Imperial College London

IFT Xmas Workshop

14th December 2023

Stochastic Gravitational Waves Backgrounds

Digging for physics in SGWBs

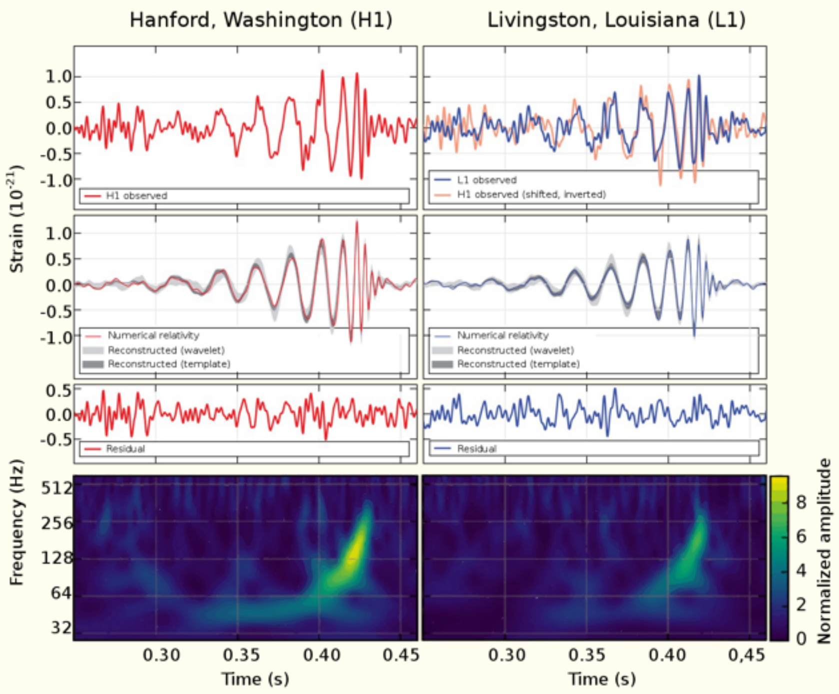
Carlo Contaldi – Imperial College London

Outline

- GW Astrophysics
- Stochastic gravitational waves backgrounds.
- Physics of SGWBs across the frequency domain.
- Observational Landscape.
- Challenges.
- New directions.

Gravitational Waves

- First direct detection of a BH merger even in 2015.
- BH binary: 36 and 31 solar masses.
- Frequency ~ 100Hz.
- Distance ~ 400 Mpc.



LIGO GW150914 discovery event

LIGO Observatory

LIGO Hanford

LIGO Livingston

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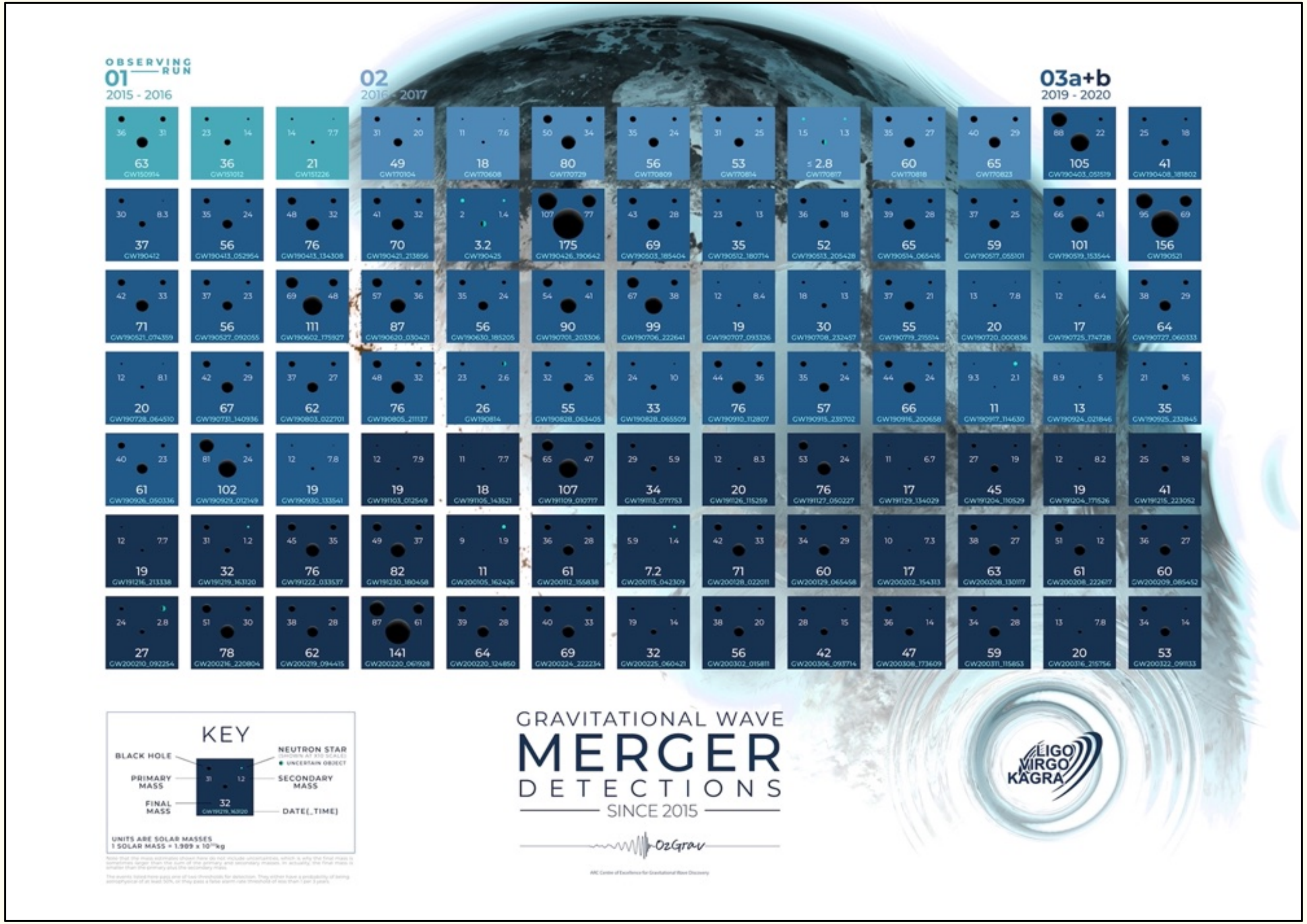

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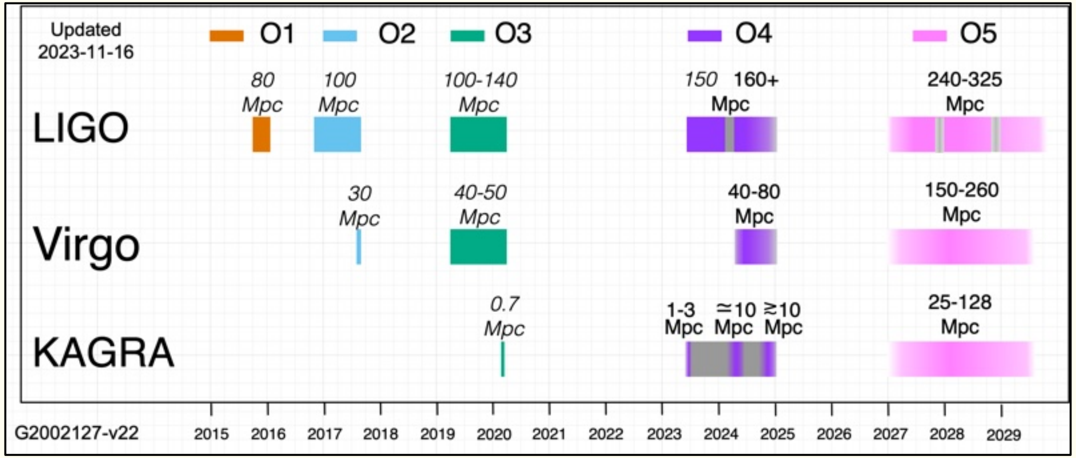
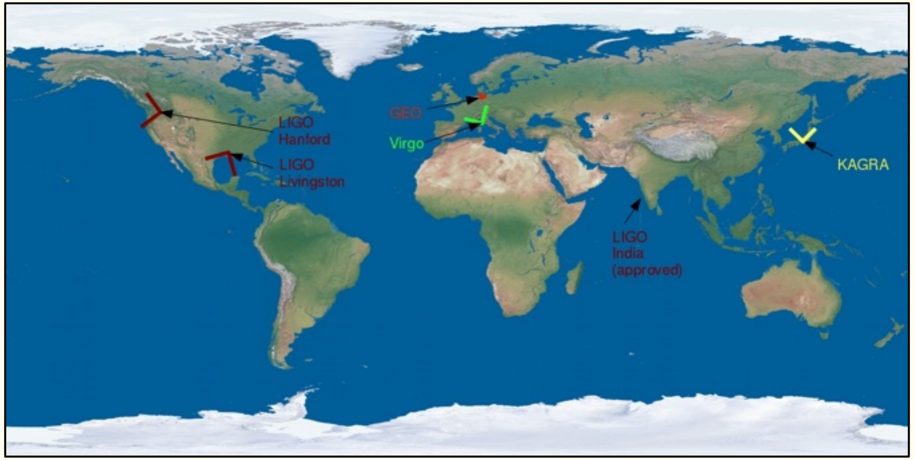


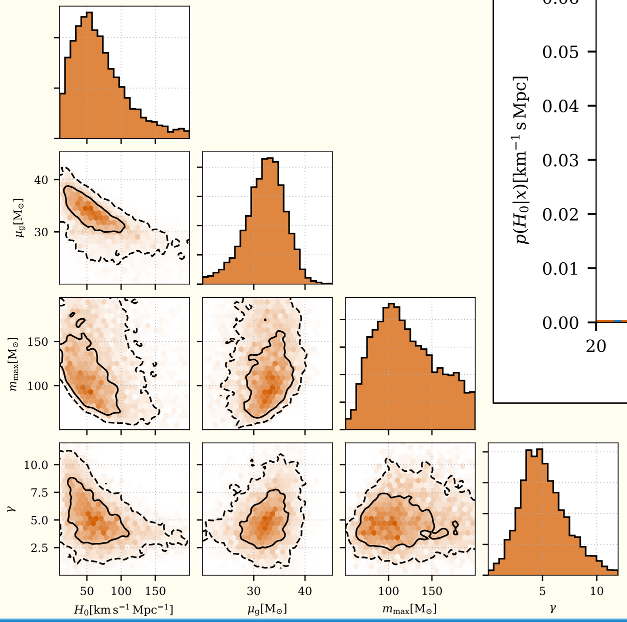


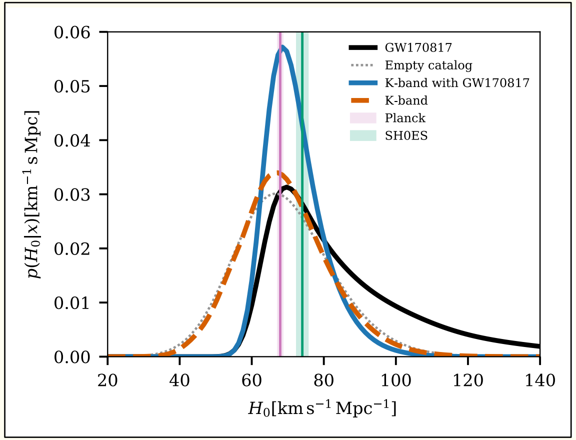
GW Astronomy

- Routine observations of transient BH-BH and BH-NS, merger events mean we are in the age of GW astronomy.
- Ongoing efforts to constrain fundamental properties of gravity, source populations, and neutron stars.
- Not yet in the ”statistical age” but this is certain for transient phenomena.

Fundamental constraints (transient events)

- Hubble rate constraints.
- Standard sirens.
  - EM counterparts.
  - BH-NS or NS-NS?
- Dark sirens.
  - Mass distributions.
  - Galaxy redshift cross-correlations.

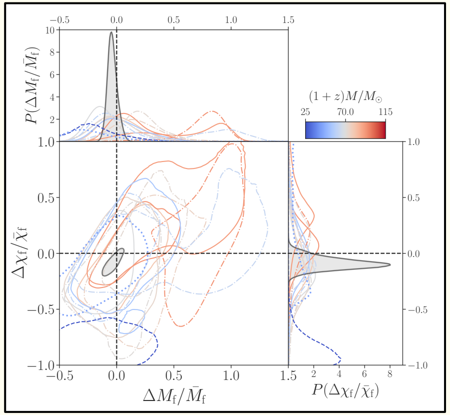


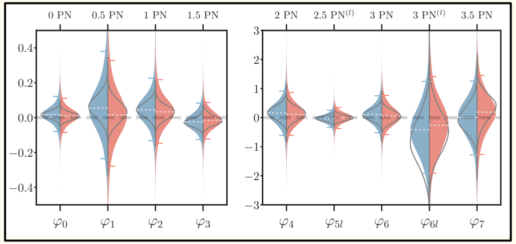


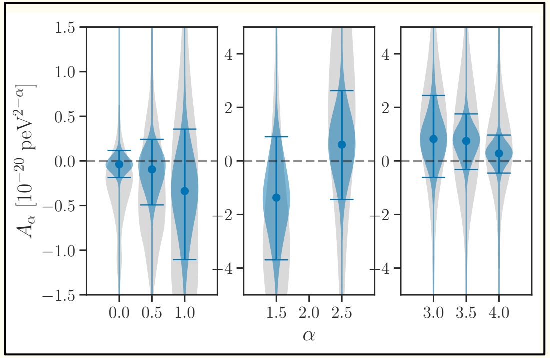
[LVK O3 results (Abbot et al. 2021)]

Fundamental constraints (transient events)

- Tests of GR.
- Parametrised models for deviation from General Relativity e.g.
  - Inspiral-Merger-Ringdown consistency (binary → final product?)
  - Post-Newtonian generalisations.
  - Dispersion relation.
  - Polarisations.
  - BH spectroscopy (ringdown analysis)







[LVK O3 results (Abbot et al. 2021)]

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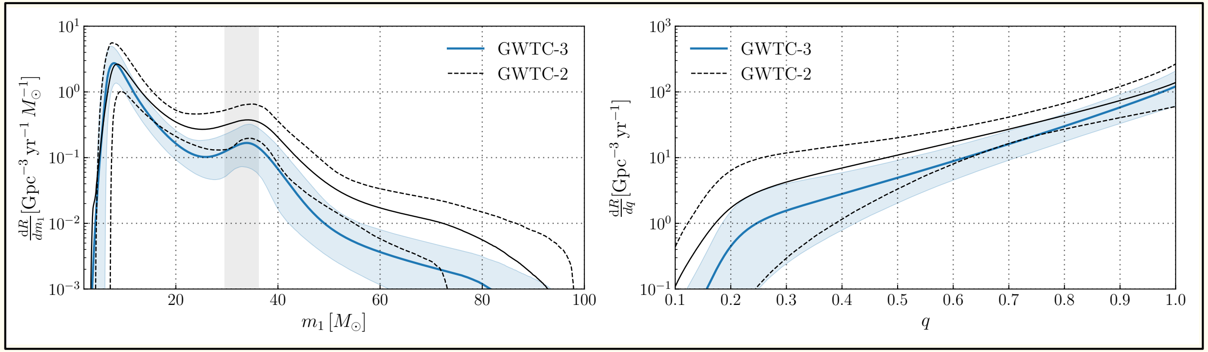
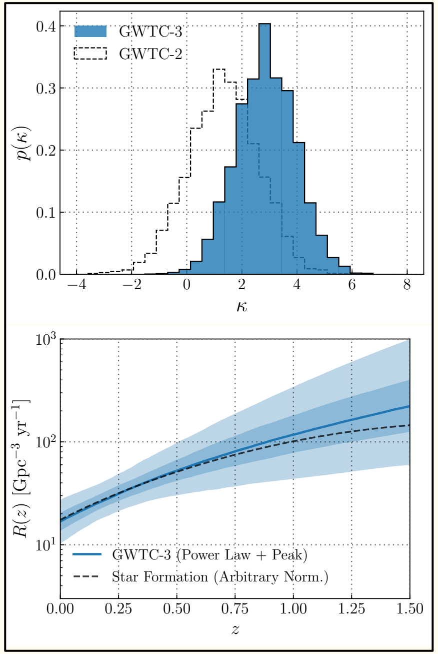
Astrophysics (transient events)

- Constraints on BH and NS mass distributions and merger rates.
- Impact forecasts on future fundamental constraints.





[LVK O3 results (Abbot et al. 2021)]

Transients → Stochastic Backgrounds

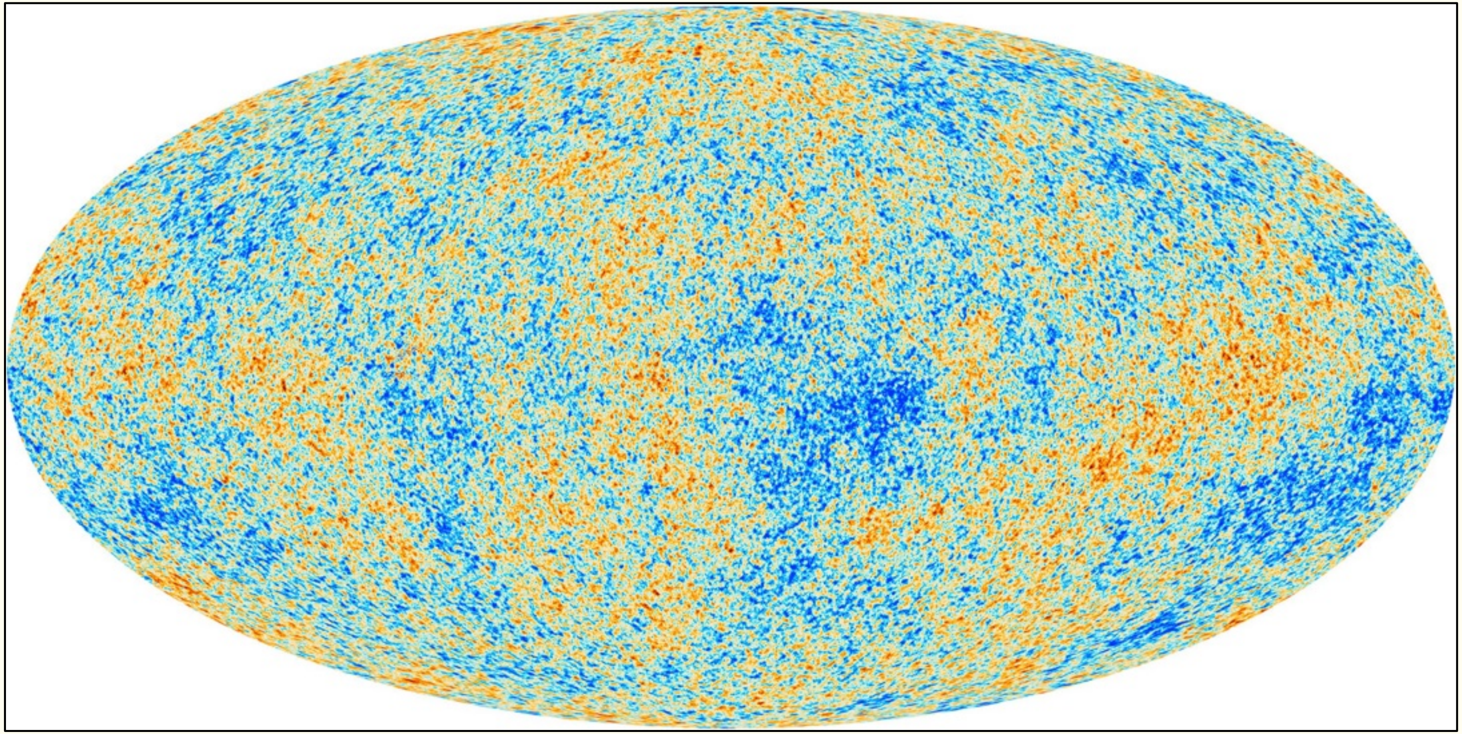
- What about the stochastic regime?
  - Signal from unresolved points sources.
  - Confusion limited.
  - Extended sources (angularly correlated).
  - Cosmological backgrounds.

$$h(t) \to \langle h(t)h(t')\rangle$$

$$A_\mu(t) \to \langle A_\mu(t)A^\mu(t')\rangle$$

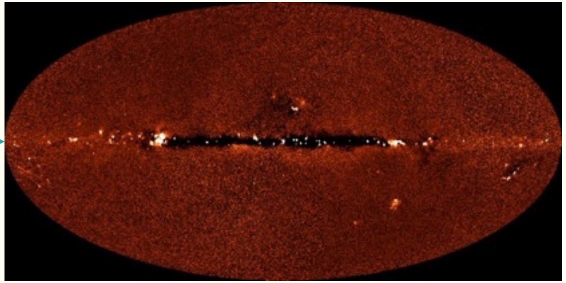
Stochastic GW Backgrounds (SGWBs)

- What signals can we expect?
- What physical mechanisms generate SGWBs?
- How well can we observe these signals?
- Can we “mine” SGWBs for statistical constraints?



ESA Planck Satellite.

Cosmic Infrared Background



DIRBE

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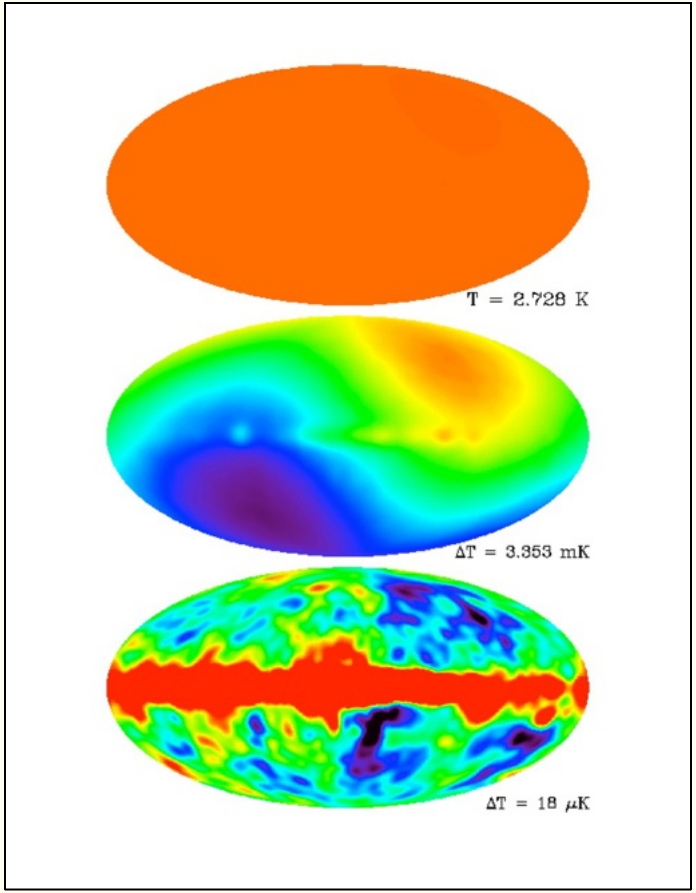
- Monopole = 1 number
- Dipole = 3 numbers
- COBE ~ 1000 numbers!



Stochastic GWs

- Deterministic
  - Amplitude and phase carry information.
- Stochastic
  - Superposition of waves with stochastic amplitudes and uncorrelated phases.

Waves: $$h_{\mu\nu}(\mathbf{x},t) = h_P(f,\hat{\mathbf{k}})\epsilon^P_{\mu\nu}(\hat{\mathbf{k}})e^{i2\pi f(t-\hat{\mathbf{k}}\cdot\mathbf{x}/c)}$$

TT-gauge tensor Polarisation:

$$\epsilon^+_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\epsilon^\times_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$h_{ij}(\mathbf{x},t) = \sum_{P=+,\times} \int d\Omega_{\hat{\mathbf{k}}} \int_{-\infty}^{\infty} df\, h_P(f,\hat{\mathbf{k}})\epsilon^P_{ij}(\hat{\mathbf{k}})e^{i2\pi f(t-\hat{\mathbf{k}}\cdot\mathbf{x}/c)}$$

Stochastic GWs

$$h_{ij}(\mathbf{x},t) = \sum_{P=+,\times} \int d\Omega_{\hat{\mathbf{k}}} \int_{-\infty}^{\infty} df\, h_P(f,\hat{\mathbf{k}})\epsilon_{ij}^P(\hat{\mathbf{k}}) e^{i2\pi f(t-\hat{\mathbf{k}}\cdot\mathbf{x}/c)}$$

- Assumptions:
  - Superposition of waves with stochastic amplitudes and uncorrelated phases.

$$\langle h_{ij}(\mathbf{x},t)\rangle = 0$$

$$\langle h_{ij}(\mathbf{x},t) h^{ij}(\mathbf{x}',t')\rangle \neq 0$$

$$\langle h_P(f,\hat{\mathbf{k}}) h^\star_{P'}(f',\hat{\mathbf{k}}')\rangle = \delta(P-P')\delta(f-f')\delta^{(2)}(\hat{\mathbf{k}}-\hat{\mathbf{k}}')I(f,\hat{\mathbf{k}})$$

[Stationary, statistically isotropic, gaussian, intensity]

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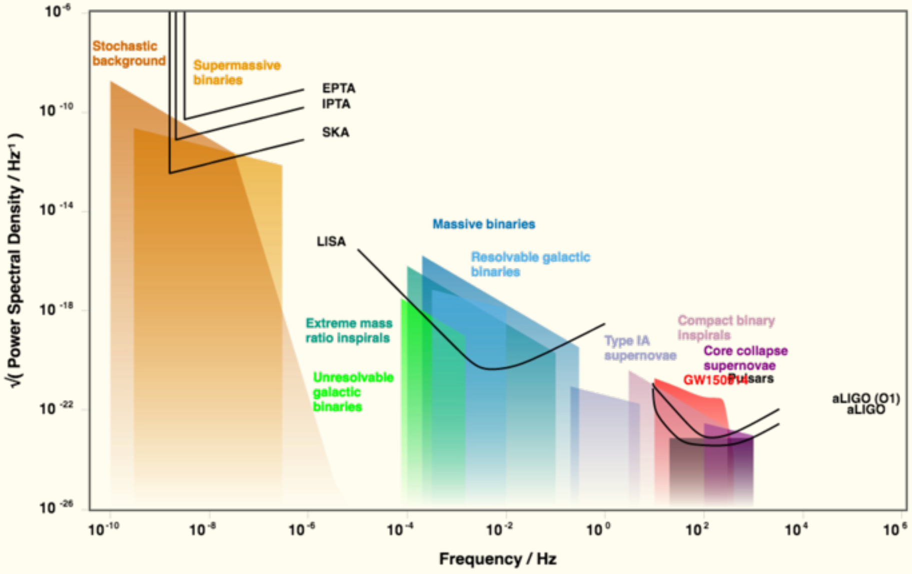
Stochastic GWs



[http://gwplotter.com]

$$\langle h_P(f,\hat{\mathbf{k}})h^\star_{P'}(f',\hat{\mathbf{k}}')\rangle = \delta(P-P')\delta(f-f')\delta^{(2)}(\hat{\mathbf{k}}-\hat{\mathbf{k}}')I(f,\hat{\mathbf{k}})$$
$$= \frac{\delta(P-P')}{2}\frac{\delta(f-f')}{2}\frac{\delta^{(2)}(\hat{\mathbf{k}}-\hat{\mathbf{k}}')}{4\pi}S_h(f,\hat{\mathbf{k}})$$

One-sided Power Spectrum:

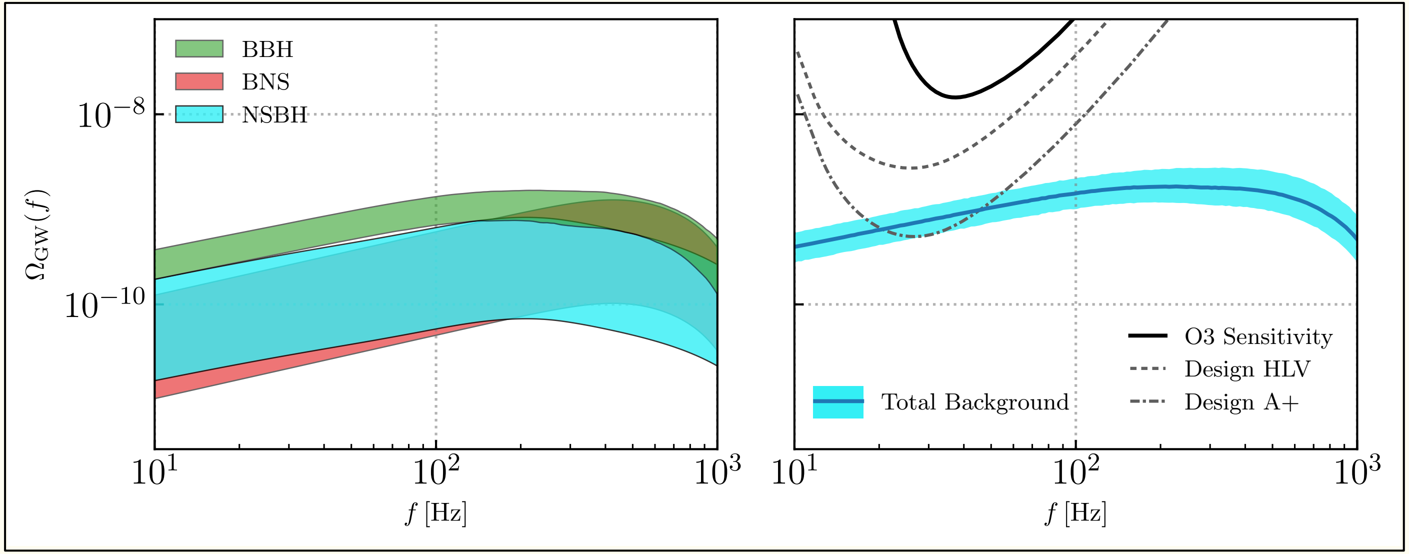
$$S_h(f,\hat{\mathbf{k}}) = 16\pi I(f,\hat{\mathbf{k}})$$

$$\rho_{\rm GW} = \frac{c^2}{32\pi G}\langle \dot{h}_{ij}\dot{h}^{ij}\rangle \equiv \frac{\pi c^2}{2G}\int d\Omega_{\hat{\mathbf{k}}}\int_0^\infty df\, f^2 I(f,\hat{\mathbf{k}})$$

Energy density:

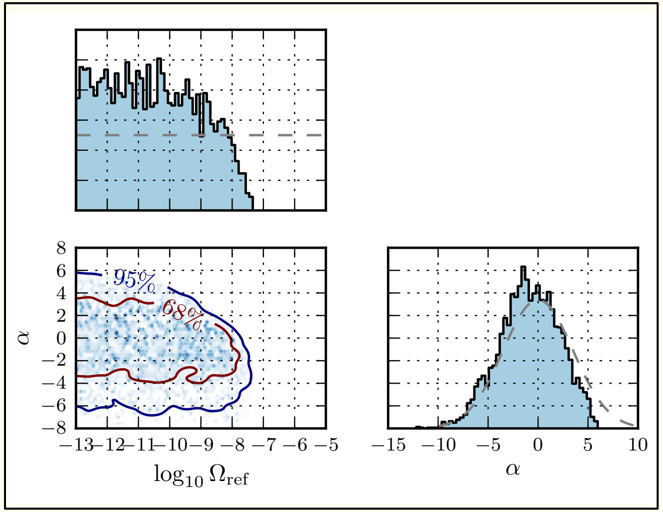
$$\Omega_{\rm GW}(f,\hat{\mathbf{k}}) = \frac{32\pi^3}{3H_0^2}f^3 I(f,\hat{\mathbf{k}})$$

- LVK prediction of isotropic SGWB from binary mergers.
- The limit is close to expected level.
- Detection in O4?
- Statistical era of population studies.



[LVK O3 2021]

SGWB constraints



[LVK O3 2021]

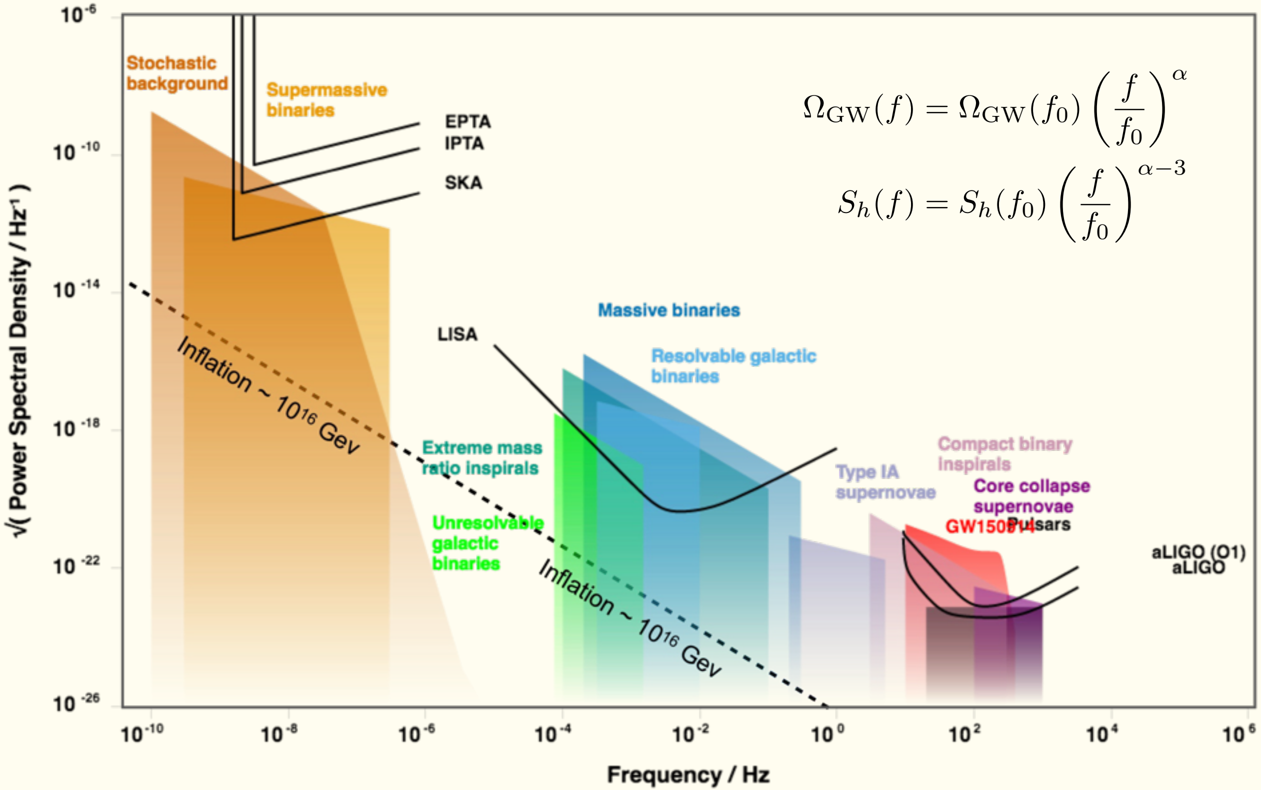
	Uniform prior			Log-uniform prior		
α	O3	O2 [43]	Improvement	O3	O2 [43]	Improvement
0	1.7×10^{-8}	6.0×10^{-8}	3.6	5.8×10^{-9}	3.5×10^{-8}	6.0
2/3	1.2×10^{-8}	4.8×10^{-8}	4.0	3.4×10^{-9}	3.0×10^{-8}	8.8
3	1.3×10^{-9}	7.9×10^{-9}	5.9	3.9×10^{-10}	5.1×10^{-9}	13.1
Marg.	2.7×10^{-8}	1.1×10^{-7}	4.1	6.6×10^{-9}	3.4×10^{-8}	5.1

Polarization	O3	O2 [43]	Improvement
Tensor	6.4×10^{-9}	3.2×10^{-8}	5.0
Vector	7.9×10^{-9}	2.9×10^{-8}	3.7
Scalar	2.1×10^{-8}	6.1×10^{-8}	2.9

GWB Type	α
Astro	3
Inspiral	2/3
Cosmo	0

$$\Omega_{\rm GW}(f) = \Omega_{\rm GW}(f_0)\left(\frac{f}{f_0}\right)^{\alpha}$$

$$S_h(f) = S_h(f_0)\left(\frac{f}{f_0}\right)^{\alpha-3}$$



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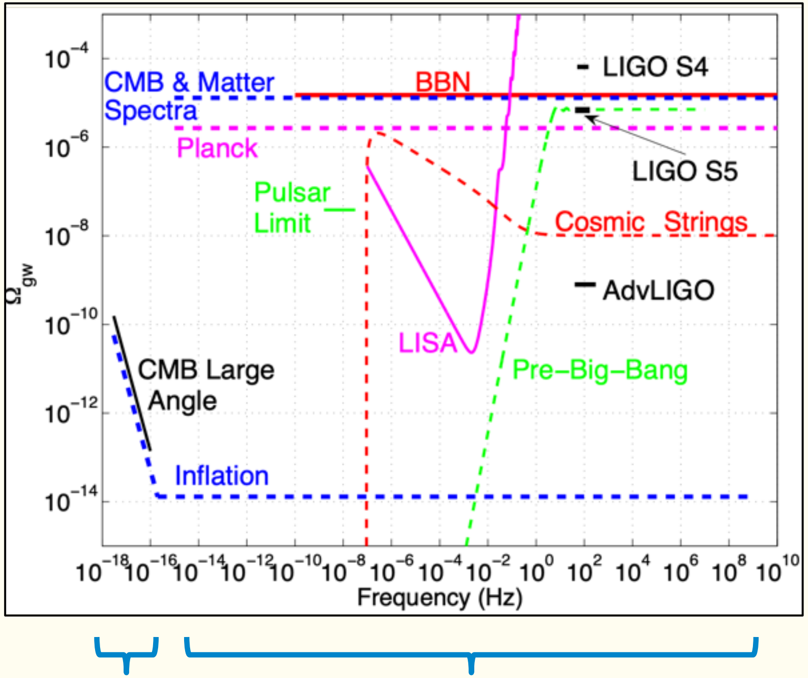
SGWB sources

- Inflation sets up a primordial, super-horizon, near-scale-invariant background of GWs.
- Similar to curvature perturbations.
- Squeezed travelling waves (zero-momentum) until horizon re-entry.
- Radiation-matter equality imprinted in sub-horizon evolution.

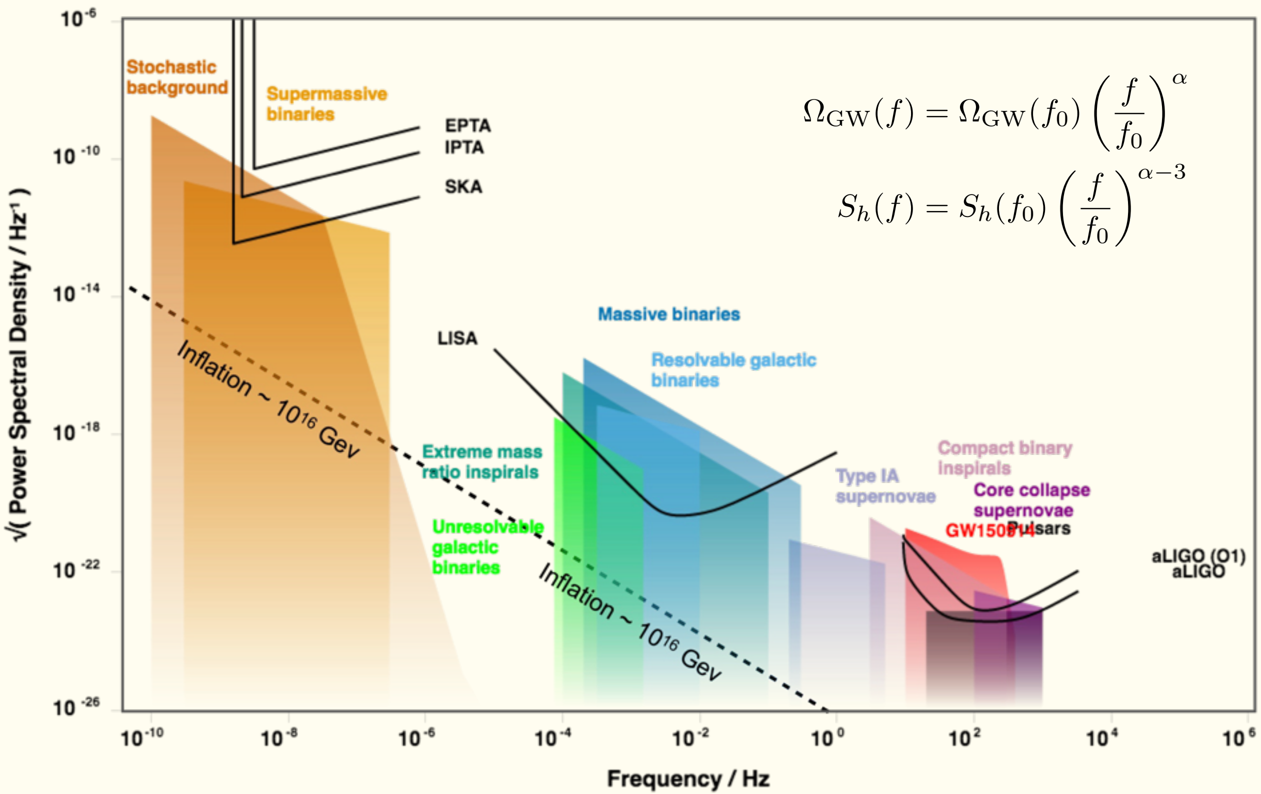
$$\Omega_{\rm GW}(f) \sim \frac{16}{9}\frac{V\phi}{M_{Pl}^4}(1-z_{eq})^{-1}$$

$$\Omega_{\rm GW}(f > f_{eq}) \sim f^{n_t}$$



[Thrane & Romano 2014]

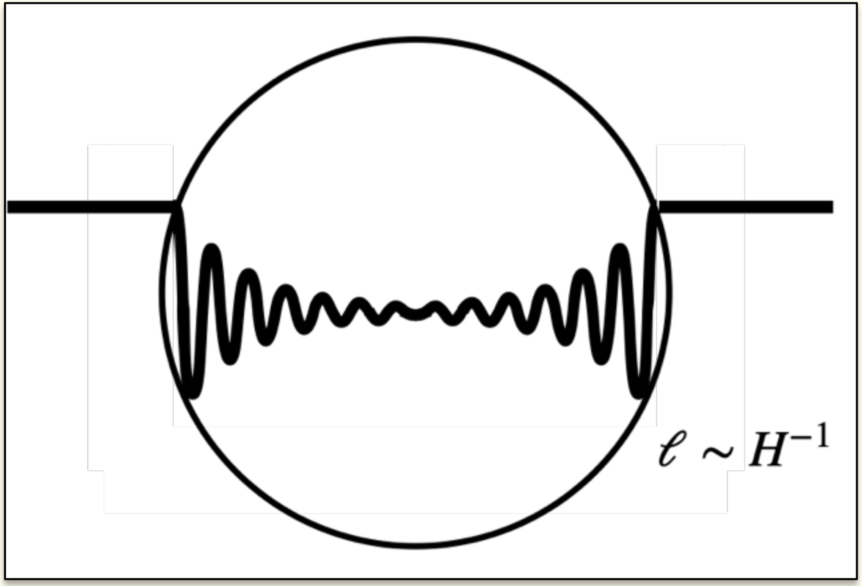
GWB Type	α
Astro	3
Inspiral	2/3
Cosmo	0

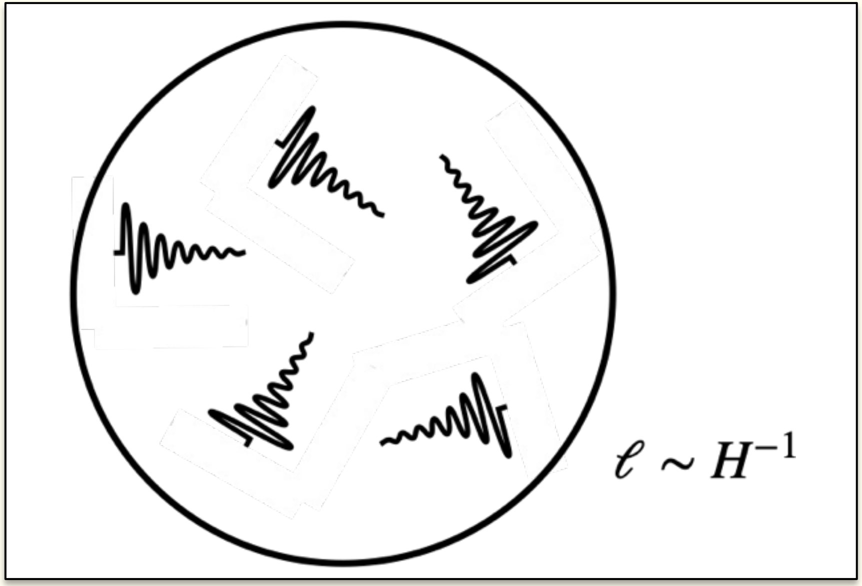


$$\Omega_{\rm GW}(f) = \Omega_{\rm GW}(f_0)\left(\frac{f}{f_0}\right)^{\alpha}$$

$$S_h(f) = S_h(f_0)\left(\frac{f}{f_0}\right)^{\alpha-3}$$

Coherent vs Incoherent?





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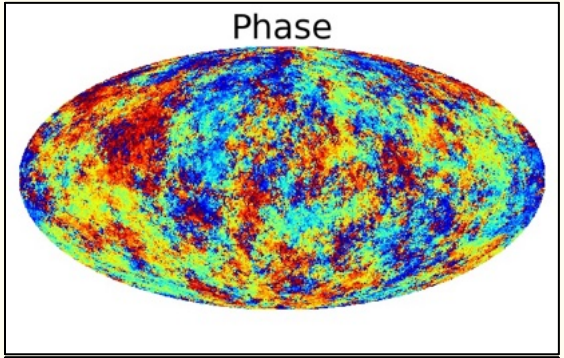
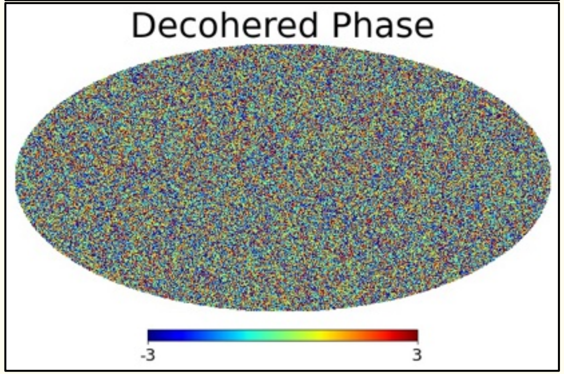
Coherent vs Incoherent?

- Squeezed super-horizon modes will be coherent on re-entry.
- "**Standing Waves**" (Grischuk 1974) correlated left and right moving travelling waves.
- Could be detected using coherent GW detectors [CC & Magueijo 2018].
- Phase coherence in h → bispectrum <h^3>? [Bartolo 2018]
- cf. angular coherence of CMB acoustic peaks.
- Large-scale structure "decoheres" primordial SGWB at observable frequencies [Margalit, Pieroni, & CC 2020].



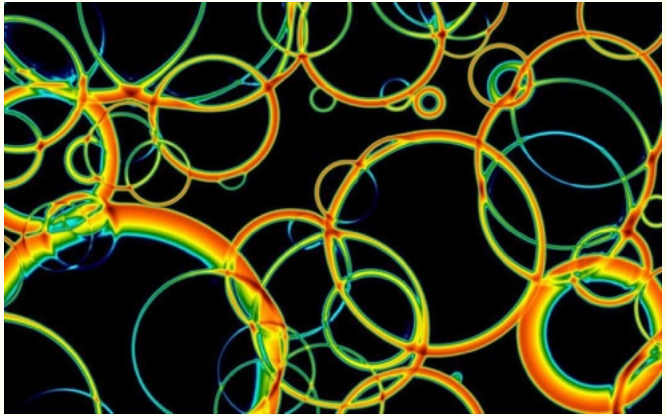


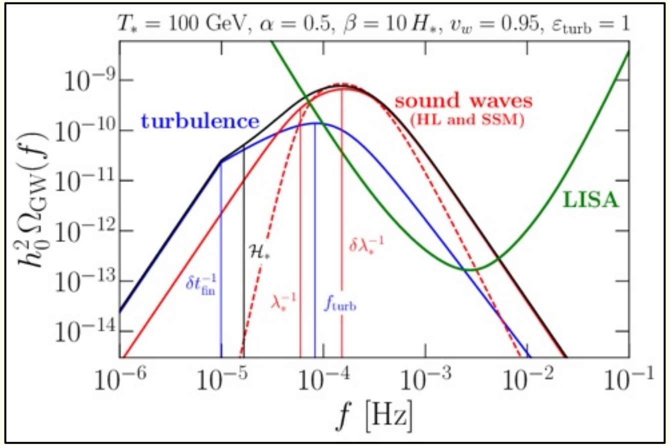
$$\delta\varphi(\hat{\mathbf{n}}) = 2\pi f \int_{\text{l.o.s.}} [\Phi(\eta, \mathbf{x}) + \Psi(\eta, \mathbf{x})] \, d\eta$$

SGWB sources (Phase Transitions)

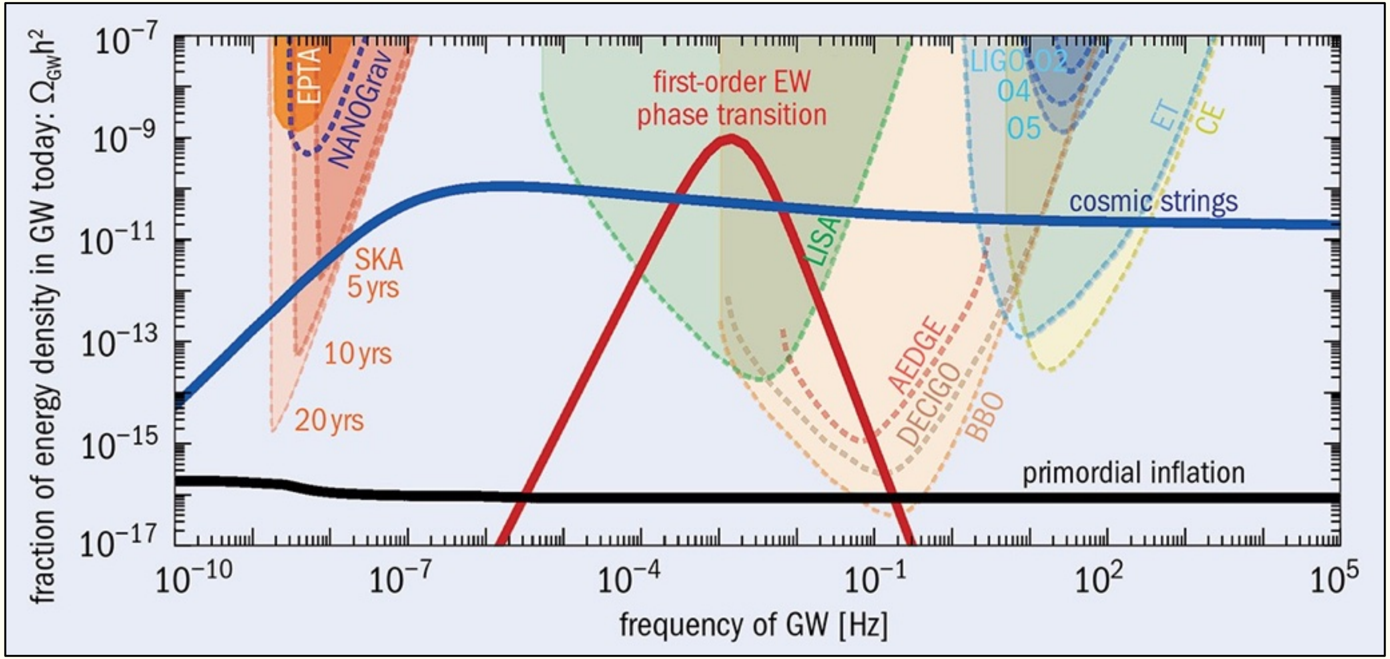
- First-order phase transitions at early times.
- E.g. QCD, EW, phase transitions.
  - Nucleate bubbles of vev.
  - Turbulent motion.
  - Bubble collision
  - Non-linear dynamics
- MeV scale → Pulsar Timing Arrays.
- GeV-TeV scale → LISA
- > TeV → AEDGE, ET?

[Mark Hindmarsh]





[Roper et al 2023]

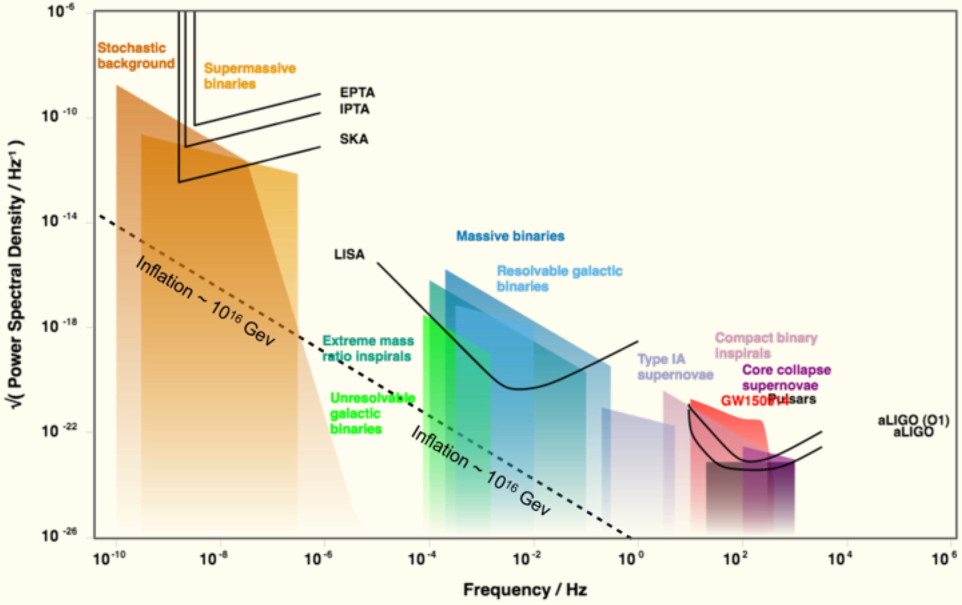


[P. Simakachorn, CERN]

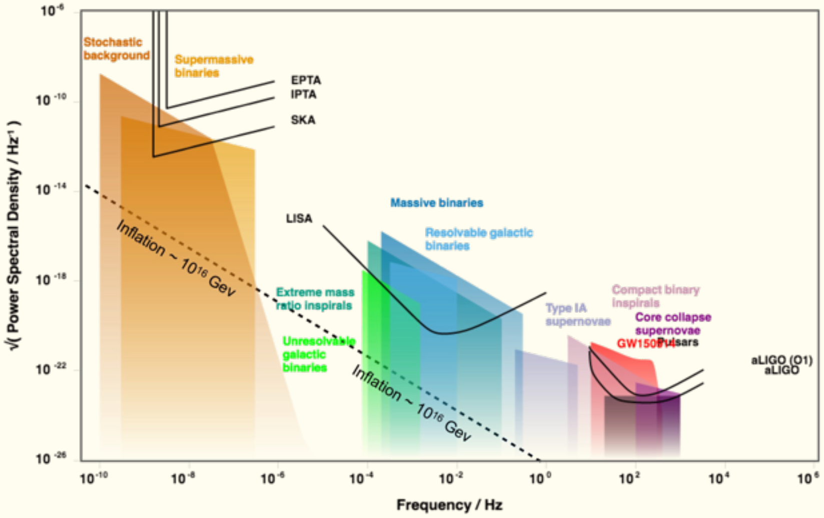
SGWB sources (Inspirals)

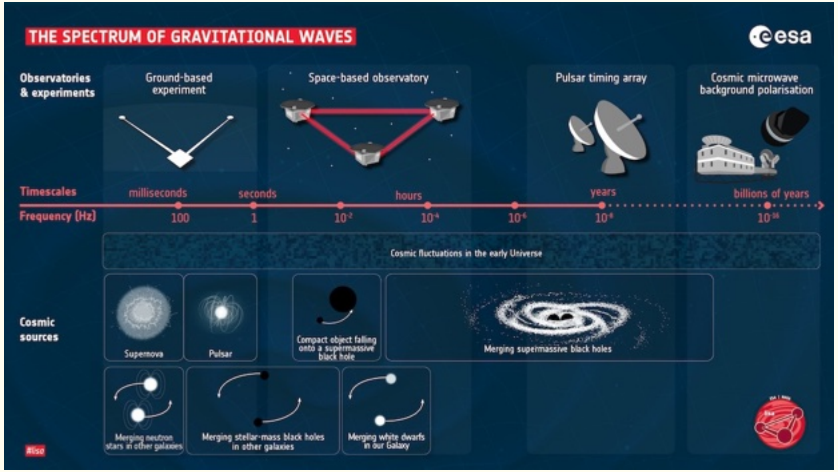
- Signal from binary inspirals.
- Galactic and extra-galactic.
- Solar masses → Super-massive.

$$\Omega_{\rm GW}(f) \sim f^{2/3} \int_0^\infty dz \int_0^\infty d\mathcal{M} \frac{dN}{dz d\mathcal{M}} \frac{1}{d_L^2(z)} \left(\frac{\mathcal{M}^5}{1+z} \right)^{1/3}$$

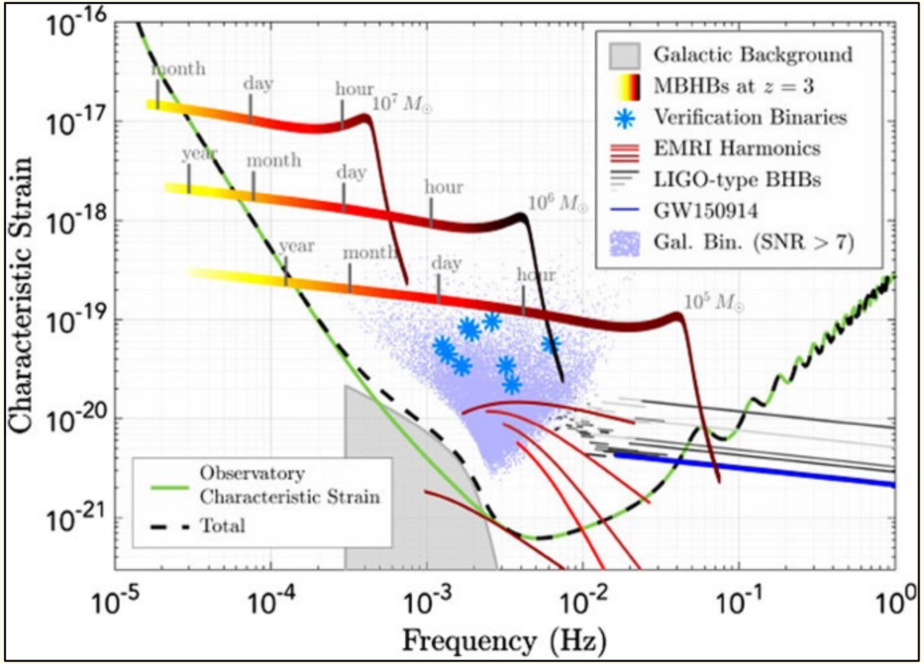


LISA Observations





- LISA SGWB
  - Huge signal from resolved and unresolved galactic binaries.
  - Unresolved signal → SGWB.
  - Superimposed over extra-galactic SGWB signal
- Foreground separation problem
  - Frequency
  - Time
  - Angular



[Sesana 2021]

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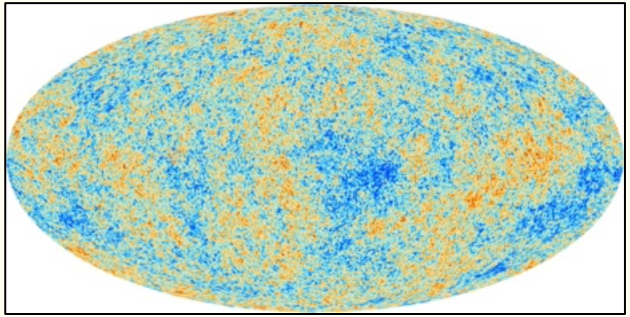
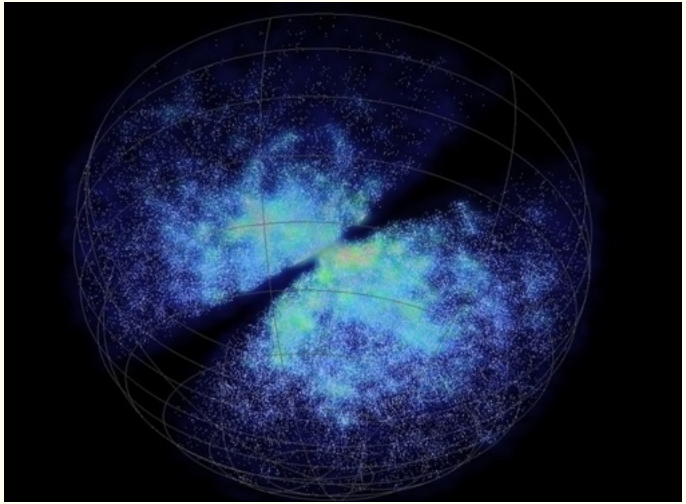
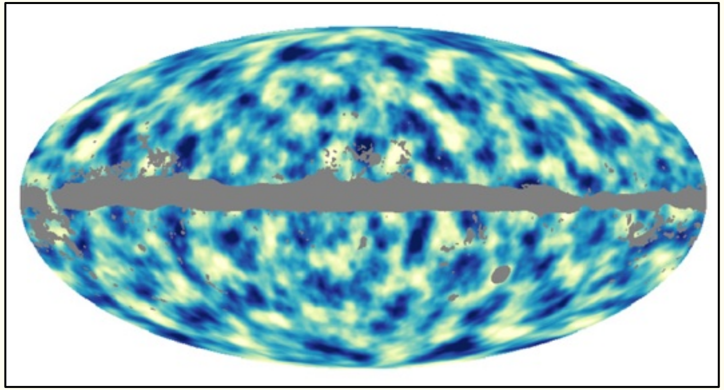
Beyond the monopole…



[Planck CMB]



[6df Redshift]



[Planck lensing]

SGWB anisotropies

- SGWBs are anisotropic
  - Kinematic dipole.
  - Doppler.
  - Intrinsic.
  - SW and ISW effects.
- Line of sight calculation cf. CMB [CC 2016].
- SGWB all tracers of inhomogeneities.
  - Primordial.
  - Cosmological.
  - Inspiral.

$$\langle h_P(f,\hat{\mathbf{k}})h^\star_{P'}(f',\hat{\mathbf{k}}')\rangle = \delta(P-P')\delta(f-f')\delta^{(2)}(\hat{\mathbf{k}}-\hat{\mathbf{k}}')I(f,\hat{\mathbf{k}})$$

$$S^{GW}(k,\eta) = 2\dot{\Phi} + \dot{\sigma}\,(\hat{p}^i\,v_i + \Pi + \Phi)$$

ISW, Doppler, Intrinsic, Sachs-Wolfe

$$C^I_\ell = \frac{2}{\pi}\int k^2\,dk\,P_\Phi(k)|\Delta^{GW}_\ell(k,\eta_0)|^2$$

[CC 2016, Uzan et al 2017,
Bartolo et al 2018]

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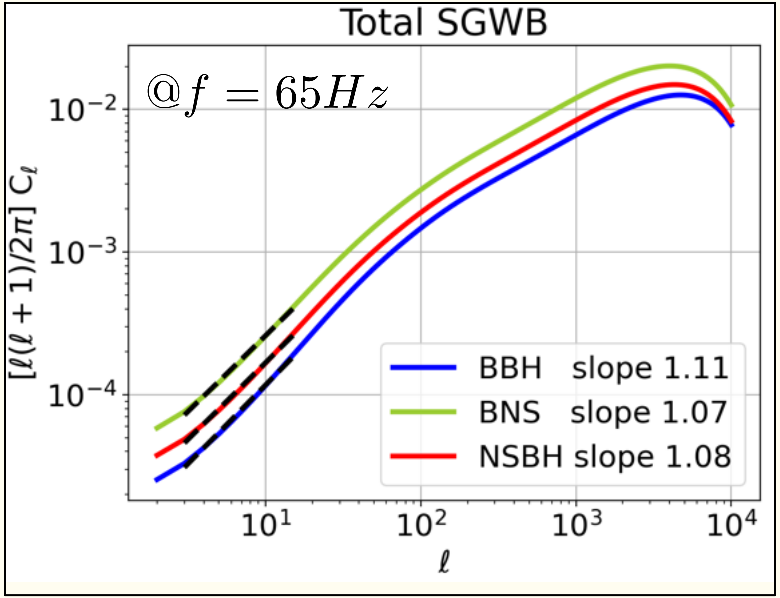
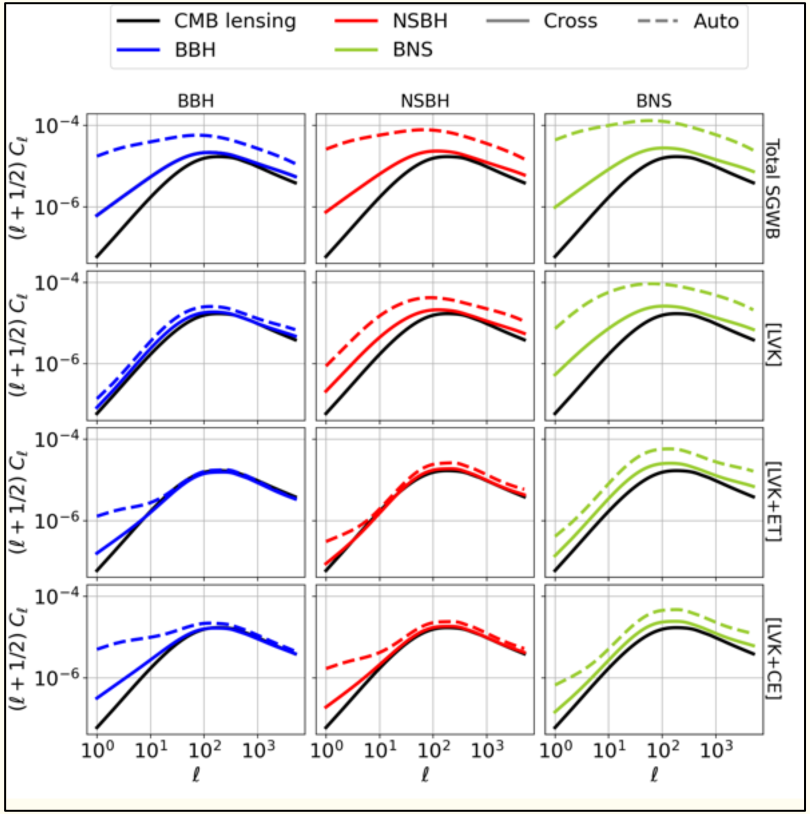
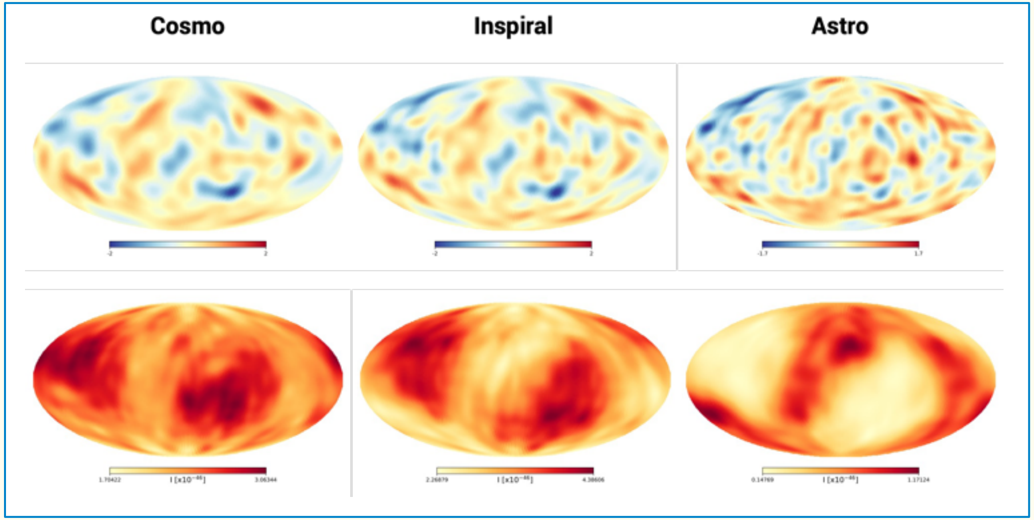


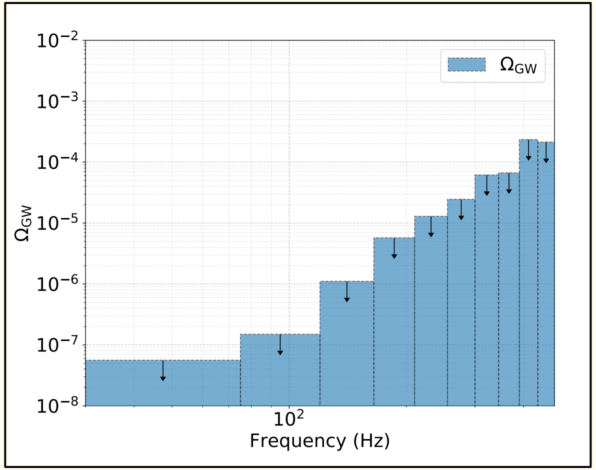
[Capurri et al. 2022]



[Capurri et al. 2022]

SGWB map-making

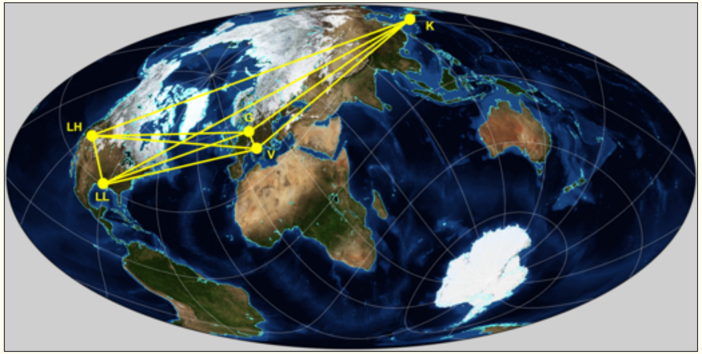




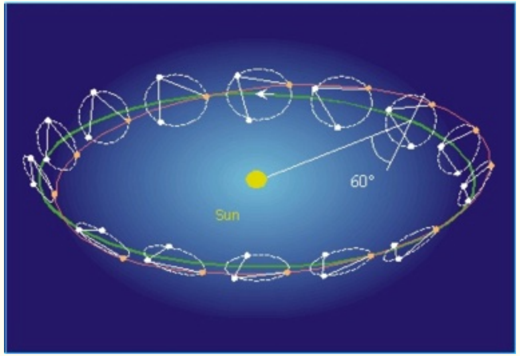
[Renzini & CC 2019]

SGWB map-making

- We know how to do it (radio-astronomy).
- Complications (observing):
  - Non-compact beams.
  - Few baselines.
  - Fixed scanning patterns.
  - Low resolution.
- Complications (signal):
  - Non-stationarity (transients).
  - Shot-noise.
  - Frequency dependence.



$$\ell \sim 2\pi \frac{bf}{c}$$



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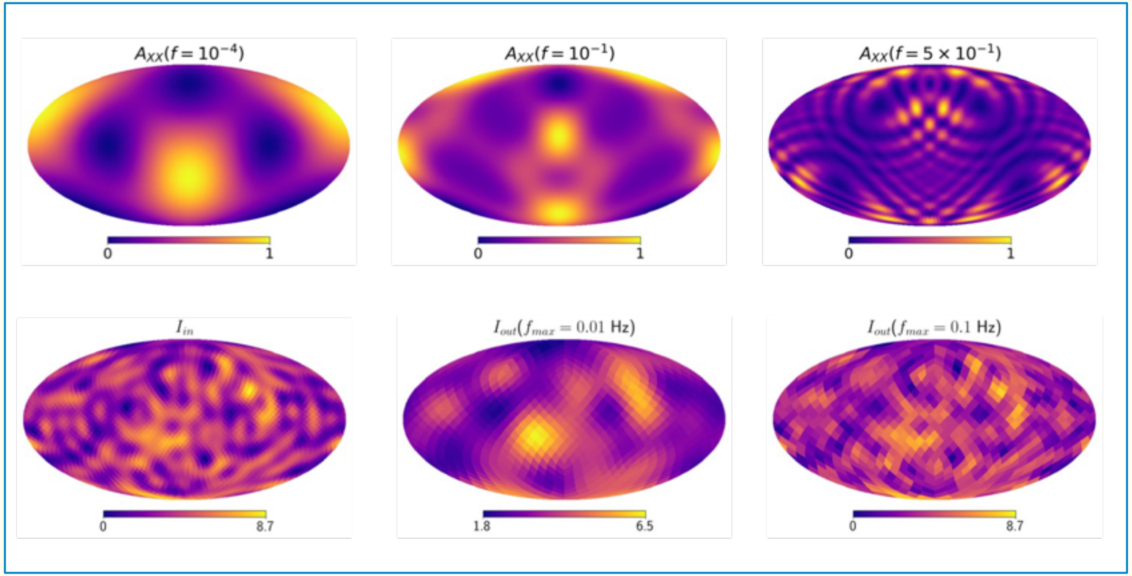
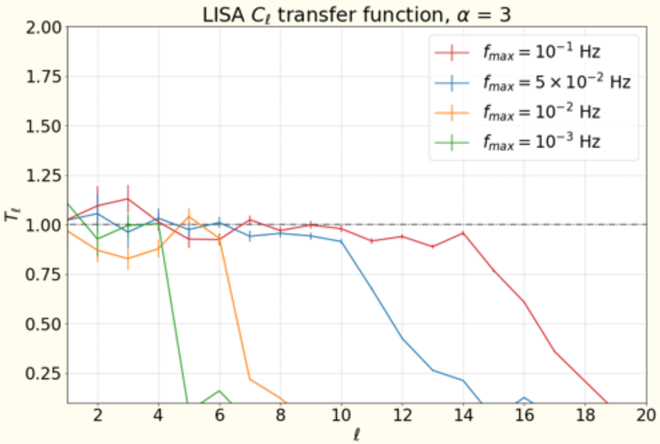
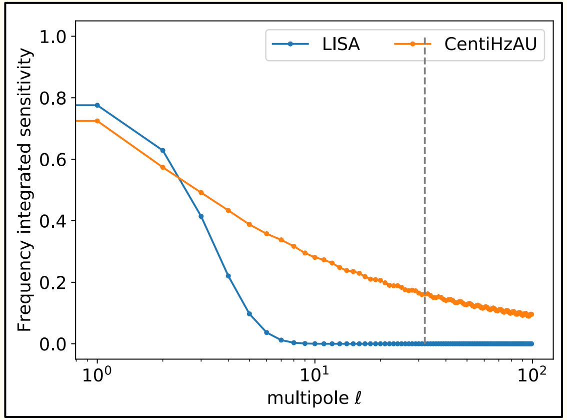


[LISA CosmoWG: CC et al. 2020]

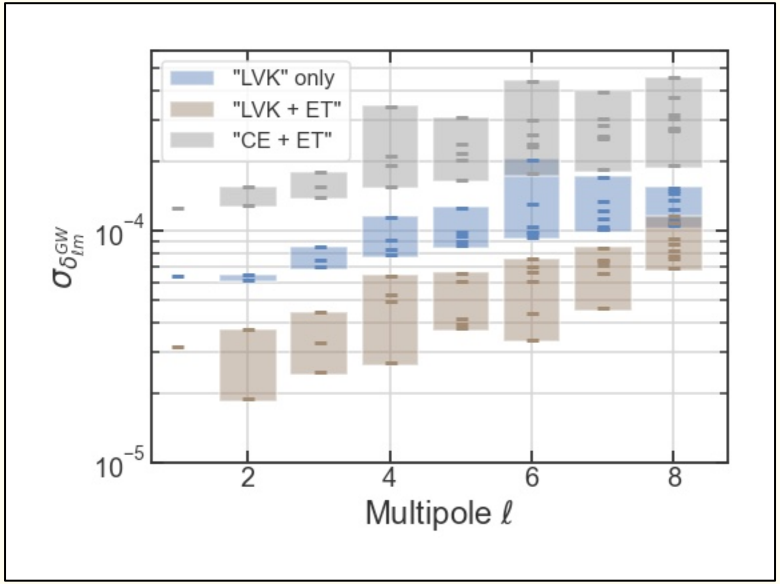




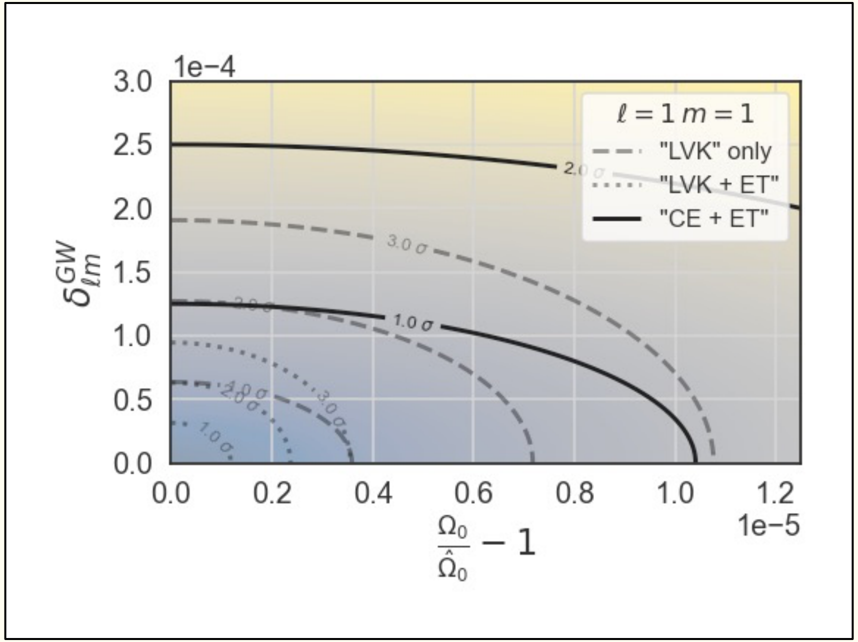
[ESA 2050 Voyage 2019]

Ground-based

- Non-trivial frequency domain structure of overlap functions make realistic map-making challenging.
- Einstein Telescope (ET) + Cosmic Explorer (CE) + 4G LVK.
- Will detect kinematic dipole but struggle to detect anisotropy of inspiral SGWB.
- But sub-percent level constraint on SGWB monopole!

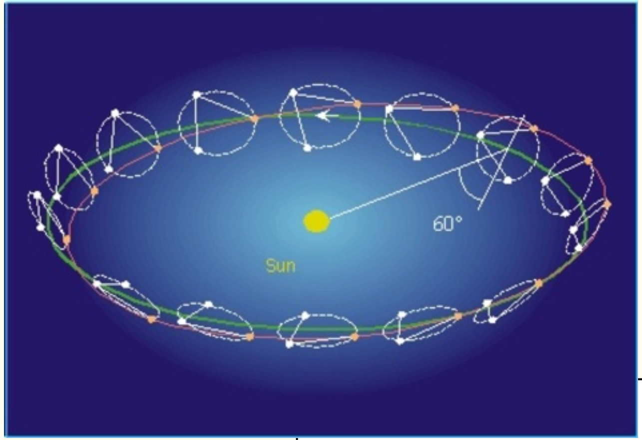


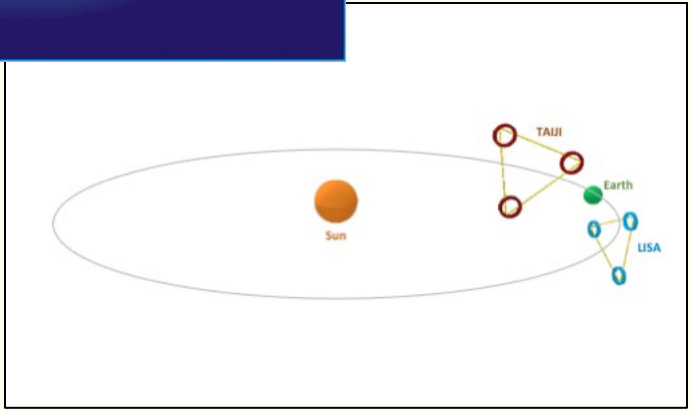
[Mentasti, CC, & Peloso 2023]


1e−4
3.0
2.5
2.0
1.5
1.0
0.5
0.0
$\delta_{\ell m}^{GW}$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1e−5
$\frac{\Omega_0}{\hat{\Omega}_0} - 1$
$\ell = 1\ m = 1$
"LVK" only
"LVK + ET"
"CE + ET"
2.0 σ
3.0 σ
1.0 σ

Space-based

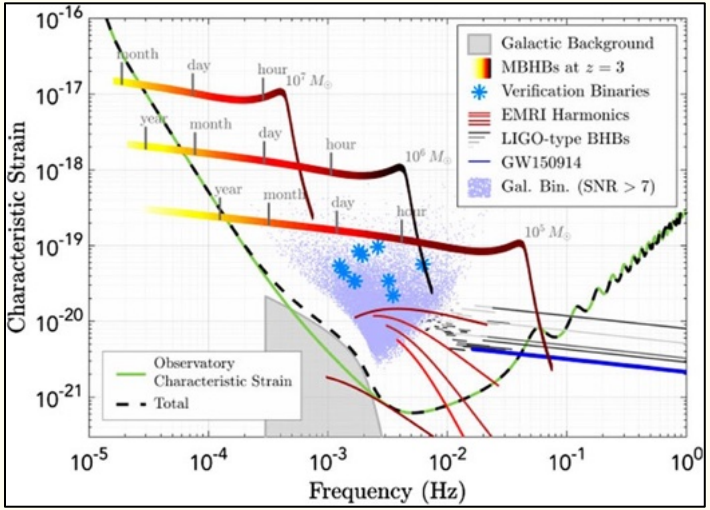
- Milli Hz frequencies.
- 10^6 km baselines.
- Galactic foreground of resolved and unresolved binaries.
- Extra-galactic SGWBs?
- Can we separate the galactic from the extragalactic background?
- LISA and Taiji?



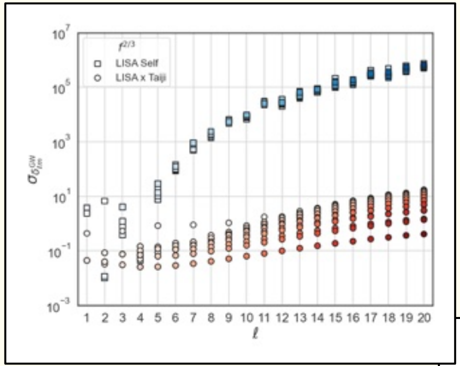


[Mentasti, CC, & Peloso in prep.]

Space-based

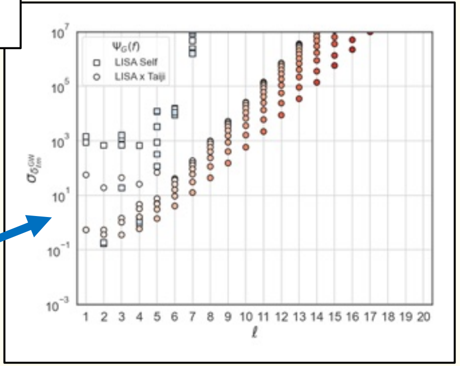


$$\Omega_{gal} \sim f^{2/3}$$



Realistic frequency spectrum and time domain scanning.

No morphology template.



[Mentasti, CC, & Peloso in prep.]

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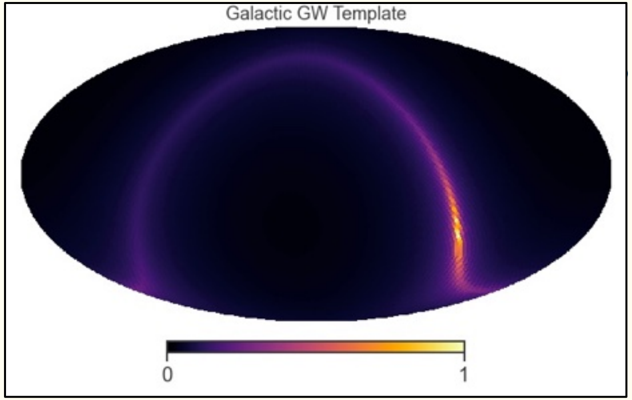
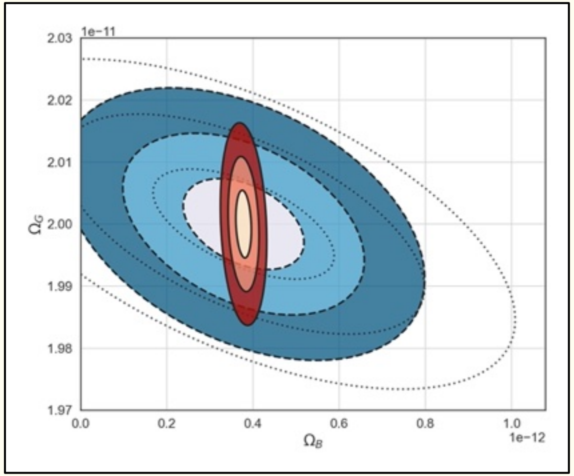
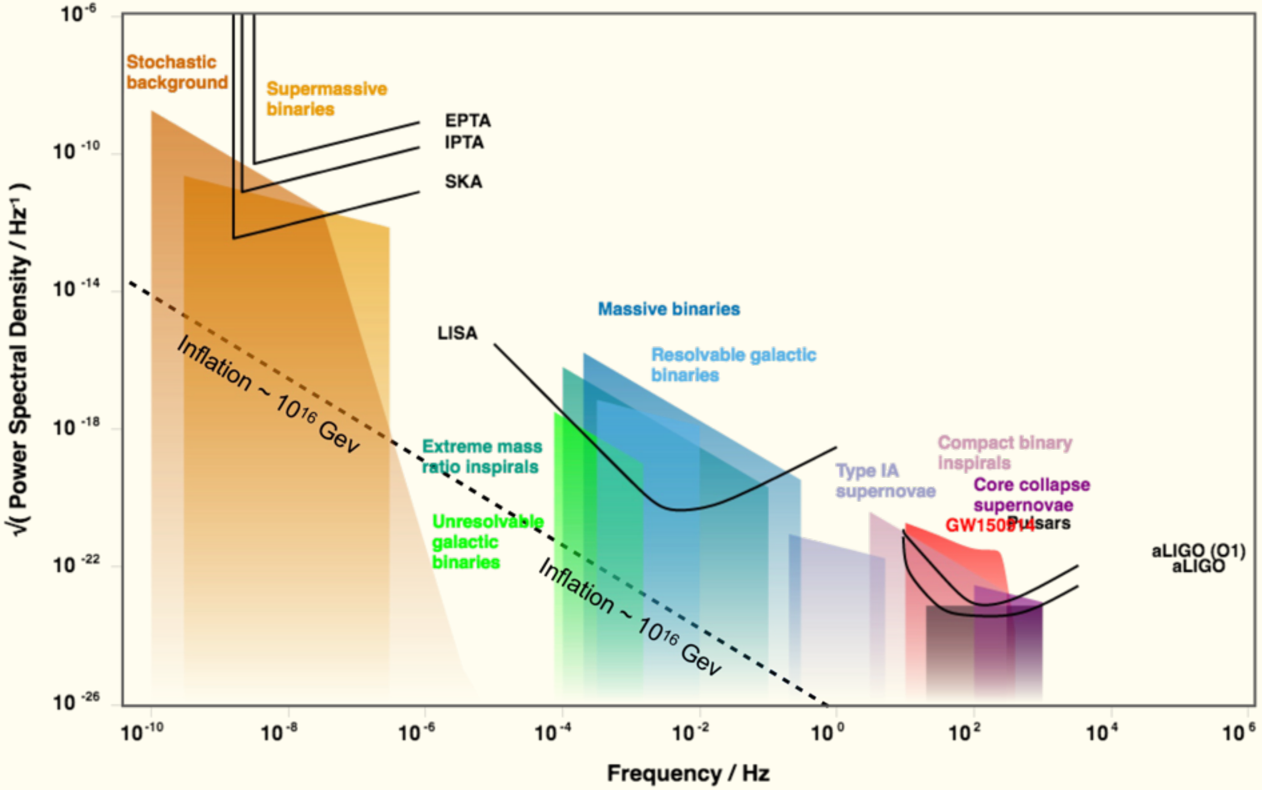
Space-based

- Template (morphology) based analysis.
- Assuming LISA noise is well understood then a full time-frequency analysis will distinguish between galactic and SGWB (cosmo $\Omega \sim 10^{-9}$) amplitudes.
- If LISA self-correlations are too big:
  - LISA-Taiji combination frequency only <u>difficult</u>.
  - LISA-Taiji combination time-frequency <u>OK</u>.





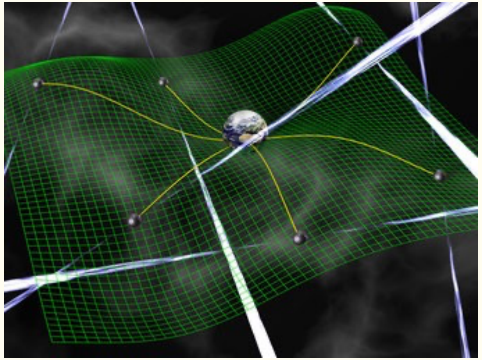
[Mentasti, CC, & Peloso in prep.]

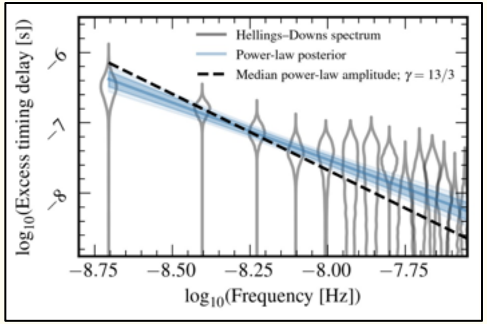


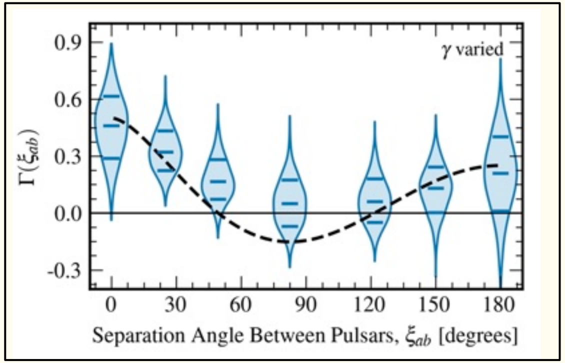
√(Power Spectral Density / Hz⁻¹)
Frequency / Hz
Stochastic background
Supermassive binaries
EPTA
IPTA
SKA
LISA
Massive binaries
Resolvable galactic binaries
Extreme mass ratio inspirals
Unresolvable galactic binaries
Inflation ~ 10¹⁶ Gev
Inflation ~ 10¹⁶ Gev
Type IA supernovae
Compact binary inspirals
Core collapse supernovae
GW150914
Pulsars
aLIGO (O1)
aLIGO

Pulsar Timing Arrays

- PTAs are detecting something!
- Hellings-Down correlation must be confirmed.
- Single source (SMBHs) or SGWB?
- Integration continues.
- Can ”map-make” but sparse reconstruction.
- Next generation surveys using radio telescope arrays – SKA.







[NANOGrav 2023]

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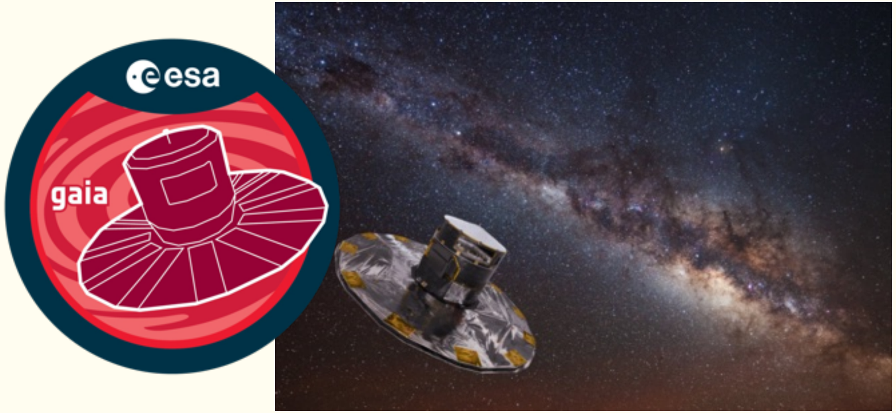
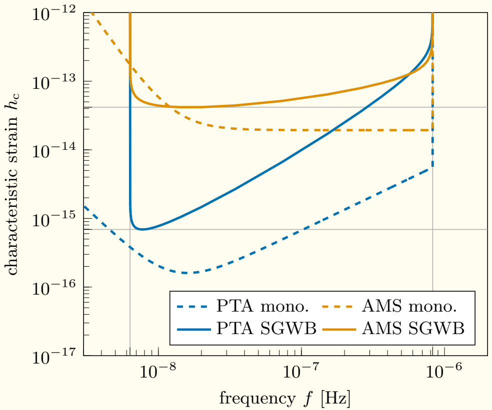
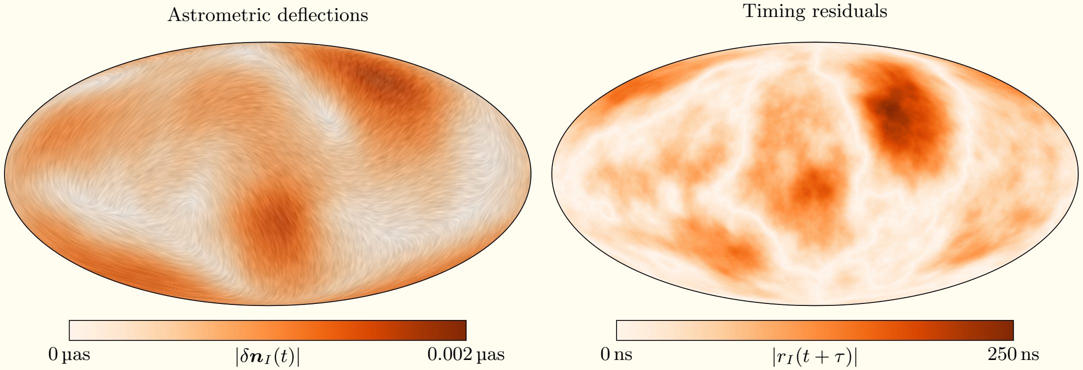
Astrometry

- Distortion of apparent position vs timing redshift (PTAs).
- Micro arcsecond resolution with 10^9 galactic sources – many baselines...







[Golat & CC 2022]

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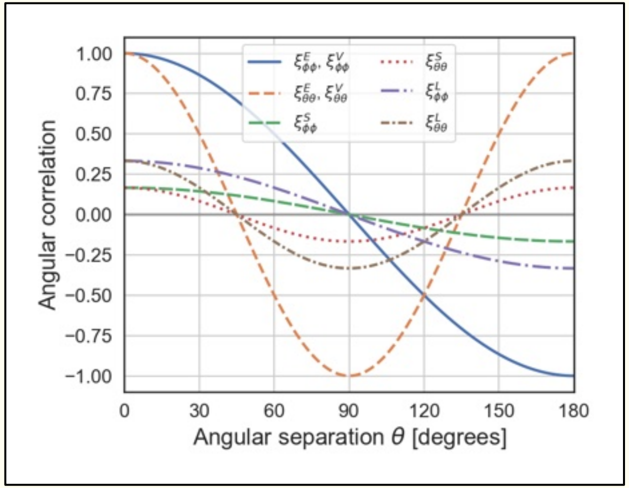
Astrometry

- Vera C. Rubin (LSST).
- 50 micro arcsec astrometry with 10^6 solar system objects (asteroids).
- Short-distance limit of astrometry.
- Correlations similar to Hellings-Down curves.

σ [mas]	$N = 1 \times 10^5$	$N = 5 \times 10^6$
50.0	9.9×10^{-1}	2.0×10^{-2}
0.1	3.9×10^{-6}	7.9×10^{-8}
0.01	3.9×10^{-8}	7.9×10^{-10}



[Mentasti & CC 2023]

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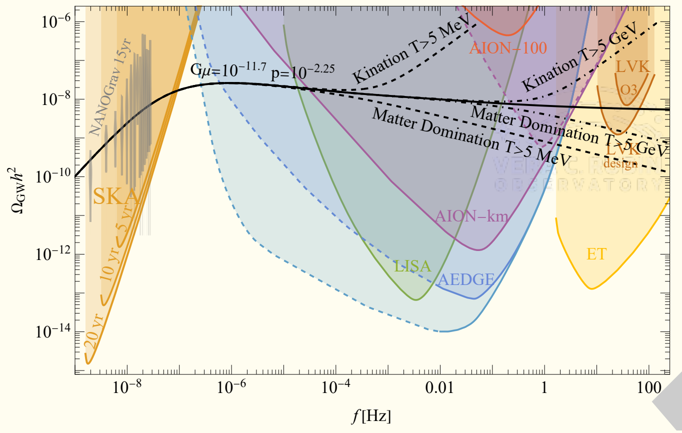
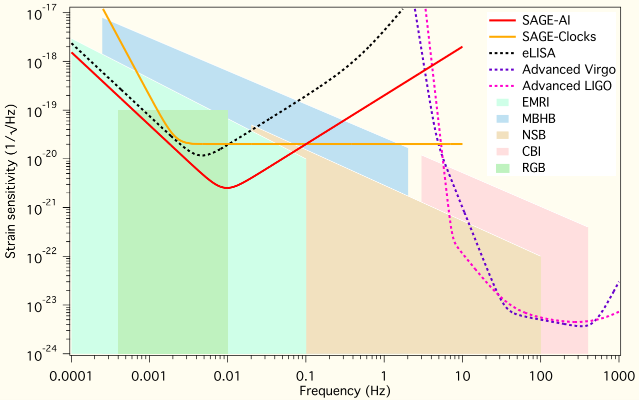
Cold atoms

- Atom Interferometry.
- Optical lattice atomic clocks.
- Phase and/or doppler shift measurements at tunable frequencies.
- AION, AEDGE, MAGIS, etc. proposals.
- Ground and space-based proposals.
- Track inspirals across 4 decades in frequency?

[Abend et al. 2023, TVLBAI Workshop]



[Tino et al. 2019]



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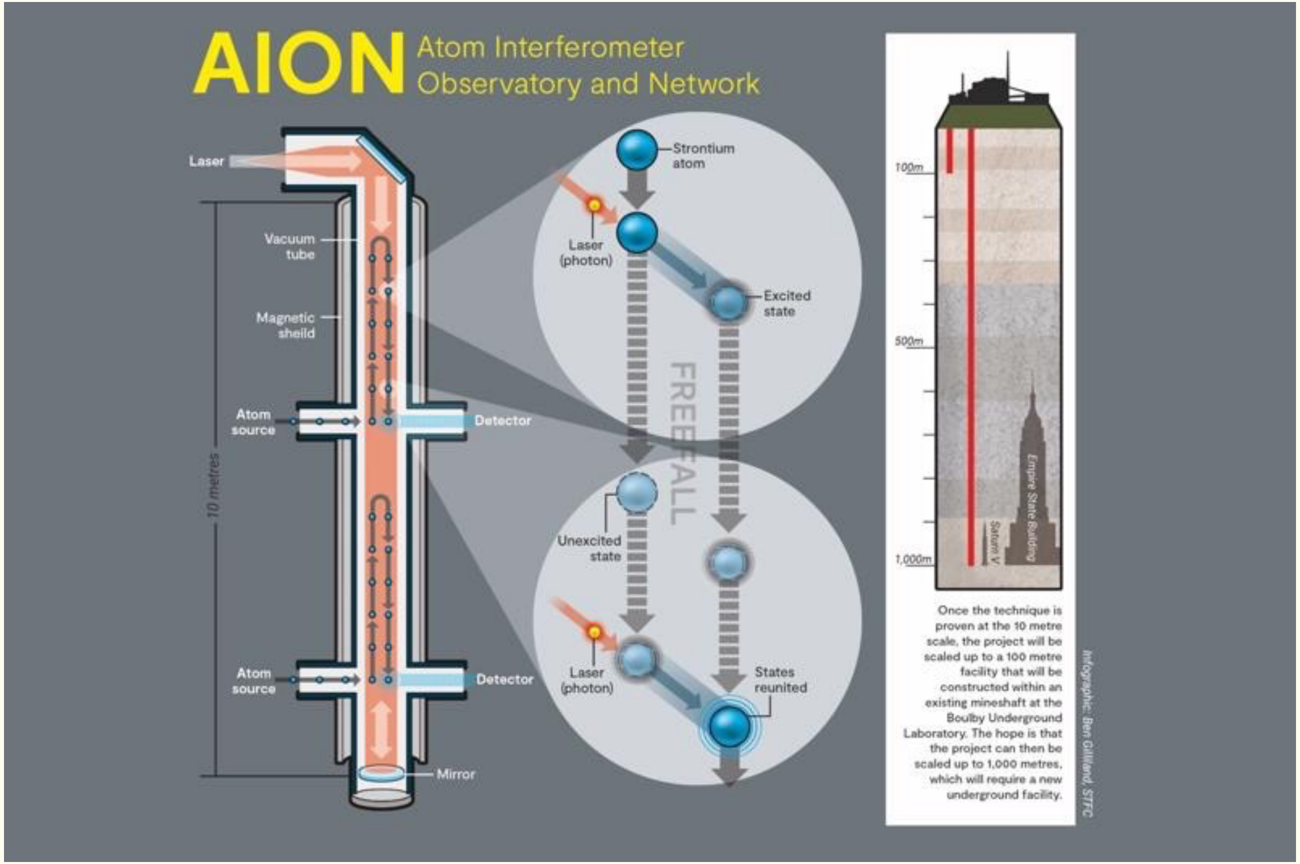

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Summary

- Foothills of the stochastic era for GWs.
- Detection of SGWB (inspirals) in LVK O4 run (late 2024?).
- Lots of physical mechanisms generate SGWBs. Rich landscape.
- Characterising angular distribution with $1^{st}/2^{nd}$ Gen detectors will be difficult.
- Next generation of detectors will exploit new methods and frequencies.
- Frequencies > kHz?