# The gravitational memory of supernova neutrinos

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*Mainak Mukhopadhyay, Carlos Cardona and Cecilia Lunardini; JCAP 07 (2021) 055, arXiv:2105.0586.*
*Mainak Mukhopadhyay, Zidu Lin, Cecilia Lunardini, arXiv:2110.14657*

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# Structure of this talk

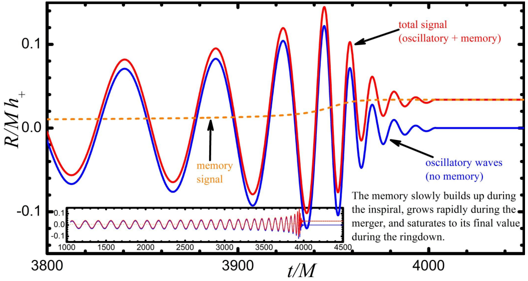


- Introduction, theoretical considerations
- A phenomenological model of the supernova neutrino memory.
    - Detectability and physics potential
- Memory-triggered SN neutrino searches
- Summary and discussion

*Introduction*

# Gravitational waveforms with memory



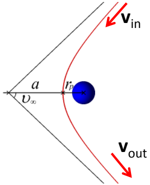



slide from M. Favata

- the GW strain converges to a non-zero value: *memory* is present

# Memory from General Relativity

- *permanent* distortion of the local space time metric
- Due to gravitationally *unbound* systems:
  - anisotropic emission of energy (mass/radiation)
- appears as a permanent change in the distance between two free falling masses: signal at GW interferometers!





figures from M. Favata

# The memory of supernova neutrinos



- the memory has never been observed
- observation requires: (i) very powerful emitter and (ii) some anisotropy
- ideal candidate: a core collapse supernova!
    - $E_{tot} \sim 3\ 10^{53}$ ergs, *most as neutrinos*
    - anisotropy at $\sim 10^{-3} - 10^{-2}$ level
    - neutrino emission timescale $\Delta t \sim O(10)$ s $\rightarrow$ sub-Hz scale

# The SN $\nu$ memory: a signal for Deci-Hz interferometers



- simulations are costly, limited to $\sim 1$ s.
- Must use phenomenology, to describe long term emission, diversity of scenarios

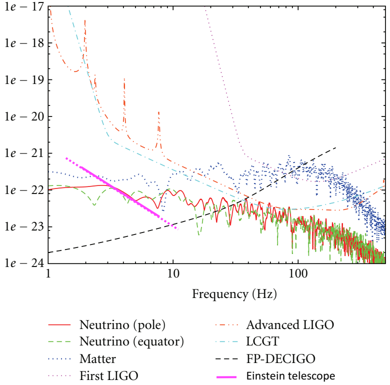



fig. from Kotake, Adv. Astron. (2012), 428757

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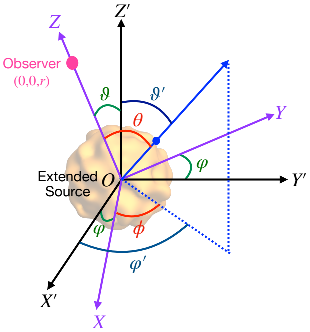
*Theoretical considerations*

# How to calculate the memory




- solving Einstein's equation, in weak-field approximation: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- longitudinal polarization ($h^{xx}_{TT} = -h^{yy}_{TT} = -h^{+}_{TT}$):

$$h^{xx}_{TT} = \frac{2G}{rc^4} \int_{-\infty}^{t-r/c} dt' \int_{4\pi} (1 + \cos\theta) \cos 2\phi \frac{dL_\nu(\Omega', t')}{d\Omega'} d\Omega' .$$

- Change of separation of free-falling masses: $\delta l_j = \frac{1}{2} h^{TT}_{jk} l^k$

# The anisotropy parameter



- For convenience, the angular dependence can be lumped into the *anisotropy parameter*:

$$\alpha(t) = \frac{1}{L_\nu(t)} \int_{4\pi} d\Omega' \; \Psi(\vartheta', \varphi') \; \frac{dL_\nu(\Omega', t)}{d\Omega'} \, ,$$

- Final form:

$$\boxed{h_{TT}^{xx} = h(t) = \frac{2G}{rc^4} \int_{-\infty}^{t-r/c} dt' L_\nu(t')\alpha(t') \, .}$$

# Phenomenology: upper bounds

- In the time domain ($\Delta h = h(+\infty) - h(-\infty)$):

$$
\begin{aligned}
|\Delta(h)| &\leq \frac{2G}{rc^4}|\alpha|_{max} E_{tot} \\
&\simeq 6.41\ 10^{-20} \left(\frac{|\alpha|_{max}}{0.04}\right) \left(\frac{E_{tot}}{3\ 10^{53}\ \mathrm{ergs}}\right) \left(\frac{r}{10\ \mathrm{kpc}}\right)^{-1} .
\end{aligned}
$$

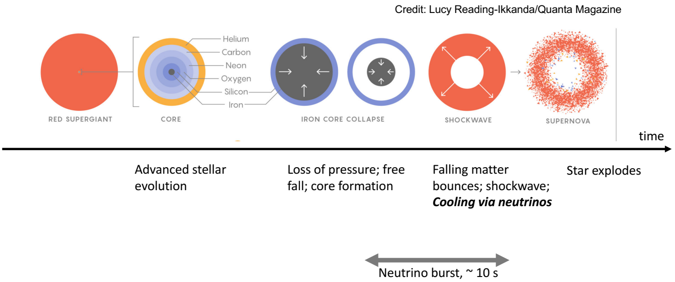
- In frequency domain: $h_c(f) \equiv 2f|\tilde{h}(f)|$ ($\tilde{h}$: Fourier transform).
- Zero frequency limit (ZFL):

$$
\lim_{f\to 0} h_c = \frac{|\Delta h|}{\pi} \lesssim 2.0\ 10^{-20} \left(\frac{|\alpha|_{max}}{0.04}\right) \left(\frac{E_{tot}}{3\ 10^{53}\ \mathrm{ergs}}\right) \left(\frac{r}{10\ \mathrm{kpc}}\right)^{-1}
$$

*A phenomenological model of the supernova neutrino memory*

# Supernova neutrinos: a mini-review






Stellar death: core collapse

- neutrinos emitted thermally, $\langle E \rangle \simeq 10 - 18$ MeV, radius $R \simeq 100$ Km.
- $E_{tot} \sim 3\ 10^{53}$ ergs emitted in $\mathcal{O}(10)$ s burst.

# Phases of neutrino emission: $L_\nu(t)$

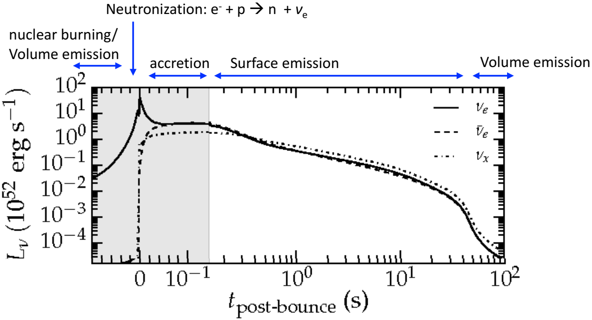



fig. from Roberts and Reddy, Handbook of Supernovae, Springer Intl., 2017

- accretion phase: $t \sim 0.003 - 0.5$ s: shockwave is stalled
- cooling phase: $t \sim 0.5 - 40$ s : shockwave re-energized by neutrino energy deposition, launches

# near-core dynamics: $\alpha(t)$

- anisotropy develops during accretion, due to:
  - convection
  - large scale sloshing motion of shock front (Standing Accretion Shock Instability, SASI)
- anisotropy during cooling phase not simulated

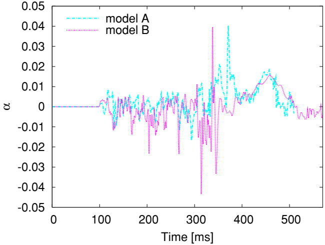



fig. from Kotake, Iwakami, Ohnishi and Yamada, Astrophys. J. 704 (2009) 951

# Building a phenomenological model



- toy $L_\nu(t)$: global shape (only valid locally) :

$$L_\nu(t) = \lambda + \beta \ \exp\left(-\chi\, t\right) \ ,$$

- toy $\alpha(t)$: multi-Gaussian+constant:

$$\alpha(t) = \kappa + \sum_{j=1}^{N} \xi_j \ \exp\left(-\frac{(t-\gamma_j)^2}{2\sigma_j^2}\right) \ ,$$

- result: analytical $h(t)$

$$h(t) = \sum_{j=1}^{N} \left\{ \left[ h_{1j} \left( \mathrm{erf}\left(\rho_j \ \tau_{1j}\right) + \mathrm{erf}\left(\rho_j (t - \tau_{1j})\right) \right) \right] + \left[ h_{2j} \left( \mathrm{erf}\left(\rho_j \ \tau_{2j}\right) + \mathrm{erf}\left(\rho_j (t - \tau_{2j})\right) \right) \right] \right\}$$
$$+ \left[ h_3 \left( \frac{\beta}{\chi} \left(1 - \exp\left(-t\chi\right)\right) + \lambda t \right) \right] \ ,$$

$$\tilde{h}(f) = \sum_{j=1}^{N} \left[ \left( h_{1j} \frac{i}{\pi f} \exp\left(\frac{-\pi^2 f^2}{\rho_j^2}\right) \exp\left(i 2\pi f \tau_{1j}\right)\right) + \left( h_{2j} \frac{i}{\pi f} \exp\left(\frac{-\pi^2 f^2}{\rho_j^2}\right) \exp\left(i 2\pi f \tau_{2j}\right)\right)\right] + \left( \sqrt{2\pi}\, h_3 \frac{\beta}{\chi} \left( \frac{1}{i 2\pi f} - \frac{1}{-\chi + i 2\pi f} \right)\right),$$

$$h_{1j} = \frac{2G}{rc^4} \sqrt{\frac{\pi}{2}} \beta \xi_j \sigma_j \exp\left(\frac{\chi}{2}(-2\gamma_j + \sigma_j^2 \chi)\right),$$

$$\rho_j = \frac{1}{\sqrt{2}\sigma_j},$$

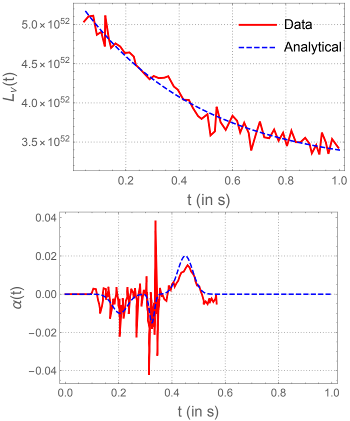
$$\tau_{1j} = \gamma_j - \sigma_j^2 \chi,$$

$$h_{2j} = \frac{2G}{rc^4} \sqrt{\frac{\pi}{2}} \lambda \xi_j \sigma_j,$$

$$\tau_{2j} = \gamma_j,$$

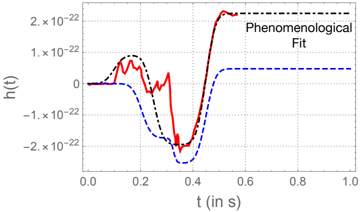
$$h_3 = \frac{2G}{rc^4} \kappa.$$

# Comparison with numerical results




**top** data: Vartanyan and Burrows, Astrophys. J. 901 (2020) 108 ; **bottom** data: Kotake, Iwakami, Ohnishi and Yamada, Astrophys. J. 704 (2009) 951.

- toy model reproduces low frequency trends (relevant for Deci-Hz detectors)




Data: Kotake, Iwakami, Ohnishi and Yamada, Astrophys. J. 704 (2009) 951.

- toy $h(t)$ reproduces numerical result
  - dashed: computed from $L(t)$ and $\alpha(t)$
  - dot-dashed: toy formula for $h(t)$ with effective parameters

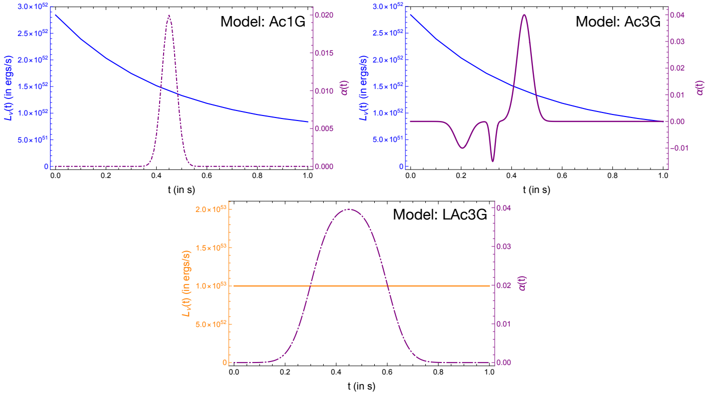
# Case studies



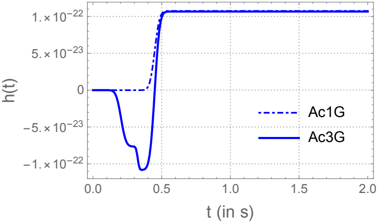
- *Accretion-only models*: zero anisotropy in cooling phase
- *Long term evolution models*: anisotropy is non-zero throughout

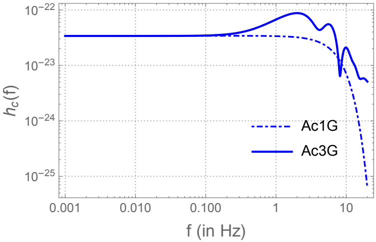
# Accretion-only models: ingredients

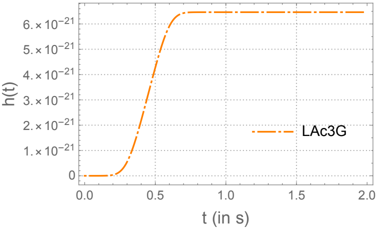


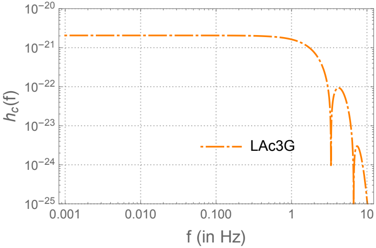


# Accretion-only models: results (D=10 kpc)



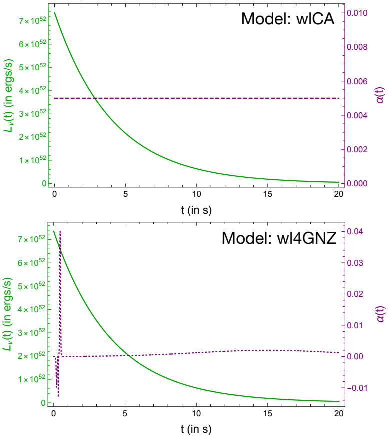



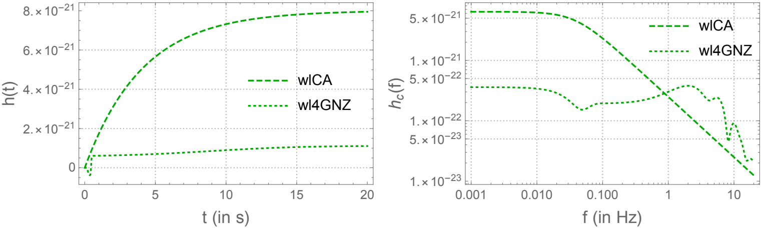
# Long term evolution models: ingredients





# Long term evolution models: results (D=10 kpc)



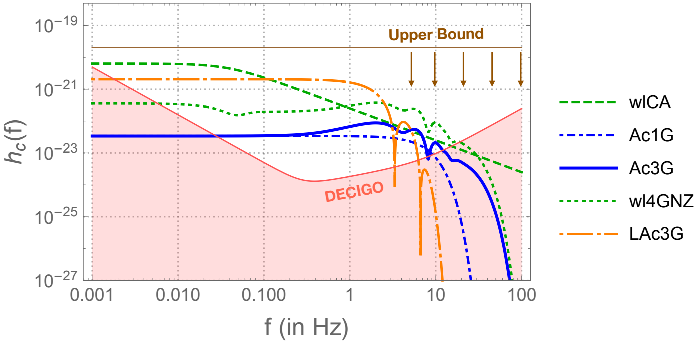


# *Detectability and physics potential*

# Memory at Deci-Hz detectors (D=10 kpc)



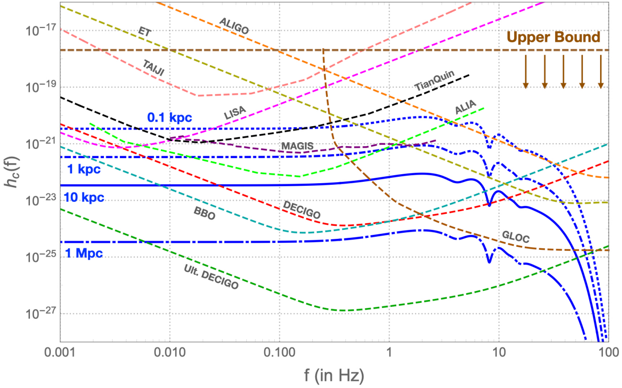
Detectable even in most pessimistic cases



# Summary of detection prospects



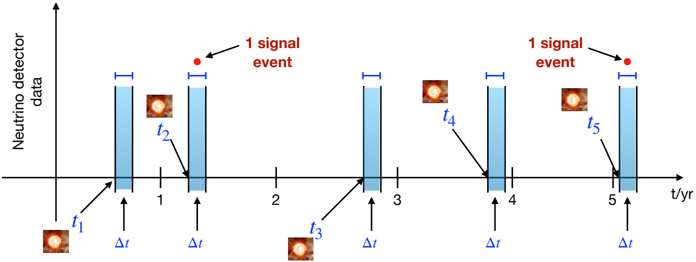
Accretion only model, Ac3G. Note sensitivity up to Mpc distance and beyond!



# *Multimessenger: memory-triggered neutrino searches*

# detecting neutrinos in *time coincidence* with memory

M. Mukhopadhyay, Z. Lin and CL, arXiv:2110.14657




- background-free SN neutrino sample from local universe!

# Probing supernovae in the local universe

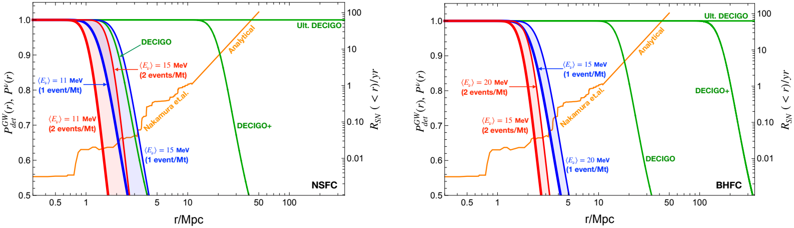


In the future, we will have SN neutrinos that are:

1. cosmological: the Diffuse Supernova Neutrino Background (DSNB)
2. galactic: supernova burst
3. *local*: memory-triggered $\nu$s from 0 - 100 Mpc
   - compare with galactic and cosmological: similarities? differences?
   - identify and study sub-populations of stars (e.g., successful vs. failed SNe)

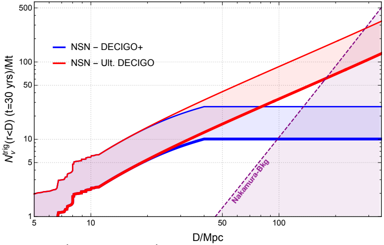
# Detection probabilities

Detection probabilities. DECIGO+ = DECIGO×10 (reduced noise).

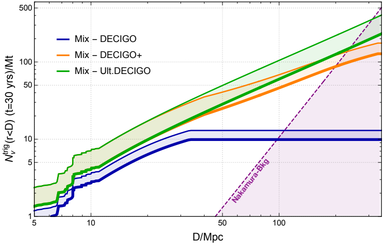


# Results

$N \sim 10 - 400$ neutrino events in 1 Mt water Cherenkov detector in 30 years




Baseline (conservative) memory scenario




Optimistic memory scenario (mix with failed SNe)

# *Summary and discussion*

# Summary and caveats

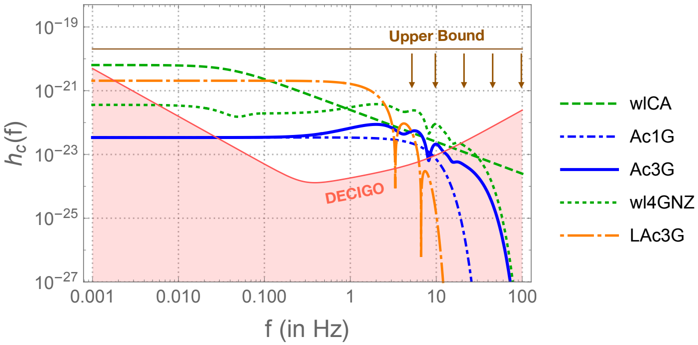


- the SN neutrino memory is detectable at (the most powerful) Deci-Hz interferometers
- A new phenomenological model is available
  - consistent with numerical simulations
  - fully analytical, useful for phenomenological studies, detector response studies, data fits, etc.
- Uncertainties:
  - $\mathcal{O}(10)$ uncertainty on $\alpha(t)$ (3D simulations result pessimistic)
  - anisotropy in cooling phase unknown
  - matter contribution to memory (sub-dominant at $f \lesssim 0.1$ Hz? )

# Physics potential

- Another General Relativity prediction will be confirmed
- A new *multimessenger* component:
  neutrinos + GW (100 Hz scale) + GW memory (0.1-10 Hz) + astro
  - potential for supernova alerts!
  - study SNe in the *local* Universe
  - test anisotropy $\rightarrow$ probe fluid dynamics in accretion phase
- memory + neutrinos: probe invisible cooling channels
  - sterile neutrinos, light scalars, invisible neutrino decay, etc.
- tests of gravity, room for theoretical developments
  - non-linear memory, quantum effects, etc.

# Thank you!


Upper Bound
DECIGO
$h_c(f)$
f (in Hz)
$10^{-19}$
$10^{-21}$
$10^{-23}$
$10^{-25}$
$10^{-27}$
0.001
0.010
0.100
1
10
100
wICA
Ac1G
Ac3G
wl4GNZ
LAc3G

*Backup*

# Alternate form of the analytical formulae



Using the approximation (accurate within 1%):

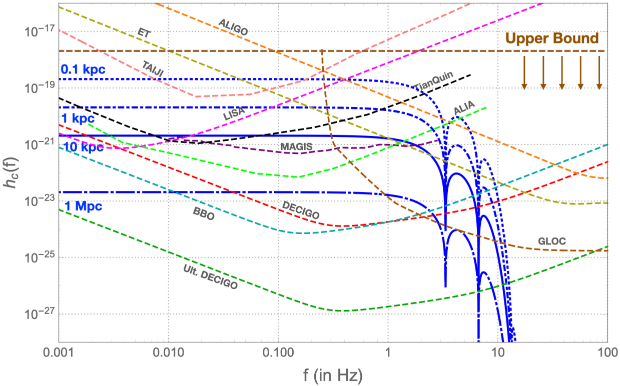
$$\mathrm{erf}\,(x) \simeq \tanh\,(mx), \;\; \text{with } m = \sqrt{\pi}\log(2)\,,$$

one can rewrite the results as:

$$h(t) = \sum_{j=1}^{N}\left[\left\{h_{1j}\left(\tanh\,(m\rho_j\;\tau_{1j}) + \tanh\,\left(m\rho_j(t-\tau_{1j})\right)\right)\right\} + \left\{h_{2j}\left(\tanh\,(m\rho_j\;\tau_{2j})\right.\right.\right.$$
$$\left.\left.\left. + \tanh\,\left(m\rho_j(t-\tau_{2j})\right)\right)\right\}\right] + h_3\left(\frac{\beta}{\chi}\,\left(1-\exp\,(-t\chi)\right) + \lambda t\right)\,,$$

$$\tilde{h}(f) = \sum_{j=1}^{N}\left[\left(h_{1j}\frac{i\pi}{m\rho_j}\,\mathrm{csch}\,\left(\frac{\pi^2 f}{m\rho_j}\right)\exp\,\left(i2\pi f\tau_{1j}\right)\right) + \left(h_{2j}\frac{i\pi}{m\rho_j}\,\mathrm{csch}\,\left(\frac{\pi^2 f}{m\rho_j}\right)\exp\,\left(i2\pi f\tau_{2j}\right)\right)\right]$$
$$+\left(\sqrt{2\pi}\;h_3\frac{\beta}{\chi}\,\left(\frac{1}{i2\pi f} - \frac{1}{-\chi + i2\pi f}\right)\right).$$

Longer accretion model, LAc3G.



# Detecting neutrinos from local SNe



signal-to-noise (SNR) ratio of GW detector:

$$\rho^2(r) = \int_{-\infty}^{\infty} d(\log f) \left( \frac{h_c(r, f)}{h_n(f)} \right)^2 .$$

The Poisson probability of observing $N \geq N_{min}$ $\nu$ events:

$$P^{\nu}(N_{min}, r) = \sum_{n=N_{min}}^{\infty} \frac{N^n(r)}{n!} e^{-N(r)} .$$

Number of neutrino events over detector lifetime, from SNe at distance $r < D$:

$$N_\nu^{trig}(D) = \Delta T \sum_{j, r_j < D} R_j N(r_j) P_{det}^{GW}(r_j)\,,$$

$R_j$= SN rate in galaxy $j$ ; $N(r_j)$= number of $\nu$ events from SN at distance $r_j$