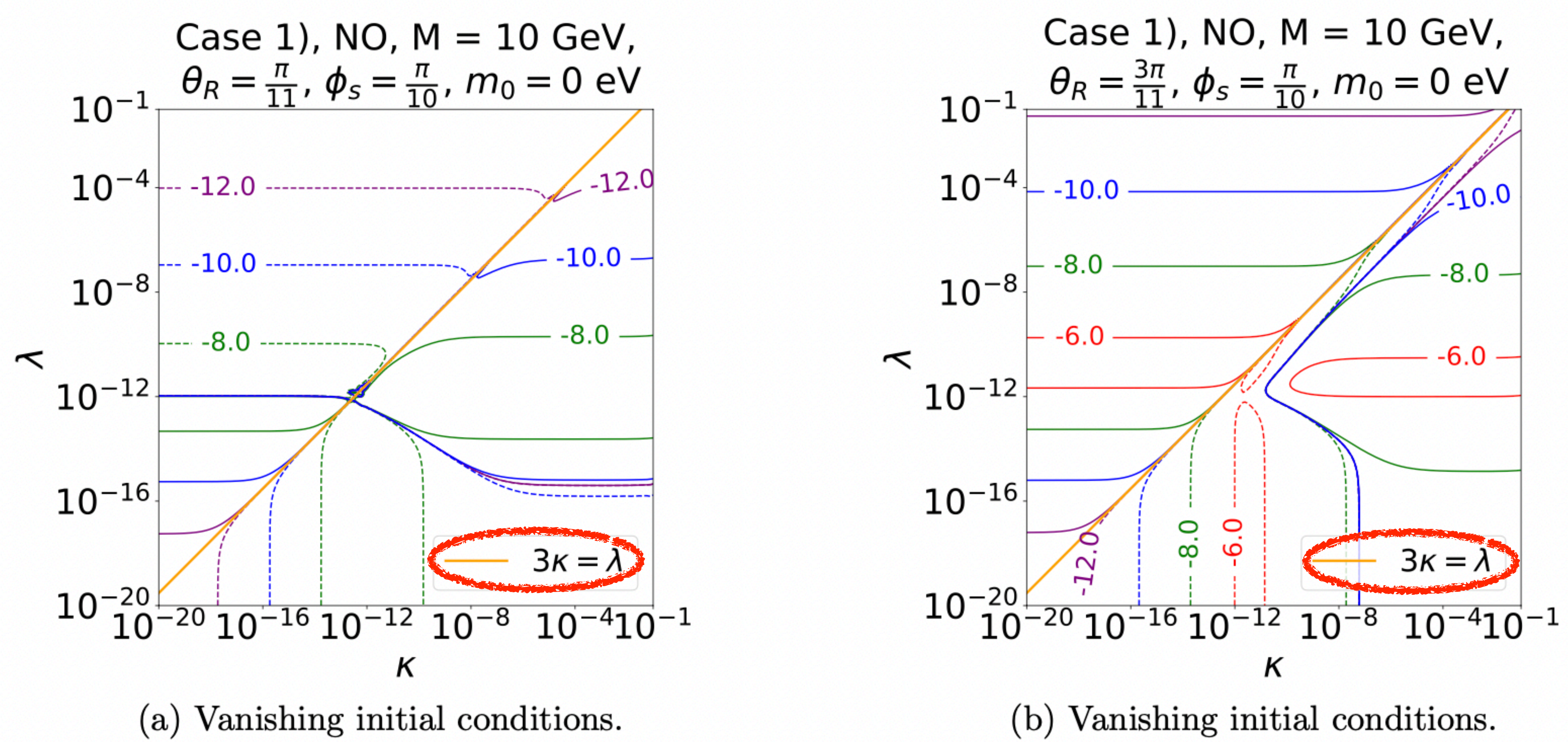
$$\Delta\sigma_{12} = (3\kappa - \lambda)(2 + \kappa + \lambda)$$

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1)

Effect of switching on λ



(a) Vanishing initial conditions. (b) Vanishing initial conditions.

$$\Delta\sigma_{12} = (3\,\kappa - \lambda)(2 + \kappa + \lambda)$$

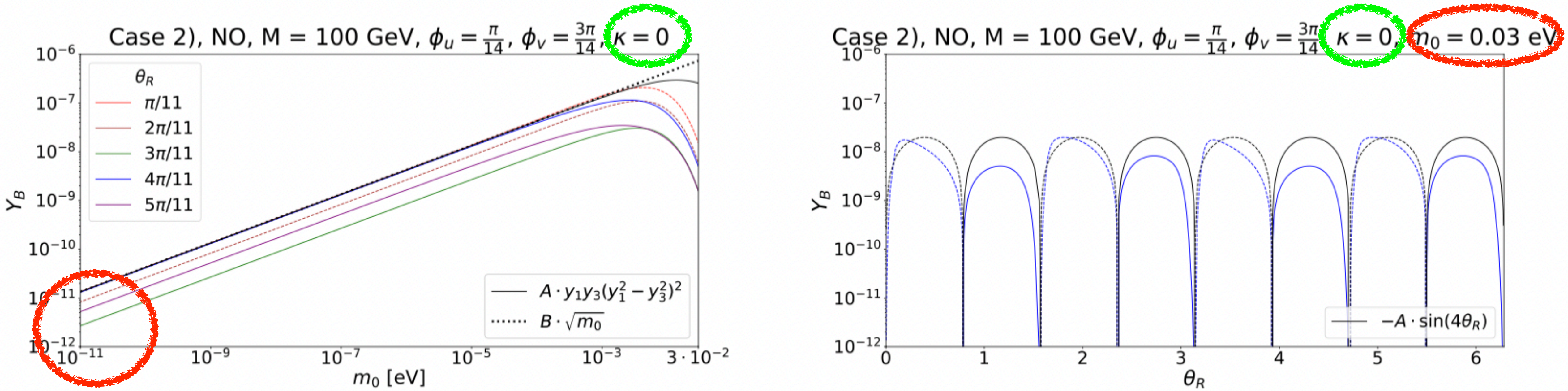
Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 2), t odd

Focus on vanishing κ and λ

$$u = 2\,s - t \qquad v = 3\,t$$



(a) Vanishing initial conditions.

(b) Vanishing initial conditions.

$$C_{\mathrm{DEG},\alpha} = \frac{1}{3}\, y_1\, y_3\, (\Delta y_{13}^2)^2\, \sin 4\,\theta_R\, \sin \phi_{u,\alpha}\,.$$

$$\phi_{u,\alpha} = \frac{\pi\, u}{n} + \rho_\alpha \frac{4\,\pi}{3} = \phi_u + \rho_\alpha \frac{4\,\pi}{3} \quad \text{with} \quad \rho_e = 0\,,\ \rho_\mu = -1\,,\ \rho_\tau = +1\,.$$

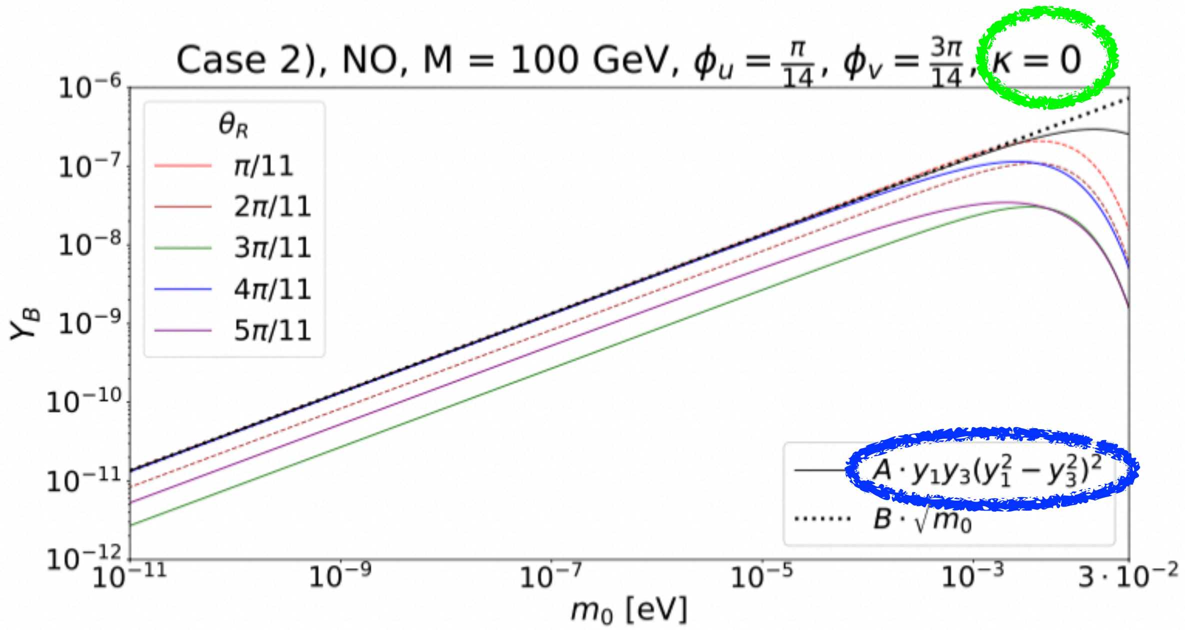
Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

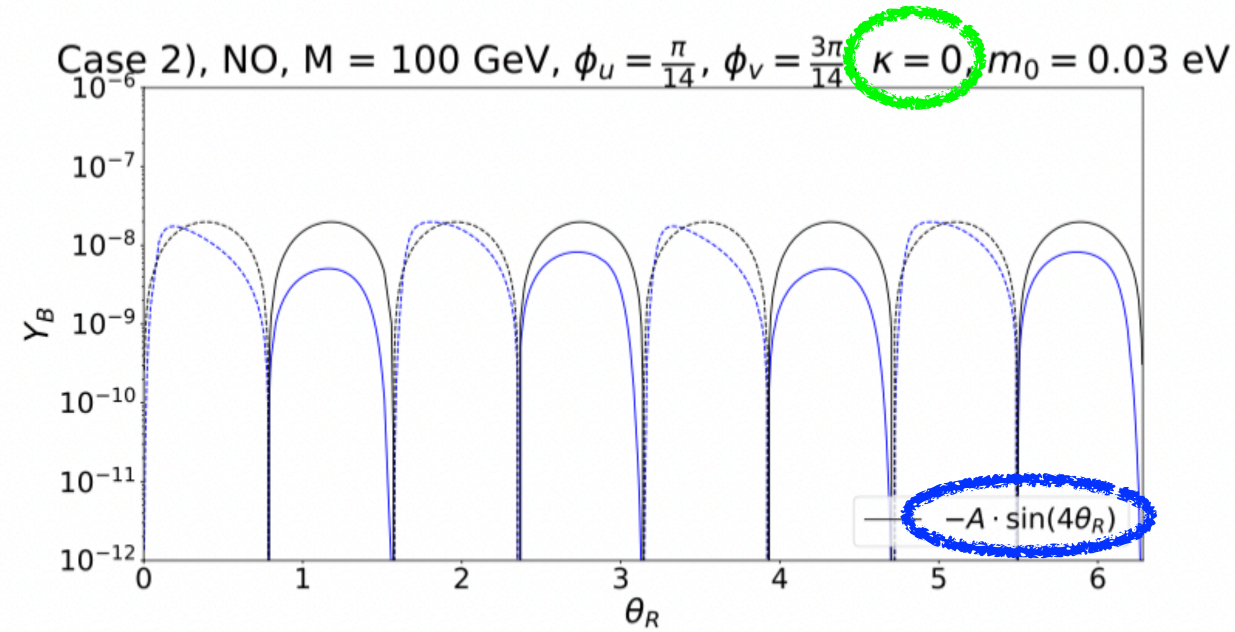
Case 2), t odd

Focus on vanishing κ and λ

$u = 2\,s - t \qquad v = 3\,t$



(a) Vanishing initial conditions.



(b) Vanishing initial conditions.

$$C_{\mathrm{DEG},\alpha} = \frac{1}{3}\, y_1\, y_3\, (\Delta y_{13}^2)^2 \sin 4\,\theta_R \sin \phi_{u,\alpha}\,.$$

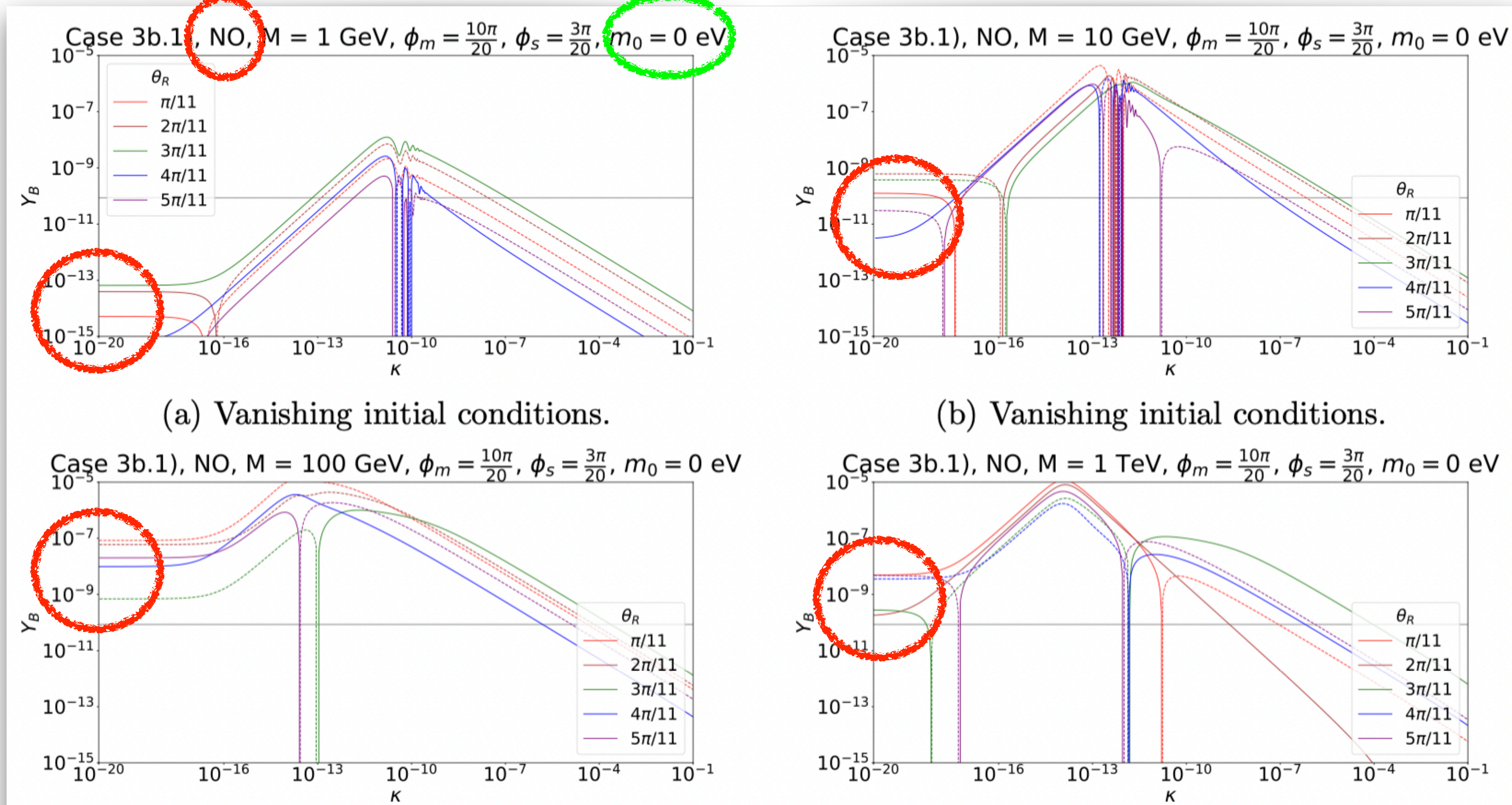
$$\phi_{u,\alpha} = \frac{\pi\, u}{n} + \rho_\alpha \frac{4\,\pi}{3} = \phi_u + \rho_\alpha \frac{4\,\pi}{3} \quad \text{with} \quad \rho_e = 0\,,\ \rho_\mu = -1\,,\ \rho_\tau = +1\,.$$

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 3 b.1), m even and s odd

Focus on vanishing κ and λ and also $m_0 = 0$



(a) Vanishing initial conditions.

(b) Vanishing initial conditions.

(c) Vanishing initial conditions.

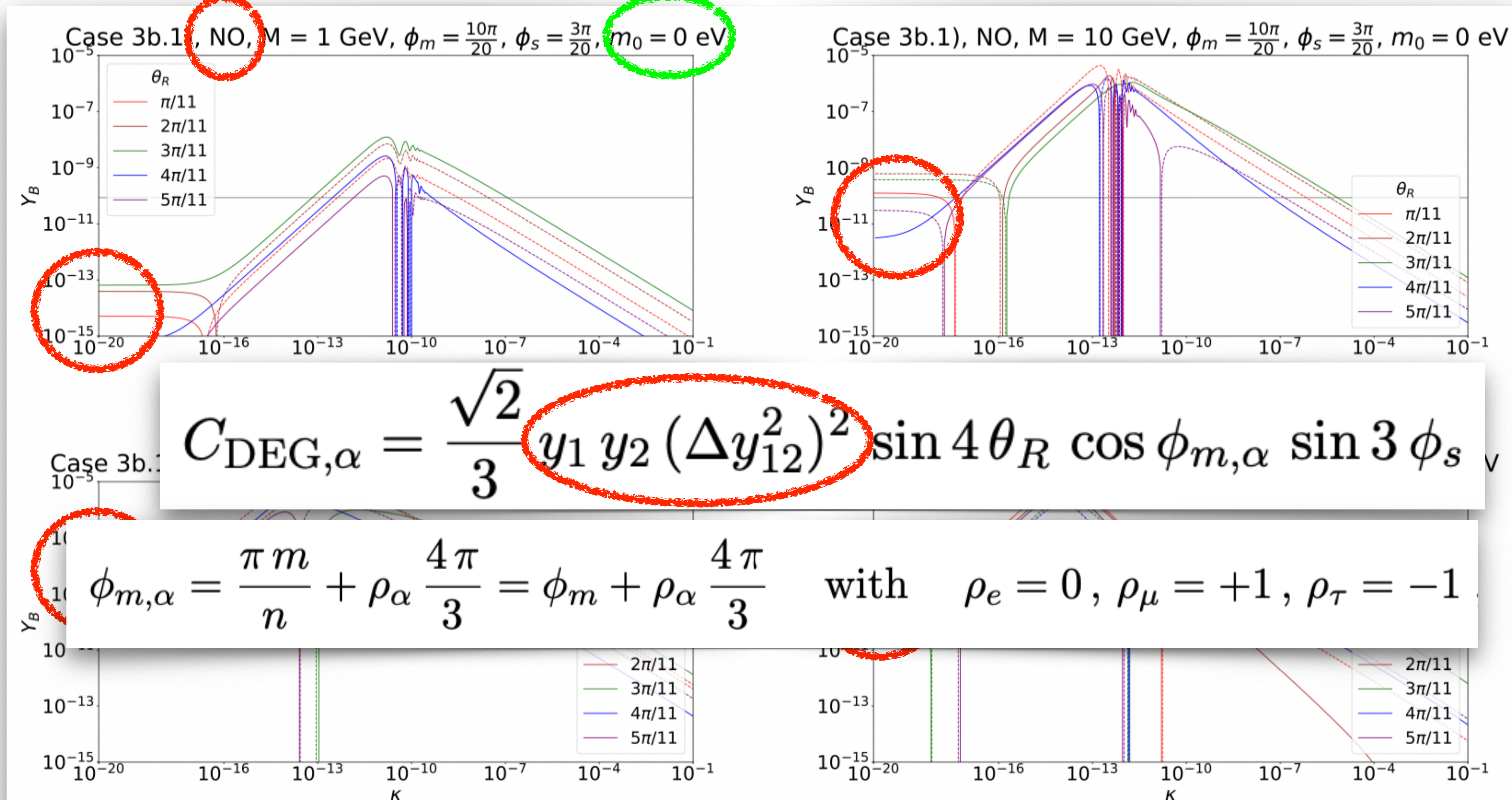
(d) Vanishing initial conditions.

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 3 b.1), *m* even and *s* odd

Focus on vanishing κ and λ and also $m_0 = 0$



$$C_{\mathrm{DEG},\alpha} = \frac{\sqrt{2}}{3} y_1\, y_2\, (\Delta y_{12}^2)^2 \sin 4\,\theta_R \,\cos\phi_{m,\alpha}\, \sin 3\,\phi_s$$

$$\phi_{m,\alpha} = \frac{\pi\, m}{n} + \rho_\alpha \frac{4\,\pi}{3} = \phi_m + \rho_\alpha \frac{4\,\pi}{3} \quad \text{with} \quad \rho_e = 0\,,\ \rho_\mu = +1\,,\ \rho_\tau = -1$$

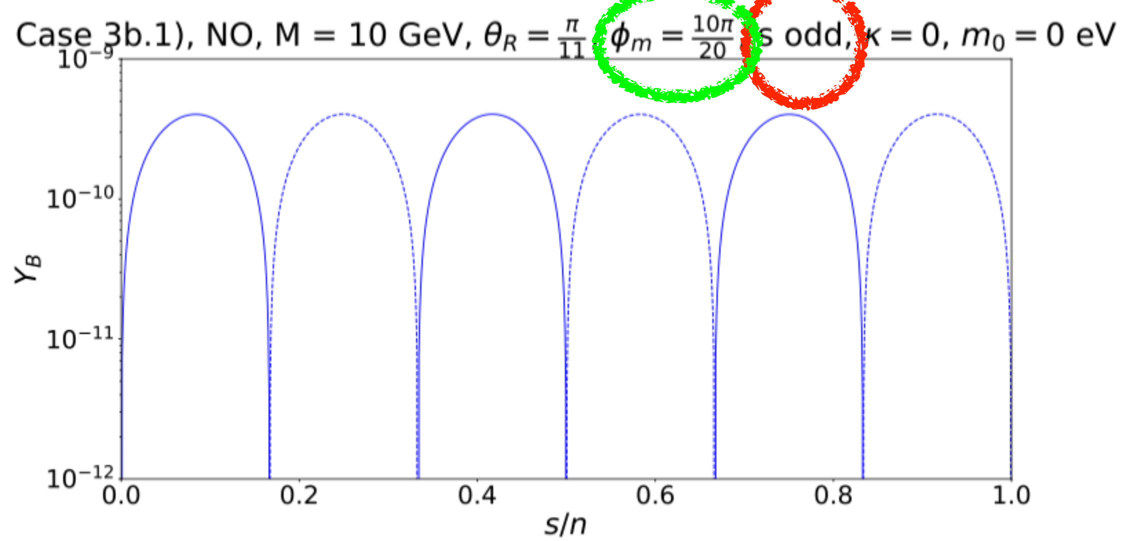
(c) Vanishing initial conditions. (d) Vanishing initial conditions.

Scenario with type I seesaw mechanism

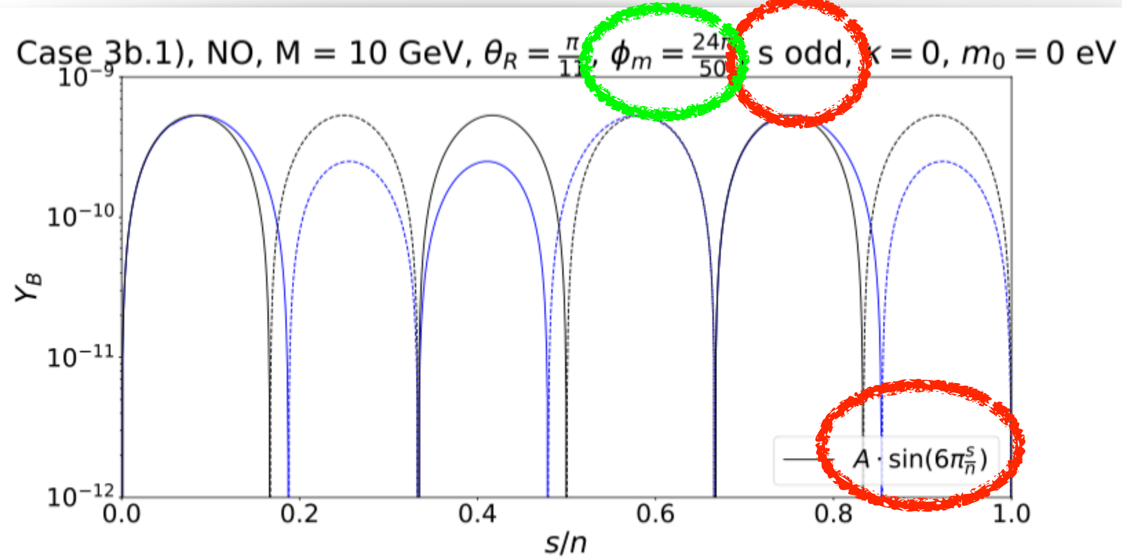
[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 3 b.1), m even and s odd

Focus on vanishing κ and λ and also $m_0 = 0$



(a) Vanishing initial conditions.



(b) Vanishing initial conditions.

$$C_{\mathrm{DEG},\alpha} = \frac{\sqrt{2}}{3}\, y_1\, y_2\, (\Delta y_{12}^2)^2\, \sin 4\,\theta_R\, \cos\phi_{m,\alpha}\, \sin 3\,\phi_s$$

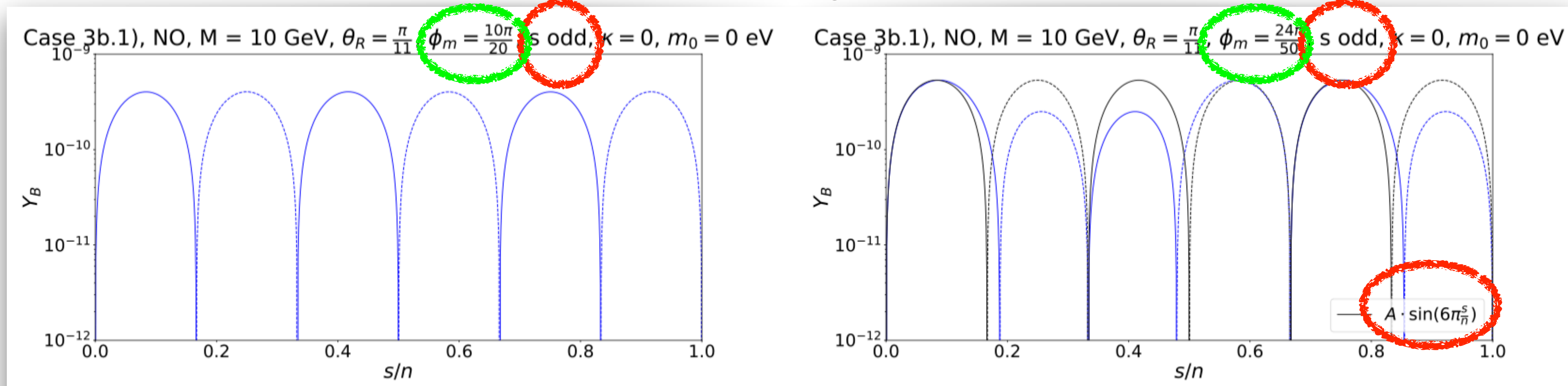
$$f_\alpha = \frac{1}{3}\left(1 + \left(\frac{\Delta y_{13}^2 - \Delta y_{12}^2\, \sin^2\theta_L}{\Sigma y^2}\right)\cos 2\,\phi_{m,\alpha} - \sqrt{2}\left(\frac{\Delta y_{12}^2}{\Sigma y^2}\right)\sin 2\,\theta_L\, \cos\phi_{m,\alpha}\, \cos 3\,\phi_s\right)$$

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 3 b.1), m even and s odd

Focus on vanishing κ and λ and also $m_0 = 0$



(a) Vanishing initial conditions. (b) Vanishing initial conditions.

If $m = \dfrac{n}{2}$

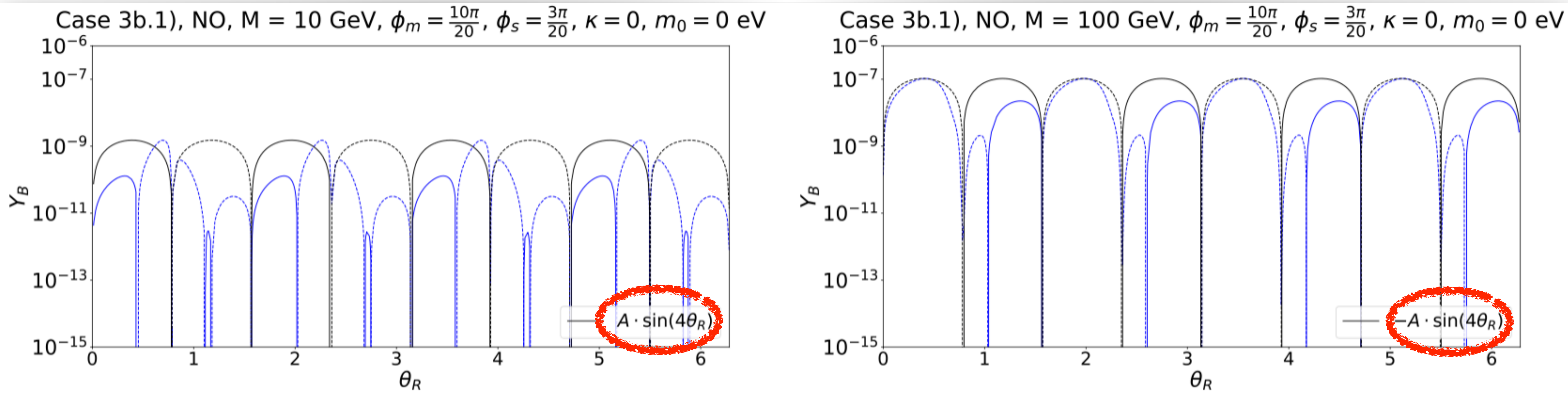
$$\sum_\alpha C_{\mathrm{DEG},\alpha}\, f_\alpha = -\left(\frac{y_1\, y_2\, (\Delta y_{12}^2)^3}{6\, \Sigma y^2}\right) \sin 2\,\theta_L\, \sin 4\,\theta_R\, \sin 6\,\phi_s$$

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 3 b.1), m even and s odd

Focus on vanishing κ and λ and also $m_0 = 0$



(a) Vanishing initial conditions. (b) Vanishing initial conditions.

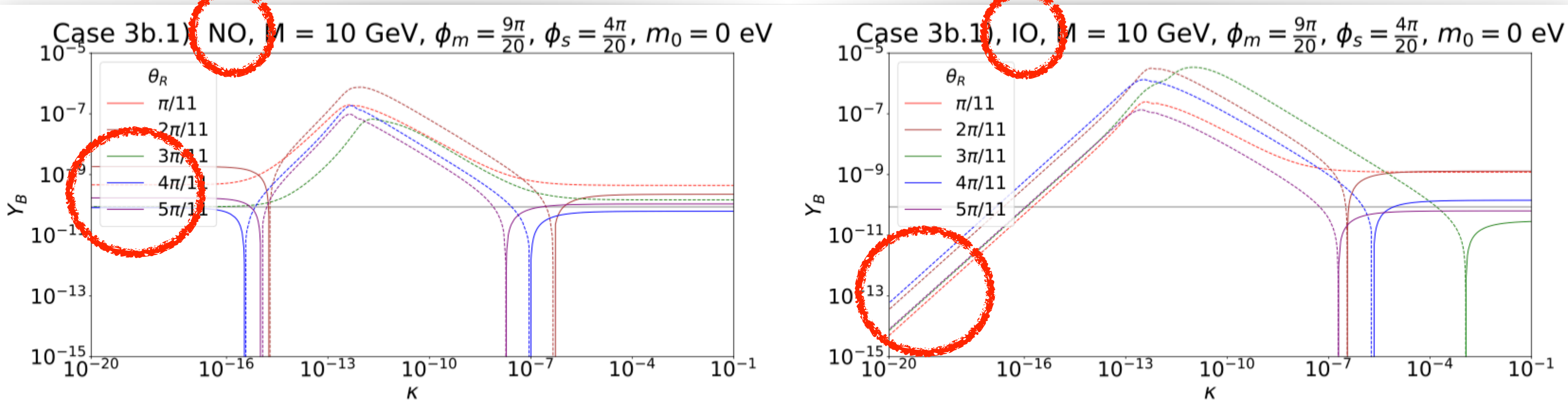
$$C_{\mathrm{DEG},\alpha} = \frac{\sqrt{2}}{3}\, y_1\, y_2\, (\Delta y_{12}^2)^2 \sin 4\,\theta_R \cos\phi_{m,\alpha} \sin 3\,\phi_s$$

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 3 b.1), m odd and s even

Focus on vanishing κ and λ and also $m_0 = 0$



(a) Vanishing initial conditions. (b) Vanishing initial conditions.

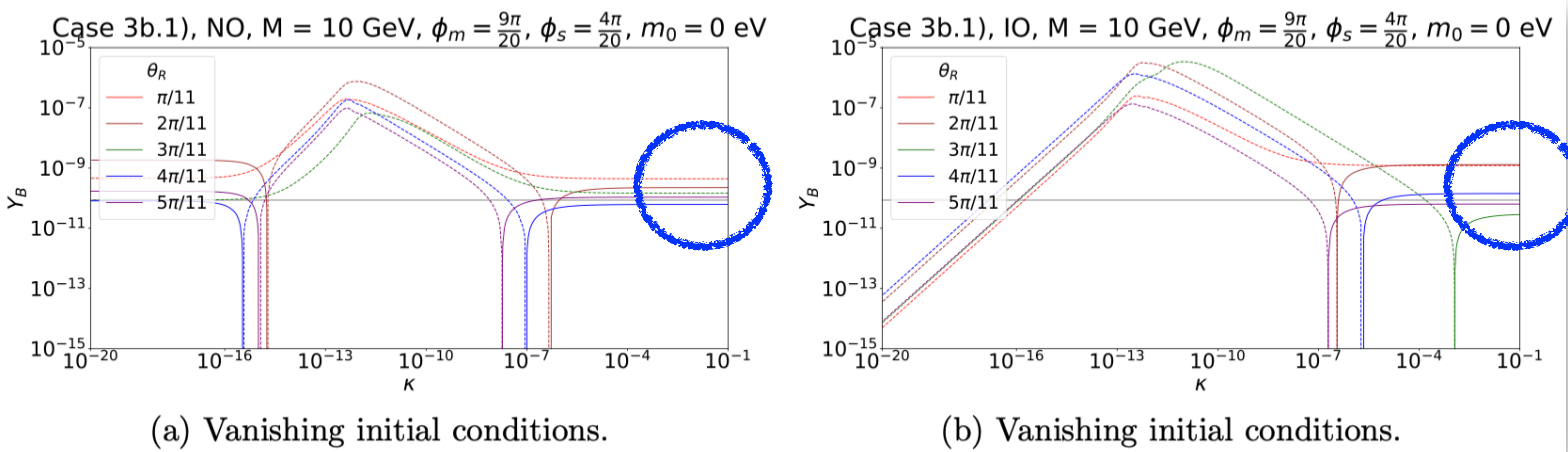
$$C_{\mathrm{DEG},\alpha} = \frac{\sqrt{2}}{3} y_1\, y_2 (\Delta y_{12}^2)^2 \sin 4\,\theta_R \cos\phi_{m,\alpha} \sin 3\,\phi_s$$

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 3 b.1), m odd and s even

Focus on vanishing κ and λ and also $m_0 = 0$



(a) Vanishing initial conditions. (b) Vanishing initial conditions.

$$C_{\mathrm{DEG},\alpha} = \frac{\sqrt{2}}{3} y_1 \, y_2 \, (\Delta y_{12}^2)^2 \, \sin 4\,\theta_R \, \cos\phi_{m,\alpha} \, \sin 3\,\phi_s$$

$$C^{(23)}_{\mathrm{DEG},\alpha} = \frac{2}{27} \Delta y_{12}^2 \left(2\,\Delta y_{23}^2 + \Delta y_{12}^2 \,(1 + 5\,\cos 2\,\theta_R)\right) \left(y_3 (y_1 \, \sin\theta_L \cos\theta_R + y_2 \, \cos\theta_L \, \sin\theta_R) \, \sin\phi_{m,\alpha} - \sqrt{2}\, y_1 \, y_2 \, \cos\phi_{m,\alpha}\right) \sin 2\,\theta_R \, \sin 3\,\phi_s .$$

Scenario with type I seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Overview over results

Type of mixing pattern	BAU non-zero for $\kappa = 0$?	BAU non-zero for large κ?	Large total mixing angle U^2 possible?
Case 1)	No, see Fig. 4	Yes, see Fig. 4	Yes, for $\cos 2\,\theta_R \approx 0$ see Fig. 9
Case 2), t even	No, see Fig. 12	No, see Fig. 12	No
Case 2), t odd	Yes, for $m_0 \neq 0$ see Fig. 17, plot (a)	Yes, see Fig. 16	Yes, for $\sin 2\,\theta_R \approx 0$ see Fig. 19
Case 3 b.1), m and s even	No, see Fig. 20	No, see Fig. 20	No
Case 3 b.1), m even, s odd	Yes, see Fig. 22 except for strong IO	No, see Fig. 22	Yes, for $\cos 2\,\theta_R \approx 0$ see Fig. 25
Case 3 b.1), m odd, s even	Yes, see Fig. 26 except for strong IO	Yes, see Fig. 26	Yes, for $\cos 2\,\theta_R \approx 0$
Case 3 b.1), m and s odd	No	No	No

Summary and Outlook

- Flavour and CP symmetries are useful for understanding fermion mixing and potentially also fermion masses
- Interesting application to scenarios that are testable (directly)
- Two examples discussed — inverse seesaw mechanism
  low-scale type I seesaw mechanism

Summary and Outlook

- Flavour and CP symmetries are useful for understanding fermion mixing and potentially also fermion masses
- Interesting application to scenarios that are testable (directly)
- Two examples discussed — inverse seesaw mechanism
  low-scale type I seesaw mechanism

- More signals are to be explored
- More options/variants of such scenarios are to be considered
- Embedding in larger framework could be interesting
- …

Many thanks for your attention!