SYMMETRIC PRODUCT ORBIFOLDS

Alejandra Castro
DAMTP

29th IFT Xmas Workshop
Madrid, December, 2023

HOW TO ENGINEER QUANTUM GRAVITY IN ADS_3/CFT_2

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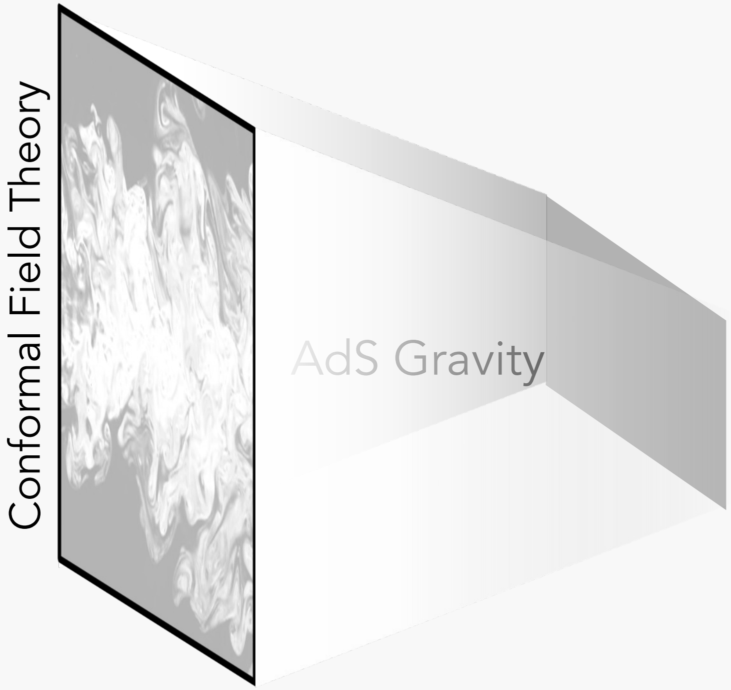
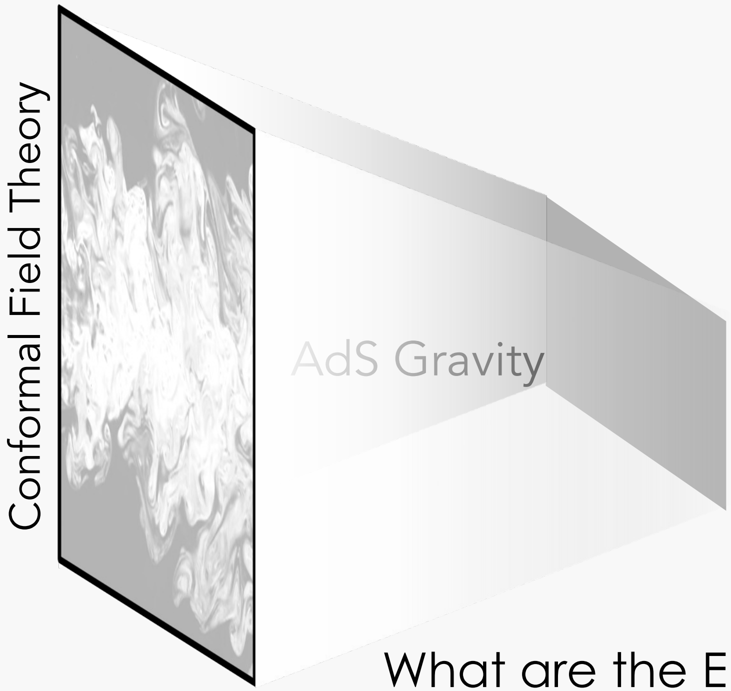
CFT_D → AdS_{D+1} Gravity



CFT_D ➔ AdS_{D+1} Gravity



What are the EFT that admit a UV completion in QG?

Landscape

What are the consequences of this connection?

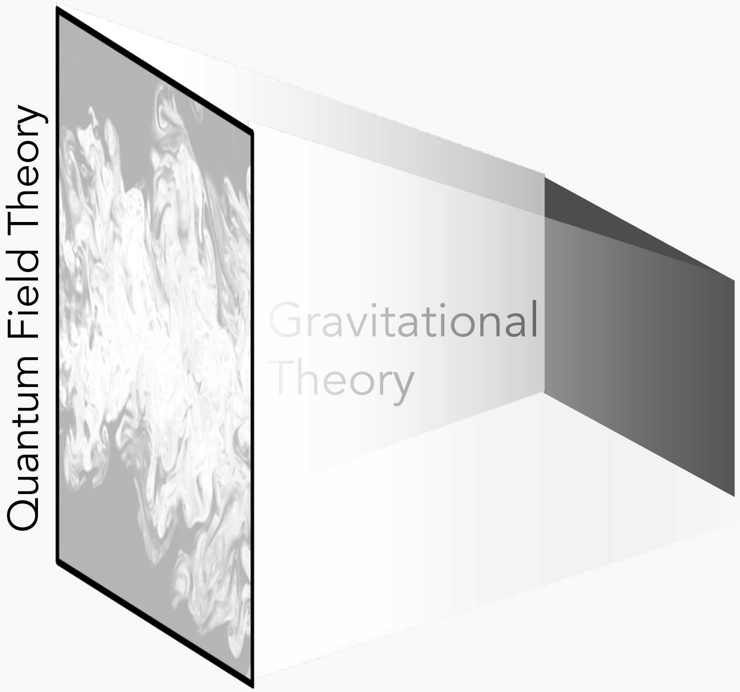
Principles & Laws in QG

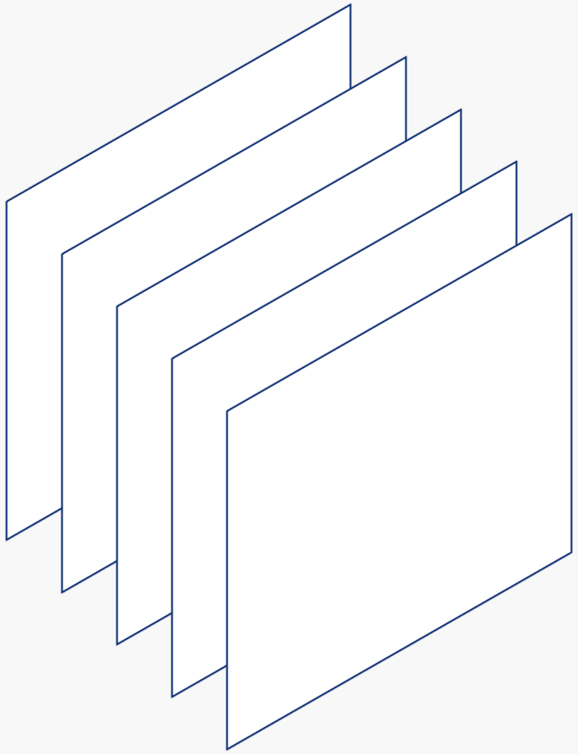
Engineering AdS_3 Quantum Gravity

Symmetric Product Orbifolds

Engineering AdS_3 Quantum Gravity

- Define gravity via the dual CFT
- Identify necessary conditions
- Possible designs we can achieve





- Implement conditions
- Precise outcomes (with surprises)
- New features in the design of AdS/CFT

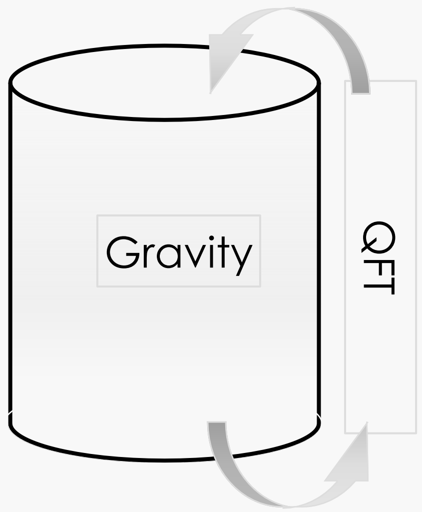
A. Belin, J. Gomes, C. Keller, AC, 2016, 2018
A. Belin, C. Keller, B. Mühlmann, AC, 2019 (x2)
A. Belin, N. Benjamin, C. Keller and S. Harrison, AC, 2020
L. Apolo, A. Belin, S. Bintanja, C. Keller, AC 2022
N. Benjamin, S. Bintanja, J. Hollander, AC 2022
L. Apolo, A. Belin, S. Bintanja, C. Keller, AC 2023

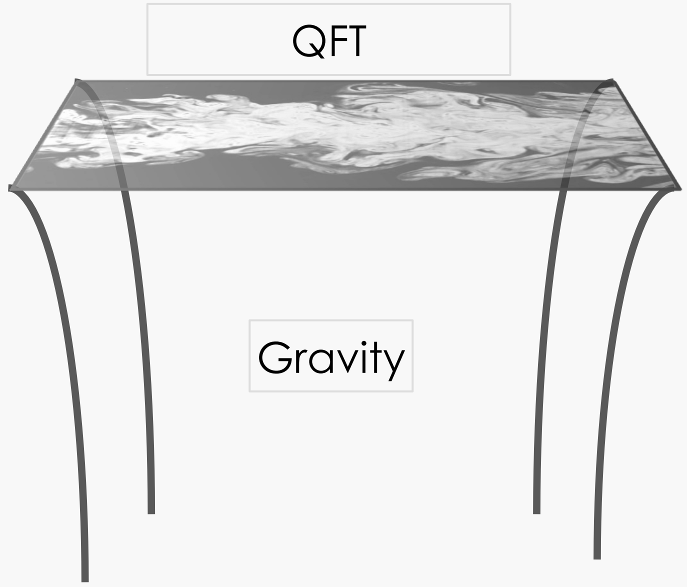
Symmetric Product Orbifolds

Engineering AdS_3 Quantum Gravity

Holographic Principle

Gravitational theory in D+1 dimensions is equivalent to quantum field theory in D dimensions





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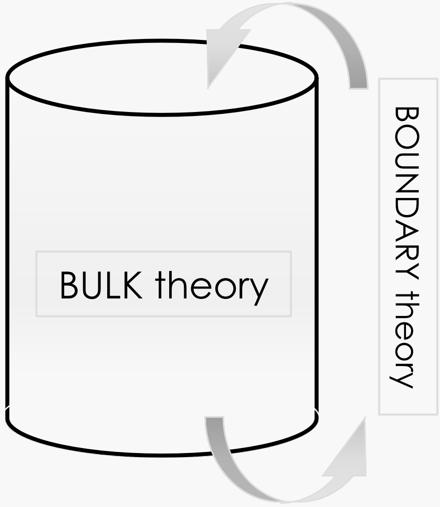

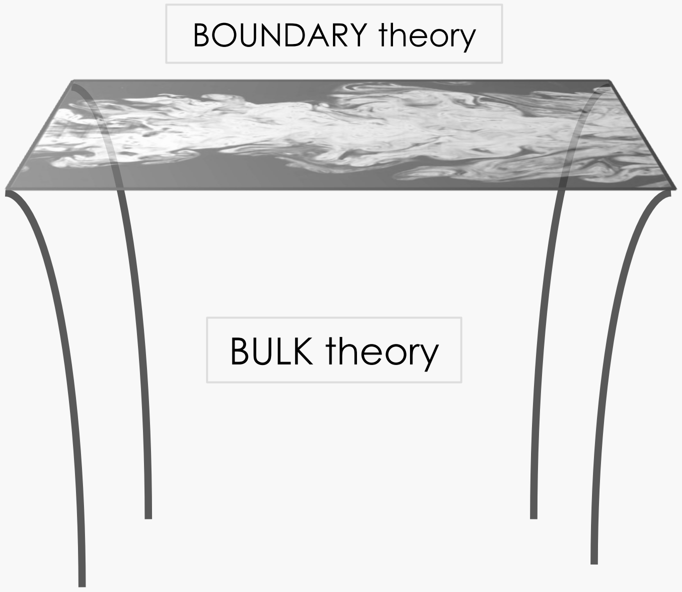

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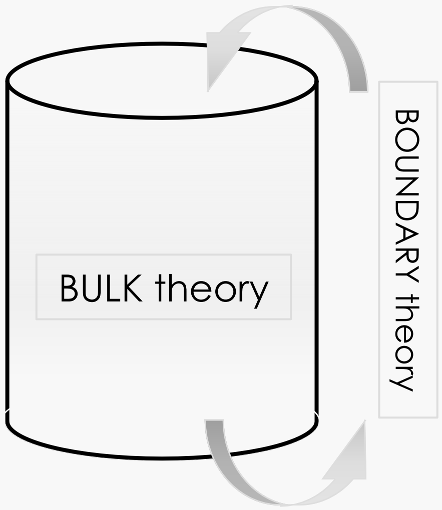
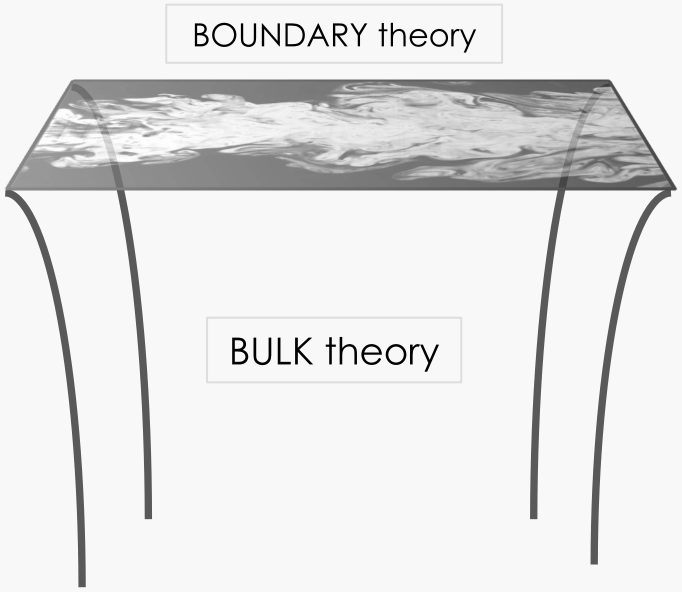




Control and precision:
AdS_{D+1}/CFT_D

AdS_{D+1}/CFT_D

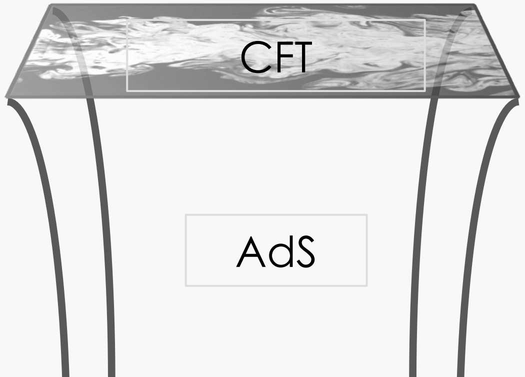
Born and refined in String Theory, the most precise incarnation states:

<u>AdS Gravity</u>

AdS_{D+1} = gravity with negative cosmological constant in D+1 spacetime dimensions.

<u>Conformal Field Theory</u>

CFT_D = quantum system invariant under dilations in D spacetime dimensions.



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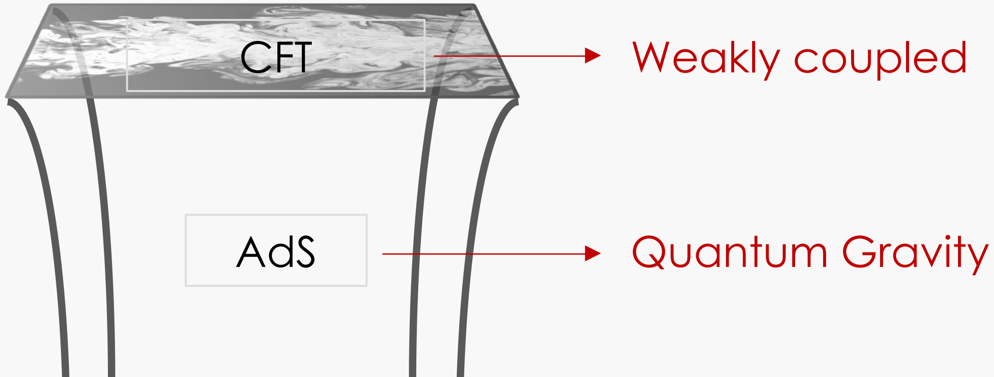

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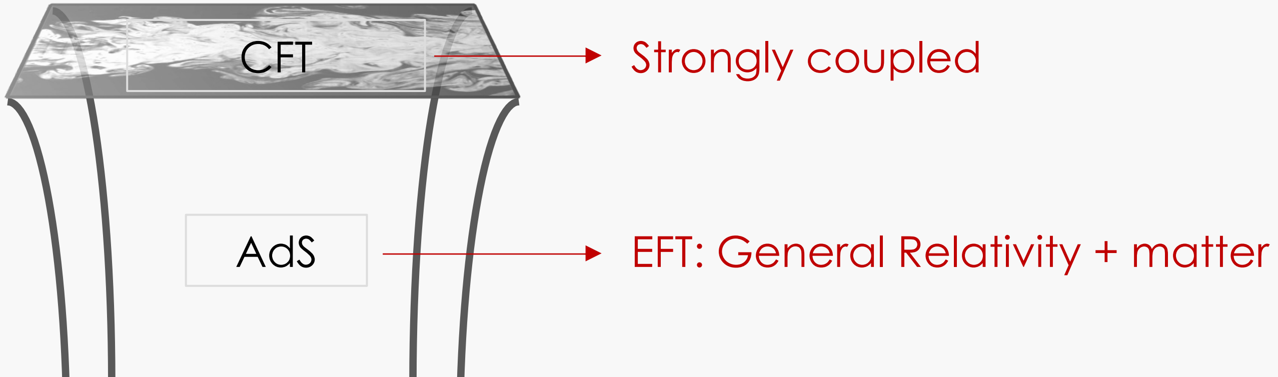
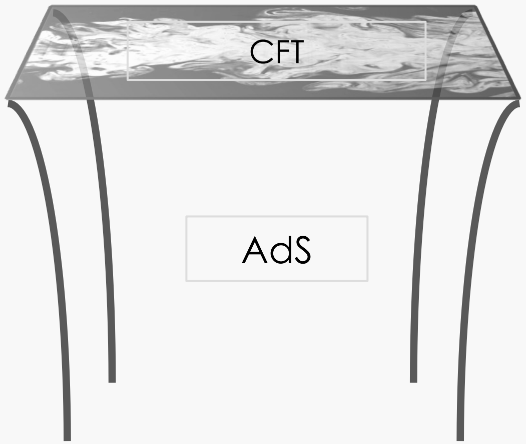
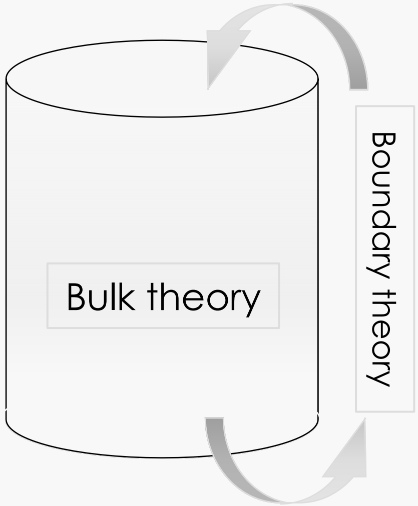


AdS_{D+1}/CFT_D

It is a **duality**:
It maps a gravitational theory to a CFT and viceversa.

It is a **framework**:
The dual CFT predicts the adequate observables in QG.

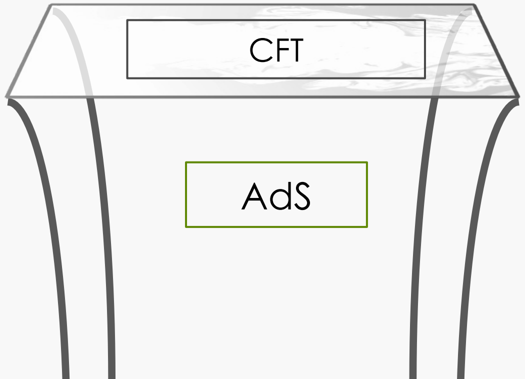




If we want to capture the fundamental mechanism of AdS/CFT, we need more than a few examples.

Possible Directions:

- Given a CFT, how to organize it such that quantum gravity is manifest?
- Which CFTs capture classical (geometric) properties of gravity?
- Select a theory of gravity, what are properties of the dual?



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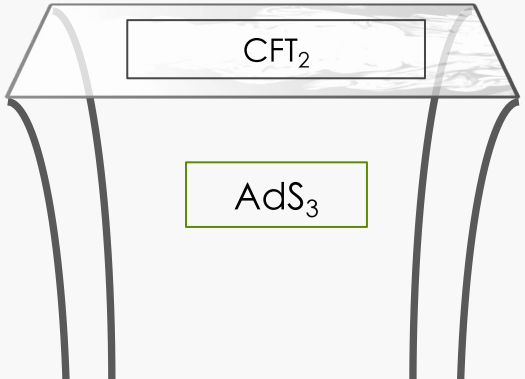
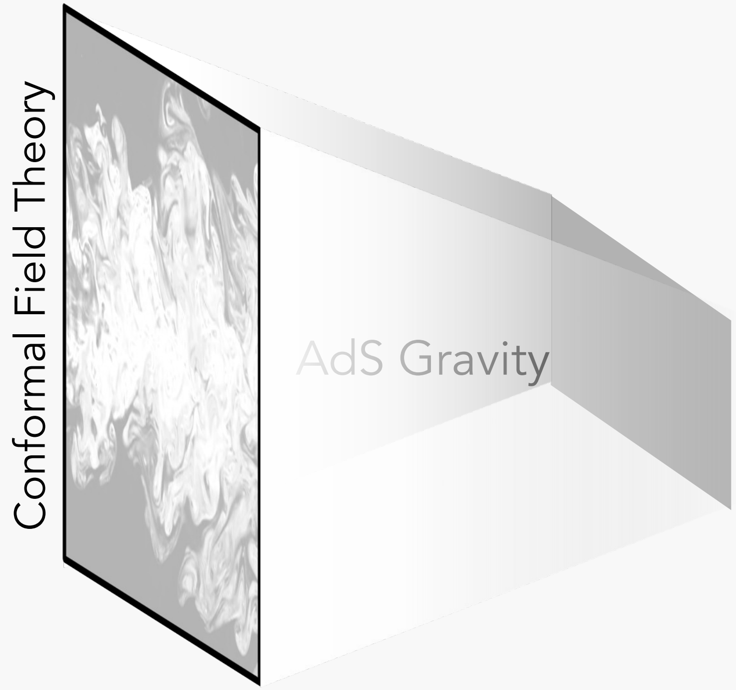


We will focus on the difficulties you encounter in AdS_3/CFT_2.
Not universal, but it illustrates the challenges.

CFT_2 ➝ AdS_3 Gravity



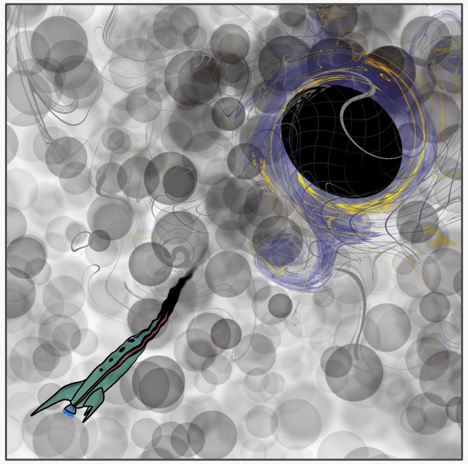
AdS$_3$ Gravity

The theory:

$$I_{3D} = \frac{1}{16\pi\, G_N}\int d^3x\,\sqrt{-g}\,\left(R + \frac{2}{\ell^2}\right)$$ + matter content: scalars, gauge fields, fermions

Basic Requirements:
Gravitational theories that have the property that their low-energy description is given by a local EFT.

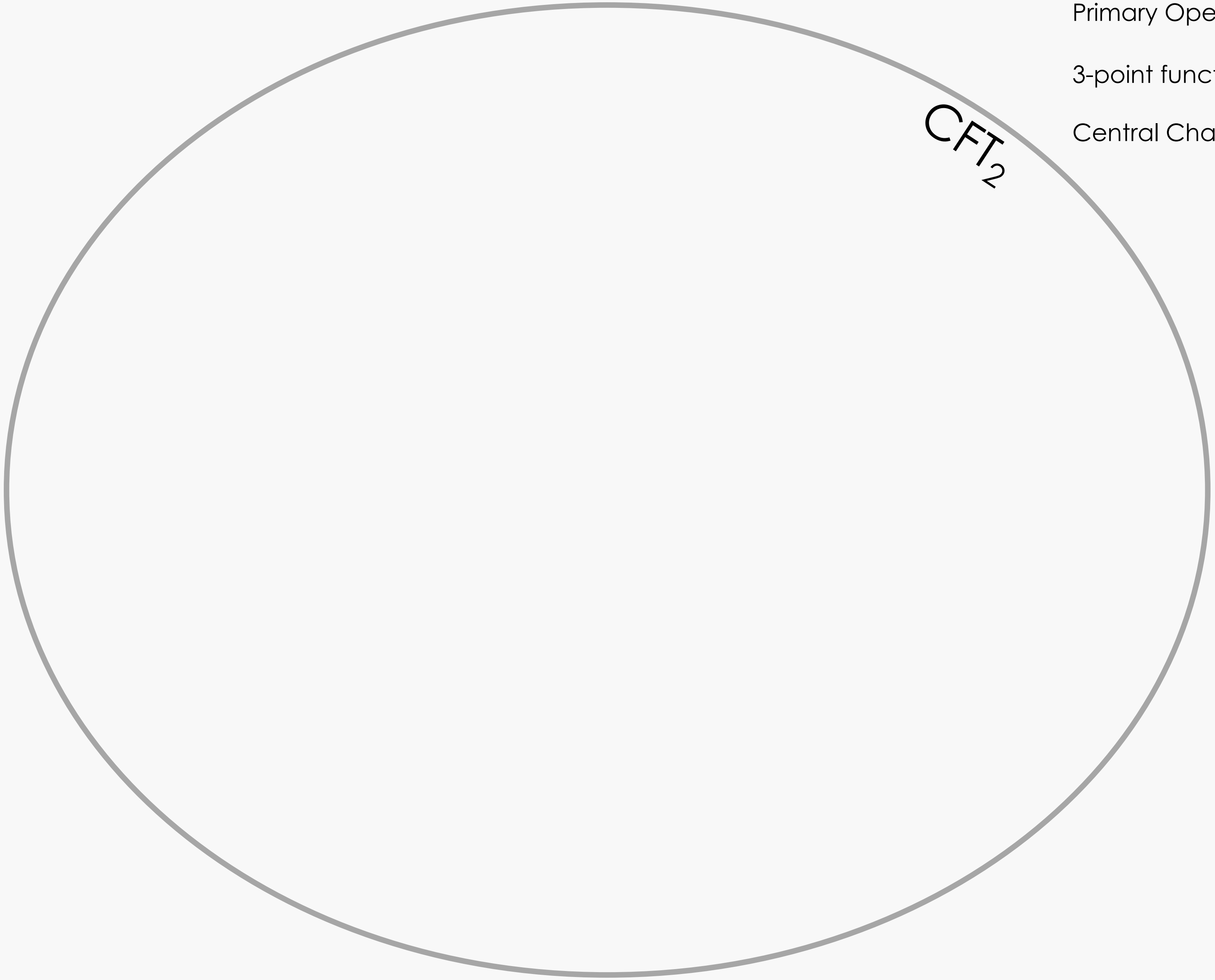
Universal entry in AdS$_3$/CFT$_2$: $c = \frac{3\ell}{2G_N} \gg 1$



Primary Operators: Δ_J (Conformal dimensions)

3-point functions: C_{IJK} (OPE coefficients)

Central Charge: c



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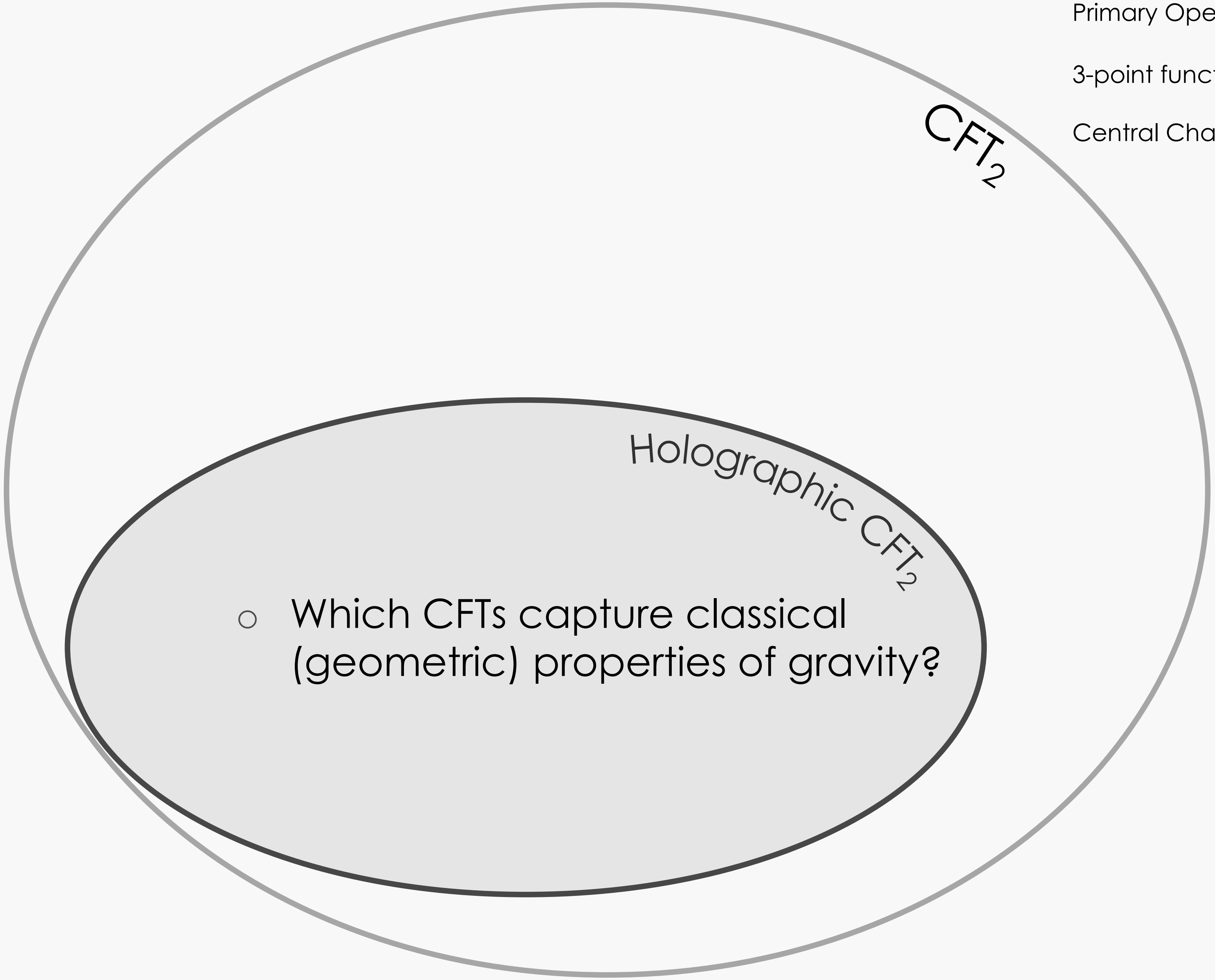

Warning: Sizes are not meaningful.

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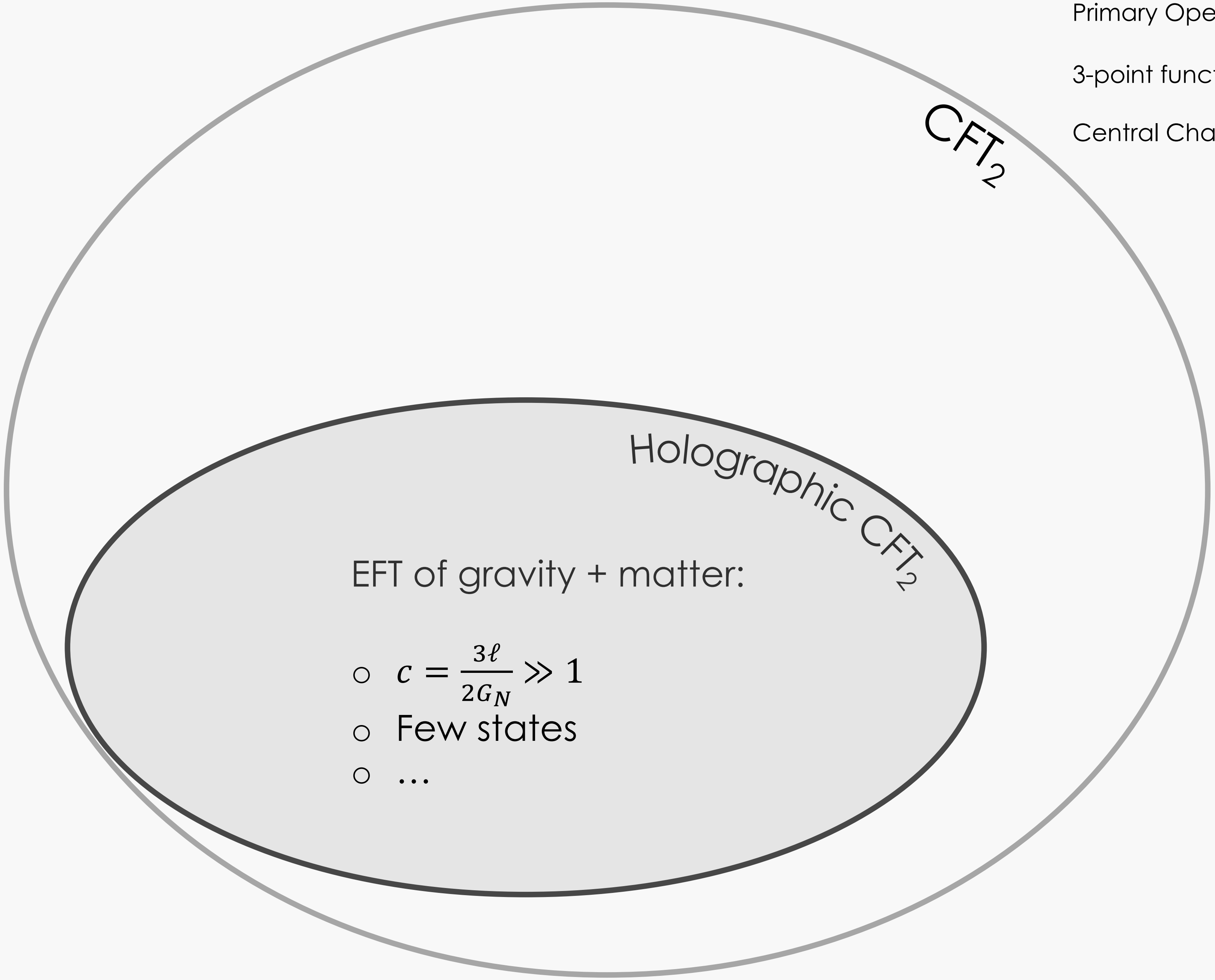

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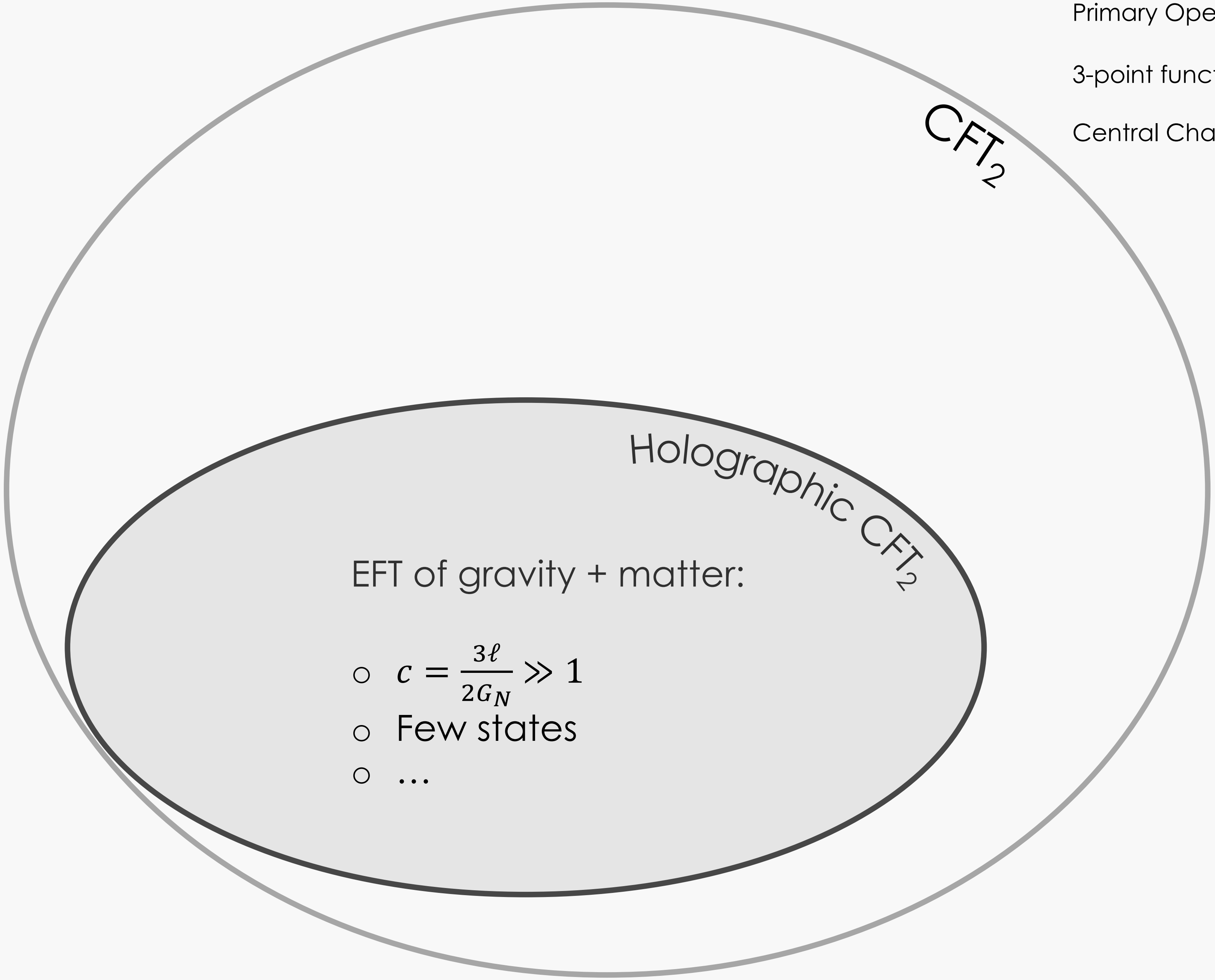

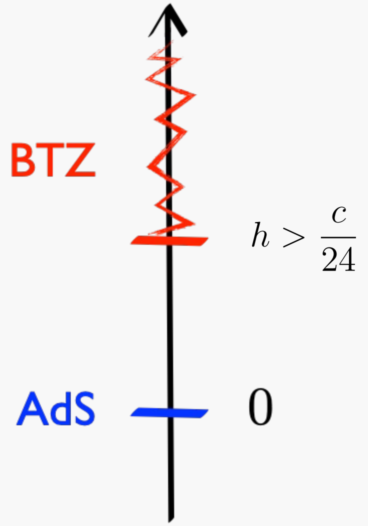

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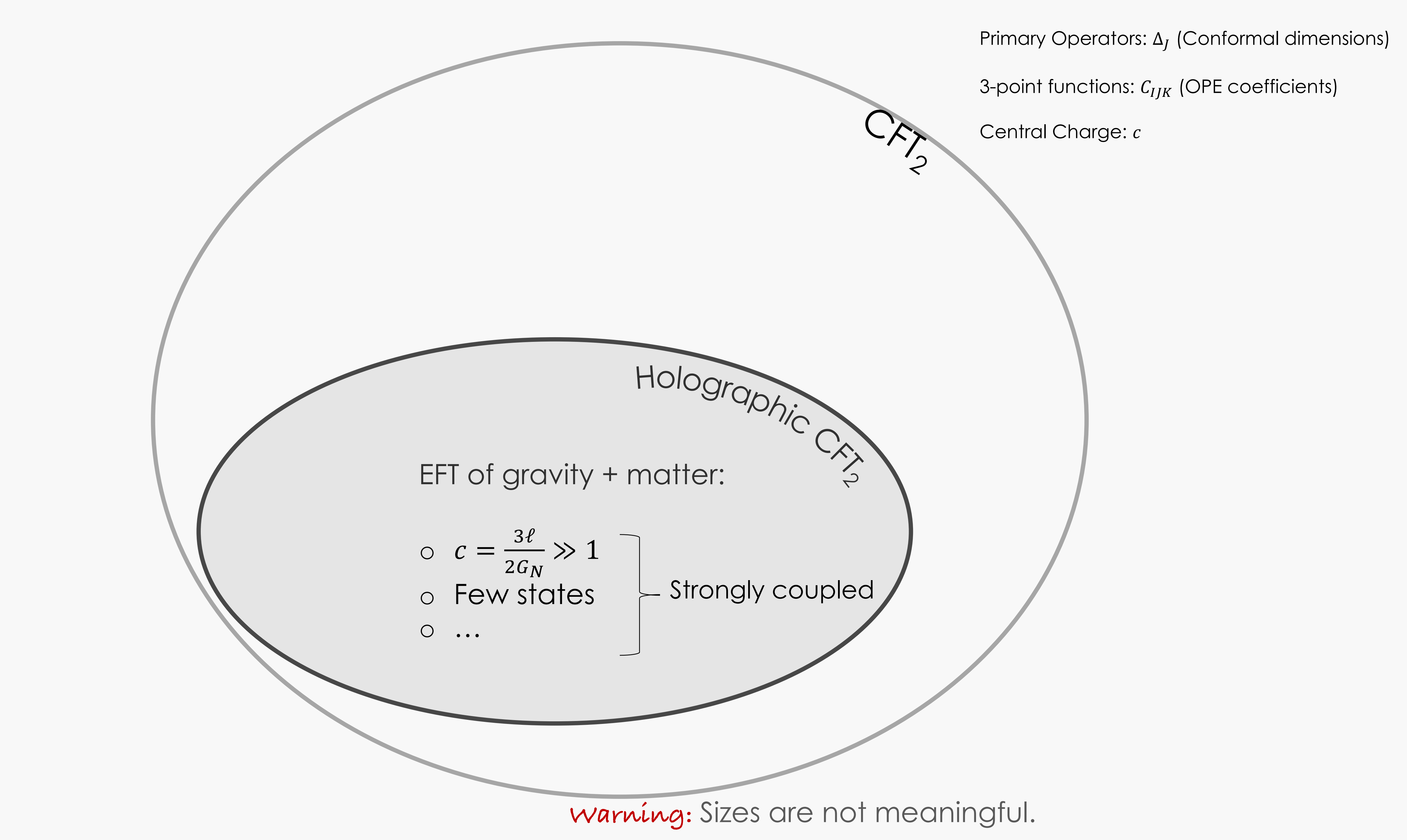

Primary Operators: Δ_J (Conformal dimensions)
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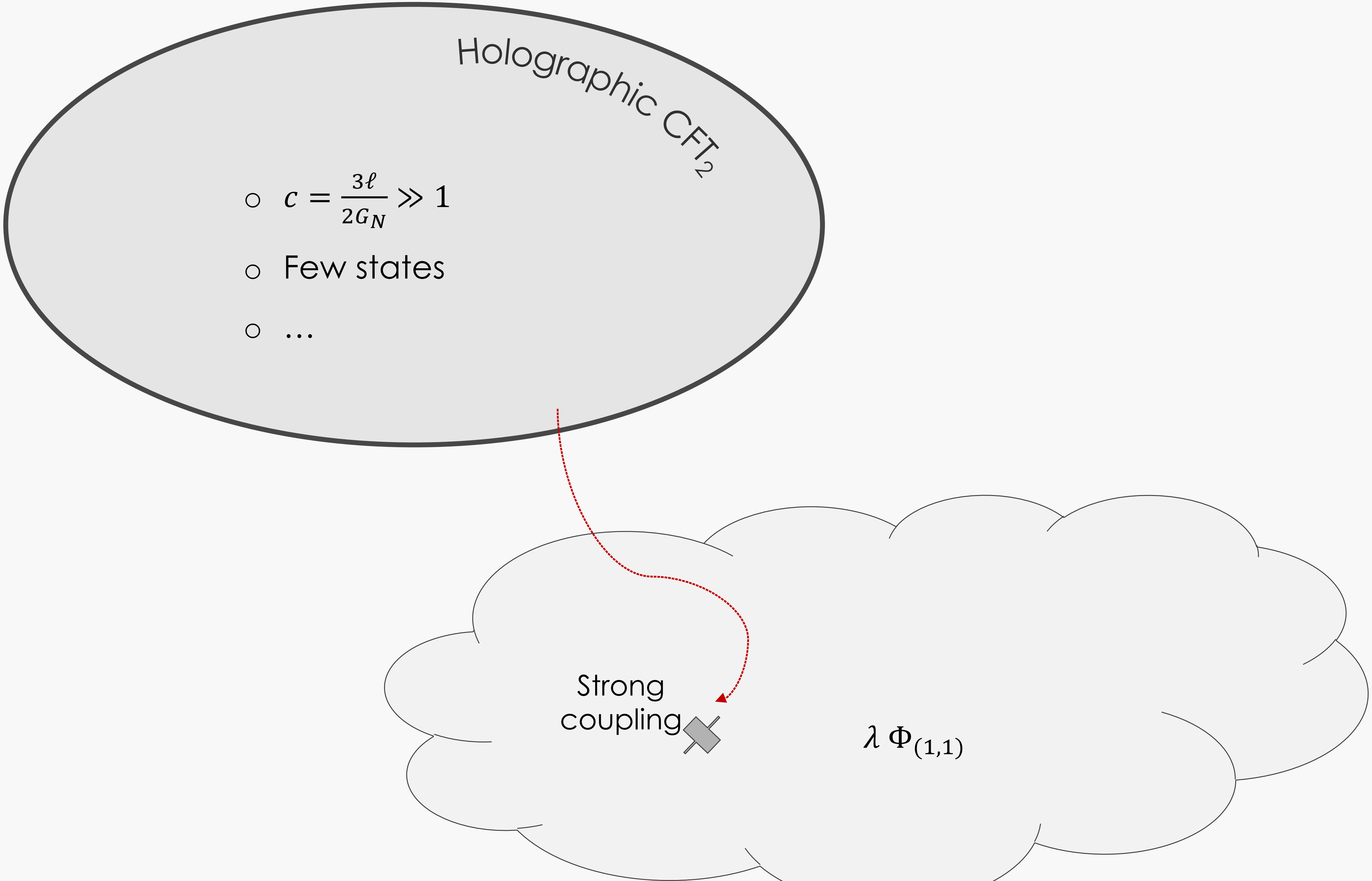
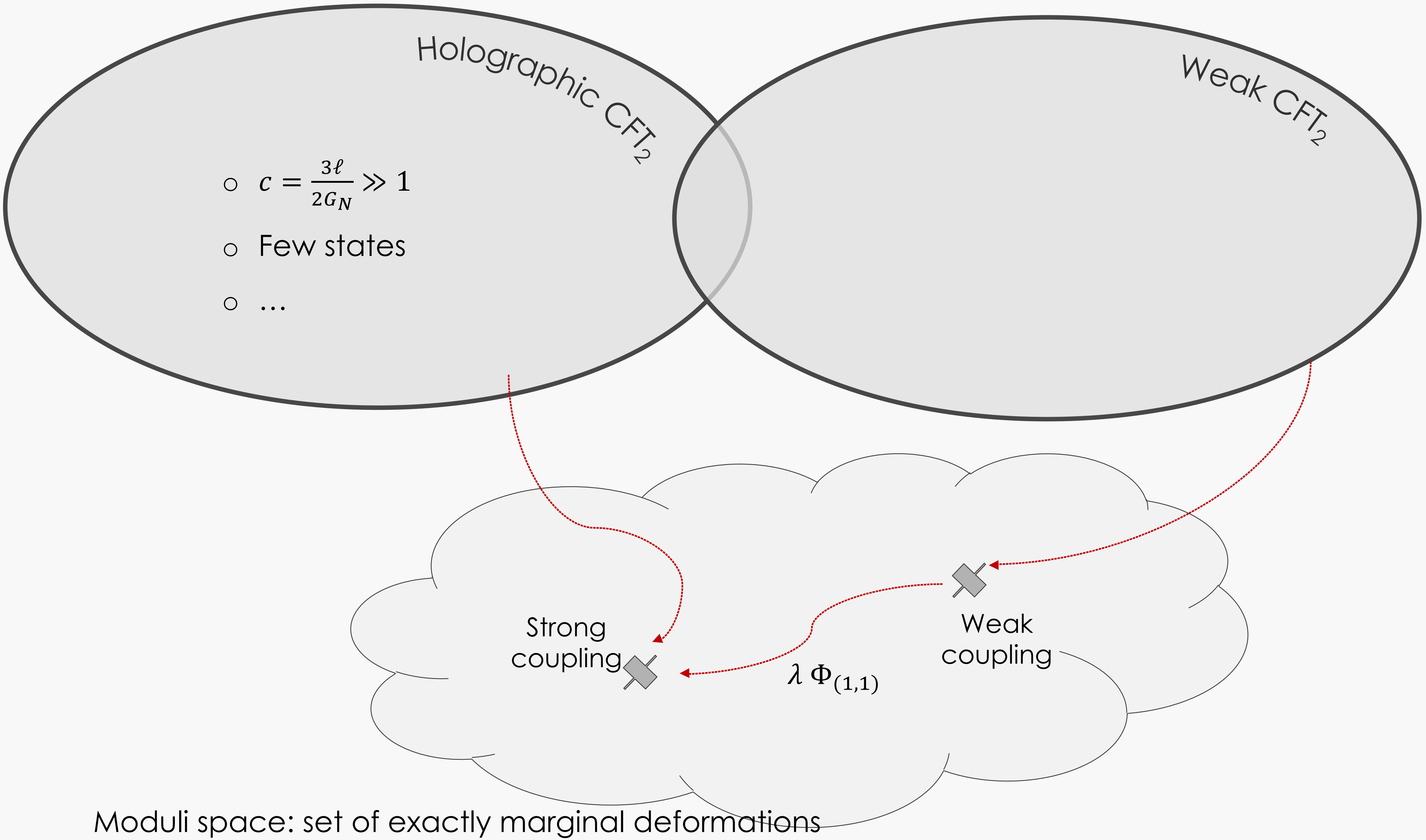
Central Charge: c
CFT_2
Holographic CFT_2
EFT of gravity + matter:
$c = \frac{3\ell}{2G_N} \gg 1$
Few states
...
Strongly coupled
Warning: Sizes are not meaningful.



Moduli space: set of exactly marginal deformations


Holographic CFT_2
$c = \frac{3\ell}{2G_N} \gg 1$
Few states
…
Weak CFT_2
Strong coupling
Weak coupling
$\lambda\, \Phi_{(1,1)}$
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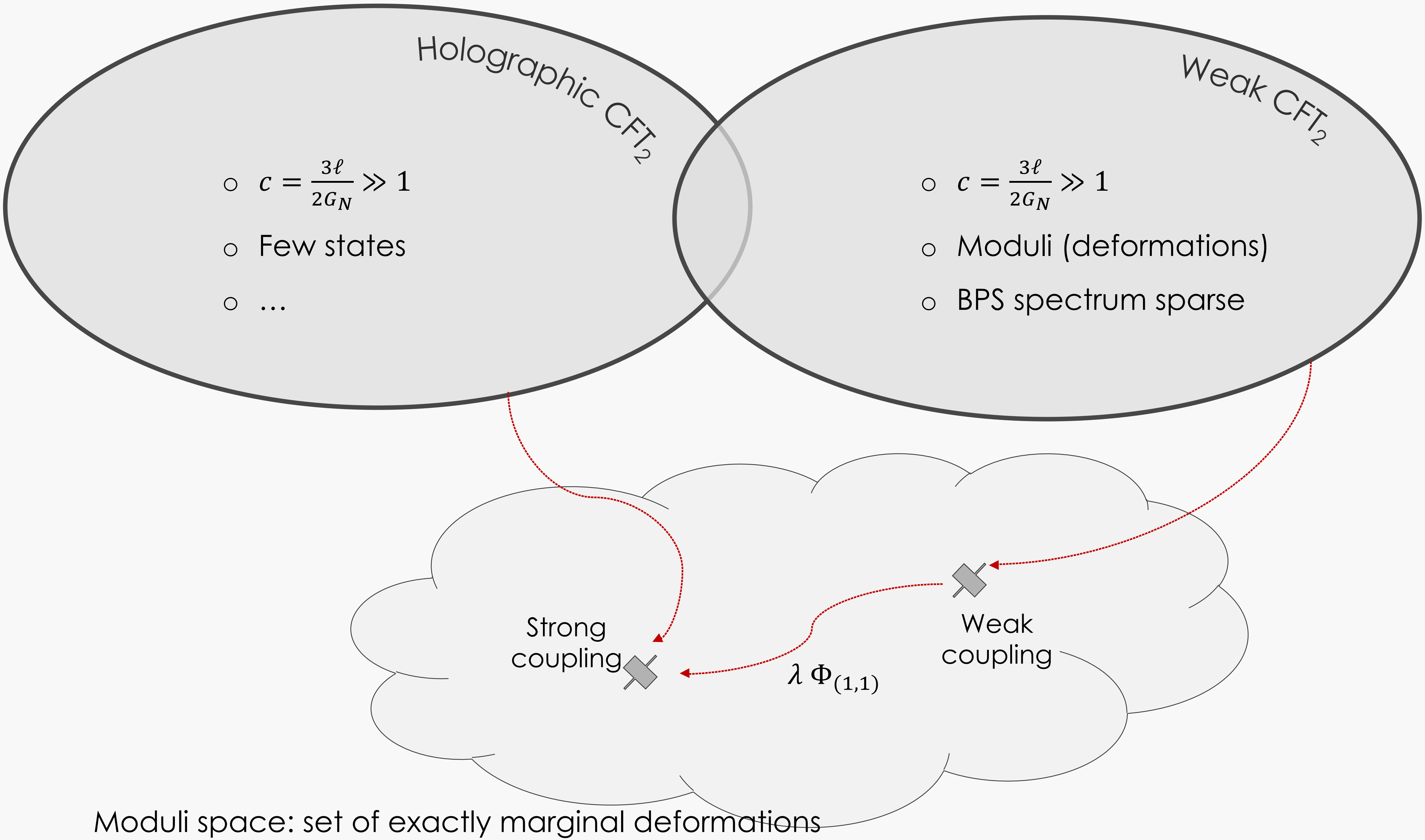

Holographic CFT$_2$
$c = \frac{3\ell}{2G_N} \gg 1$
Few states
…
Weak CFT$_2$
$c = \frac{3\ell}{2G_N} \gg 1$
Moduli (deformations)
BPS spectrum sparse
Strong coupling
Weak coupling
$\lambda\, \Phi_{(1,1)}$
Moduli space: set of exactly marginal deformations

Symmetric Product Orbifolds

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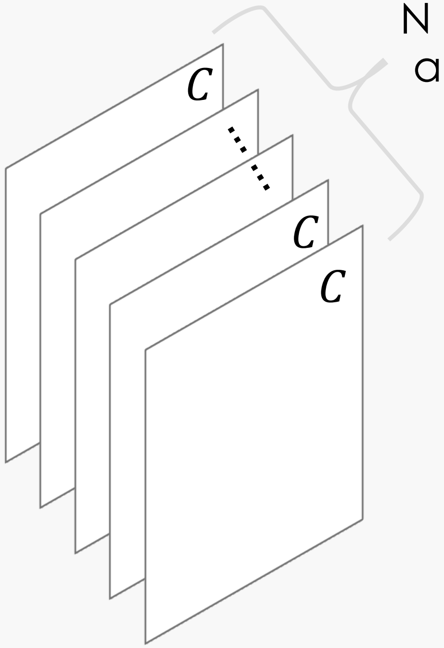
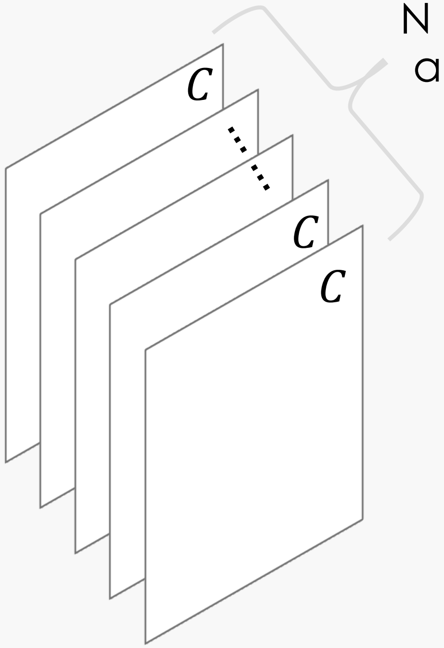


N copies of a unitary and compact CFT_2.
C = seed theory

$$Sym^N(C) = \frac{C^{\otimes N}}{S_N}$$

Orbifold by the permutation group S_N

Symmetric Product Orbifolds



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Orbifold by the permutation group S_N

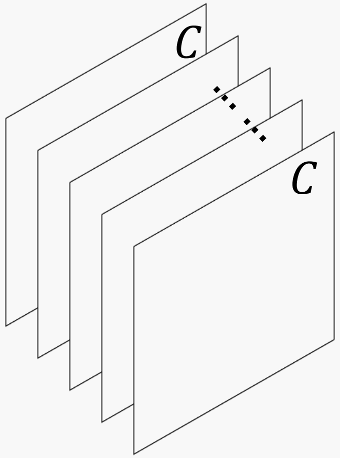
$$Sym^N(\mathcal{C}) = \frac{\mathcal{C}^{\otimes N}}{S_N}$$

The orbifold introduces two class of states:

- untwisted sector: it keeps states that are invariant under S_N.
- twisted sectors: new states labelled by conjugacy classes of S_N.

Symmetric Product Orbifolds

- Appeal: Mathematical and analytic control, e.g., DMVV formula.
- Familiarity: D1D5 CFT.
- Universality: large-N behavior is robust.
- Utility: compelling features for AdS/CFT.



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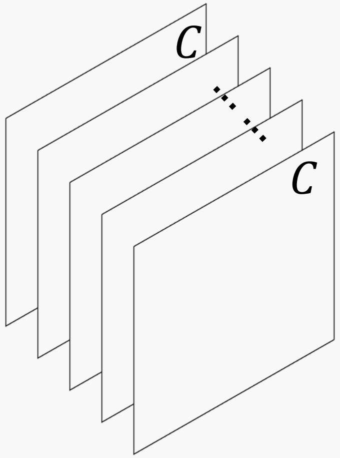
Today: non-universal properties. Demonstrate that there are different classes, and their features challenge the lore of AdS/CFT.



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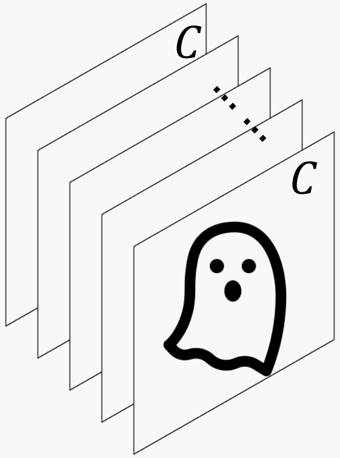


Universal Aspects

All symmetric product orbifolds satisfy:

- Correlation functions comply with large-N factorization.

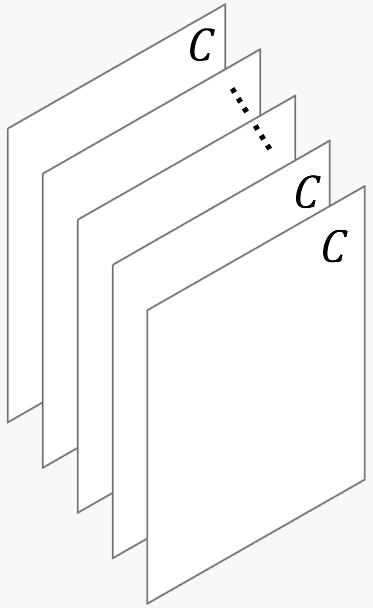
  [Pakman et.al., Mathur et.al., Belin et.al., Hael et.al., …]

- Hawking-Page transition at large-N.

  [Keller 2011; Hartman, Keller, Stoica 2014; Benjamin et.al. 2015]

- Higher spin currents due to orbifold structure.
- Universal Hagedorn growth of light states.

  [Keller 2011]



$$Sym^N(C) = \frac{C^{\otimes N}}{S_N}$$

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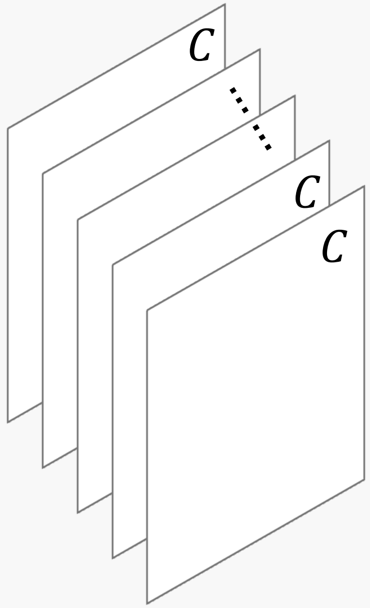

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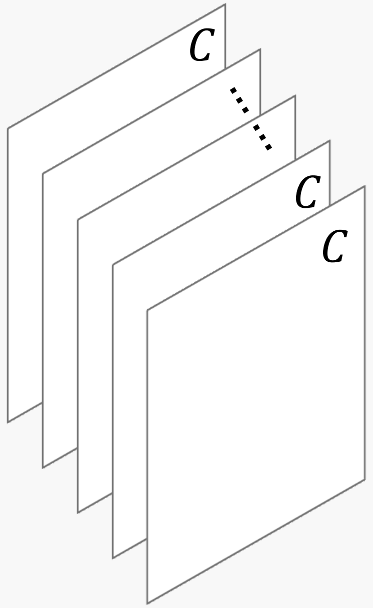
AdS/CFT interpretation: Dual of $Sym^N(C)$ looks like a tensionless string theory (or higher spin gravity).



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- Higher spin currents due to orbifold structure.
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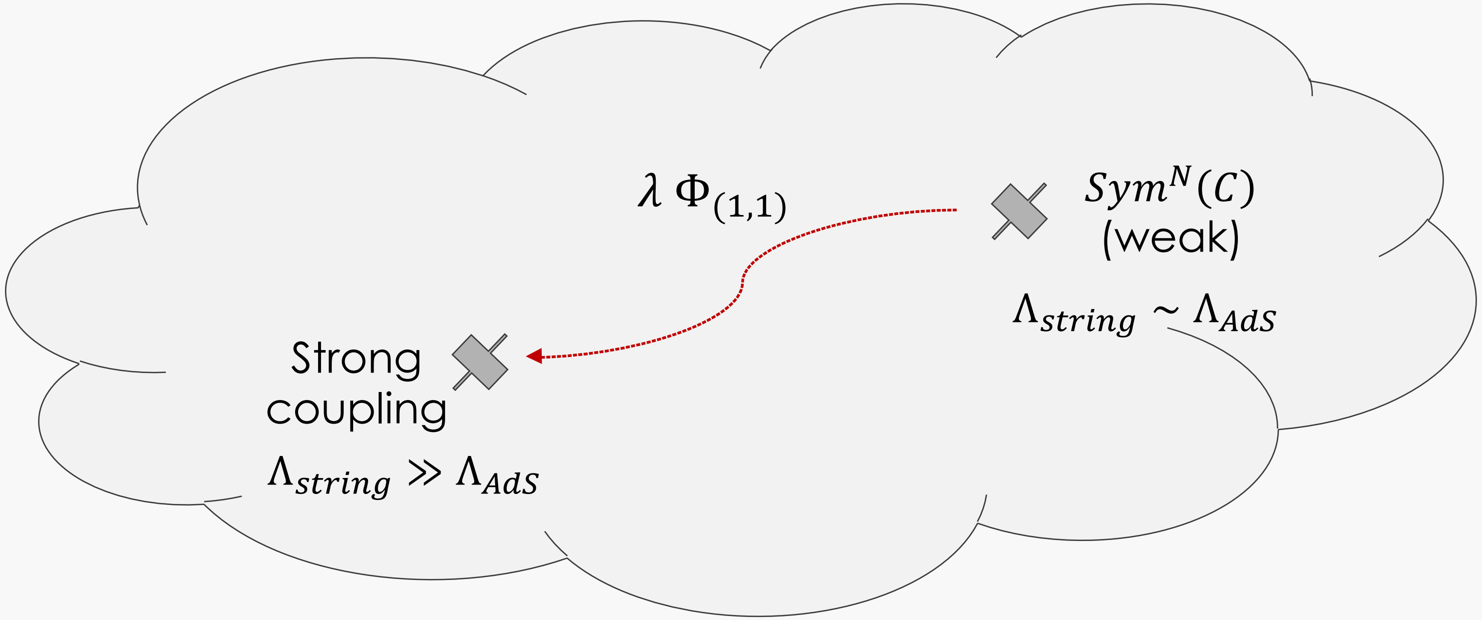
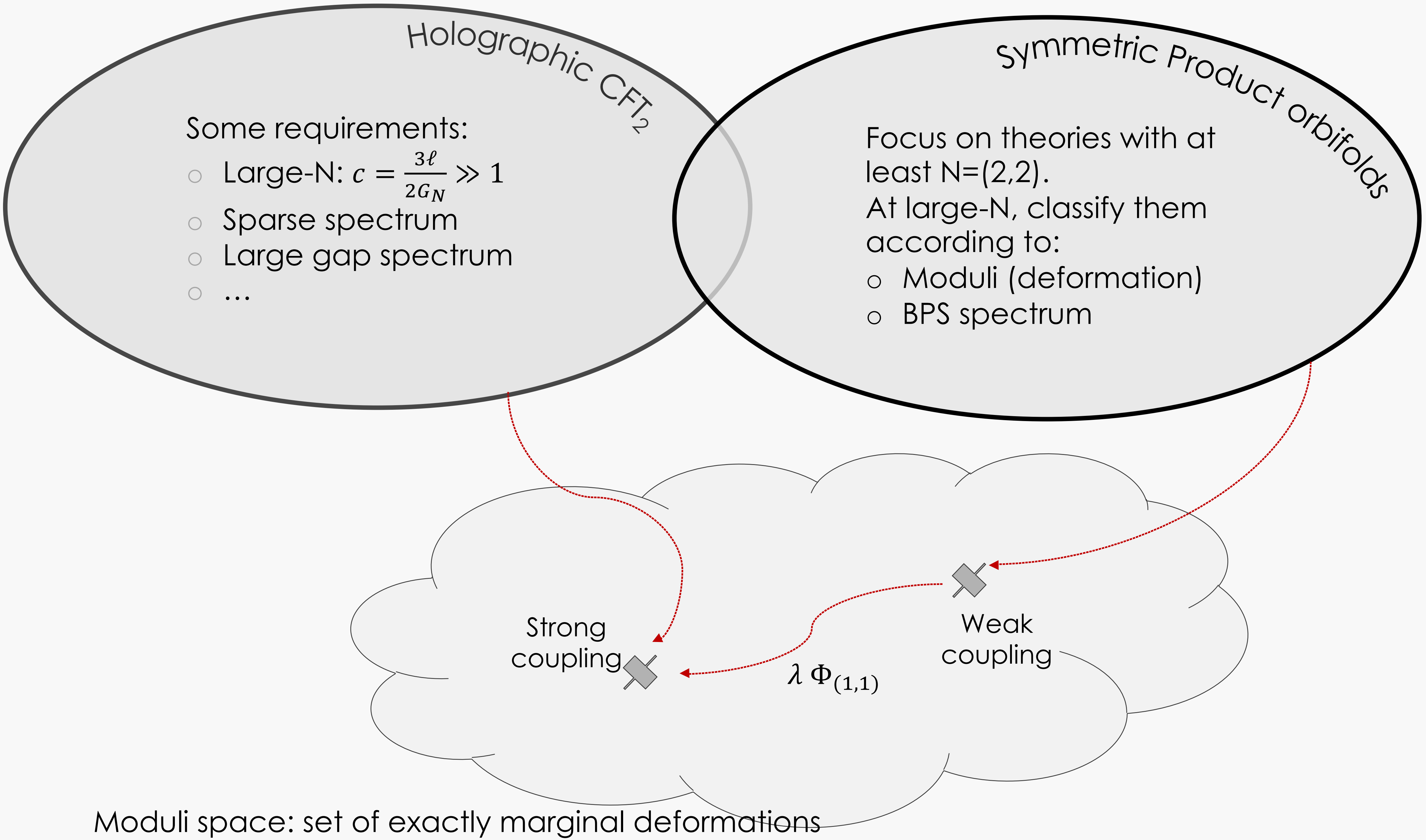
Question: Which $Sym^N(C)$ could admit in their moduli space a dual supergravity point?

Strategy: Impose necessary conditions. Identify which $Sym^N(C)$ comply with them.



Moduli space: set of exactly marginal deformations


Holographic CFT_2
Some requirements:
Large-N: $c = \frac{3\ell}{2G_N} \gg 1$
Sparse spectrum
Large gap spectrum
...
Symmetric Product orbifolds
Focus on theories with at least N=(2,2).
At large-N, classify them according to:
Moduli (deformation)
BPS spectrum
Strong coupling
Weak coupling
$\lambda\, \Phi_{(1,1)}$
Moduli space: set of exactly marginal deformations

Neccesary conditions

- Criterion 1: Existence of suitable moduli (single trace, twisted, BPS).
- Criterion 2: Sparseness condition on the elliptic genera (index that captures BPS states).

Based on these two criteria, we will classify $Sym^N(C)$ theories, and label them as

Type I: Both criteria	Type II: Only criterion 1	Type III: Neither criteria	Type IV: Only criterion 2

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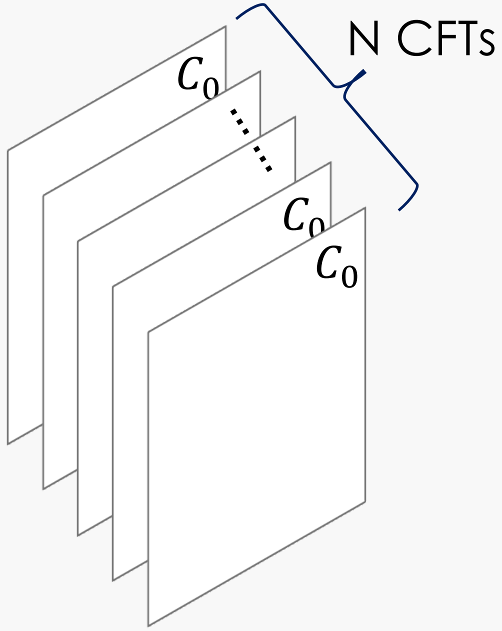
1. We proved that both criteria (independently) imply that seed theory must have

$$1 \leq c_0 \leq 6$$



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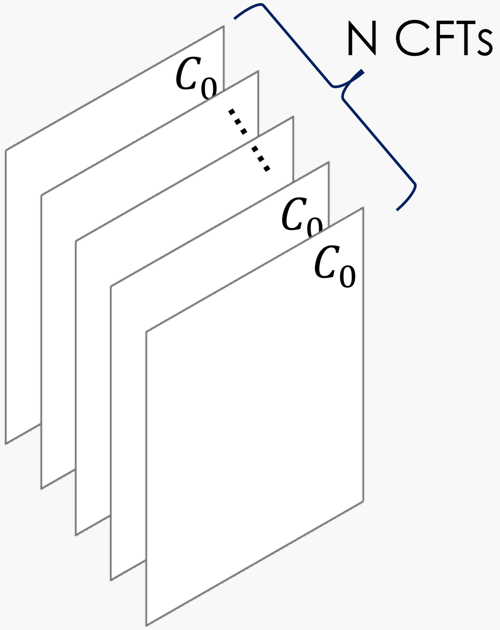

2. Criterion 2 can be done systematically and is exhaustive.

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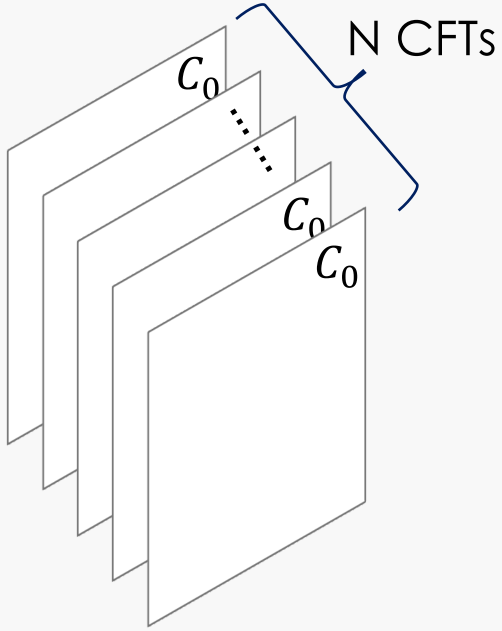

2. Criterion 2 can be done systematically and is exhaustive.

3. If Criterion 2 is satisfied, we proved that one always gets

$d(\Delta) \sim e^{\sqrt{\Delta}}$ where $\Delta \gg 1,\ \Delta \sim O(N^0)$

Classification

Type I: Both criteria	→	**Needles in a haystack.** Comply with necessary conditions to lead to a holographic CFT.
Type II: Only criterion 1	→	**Strange and counter-intuitive.** Moduli exists, but Hagedorn behavior persists.
Type III: Neither criteria	→	**Generic, most abundant.** They will never lead to a supergravity point in moduli space.
Type IV: Only criterion 2	→	**Unicorns.** No unitary example yet. Modular invariance does not rule it out.

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	Theory	Sparse?	Moduli?	Composition
	$A_6 \otimes A_{41}$	✓	✓	(11,88), (22,22)
	$A_7 \otimes A_{23}$	✓	✓	(11,55),(22,22)
	$A_8 \otimes A_{17}$	✓	✓	(11,44),(22,22)
	$A_9 \otimes A_{14}$	✓	✓	(22,22)
	$A_{11} \otimes A_{11}$	✓	✓	(11,33),(33,11),(22,22)
	$A_6 \otimes D_{22}$	✗	✗	
	$A_7 \otimes D_{13}$	✗	✓	(11,55)
	$A_{23} \otimes D_5$	✗	✓	(55,11)
	$A_8 \otimes D_{10}$	✗	✗	
	$A_{14} \otimes D_6$	✗	✗	
	$A_{11} \otimes D_7$	✓	✓	(11,33),(33,11)
	$A_8 \otimes E_7$	✗	✗	
Type II	$A_{11} \otimes E_6$	✗	✓	(33,11)
	$D_5 \otimes D_{13}$	✗	✓	(11,55)
	$D_7 \otimes D_7$	✓	✓	(11,33),(33,11)
Type I	$D_7 \otimes E_6$	✓	✓	(33,11)
	$E_6 \otimes E_6$	✗	✗	
	$A_2 \otimes A_5 \otimes A_5$	✓	✓	(11,11,22),(11,22,11)
	$A_2 \otimes A_5 \otimes D_4$	✓	✓	(11,22,11)
	$A_2 \otimes D_4 \otimes D_4$	✗	✗	
	$A_3 \otimes A_3 \otimes A_5$	✓	✓	(11,11,22)
	$A_3 \otimes A_3 \otimes D_4$	✗	✗	

Examples of theories where the seed has $c_0 = 5$

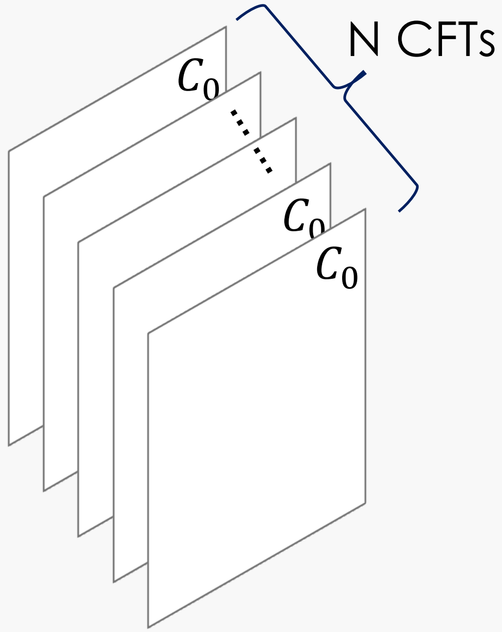
Comparisson

Type I:
Both criteria → Needles in a haystack.
Comply with necessary conditions to lead to a holographic CFT.

Type II:
Only criterion 1 → Strange and counter-intuitive.
Moduli exists, but Hagedorn behavior persists 👻

- Evaluated anomalous dimension of several holomorphic operators (currents).
- Type I and II theories exhibit no difference at leading order in perturbation theory. 😖
- What is the key feature that guarantees a supergravity point in moduli space?

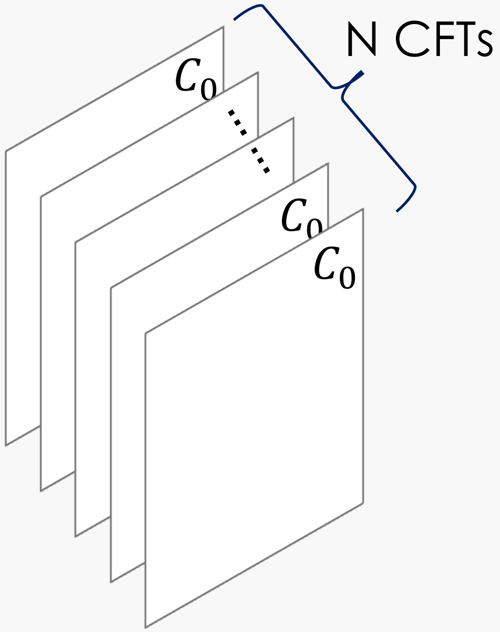
Summary



Summary

Comments:

- Only consider CFTs that are unitary and compact.
- Assume that the elliptic genus does not vanish.
- D1D5 on K3 sits at $c_0 = 6$.
- Search between $1 \leq c_0 < 3$ is exhaustive: N=2 Minimal Models.
- Search between $3 \leq c_0 \leq 6$ is not exhaustive (but systematic).



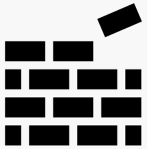
Type I: Examples

Series	k	untwisted moduli	twisted moduli	single trace twisted
A_2	1	1	28	1 twist 5, 1 twist 7
A_3	2	3	26	1 twist 3, 1 twist 4, 1 twist 5
A_5	4	9	24	1 twist 2, 1 twist 3, 1 twist 4
A_{k+1}	odd, ≥ 3	$P(k+2)-2$	9	1 twist 3
A_{k+1}	even, ≥ 6	$P(k+2)-2$	$10+\sum_{r=1}^{\frac{k}{2}+2} P(r)$	1 twist 2, 1 twist 3
D_4	4	6	20	1 twist 2, 2 twist 3, 1 twist 4
$D_{\frac{k}{2}+2}$	0 mod 4, ≥ 8	$P(\frac{k}{2}+1)+P(\frac{k}{4}+1)$	$8+\sum_{r=1}^{\frac{k}{4}+1} P(r)$	1 twist 2, 1 twist 3
$D_{\frac{k}{2}+2}$	2 mod 4, ≥ 6	$P(\frac{k}{2}+1)$	7	1 twist 3
E_6	10	4	5	1 twist 2
E_7	16	6	5	1 twist 2
E_8	28	6	5	1 twist 2

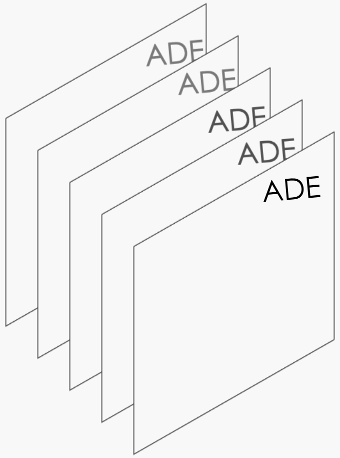
N=2 Virasoro Minimal Models

$$c_0 = \frac{3k}{k+2} < 3$$

where $k = 1, 2, \ldots$



Necessary conditions:
- Criterion 1: Exactly marginal operator
- Criterion 2: Sparse spectrum for elliptic genera



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Responsible of lifting most states.
Breaks higher spin symmetry

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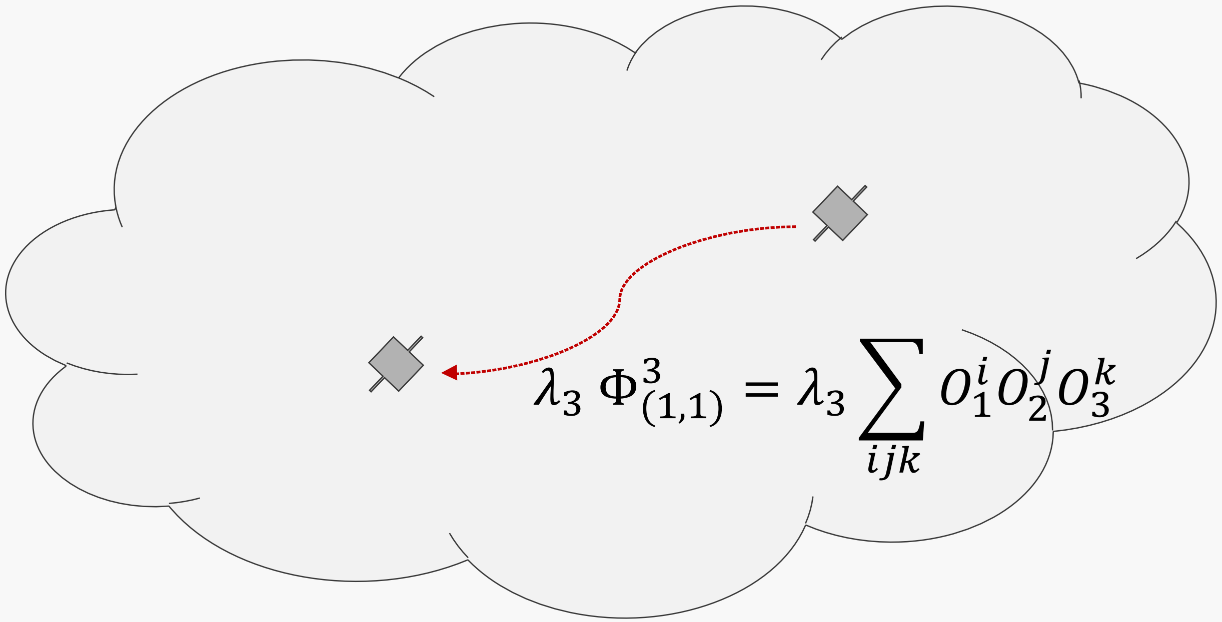
Multi-trace deformations.
Formally known, first explicit example of CFT with these BPS deformations.
They can turn on couplings, which hints to strongly coupled matter.

Destroy Factorization

Consider any CFT that complies with Large-N Factorization



To make sure that the deformation is robust, we ask that $\Phi^3_{(1,1)}$ is:

- Marginal
- ½-BPS

Note: it can be either twisted or untwisted.

The coupling λ is independent of N.

Destroy Factorization

Consider any CFT that complies with Large-N Factorization.
Next deform the theory by $\Phi^3_{(1,1)} = \sum_{ijk} O_1^i O_2^j O_3^k$

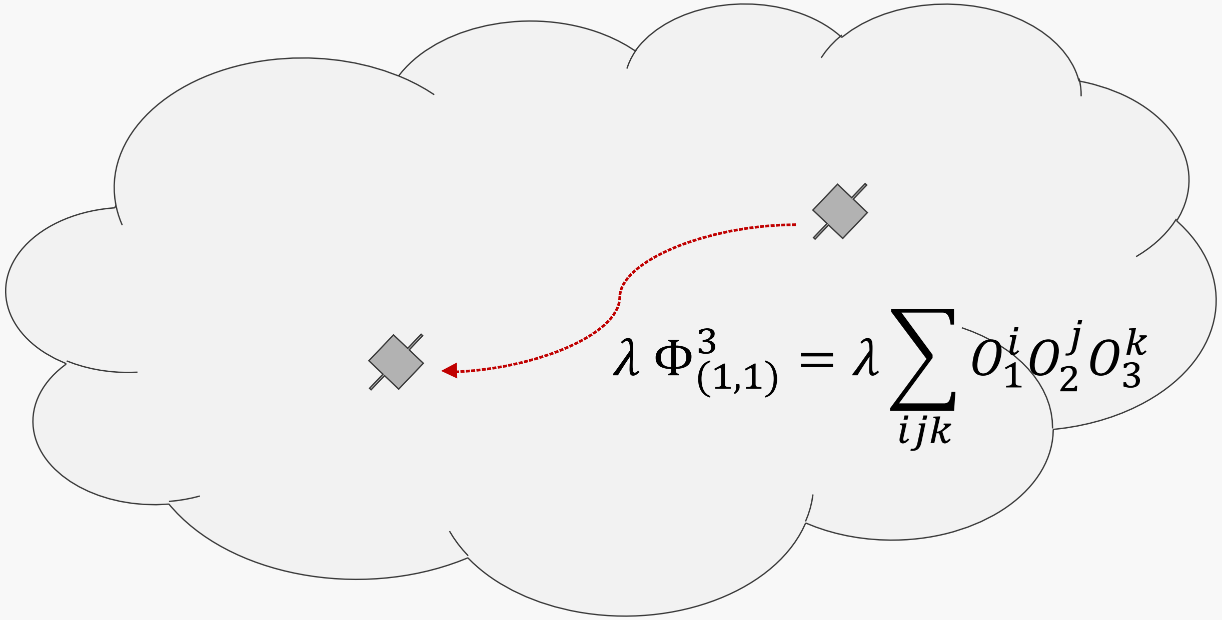
$$\langle O_1(z) O_2(w) O_3(w) \rangle_\lambda = \langle O_1(z) O_2(w) O_3(w) \rangle_0 + \lambda_3 \int \left\langle O_1(z) O_2(w) O_3(w) \Phi^3_{1,1}(w) \right\rangle_0 dw + \cdots$$

Suppressed by $O\left(N^{-\frac{1}{2}}\right)$
Large-N factorization

Leading order $O(N^0)$
Introduced a new coupling!

Destroy Factorization

Consider any CFT that complies with Large-N Factorization



$$\langle O_1 O_2 O_3 \rangle_\lambda \sim \lambda\, N^0 + \cdots$$

- Breaks large-N factorization
- Interactions that are not controlled by G_N
- Type I theories have these deformations
- Argument is general: applies to $\mathrm{CFT_D}$

The coupling λ is independent of N.
This deformation does not affect the convergence of the large-N limit (observables finite).

AdS_3/CFT_2

Typical theory: Couplings controlled by G_N

$$I_{3D} = \frac{1}{16\pi\, G_N}\int d^3x\,\sqrt{-g}\,\left(R + \frac{2}{\ell^2} - (\partial\Phi)^2 - \frac{\hat{\lambda}_3}{\ell^2}\Phi^3\right) + \cdots$$

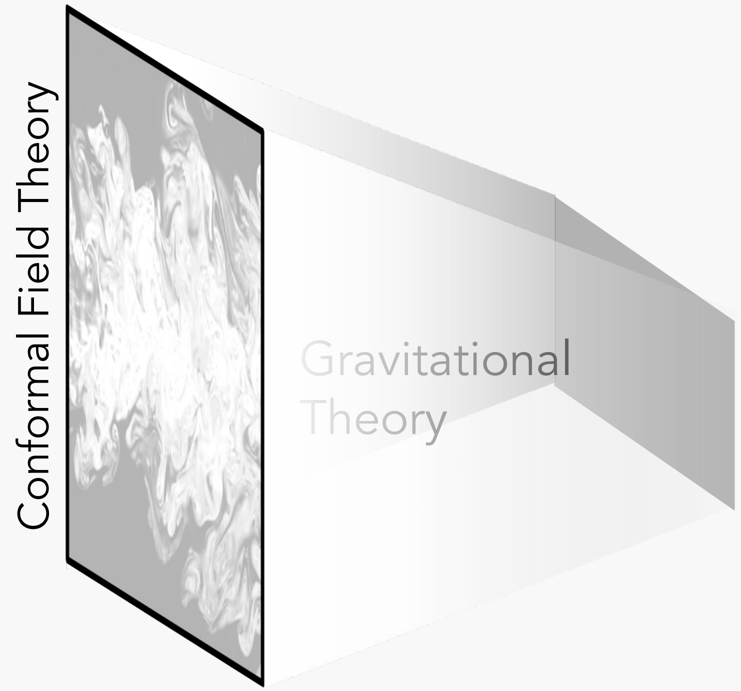
New flavors: multi-trace deformations could turn on independent couplings.

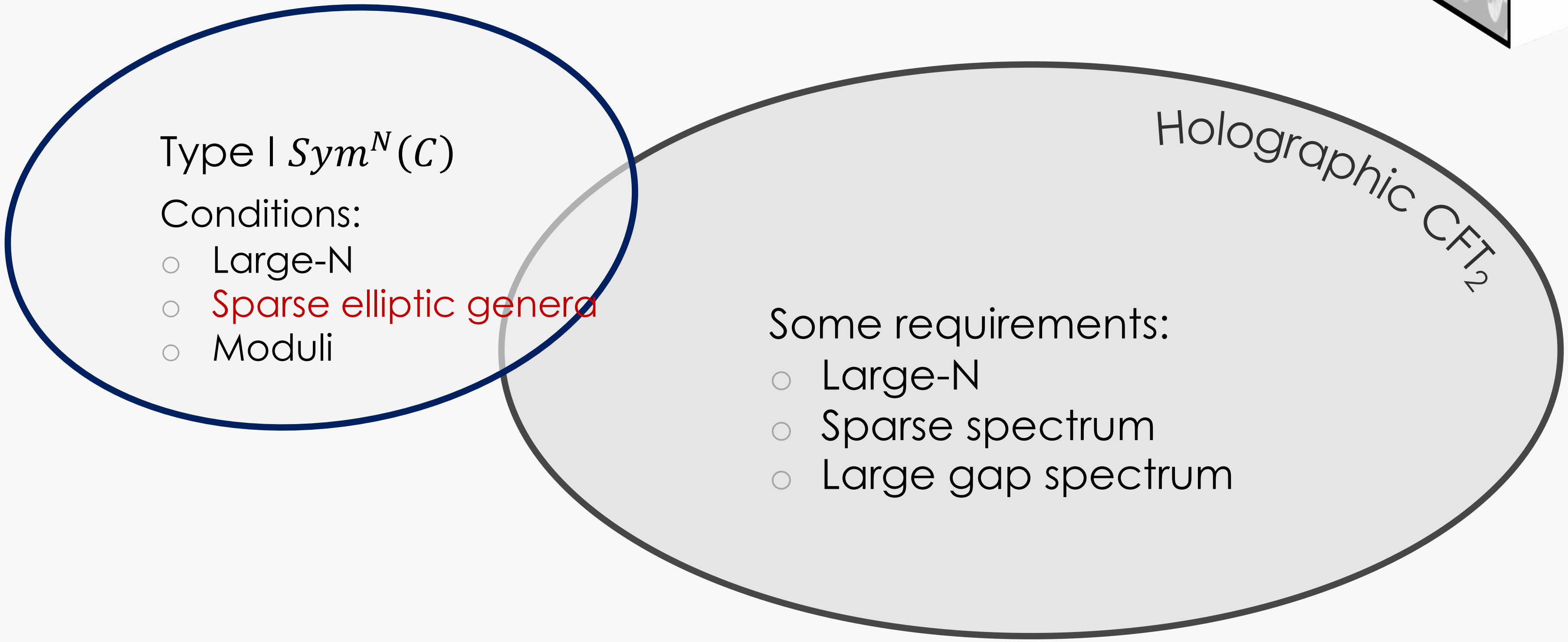
$$I_{3D} = \frac{1}{16\pi\, G_N}\int d^3x\,\sqrt{-g}\,\left(R + \frac{2}{\ell^2}\right) + \int d^3x\,\sqrt{-g}\,\left(-(\partial\Phi)^2 - \frac{\lambda_3}{\ell}\Phi^3\right)$$

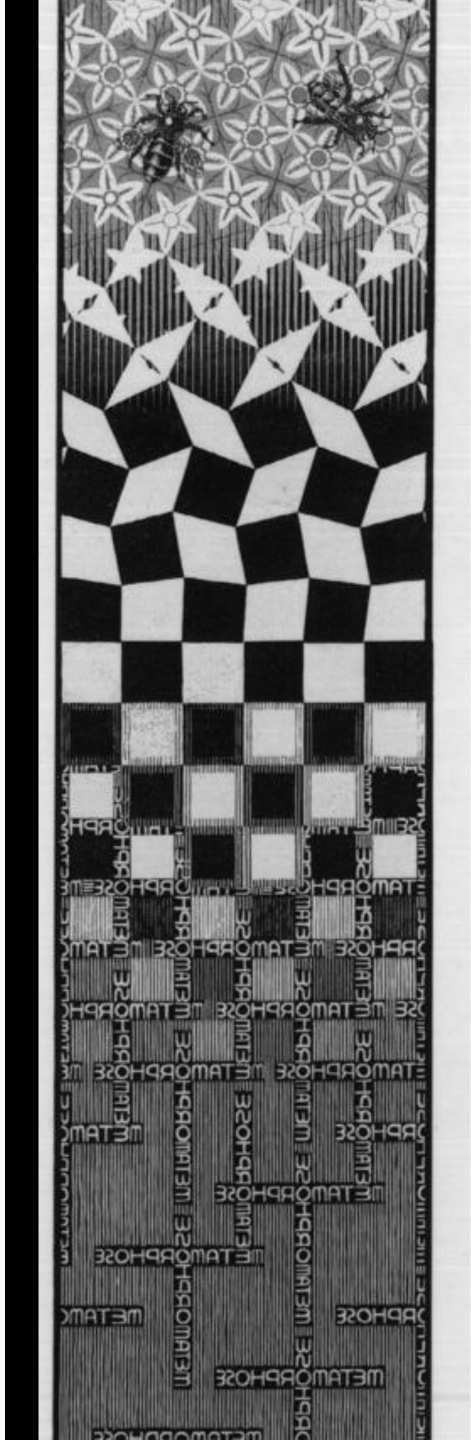
Outlook

Quantify the space of type I theories:

- Different from known examples
- Systematic and tractable
- Infinite family







- Which CFTs capture classical (geometric) properties of gravity?
- What are possible theories of quantum gravity that can be designed?
- What are the materials needed to assemble them?

Next steps:

- Multi-trace deformations (to appear by Apolo, Belin and Bintanja).
- String theory and supergravity description.
- Heavy states: contrast black holes among type I, II and III.
- Mock Modularity and asymptotic expansions.
- Type I vs II: lifting of generic operators.