OBSERVABLE LEPTOGENESIS

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What's new ?

- Precise analytic understanding of numerical scan for successful baryogenesis via heavy neutral lepton (HNL) oscillations in the minimal seesaw (2 HNL)

- Inclusion of LNV rates, suppressed by $(M/T)^2$

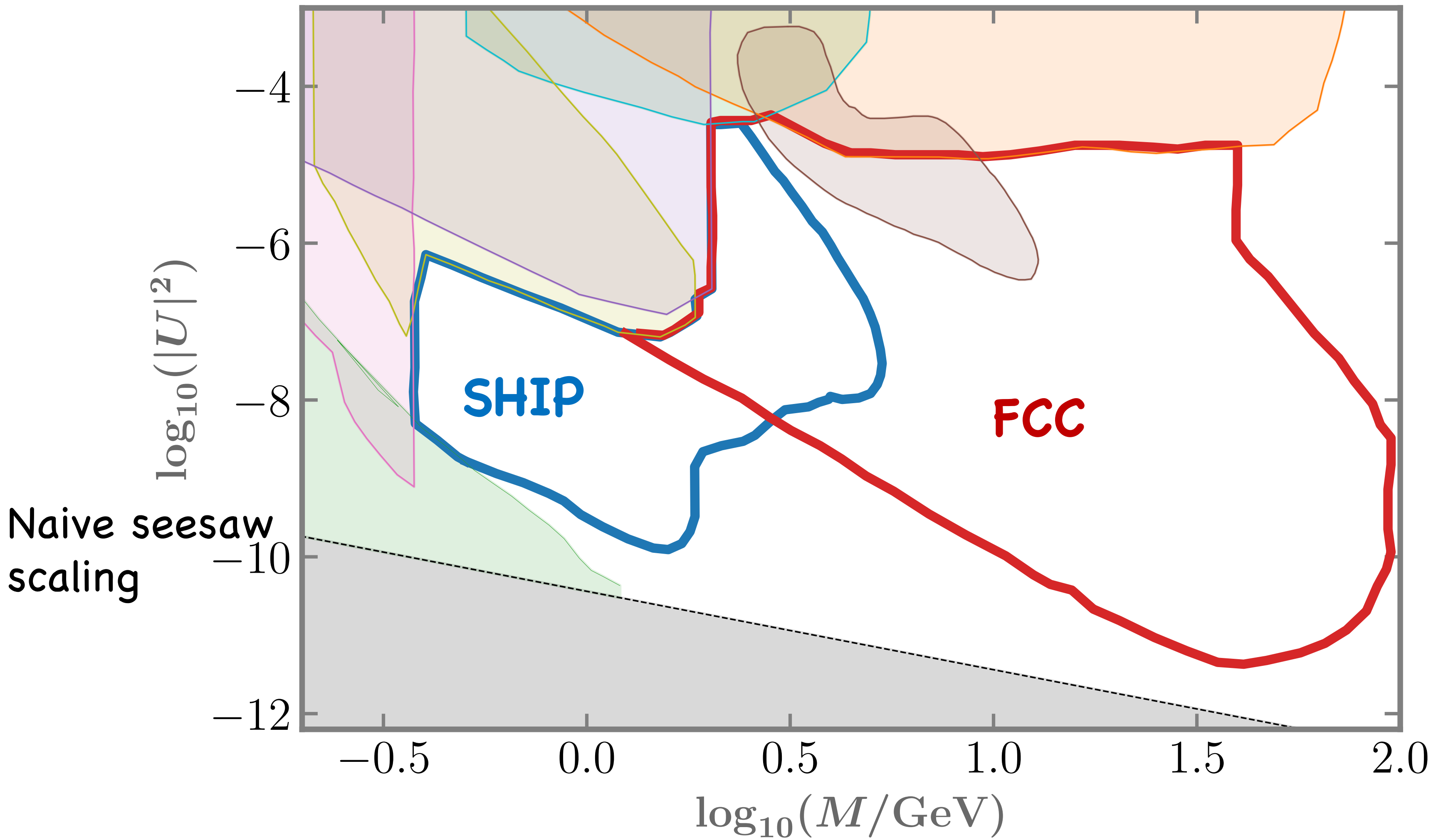
- Focus on parameter region testable at SHiP, FCC-ee and correlations with other observables

Minimal type I seesaw model with 2 HNL

$$\mathcal{L} = \mathcal{L}_{\rm SM} + i\overline{N}_i\gamma^\mu\partial_\mu N_i - \left(Y_{\alpha i}\overline{L}_\alpha N_i\Phi + \frac{M_i}{2}\overline{N}_i^c N_i + h.c.\right)$$

- $m_\nu = v^2\, Y\, M^{-1}\, Y^T$, $v = \langle\Phi\rangle$
- Low scale: $M \in [0.1 - 100]$ GeV
- Naive seesaw scaling of active-HNL mixing:

$$U = v\, Y/M = O(\sqrt{m_\nu/M})$$



SHIP
FCC
Naive seesaw
scaling
−4
−6
−8
−10
−12
−0.5
0.0
0.5
1.0
1.5
2.0
$\log_{10}(|U|^2)$
$\log_{10}(M/\mathrm{GeV})$

Approximately conserved lepton number limit

$$M = \begin{pmatrix} \mu_1 & \Lambda \\ \Lambda & \mu_2 \end{pmatrix}. \quad Y = \begin{pmatrix} y_e e^{i\beta_e} & y'_e e^{i\beta'_e} \\ y_\mu e^{i\beta_\mu} & y'_\mu e^{i\beta'_\mu} \\ y_\tau e^{i\beta_\tau} & y'_\tau e^{i\beta'_\tau} \end{pmatrix},$$

- $y'_\alpha \ll y_\alpha$, $\mu_i \ll \Lambda$

- Only three independent phases, $\mu_1 = \mu_2 \equiv \mu$ can be chosen real

- $\Lambda = (M_1 + M_2)/2 = M$, $\mu = (M_2 - M_1)/2 = \Delta M/2$

Leptogenesis via HNL oscillations

Akhmedov, Rubakov, Smirnov 1998, Asaka,Shaposhnikov 2005, etc

Sakharov conditions for baryogenesis:

- New CP violating phases in Y, M
- B violated by sphaleron processes at $T > T_{EW}$
- At least one sterile neutrino does **NOT** equilibrate by T_{EW} , i.e. for some rate

$$\Gamma_i\,(T_{EW}) \leq H_u(T_{EW}) = T_{EW}^2\,/M_P^*$$

Fulfilled for M = O(GeV), Y ~ 10^{-6} – 10^{-7} , in the correct range to explain neutrino masses !

Density matrix formalism

Raffelt & Sigl, 1993

$$\dot{\rho} = -i[H,\rho] - \frac{1}{2}\{\Gamma^a,\rho\} + \frac{1}{2}\{\Gamma^p,\rho_{eq}-\rho\}$$

- Hamiltonian term : $H = \frac{M^2}{2k^0} + \frac{T^2}{8k^0}Y^\dagger Y$
- Annihilation and production rates of the N's: Γ^a, Γ^p
- For antineutrinos: $\bar{\rho}$, H ⟹ H*
- Diagonal density matrix for SM leptons, which are in equilibrium, with chemical potential

$$f_\alpha(k^0) = \frac{1}{e^{(k^0-\mu_\alpha)/T}+1}$$

- For antileptons $\mu_\alpha \Longrightarrow -\mu_\alpha$
- Symplifying approximations:
  - momentum averaged equations
  - linearized in μ_α

Time scales and slow modes

$$\dot{\rho} = -i[H,\rho] - \frac{1}{2}\{\Gamma^a,\rho\} + \frac{1}{2}\{\Gamma^p,\rho_{eq}-\rho\}$$

- Annihilation and production rates of the N's: at T >> M,

Ghiglieri, Laine, 2017

$$\Gamma(T) \propto \mathrm{Tr}[YY^\dagger]\,T$$

- Flavoured rates: $\Gamma_\alpha(T) \propto \epsilon_\alpha \Gamma(T)$ $\qquad \epsilon_\alpha \equiv \frac{(YY^\dagger)_{\alpha\alpha}}{\mathrm{Tr}[YY^\dagger]}$

- Oscillation rate: $\Gamma_{osc}(T) \propto \frac{\Delta M^2}{T}$

Time scales and regimes

- Asymmetry generated mostly at T_{osc}, defined as:

$$\Gamma_{osc}(T_{osc}) = H_u(T_{osc})$$

- Fast oscillations: $\Gamma_{osc}(T) \gg \Gamma(T)$ at $T = T_{osc}$

- Intermediate regime: $\Gamma_{osc}(T) \ll \Gamma(T)$ at $T = T_{osc}$, but
  $\Gamma_{osc}(T) > \Gamma(T)$ at $T = T_{EW}$

- Overdamped regime: $\Gamma_{osc}(T) \ll \Gamma(T)$ at $T = T_{osc}, T_{EW}$

- **Slow modes at T_{EW} :**

- Weak washout: $\Gamma_\alpha(T_{EW}) < \Gamma(T_{EW}) < H(T_{EW})$

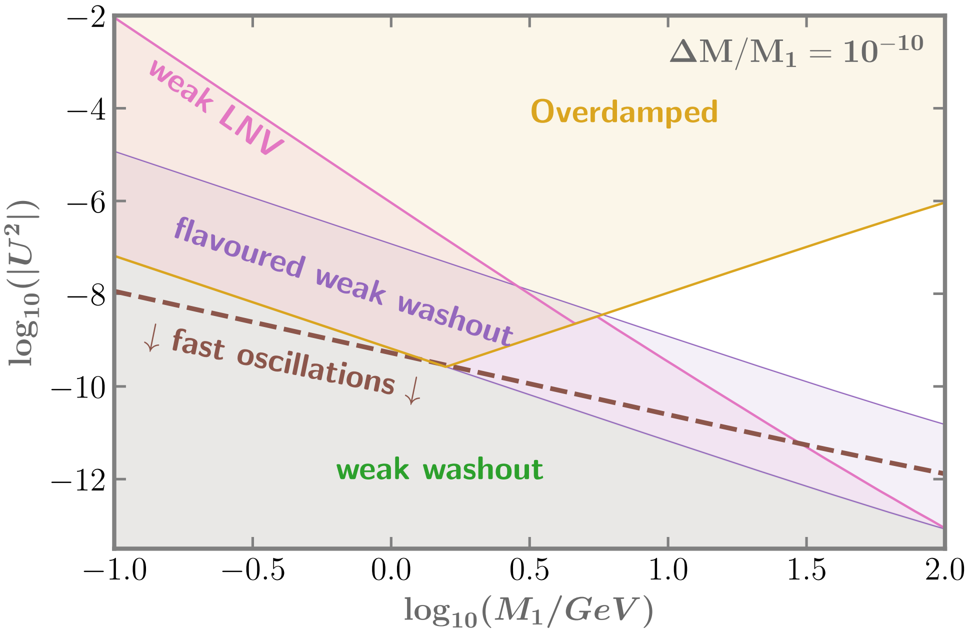
- Flavoured weak washout: $\Gamma_\alpha(T_{EW}) < H(T_{EW}) < \Gamma(T_{EW})$

- Overdamped regime: when $\epsilon \propto \dfrac{\Delta M^2/T}{\Gamma(T)} \ll 1$ at $T \geq T_{EW}$,

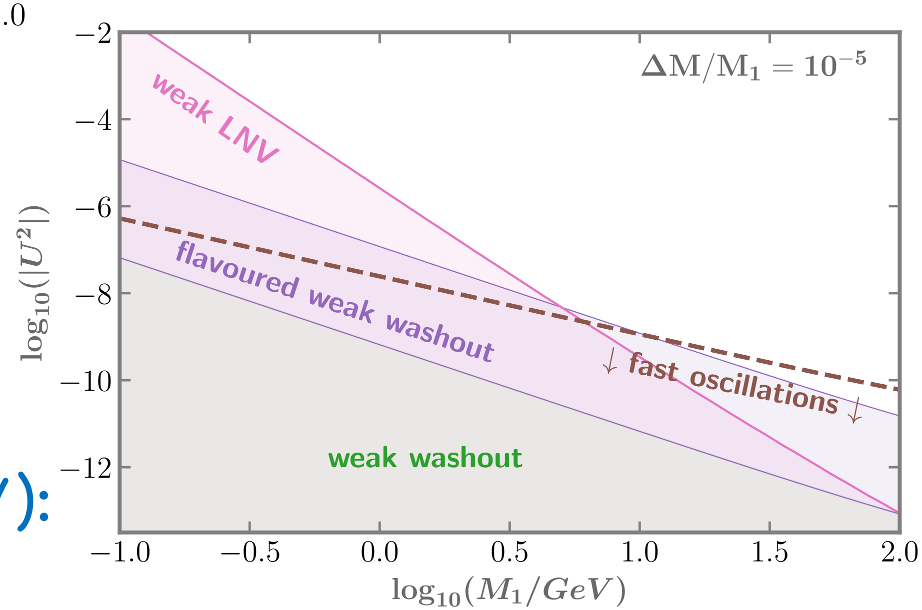
$$\Gamma_{ov}(T_{EW}) \propto [\epsilon(T_{EW})]^2\, \Gamma(T_{EW}) < H(T_{EW})$$

- Weak LNV regime: $\Gamma_M(T_{EW}) \propto (M/T_{EW})^2\, \Gamma(T_{EW}) < H(T_{EW})$

- Fast oscillations: $\Gamma_{osc}(T) >> \Gamma(T)$ at $T = T_{osc}$



Asymmetry exponentially washed out in white regions



- Overdamped:
  lower bound on U^2

- Weak washout (flavoured, LNV):
  upper bound on U^2

CP violating flavour basis invariants

- All CP violating observables must be proportional to a combination of CP weak basis invariants
- Change of weak basis: Hernández et al., 2015

$$Y \to V^\dagger Y W, \qquad Y_\ell \to V^\dagger Y_\ell U, \qquad M_R \to W^T Y W$$

- Define Hermitian matrices:

$$h = Y^\dagger Y, \bar{h} = Y^\dagger Y_\ell Y_\ell^\dagger Y, H_M = M_R^\dagger M_R \to W^\dagger (h, \bar{h}, H_M) W$$

- LNC CP invariants: independent of HNL Majorana character

$$I_0 = \mathrm{Im}\left(\mathrm{Tr}\left[h\, H_M \bar{h}\,\right]\right)$$

- In the basis where M_R, Y_l are diagonal:

$$I_0 = \sum_\alpha y_{\ell_\alpha}^2 \sum_{i<j} (M_j^2 - M_i^2)\, \mathrm{Im}\left[Y_{\alpha j}^* Y_{\alpha i} \left(Y^\dagger Y\right)_{ij}\right] \equiv \sum_\alpha y_{\ell_\alpha}^2 \Delta_\alpha$$

$$\sum_\alpha \Delta_\alpha = 0$$

- Weak washout regime:

$$\Delta^{w}_{LNC} \propto \sum_{\alpha} \Gamma_\alpha \sum_{i<j} g(M_i, M_j) \mathrm{Im}\left[Y^*_{\alpha j} Y_{\alpha i} \left(Y^\dagger Y\right)_{ij}\right]$$

- Flavoured weak washout: weakly coupled flavour α at T_{EW}

$$\Delta^{\alpha}_{LNC} \propto \sum_{i<j} f(M_i, M_j) \mathrm{Im}\left[Y^*_{\alpha j} Y_{\alpha i} \left(Y^\dagger Y\right)_{ij}\right]$$

$g(M_i, M_j), f(M_i, M_j)$: antisymmetric functions of M_i,M_j

- Overdamped regime (new): oscillations cutoff by Γ_α

$$\Delta^{\mathrm{ov}}_{\mathrm{LNC}} \propto \sum_{\alpha} \frac{\Delta_\alpha}{\Gamma_\alpha}$$

- Additional normalization factors and mass functions

$$g(M_i, M_j), f(M_i, M_j)$$

determined so as to match the analytical solutions

- LNV CP invariants: sensitive to Majorana character of HNLs

Branco et al., 2001

$$I_1 = \mathrm{Im}\,\{\mathrm{Tr}\,[h\, H_M M^* h^* M\,]\}$$

$$I_1 = \sum_\alpha \sum_{i<j} \left(M_j^2 - M_i^2\right) M_i M_j \mathrm{Im}\left[Y_{\alpha j} Y^*_{\alpha i} \left(Y^\dagger Y\right)_{ij}\right] \equiv \sum_\alpha \Delta_\alpha^M$$

- Overdamped regime: $\Delta_{\mathrm{LNV}}^{\mathrm{ov}} = \frac{1}{[\mathrm{Tr}\,(Y^\dagger Y)]^2} \sum_\alpha \Delta_\alpha^M$
- Flavoured weak washout regime: $\Delta_{\mathrm{LNV}}^{\mathrm{int}\,(\alpha)} = \frac{\Delta_\alpha^M}{[\mathrm{Tr}\,(Y^\dagger Y)]^2}$
- Weak LNV regime:

$$\Delta_{\mathrm{LNV}}^{osc} = \frac{1}{\mathrm{Tr}\,(Y^\dagger Y)} \sum_\alpha \sum_{i<j} \mathrm{Im}\left[Y_{\alpha j} Y^*_{\alpha i} \left(Y^\dagger Y\right)_{ij}\right] g_M(M_i, M_j)$$

$g_M(M_i, M_j)$ antisymmetric

CP invariants in terms of neutrino masses and U_{PMNS}

$$-(m_\nu)_{\alpha\beta} = \frac{v^2}{\Lambda}\left(Y_{\alpha 1}Y_{\beta 2} + Y_{\alpha 2}Y_{\beta 1} - Y_{\alpha 1}Y_{\beta 1}\frac{\mu_2}{\Lambda}\right) = \left(U^* m U^\dagger\right)_{\alpha\beta}$$

- $Y_{\beta 2}$ and μ_2 violate LN
- Parametrization equivalent to Casas-Ibarra in the symmetry protected limit ($y'/y \approx e^{-2\mathrm{Im}[z]}$, θ=2Re[z]) Gavela et al. 2009
- $$Y_{\alpha 1} = \frac{e^{-i\theta/2} y}{\sqrt{2}}\left(U^*_{\alpha 3}\sqrt{1+\rho} + U^*_{\alpha 2}\sqrt{1-\rho}\right)$$

  $$Y_{\alpha 2} = \frac{e^{i\theta/2} y'}{\sqrt{2}}\left(U^*_{\alpha 3}\sqrt{1+\rho} - U^*_{\alpha 2}\sqrt{1-\rho}\right) + \frac{\Delta M}{4M} Y_{\alpha 1}$$

  NH

  $$\rho = \frac{\sqrt{\Delta m^2_{\rm atm}} - \sqrt{\Delta m^2_{\rm sol}}}{\sqrt{\Delta m^2_{\rm atm}} + \sqrt{\Delta m^2_{\rm sol}}}, \qquad y' = \frac{M}{2v^2 y}\left(\sqrt{\Delta m^2_{\rm atm}} + \sqrt{\Delta m^2_{\rm sol}}\right).$$

- Free parameters: M, **Δ**M, y, and 3 phases: δ, ϕ (U_{PMNS}), θ

$$U^2 \equiv \sum_\alpha |U_{\alpha i}|^2 \approx \frac{y^2 v^2}{2M^2}, |U_{\alpha 1}| \simeq |U_{\alpha 2}|$$

- $\Delta^{\rm ov}_{\rm LNV}$ and $\Delta^{\rm osc}_{\rm LNV}$ only depend on $\boldsymbol{\theta}$, for both NH and IH
- At leading order in y'/y, **Δ**M/M and

$$r \equiv \frac{\sqrt{\Delta m^2_{\rm sol}}}{\sqrt{\Delta m^2_{\rm atm}}} \sim \theta_{13} \sim |\theta_{23} - \pi/4| \sim 10^{-1}$$

$$\frac{\Delta^{\rm ov}_{\rm LNC}}{M_2^2 - M_1^2} \approx -\frac{v^2\sqrt{\Delta m^2_{\rm atm}}}{8M^3U^4} s_\theta,$$

$$\frac{\Delta^{\rm ov}_{\rm LNV}}{M_1 M_2 (M_2^2 - M_1^2)} \approx -\frac{\sqrt{\Delta m^2_{\rm atm}}}{4MU^2} s_\theta,$$

$$\frac{\Delta^{e}_{\rm LNC}}{M_2^2 - M_1^2} \approx U^2 M^3 \frac{\sqrt{\Delta m^2_{\rm atm}}}{v^4}\, r\, s^2_{12} s_\theta,$$

$$\frac{\Delta^{\mu}_{\rm LNC}}{M_2^2 - M_1^2} \approx -\frac{\Delta^{\tau}_{\rm LNC}}{M_2^2 - M_1^2} \approx \frac{U^2 M^3}{2} \frac{\sqrt{\Delta m^2_{\rm atm}}}{v^4} \sqrt{r}\, c_{12} \sin(\theta - \phi)$$

Analytical solutions

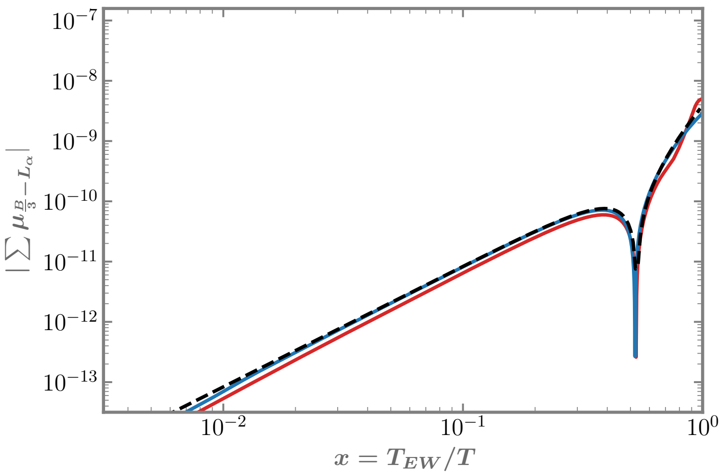
- Linearized eqs.

- Perturbing in y' and in $(M/T)^2$, in the adiabatic approximation if $\epsilon = \Gamma_{osc}(x)/\Gamma(x)$ is << 1 (overdamped) or >> 1 (fast oscillations).

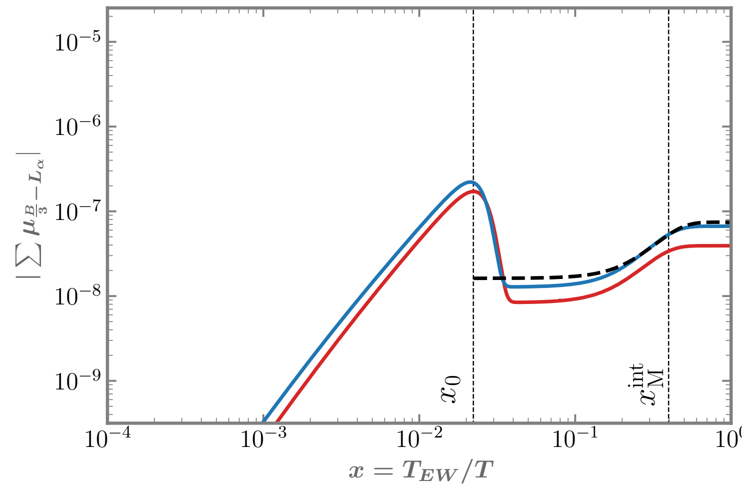
- Intermediate regime: evolve as overdamped up to x_0 / $\Gamma_{ov}(x_0) \propto [\epsilon(x_0)]^2\, \Gamma(x_0) = H(x_0)$ and then project into slow modes.

Comparison

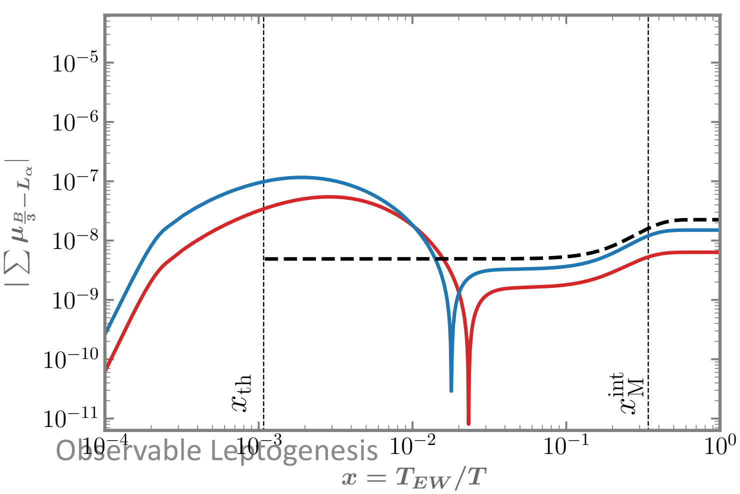
Overdamped weak LNV



Intermediate slow flavour α



Fast oscillation slow flavour α



---- analytical solution

—— numerical solution same approximations

—— full numerical solution

Parameter scan

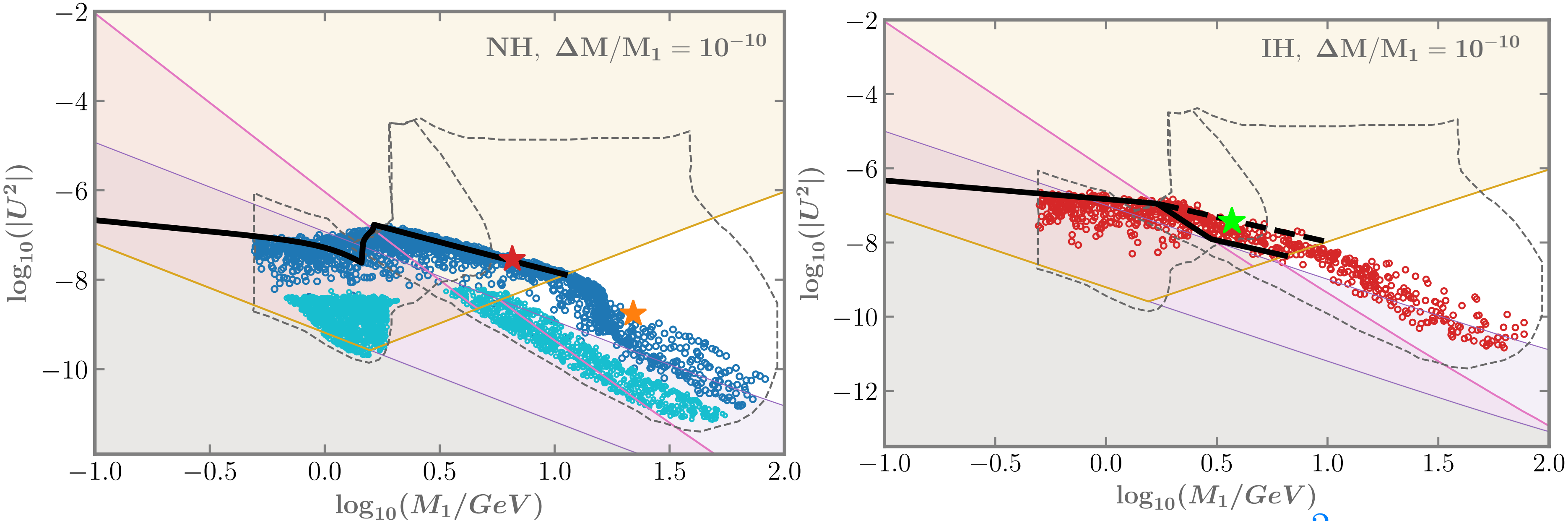
- Nested sampling algorithm UltraNest

$$\log(\mathcal{L}) = -\frac{1}{2}\left(\frac{Y_B(T_{\mathrm{EW}}) - Y_B^{\mathrm{exp}}}{\sigma_{Y_B^{\mathrm{exp}}}}\right) \qquad Y_B^{\mathrm{exp}} = (8.66 \pm 0.05) \times 10^{-11}$$

- Priors:

$\log_{10}(M_1)$	$\log_{10}(\Delta M/M_1)$	$\log_{10}(y)$	θ	δ	α
$[-1, 2]$	$[-14, -1]$	$[-8, -4]$	$[0, 2\pi]$	$[0, 2\pi]$	$[0, 2\pi]$

- y'/y < 0.1 , to ensure approximate LNC limit
- Restricted to region testable at SHIP, FCC-ee.
- Additional scan for NH with log priors in all angles in range [-5,-2]



- Overdamped regime: $(U^2)_{\rm ov} \geq 8 \times 10^9 \left(\frac{\Delta M}{M} \frac{M}{1\rm GeV} \right)^2$
- For M ≲ O(1 GeV), weak LNV:

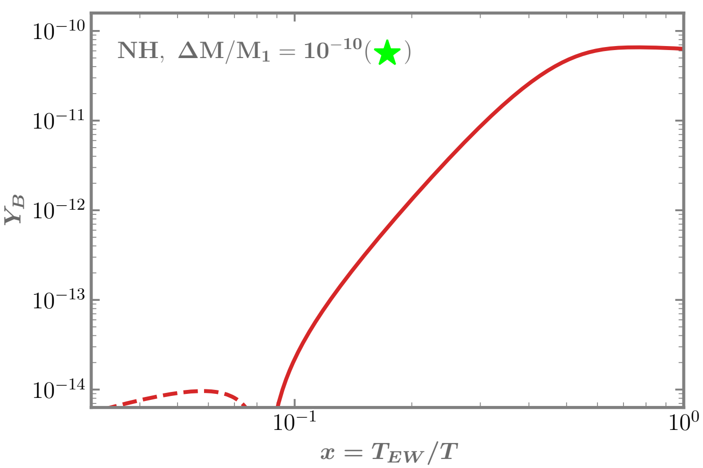
$$(Y_B)^{\rm wLNV}_{\rm ov} \simeq 2 \times 10^{-1} \frac{\Delta M}{M} \left(\frac{10^{-7}}{U^2} \right) \frac{1\,\rm GeV}{M} \left(\left(\frac{M}{1\,\rm GeV} \right)^4 f^{\rm H}_{\rm LNV} - \left(\frac{10^{-7}}{U^2} \right) f^{\rm H}_{\rm LNC} \right)$$

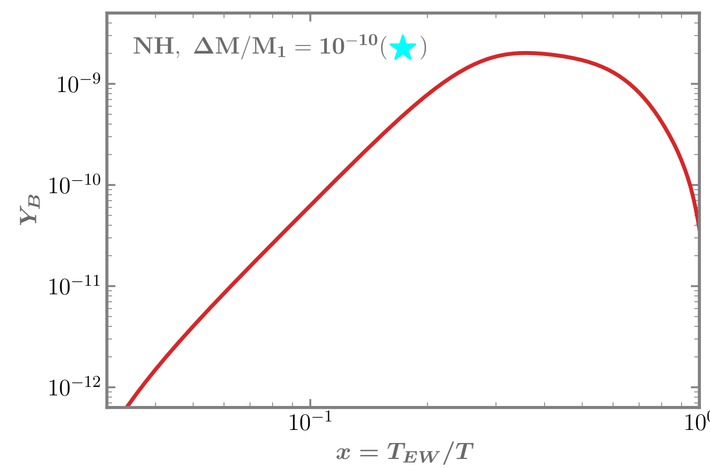
- $f^{\rm H}_{\rm LNC/LNV}$ are the angular part of the CP invariants:

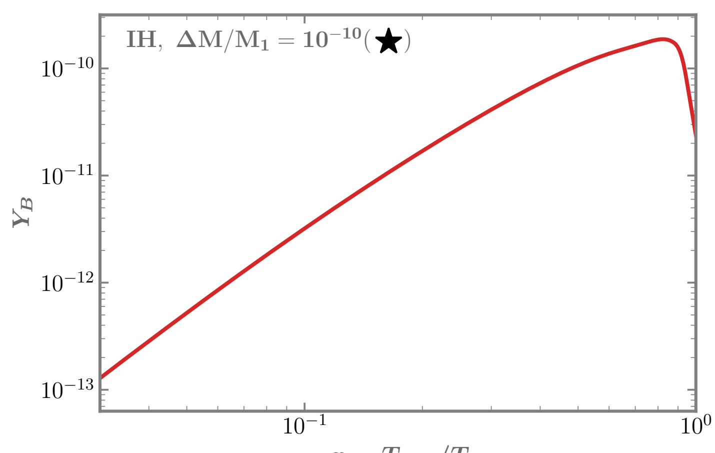
$$f^{\rm IH}_{\rm LNC} = \frac{(1 + 3c_\phi \sin 2\theta_{12})(c_\theta s_\phi \sin 2\theta_{12} + s_\theta \cos 2\theta_{12})}{1 - c_\phi^2 \sin^2 2\theta_{12}}$$

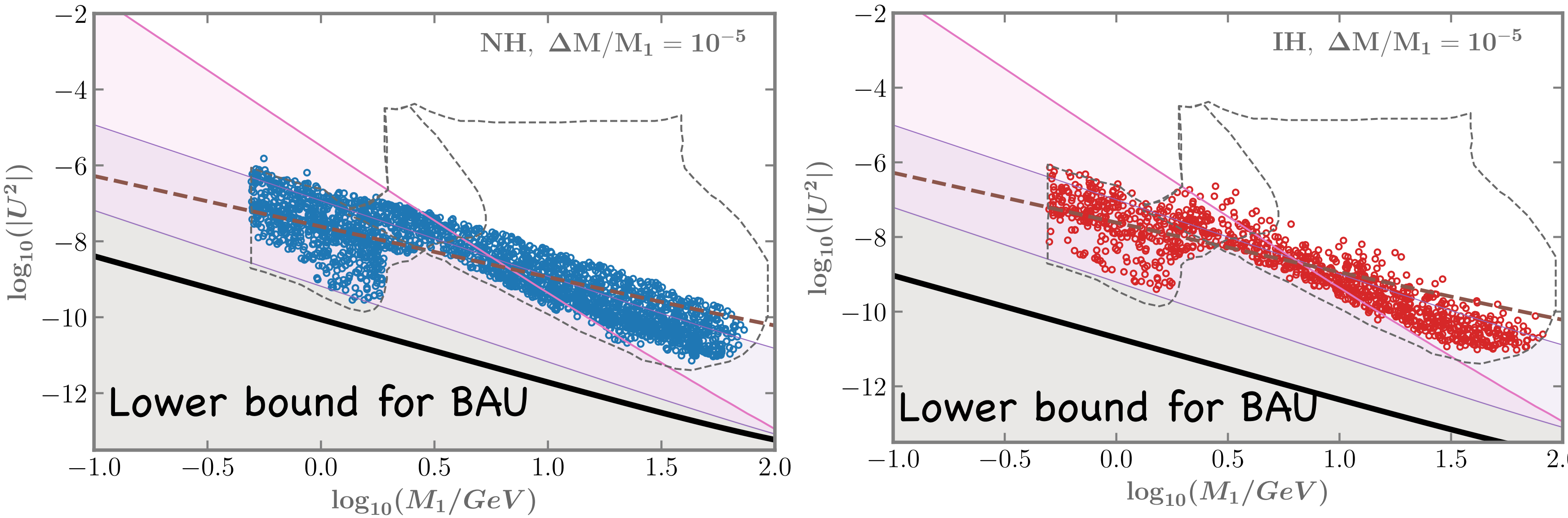
$$f^{\rm NH}_{\rm LNC} = f^{\rm NH}_{\rm LNV} = 2/r^2 f^{\rm IH}_{\rm LNV} = s_\theta$$

- Upper bound on U^2 from imposing $Y_B = Y_B^{EXP}$, and overdamped condition
- For much smaller mixings, it is required a suppression of the $f^{H}_{LNC/LNV}$ to match the BAU (difficult to find in the scan) or exponential washout of the asymmetry.









- Fast oscillation/intermediate regime:

$$(U^2)_{\rm osc/int} \simeq 10^{-6} \left(\frac{\Delta M}{M}\right)^{1/3} \left(\frac{1\,{\rm GeV}}{M}\right)^{4/3}$$

- One flavour α remains weak at T_{EW}

$$10^{-9}\left(\frac{1{\rm GeV}}{M}\right)^2 \frac{1}{{\rm Max}(\epsilon_\alpha)} \leq (U^2)_{\rm fw} \leq 10^{-9}\left(\frac{1{\rm GeV}}{M}\right)^2 \frac{1}{{\rm Min}(\epsilon_\alpha)}$$

- NH : $\alpha = e$ ($\delta + \phi \approx \pi$) , IH: $\alpha = e, \mu,\tau$, depending on (δ,ϕ)

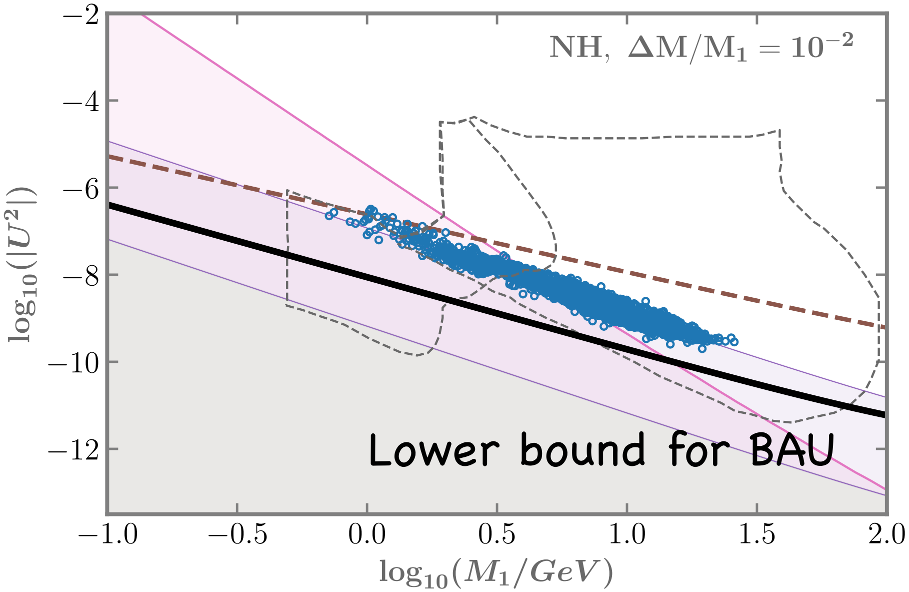
$${\rm Min}(\epsilon_e)_{\rm NH} \simeq {\rm Min}(\epsilon_\alpha)_{\rm IH} = 5 \times 10^{-3}$$

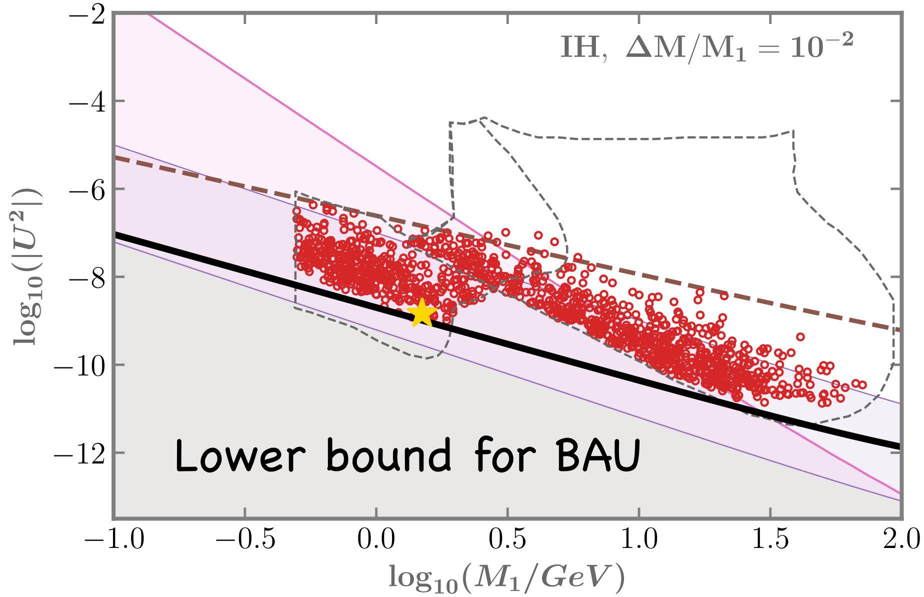
$$(Y_B)_{\rm fw-int} \simeq 9.5 \times 10^{-9} \eta \, \tilde{f}^{\alpha}_{\rm NH/IH} \left(\frac{\Delta M}{M}\right)^{-1/5} \left(\frac{\rm 1GeV}{\rm M}\right)^{1/5} \left(\frac{10^{-9}}{U^2}\right)^{2/5},$$

$$\tilde{f}^{e}_{\rm NH} = r s_{12}^2 s_\theta \quad , \tilde{f}^{\mu}_{\rm IH} = \tilde{f}^{\tau}_{\rm IH} = -\tilde{f}^{e}_{\rm IH}/2 = -\frac{1}{4}(\sin 2\theta_{12} s_\phi c_\theta + \cos 2\theta_{12} s_\theta)$$

- $\eta = 4(1)$ if the LNV mode becomes strong (not) before T_{EW}
- Upper bound on U^2 determined from the flavoured weak washout condition
- For $\tilde{f}^{\alpha}_{\rm NH/IH} = \mathcal{O}(1)$, $Y_B >> Y_B^{EXP}$, so a significant suppression of the angular factors or a exponential washout is needed
- Y_B in weak LNV regime too small
- Lower bound on U^2 to get successful baryogenesis in the fast oscillation regime:

$$(U^2)^{\rm FW-osc}_{\rm Y_B} \geq 18(3.7) \times 10^{-8} \eta \left(\frac{\Delta M}{M}\right)^{2/3} \left(\frac{\rm 1\,GeV}{M}\right)^{5/3} \rm NH(IH)$$

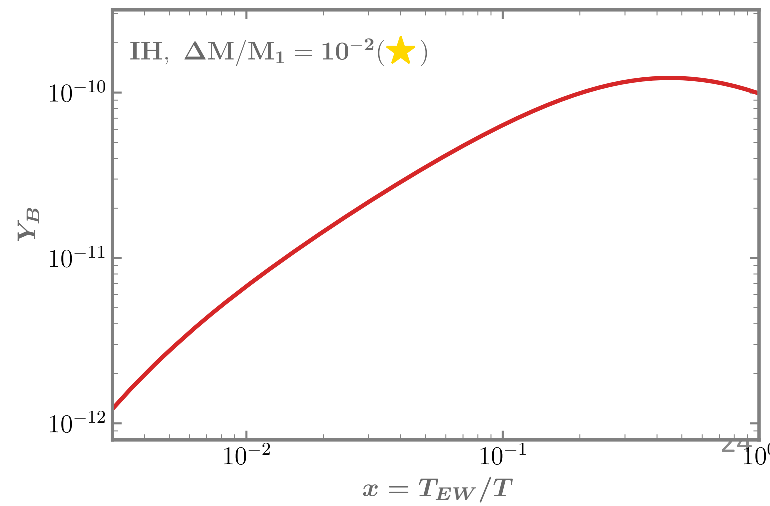




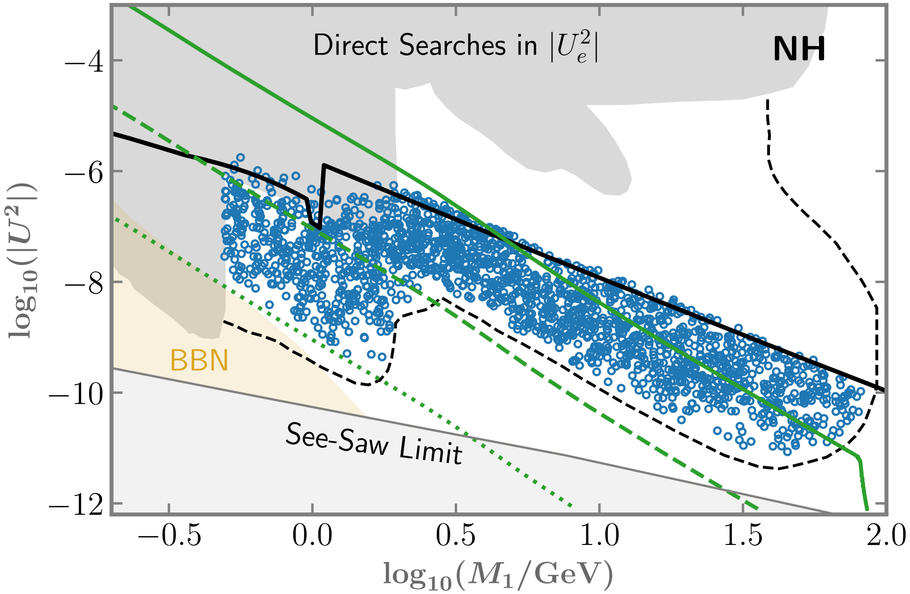
- Only fast oscillation, one slow flavour at T_{EW}:

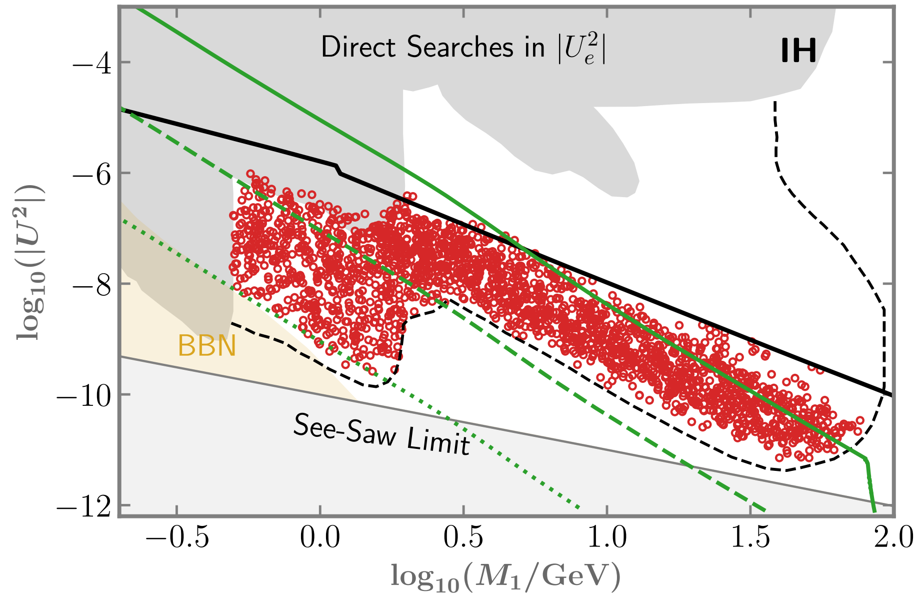
$$(Y_B)_{\text{fw-osc}} = -4.3 \times 10^{-12} \eta \tilde{f}^{\alpha}_{\text{NH/IH}} \left(\frac{U^2}{10^{-9}}\right) \left(\frac{\Delta M}{M}\right)^{-2/3} \left(\frac{M}{1\text{GeV}}\right)^{5/3}$$

- Y_B in weak LNV regime too small



Full scan





Absolute upper bound on U² from the overdamped regime:

- Weak LNV
  M ≲ O(1 GeV)

$$(U^2)^{\mathrm{wLNV}}_{\mathrm{ov}} \lesssim 4(17) \times 10^{-7} \left(\frac{1\,\mathrm{GeV}}{M}\right)^{4/3} \quad \mathrm{NH(IH)}$$

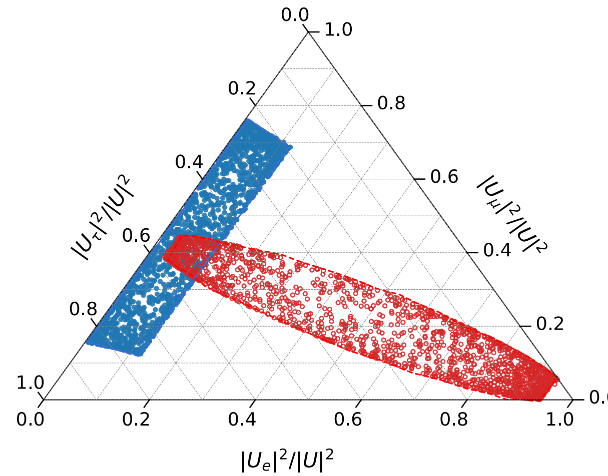
- Strong LNV
  M ≳ O(1 GeV)

$$(U^2)^{\mathrm{sLNV}}_{\mathrm{ov}} \lesssim 16(2.3) \times 10^{-7} \left(\frac{1\,\mathrm{GeV}}{M}\right)^{28/13} \quad \mathrm{NH(IH)}$$

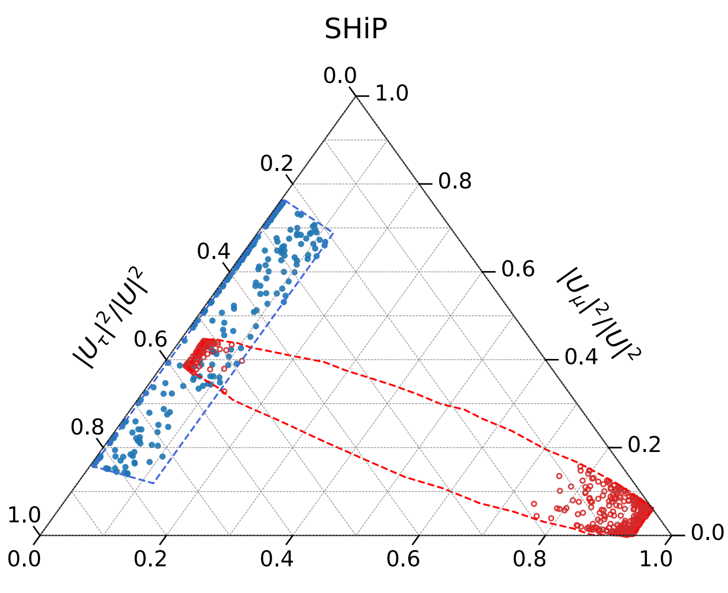
Relation to other observables

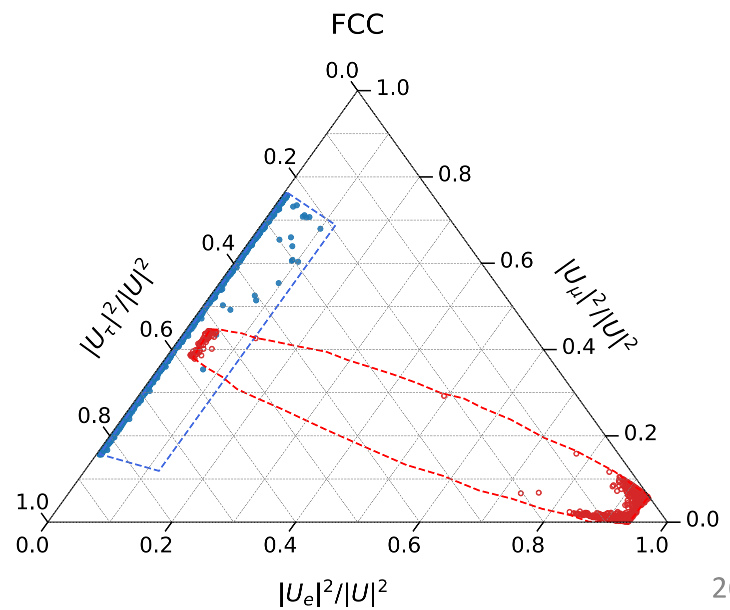
1. HNL flavour mixing

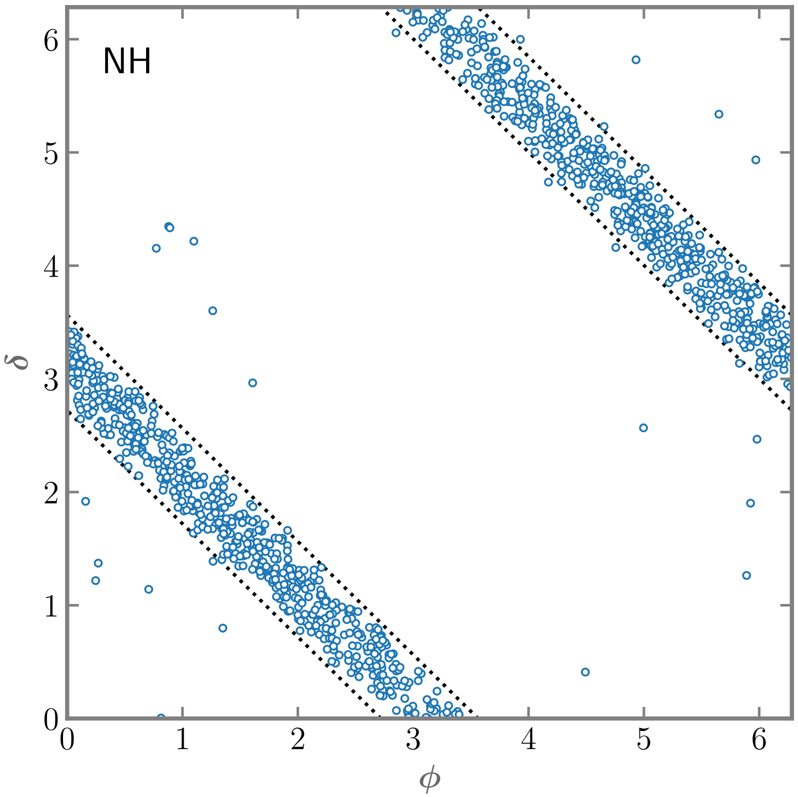
- Full scan: NH and IH

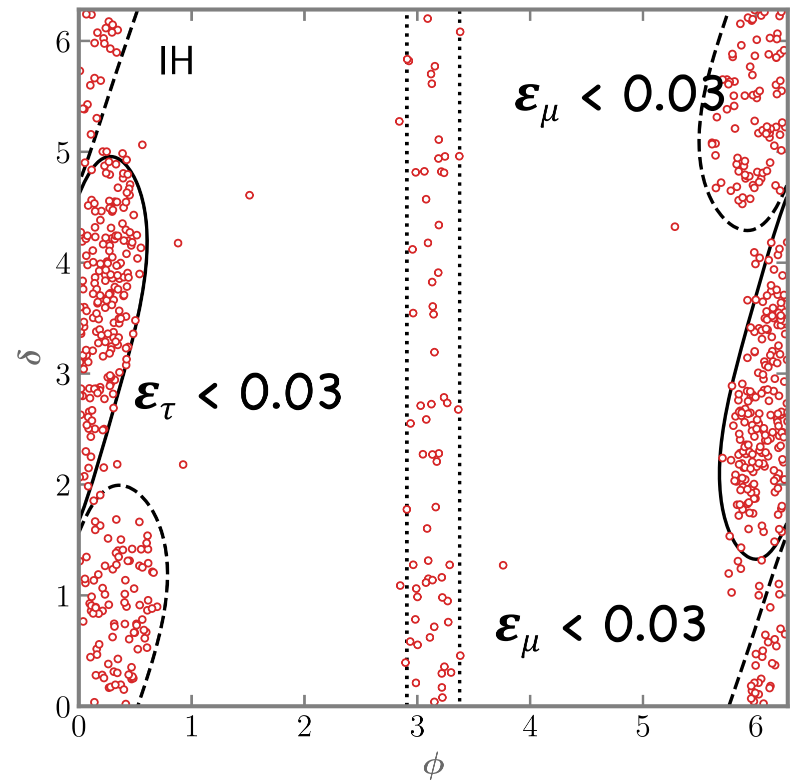


- $\Delta M/M = 10^{-2}$









ε_e < 0.01 ($\delta+\phi \approx \pi$)

ε_e < 0.05

($\phi \approx \pi$)

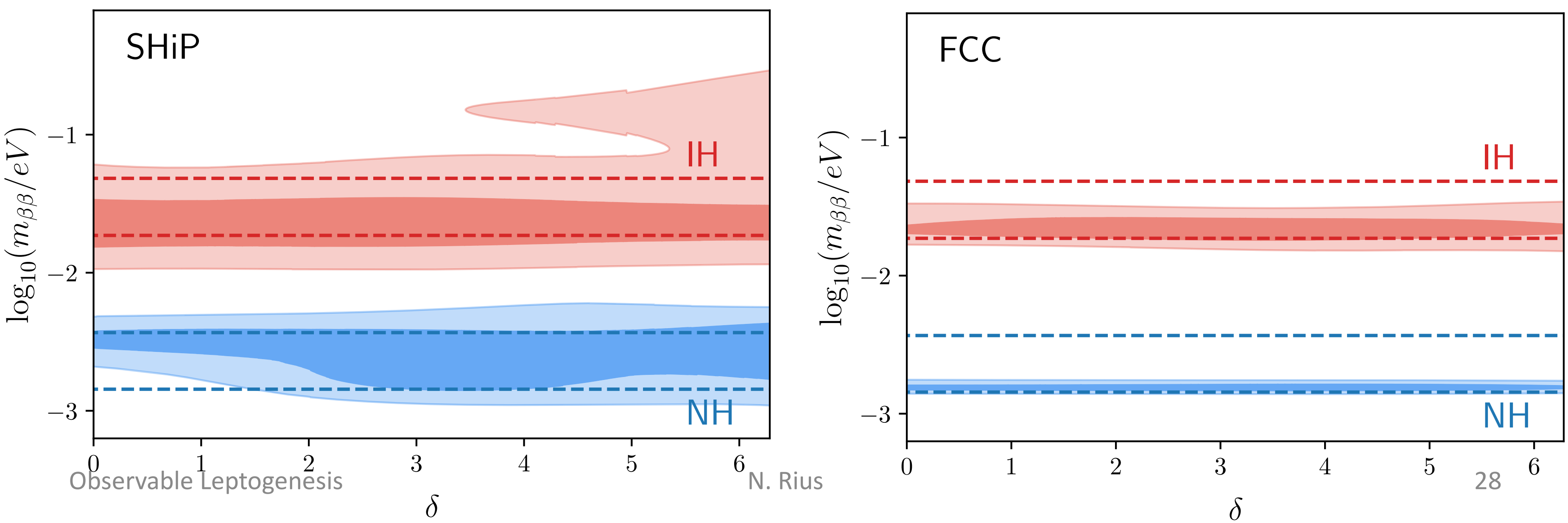
$$\epsilon_\alpha \equiv \frac{(YY^\dagger)_{\alpha\alpha}}{\mathrm{Tr}[YY^\dagger]}$$

2. Neutrinoless double beta decay: $\Delta M/M = 10^{-2}$

Effect of HNL only in SHIP range

$$m_{\beta\beta}^{NH} = \Bigg| \sqrt{\Delta m^2_{\rm atm}} \left(c_{12}^2 c_{13}^2 r - e^{-2i(\delta+\phi)} s_{13}^2\right) - 2e^{i\theta} U^2 \Delta M f(A) \left(\frac{0.9\mathrm{GeV}}{M}\right)^2 \left(r s_{12}^2 + 2\sqrt{r} s_{12} s_{13} e^{-i(\delta+\phi)} + s_{13}^2 e^{-2i(\delta+\phi)}\right)\Bigg| \, ,$$

$$m_{\beta\beta}^{IH} = \Bigg| \sqrt{\Delta m^2_{\rm atm}} c_{13}^2 \left(c_{12}^2 - s_{12}^2 e^{2i\phi} + \mathcal{O}\left(r^2\right)\right) - e^{i\theta} U^2 \Delta M f(A) \left(\frac{0.9\mathrm{GeV}}{M}\right)^2 \left(c_{12} + s_{12}^{i\phi}\right)^2 \left(1 + \mathcal{O}\left(r^2\right)\right)\Bigg|$$



3. Challenge: to determine ΔM smaller than 10^{-2} GeV

Coherent HNL oscillations if ΔM $\approx \Gamma$?

Thank you !

Analytical approximation

- Linearized kinetic eqs. can be written as:

$$\frac{dr(x)}{dx} = A(x)r(x) + h(x) \qquad x = T_{EW}/T$$

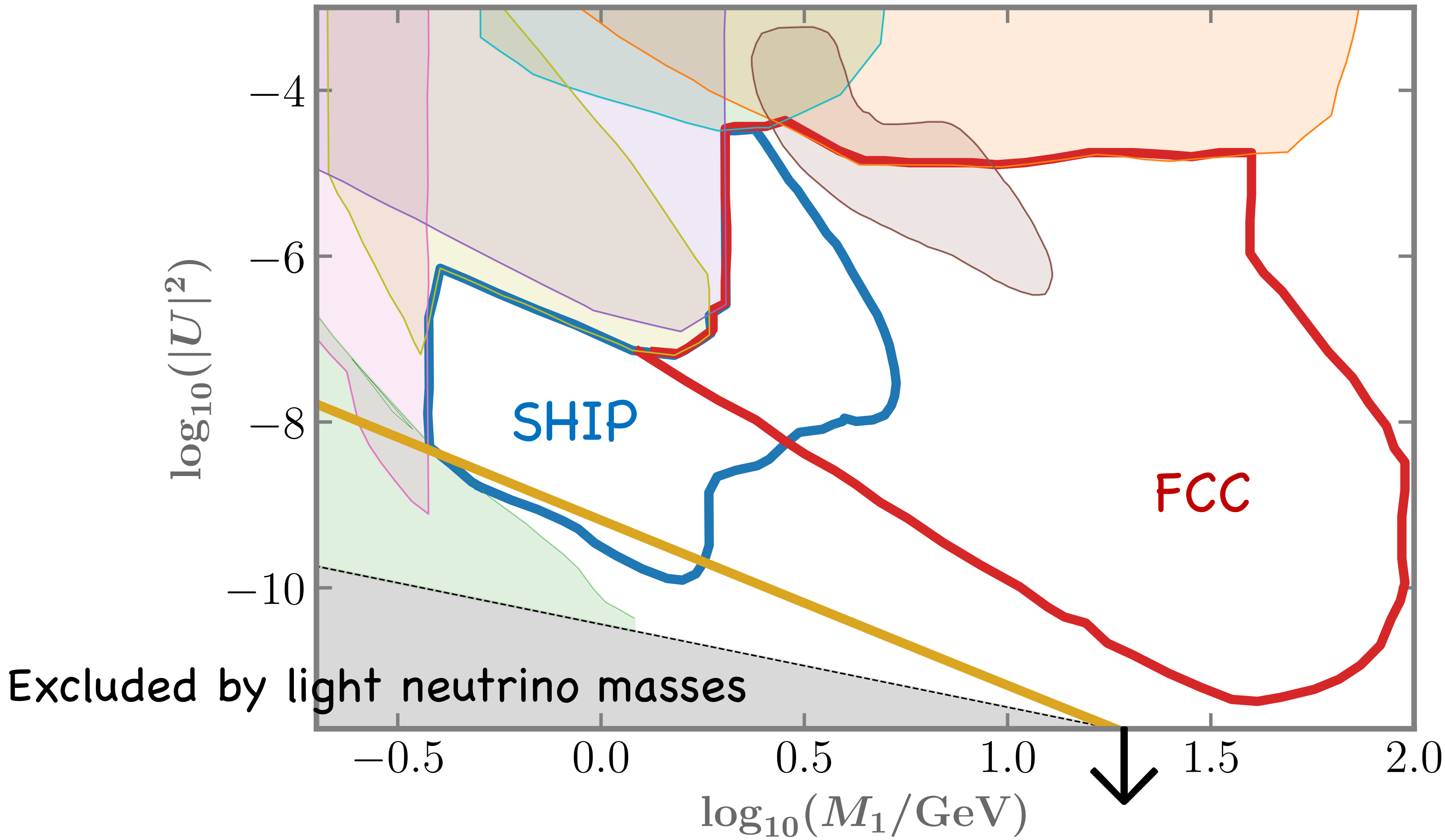
- r(x) is an 11-dimensional vector containing the 8 neutrino and antineutrino density matrix elements and the SM lepton chemical potentials.
- Perturbing around the symmetric texture for Y, and in M/T, it is possible to solve the zeroth order, $A^0(x)$ in the adiabatic approximation if $\epsilon = \Gamma_{osc}(x)/\Gamma(x)$ is << 1 (overdamped) or >>1 (fast oscilaltions).
- Intermediate regime: evolve as overdamped up to x_0 / $\Gamma_{ov}(x_0) \propto [\epsilon(x_0)]^2\, \Gamma(x_0) = H(x_0)$ and then project into slow modes.

- Thermalization rates: $\Lambda(x) = \int_0^x \lambda(z) dz.$
- $\lambda(x)$ are the eigenvalues of $A^0(x)$
- Solution approaches equilibrium limit (all μ_α =0 and thus Y_B=0) exponentially:

$$\propto e^{-\Lambda_i(x)} \equiv \exp\left(- \int_0^x dz |\mathrm{Re}(\lambda_i(z))| \right)$$

- Oscillation rate related to Im[$\lambda(x)$] , $\propto e^{-i\Lambda_{\rm osc}(x)}$

$$\Lambda_{\rm osc}(x) = \int_0^x dz |\mathrm{Im}(\lambda_i(z))| \propto x^3 \frac{|M_2^2 - M_1^2| M_P^*}{T_{EW}^3}$$



$\Gamma\,(T_{EW}) = H_u(T_{EW})$