A survey of some recent developments in black hole physics

[Side note: a.k.a. ‘ TBA ’]

Óscar Dias

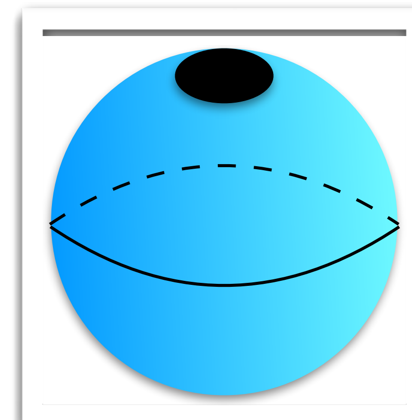
29th IFT Xmas Workshop

IFT-UAM, Madrid, Dec 2023

Outline:

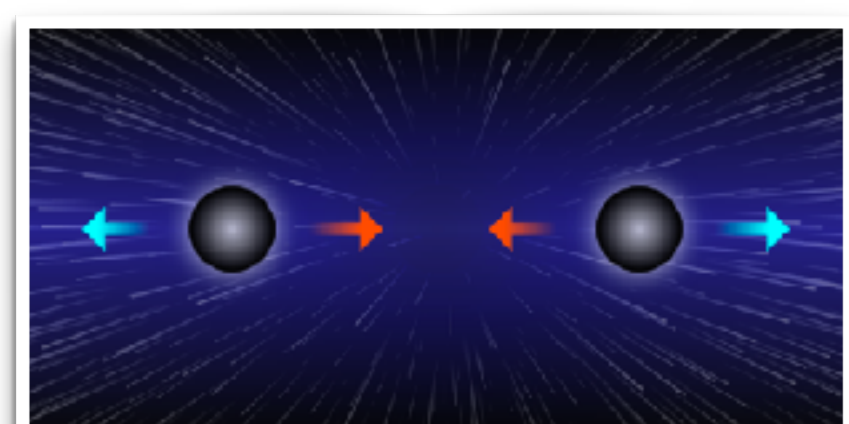
- Kerr-AdS, superradiance,
  missing CFT states & resonators

- Localized black holes
  in Gravity & Supergravity

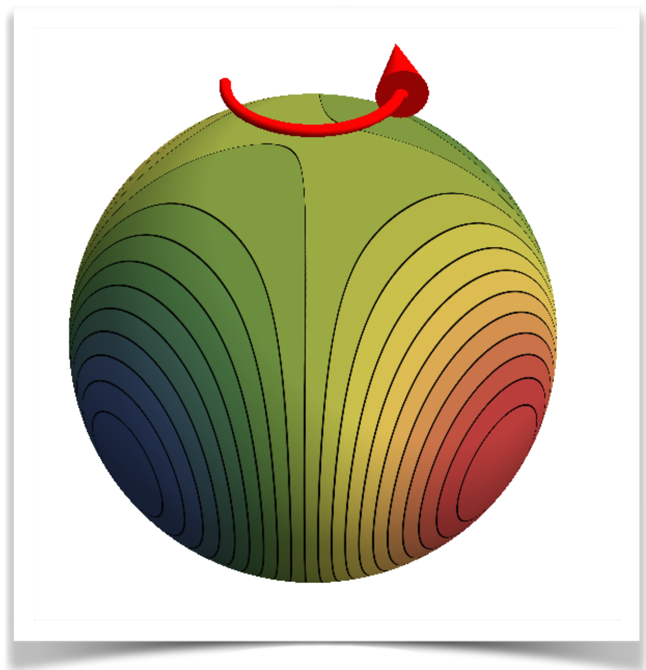


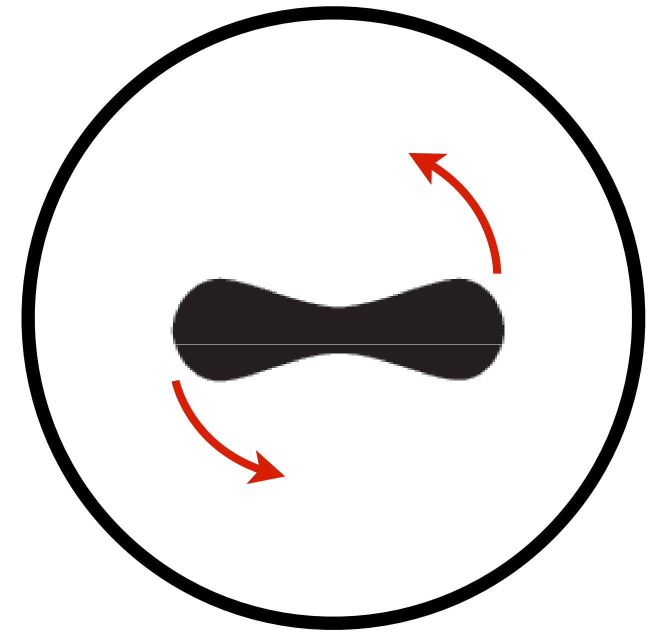
- More missing CFT states
  in N=4 SYM / IIB Sugra

- Black hole binaries
  in an expanding Universe



1. Kerr-AdS, superradiance, missing CFT states & resonators





OD, Gary Horowitz, Jorge Santos, 1105.4167, 1109.1825

OD, Jorge Santos, 1302.1580

Vitor Cardoso, OD, Gavin Hartnett, Luis Lehner, Jorge Santos, 1312.5323

Gary Horowitz, Jorge Santos, 1408.5906

Benjamin Niehoff, Jorge Santos, Benson Way, 1510.00709

OD, Jorge Santos, Benson Way, 1505.04793

Green, Hollands, Ishisbashi, Wald, 1512.02644

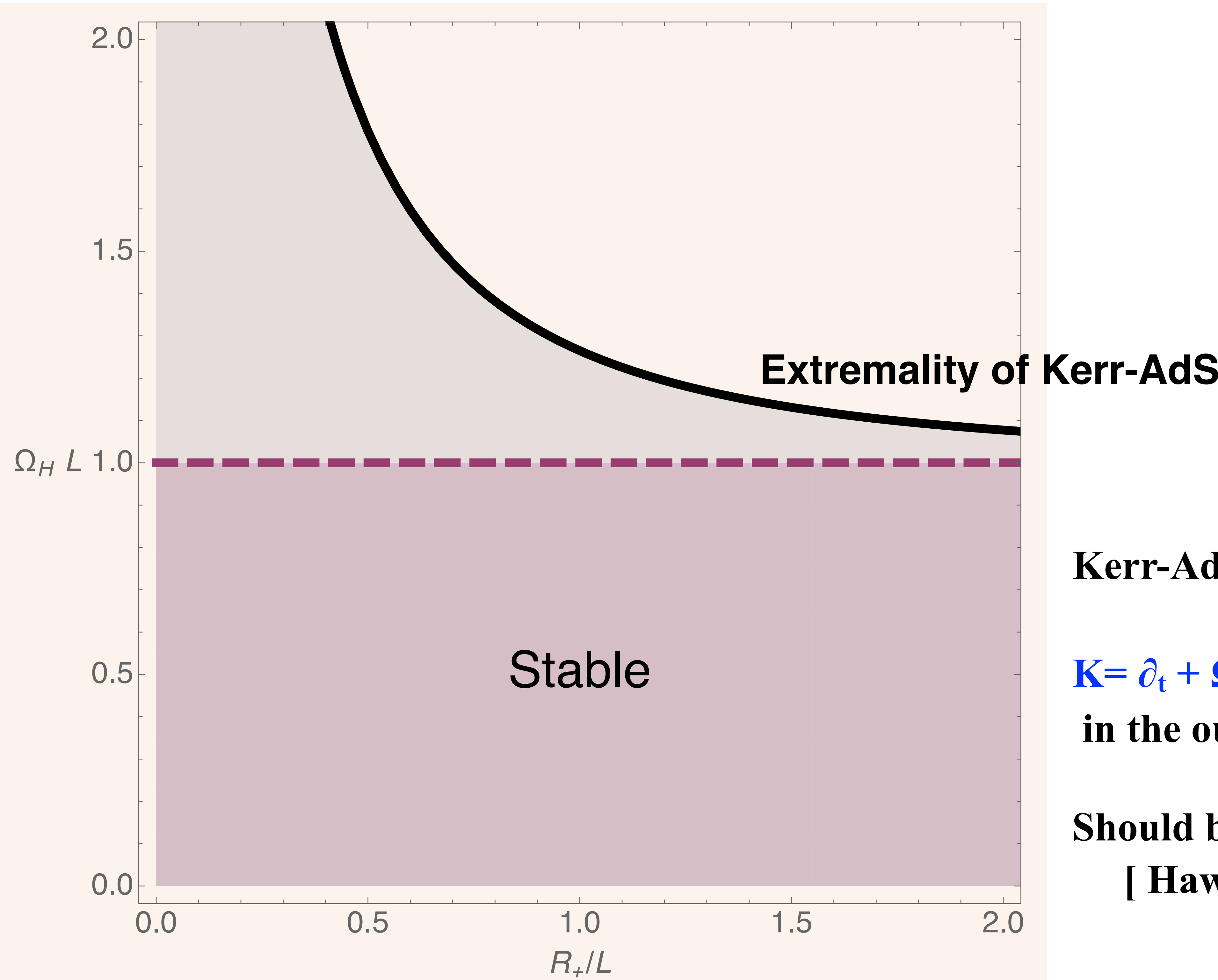
OD, Jorge Santos, 1602.03890

Paul Chesler, 1801.09711, 2109.06901

Kim, Kundu, Lee, Lee, Minwalla, Patel, 2305.08922

→ Kerr-AdS$_4$ black holes & their Superradiant instability

2 parameters: $(m/L, a/L) \Leftrightarrow (R_+/L, \Omega_H L)$



Kerr-AdS with $\Omega_H L \le 1$:

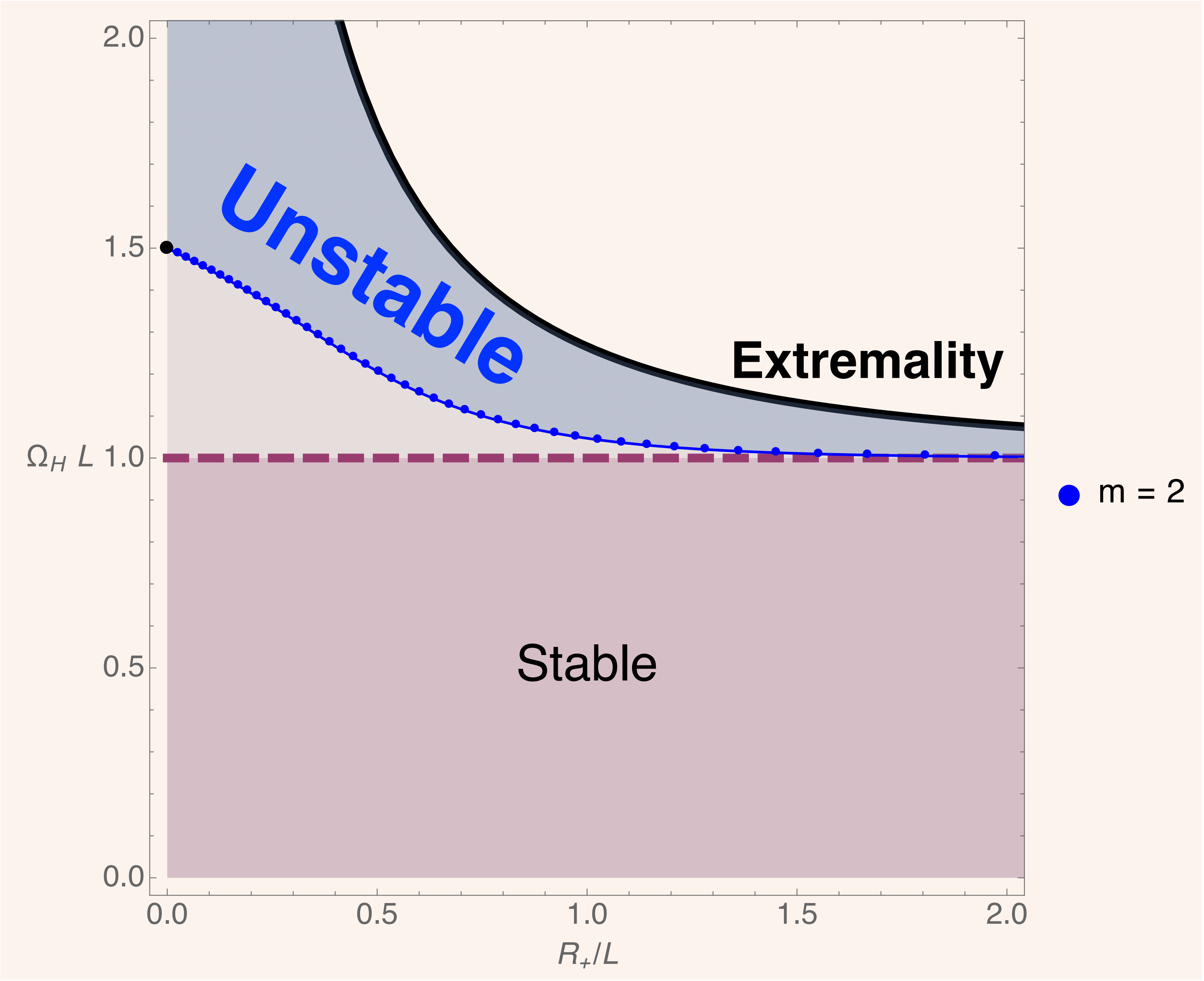
$K= \partial_t + \Omega_H \partial_\varphi$ is timelike everywhere in the outer domain

Should be stable
[Hawking, Reall ’00]

¿What about BHs with $\Omega_H L > 1$?

There is <u>no</u> Killing vector that is timelike everywhere in the outer domain

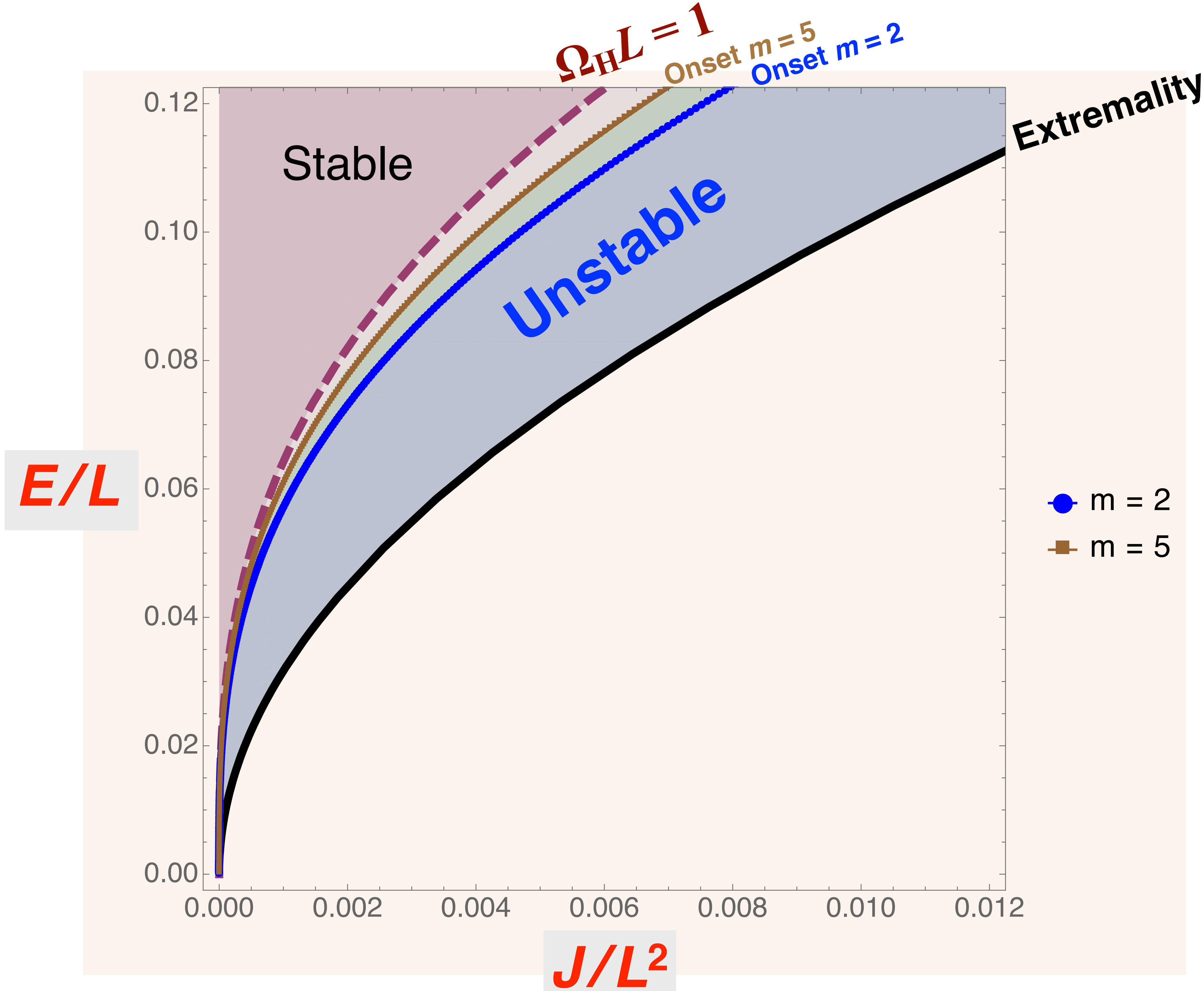
∂_t and ∂_φ are Killing fields => $\delta g = e^{-i\,\omega\, t + i\, m\, \varphi}$



Modes with m>0 are unstable (Im ω >0) to superradiance if Re ω ≤ m Ω_H

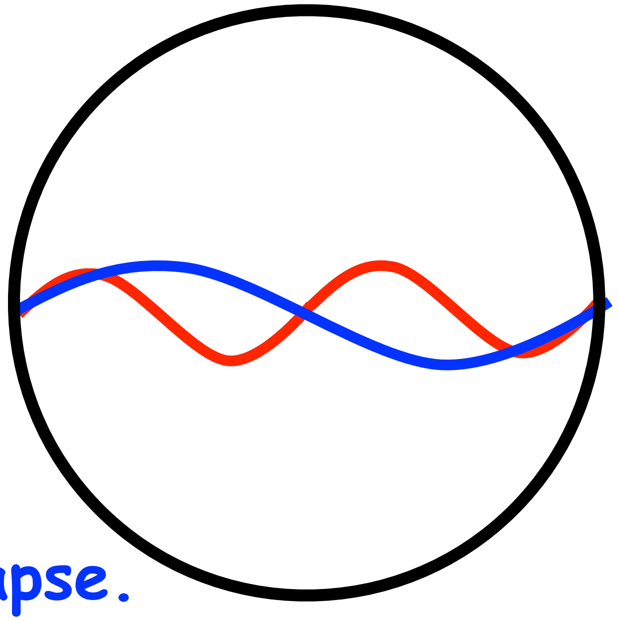
Onset, Im ω =0, saturates inequality

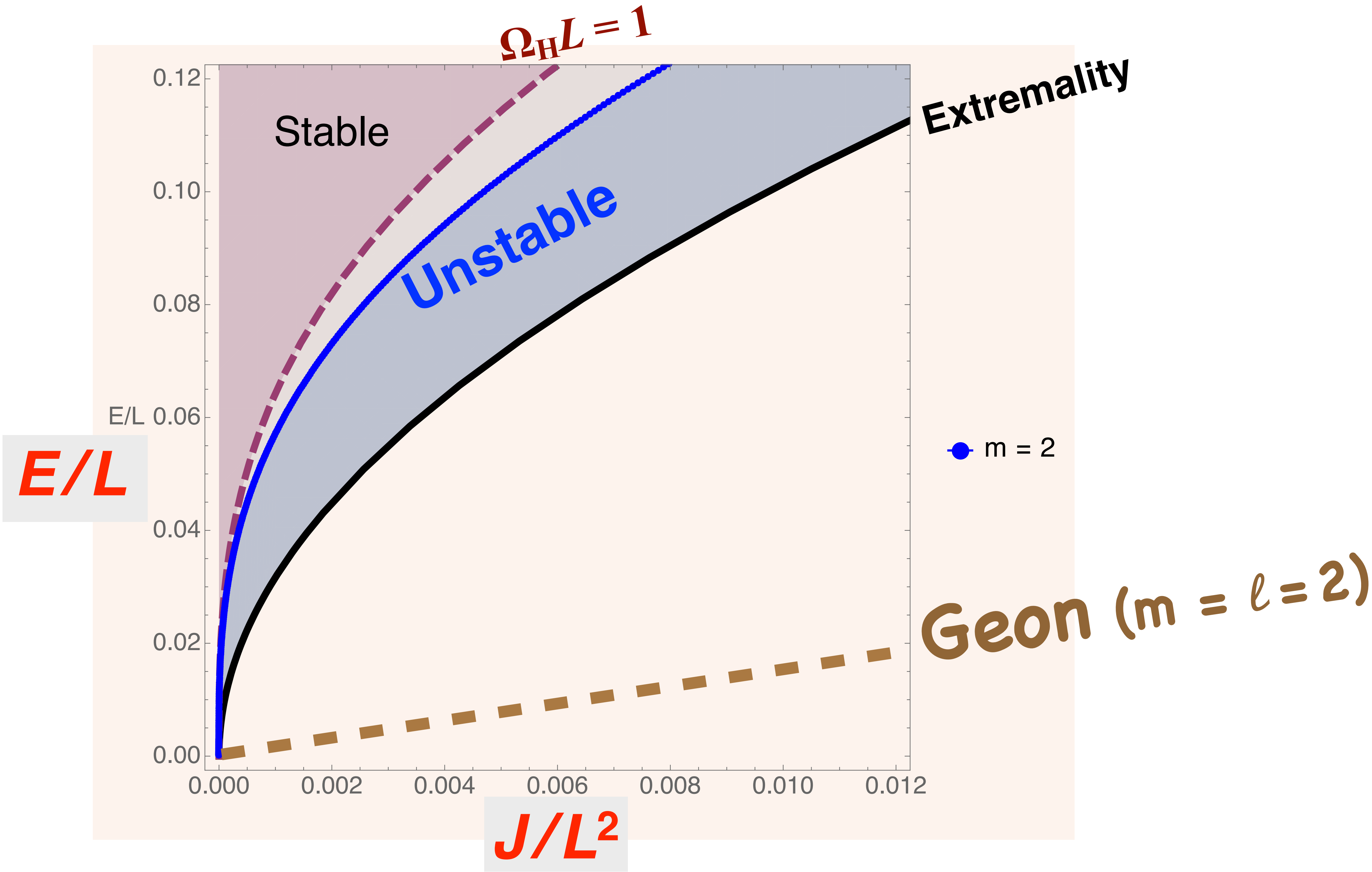
- **Higher m modes appear closer to $\Omega_{\rm H}L = 1$**
- **$\Omega_{\rm H}L = 1$ is reached as $m \to +\infty$**

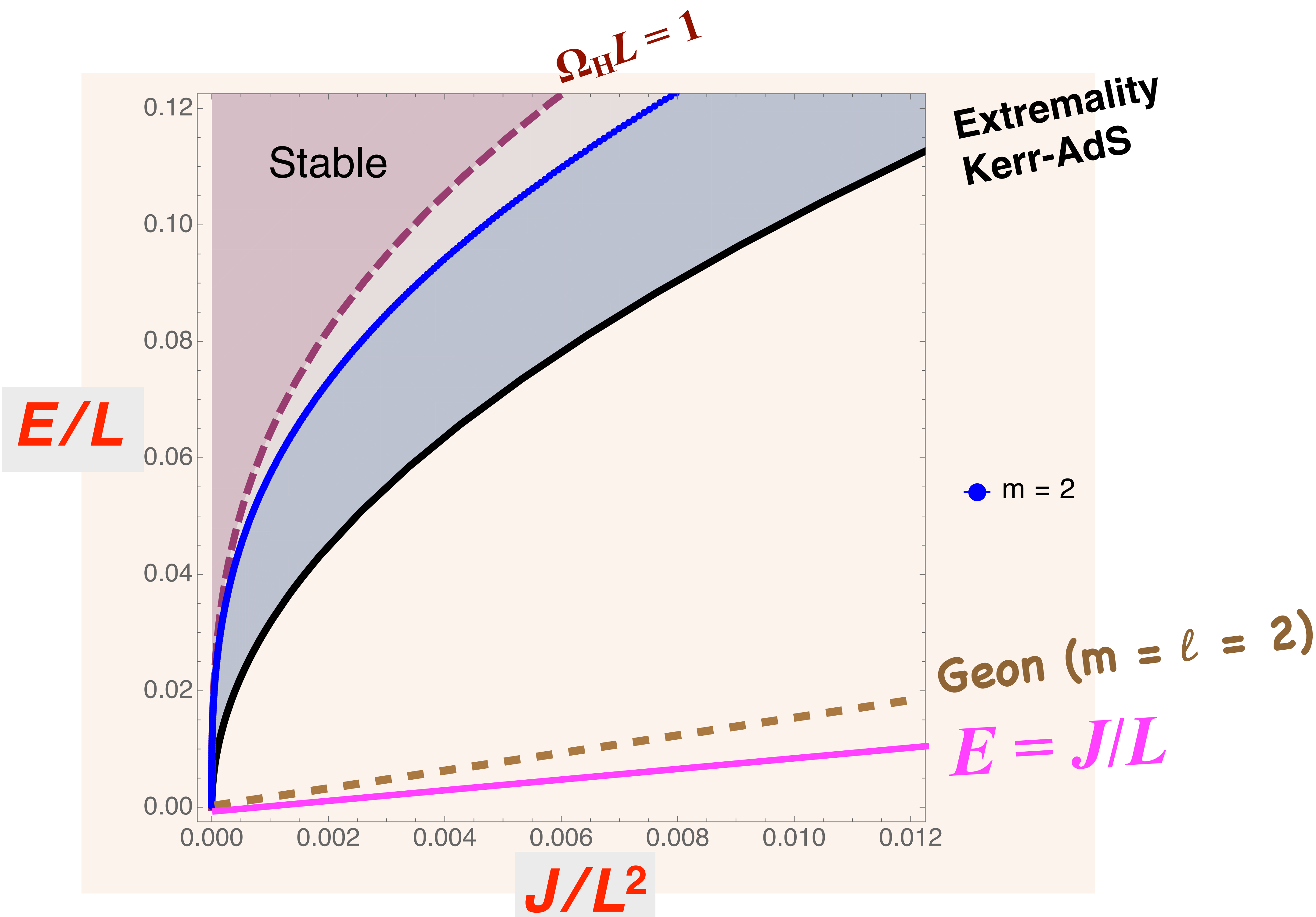


→ Geon (horizonless solution) also present

- Key player: Geon
  - Backreaction of a (single) normal mode of AdS
  - Stationary solution (no formation of BH)
  - Centrifugal force balances self gravity against collapse.





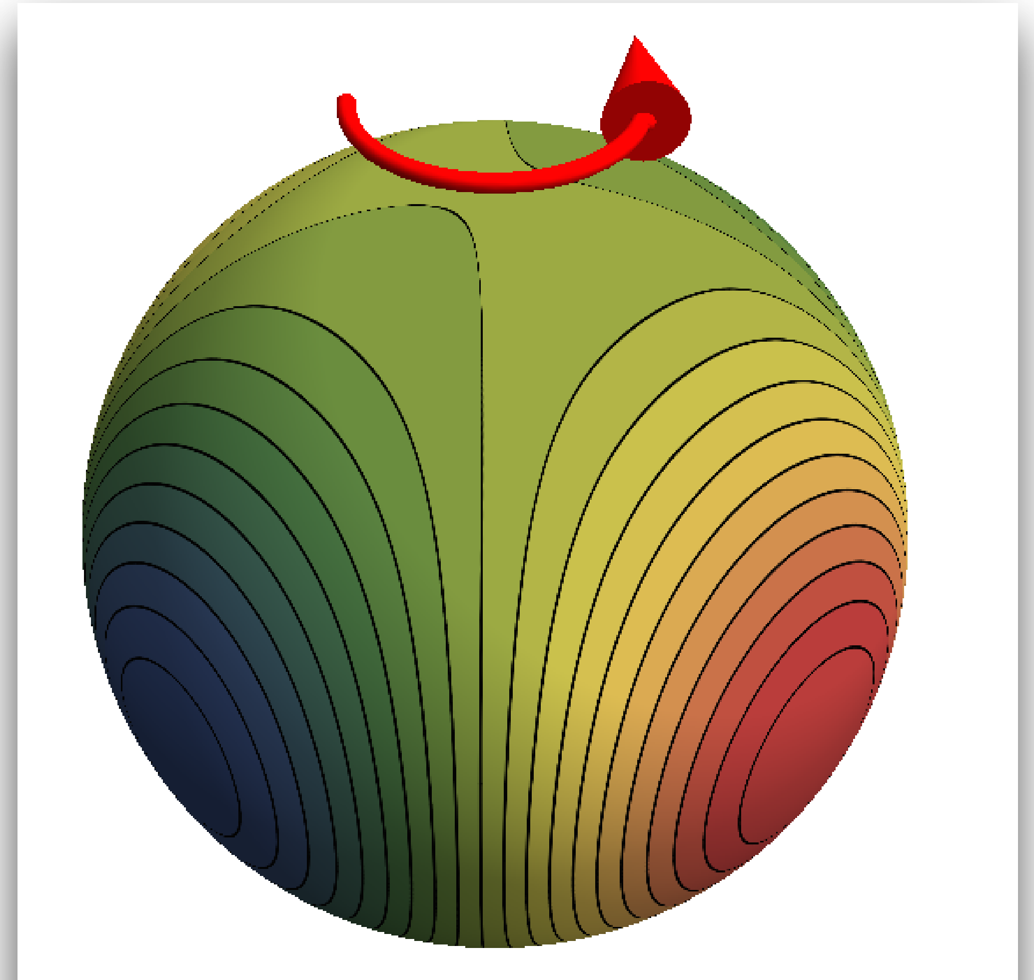


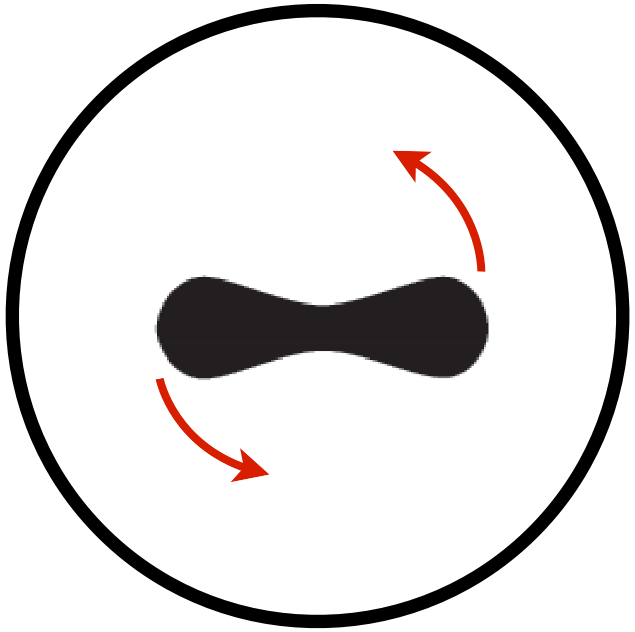
- In the limit $m \to +\infty$ geons approach $E = J/L$
- CFT states are expected to saturate the bound $E \geq J/L$, … but Kerr-AdS BHs do not,

  => suggests that another BH should fill the gap —> these are the black resonators

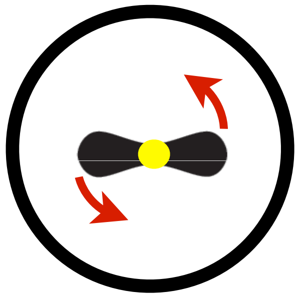
Some properties of geons & black resonators:

- **Geon: regular horizonless solutions of Einstein-AdS**
- **Geon: Obeys the first law:** $m\, dE = \omega\, dJ$
- **Both Invariant under single helical Killing vector field:** $$K = \partial_t + \frac{\omega}{m}\partial_\varphi$$
  which is timelike near the poles but spacelike near the equator.
- **Time periodic but NOT time independent neither axisymmetric**
- **Boundary stress-tensor has regions of negative and positive energy density around the equator:**





→ Constructing black resonators using a thermodynamic model



- Leading order thermodynamics: model black resonator as a non-interacting mixture of a Kerr BH and a geon.

- Assume absence of interaction: E, J of resonator are simply the sum of the charges of individual constituents: $E = E_K + E_g, \quad J = J_K + J_g$

- Geon's KVF must coincide with the horizon generator of the black resonator: angular velocity of the later must be $\Omega_H = \frac{\omega}{m}$

- This equilibrium condition also follows from maximizing entropy for a given total E, J.

- Combine these conditions with the leading order thermodynamics of the two components. At leading order, geon component carries all J & Kerr component stores all S.

$$\{J_g, E_g\} = \left\{J, \frac{\omega}{m} J\right\}, \quad \{J_K, E_K\} = \left\{0, E - \frac{\omega}{m} J\right\}$$

$$S = 4\pi \left(E - \frac{\omega}{m} J\right)^2, \quad T_H = \frac{1}{8\pi} \left(E - \frac{\omega}{m} J\right)^{-1}.$$

→ What is the endpoint of superradiant instability ?

- $S_{\text{resonator}\,|\,m=2} > S_{\text{Kerr-AdS}}$ for same $\{E, J\}$ but all black resonators have $\Omega_H L > 1$:

  they are still superradiantly unstable to higher m-modes

- Two possibilities for the endpoint of superradiant instability:

1) there is a limiting black resonator with $\Omega_H L = 1$?

   => perturbatively construct it as: Kerr-AdS at the core of a limiting geon with $\Omega\, L = 1$.

   Such geon would saturate the minimum energy bound, $E=J/L$ => it is a SUSY solution.

   BUT (regular) geon cannot exist: only SUSY vacuum solution with AdS asymptotics is … AdS

   Niehoff, Santos, Way, 1510.00709

… no other classical candidate solution for the endpoint of superradiance so we conjectured:

2) Time evolution develops cascade of higher & higher m-structure ($m \nearrow$ => $S_{\text{resonator}} \nearrow$)

   till GR breaks down & quantum effects kick-in => cosmic censorship violation !

   OD, Horowitz, Santos, 1105.4167

Available time evolutions are consistent with this conjecture (but only see early cascade)

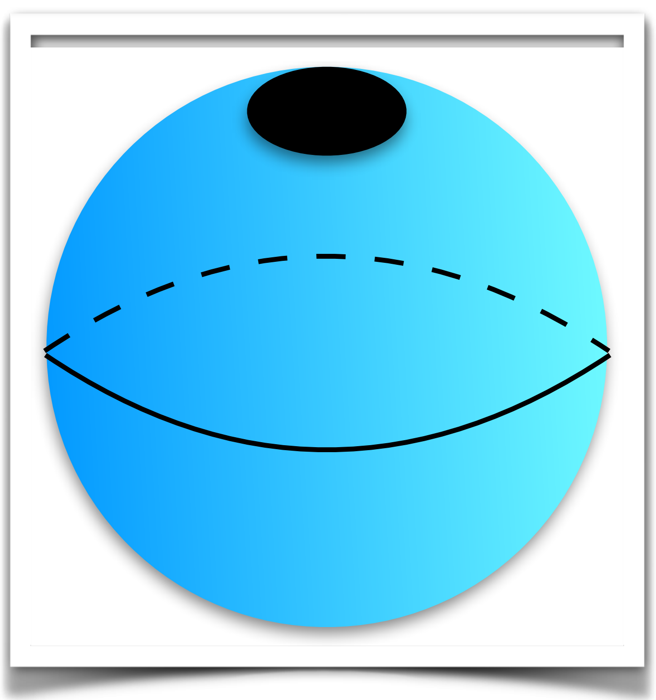
Chesler, 1801.09711, 2109.06901

Grey Galaxy: Endpoint seems to be Rotating BH at core region + quantum gas of gravitons in far-region

Minwalla et al, 2305.08922

2. Localized black holes
in Gravity & Supergravity

$AdS_5 \times S^5$ black holes of sugra IIB



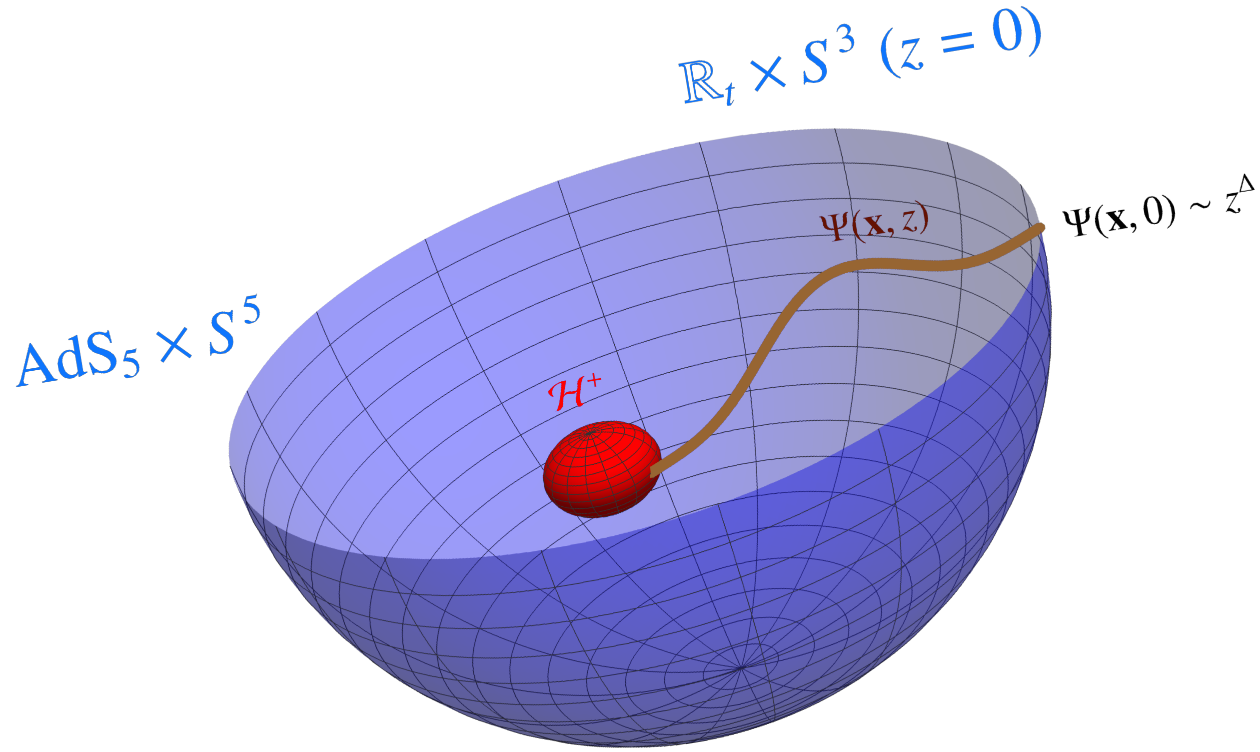
OD, Jorge Santos, Benson Way,
1605.04911 & 1501.06574 & 1702.07718

→ Recalling the primordial days: AdS_5 / CFT_4

Type IIB supergravity theory on **$AdS_5 \times S^5$** with radius **L** and **N** units of **$F_{(5)}$** flux on **S^5**

↕

Large **N** and strong t'Hooft coupling $\lambda = g^2_{\rm YM} N$ limit of $\mathcal{N}=4$ Super Yang-Mills (SYM) theory with gauge group SU(**N**) & YM coupling **g_{YM}**



- **Type IIB supergravity (only with g and $F_{(5)}$):**

$$R_{MN} - \frac{1}{48} F_{MPQRS} F_N{}^{PQRS} = 0, \qquad \nabla_M F^{MPQRS} = 0\,, \qquad F_{(5)} = \star F_{(5)}$$

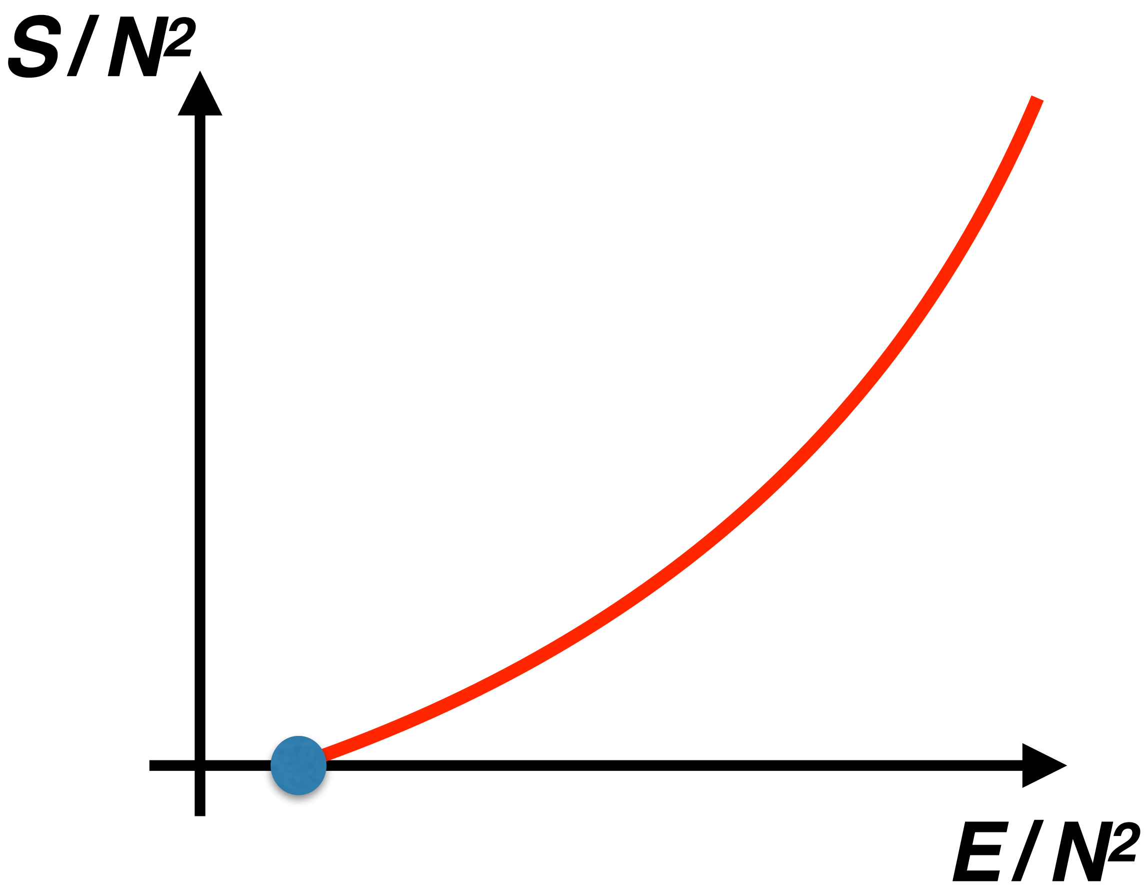
- **Freund-Rubin (80's): any soln of Einstein-AdS_5 can be oxidised to 10D via:**

$$\mathrm{d}s^2 = g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + L^2 \mathrm{d}\Omega_5^2\,, \qquad F_{(5)} = \mathrm{Vol}_{\mathrm{AdS}_5} + \mathrm{Vol}_{\mathrm{S}^5}$$

→ Thermal Phases of $AdS_5 \times S^5$ and their competition

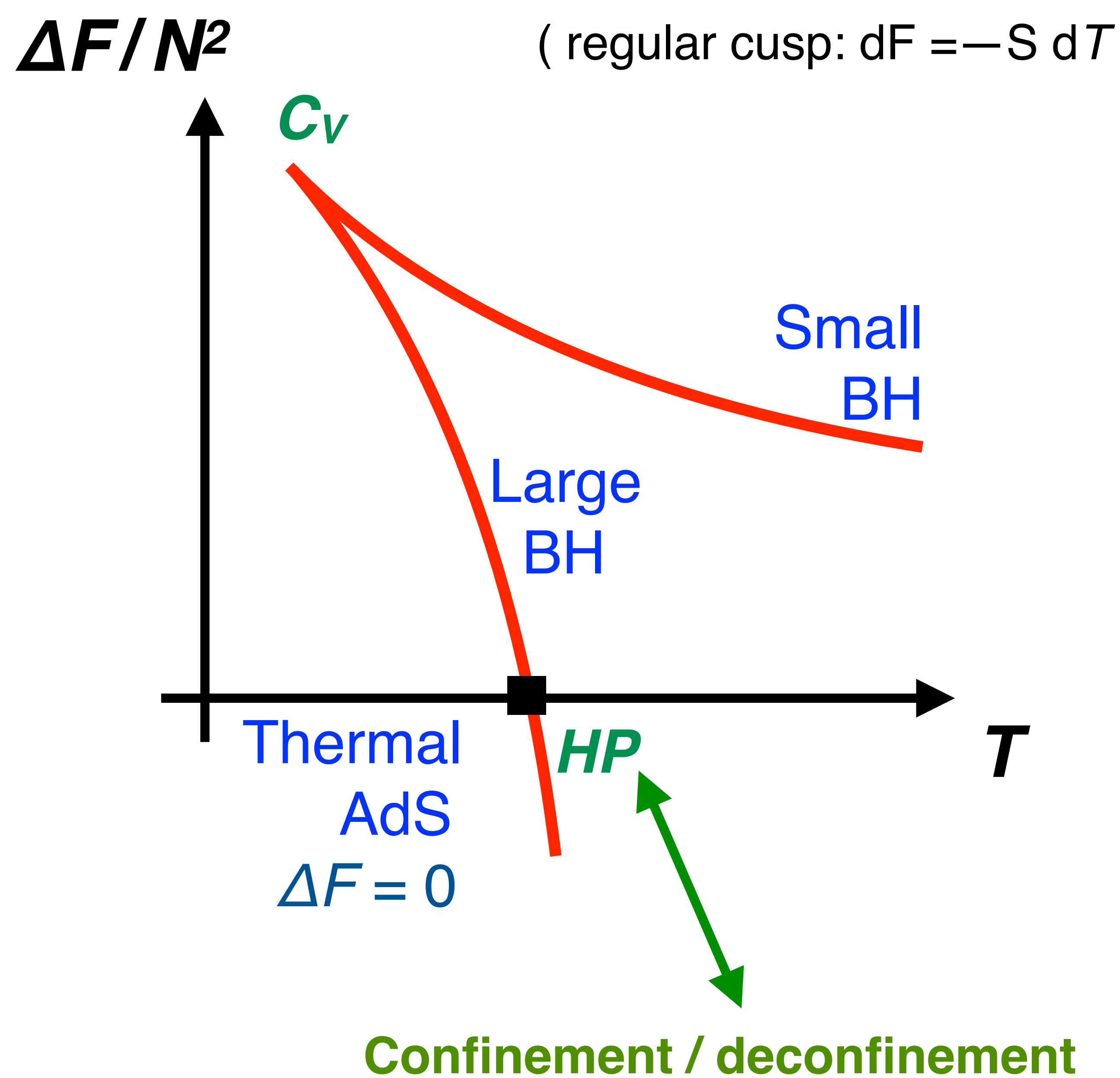
Microcanonical ensemble:

(fixed E)



Canonical ensemble:

(fixed T)



Thermal $AdS_5 \times S^5$

&

Schw $AdS_5 \times S^5$

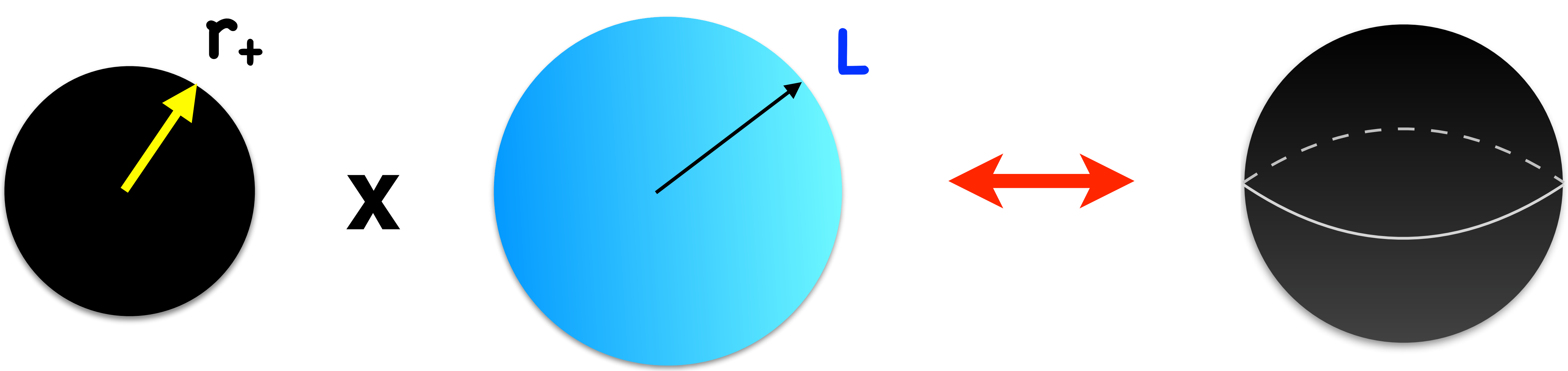
→ Are these 2 the only solutions with $AdS_5 \times S^5$ asymptotics ?

No … because we can have **hierarchy of scales**:

Schwarzschild-$AdS_5 \times S^5$:

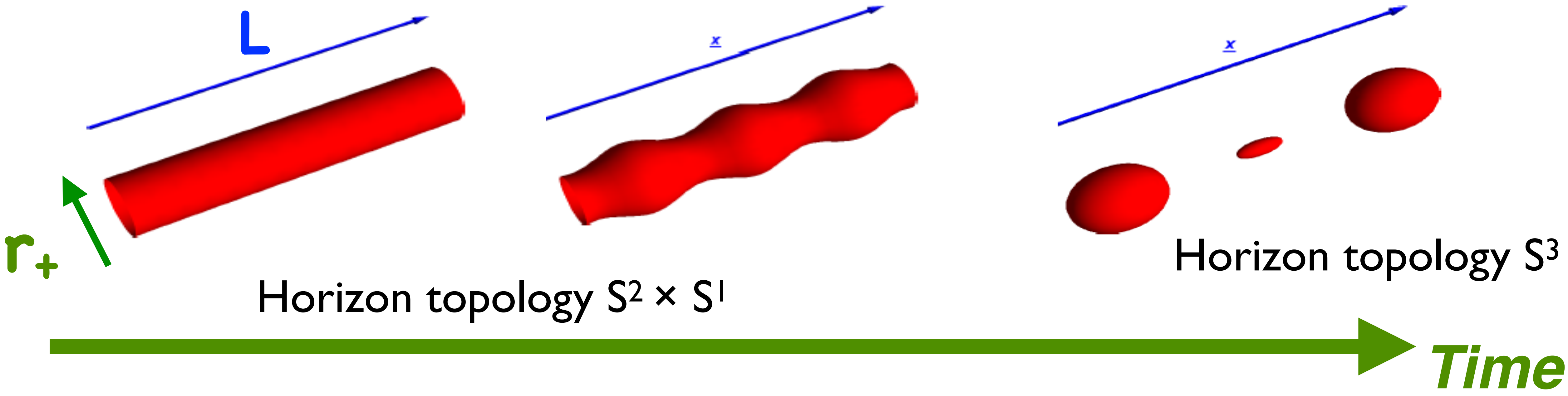
$$f(r) = 1 + \frac{r^2}{L^2} - \frac{r_+^2}{r^2}\left(\frac{r_+^2}{L^2} + 1\right)$$

… Two scales: **horizon radius r_+ and S^5 radius L**



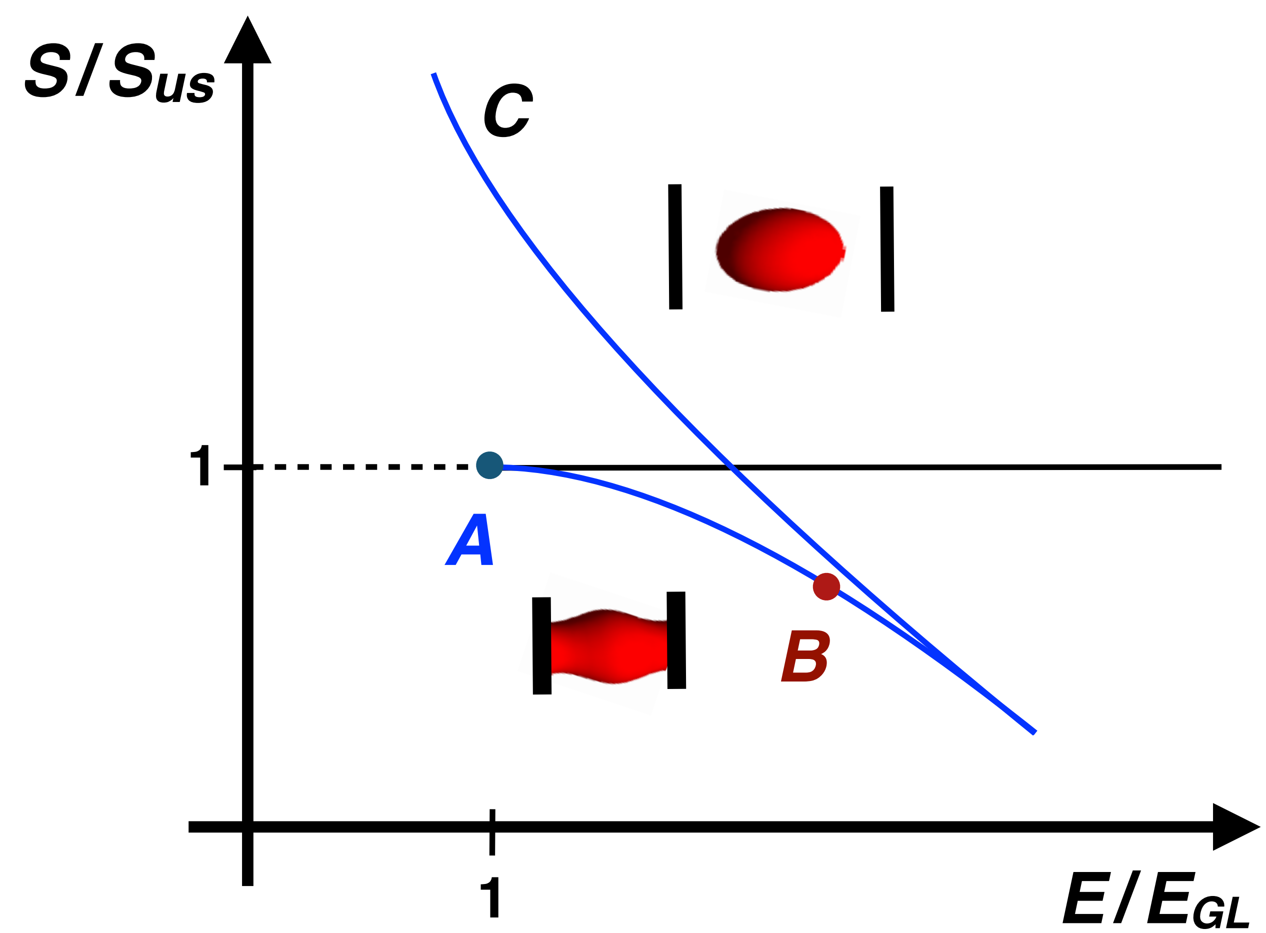
Horizon topology $S^3 \times S^5$

- **Recall *Gregory-Laflamme* instability** on a **black string Mink $_4$ x S^1** with $r_+ << L$

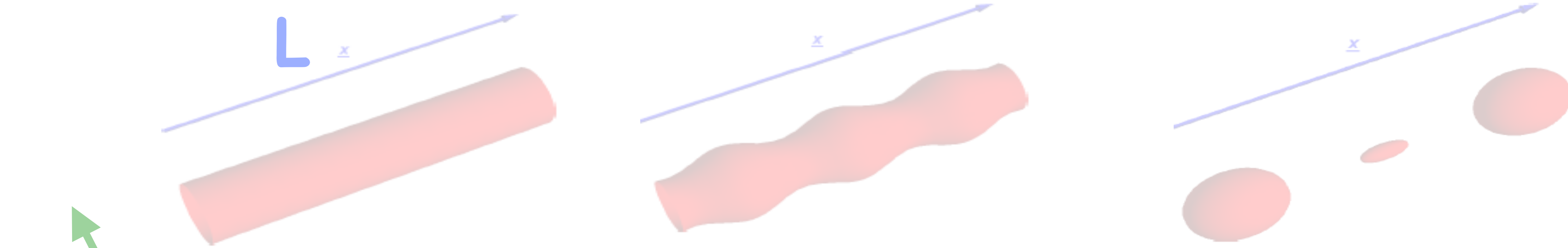


- **Hierarchy of scales => GL instability => new phases:**

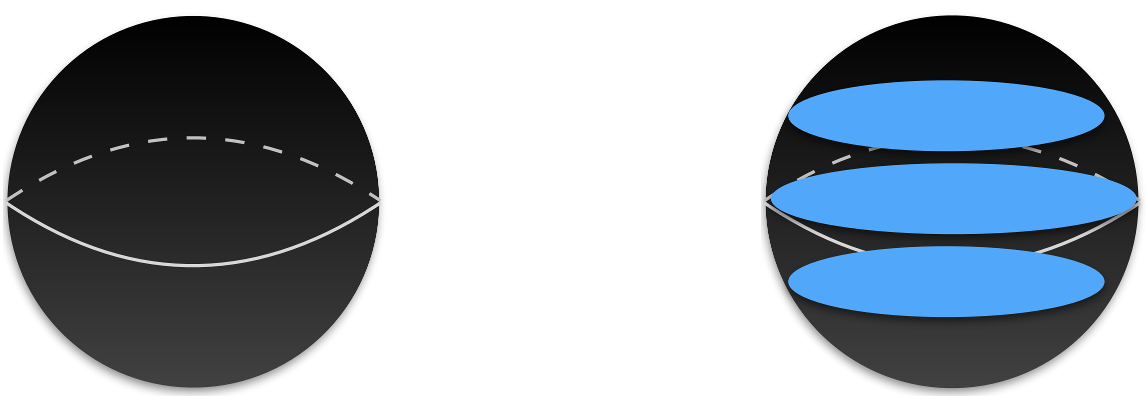
Gregory, Laflamme
Gubser,
Harmark, Obers, Kol
Wiseman, Kudoh,
Pretorius, Lehner



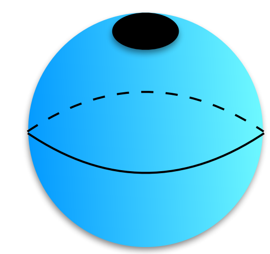
- Recall *Gregory-Laflamme* **instability** on a **black string Mink$_4$ x S^1** with $r_+ << L$



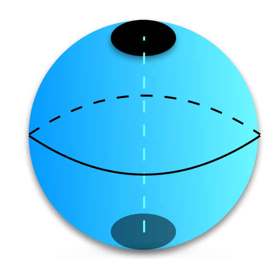
- **Expect that for $r_+ << L$ Schwarzschild-AdS$_5$ xS^5:**



Horizon topology $S^3 \times S^5$



Horizon topology S^8



Lumpy BHs

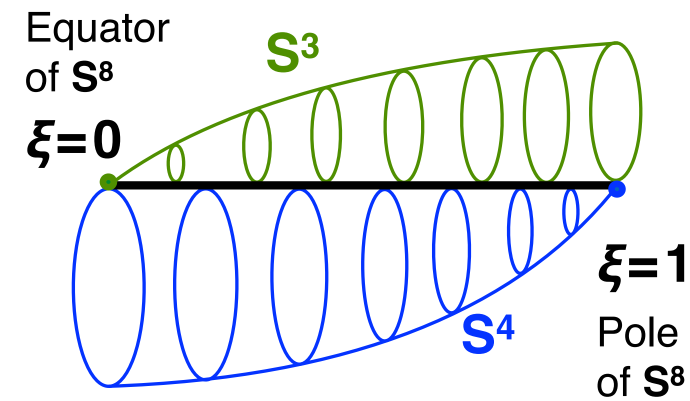
Localised BHs

[Banks, Douglas, Horowitz, Martinec, 1998]
[Peet, Ross, 1998]

Use **patching technique**, ie an ansatz that:

1) **near the horizon** it's adapted to S^8 horizon topology (10D Schw in isotropic coord)

Near-horizon region (S^8):



$$ds^2_{S^8} = d\theta^2 + \sin^2\theta\, d\Omega_3^2 + \cos^2\theta\, d\Omega_4^2$$

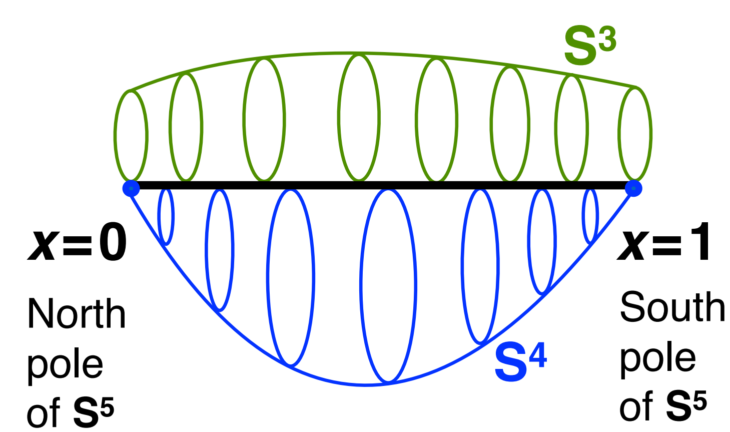
$$\xi = \sqrt{1-\sin\theta}$$

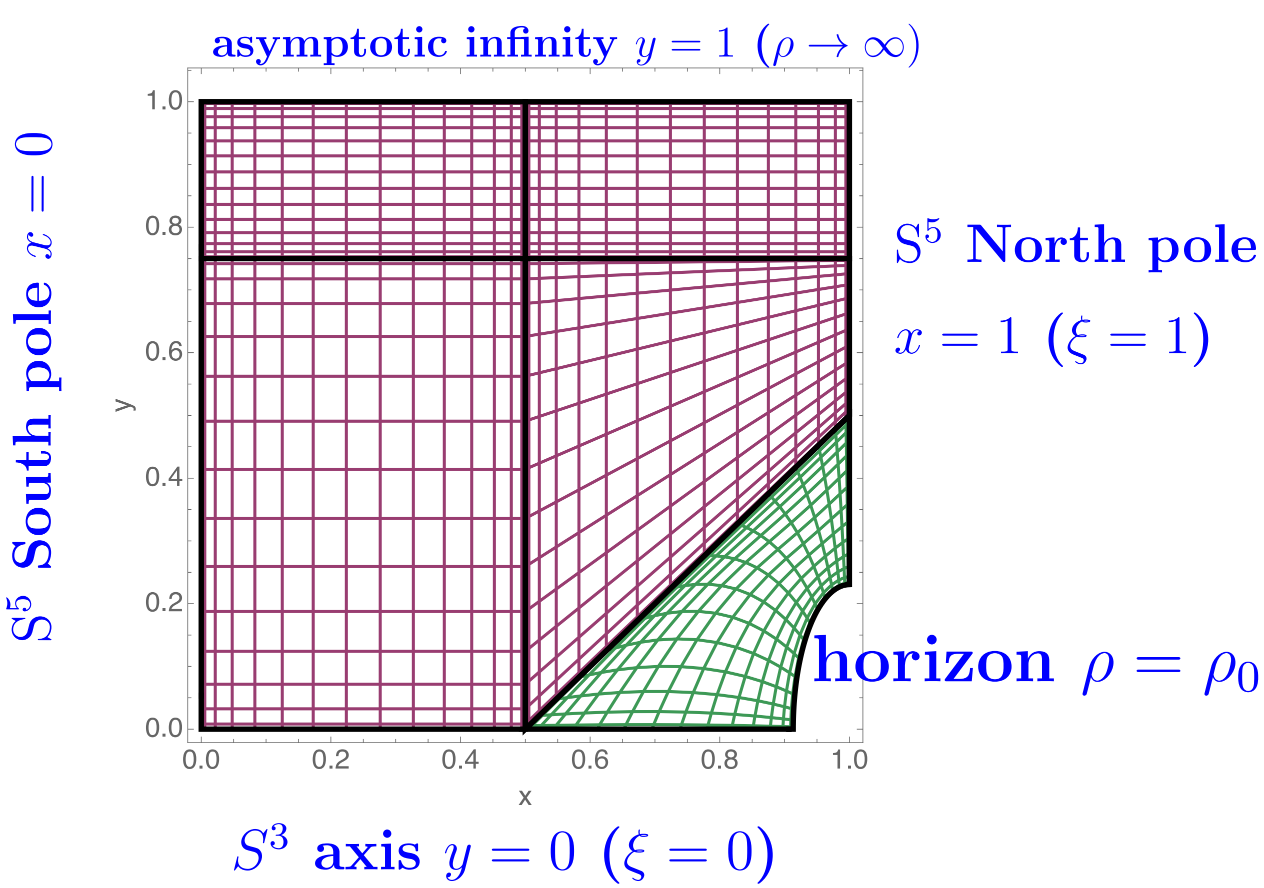
2) **asymptotes to $AdS_5 \times S^5$** with $R_t \times SO(4) \times SO(5)$ symmetry

[largest subgroup of SO(6)]

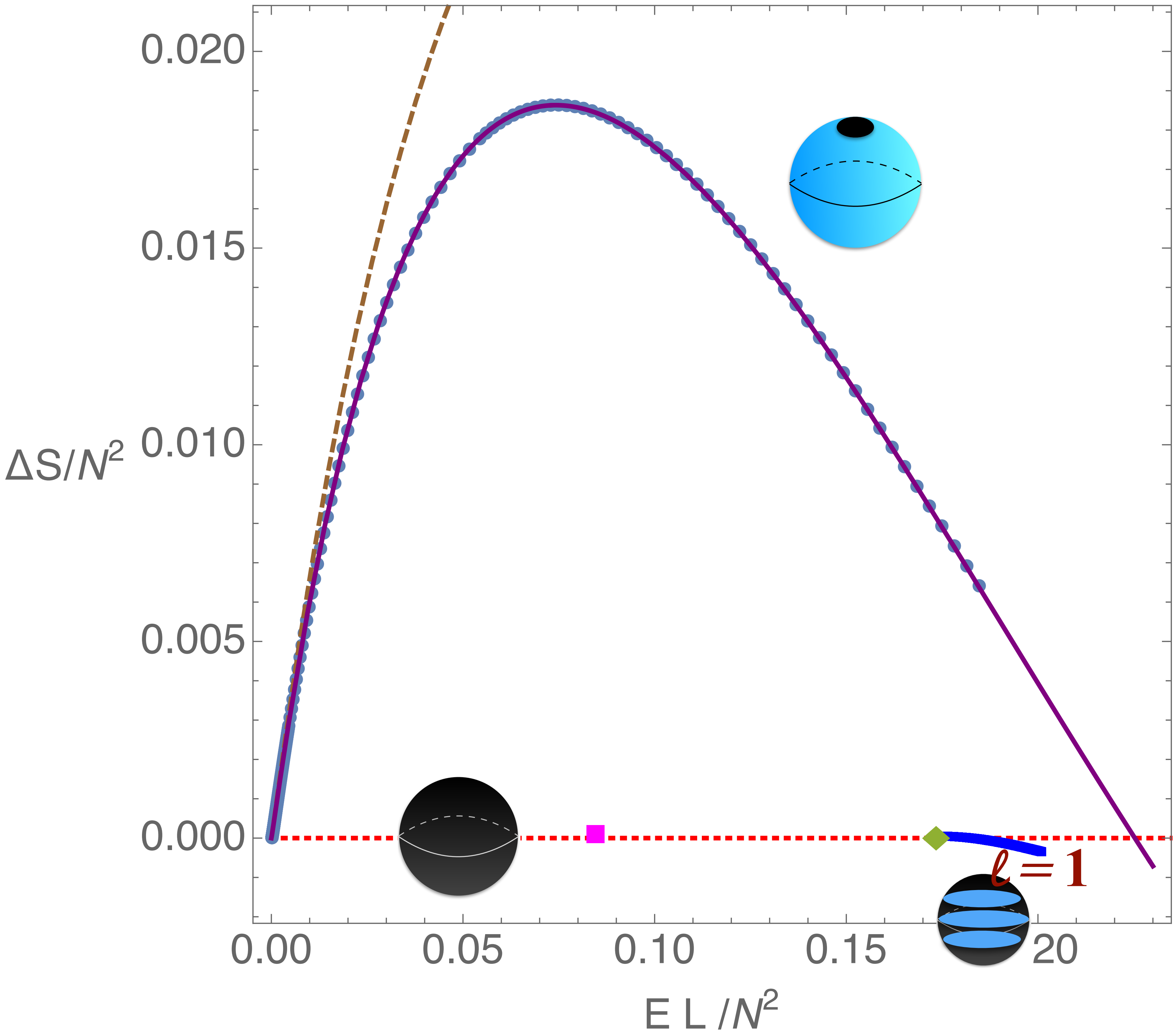
ie it's adapted to $S^3 \times S^5$ topology

Asymptotic region ($S^3 \times S^5$):





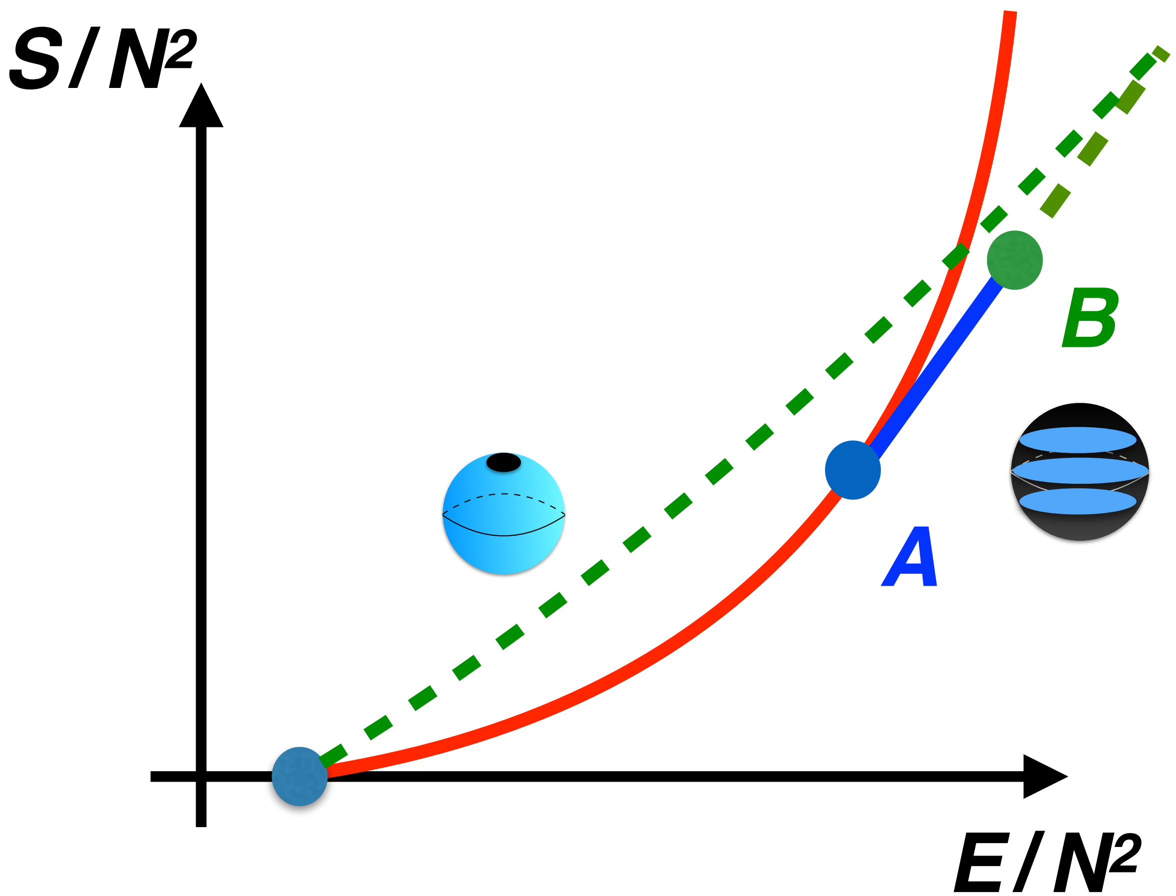
→ Complete Phase diagram (Microcanonical ensemble):



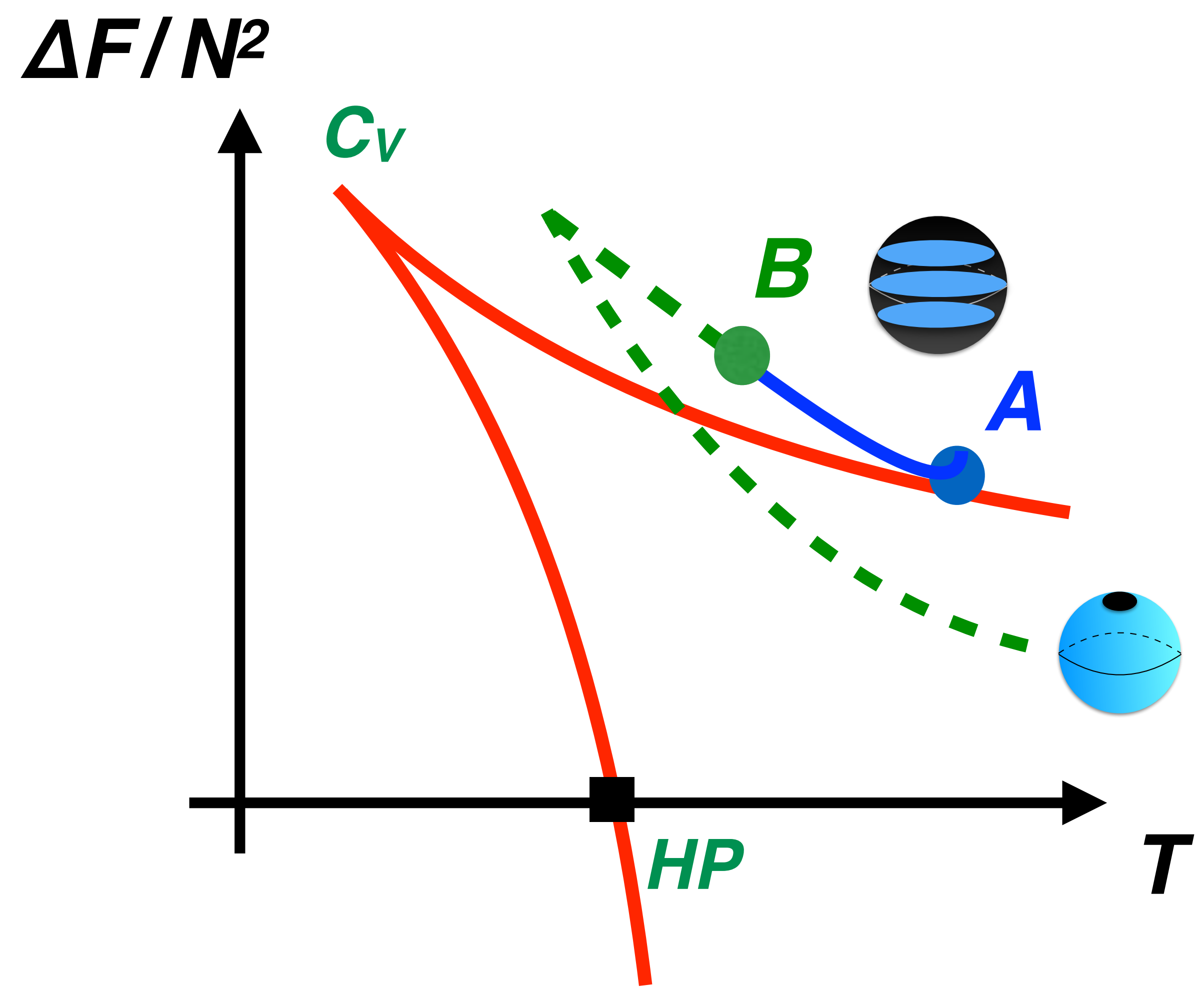
(**Thermodynamics:** use **Kaluza-Klein holography** & **holographic renormalisation**)

→ Update: **Thermal Phases of $AdS_5 \times S^5$ & their competition**

Microcanonical ensemble:
(fixed E)



Canonical ensemble:
(fixed T)

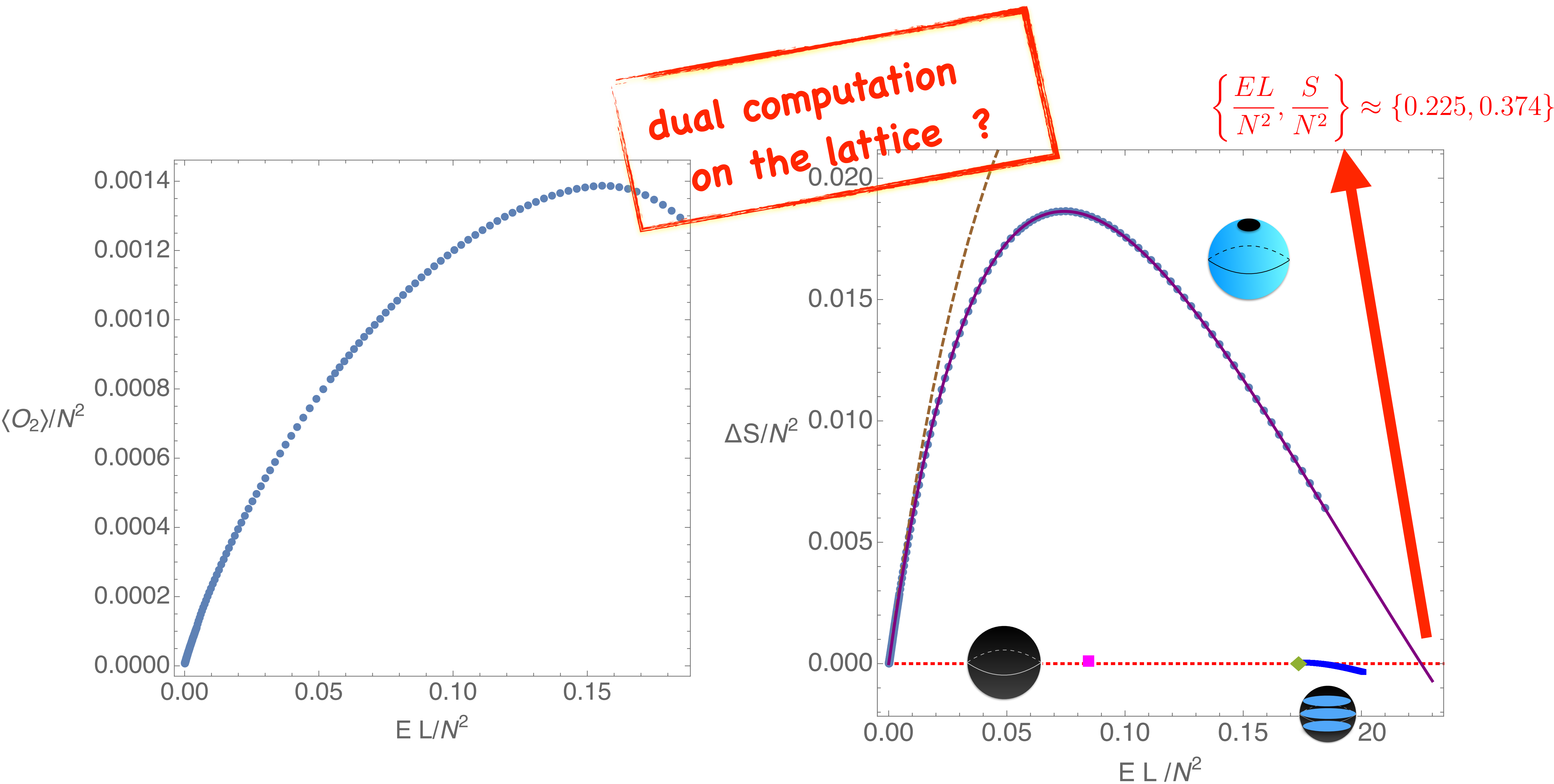


→ **CFT dual interpretation ?** **Spontaneous symmetry breaking** of the **SO(6) R-symmetry** of the scalar sector of $\mathcal{N}$=4 SYM **down to SO(5).**

=> **condensation** of an **infinite tower** of **scalar operators** with **increasing Δ.**

Lowest has Δ = 2 and vev:

$$\langle \mathcal{O}_2 \rangle = -\frac{N^2}{\pi^2}\frac{1}{8}\sqrt{\frac{5}{3}}\,\beta_2$$



3. More missing CFT states in N=4 SYM / IIB Sugra

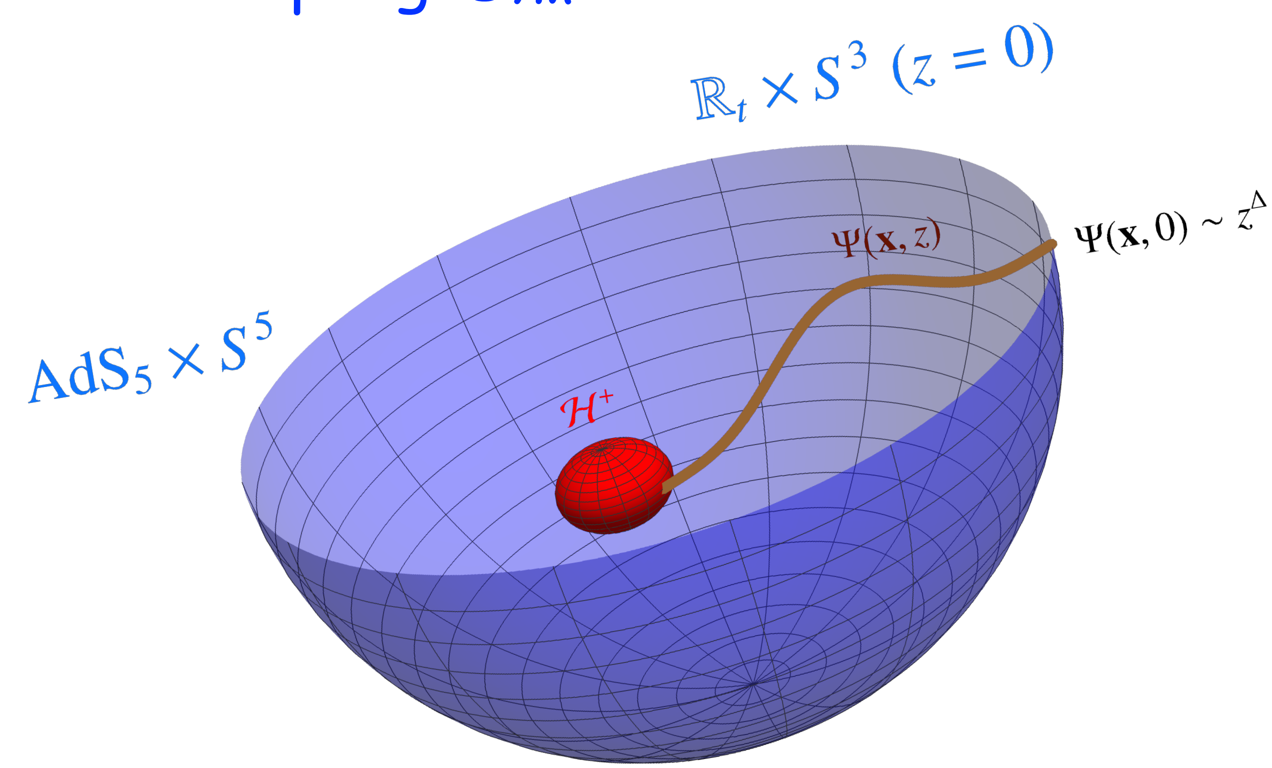
[Bhattacharyya, Minwalla, Papadodimas (2011)]
[Markeviciute, Santos (2016, 2018)]
[OD, Mitra, Santos **2207.07134** + 2024]

→ AdS_5 / CFT_4 duality

Type IIB supergravity theory on **$AdS_5 \times S^5$** with radius **L** and **N** units of **$F_{(5)}$** flux on **S^5**

↕

Large **N** and strong t'Hooft coupling $\lambda = g^2_{\mathrm{YM}} N$ limit of $\mathcal{N} = 4$ Super Yang-Mills (SYM) theory with gauge group SU(**N**) & YM coupling **g_{YM}**



Thermal states of $\mathcal{N} = 4$ **SYM**
with **temperature T**, **chemical potentials** **μ_j** and **energies** **$O(N^2)$**
living on the **Einstein static Universe** $\mathbb{R}_t \times S^3$

↔

Asymptotically **global** **$AdS_5 \times S^5$** **BHs** of IIB supergravity
with Hawking **temperature T** and **chemical potentials** **μ_j**

→ Motivations

- We should **find <u>all</u> the BHs** and **map** them into **thermal states in the dual SYM**
  => identify the **dominant phases** (as saddle points) in the **thermodynamic ensembles**.

- Necessary to **reproduce microscopically** the Bekenstein-Hawking **entropy** of AdS BHs.

  [See Benini's review talk at Strings 2022]

- Contribute to understand a **puzzle of SO(6) gauged supergravity**: its most general SUSY BH known so far — Kunduri-Lucietti-Reall BH — has **<u>only</u> 4 independent parameters**.

- However, asymptotically $AdS_5 \times S^5$ BHs are characterized by **6 conserved charges** with the BPS relation constraint $\mathbf{E= Q_1+Q_2+Q_3+J_1/L+J_2/L}$

  => the **most general SUSY BH** should be a **5-parameter solution**.

  From dual CFT perspective, most general SUSY states **also** expected to be characterized by **5 parameters**.

  [Gutowski, Reall '04]
  [Kunduri, Lucietti, Reall '06]

  So, what is the missing gravitational parameter?

→ AIM: can we **identify new thermal phases with a finite chemical potential that can dominate some thermodynamic ensembles ?**

→ Strategy to find more BHs dual to thermal SYM phases

- **The massless bosonic fields of type IIB supergravity:**
  metric tensor g_{ab}, dilaton Φ, axion C, NS-NS antisymmetric 2-tensor $B_{(2)}$,
  RR 2-form potential $C_{(2)}$, and RR 4-form $C_{(4)}$ with a 5-form field strength $F_{(5)} = dC_{(4)}$
  satisfying a **self-duality condition.**
  Fermionic superpartners: complex Weyl gravitino & complex Weyl dilatino.

- Useful: **dimensional reduction of IIB along S^5** yields 5d N=8 gauged supergravity.
  It's believed (not proven) to be a consistent reduction of IIB on **$AdS_5 \times S^5$**.

Gunaydin, Romans, Warner (1986)

- But IIB with only $g_{ab}, F_{(5)}$ **(relevant for AdS/CFT: source D3's) can be consistently dim reduced** along the S^5 to yield 5d SO(6) gauged supergravity.

Cvetic-Lü-Pope-Sadrzadeh-Tran [hep-th/0003103]

- It's itself **a consistent truncation of** gauged N =8 SUGRA where we set to 0 some 5D scalars and gauge fields.

  5D Bosons that survive (**graviton g_{ab}**, 15 **gauge fields A^{ij}** & 20 **scalars**)
  descend uniquely from 10D $\{g_{ab}, F_{(5)}\}$ of IIB

→ $U(1)^3$ gauged supergravity:

"*Cut even more (no mercy!)*":

Consistent truncation of SO(6) gauged SUGRA down to the $U(1)^3$ **Cartan subgroup of SO(6) with associated gauge fields** $\{A_{(1)}{}^K\}$ (K=1,2,3): **$U(1)^3$ gauged supergravity.**

The 5D **field content [that** descend from $\{g_{ab}, F_{(5)}\}$ of IIB **]** :

Graviton g_{ab} + 2 neutral scalars $\{\varphi_1, \varphi_2\}$ **+ 3 U(1)'s gauge fields** $\{A_{(1)}{}^K\}$,

+ 3 complex scalar fields $\{\Phi_K\}$ minimally coupled to $\{A_{(1)}{}^K\}$ with **charge** $qL = 2$.

All 5 scalars have **mass** $m^2L^2 = -2$ => **saturate AdS$_5$ Breitenlöhner-Freedman (BF) bound.**

a.k.a.

[Bhattacharyya, Minwalla, Papadodimas (2011)]
[Markeviciute, Santos (2016, 2018)]
[OD, Mitra, Santos 2022 + 2024]

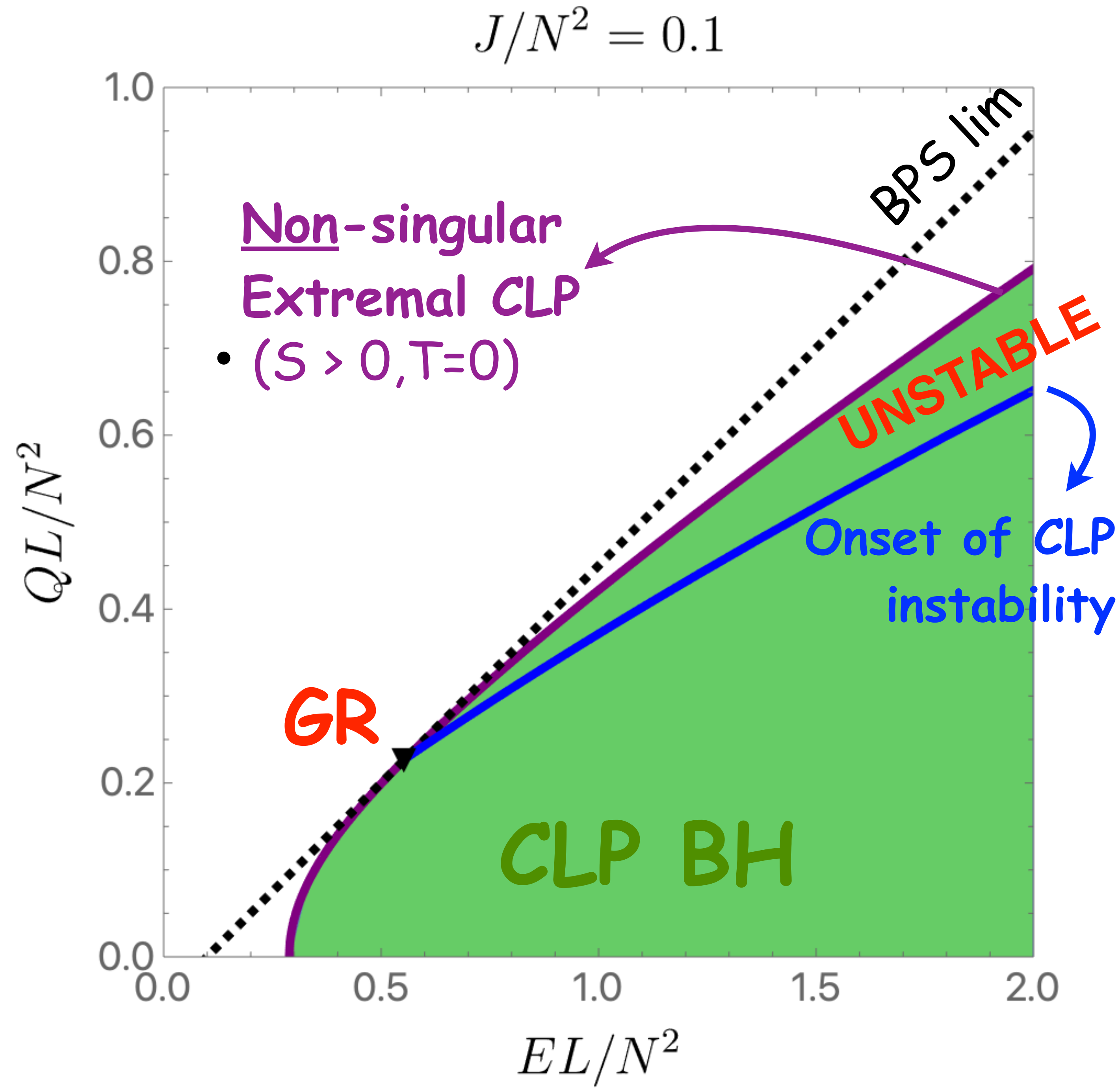
➞ Microcononical phase diagram (of truncation with three equal Q's and equal J's)

- Set $Q_1=Q_2=Q_3=Q$. When $J_1=J_2=J$: co-homogeneity one (coupled nonlinear ODEs)
- **BPS relation** is **$E = 3Q + 2J/L$**
- $\Phi_{1,2,3}=0$: "**Kerr-Newman—AdS$_5$**" (CLP) BHs of theory but with **non-trivial neutral scalar fields** $\{\varphi_1, \varphi_2\}$ supporting them.
- At fixed J, ∃ a single point **GR** where **extremal CLP** is **SUSY**: the (1-parameter) Gutowski, Reall '04 BH. (Kunduri-Lucietti-Reall BH: arbitrary $Q_{1,2,3}$, $J_{1,2}$)
- **Scalar condensation instability of 'Bald' CLP BHs ($\Phi = 0$):**

$$\delta\Phi \sim e^{-i\omega t}\psi$$

=> Unstable when Im (ω L) >0

Due to the **violation of the near-horizon AdS2 Breitenlöhner-Freedman (BF) bound**

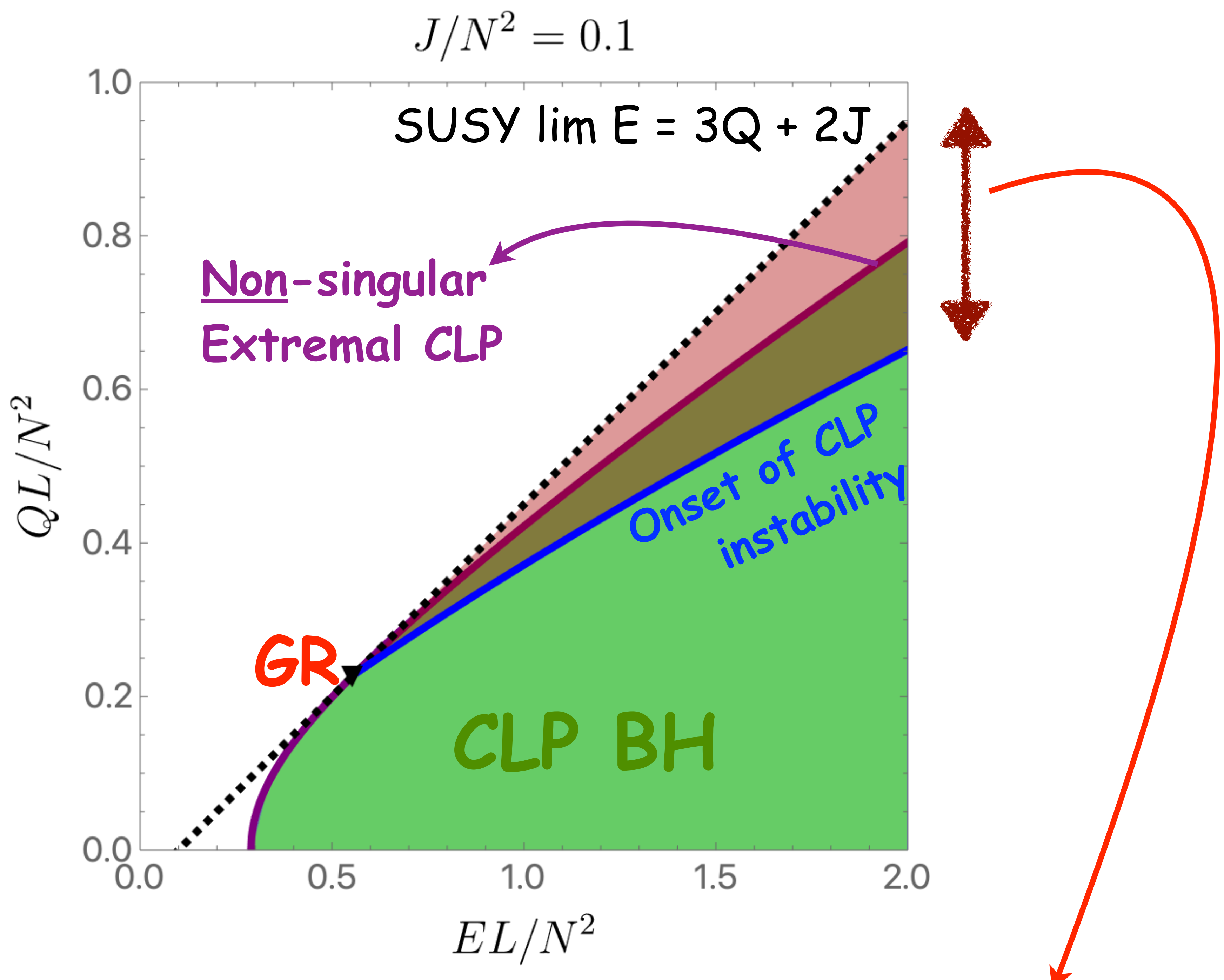


=> Suggests ∃ of hairy BHs that have a charged scalar field condensate: $\langle\Phi\rangle \neq 0$

Behrndt-Cvetic-Sabra 1998, Cvetic-Lü-Pope (CLP) 2004, Wu 2011

→ Microcononical phase diagram (of truncation with three equal Q's and equal J's)

• Work at **finite temperature** & approach T →0: **find (evidence for) novel SUSY BHs !**



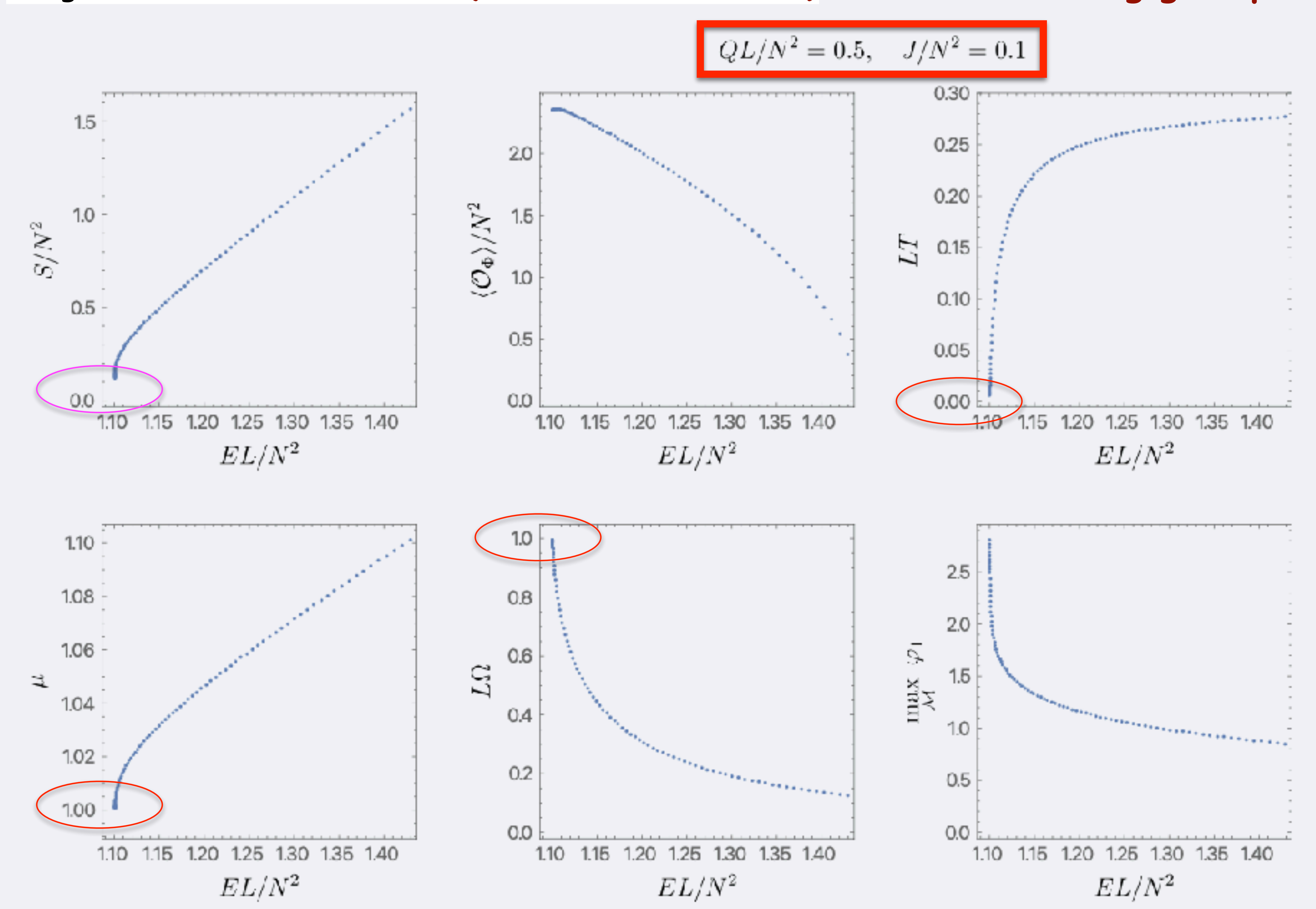
• **Hairy BHs with a charged scalar condensate**

have higher entropy S than CLP for given {E,Q,J}.

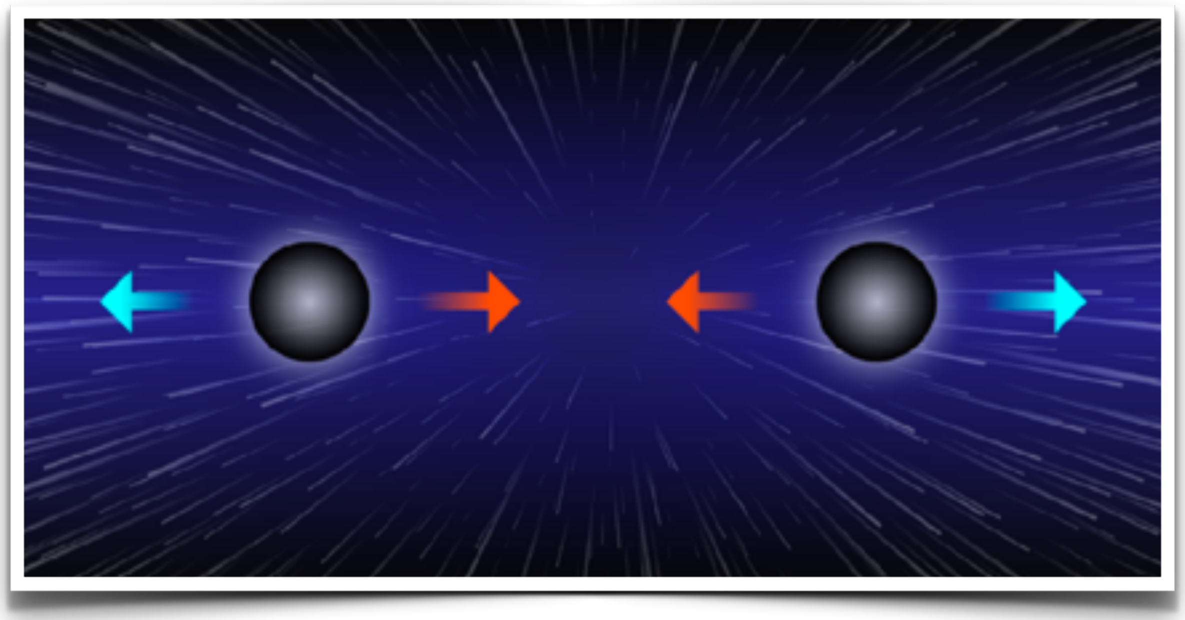
• **Hairy BHs** have a **non-singular (?)** BPS lim (where $TL \to 0$, $\mu \to 1^+$, $\Omega L \to 1^-$):

might be **novel SUSY BHs (this time with hair) => can be missing grav parameter!**

- **Hairy BHs with a charged scalar condensate**
  have higher entropy S than CLP for given {E,Q,J}.
- **Hairy BHs** have a **non-singular (?)** BPS lim (where **TL → 0, μ → 1⁺, ΩL → 1⁻**):
  might be **novel SUSY BHs (this time with hair) => can be missing grav parameter!**



4. Black hole binaries in an expanding Universe



Our Universe appears to be expanding due to the presence of a positive cosmological constant…

So we should ask:

What is the phase space of **stationary black hole** solutions of the Einstein equation in **de Sitter**?

Are there **other solutions** besides de Sitter Schwarzschild and de Sitter Kerr?

OD, Gary Gibbons, Jorge Santos, Benson Way, 2303.07361

Start with **Newtonian** analysis: consider a configuration of **N small BHs in de Sitter** space

$$\Lambda \equiv 3/\ell^2 > 0.$$

- **Newton-Hooke** equations of motion: $$m_a \frac{\mathrm{d}^2\mathbf{x}_a}{\mathrm{d}t^2} - m_a \frac{\mathbf{x}_a}{\ell^2} = -\sum_{b\neq a}^{b=N} \frac{m_a m_b(\mathbf{x}_a - \mathbf{x}_b)}{|\mathbf{x}_a - \mathbf{x}_b|^3}$$

- **Static** solutions exist when: $$\frac{\mathbf{x}_a}{\ell^2} = \sum_{b\neq a}^{b=N} \frac{m_b(\mathbf{x}_a - \mathbf{x}_b)}{|\mathbf{x}_a - \mathbf{x}_b|^3} \qquad (1)$$

- **Two equal mass BHs** aligned along z axis and separated by a **distance d:**

$$N = 2, \quad x_1 = -x_2 = \frac{d}{2}\,\hat{e}_z, \quad m_a = m_b = M$$

- Then (1) yields: $$\frac{d^3}{\ell^3} = \frac{r_+}{\ell} \quad \Rightarrow \quad \frac{d}{\ell} = \frac{1}{(4\pi\ell\, T_+)^{1/3}} \qquad (2)$$

$$r_+ = 2M$$
$$T_+ = (4\pi r_+)^{-1}$$

- Require **validity** of **Newton** + **Hooke** approxs + **BHs inside** a <u>**single**</u> **cosmological** horizon:

$$r_+ \ll d\,, \quad d \ll \ell \quad \text{and} \quad r_+ \ll \ell$$

- If the distance between BHs is (2), first 2 conditions are obeyed if we assume the 3rd

=> **static de Sitter binaries** with **small BHs** are **consistent** with **Newton-Hooke theory**.

- On the other hand, some **mathematical theorems** in the literature **claim uniqueness** of Schwarzschild/Kerr solutions in de Sitter!

- Solve the Einstein equations to **settle the issue**!

- Use **patching technique**, ie an ansatz that:

  1) near the **two event horizons** looks like the **Israel-Khan** solution (but **<u>without</u> conical singularity**) &

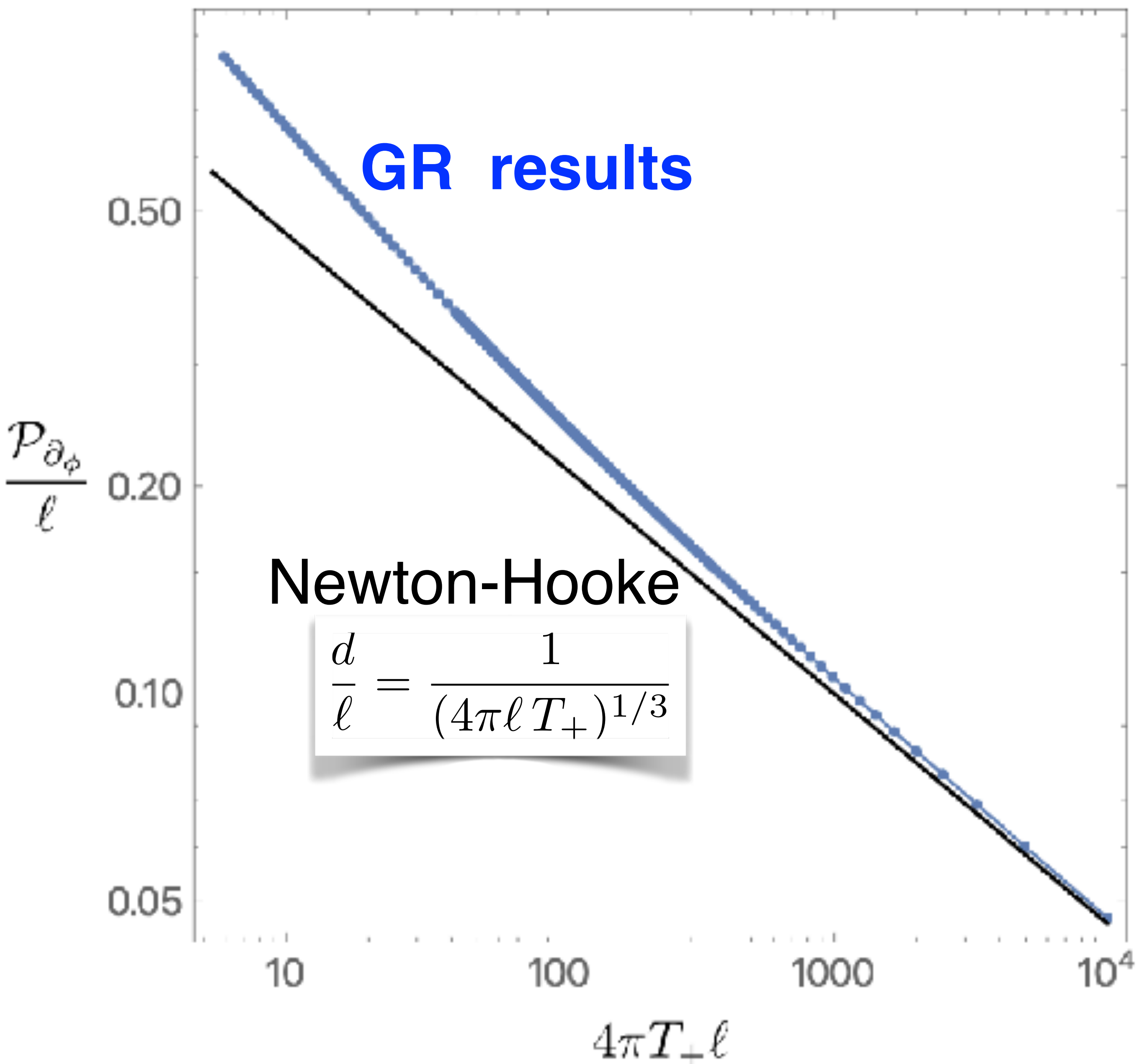
  2) **near** the (**<u>single</u>**) **cosmological horizon** looks like de **Sitter space** (in the static patch)

- We find that **regular static BH binaries do exist in de Sitter.**

- Not in conflict with available Uniqueness theorems:

  we have (explicitly identified) **assumptions** of these theorems that can be **evaded**

Proper distance between the **BH horizons** versus the BH **temperature:**



Total BH entropy versus the **cosmological horizon entropy:**