Neutrino clusters and Neutrino stars

A. Yu. Smirnov

Max-Planck Institut fur Kernphysik, Heidelberg, Germany

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Consequences and implications

A.Y.S, and Xun-Jie Xu,
2201.00939 [hep-ph]
+ update + ...

History

Neutrino bound states and systems

M. Markov, Phys.Lett. 10, 122 (1964): Neutrino superstars

Massive neutrinos + gravity, analogy with neutron stars, $n \to \nu$

$$R = \sqrt{\frac{8\pi}{G_N}} \frac{1}{m_\nu^2}$$

For m_ν = MeV → $M = 10^6 M_{sun}$, $R = 10^{12}$ cm

$m_\nu = 0.05$ eV: $M = 4 \times 10^{20} M_{sun}$, $R = 5 \times 10^{26}$ cm

R. D.Viollier et al, Phys.Lett. B306, 79 (1993) ,….

Gravity, $m_\nu = (10 - 100)$ keV:

$M = (10^8 - 10^{10}) M_{sun}$, $R = (10^{14} - 10^{16})$ cm

- essentially, warm DM

Neutrino bound states and systems

G. J. Stephenson et al, Int. J. Mod. Phys. A13, 2765 (1998) ...

Long range scalar Yukawa forces with coupling y,
m_ν = 13 eV, motivated by ^3H exp. anomaly, negative m^2

$$G_N \rightarrow G_\nu = \frac{y^2}{4\pi m_\nu^2}$$

Hadrodynamics, Thomas - Fermi approximation ...

Equations for final configurations → Density profiles,
Formation of clouds in the Universe.

$$R = 4\sqrt{2}\,\pi \frac{1}{y\, m_\nu}$$

$M = (10^8 - 10^{10})\, M_{sun}$, $R = 10^{13}$ cm, central density: $10^{15}\ cm^{-3}$

Dark Matter Nuggets

M.B. Wise and Y. Zhang, Phys. Rev. D 90, 055030 (2014), JHEP 02, 023 (2015)

M.I. Gresham, H.K. Lou and K.M. Zurek, Phys. Rev. D96, 096012 (2017), Phys. Rev. D 98, 096001 (2018)

Dirac fermions with m_D ~ 100 GeV and coupling constant with scalar $\alpha_\phi = 0.01 - 0.1$

➡ Applications to asymmetric dark matter

Description is similar to that by Stephenson et al:

System of equations for scalar field and Fermi momentum of DM

Solved numerically in *M.I. Gresham*

Dependences of properties of nuggets on N and m_ϕ:

With increase of N radius N(R) first decreases, reaches minimum and then increases. The increase is due to relativistic regime

In relativistic case $R > 1/m_\phi$ is possible - "saturation" regime

Equations of motion and equations for stars

Scalar interaction. Equations of motion

$L = \;.... y \bar{\nu}\nu \phi - \frac{1}{2} m_\phi^2 \phi^2 - m_\nu \bar{\nu}\nu + \;...$

where ϕ – scalar with mass m_ϕ

m_ν - neutrino mass

y - effective coupling   pheno bound $y < 10^{-7}$

Equations of motion:

$$i\partial\!\!\!/ \nu - m^* \nu = 0 \qquad (*)$$

$$(\partial^2 + m_\phi^2)\phi + y\bar{\nu}\nu = 0 \qquad (**)$$

$m^* = m_\nu + y\phi$ - effective mass of neutrino in medium

V - potential

$m_\phi \ll m_\nu$ ϕ – classical field, ν - particles, quanta

in eq. (**) should take expectation value

$\bar{\nu}\nu \rightarrow \langle \bar{\nu}\nu \rangle$ Source of field

State of system

Interacting neutrino gas with density n and distribution over momentum $f(p, t, x)$

Using Eq. (*)

$$< \bar{\nu}\nu > = n^* = \frac{1}{2\pi^2}\int p^2 dp \frac{m^*}{E_p} f(p)$$

$$E_p = \sqrt{p^2 + m^{*2}} \qquad (m^* = m_\nu + y\phi)$$

In non-relativistic limit $p \ll m^*, m_\nu$ $n^* \to n$

In the relativistic case $p \gg m^*$ - chiral suppression $n^* \ll n$

The field (potential) is suppressed, attraction force is suppressed

Difference from gravity – no collapse

Ground state

Degenerate Fermi gas distribution of neutrinos over p

$$f(p) = \begin{cases} 1, & p < p_F \\ 0, & p > p_F \end{cases}$$

p_F - Fermi momentum

$$n = \frac{p_F^3}{6\pi^2}$$

$$P_{deg} = \frac{p_F^5}{30\pi^2 m_\nu}$$

Equilibrium condition

Statistical system

Attraction by Yukawa forces ↔ Pressure of neutrino gas

Equilibrium

Global characteristics can be obtained considering uniform sphere

$$P_{deg} = - P_{Yuk}$$

In non-relativistic case: Hydrostatic equilibrium

$$F_{deg}(r) = - F_{yuk}(r) \qquad F_{deg}(r) = dP_{deg}(r)/\,dr$$

General: chemical equilibrium

$$d\mu\,/dr = 0 \qquad dE_F\,/dr = 0 \quad \text{(degenerate gas)}$$

Final stable configuration

Non-relativistic case

In non-relativistic case: Hydrostatic equilibrium

$$F_{deg}(r) = - F_{yuk}(r)$$

$$F_{deg}(r) = \frac{(6\pi^2)^{2/3}}{5m_\nu}\, n(r)^{5/3} \qquad F_{yuk}(r) = - \frac{y^2}{4\pi\, r^2}\, n(r)\, N(r)$$

Reduced to the Lane-Emden equation (after differentiation over r)

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\,\frac{dn^{2/3}}{dr}\right) = - \kappa y^2\, n$$

$$\kappa = \frac{2\, m_\nu}{(6\pi^2)^{2/3}}$$

$\gamma = 3/2$ - solution with finite radius

Boundary condition in the center:

$n(0) = n_0$ or $p_F(0) = p_{F0}$

Global characteristics

R N p_{F0} $m_\phi = 0$

Radius $$R = \frac{20}{y\sqrt{m_\nu p_{F0}}}$$

At the transition between the non-relativistic and relativistic cases

$p_{F0} \sim m_\nu$ $$R \sim \frac{20}{y\, m_\nu} \sim R_{min}$$ (as in the gravitational case)

$$R = \frac{90.4}{y^2\, m_\nu} \frac{1}{N^{1/3}}$$ radius decreases with increase of N

Fermi momentum and density in the center

$$p_{F0} = 0.0485\, m_\nu\, y^2 N^{2/3}$$

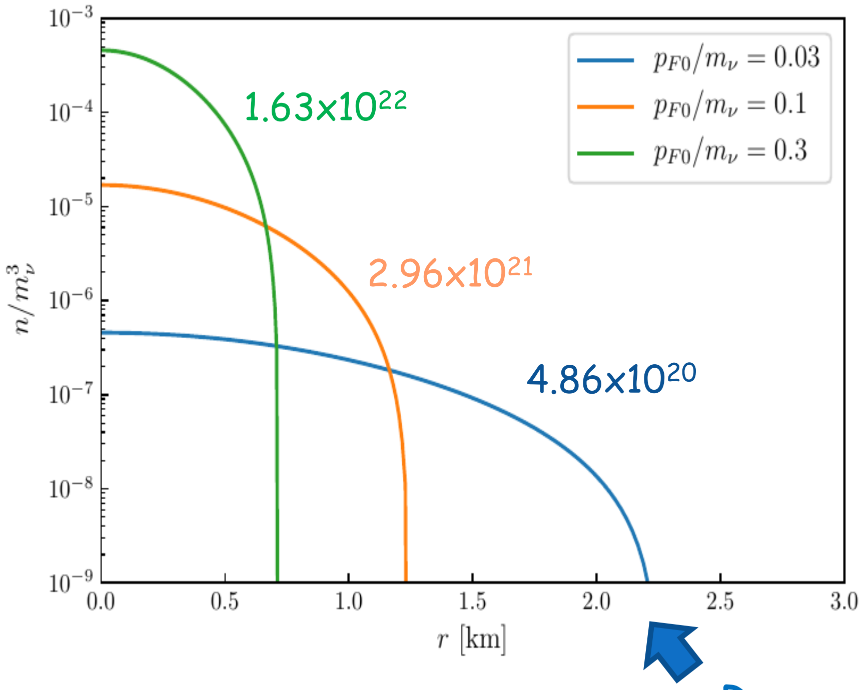
$n_{\nu 0} = 2\ 10^{-6}\, m_\nu^{\ 3} y^6 N^2$ → fast increase with y and m_ν

Number density distributions

Non-relativistic case

Density distributions in the clusters for different values of p_{F0}/m_ν - non-relativistic parameter

$y = 10^{-7}$, $m_\nu = 0.1$ eV, $m_\phi = 0$



p_{F0}/m_ν - gives the central density

Shown are the corresponding total numbers of neutrinos N

with increase of N, radius decreases and central density increases fast

in agreement with analytic results

Dependence on coupling – scaling:

$N \sim 1/y^3$ $R \sim 1/y$

Non-degenerate case

$$f(p) = \frac{1}{\exp(p/T) + 1}$$

$$n = \frac{I_3}{2\pi^2} T^3$$

Results from degenerate case (upto numerical coefficients) by substitution

$$p_{F0} \rightarrow T$$

Radius is bigger:

$$R_T = 1.9\, R_{deg}$$

Relativistic clusters . Equilibrium

With increase of N the Fermi momentum increases as $p_{F0} \sim N^{2/3}$

Transition to relativistic case at $p_{F0} \sim m_\nu$

Correct criteria of relativistic case: $p_{F0} > m^*$

Chemical equilibrium condition

$$d\mu/dr = dE_F/dr = d(p_F^2 + m^{*2})^{1/2}/dr = 0$$

$$\Rightarrow \quad m^* \frac{dm^*}{dr} = - p_F \frac{dp_F}{dr}$$

$F_{yuk}(r)$ $\qquad$ $F_{deg}(r)$

- generalization of the Hydrostatic equilibrium.

Indeed $\quad F_{yuk}(r) = dV/dr = y\, d\phi/dr = \dfrac{dm^*}{dr}$

Relativistic equations

Static case

$$(\nabla^2 - m_\phi^2)\, m^* = y\, n^*$$

$$m^* \frac{dm^*}{dr} = -\, p_F \frac{dp_F}{dr}$$

(*)

$$n^* = \frac{1}{2\pi^2} \int_0^{p_F} \frac{m^*}{\sqrt{p^2 + m^{*2}}}\, p^2 dp$$

Equations for m^* (instead of ϕ) and p_F (neutrino density)

Boundary conditions:

$p_F(0) = p_{F0}$ - External (given) parameter

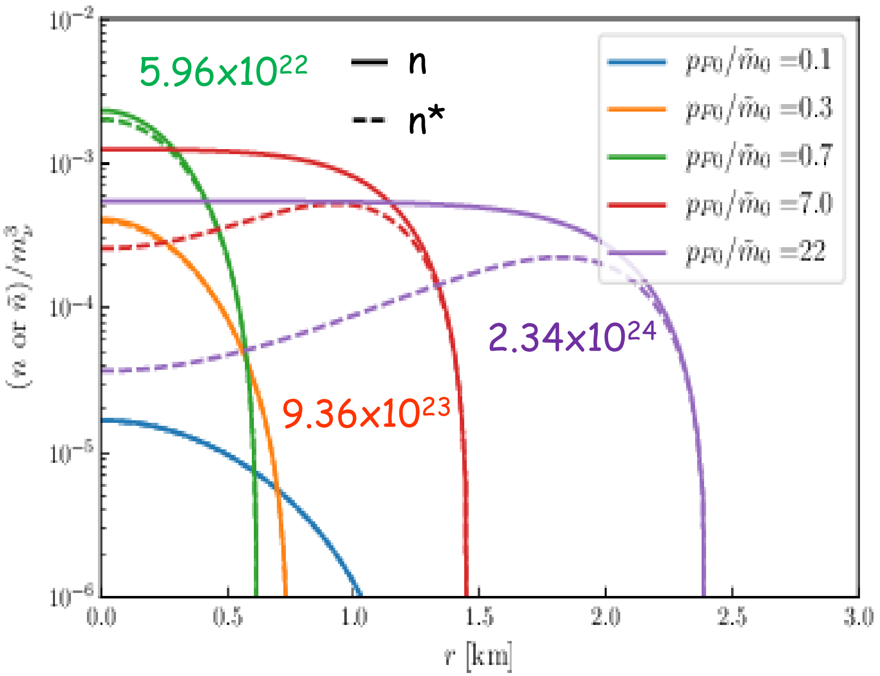
$m^*(0) = m^*_0$ m^*_0 is tuned in such a way that at $r \to$ infty $m^* \to m_\nu$

In non-relativistic case (*) is reduced to the Lane-Emden equation

Relativistic case

Density and effective density distributions in the stars for different Values of p_{F0}/m_ν (corresponding values of N indicated)

$y = 10^{-7}$, $m_\nu = 0.1$ eV, $m_\phi = 0$



Below N = $6\ 10^{22}$ with increase of N
- R decreases
- n_0 increases

Above N = $6\ 10^{22}$ with increase of N
- R increases
- n_0 decreases

$n^*/n \sim \langle m^*/E \rangle \sim \langle m^*/p_{F0} \rangle$

Characteristics of neutrino clusters

for different values of N

$y = 10^{-7}$, $m_\nu = 0.1$ eV, $m_\phi = 0$

N	2.96×10^{21}	1.63×10^{22}	5.96×10^{22}	9.36×10^{23}	2.34×10^{24}
m_ν^*/m_ν	0.991	0.922	0.688	0.060	0.014
R, km	1.25	0.75	0.62	1.46	2.41
n^0, cm^{-3}	2.0×10^6	4.9×10^7	3.7×10^8	1.5×10^8	6.1×10^7

non-relativistic ⟷ ultra relativistic

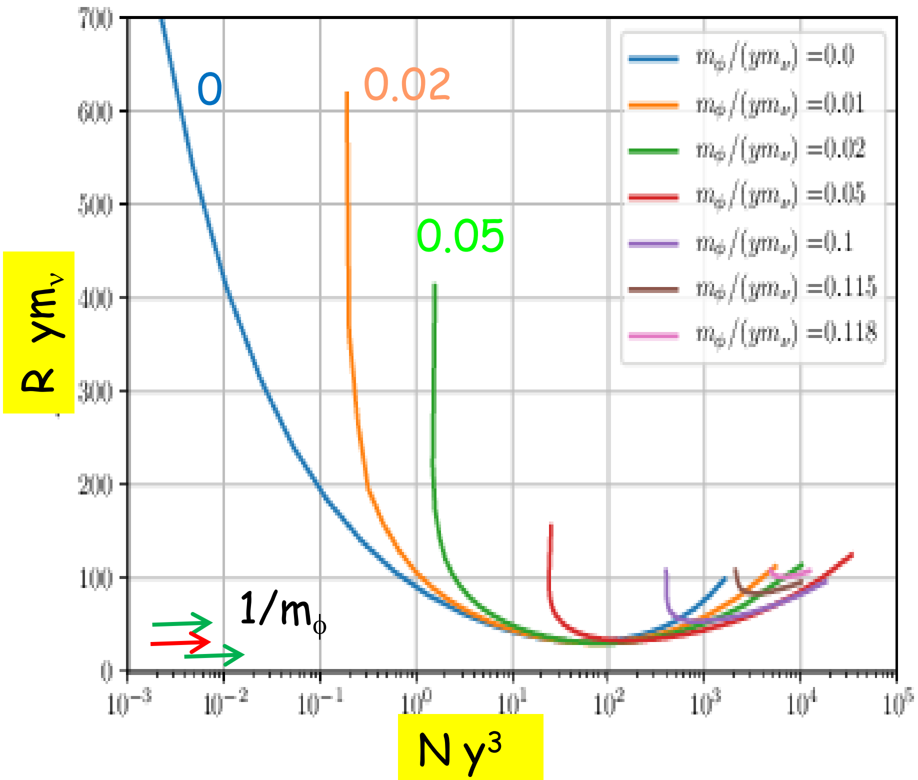
$m_\nu^* = m_\nu + V$ - the effective neutrino mass in medium,

n^0 - central density

Dependence on coupling – scaling: $N \sim 1/y^3$ $R \sim 1/y$

For $y = 10^{-14}$ and $n_0 = 4\ 10^8$ cm^{-3}: $R = 2.4\ 10^{12}$ cm, $N = 2.3\ 10^{45}$,
$M = 3.3\ 10^{12}$ g

Dependence on mass of a mediator



Radius as function of number of neutrinos for different m_ϕ/m_ν

Radius of interaction $r_\phi = 1/m_\phi$

★ Lover bound on N, for non-zero m_ϕ which increases with m_ϕ

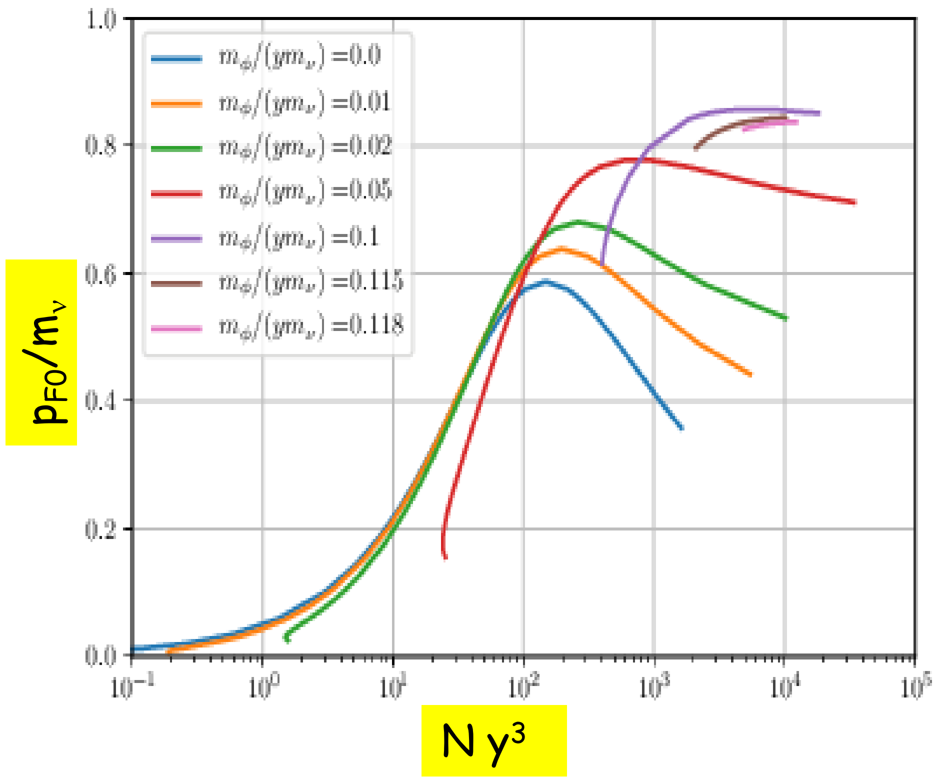
★ Minimal radius: increases with m_ϕ and shifts to larger N

With increase of m_ϕ

in non- relativistic range binding effect becomes weaker, R increases

in relativistic range - R decreases as a result of shift of minimum

Properties of neutrino clusters



Fermi momentum in center as function of number of neutrinos for different values of m_ϕ/m_ν

With increase of m_ϕ

Maximum of p_{F0} increases and shifts to larger N

Absolute maximum of p_{F0} and therefore central density determined by value of neutrino mass

$$n_\nu^{max} = 4\ 10^8\ \text{cm}^{-3} \left(\frac{m_\nu}{0.1\ \text{eV}}\right)$$

Strength of interaction. Bounds

$$S_\phi = \frac{y^2 m_\nu^2}{m_\phi^2}$$

Numerically, stable solution exist for

$S_\phi^{-1/2} < 0.12$ ⇒ $S_\phi > 70$

Stronger bound follows from condition of formation of clusters

Radius of cluster R can be smaller, comparable or bigger than radius of interactions $R_\phi = 1/m_\phi$

$$\frac{R}{R_\phi} = Y S_\phi^{-1/2}$$

Y vertical value in Fig for R

For $S_\phi^{-1/2} = 0.1$, $Y = (55 - 100)$, $R/R_\phi = (5 - 10)$ (relativistic case)

Formation of clusters and applications

Maximal density and fragmentation

Absolute maximal central density of stable configuration
$n_{\nu 0}^{max}$ depends on neutrino mass only

General picture:

Expansion of uniform cosmic neutrino background
→ decrease of its density and temperature

When $n_\nu(T) \sim n_{\nu 0}^{max}$ fragmentation may start

After separation the pre clusters may slightly shrink
and their central density – increase. Therefore

Fragmentation density

$n_{frag} < n_{\nu 0}^{max} \sim 10^9 \text{ cm}^{-3}$ $(m_\nu = 0.1 \text{ eV})$

This density is realized in the epoch

$$z_f + 1 = \left(\frac{n_{frag}}{n_\nu(0)} \right)^{1/3} \sim 200$$

Corresponds to $p_\nu \sim 0.1$ eV

Fragmentation may start when $p_\nu \sim m_\nu$

Evolution of the effective mass

For uniform and configuration the eq. of motion gives

$$\phi = -\frac{y \langle \bar{\nu}\nu \rangle}{m_\phi^2} = -\frac{y\, n^*}{m_\phi^2}$$

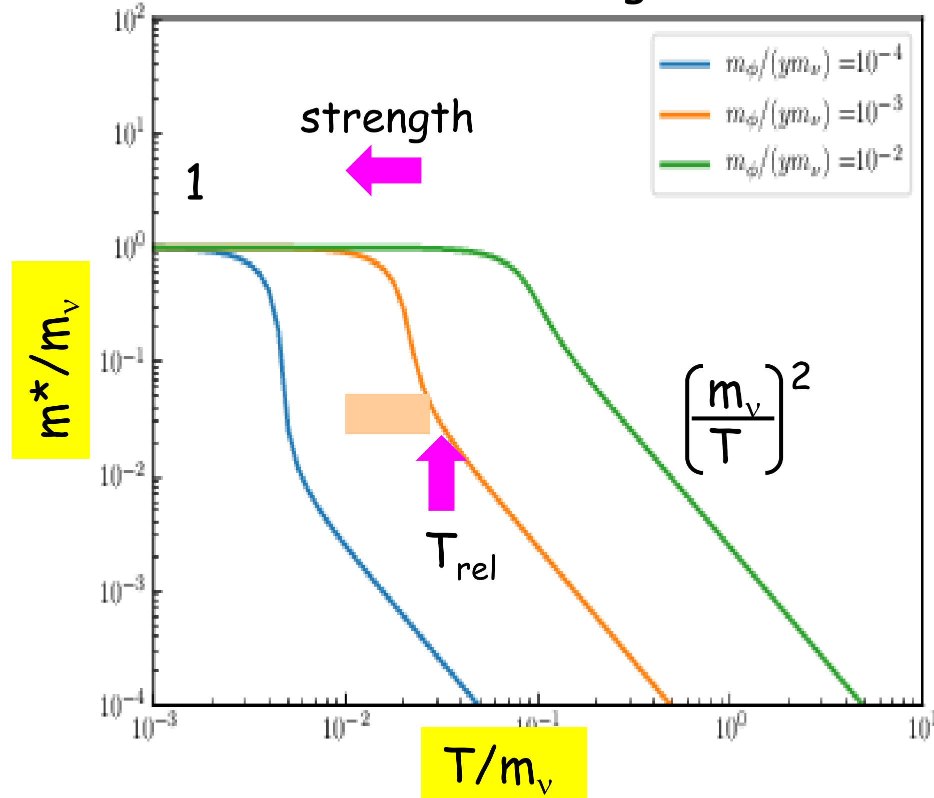
$$m^* = m_\nu + y\phi = m_\nu - \frac{y^2\, n^*(m^*)}{m_\phi^2}$$

For thermal distribution

$$m^* = m_\nu - m^* \frac{y^2\, T^2}{24\, m_\phi^2}$$

transition region

For different strength



non-relativistic

$m^* \to m_\nu$

relativistic

$m^* \to 0$

$T_{rel}/m_\nu = (24/S_\phi)^{1/3}$

Energy of the system per neutrino

$\varepsilon_\alpha = \rho_\alpha / n_\nu$ α = tot, ν , ϕ

ρ_α - energy density in α component

in the uniform static medium (all derivatives are zero)

Scalar field

$$\rho_\phi = \tfrac{1}{2} m_\phi^2 \phi^2 = (m^* - m_\nu)^2 \frac{m_\phi^2}{2 y^2}$$

$$\varepsilon_\phi = m_\nu \begin{cases} 1/\chi & \text{relativistic limit} \\ \chi/4 & \text{non-relativistic limit} \end{cases}$$

$$\chi = \frac{S_\nu I_3}{\pi^2} \left(\frac{T}{m_\nu} \right)^3$$

$I_3 = 1.80$

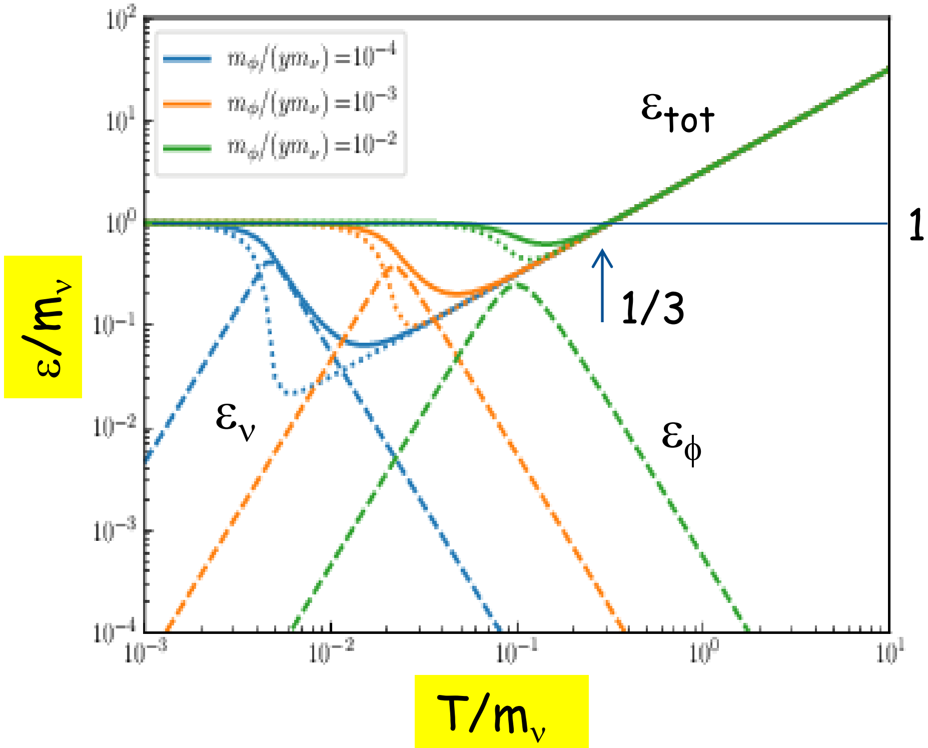
Neutrinos

$$\varepsilon_\nu = \langle E_\nu \rangle = \langle \sqrt{p^2 + m^{*2}} \rangle$$

$$\varepsilon_\nu = \begin{cases} 3.15T & \text{relativistic limit} \\ m_\nu & \text{non-relativistic limit} \end{cases}$$

Total energy $\varepsilon_{tot} = \varepsilon_\nu + \varepsilon_\phi$

Evolution of energies. Dip



Dependence of energy per neutrino on T/m_ν for different values of m_ϕ/ym_ν
ε_ϕ - dashed, ε_ν – dotted, ε_{tot} - solid

For large enough strength

$$S_\phi > S_\phi^{min} \sim 600$$

the dip develops in $\varepsilon^{tot}(T)$ dependence with

$$\varepsilon_{tot} < m_\nu \qquad (*)$$

at $T \sim m_\nu/3$, when neutrinos become non-relativistic (correspond to transition region in $m^*(T)$ dependence)

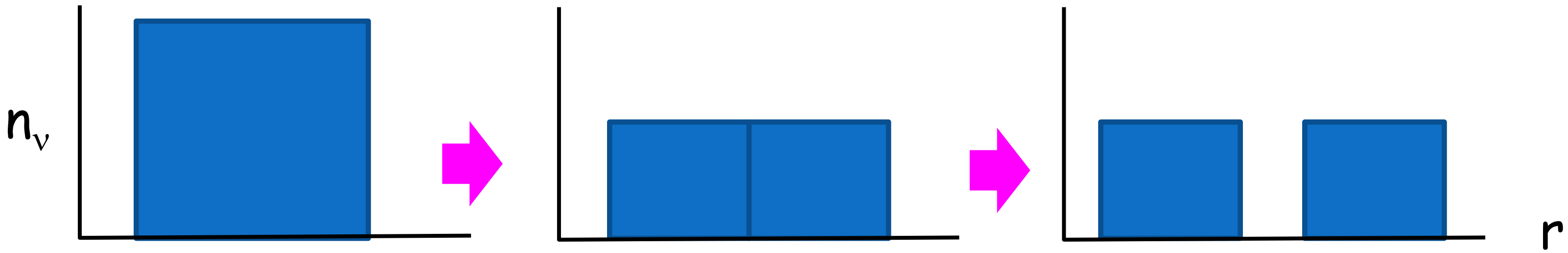
(*) implies existence of bound state with $m_\nu - \varepsilon_{tot}$ being the binding energy

With increase of strength the minimum of dip sifts to lower

Instability and Fragmentation

T_{min} - temperature of minimum of the dip

Below T_{min} further expansion and cooling would require increase of energy of the system → fragmentation without further decrease of T and density is energetically more profitable:



$T_U > T_{min}$ $T_U = T_{min}$ $T_U < T_{min}$

T_U - temperature in the Universe

Fragmentation stars at $z_f \sim 200$

The size of the Universe that epoch:	$D_U(200) = 20$ Mpc
The radius of the biggest structures:	$R_f \sim D_U(200)/4 = 5$ Mpc
Distance between structures:	$d_f(200) \sim D_U(200)/2 = 10$ Mpc
Present distance (size of voids)	$d(0) \sim z_f d_f = 2000$ Mpc

$N_f = 1.2\ 10^{85}$, $M_f = 4\ 10^{17}\ M_{sun}$

Fragmentation and final configurations

Bounds on possible structures

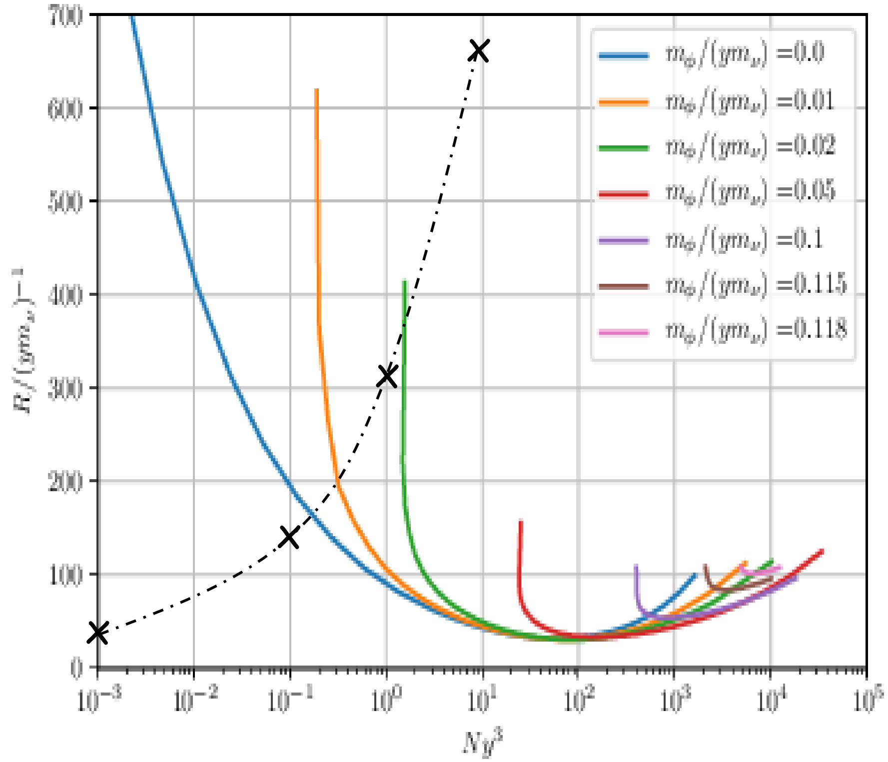
Relation $N_f = \frac{4\pi}{3} R_f^3 \, n_f$

can be rewritten multiplying by y^3 as

$$Y = m_\nu \left(\frac{3}{4\pi n_f}\right)^{1/3} X^3$$

where X and Y are coordinates of Fig

$X = N y^3 \qquad Y = R y m_\nu$



Eq. (*) determines a line (dashed) in the Fig. Which is determined by m_ν and n_f only

$$y = \frac{1}{R_f}\left(\frac{3X_g}{4\pi n_f}\right) \qquad m_\phi = y m_\nu S_\phi^{-1/2}$$

Its crossing with line of a given S_ϕ at X_g gives depending on R_f

For $S_\phi^{-1/2} = 0.01$ and $R_f = 10$ kpc
$y = 1.4\ 10^{-22}$, $m_\phi = 1.4\ 10^{-26}$ eV

Observational consequences

Further disintegrations : other perturbations, DM halos, gravity,

Ratio of distances between clusters d and radiuses of clusters

$$d/R = 10^{-2} d_0 m_\nu y^2 N^{2/3}$$

d_0 - distance between neutrinos without clustering

d/R ~ 100 does not depend on y for stable configuration

Affects detection of relic neutrinos depending on sizes of stars

Summary

If neutrinos interact with light or massless scalar boson with $y < 10^{-7}$ and $m_{\phi} < 10^{-10}$ eV, formation of stable bound systems of neutrinos become possible

Final stable configurations: degenerate (close to degenerate) Fermi gas with the following features

Due to chiral suppression of attraction, existence of relativistic regime in which dependence of characteristics is opposite to the non-relativistic case. Absence of collapse.

- Existence of minimal radius determined by, $1/y\, m_{\nu}$
- in relativistic regime R can be bigger than radius of interactions $R_{\phi} = 1/m_{\phi}$
- the lower bound on N, for a given strength
- upper bound on central density

Extremes in the transition region

Formation: via development of instabilities and fragmentation of the uniform relic neutrino background at $z < 200$

Affects programs of detection of relic neutrinos

Backup

Equation for m*

static case **

$$(\nabla^2 - m_\phi^2)(m^* - m_\nu) = y\, n^*(m^*)$$

$$n^* = \frac{1}{2\pi^2}\int_0^{p_F} \frac{m^*}{\sqrt{p^2 + m^{*2}}}\, p^2 dp$$

Equations for m* (instead of ϕ)

G. J. Stephenson, et al

Boundary conditions:

$m^*(0) = m^*_0$

$r \to$ infty $m^* \to m_\nu$

Similar to gravity in non-relativistic limit, but in relativistic case
- chiral suppression of attraction with various consequences:
absence of collapce, increase of size with N, etc.

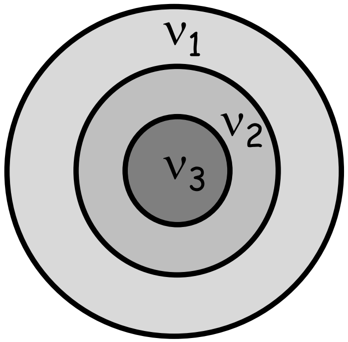
Properties [R, n(r) ..] depend on y, m_ϕ, m_ν

They (dependences) differ in non-relativistic and relativistic cases

Characteristics of nu clusters

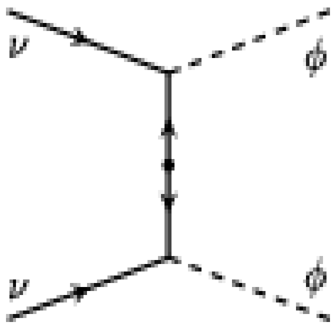
For fixed N

$$R \sim \frac{1}{y^2 m_\nu}$$

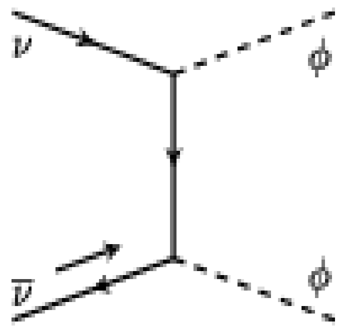


Radiation, cooling

$\nu\nu$ annihilation
(only for Majorana ν)



$\nu\overline{\nu}$ annihilation
(for Dirac/Majorana ν)



Formation of neutrino stars

From the cosmological neutrino background

For $y < 10^{-7}$ cooling mechanisms: ϕ –emission (bremstrachlung), annihilation $\nu\nu \rightarrow \phi\phi$, are negligible

Formation of ν-stars in analogy to formation of DM halos?

In terms of effective neutrino mass $m^* = m_\nu + y\phi$

At early epoch (large n) $m^* \ll m_\nu$

G. J. Stephenson ,et al.

With decrease of density $m^* \rightarrow m_\nu$ due to decrease of kinetic energy $\rightarrow$ formation of degenerate neutrino gas

Phase transition

In terns of maximal density

Neutrino clusters and detection of relic neutrinos