(GLOBAL) LEPTON NUMBER SYMMETRY AND NEUTRINO MASSES

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Accidental symmetries of the SM

The Standard Model has accidental perturbative symmetries, arising from: gauge group + field content + renormalizability



Non perturbative effects violate both B and L, but preserve

$$\partial_{\mu}J_{B}^{\mu} = \partial_{\mu}J_{L}^{\mu} = \frac{N_{f}}{32\pi^{2}}\epsilon^{\mu\nu\sigma\tau} \left(-g_{W}^{2}\operatorname{Tr}W_{\mu\nu}W_{\sigma\tau} + g_{Y}^{2}B_{\mu\nu}B_{\sigma\tau}\right)$$

G. 't Hooft, Phys. Rev. Lett. 37 (1976) 8; Phys. Rev. D 14 (1976) 3432

Accidental symmetries: experimental status





Massive neutrinos violate it if they are Majorana particles

SM as an effective theory

Relaxing the renormalizability condition there is only one dim=5 gauge invariant operator (Weinberg operator) S. Weinberg, Phys. Rev. Lett. 43 (1979) 1566



Unveiling neutrino mass generation mechanism



A suppression: naive Seesaw scaling Seesaw scaling $m_{\nu} = -v^2 F \frac{1}{\pi} F^T$ $|U_{\alpha i}| \lesssim \sqrt{\frac{m_{\nu}}{M}} \lesssim 10^{-5} \sqrt{\frac{\text{GeV}}{M}}$ In the absence of any structure in the F and M matrices Experimentally 10⁻⁵ excluded 10⁻⁷ |U_{ei}|2 10⁻⁹ Naive seesaw scaling 10⁻¹¹ 5 0.1 0.5 1 10 50 M_i[GeV] Michele Lucente - RWTH Aachen University NuTs 2022 6

Symmetries: L number has a special role

Theorem: SM + fermionic gauge singlets

K. Moffat, S. Pascoli and C. Weiland, arXiv:1712.07611 [hep-ph]

"The most general gauge-singlet neutrino extensions of the SM with no cancellation between different orders of the seesaw expansion, no fine-tuned cancellations between different radiative orders and which lead to three massless neutrinos are lepton number conserving"

In the SM extended with fermionic gauge singlets (e.g. Right-Handed neutrinos)



Unless there are accidental cancellations in m_v , the rate for Lepton number violating events is proportional to the small active neutrino masses

The theorem extends and generalises previous results: G. Ingelman and J. Rathsman, Z. Phys. C 60 (1993) 243; J. Gluza, hep-ph/0201002; J. Kersten and A. Y. Smirnov, arXiv:0705.3221 [hep-ph]

Accidental cancellations: quantify fine tuning

If a symmetry is present in the Lagrangian, it will be manifest at any order in perturbation theory



The neutrino mass scale is stable under radiative corrections

Compute neutrino masses m_v at 1-loop, and quantify the level of fine-tuning of a solution as

$$f.t.(m_{\nu}) = \sqrt{\sum_{i=1}^{3} \left(\frac{m_i^{\text{loop}} - m_i^{\text{tree}}}{m_i^{\text{loop}}}\right)^2}$$

m_i ^{loop} 1-loop neutrino mass spectrum

m_i tree tree-level neutrino mass spectrum

Fermionic singlet extensions of the SM

SM + *n* gauge singlet fermions *N*₁

$$\mathcal{L} = \mathcal{L}_{SM} + i\overline{N_{I}}\partial N_{I} - \left(\begin{matrix} F_{\alpha I}\overline{\ell_{L}^{\alpha}}\widetilde{\phi}N_{I} + \frac{M_{IJ}}{2}\overline{N_{I}^{c}}N_{J} + h.c. \end{matrix}\right) \\ \uparrow & \uparrow & \uparrow & \uparrow \\ 3 \times n \text{ matrix} \\ Yukawa \text{ couplings} & n \times n \text{ matrix} \\ Majorana \text{ mass} \\ \text{couplings} \end{matrix}$$

After electroweak phase transition $\langle \Phi \rangle = v \approx 174$ GeV

$$-\mathcal{L}_{m}^{\nu} = \frac{1}{2} \left(\begin{array}{cc} \overline{\nu_{L}} & \overline{N^{c}} \end{array} \right) \underbrace{\left(\begin{array}{cc} \delta m_{\nu}^{\text{loop}} & vF \\ vF^{T} & M \end{array} \right)}_{\mathcal{M}} \left(\begin{array}{c} \nu_{L}^{c} \\ N \end{array} \right) + h.c.$$

$$(3+n) \text{ dimensional mass matrix}$$



L symmetry and Majorana fields

Majorana fermions violate all global symmetries, including L

How to preserve lepton number with Majorana states?

	Pair two states to form a Dirac state (equal masses, maximal mixing, opposite CP)	Decouple a state	Have a massless state
Exact symmetry	$M_1 = M_2$ $\mathcal{U}_{\alpha 1} = i \mathcal{U}_{\alpha 2}$	$\mathcal{U}_{\alpha i}=0$	$M_i = 0$
Approximate symmetry	$\frac{M_2 - M_1}{M_1 + M_2} \ll 1$ $\mathcal{U}_{\alpha 1} \simeq i \ \mathcal{U}_{\alpha 2}$	$ \mathcal{U}_{lpha,i} \ll \mathcal{U}_{lpha,j eq i} $	$M_i \ll M_{j \neq i}$

NEUTRINOLESS DOUBLE BETA DECAY

Double beta decay

2 β decay: 2nd order weak process $\mathcal{N}(A, Z) \rightarrow \mathcal{N}(A, Z+2) + 2e^- + 2\overline{\nu_e}$

Only relevant when the single $\beta_{48Ca, 76Ge, 82Se, 96Zr, 100Mo, 116Cd, 130Te, 136Xe, 150Nd}$ decay is kinematically forbidden



Figure from P. Lipari, Introduction to neutrino physics, in 2001 CERN-CLAF School of high-energy physics

Neutrinoless double beta decay: ΔL = 2

W. H. Furry, Phys. Rev. 56 (1939) 1184

If neutrinos are Majorana particles 0v2β is possible

 $\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + 2e^{-}$



Figure modified from F. T. Avignone III, S. R. Elliott and J. Engel, arXiv:0708.1033 [nucl-ex]

The black box theorem

J. Schechter and J. W. F. Valle, Phys. Rev. D 25 (1982) 2951; E. Takasugi, Phys. Lett. 149B (1984) 372; see also M. Duerr, M. Lindner and A. Merle, arXiv:1105.0901 [hep-ph]



186 **Experimental status: minimal SM**

The amplitude for light neutrino exchange is proportional to $m_{2\beta} = \left| \sum_{i} U_{ei}^2 m_i \right|$

From current knowledge on neutrino oscillation parameters it is possible to compute m₂ as a function of unknown lightest neutrino mass, ordering and CP phases

I	Excluded at 90% confidence level			C	urrent bou	inds
	Combination (CUORE, EXO-200, GERDA, KamLAND-Zen, NEMC	9-3)	Isotope	$T_{1/2}^{0\nu} (\times 10^{25} \text{ y})$	$\langle m_{\beta\beta} \rangle ~(\mathrm{eV})$	Experiment
10 ⁻¹			⁴⁸ Ca	$> 5.8 \times 10^{-3}$	< 3.5 - 22	ELEGANT-IV
			⁷⁶ Ge	> 8.0	< 0.12 - 0.26	GERDA
	Inverted Hierarchy			> 1.9	< 0.24 - 0.52	Majorana Demonstrator
e S		Planck	⁸² Se	$> 3.6 \times 10^{-2}$	< 0.89 - 2.43	NEMO-3
<u>_</u> 10 ⁻²		95% limit	96 Zr	$> 9.2 \times 10^{-4}$	< 7.2 - 19.5	NEMO-3
ີ່			¹⁰⁰ Mo	$> 1.1 \times 10^{-1}$	< 0.33 - 0.62	NEMO-3
	Normal Hierarchy		¹¹⁶ Cd	$> 1.0 \times 10^{-2}$	< 1.4 - 2.5	NEMO-3
3			¹²⁸ Te	$> 1.1 \times 10^{-2}$		
10 ⁻			¹³⁰ Te	> 1.5	< 0.11 - 0.52	CUORE
	Ē		¹³⁶ Xe	> 10.7	< 0.061 - 0.165	KamLAND-Zen
				> 1.8	< 0.15 - 0.40	EXO-200
10 ⁻⁴			¹⁵⁰ Nd	$> 2.0 \times 10^{-3}$	< 1.6 - 5.3	NEMO-3
1(10^{-4} 10^{-3} 10^{-2} 10^{-1} m_0^{-1} (eV)		1	Table from N	/I. J. Dolinski, A	. W. P. Poon and

Figure from P. Guzowski, L. Barnes, J. Evans, G. Karagiorgi, N. McCabe and S. Soldner-Rembold, arXiv:1504.03600 [hep-ex]

W. Rodejohann, arXiv:1902.04097 [nucl-ex]

Contribution of heavy neutrinos

Heavy Majorana neutrinos contribute as well to 0v2ß amplitude

F. L. Bezrukov, hep-ph/0505247; M. Blennow, E. Fernandez-Martinez, J. Lopez-Pavon and J. Menendez, arXiv:1005.3240 [hep-ph]; A. Abada and M.L., arXiv:1401.1507 [hep-ph]; A. Faessler, M. González, S. Kovalenko and F. Šimkovic, arXiv:1408.6077 [hep-ph]; A. Abada, V. De Romeri, M.L., A. M. Teixeira and T. Toma, arXiv:1712.03984 [hep-ph]; A. Babič, S. Kovalenko, M. I. Krivoruchenko and F. Šimkovic, arXiv:1804.04218 [hep-ph]



Heavy neutrinos at GeV scale



see also J. Lopez-Pavon, S. Pascoli and C. f. Wong, arXiv:1209.5342 [hep-ph]; J. Lopez-Pavon, E. Molinaro and S. T. Petcov, arXiv:1506.05296 [hep-ph]

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Extracting contraints on heavy neutrinos



TAU AND MESON DECAY

L-violating **t** and meson decay

Heavy Majorana neutrinos can mediate L-violating decays of pseudo-scalar mesons and τ lepton

 $M_1(p, m_{M_1}) \to \ell_{\alpha}(k_1, m_{\ell_{\alpha}}) \ell_{\beta}(k_2, m_{\ell_{\beta}}) M_2(k_3, m_{M_2})$



Negligible amplitude unless the intermediate state can go on-shell

$$\frac{1}{\left(m_{ij}^2 - m_4^2\right)^2 + m_4^2\Gamma_4^2} \quad \rightarrow \quad \frac{\pi}{m_4\Gamma_4}\delta\left(m_{ij}^2 - m_4^2\right)$$

Lifetime limitations

In the resonant regime
$$i\mathcal{M}\propto \frac{M_{\nu_s}}{\Gamma_{\nu_s}}\equiv M_{\nu_s}\tau_{\nu_s}$$

But too long-lived heavy neutrinos decay outside the detector



Asking for observable (inside detector) decays imposes a further constraint

Current bounds

Tables (and list of references) from A. Abada, V. De Romeri, M.L., A. M. Teixeira and T. Toma, arXiv:1712.03984 [hep-ph]

Meson decay

LNV docay	Current bound				
	$\ell_{\alpha} = e, \ \ell_{\beta} = e$	$\ell_{\alpha} = e, \ \ell_{\beta} = \mu$	$\ell_{\alpha} = \mu, \ \ell_{\beta} = \mu$		
$K^- \to \ell_\alpha^- \ell_\beta^- \pi^+$	6.4×10^{-10} [41]	5.0×10^{-10} [41]	1.1×10^{-9} [41]		
$D^- \to \ell_\alpha^- \ell_\beta^- \pi^+$	$1.1 \times 10^{-6} [41]$	2.0×10^{-6} [78]	2.2×10^{-8} [79]		
$D^- \to \ell_\alpha^- \ell_\beta^- K^+$	9.0×10^{-7} [78]	$1.9 \times 10^{-6} \ [78]$	1.0×10^{-5} [78]		
$D^- \to \ell_\alpha^- \ell_\beta^- \rho^+$			5.6×10^{-4} [41]		
$D^- \to \ell_\alpha^- \ell_\beta^- K^{*+}$			8.5×10^{-4} [41]		
$D_s^- \to \ell_\alpha^- \ell_\beta^- \pi^+$	$4.1 \times 10^{-6} [41]$	8.4×10^{-6} [78]	1.2×10^{-7} [79]		
$D_s^- \to \ell_\alpha^- \ell_\beta^- K^+$	5.2×10^{-6} [78]	$6.1 \times 10^{-6} \ [78]$	1.3×10^{-5} [78]		
$D_s^- \to \ell_\alpha^- \ell_\beta^- K^{*+}$			1.4×10^{-3} [41]		
$B^- \to \ell_\alpha^- \ell_\beta^- \pi^+$	$2.3 \times 10^{-8} [80]$	$1.5 \times 10^{-7} [81]$	4.0×10^{-9} [82]		
$B^- \to \ell_\alpha^- \ell_\beta^- K^+$	$3.0 \times 10^{-8} [80]$	$1.6 \times 10^{-7} [81]$	4.1×10^{-8} [83]		
$B^- \to \ell_\alpha^- \ell_\beta^- \rho^+$	1.7×10^{-7} [81]	$4.7 \times 10^{-7} [81]$	4.2×10^{-7} [81]		
$B^- \to \ell_\alpha^- \ell_\beta^- D^+$	$2.6 \times 10^{-6} [84]$	$1.8 \times 10^{-6} \ [84]$	6.9×10^{-7} [85]		
$B^- \to \ell_\alpha^- \ell_\beta^- D^{*+}$			2.4×10^{-6} [41]		
$B^- \to \ell_\alpha^- \ell_\beta^- D_s^+$			5.8×10^{-7} [41]		
$B^- \to \ell_\alpha^- \ell_\beta^- K^{*+}$	$4.0 \times 10^{-7} [81]$	$3.0 \times 10^{-7} [81]$	5.9×10^{-7} [81]		
LNV matrix m_{ν}	$m_{ u}^{ee}$	$m_{ u}^{e\mu}$	$m_{ u}^{\mu\mu}$		

LNV decay	Current bound		
	$\ell = e$	$\ell=\mu$	
$\tau^- \to \ell^+ \pi^- \pi^-$	2.0×10^{-8}	3.9×10^{-8}	
$\tau^- \to \ell^+ \pi^- K^-$	3.2×10^{-8}	4.8×10^{-8}	
$\tau^- \to \ell^+ K^- K^-$	3.3×10^{-8}	4.7×10^{-8}	
LNV matrix m_{ν}	$m_{\nu}^{e\tau}$	$m^{\mu au}_{ u}$	

Results from

Belle [84], BABAR [78,80,81] and LHCb [79,82,83,85];

summarised in PDG [41]

upper bounds from the Belle

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τ decay

Constraints: single intermediate state

Figures from A. Abada, V. De Romeri, M.L., A. M. Teixeira and T. Toma, arXiv:1712.03984 [hep-ph]; see also A. Atre, T. Han, S. Pascoli and B. Zhang, arXiv:0901.3589 [hep-ph]



Dashed lines: the on-shell heavy neutrino travels for less than 10 m

Some predictions: single intermediate state

Comprehensive analysis for τ and pseudo-scalar mesons in 1712.03984

(all possible initial and 3-body final states)



Multiple intermediate states: interference

A. Abada, C. Hati, X. Marcano and A. M. Teixeira, arXiv:1904.05367 [hep-ph]

If more than one heavy neutrino mediate the process, and

$\Delta M \ll M$ and $\Delta M < \Gamma_N$

interference effects arise due to the CP-violating phases



26

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LHC SEARCHES

LNV at LHC

Heavy neutrinos in pp collisions produced through a variety of mechanisms



see also Y. Cai, T. Han, T. Li and R. Ruiz, arXiv:1711.02180 [hep-ph]

LNV can manifest with clean experimental signatures:

e.g. two same-sign leptons (any flavour combination of e and μ) and at least one jet



Figure from CMS Collaboration, arXiv:1806.10905 [hep-ex]

Current bounds: single mediator

CMS Collaboration, arXiv:1806.10905 [hep-ex]; see also ATLAS Collaboration, arXiv:1506.06020 [hep-ex]



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LNV/LNC oscillations

Y. Nir, Conf. Proc. C9207131, 81 (1992); G. Anamiati, M. Hirsch and E. Nardi, arXiv:1607.05641 [hep-ph]

Flavour eigenstate = coherent superposition of mass eigenstates

$$\begin{cases} N_{\ell} = \frac{1}{\sqrt{2}}(N_{+} - iN_{-}) \\ N_{\bar{\ell}} = \frac{1}{\sqrt{2}}(N_{+} + iN_{-}) \end{cases} \text{ evolution} \qquad \begin{cases} N_{\ell}(t) = g_{+}(t)N_{\ell} + g_{-}(t)N_{\bar{\ell}} \\ N_{\bar{\ell}}(t) = g_{-}(t)N_{\ell} + g_{+}(t)N_{\bar{\ell}} \end{cases} \\ g_{+}(t) = e^{-iMt}e^{-\frac{\Gamma}{2}t}\cos\left(\frac{\Delta M}{2}t\right) \\ g_{-}(t) = i e^{-iMt}e^{-\frac{\Gamma}{2}t}\sin\left(\frac{\Delta M}{2}t\right) \end{cases} \Delta M = M^{+} - M^{-} \end{cases}$$
$$Timescales \qquad \qquad \Delta M \gg \Gamma \quad \text{decay after decoherence (Majorana limit)} \\ \Delta M \approx \Gamma \quad \text{oscillations} \\ \Delta M \ll \Gamma \quad \text{oscillations do not develop (Dirac limit)} \end{cases}$$
$$R_{\ell\ell}(t_{1}, t_{2}) = \frac{\int_{t_{1}}^{t_{2}}|g_{-}(t)|^{2}dt}{\int_{t_{1}}^{t_{2}}|g_{+}(t)|^{2}dt} = \frac{\#(\ell^{+}\ell^{+}) + \#(\ell^{-}\ell^{-})}{\#(\ell^{+}\ell^{-})} \qquad R_{ll}(0, \infty) = \frac{\Delta M^{2}}{2\Gamma^{2} + \Delta M^{2}}$$

Are these oscillations observable?

S. Antusch, E. Cazzato and O. Fischer, arXiv:1709.03797 [hep-ph]





However, for heavy neutrinos with γ =50

very small track separation of decay products

Richard Ruiz, private communication

• very forward rapidity

Why to look for LNV if $m_v \simeq 0$?

Equivalence between L conservation and massless neutrinos only holds in SM + singlet fermions

E.g. Left-right symmetric model





A LNV observation at LHC likely falsifies high-scale leptogenesis

EARLY UNIVERSE

Low Scale Leptogenesis with 2 RHN

The approximate L conservation forces the HNL to be degenerate in mass

This allows to account for BAU via freeze-in leptogenesis with low-scale NHL



Figure from A. Abada, G. Arcadi, V. Domcke and M. Lucente, arXiv:1507.06215 [hep-ph]

Many studies, e.g. E. K. Akhmedov, V. A. Rubakov and A. Y. Smirnov, hep-ph/9803255; T. Asaka and M. Shaposhnikov, hep-ph/0505013; M. Shaposhnikov, arXiv:0804.4542 [hep-ph]; T. Asaka and H. Ishida, arXiv:1004.5491 [hep-ph]; T. Asaka, S. Eijima and H. Ishida, arXiv:1112.5565 [hep-ph]; L. Canetti, M. Drewes and M. Shaposhnikov, arXiv:1204.4186 [hep-ph]; L. Canetti, M. Drewes, T. Frossard and M. Shaposhnikov, arXiv:1208.4607 [hep-ph]; P. Hernández, M. Kekic, J. López-Pavón, J. Racker and N. Rius, arXiv:1508.03676 [hep-ph]...

Low Scale Leptogenesis with 3 RHN

Mass spectrum with 3 right-handed neutrinos and B - L symmetry



If the vacuum mass of the decoupled state is heavier than the pseudo-Dirac one, there is **necessarily** a level crossing at some finite temperature

A. Abada, G. Arcadi, V. Domcke, M. Drewes, J. Klaric and M. Lucente, arXiv:1810.12463 [hep-ph]

T_{EW} = 140 GeV R: sterile neutrinos density matrix x μ_{lpha} : active flavours chemical potentials = $\overline{T}_{\rm EW}$ Active flavours asymmetries Sterile neutrinos abundances equilibrium value 10⁻⁵ 0.100 10⁻⁷ $\Delta\mu_{lpha}$ ື 🗠 0.010 10⁻⁹ е 0.001 μ **10⁻¹¹** τ **10⁻⁴** 0.500 0.0050.010 0.0500.100 0.500 0.001 0.001 0.0050.010 0.0500.100 1 Х Energy eigenvalues Total asymmetries 10^{-7} **10⁻⁴** 10^{-8} **10⁻⁶** $\Sigma_{\alpha} \Delta \mu_{\alpha}; \Sigma_{j} \Delta R_{ii}$ 10⁻⁹ ۸_i(⟨H⟩) [GeV] **10⁻⁸** Level crossing **10**⁻¹⁰ at x ~ 0.13 **10⁻¹⁰ 10⁻¹¹ 10⁻¹²** active **10**⁻¹² sterile **10⁻¹⁴ 10⁻¹³** 0.0050.010 0.0500.100 0.500 0.001 0.0050.010 0.0500.100 0.500 0.001 Michele Lucente - RWTH Aachen **X** Jniversity NuTs 2022 37 Х

Level crossing: resonant asymmetry production

Conclusion

Lepton number is an accidental symmetry of the SM: test its conservation!

It is violated by non-renormalizable SM operators: EFT generation of v masses

LNV phenomenology is generally connected with v mass generation mechanism

LNV rates depend in general on the interference of multiple virtual states



LNV observation could signal the existence of new gauge bosons and/or falsify high-scale leptogenesis

A global B-L symmetry has non-trivial consequences in the early Universe

Backup

Temperature of level crossing

The level crossing temperature in the SM + 3 RHN can be estimated at

$$T_{\text{crossing}} \approx \frac{2\sqrt{2}\bar{M}\sqrt{{\mu'}^2 - 1}}{\sqrt{\sum_a |F_a|^2}} = 2.8 \times 10^5 \text{ GeV}\left(\frac{\bar{M}}{\text{GeV}}\right) \frac{\sqrt{{\mu'}^2 - 1}}{\sqrt{\sum_a |(F_a/10^{-5})|^2}}$$

where the Majorana and Yukawa matrices are parameterised as

$$M_{M} = \bar{M} \begin{pmatrix} 1 - \mu & 0 & 0 \\ 0 & 1 + \mu & 0 \\ 0 & 0 & \mu' \end{pmatrix} \qquad F = \frac{1}{\sqrt{2}} \begin{pmatrix} F_{e}(1 + \epsilon_{e}) & iF_{e}(1 - \epsilon_{e}) & F_{e}\epsilon'_{e} \\ F_{\mu}(1 + \epsilon_{\mu}) & iF_{\mu}(1 - \epsilon_{\mu}) & F_{\mu}\epsilon'_{\mu} \\ F_{\tau}(1 + \epsilon_{\tau}) & iF_{\tau}(1 - \epsilon_{\tau}) & F_{\tau}\epsilon'_{\tau} \end{pmatrix}$$

Momentum averaged effective Hamiltonian for the heavy neutrinos

$$\langle H \rangle = \langle H_0 + V_N \rangle = \frac{\pi^2}{36\,\zeta(3)} \left(\frac{\operatorname{diag}(0, M_2^2 - M_1^2, M_3^2 - M_1^2)}{T} + \frac{T}{8}F^{\dagger}F \right)$$

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Quantum kinetic equations for freeze-in leptogenesis

$$\begin{aligned} \frac{dR_N}{dt} &= -i\left[\langle H \rangle, R_N\right] - \frac{1}{2} \langle \gamma^{(0)} \rangle \left\{ F^{\dagger}F, R_N - I \right\} - \frac{1}{2} \langle \gamma^{(1b)} \rangle \left\{ F^{\dagger}\mu F, R_N \right\} + \langle \gamma^{(1a)} \rangle F^{\dagger}\mu F + \\ &- \frac{1}{2} \langle \widetilde{\gamma}^{(0)} \rangle \left\{ M_M F^T F^* M_M, R_N - I \right\} + \frac{1}{2} \langle \widetilde{\gamma}^{(1b)} \rangle \left\{ M_M F^T \mu F^* M_M, R_N \right\} + \\ &- \langle \widetilde{\gamma}^{(1a)} \rangle M_M F^T \mu F^* M_M \,, \end{aligned}$$

$$\begin{aligned} \frac{d\mu_{\Delta a}}{dt} &= -\frac{9\zeta(3)}{2N_D \pi^2} \left\{ \langle \gamma^{(0)} \rangle \left(FR_N F^{\dagger} - F^* R_{\bar{N}} F^T \right) - 2 \langle \gamma^{(1a)} \rangle \mu F F^{\dagger} + \\ &+ \langle \gamma^{(1b)} \rangle \mu \left(FR_N F^{\dagger} + F^* R_{\bar{N}} F^T \right) \\ &+ \langle \widetilde{\gamma}^{(0)} \rangle \left(F^* M_M R_{\bar{N}} M_M F^T - F M_M R_N M_M F^{\dagger} \right) - 2 \langle \widetilde{\gamma}^{(1a)} \rangle \mu F^* M_M^2 F^T \\ &+ \langle \widetilde{\gamma}^{(1b)} \rangle \mu \left(F^* M_M R_{\bar{N}} M_M F^T + F M_M R_N M_M F^{\dagger} \right) \right\}_{aa} \,, \end{aligned}$$

G. Sigl and G. Raffelt, Nucl. Phys. B406 (1993) 423; E. K. Akhmedov, V. A. Rubakov and A. Yu. Smirnov, 9803255; T. Asaka and M. Shaposhnikov, 0505013; L. Canetti, M. Drewes, T. Frossard and M. Shaposhnikov, 1208.4607; T. Asaka, S. Eijima and H. Ishida, 1112.5565; P. Hernandez, M. Kekic, J. Lopez-Pavon, J. Racker and J. Salvado, 1606.06719; S. Antusch, E. Cazzato, M. Drewes, O. Fischer, B. Garbrecht, D. Gueter et al., 1710.03744; A. Abada, G. Arcadi, V. Domcke, M. Drewes, J. Klaric and M. Lucente, 1810.12463

Some SM extensions with L symmetry

Linear seesaw

E. K. Akhmedov, M. Lindner, E. Schnapka and J. W. F. Valle, hep-ph/9507275 and hep-ph/ 9509255; S. M. Barr, hep-ph/0309152; M. Malinsky, J. C. Romao and J. W. F. Valle, hep-ph/ 0506296; M. B. Gavela, T. Hambye, D. Hernandez and P. Hernandez, arXiv:0906.1461 [hep-ph]

Inverse seesaw

D. Wyler and L. Wolfenstein, Nucl. Phys. B 218 (1983) 205; R. N. Mohapatra, Phys. Rev. Lett. 56 (1986) 561; R.
N. Mohapatra and J. W. F. Valle, Phys. Rev. D 34 (1986) 1642; J. Bernabeu, A. Santamaria, J. Vidal, A. Mendez and J. W. F. Valle, Phys. Lett. B 187 (1987) 303; M. C. Gonzalez-Garcia and J. W. F. Valle, Phys. Lett. B 216 (1989) 360; F. Deppisch and J. W. F. Valle, hep-ph/0406040; A. Abada and M. Lucente, arXiv:1401.1507 [hep-ph]

Supersymmetry with R-parity violation

J. R. Ellis, G. Gelmini, C. Jarlskog, G. G. Ross and J. W. F. Valle, Phys. Lett. 150B (1985) 142; G. G. Ross and J.
W. F. Valle, Phys. Lett. 151B (1985) 375; J. C. Romao, M. A. Diaz, M. Hirsch, W. Porod and J. W. F. Valle, hep-ph/ 9907499; A. Abada and M. Losada, hep-ph/9908352; M. Hirsch, M. A. Diaz, W. Porod, J. C. Romao and J. W. F.
Valle, Phys. Rev. D62 (2000) 113008 [hep-ph/0004115]; A. Abada, S. Davidson and M. Losada, hep-ph/0111332

Scale invariance V. V. Khoze and G. Ro, arXiv:1307.3764 [hep-ph]

Technicolor-inspired

T. Appelquist and R. Shrock, hep-ph/0204141; T. Appelquist and R. Shrock, hep-ph/0301108

vMSM M. Shaposhnikov, hep-ph/0605047

Low-scale seesaw realisations

A. Ibarra, E. Molinaro and S. T. Petcov, arXiv:1007.2378 [hep-ph]; A. Ibarra, E. Molinaro and S. T. Petcov, arXiv:1103.6217 [hep-ph]; D. N. Dinh, A. Ibarra, E. Molinaro and S. T. Petcov, arXiv:1205.4671 [hep-ph]