Massive Gravity in the Swampland

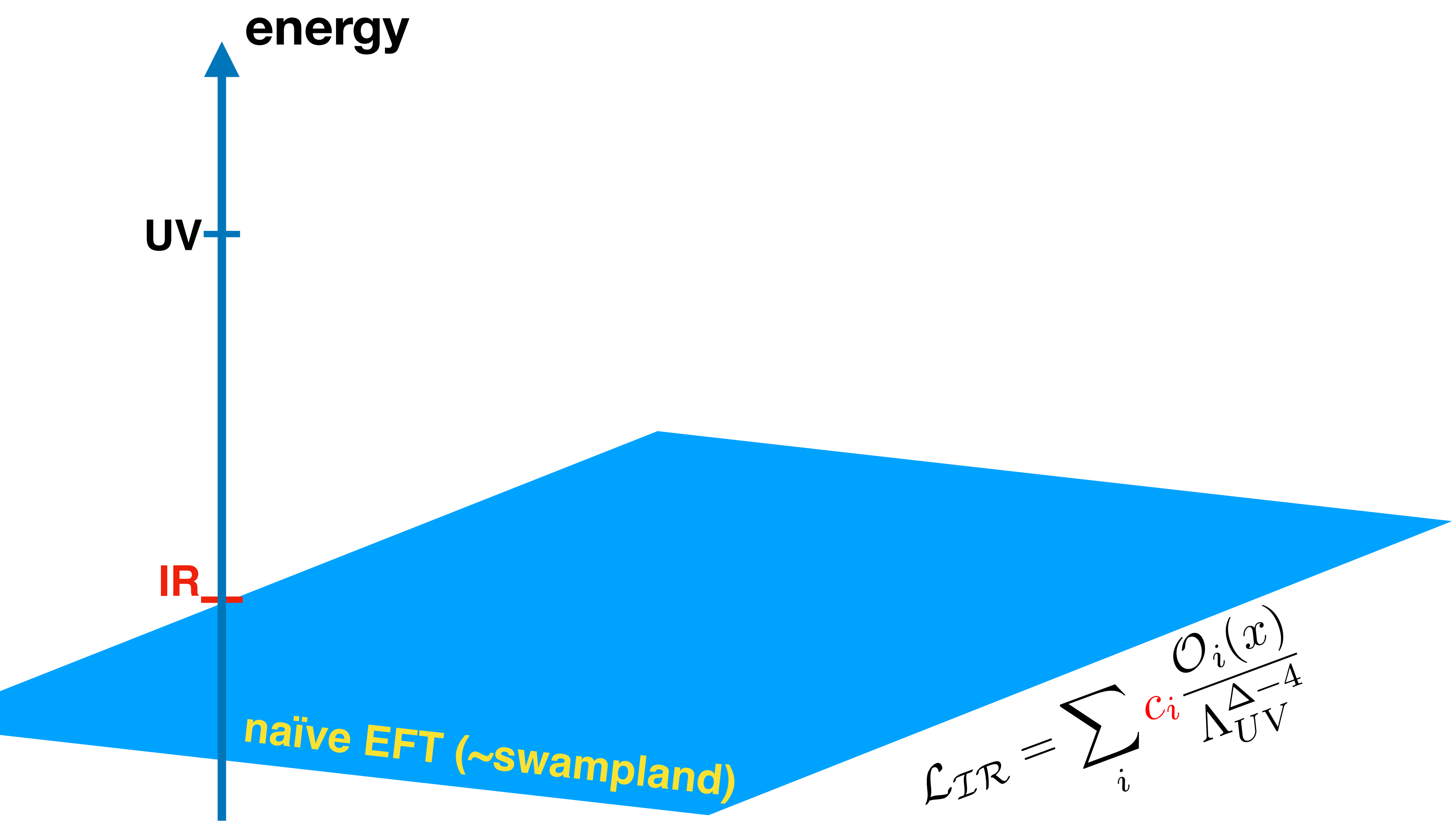
Brando Bellazzini

based on arXiv: 2304.02550 by B.B., G. Isabella, S. Ricossa, F. Riva

XXIX XMass workshop @ IFT, Madrid. 13 Dec. 2023

QFT and String Swampland

What's the landscape of consistent EFTs?



$$\mathcal{L}_{IR} = \sum_i c_i \frac{\mathcal{O}_i(x)}{\Lambda_{UV}^{\Delta-4}}$$

QFT and String Swampland

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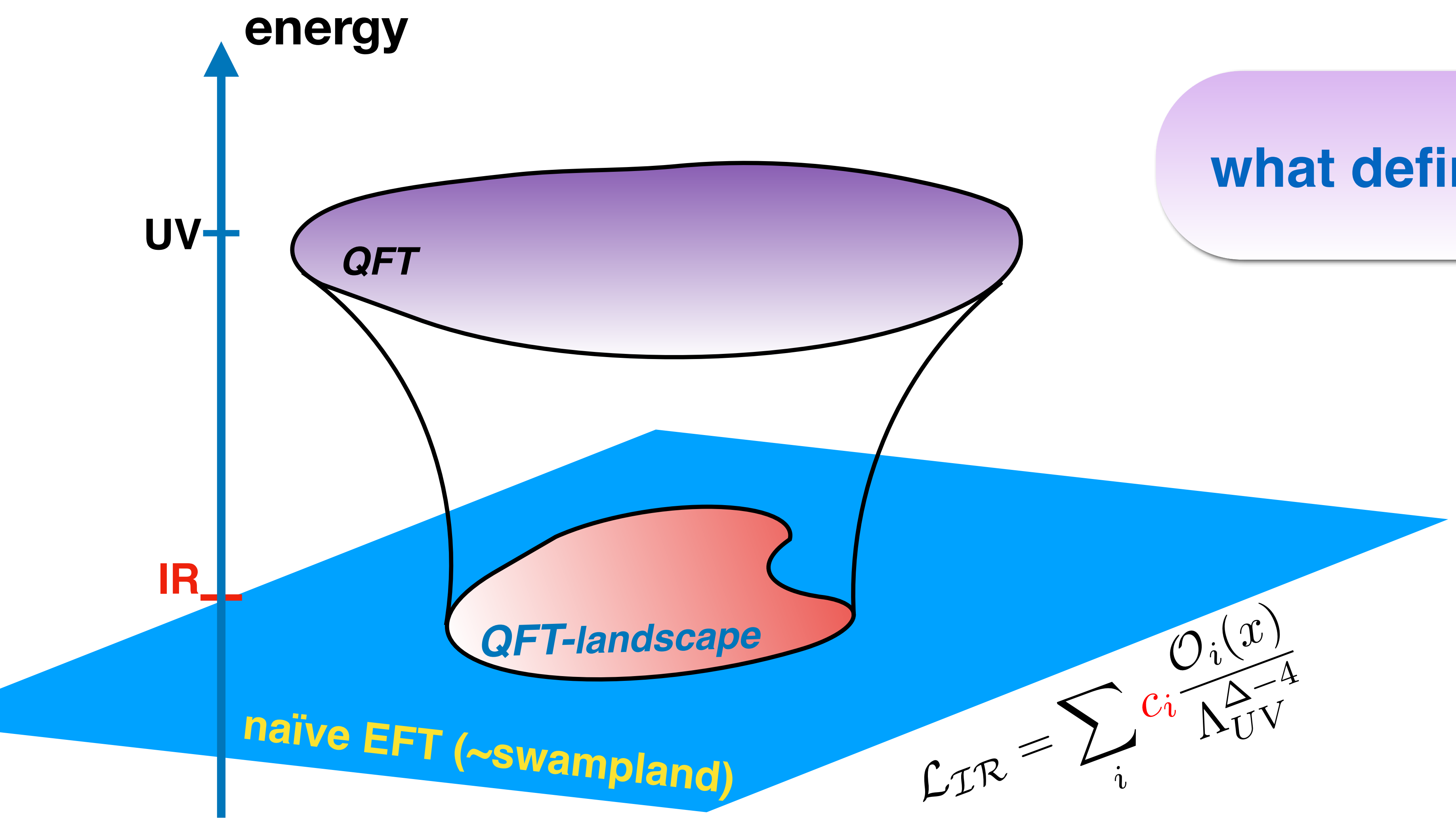
what defines the boundary?



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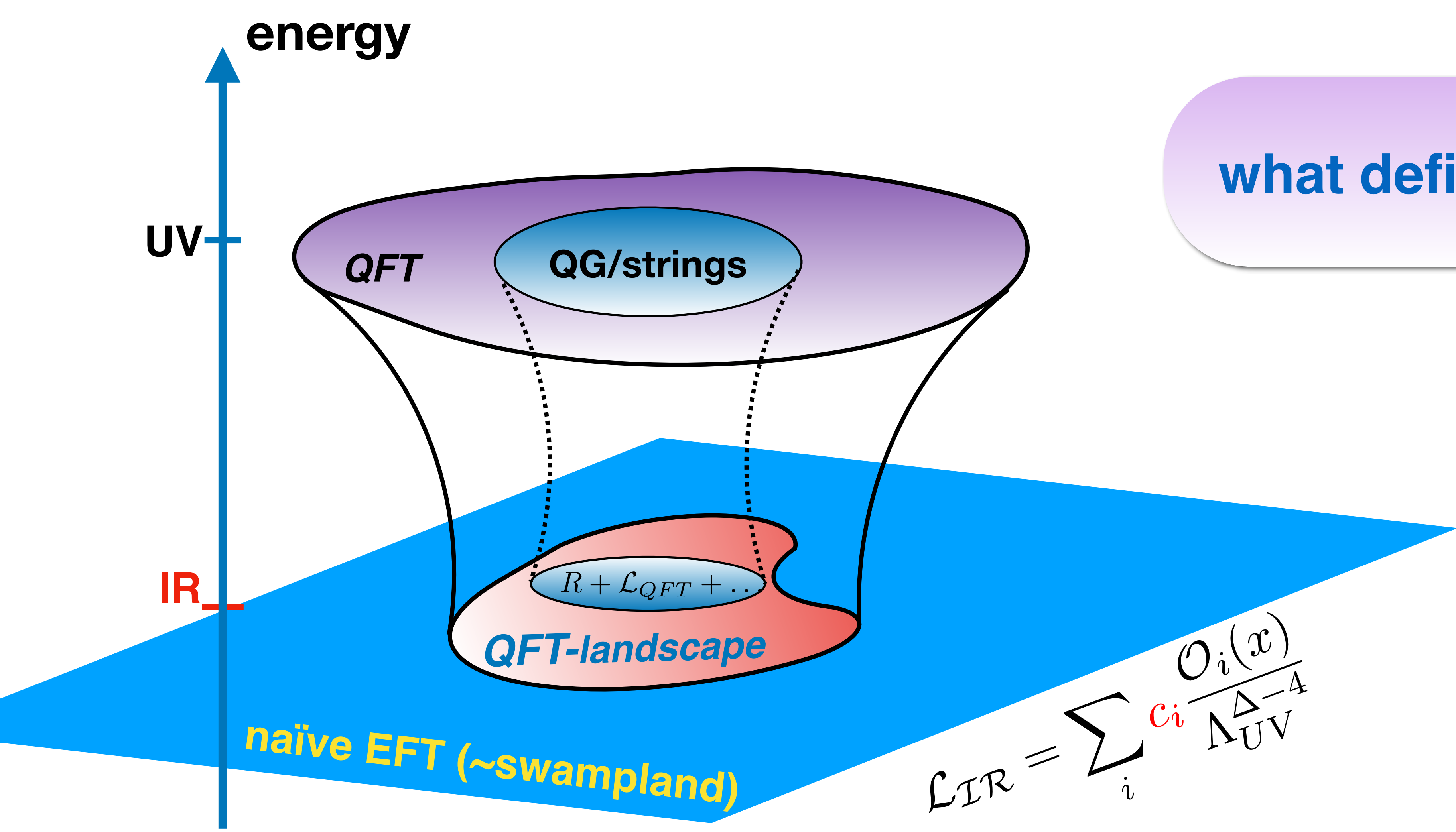

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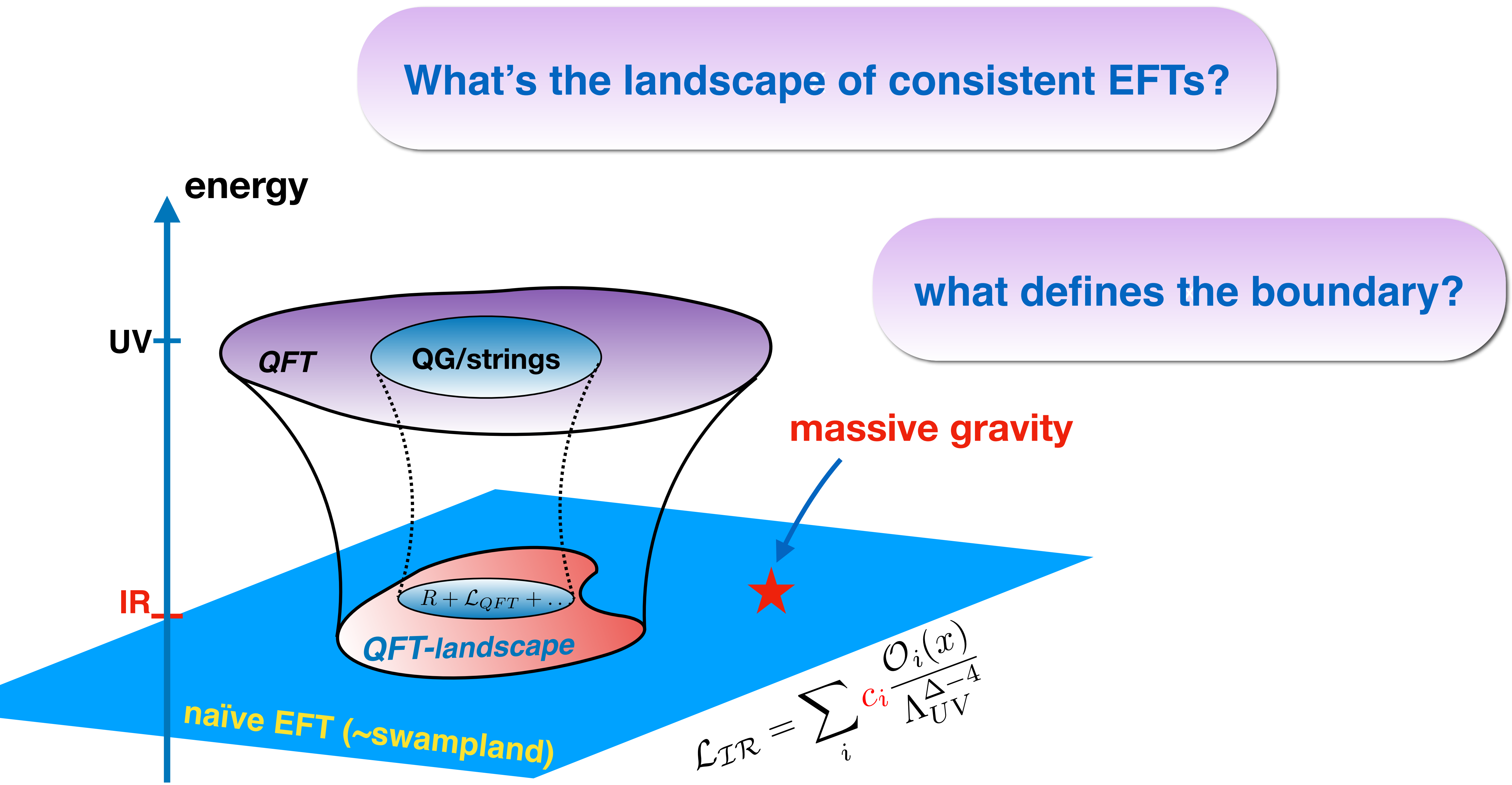
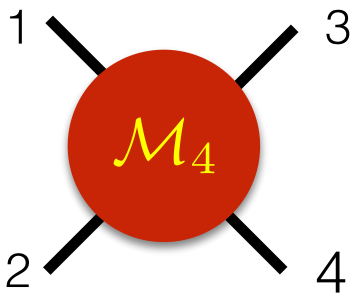


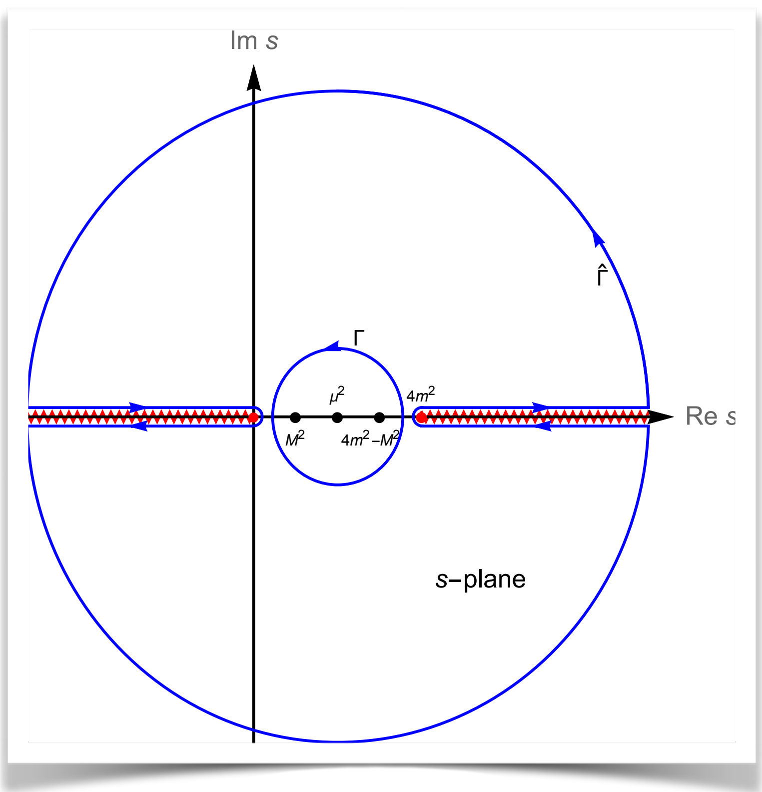
- Not every IR theory can be embedded in a consistent UV-theory
- This talk: massive gravity (and any isolated massive spin J>=2) is in the swampland

Positivity Bounds

UV-IR connection



Analyticity, Crossing, Unitarity, Locality



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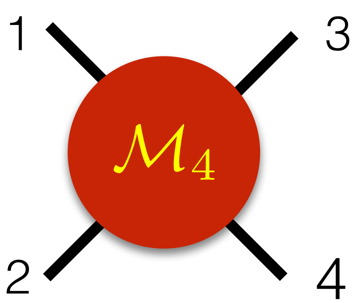

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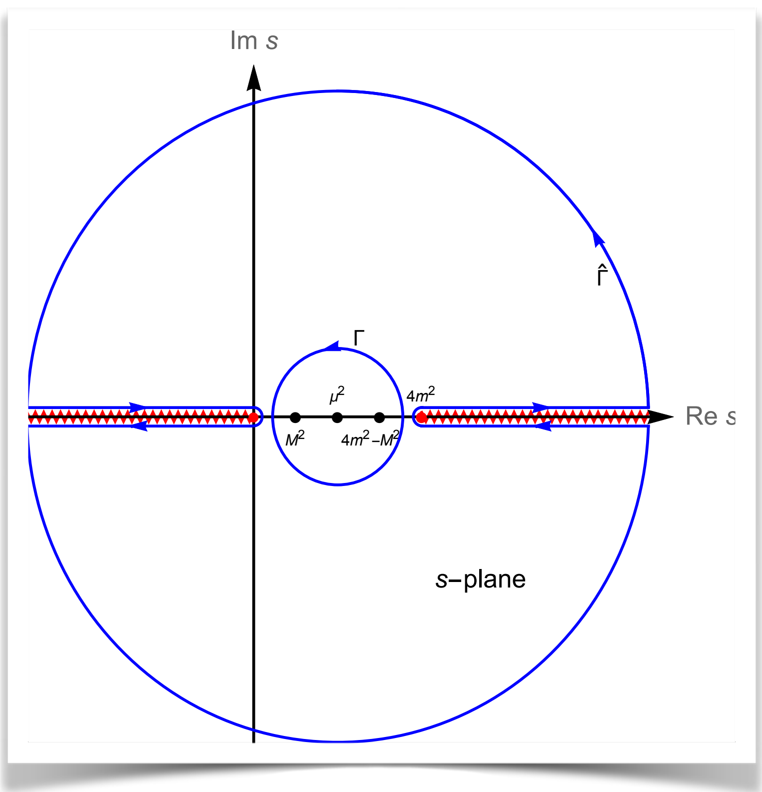

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IR-side UV-side

s^2-terms are strictly positive

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi hep-th/0602178

BB 1605.06111

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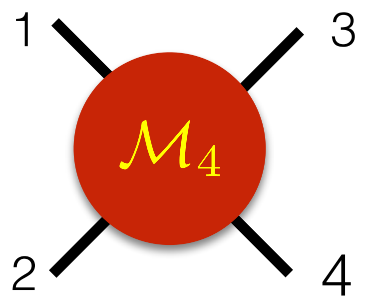

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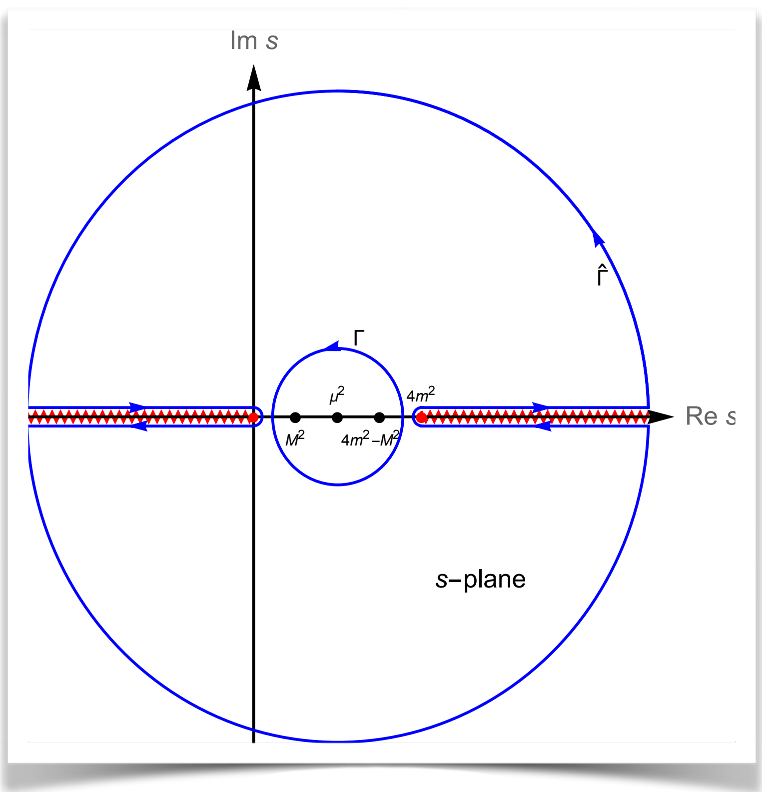

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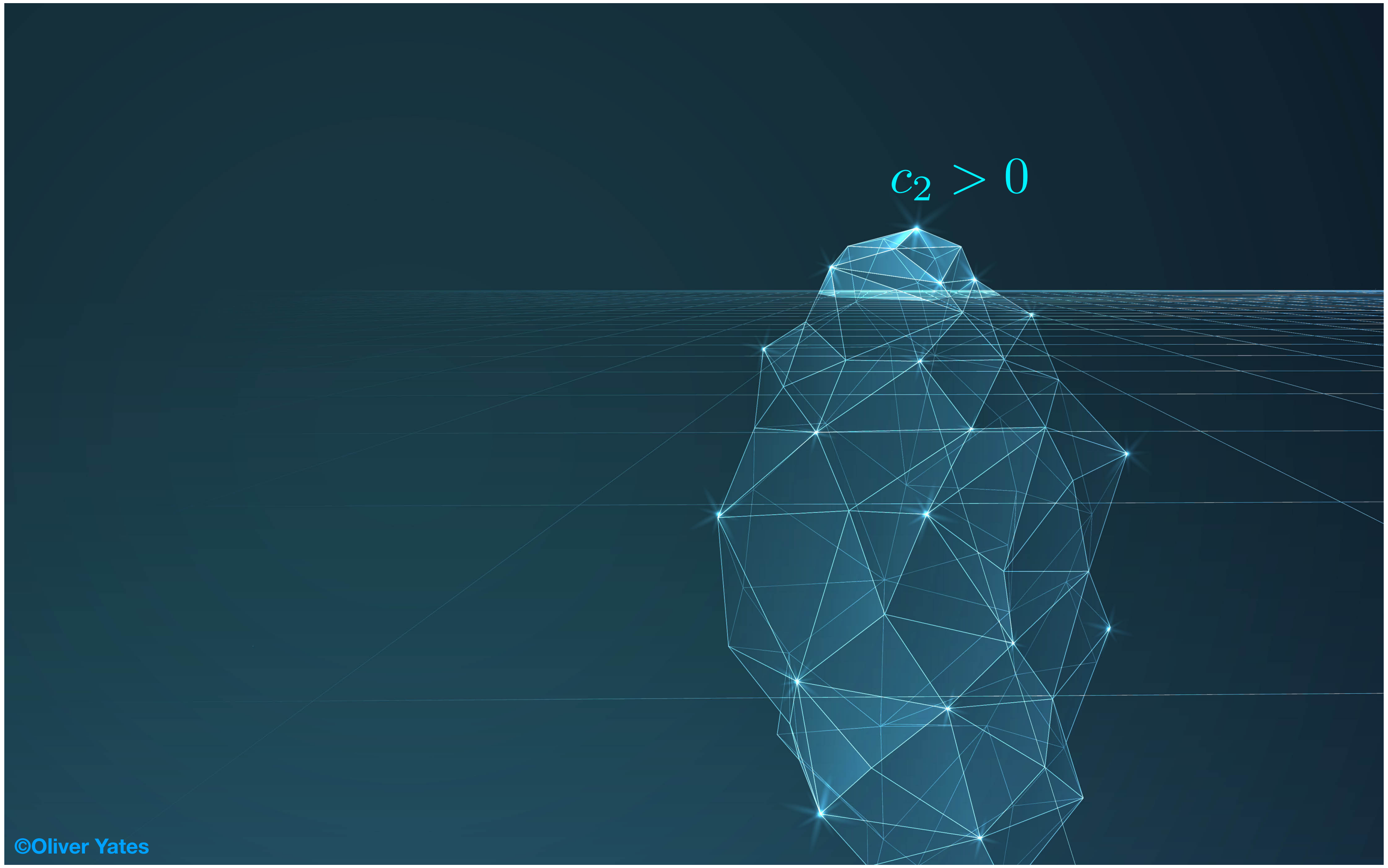
paradigmatic example

$$\pi \to \pi + \mathrm{const} \qquad \mathcal{L} = \frac{1}{2}(\partial_\mu\pi)^2 + \frac{c}{\Lambda^4}(\partial_\mu\pi)^4 + \ldots \rightarrow \mathcal{M}(\pi\pi \to \pi\pi)(s, t=0) = cs^2 \rightarrow c > 0$$

c<0 in the swampland

Positivity Bounds

$$\mathcal{M}(s)\big|_{\rm EFT} = c_0 + c_2 s^2 + c_4 s^4 + \ldots c_{21} s^2 t + \ldots$$



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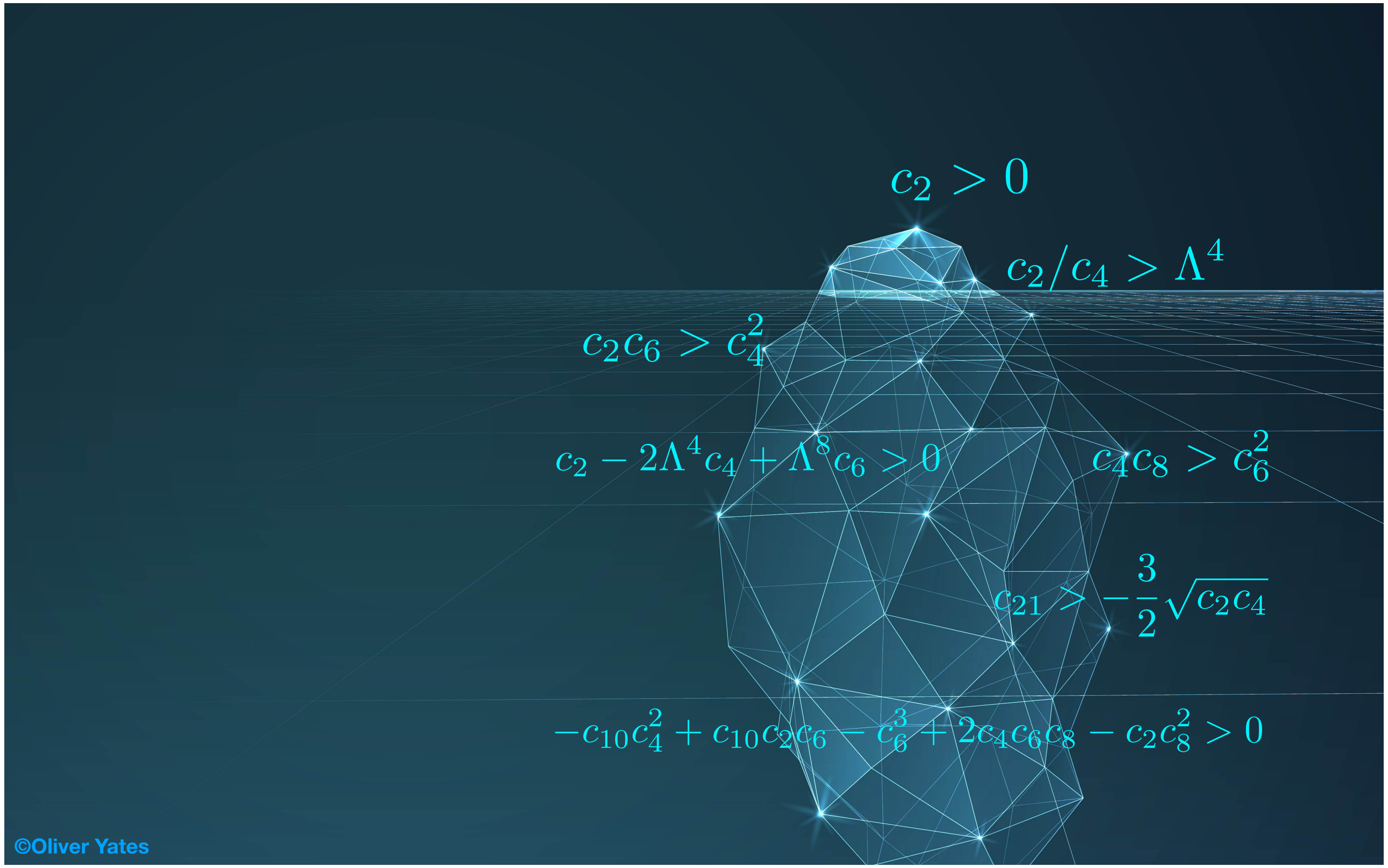

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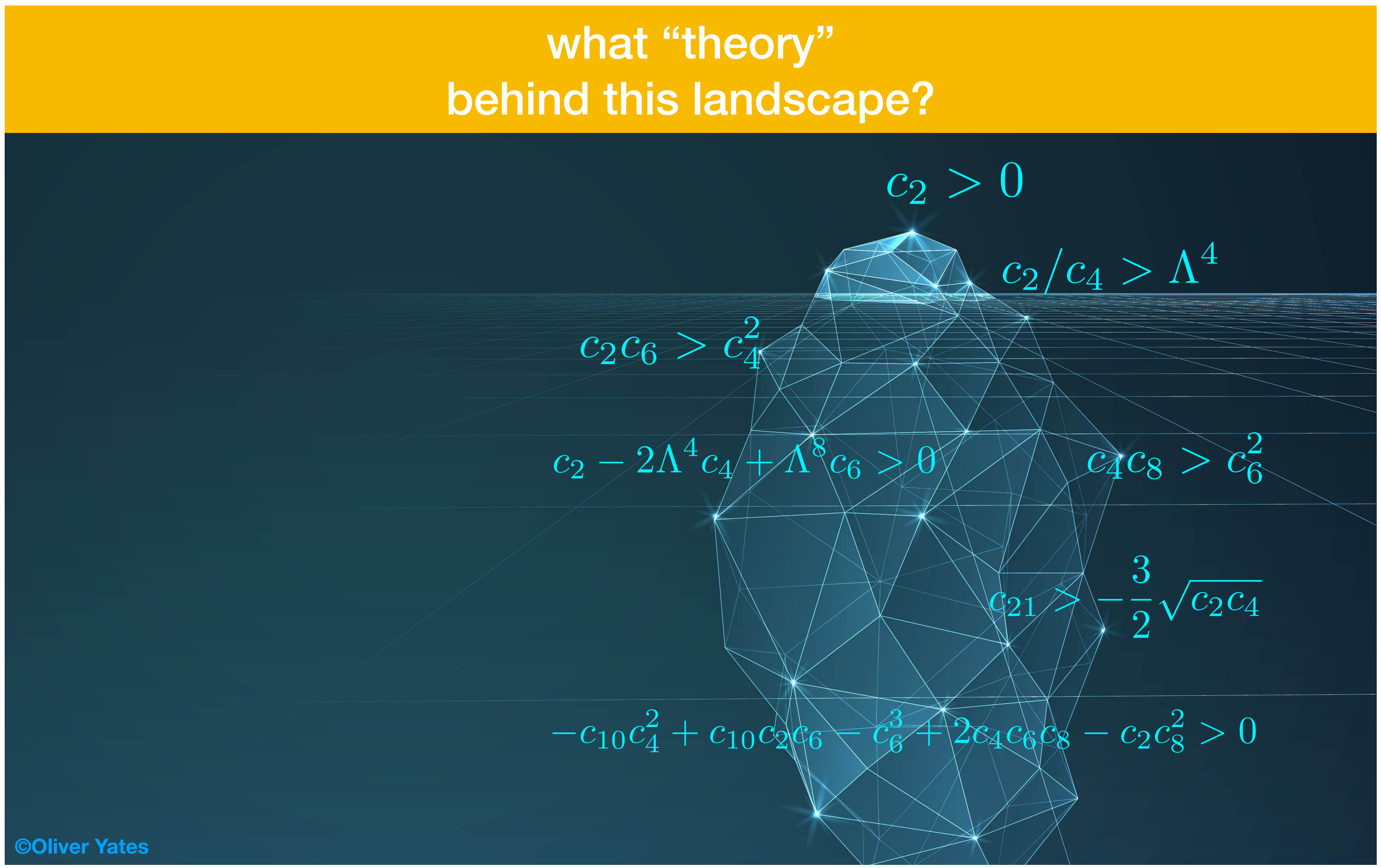
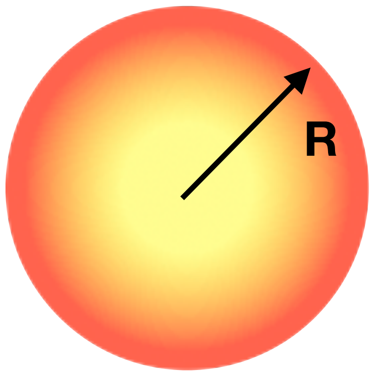


MOMENTS: AN ANALOGY WITH STARS



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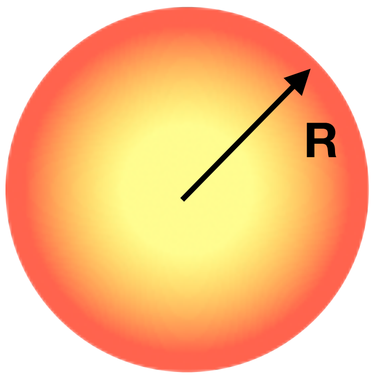

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double-sided bounds

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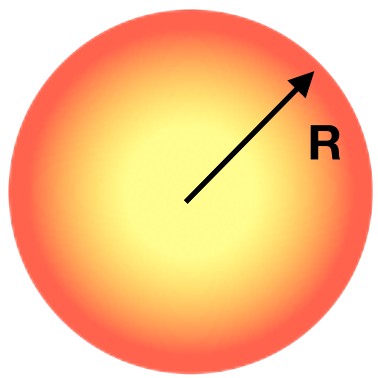

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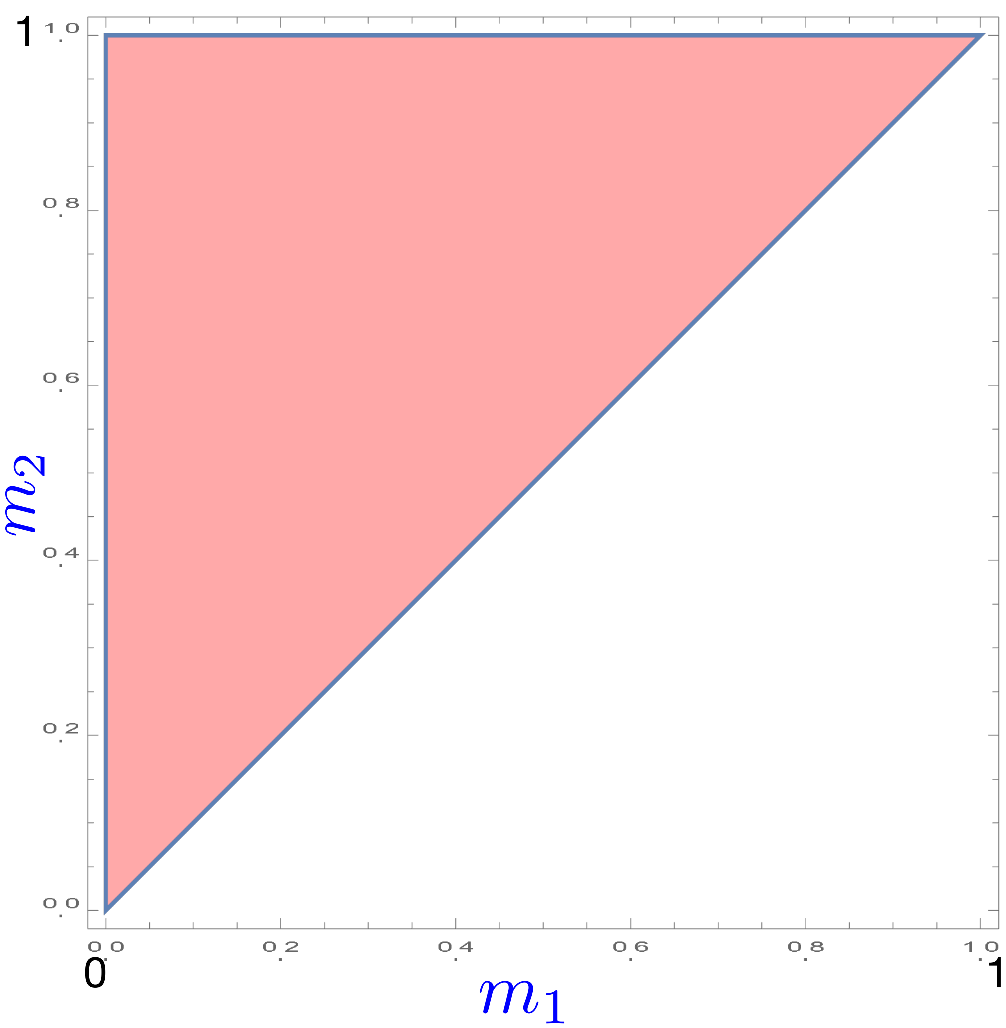

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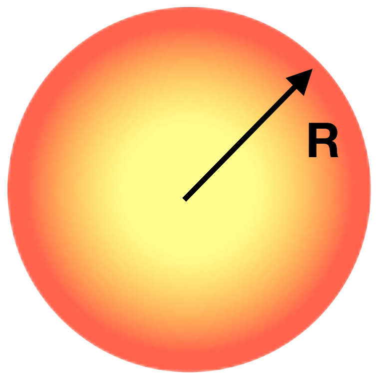

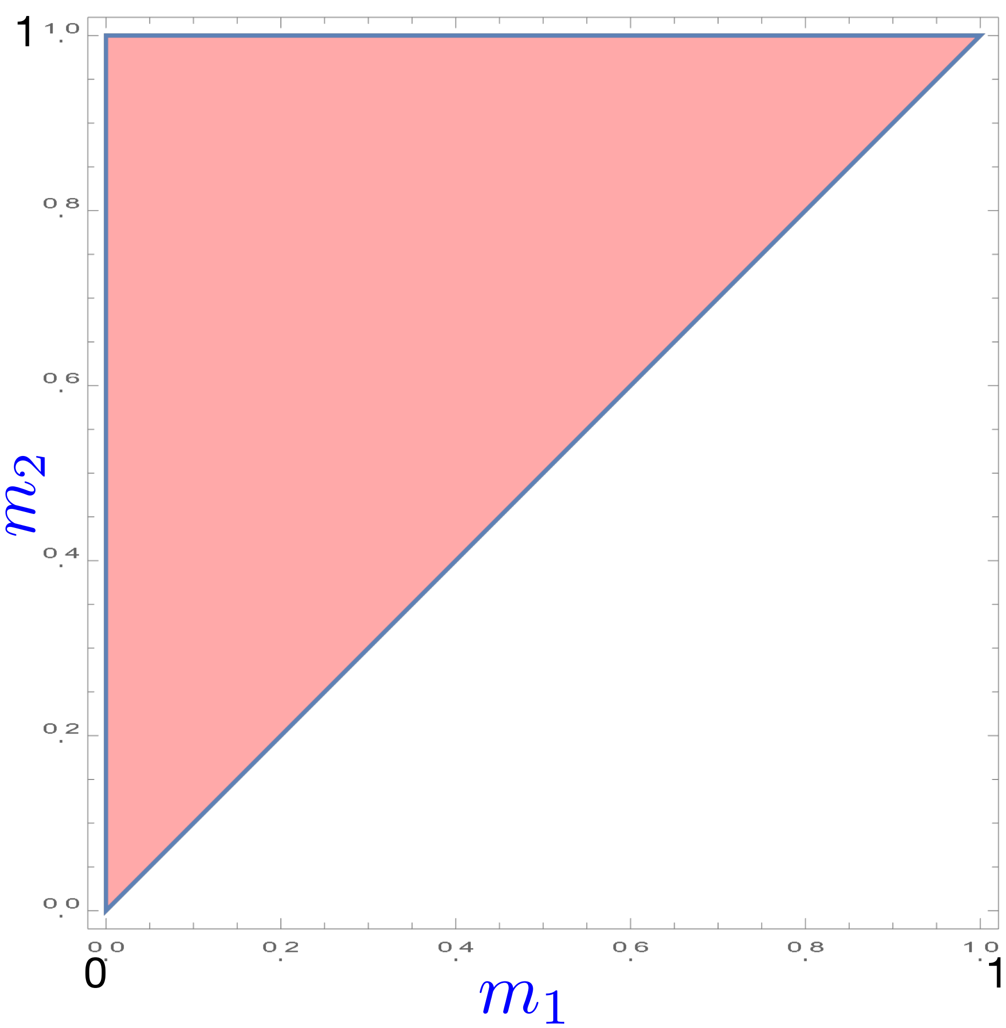

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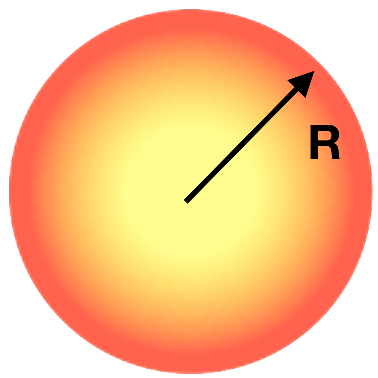

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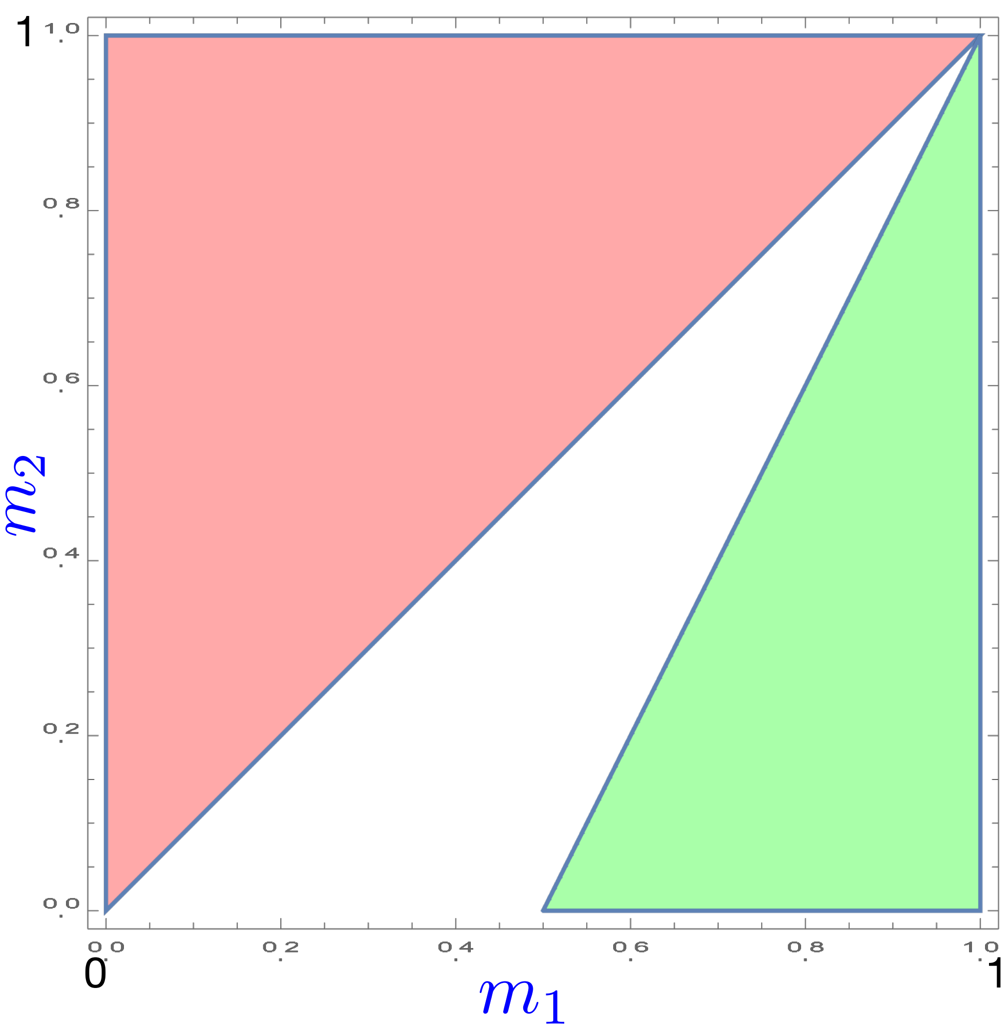

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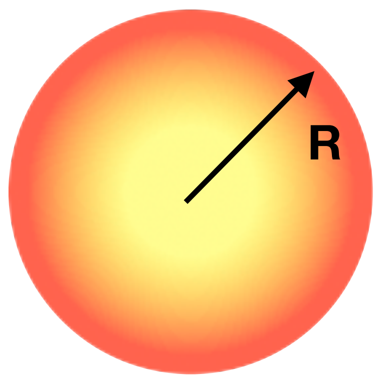

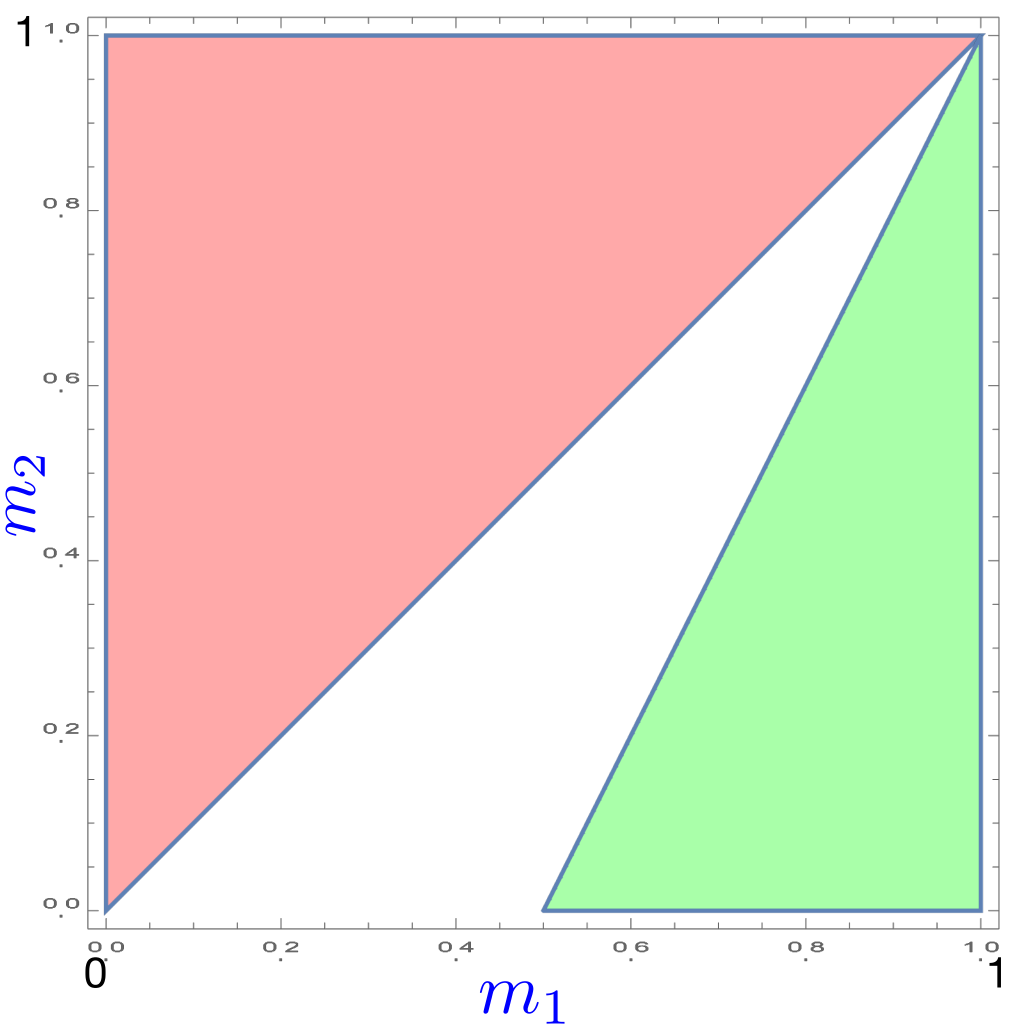

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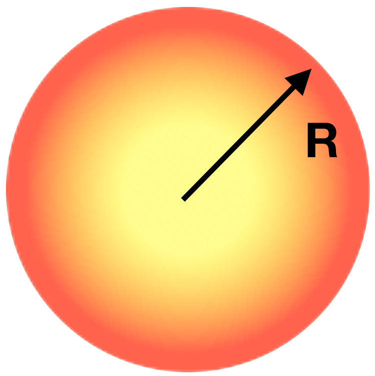

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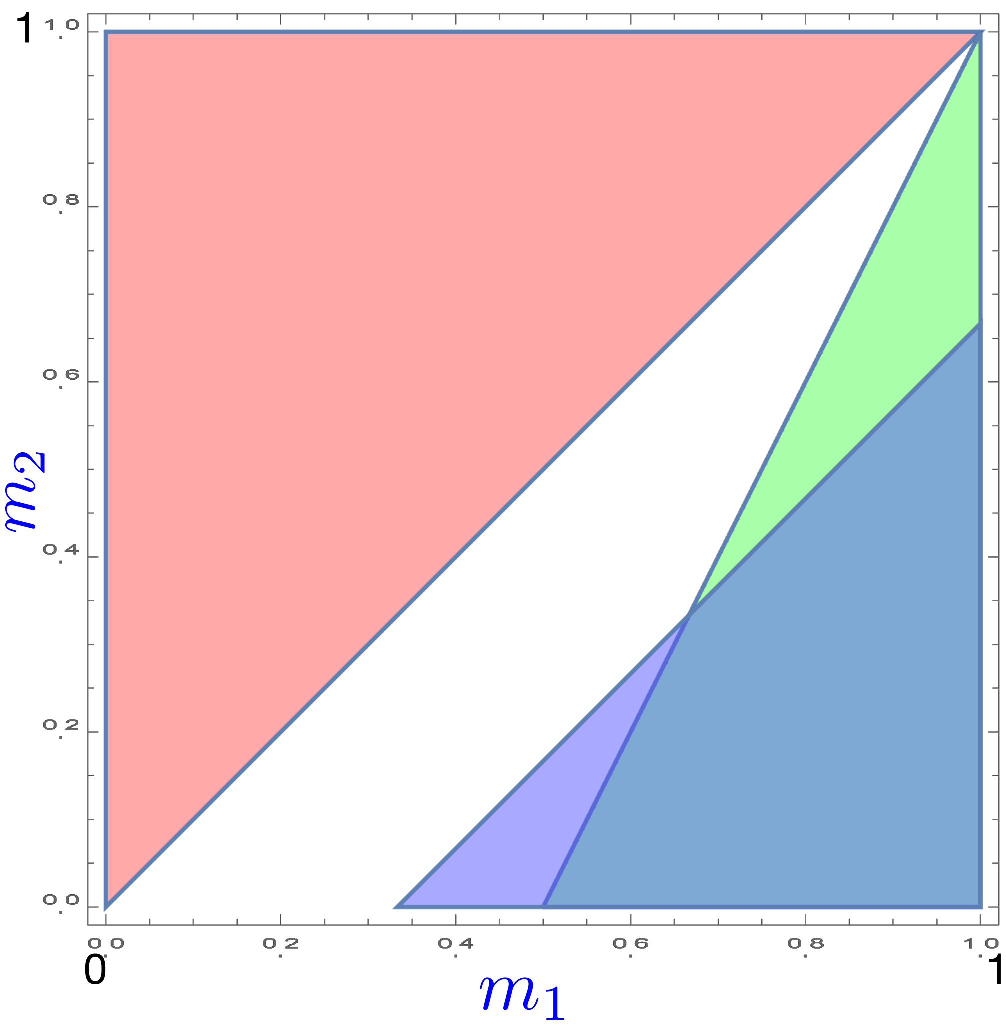

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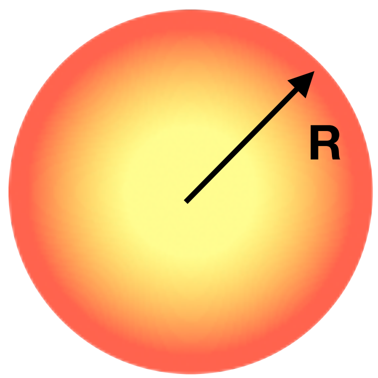

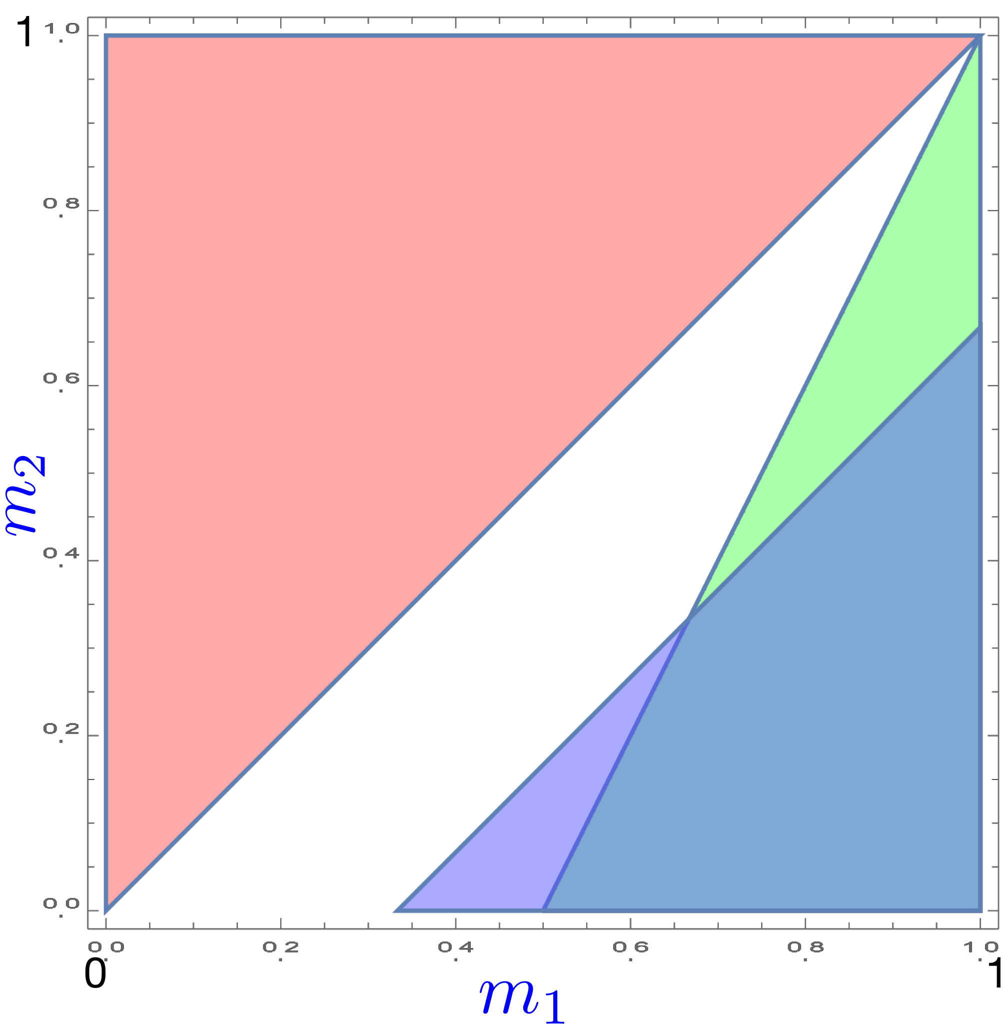

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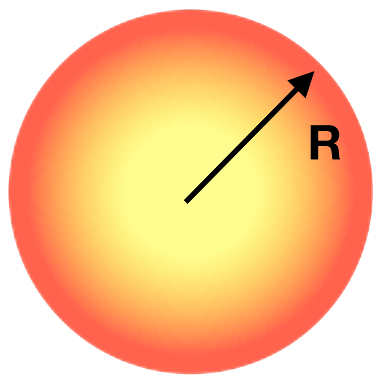

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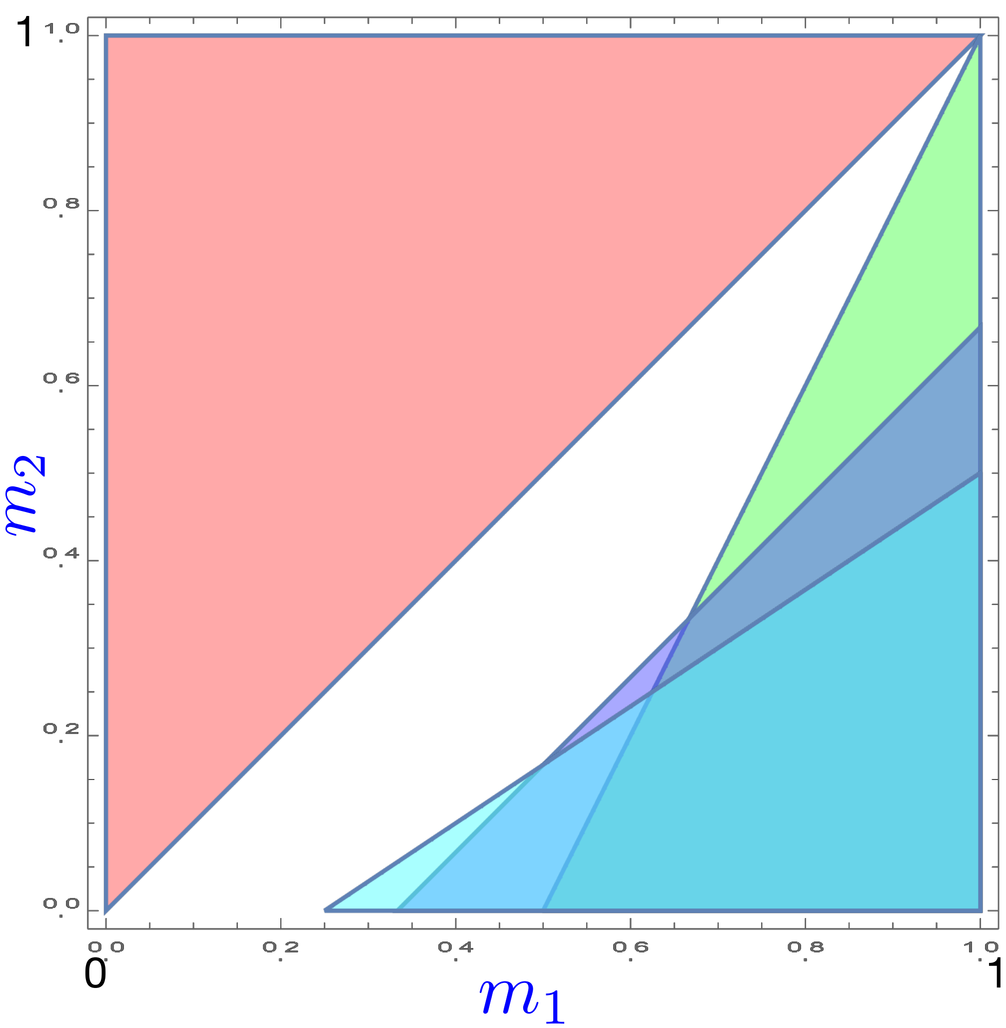

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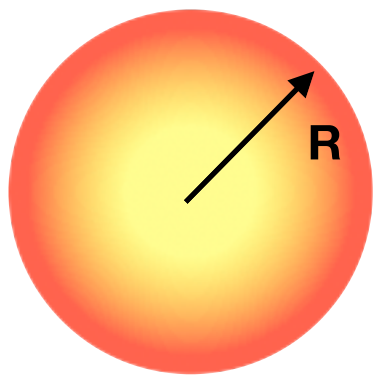

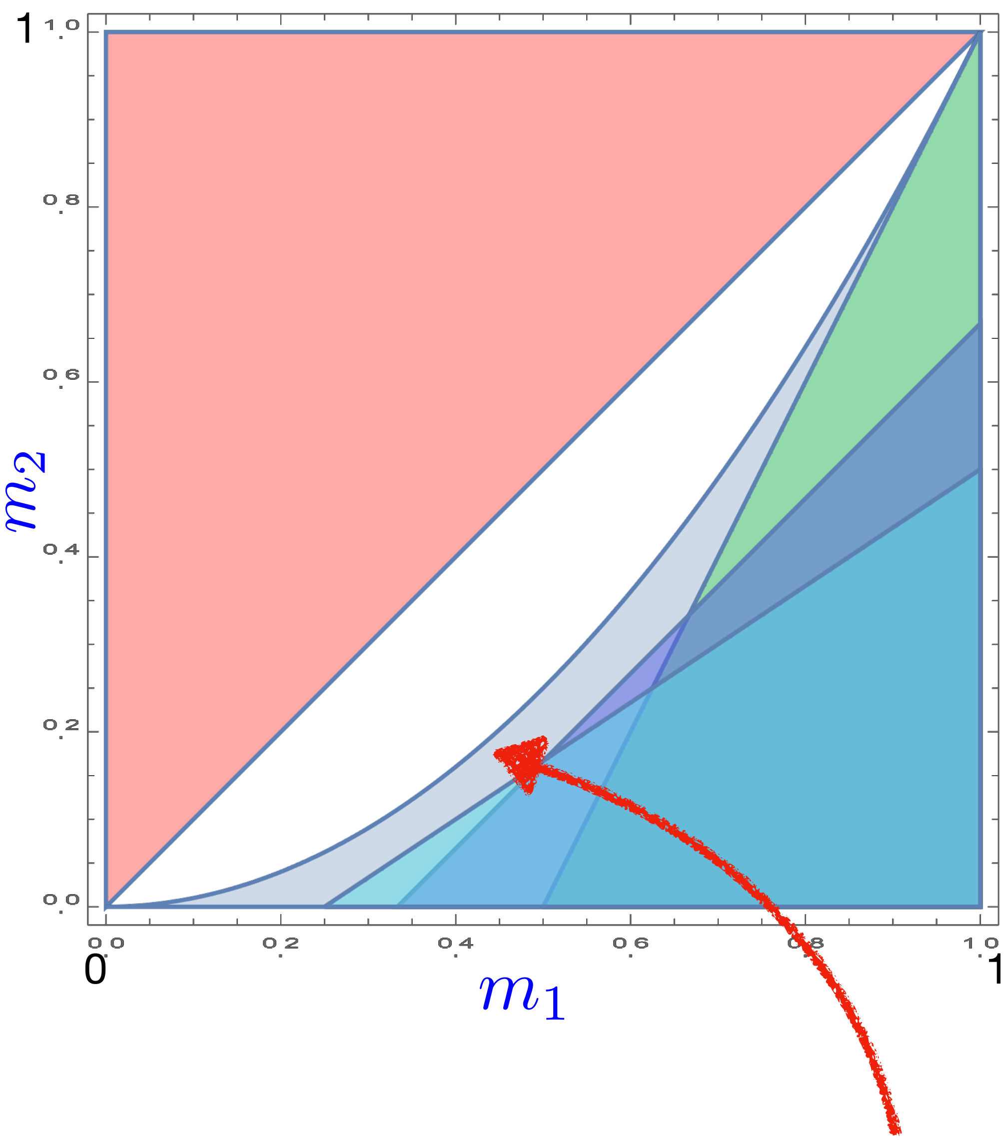

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non-linear bounds (EFT-hedron approach) = inf-many linear bounds

main lessons so far:

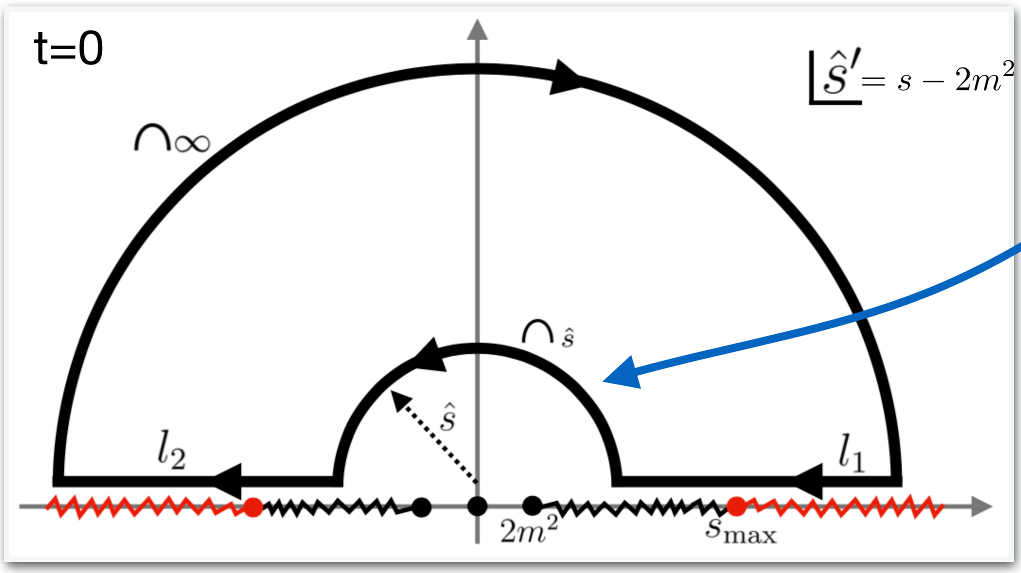
1) all bounds <—> all positive functions in [0,R]

2) it's possible to project on a finite subspace

3) non-linear bounds= infinitely many linear

back to the amplitude

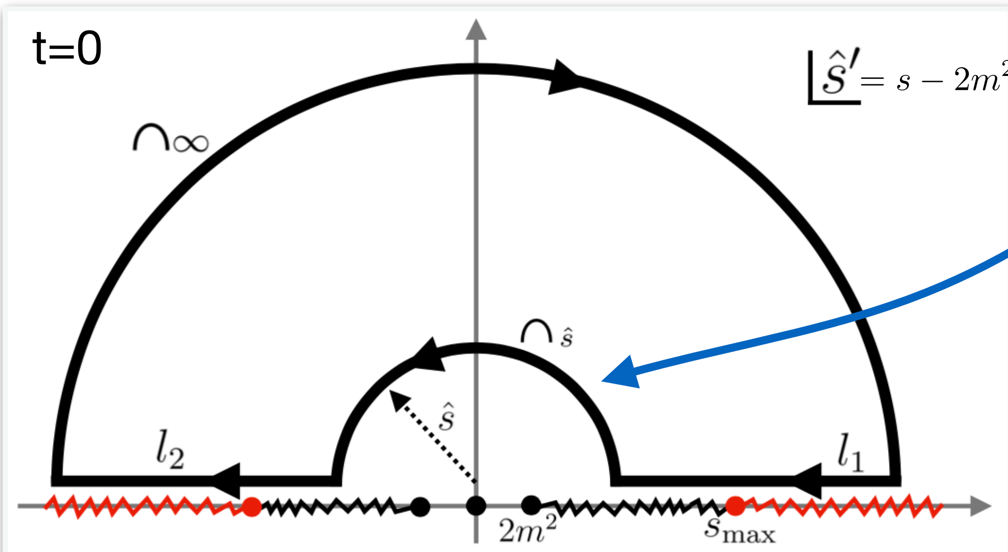
Moments=Arcs



$$a_n(\hat{s}) \equiv \int_{\cap_{\hat{s}}} \frac{d\hat{s}'}{\pi i} \frac{\hat{\mathcal{M}}(\hat{s}')}{\hat{s}'^{2n+3}}$$

"Arcs" IR-rep. $n \in \mathbb{N}$

Moments=Arcs



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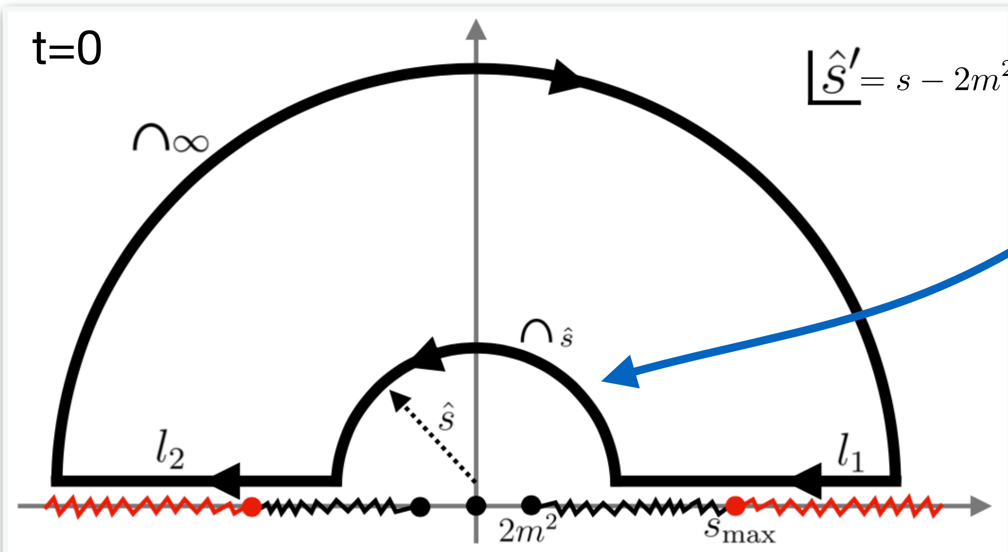
UV-rep.

Moments=Arcs



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theory space = most general positive function in [0,1]

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Bernstein pol.

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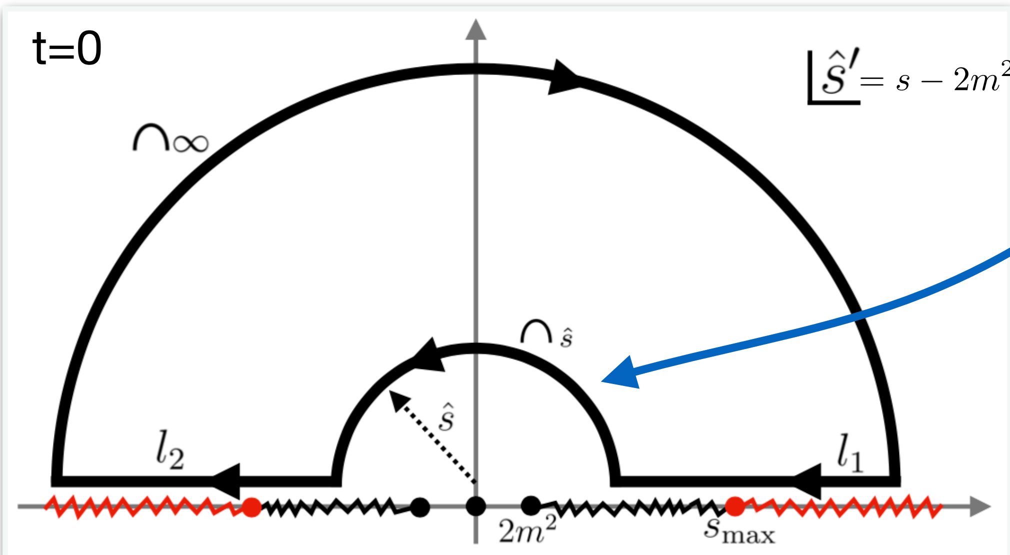

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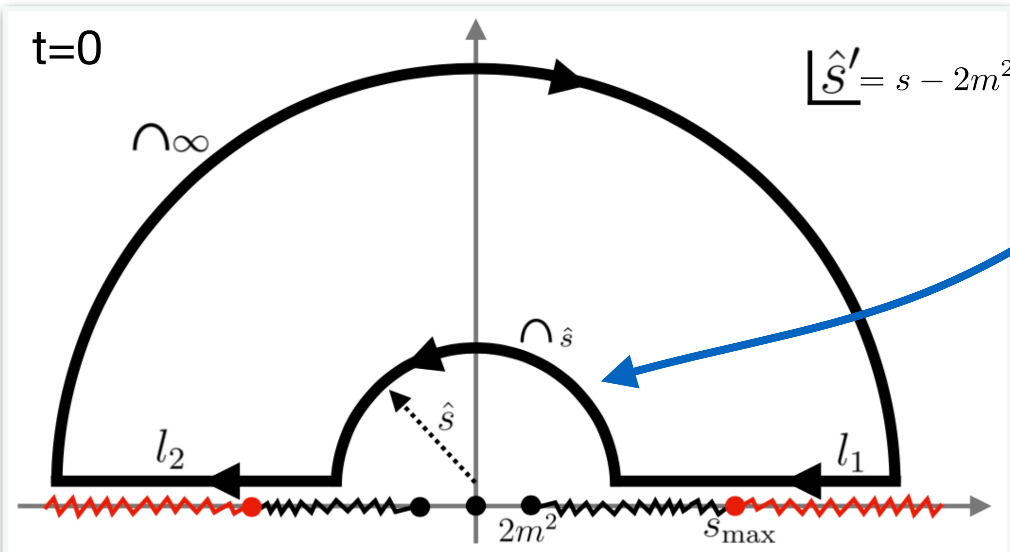
All bounds: $$\frac{1}{\hat{s}^{2n+2}} \int_0^1 x^n (1-x)^k d\mu(x, \hat{s}) > 0$$

Moments=Arcs



$$a_n(\hat{s}) \equiv \int_{\cap_{\hat{s}}} \frac{d\hat{s}'}{\pi i} \frac{\hat{\mathcal{M}}(\hat{s}')}{\hat{s}'^{2n+3}}$$

"Arcs" IR-rep. $n \in \mathbb{N}$

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UV-rep.

theory space = most general positive function in [0,1]

$$\int_0^1 \underbrace{f(x)}_{>0} d\mu(x) \longleftrightarrow f(x) = \sum_{n,m} \underbrace{x^m (1-x)^{n-m}}_{>0} \underbrace{c_{nm}}_{>0}$$

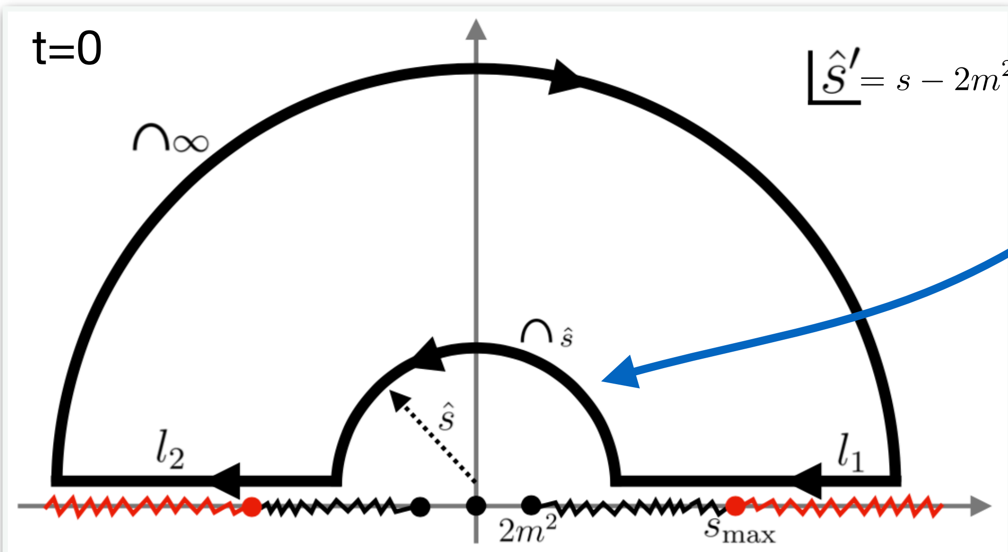
Bernstein pol.

All bounds:

$$\underbrace{\frac{1}{\hat{s}^{2n+2}} \int_0^1 x^n (1-x)^k d\mu(x, \hat{s})}_{=\,(-\Delta)^k a_n} > 0$$

$$\Delta a_n \equiv \hat{s}^2 a_{n+1} - a_n \qquad (-\Delta)^k a_n > 0$$

Moments=Arcs



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"Hausdorff's moment theorem"

"complete monotonic series"

Example @ tree-level

all bounds

$$\Delta^0 a_n = a_n > 0$$

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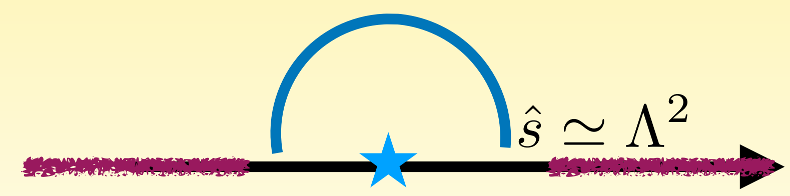
tree-level

$$\frac{\mathcal{M}(s)\big|_{\text{EFT}}}{s^{2n+3}} = \frac{c_0 + c_2 s^2 + c_4 s^4 + \ldots}{s^{2n+3}}$$



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⋮

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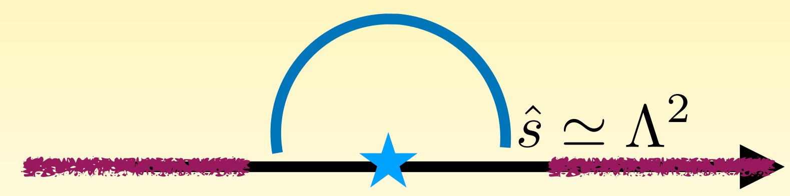

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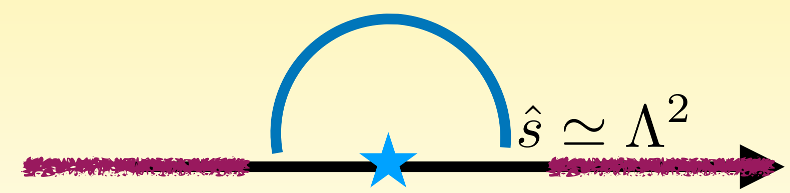

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sharp upper bound on the cutoff

$$\Lambda^4 < \frac{c_n}{c_{n+2}}$$

even for very weakly coupled theory c<<1

much more stringent strong-coupling cutoff

$$\mathcal{M}\big|_{\mathrm{EFT}} = g^2 \left(1 + \bar{c}_2 s^2 + \bar{c}_4 s^4 + \ldots\right)$$

bound independent of g<<1

Example: Massive Higher Spins

Higher Spins J>2

$$\epsilon^{(0)}_{\mu_1\mu_2\cdots\mu_J} \sim (E/m)^J$$

$$\epsilon^{(\pm 1)}_{\mu_1\mu_2\cdots\mu_J} \sim (E/m)^{J-1}$$

$$\vdots$$

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$$\rightarrow \begin{cases} \mathcal{M}^{(J-\text{even})}_{\text{long.}} = g^2(m_J/\Lambda_J)\left[\left(\frac{E}{m_J}\right)^{3J} + \ldots + \frac{E^4}{m_J^4} + \ldots\right] \\ \mathcal{M}^{(J-\text{odd})}_{\text{long.}} = g^2(m_J/\Lambda_J)\left[\left(\frac{E}{m_J}\right)^{3J+1} + \ldots + \frac{E^4}{m_J^4} + \ldots\right] \end{cases}$$

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Main Lesson: EFTs isolated Massive Higher spin (J>2) in the swampland

1903.08664 B.B., F. Riva, J. Serra, F. Sgarlata

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caveat: **trivial forward amplitude** $\mathcal{M}(t \to 0) \approx 0$

Higher Spins: J=2

Massive Spin-2 borderline: $\mathcal{M}^{J=2} \sim t s^2 \to f(c_3, d_5) m^2 s^2$

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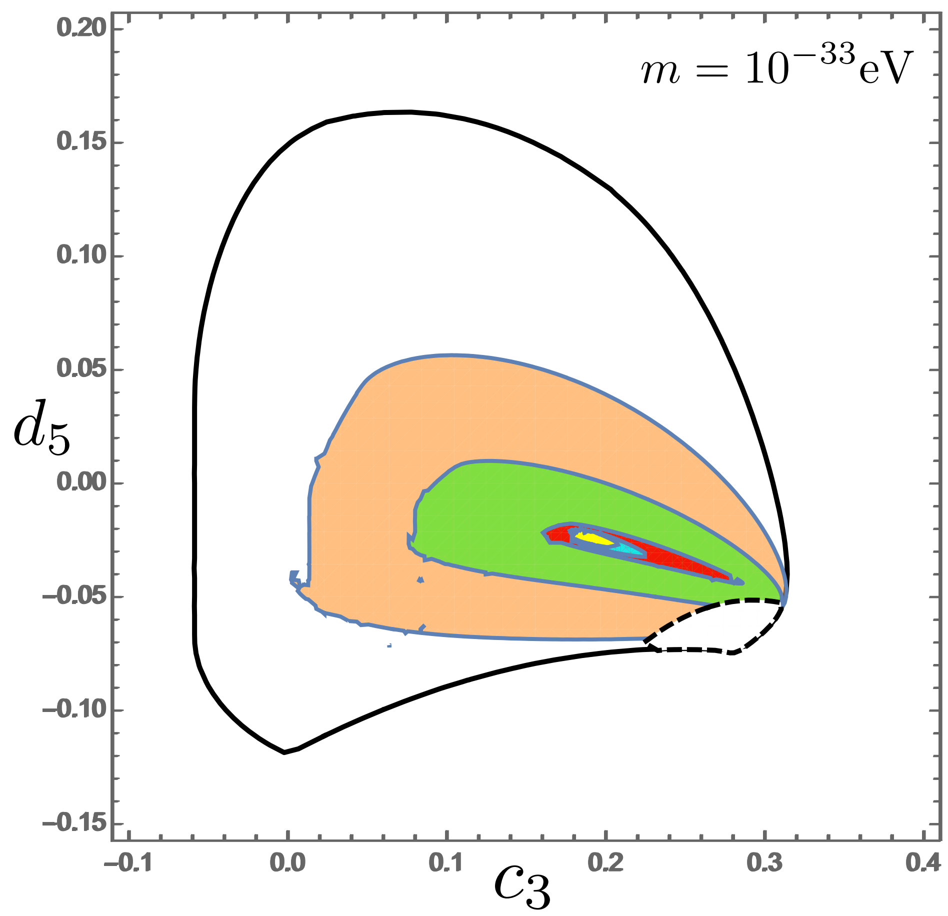

Cheung Remmen 1601.04068

$\Lambda \sim m$

$\Lambda \sim 10^{-18}\mathrm{eV} \sim (3 \cdot 10^7 \mathrm{km})^{-1}$

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1710.02539 B.B. , F.Riva, J.Serra, F. Sgarlata

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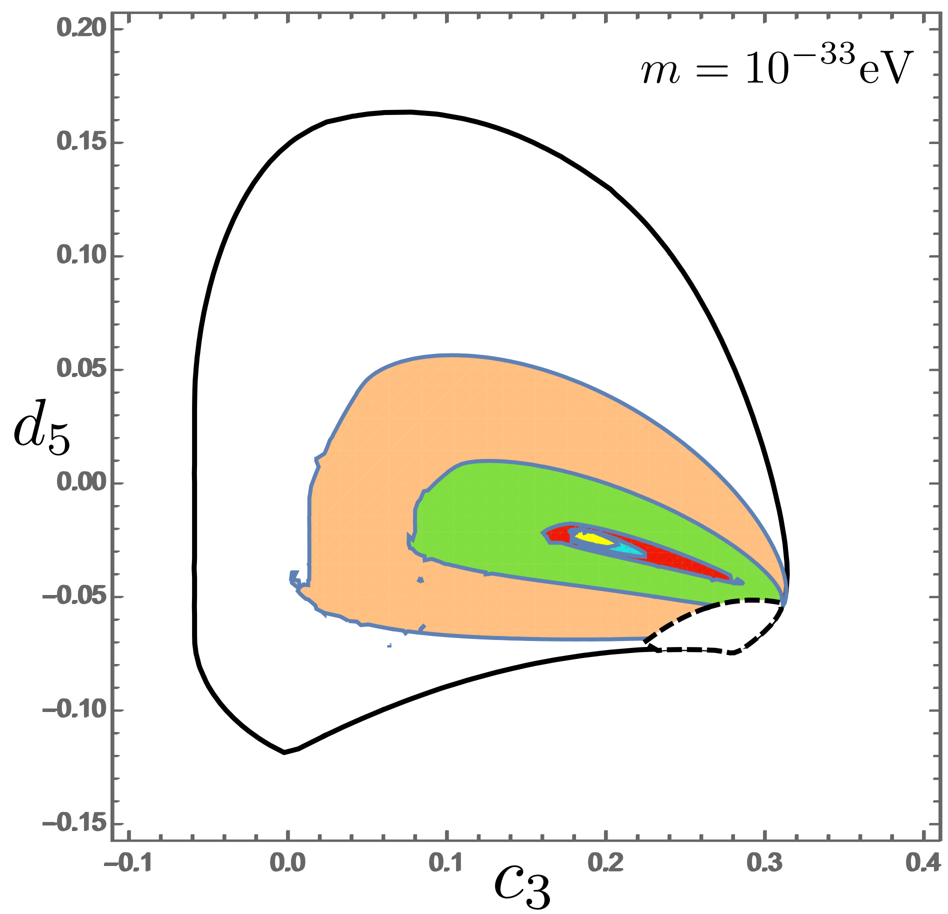

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need to go beyond forward limit

Beyond Forward

Finite-t

$$\mathcal{M}(s)\Big|_{\rm EFT} = c_0 + c_2 s^2 + c_4 s^4 + \ldots + c_{21} s^2 t + c_{22} s^2 t^2 + \ldots$$

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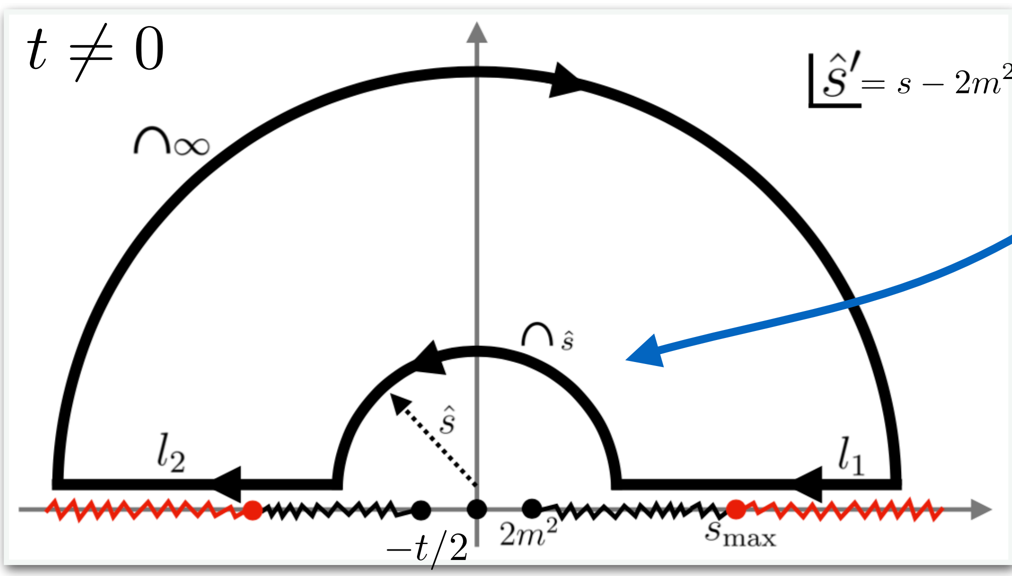

$$a_n(\hat{s},t) = \frac{2}{\pi}\int_{\hat{s}}^{\infty} d\hat{s}' \frac{\mathrm{Im}\hat{\mathcal{M}}(\hat{s}',t)}{(\hat{s}'+\frac{t}{2})^{2n+3}}\,, \qquad n \in \frac{\mathbb{N}}{2}$$

UV-rep.

$$= \sum_p t^p c_p^{n,m} \mu_{n,m}$$

2D-moments

$$\mu_{n,m} = \int d\mu(x,J^2)\, x^n \left(J^2\right)^m$$

$$d\mu(x,J^2) = \sum_{J_\ell^2=\ell(\ell+1)} \frac{1}{\pi}\mathrm{Im} f_{\ell(J^2)}(\hat{s}/\sqrt{x})\delta(J^2-J_\ell^2) > 0$$

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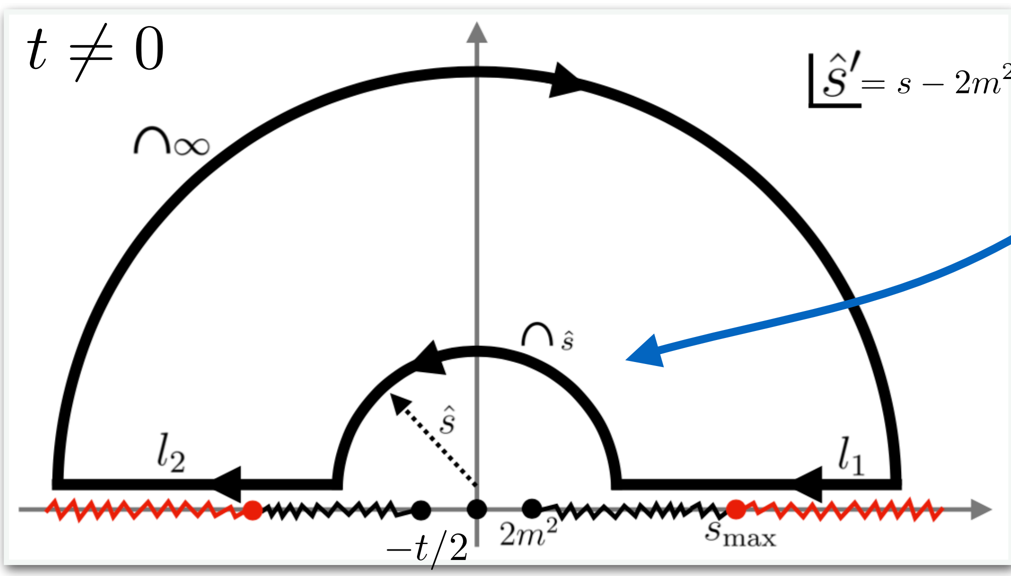

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example:

$$a_0(\hat{s},t)\hat{s}^2 = \underbrace{\mu_{0,0}}_{c_2} + \frac{t}{\hat{s}}\Big(\underbrace{\mu_{\frac{1}{2},1} - \frac{3}{2}\mu_{1/2,0}}_{c_{2,1}}\Big) + \frac{t^2}{\hat{s}^2}\Big(\underbrace{\frac{3}{2}\mu_{1,0} - 2\mu_{1,1} + \frac{1}{4}\mu_{1,2}}_{c_{2,2}}\Big) + \ldots$$

FINITE-T

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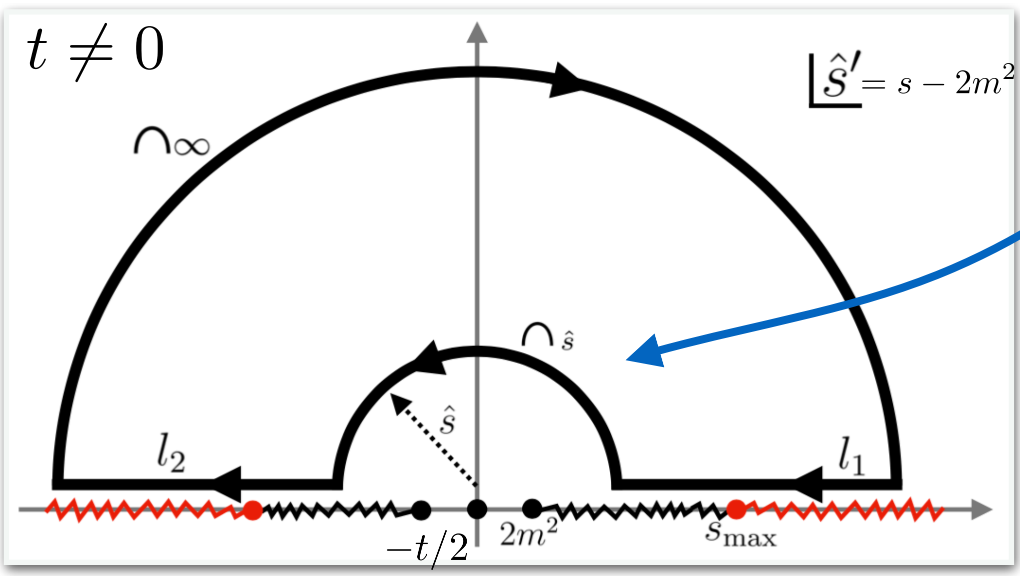

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$\longleftrightarrow$ $-\frac{3}{2} < \frac{c_{21}\hat{s}}{c_2} < 5.3$

2011.02957 S. Caron-huot, van Duong
2011.02400 A. Tolley, Z. Wang, S.Y. Zhou
2102.08951 Caron-huot, Mazac, Rastelli, Simmons-Duffin
2112.12561 B.B,, Riembau, Riva
Huang, Rodina, Zhiboedov, Arkani-Hamed, Pomarol…. etc

|Inelastic| < Elastic

&

finite-T and Crossing

FINITE-T AND CROSSING

Massive particle crossing: **very complicated**

(2J+1)^4 helicities mix!

$$\mathcal{M}_{\lambda_1\lambda_2}^{\lambda_3\lambda_4}(s,t) = \sum_{\lambda_i'=-S}^{S} X_{\lambda_1\lambda_2\lambda_3\lambda_4}^{\lambda_1'\lambda_2'\lambda_3'\lambda_4'}(s,t)\mathcal{M}_{\lambda_1'\bar{\lambda}_4'}^{\lambda_3'\bar{\lambda}_2'}(u,t)$$

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Massless, or |t|<<s, particle crossing: **very simple!**

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FINITE-T AND CROSSING

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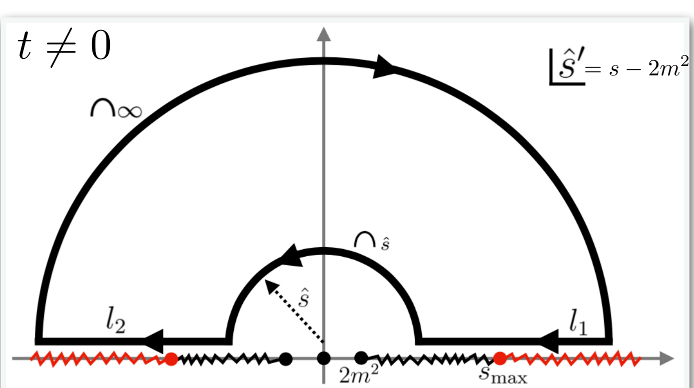


EFTs with gap M to new states $m^2 \ll |t| \ll s \lesssim M^2$

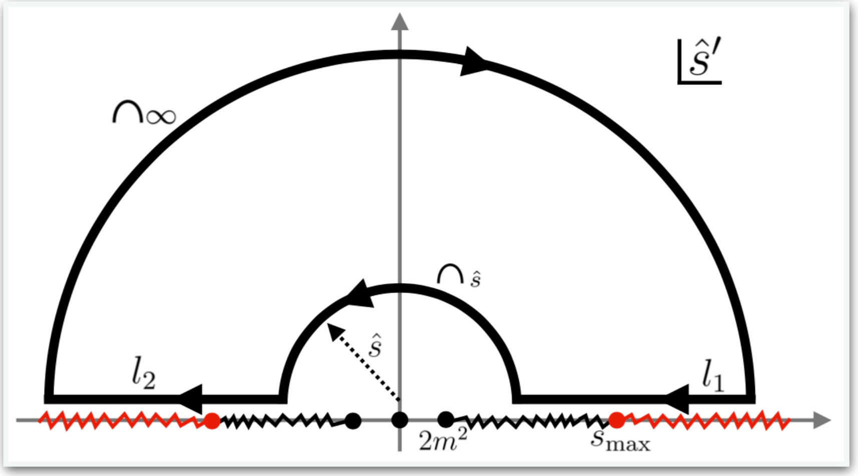
Crossing nice, error bounded by EFT-calculable quantity

$$|\text{error}| < \frac{\sqrt{|t|}m}{M^2} c^{\lambda'}_{\lambda_1\lambda_2}\mathcal{A}_{\lambda_1'\lambda_2'}(0)/\mathcal{A}_{\lambda_1\lambda_2}(0)$$

|Inelastic| < Elastic

$$A^{(2)}_{\lambda_1\lambda_2} = \int_{M^2}^{\infty} \frac{ds}{2\pi} \frac{\left\langle 3^{\lambda_1} 4^{\lambda_2} \right| \mathcal{M}^\dagger \mathcal{M} \left| 1^{\lambda_1} 2^{\lambda_2} \right\rangle + \left\langle 3^{\lambda_1} 4^{\bar{\lambda}_2} \right| \mathcal{M}^\dagger \mathcal{M} \left| 1^{\lambda_1} 2^{\bar{\lambda}_2} \right\rangle}{(s - 2m^2 + t/2)^3}$$

$$t \neq 0$$



lack of positivity at t-finite?

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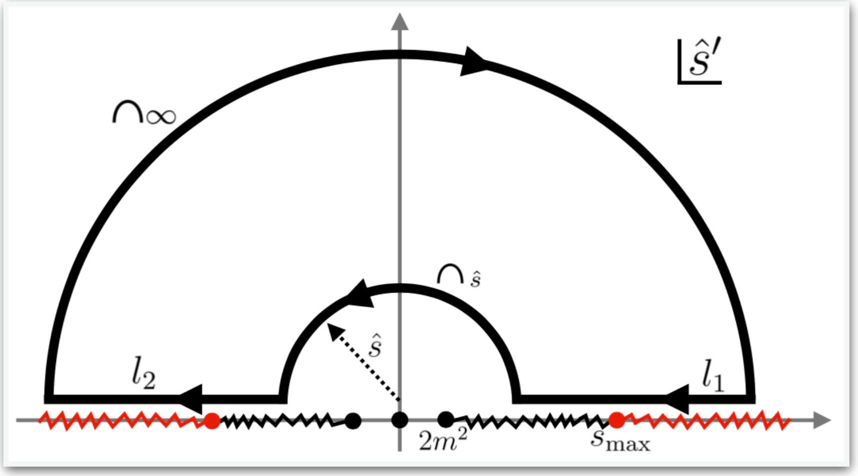

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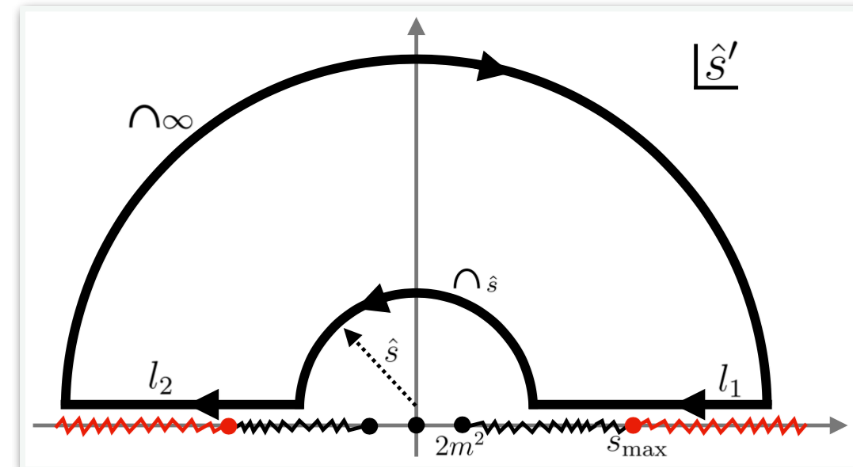

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$$\begin{array}{c|c} & |\rightarrow\leftarrow\rangle \quad |\nearrow\swarrow\rangle \\ \hline |\rightarrow\leftarrow\rangle, |\nearrow\swarrow\rangle & \mathcal{M}^\dagger \mathcal{M} \succ 0 \end{array} \implies \left| \langle 3^{\lambda_1} 4^{\lambda_2} | \mathcal{M}^\dagger \mathcal{M} | 1^{\lambda_1} 2^{\lambda_2} \rangle \right| \leq \langle 1^{\lambda_1} 2^{\lambda_2} | \mathcal{M}^\dagger \mathcal{M} | 1^{\lambda_1} 2^{\lambda_2} \rangle$$

positive definite matrix

|Inelastic| < Elastic

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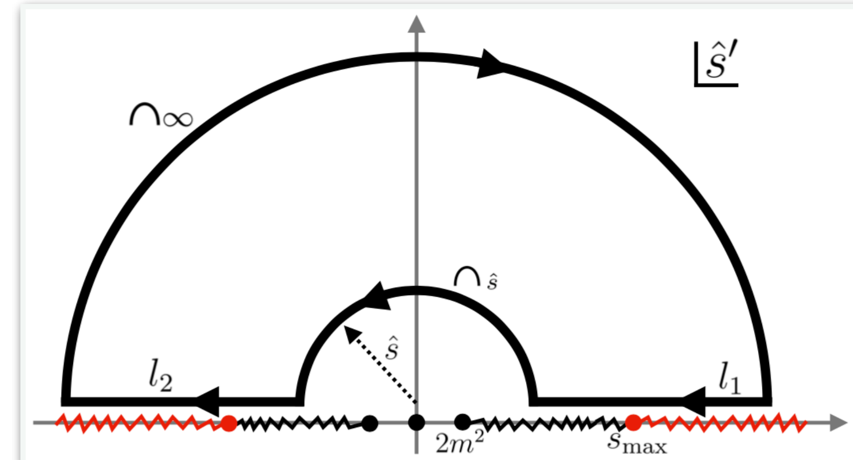

lack of positivity at t-finite? NO!

$|\rightarrow\leftarrow\rangle$ $|\nearrow\swarrow\rangle$

$|\rightarrow\leftarrow\rangle$, $|\nearrow\swarrow\rangle$

$$\mathcal{M}^\dagger\mathcal{M} \succ 0$$

positive definite matrix

$$\Longrightarrow \quad \left|\langle 3^{\lambda_1}4^{\lambda_2}|\mathcal{M}^\dagger\mathcal{M}|1^{\lambda_1}2^{\lambda_2}\rangle\right| \leq \langle 1^{\lambda_1}2^{\lambda_2}|\mathcal{M}^\dagger\mathcal{M}|1^{\lambda_1}2^{\lambda_2}\rangle$$

$$\Downarrow$$

finite-t arc bounded by t=0:

$$\frac{\left|\mathcal{A}^{(2)}_{\lambda_1\lambda_2}(t)\right|}{\mathcal{A}^{(2)}_{\lambda_1\lambda_2}(0)} \leq \frac{1}{\left(1+\frac{t/2}{M^2-2m^2}\right)^3}$$

2304.02550 B.B., Isabella, Ricossa, Riva up to small EFT-controlled error

Back to Massive Gravity

$$\mathcal{L} = \sqrt{-g}\frac{m_{\rm Pl}^2}{2}\left[R - \frac{m^2}{4}\left(h^2 + \ldots + c_3 h^3 + \ldots d_5 h^4\right)\right]$$

5 d.o.f, 2 free parameters

$$h_{\mu\nu} \to h_{\mu\nu}^{(\pm 2)} + \partial_{(\mu} A_{\nu)}^{(\pm 1)} + \partial_\mu \partial_\nu \pi^{(0)}$$

$$\Lambda_3 = (m^2 M_{\rm Pl})^{1/3} \approx 10^{-13}\,{\rm eV} \approx (1000\,{\rm km})^{-1}$$

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$$\mathcal{A}_{00} \xrightarrow[m^2 \ll |t|]{} \frac{t}{6\Lambda_3^6}\left(1 - 4c_3 + 36c_3^2 + 64d_5\right)$$

$$\mathcal{A}_{0+} \xrightarrow[m^2 \ll |t|]{} \frac{t}{96\Lambda_3^6}\left(1 + 24c_3 + 144c_3^2 + 384d_5\right)$$

$$\mathcal{A}_{++} \xrightarrow[m^2 \ll |t|]{} \frac{9t}{64\Lambda_3^6}\left(1 - 4c_3\right)^2 ,$$

vs

$$\mathcal{A}_{00} \xrightarrow[t=0]{} \frac{2m^2}{9\Lambda_3^6}\left(7 - 6c_3 - 18c_3^2 + 48d_5\right)$$

$$\mathcal{A}_{0+} \xrightarrow[t=0]{} \frac{m^2}{48\Lambda_3^6}\left(91 - 312c_3 + 432c_3^2 + 384d_5\right)$$

$$\mathcal{A}_{++} \xrightarrow[t=0]{} \frac{m^2}{8\Lambda_3^6}\left(7 - 24c_3^2 + 48d_5\right) .$$

Back to Massive Gravity

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\end{aligned}$$

vs

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\end{aligned}$$

TENSION!

$$\frac{|\mathcal{A}_{\lambda_1\lambda_2}(t)|}{\mathcal{A}_{\lambda_1\lambda_2}(0)} \leq \frac{1}{\left(1 + \frac{t/2}{M^2 - 2m^2}\right)^3}$$

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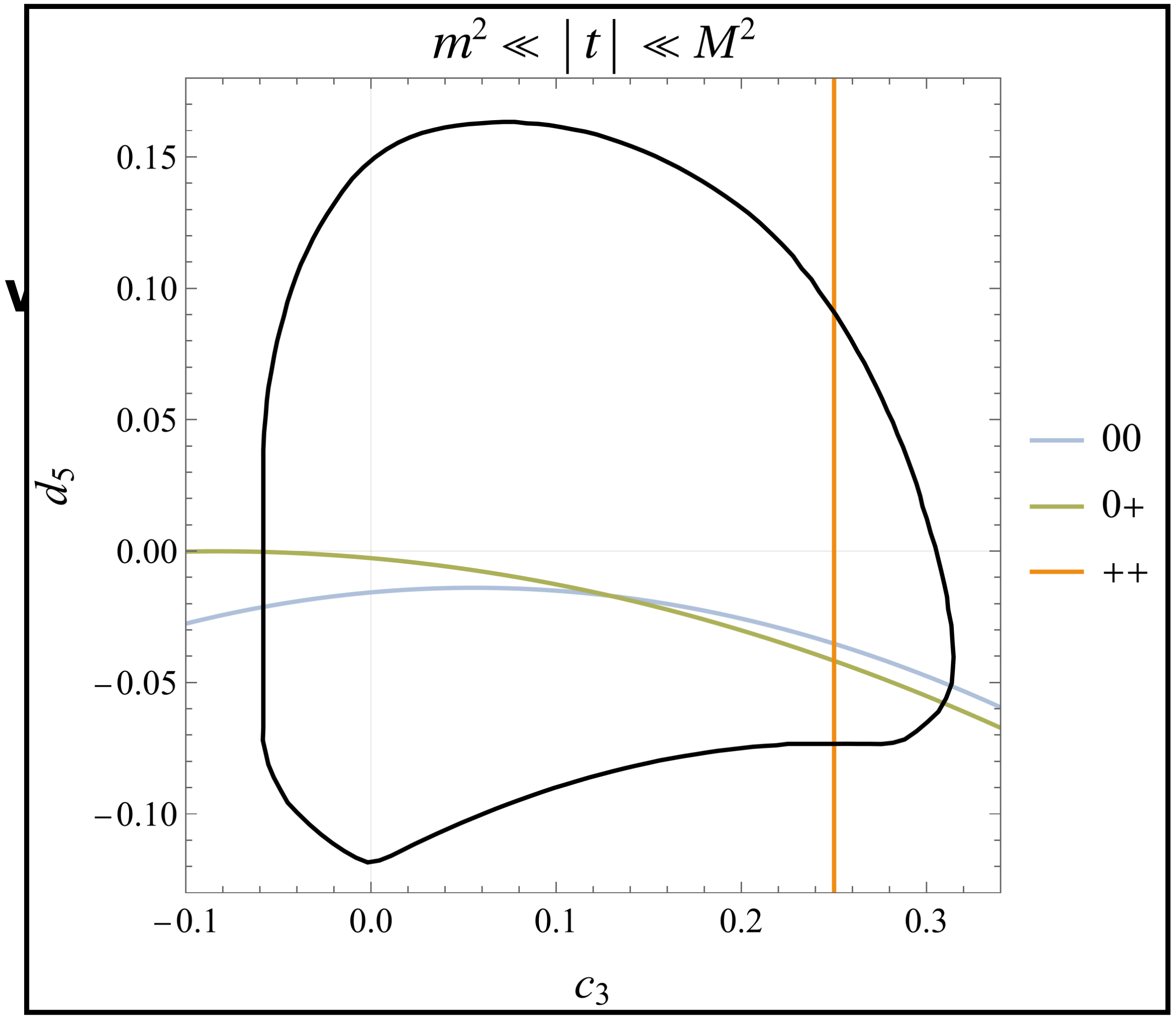


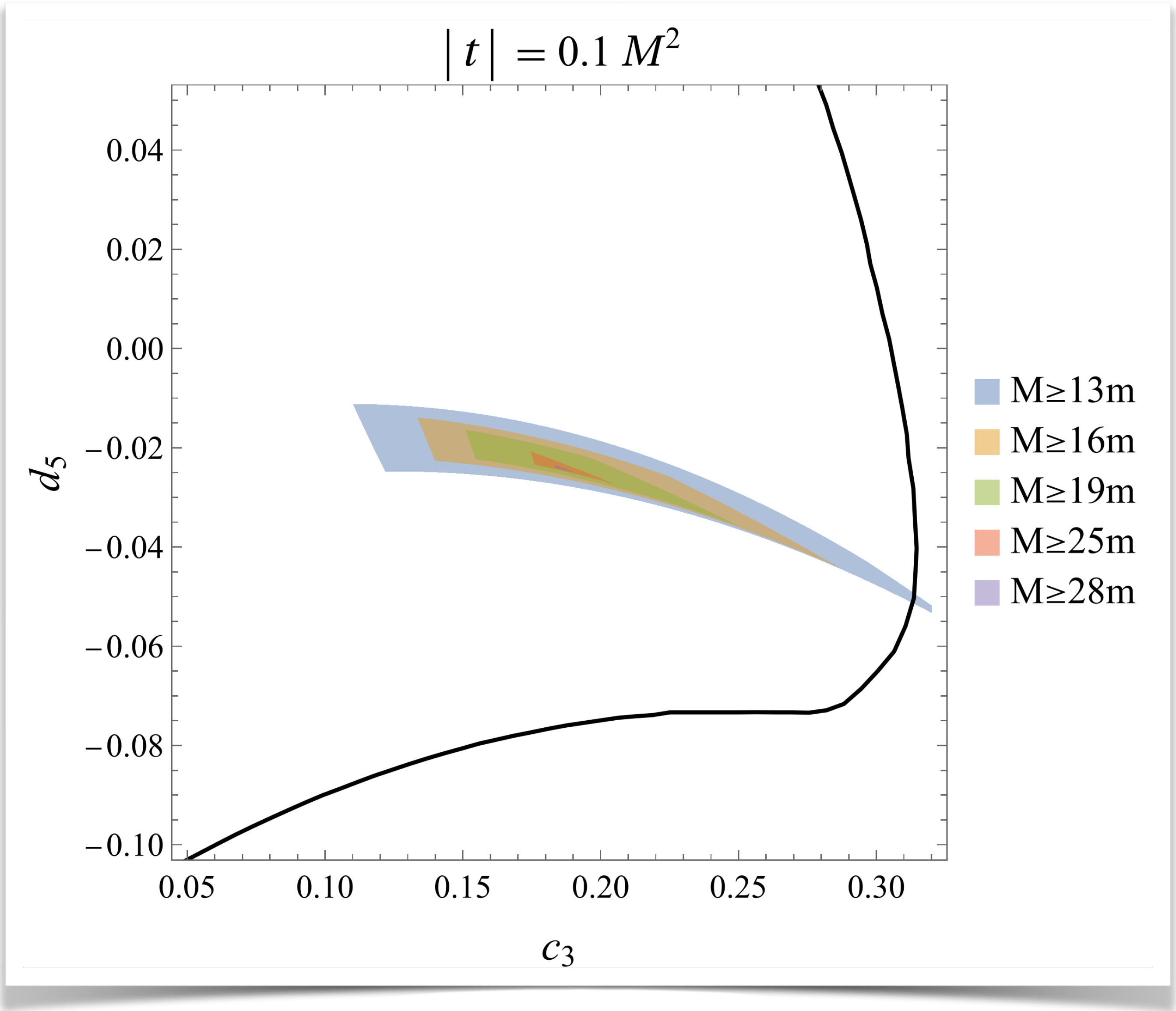
TENSION!

$$\frac{|\mathcal{A}_{\lambda_1\lambda_2}(t)|}{\mathcal{A}_{\lambda_1\lambda_2}(0)} \leq \frac{1}{\left(1 + \frac{t/2}{M^2 - 2m^2}\right)^3}$$

no intersection —> M bounded above by few m

(assuming analyticity in s cut-plane for |t| within EFT)

Massive Gravity Cutoff



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$$\frac{|\mathcal{A}_{\lambda_1\lambda_2}(t)|}{\mathcal{A}_{\lambda_1\lambda_2}(0)} \leq \frac{1}{\left(1+\frac{t/2}{M^2-2m^2}\right)^3}$$

no parametric separation with m

$$\Longrightarrow \quad M < 30m \sim o(10)H$$

teeny tiny range of validity
(60 orders worse than GR!)

no physical system in the universe where this EFT can be used

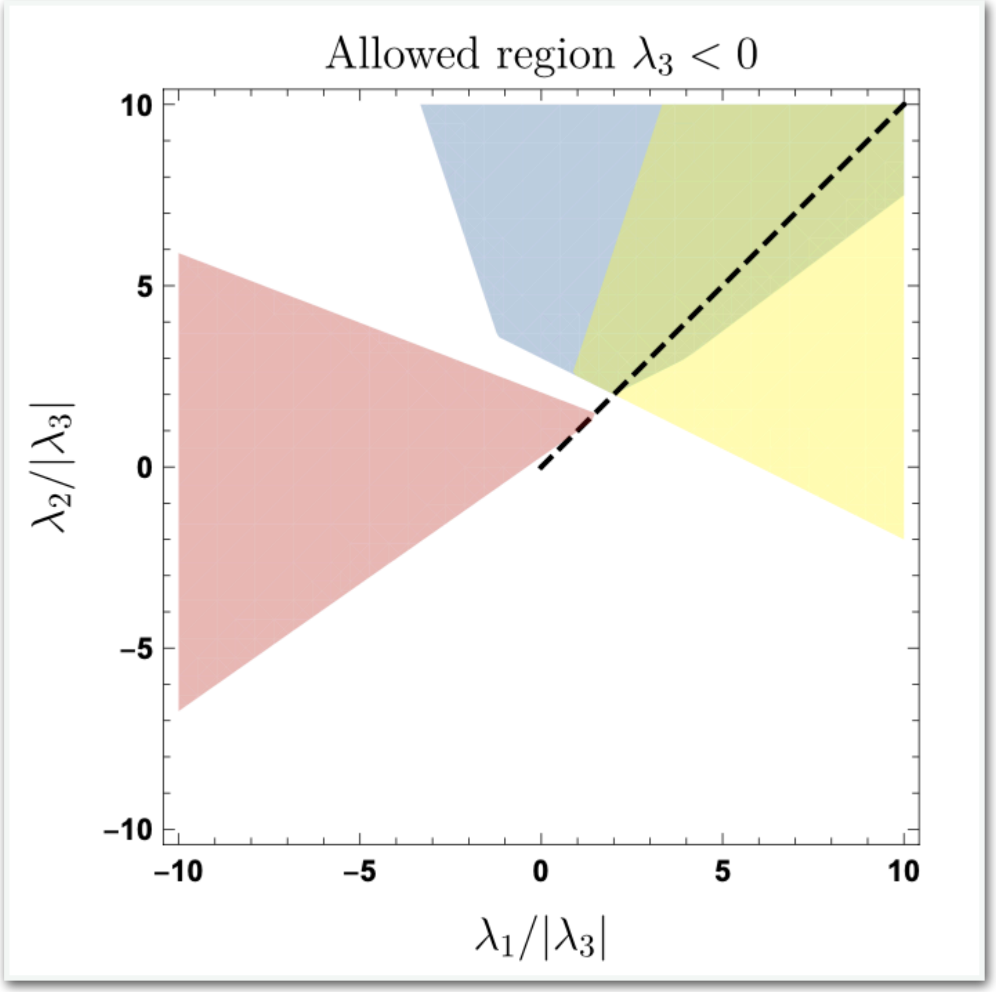
Isolated J>2 Spins?

suffer of the same problem:
longitudinal-helicities grow too fast with energy, either in s or t or both

$$\mathcal{M}^{(J-\text{even})}_{\text{long.}} = g^2(m_J/\Lambda_J)\left[\left(\frac{E}{m_J}\right)^{3J} + \ldots + \frac{E^4}{m_J^4} + \ldots\right] \qquad \mathcal{M}^{(J-\text{odd})}_{\text{long.}} = g^2(m_J/\Lambda_J)\left[\left(\frac{E}{m_J}\right)^{3J+1} + \ldots + \frac{E^4}{m_J^4} + \ldots\right]$$

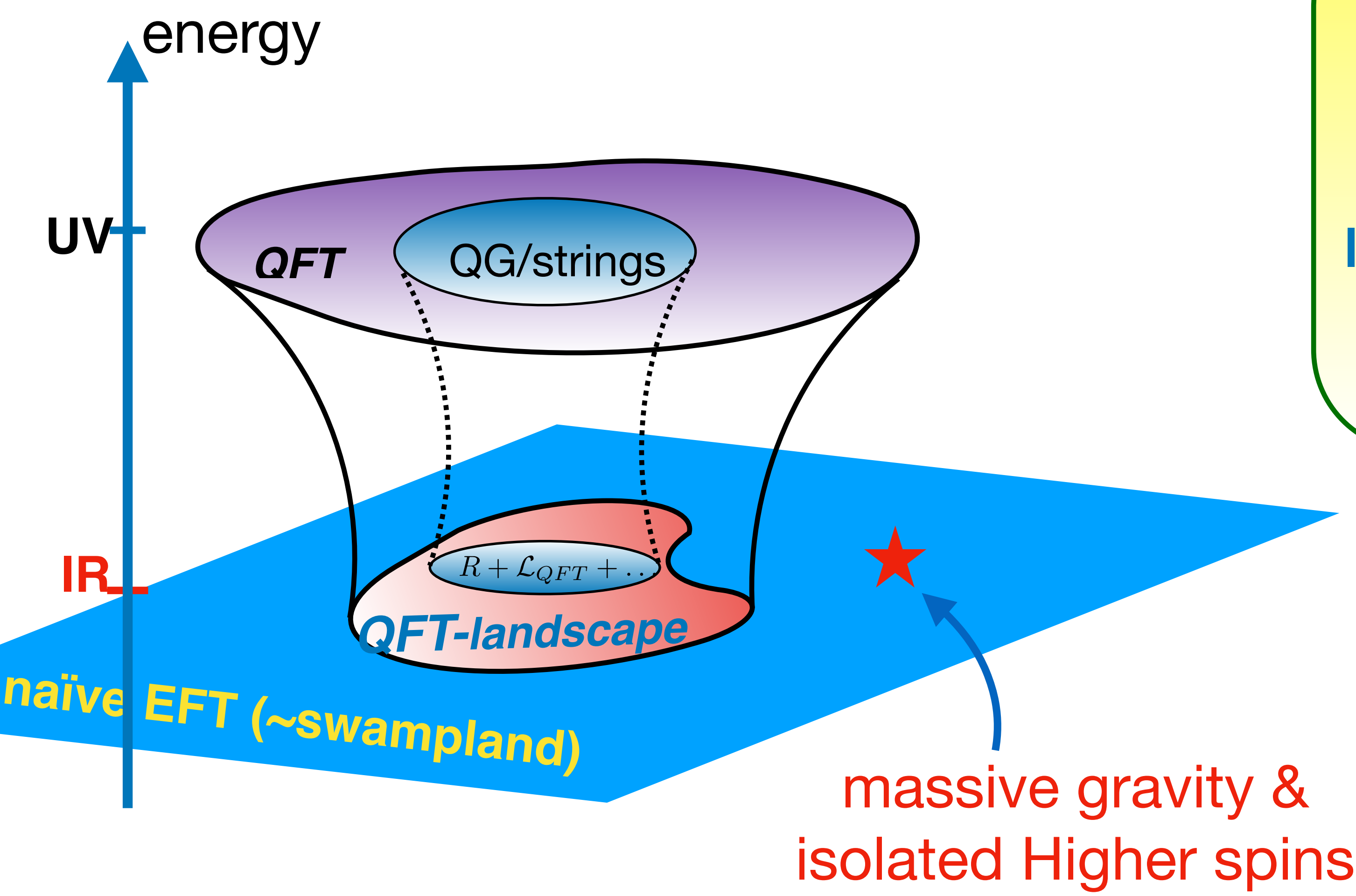
example J=3: fails positive already @ t=0

1903.08664 B.B., F. Riva, J. Serra, F. Sgarlata



Conclusions

Conclusions



I. positivity bounds shape the swampland-landscape boundary

II. isolated (M>>m) higher-spins can't be UV completed
(corollary: massive gravity is not positive!)

Open Question for Christmas:

do massive higher-spins only come in towers of infinitely many states ?
e.g. KK tower, Regge states, QCD resonances,… (no parametric separation)

thank you!

Image Credits

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