Emergent higher-form symmetries in phases of matter

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Based on [arXiv:2210.14802] with

Richard Davison (Heriot-Watt U) and **Eric Mefford** (U of Victoria)

References

- *Zero sound and higher-form symmetries in compressible holographic phases*, with Richard Davison and Eric Mefford, [arXiv:2210.14802].
- *Superfluids as Higher-form Anomalies*, Luca Delacrétaz, Diego Hofman and Grégoire Mathys, [arXiv: 1908.06977].
- *Deconstructing holographic liquids*, Dominic Nickel and Dam T Son, [arXiv: 1009.3094].

Dynamics of topological defects

Normal phase $\langle \Phi \rangle = 0$

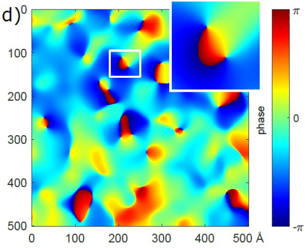
Restoration of symmetry

Individual topological defects proliferate

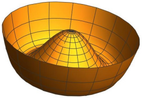
Berezinsky-Kosterlitz-Thouless transition: topological defects condense

Pairs of topological defects form

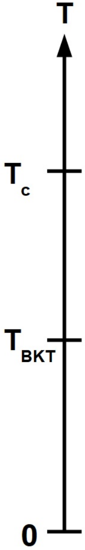
Symmetry spontaneously broken $\langle \Phi \rangle \neq 0$

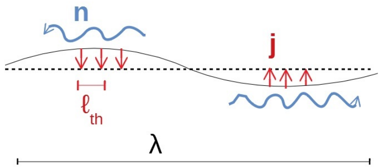


[Fang et al'19]



$T > T_c$: Low energy dynamics in the normal phase





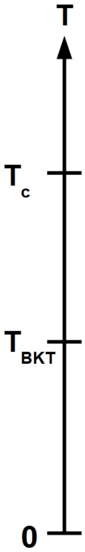
- Resolving molecular dynamics not efficient or needed for low energy dynamics $\lambda \gg \ell_{th}$: Effective Field Theory (EFT) approach $\Rightarrow$ **hydrodynamics**.

- Also describes e.g. Quark-Gluon Plasma or electron flows in Graphene.

- Systematic construction from **global symmetries** (translations, rotations, U(1), etc.)

$$\Rightarrow \text{conserved currents:} \quad \partial_\mu J^\mu = 0 \,, \qquad \mu = t, x^i .$$

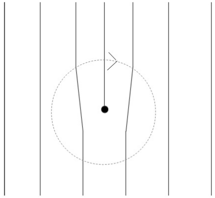
$T < T_c$: Low energy dynamics in the broken symmetry phase



- Include **gapless Goldstone** field in the EFT.
- In the absence of defects, well-known: e.g. superfluid/crystal hydrodynamics, pion hydrodynamics (chiral limit of QCD).
- Defects: **non-trivial winding**

$$\int_{\mathcal{C}} \vec{\nabla}\phi \cdot d\vec{\ell} = 2\pi N_w$$

$\Rightarrow$ Goldstone field **multi-valued**



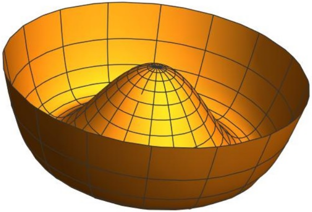
- Bad starting point for EFTs.
- Tracking motion of many defects: goes contrary to EFT spirit.

Zero temperature superfluids

- $T = 0$ low-energy EFT [Son'02]:

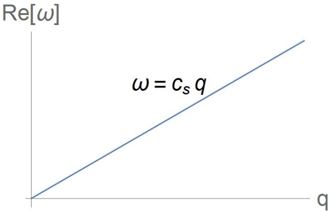
$$\mathcal{L}_\varphi = -\frac{\chi_{nn}}{2}(\partial_t\varphi)^2 + \frac{n_s}{2}(\partial^i\varphi)^2$$

$\chi_{nn} \equiv \partial n/\partial\mu$: charge static susceptibility,
$n_s \equiv \partial^2 f/\partial|\partial^i\varphi|^2$: superfluid density.



- **Linear** dispersion relation:

$$\omega_\pm = \pm c_s q\,, \quad c_s^2 = \frac{n_s}{\chi_{nn}}$$



- Higher energies: outside of regime of validity of EFT, extra gapped dofs (rotons, vortices...).

Zero temperature superfluids

- The **Goldstone shifts** under U(1) gauge transformations:

$$\varphi \mapsto \varphi + \lambda$$

- Noether: U(1) charge conservation

$$\nabla_\mu J^\mu = 0\,, \quad (J^t, J^i) = \left(-\chi_{nn}\partial_t\varphi\,, n_s\partial^i\varphi\right)$$

- Constitutive relations + Josephson relation

$$J^t = -\chi_{nn}\partial_t\varphi \quad \Leftrightarrow \quad \partial_t\varphi = -\mu$$

- No vortices: **conserved winding number**

$$N_w = \frac{1}{2\pi}\oint d\varphi \quad \Rightarrow \quad \partial_{[\mu}\partial_{\nu]}\varphi = 0 \quad \text{(Stokes' theorem)}$$

Emergent higher-form symmetry

- External gauge field $\Rightarrow$ **covariant derivative (gauge invariance)**

$$\mathcal{L}_\varphi \mapsto \mathcal{L}_\varphi - A_\mu J^\mu \quad \Rightarrow \quad \partial_\mu \varphi \mapsto D_\mu \varphi = \partial_\mu \varphi - a\, A_\mu$$

a: charge of the condensed operator.

- **Hodge dualize**, [Delacrétaz, Hofman & Mathys'19]

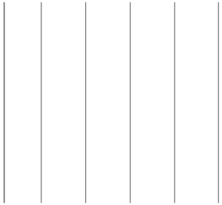
$$(\star K)_\mu \equiv D_\mu \varphi$$

$\star K$: d-form in $d+1$ dimensions.

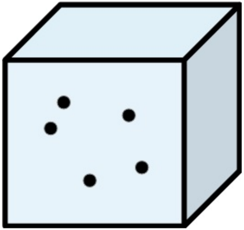
- No vortices:

$$\partial_{[\mu}\partial_{\nu]}\varphi = 0 \quad \Rightarrow \quad \boxed{d \star K = -aF}\,, \quad F \equiv dA\,.$$

- $\mathrm{U}(1)_w^{d-1}$ symmetry [Gaiotto et al'14] :
Conservation equation for the number of winding hyperplanes.



Ordinary symmetries vs higher-form symmetries

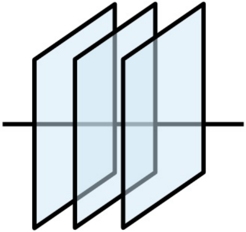


0-form symmetry in D=3+1

1-form current J^μ

$\nabla_\mu J^\mu = 0$

counts **particles** $\langle J^t \rangle = n$



2-form symmetry in D=3+1

3-form current $(\star K)^\mu = D^\mu \varphi$

Anomalous conservation law
$\nabla_\kappa K^{\mu\nu\kappa} = \frac{a}{2}\epsilon^{\kappa\lambda\mu\nu}F_{\kappa\lambda}$

counts **planes** $\langle (\star K)^i \rangle = \tilde{\rho}^i$

Maxwell E&M

Maxwell equations + Bianchi identity

$$\nabla_\mu F^{\mu\nu} = j^\nu_{el}\,, \quad \epsilon^{\lambda\mu\nu}\nabla_\lambda F_{\mu\nu} = 0$$

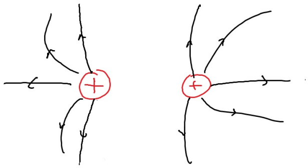
- Define two 2-form currents: $J^{\mu\nu}_{el} = F^{\mu\nu}$, $J^{\mu\nu}_{mag} = (\star F)^{\mu\nu}$, corresponding to $\mathrm{U}^{el}_1(1)$ and $\mathrm{U}^{mag}_1(1)$, [Gaiotto et al'14].

- If $\langle j^\mu_{el}\rangle = 0$: both are conserved

$$\nabla_\mu J^{\mu\nu}_{el} = 0 \qquad \nabla_\mu J^{\mu\nu}_{mag} = 0$$

  Conservation of electric and magnetic field lines.

- Couple to charged matter $\langle j^\mu_{el}\rangle \neq 0$: $\mathrm{U}^{el}_1(1)$ broken, electric field lines end on charges.



- MHD: conservation of $T^{\mu\nu}$ + magnetic higher form symmetry $J^{\mu\nu}_{mag} = (\star F)^{\mu\nu}$, [Grozdanov, Hofman & Iqbal'16, Armas & Jain'18].

$$\boxed{d\star K = -\, a \, F\,.}$$

- The source for the 0-form U(1) appears on the rhs of the conservation equation of the $d-1$-form density: **mixed 't Hooft anomaly**, [Delacrétaz et al'19].
- Anomaly coefficient fixed in the UV (UV-IR mixing).
- Anomalies
  - Well-known from QED (axial anomaly)
  - Source hydrodynamic terms at first order in gradients [Son & Surowka'09]: chiral magnetic/vortical effect.
  - Mixed axial-gravitational anomalies (review [Landsteiner'16]), measured in Weyl semi-metals [Gooth et al'17].

Hydrodynamics with anomalous higher-form symmetries

- Here, the anomaly sources **ideal, zeroth-order terms**:

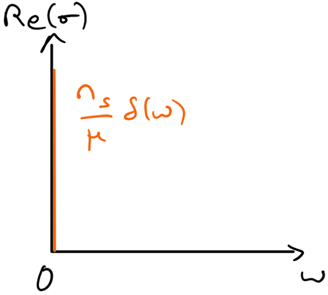
$$(\star K)^t = a\mu\,, \quad J^i = a\tilde{\mu}^i$$

- Anomaly implies **gapless superfluid sound modes**

$$\omega_\pm = \pm a \frac{\sqrt{n_s}}{\sqrt{\chi_{nn}}} q + \ldots, \quad n_s \equiv \frac{1}{\tilde{\chi}}\,.$$

- **Divergent dc conductivity**:

$$\sigma(\omega) \equiv \frac{i}{\omega} G^R_{J_x J_x}(\omega, q=0) = \frac{i}{\omega}\frac{a^2 n_s}{\mu}\,,$$

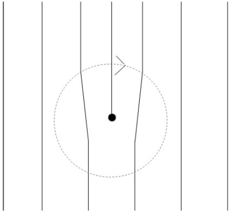


$T > T_{BKT}$: Broken anomalous higher-form symmetries

- At $T > T_{BKT}$, **vortices** condense: $\langle J_v^\mu \rangle \neq 0$

$$\partial_\mu K^{\mu\nu} = -a\epsilon^{\lambda\mu\nu} F_{\lambda\mu} + J_v^\mu$$

Explicitly breaks winding conservation



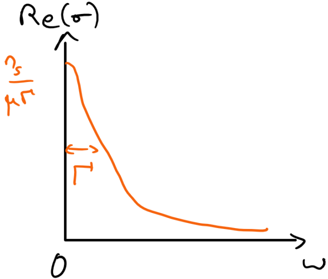
- Constitutive relation:

$$\langle J_v^\mu \rangle = -\Gamma u_\mu K^{\mu\nu} + \dots$$

- Gaps one of the sound modes:
  $\omega_+ = -i\Gamma + \mathcal{O}(q^2)$, $\omega_- = \mathcal{O}(q^2)$.

- Finite conductivity:

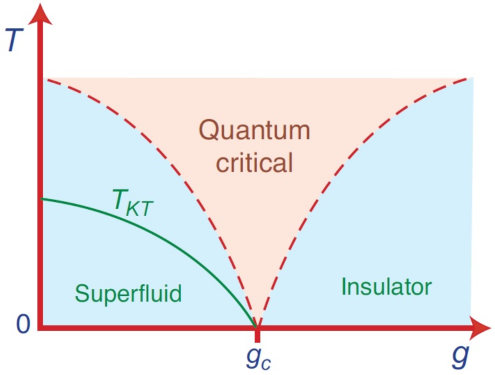
$$\sigma(\omega) = \frac{n_s}{\mu} \frac{1}{\Gamma - i\omega}$$



Conclusion and outlook part I

- Higher-form symmetries are useful to avoid using multi-valued fields in phases with SSB.
- The anomaly plays a crucial role. What about other SSB phases (translations, etc.)?
- When vortices condense, treat breaking of symmetry more systematically within hydro (similar to momentum relaxing hydro): analogous to formulating Navier-Stokes equations.
- Use it to simulate fluids of defects: BECs in cold atom systems, etc.

Quantum critical points



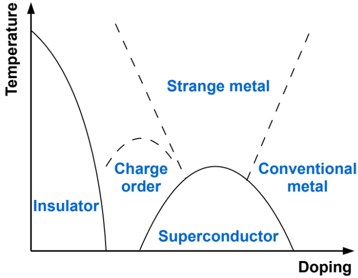
[SACHDEV'08]

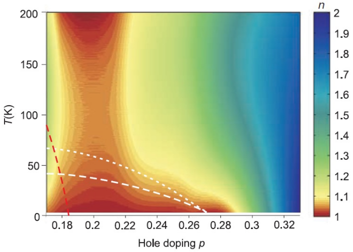
- QCPs: Mediate phase transition at fixed (zero) temperature as a function of external parameter (magnetic field, pressure, doping...).

- Temperature is the only scale $\Rightarrow$ scale-invariant physics

$$[\sigma] = d - 2 \quad \Rightarrow \quad \sigma(\omega, T) = \Sigma(\omega/T)$$

Quantum criticality and strange metals





[Cooper et al'09]

Conondrum: strange metals reminiscent of QCPs, but

- No order parameter clearly associated to strange metal phase.
- $\rho_{dc} \equiv 1/\sigma_{dc} \sim 1/T$ incompatible with scale invariance.

Outside Landau paradigm? Unconventional quantum criticality?

Deconfined Quantum criticality

- DQCPs: different order parameters on either side of the QCP
- Emergent dofs at the QCP (typically emergent gauge fields): emergent topological conservation law

$$S[z,a] = \int d^3x \left[|(\partial - ia)z|^2 + r|z|^2 + \left(u|z|^2\right)^2 + \frac{1}{g^2} f^2 \right]$$

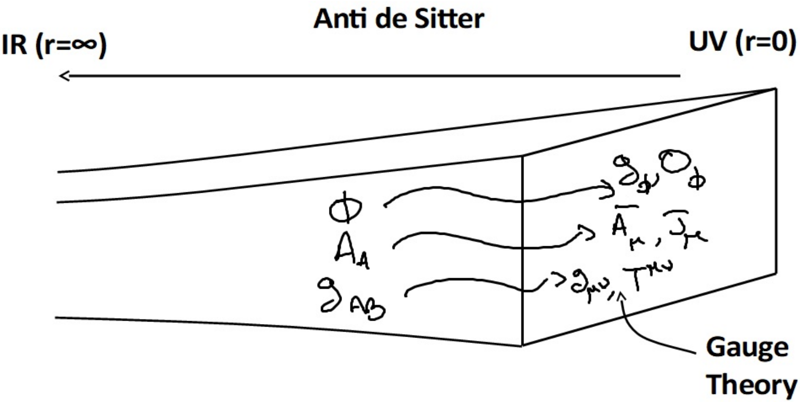
- z: spinon; a emergent gauge field;
- Maps back to Wilson-Fisher type action through $\Phi = z_\alpha \sigma^{\alpha\beta} z_\beta$
- In the DQCP scenario, the a's become dynamical at the QCP and gauge the U(1) redundancy of the spinon description.

Deconfined Quantum criticality

- Scenarii with deconfined gauge fields put forward to describe the pseudogap phase of high T_c superconductors (review [Sachdev & Chowdhury'16]).

- AdS/CFT allows to construct whole families of QCPs with unconventional scaling properties:
  - Emergent dofs and symmetries?
  - New effective actions?
  - Relation to nature of charged black hole horizons?
  - Relevance for strange metals?

Holographic duality primer



$$Z_{QFT}[g_\phi, \bar{A}_\mu, g_{\mu\nu}] = Z_{gravity}[g_\phi, \bar{A}_\mu, g_{\mu\nu}]$$

Holographic quantum critical phases

- Consider deforming holographic CFT by relevant scalar operator

$$\mathcal{L} = R - \frac{1}{2}\partial\phi^2 - \frac{1}{4}Z(\phi)F^2 - V(\phi)\,, \quad V(\phi \to 0) \to -2\Lambda\,, \quad Z(\phi \to 0) \to 1$$

- In the IR, $\phi \to \infty$. Pick scalar couplings such that

$$V(\phi \to +\infty) \to V_o e^{-\delta\phi}\,, \quad Z(\phi \to \infty) \to Z_o e^{\gamma\phi}$$

holographic quantum critical phases [Charmousis, B.G., Kim, Kiritsis & Meyer'10] (a story similar to what follows applies to probe branes).

- **Hyperscaling-violating** scaling solutions in the IR

$$ds^2_{IR} = \zeta^{\theta-2}\left(-dt^2 + d\zeta^2 + d\vec{x}^2\right)\,, \quad \phi_{IR} = \kappa(\theta)\log\zeta\,.$$

- Vanishing ground state entropy $s \sim T^{(2-\theta)}$.

Holographic quantum critical phases

- Zero density: Maxwell field in background with UV AdS_4 and IR $\theta \neq 0$

$$S_M = -\int d^{3+1}x\sqrt{-g}\,\frac{1}{4}Z(\phi)F_{AB}F^{AB}$$

- Low temperatures, assume

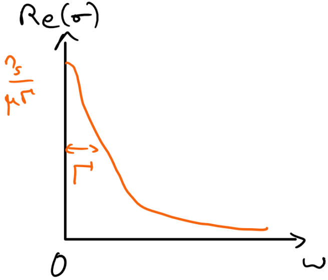
$$Z(\phi(\zeta)) \sim \zeta^{\Delta_\chi - 1}\,, \qquad \Delta_\chi < 0 \quad \Rightarrow \quad \sigma_{dc} \sim T^{\Delta_\chi - 1} \neq T^{d-2=0}$$

- Compute ac conductivity:

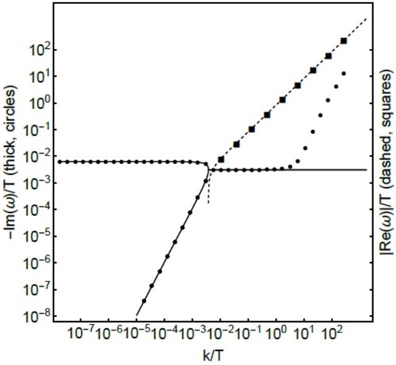
$$\sigma(\omega) = \frac{i}{\omega} G^R_{JJ}(\omega, q = 0) = \frac{\sigma_{dc}}{1 - i\omega\tau}\,,$$

$$T\tau \sim T^{\Delta_\chi} \gg 1$$

- **Sharp Drude-like peak**, similar to superfluids with condensed vortices.



Spatially resolved transport



- **Pole collision** between gapped and diffusion pole

$$\omega = -iD_n q^2 + \dots\,, \quad + \quad \omega = -i/\tau + iD_n q^2 + \dots$$

$$\Downarrow$$

$$\omega = \pm c_s q - \frac{i}{2\tau} + \dots$$

Effective action at $T \neq 0$

- Split bulk action into UV ($0 \leq r < r_\star$) and IR pieces ($r_\star \leq r < r_h$), [Nickel & Son'10]:

$$S[\varphi, \bar{A}_\mu, a_\mu] = \frac{1}{2}\int d^{2+1}x \left[-\chi_{nn}\left(\partial_t\varphi - \bar{A}_t + a_t\right)^2 + \chi_{JJ}\left(\partial_x\varphi - \bar{A}_x + a_x\right)^2\right]$$

$$-\frac{1}{2}\int d\omega dq \, \frac{f_{tx}^2}{-i\omega\sigma_{dc}}$$

$$\varphi \equiv \int_{r_\star}^{0} dr\, A_r\,, \quad \bar{A}_\mu \equiv A_\mu(r=0)\,, \quad a_\mu = A_\mu(r_\star)$$

- Looks like the action for an **ideal superfluid**, up to a_μ.

- Emergent gauge field a_μ, with a nonlocal kinetic term.

Effective theory at $T \neq 0$

- Now integrate out a_μ in $S_{UV} + S_{IR}$:

$$j^t = \chi_{nn}\left(\partial^t\varphi - \bar{A}^t\right) = \chi_{nn}\mu\,, \qquad j^i = \frac{\sigma_{dc}\partial_t}{\left(1+\chi_{JJ}^{-1}\sigma_{dc}\partial_t\right)}\left(\partial^i\varphi - \bar{A}^i\right)$$

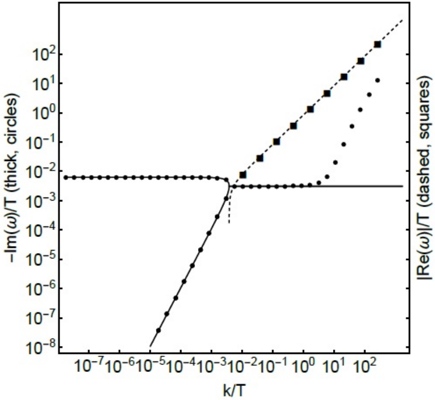
- $\omega \ll \chi_{JJ}\sigma_{dc}^{-1} \ll T$: **diffusive** hydro

$$j^i = -\sigma_{dc}\left(\partial^i\mu - E^i\right)\,,$$

$$\mu = \partial^t\varphi - \bar{A}^t$$

- $\chi_{JJ}\sigma_{dc}^{-1} \ll \omega \ll T$: **'superfluid'**

$$j^i = \chi_{JJ}\left(\partial^i\varphi - \bar{A}^i\right)\,,$$



Emergent higher-form symmetry

$$j^t = \chi_{nn}\left(\partial^t\varphi - \bar{A}^t\right)\,, \qquad j^i = \frac{\sigma_{dc}\partial_t}{(1+\chi_{JJ}^{-1}\sigma_{dc}\partial_t)}\left(\partial^i\varphi - \bar{A}^i\right)$$

$$\Downarrow$$

$$\partial_t j^i + \chi_{JJ}\partial^i\mu = \chi_{JJ}\,E^i - j^i/\tau, \quad \tau \equiv \sigma_{dc}\chi_{JJ}^{-1} \gg 1/T$$

- ‘t Hooft anomaly.

- Define

$$(\star K)_\mu = \partial_\mu\varphi - \bar{A}_\mu + a_\mu$$

K obeys the equation

$$d\star K = -\bar{F} + f$$

The emergent, dynamical gauge field a is responsible for the relaxation of K ($\neq$ from a superfluid).

Effective action at $T = 0$

- Does the symmetry persist at $T = 0$? Repeat the calculation:

$$S_{\mathit{eff}} = \frac{1}{2}\int d^{2+1}x \left[-\chi_{nn}\left(\partial_t\varphi - \bar{A}_t + a_t\right)^2 + \chi_{JJ}\left(\partial_x\varphi - \bar{A}_x + a_x\right)^2\right]$$

$$-\frac{1}{2}\int d\omega dq \, \frac{f_{ti}f^{ti} - c_{IR}^2 f_{ij}f^{ij}}{i(\omega^2 - c_{IR}^2 q^2)^{1-\Delta_\chi/2}} \, ,$$

- Collective mode

$$\omega = c_s q - i\# q^{1-\Delta_\chi} + \ldots$$

Different attenuation from a $T = 0$ superfluid. **Holographic 'zero sound'** [Karch, Son & Starinets'08].

Summary zero density

$$S_M = -\int d^{3+1}x\sqrt{-g}\frac{1}{4}Z(\phi)F_{AB}F^{AB}\,, \quad Z(\phi(\zeta)) \sim \zeta^{\Delta_\chi - 1}\,, \qquad \Delta_\chi < 0\,.$$

- Effective action in terms of a superfluid-like scalar coupled to an emergent gauge field a_μ with nonlocal action: evades scale invariance.

$$\sigma_{dc} \sim T^{\Delta_\chi - 1}$$

- $T = 0$: 'zero sound' mode with anomalous attenuation.
- $T \neq 0$: crossover from diffusive+gapped mode to propagating modes.
- Can be reformulated in terms of relaxed higher-form symmetry

$$d \star K = -\bar{F} + f\,, \quad f = da$$

Outlook zero density

$$S_M = \int d^{3+1}x\sqrt{-g}\frac{1}{4}Z(\phi)F_{AB}F^{AB}\,, \quad Z(\phi(\zeta)) \sim \zeta^{\Delta_\chi - 1}\,, \qquad \Delta_\chi < 0\,.$$

- Does the symmetry survive at nonlinear level?
- The same physics underlies probe brane models and the zero sound mode there [Karch et al'08; Nickel & Son'10; Hoyos, O'Bannon & Wu'10; Davison & Starinets'11; Chen & Lucas'17; Gushterov, O'Bannon & Rodgers'18], as well as higher-derivative Maxwell theories [Witczak-Krempa & Sachdev'12, Witczak-Krempa'13], [Grozdanov, Lucas & Poovuttikul'18].
- (Some version of it) plausibly also underlies higher-derivative gravity theories [Kaplis, Grozdanov & Starinets'16]

Finite density in a nutshell

- $T \neq 0$, states with emergent $z = 1$ and $\theta \neq 0$ contain collective excitation similar to zero density.

- Reflects dynamics of the incoherent current

$$\delta j^x_{\text{inc}} \equiv \delta j^x - \rho \delta u^x \,, \qquad \chi_{J_{inc}P} = 0 \,.$$

- Different from phase-relaxed superfluid: long-lived mode affects all thermoelectric conductivities.

- $T \to 0$:

$$\chi_{J_{inc}J_{inc}} \sim \chi_{n_{inc}n_{inc}} \to 0$$

- $\Rightarrow$ Collective mode dissolves into branch cut at $T = 0$. Fate of the emergent higher-form symmetry?

Final comments

- In these holographic states, the effective gauge coupling in the bulk vanishes in the IR: higher-derivative terms might be important [Goldstein, Kachru, Prakash & Trivedi'09]. Restore holographic zero sound?

- Effective holographic action? Emergent gauge field, metric? Complicated due to need to integrate out metric dofs.

- Scaling theories with large anomalous dimensions were constructed [Goutéraux'13,'14; Karch'14; Davison, Hartnoll & Goutéraux'15; Davison, Goutéraux & Gentle'18] to reproduce the low T scalings of currents

$$[s] = d - \theta\,, \qquad [n]_{IR} = d - \theta + \Phi$$

  θ: effective spatial dimensionality [Kanitscheider & Skenderis'09; Goutéraux & Kiritsis'11; Goutéraux, Skenderis, Smolic, Smolic & Taylor'12].
  Φ: Anomalous charge dimension?

- Reflects presence of emergent dofs coupling to J^μ?

Final comments

- In DQCPs, emergent gauge fields lead to large anomalous dimensions, [Senthil, Vishwanath, Balents, Sachdev & Fisher'03].

- Emergent gauge fields often associated with emergent higher-form symmetries, anomalies and fractionalized dofs [Sachev'18], [Else, Senthil & Thorngren'20], which affect Luttinger theorem

$$j^t = Vol_{FS} + n_{egf}$$

- In holography, charged horizons dubbed fractionalized since no FS in correlators, [Huijse & Sachdev'11, Hartnoll'11,...]. Make this more precise? Deconfined nature of horizon dofs?

- Use holographically-derived EFTs to study unconventional quantum critical phases in cond mat?