Enhancing Cosmological Model Selection with Interpretable Machine Learning

Indira Ocampo Justiniano

IFT UAM CSIC

AI goes MAD

Madrid, 14th of October, 2024

Instituto de Física Teórica UAM-CSIC

Fundación "la Caixa"

CSIC
Consejo Superior de Investigaciones Científicas

esa

Motivation

Problems in Cosmology with the Standard Model **ΛCDM**

Model-independent (agnostic) framework for analyzing data

Machine learning techniques for discriminating models

Performs well, BUT: **ML = Black Box**

Towards Feature Importance and **Interpretability**

IFT-UAM/CSIC-24-86

Enhancing Cosmological Model Selection with Interpretable Machine Learning

Indira Ocampo,[1,*] George Alestas,[1,†] Savvas Nesseris,[1,‡] and Domenico Sapone[2,§]

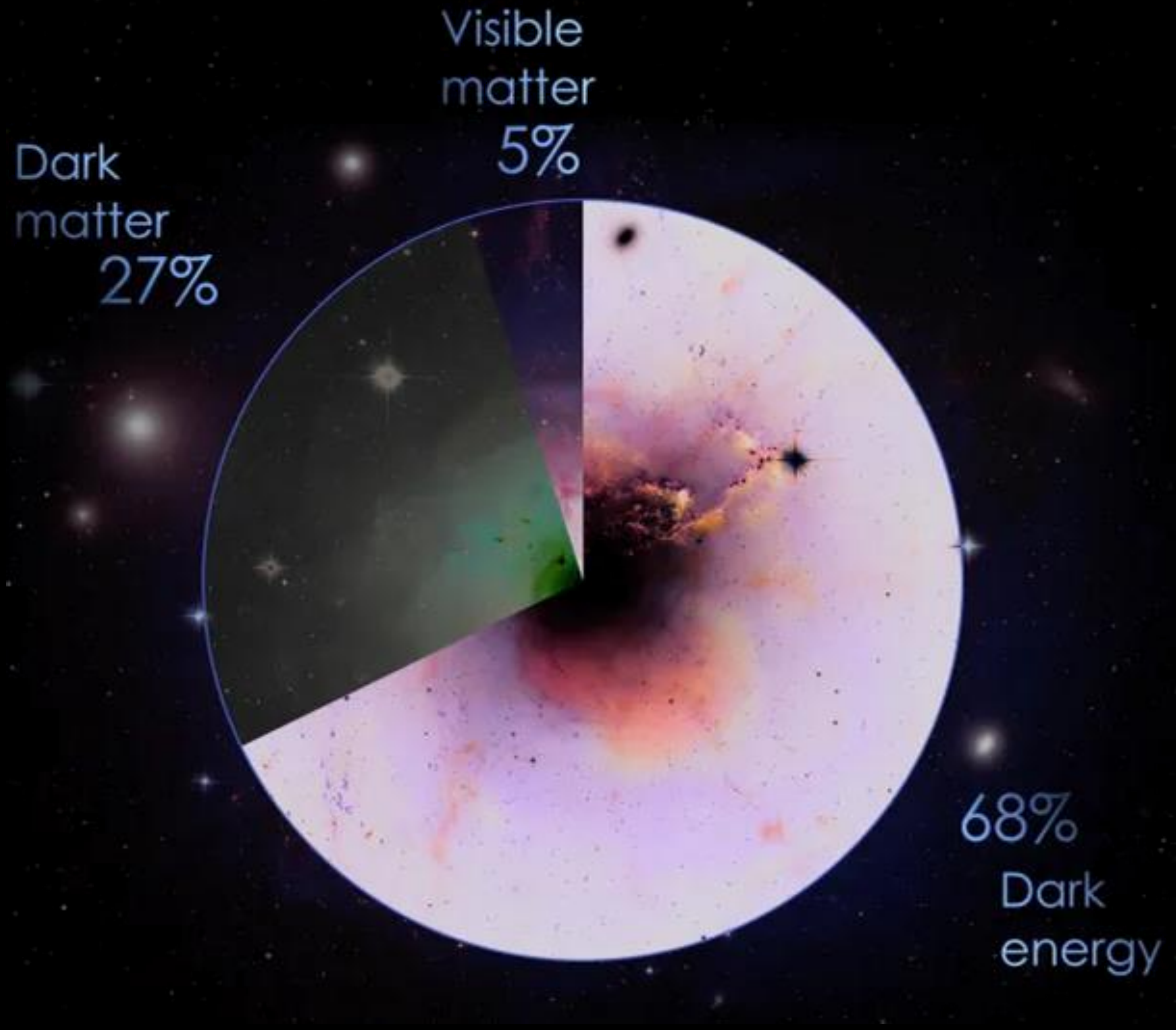
[1]Instituto de Física Teórica UAM-CSIC, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain.
[2]Departamento de Física, FCFM, Universidad de Chile, Santiago, Chile.
(Dated: June 13, 2024)

We propose a novel approach using neural networks (NNs) to differentiate between cosmological models, especially in the case where they are nested and the additional model parameters are close to zero, making it difficult to discriminate them with traditional approaches. Our method complements Bayesian analyses for cosmological model selection, which heavily depend on the chosen priors and average the unnormalized posterior over potentially large prior volumes. By analyzing simulated realistic data sets of the growth rate of the large scale structure (LSS) of the Universe, based on current galaxy-clustering survey specifications, for the cosmological constant and cold dark matter (ΛCDM) model and the Hu-Sawicki $f(R)$ model, we demonstrate the potential of NNs to enhance the extraction of meaningful information from cosmological LSS data. We find that the NN can successfully distinguish between ΛCDM and the $f(R)$ models, by predicting the correct model with approximately 97% overall accuracy, thus demonstrating that NNs can maximise the potential of current and next generation surveys to probe for deviations from general relativity.

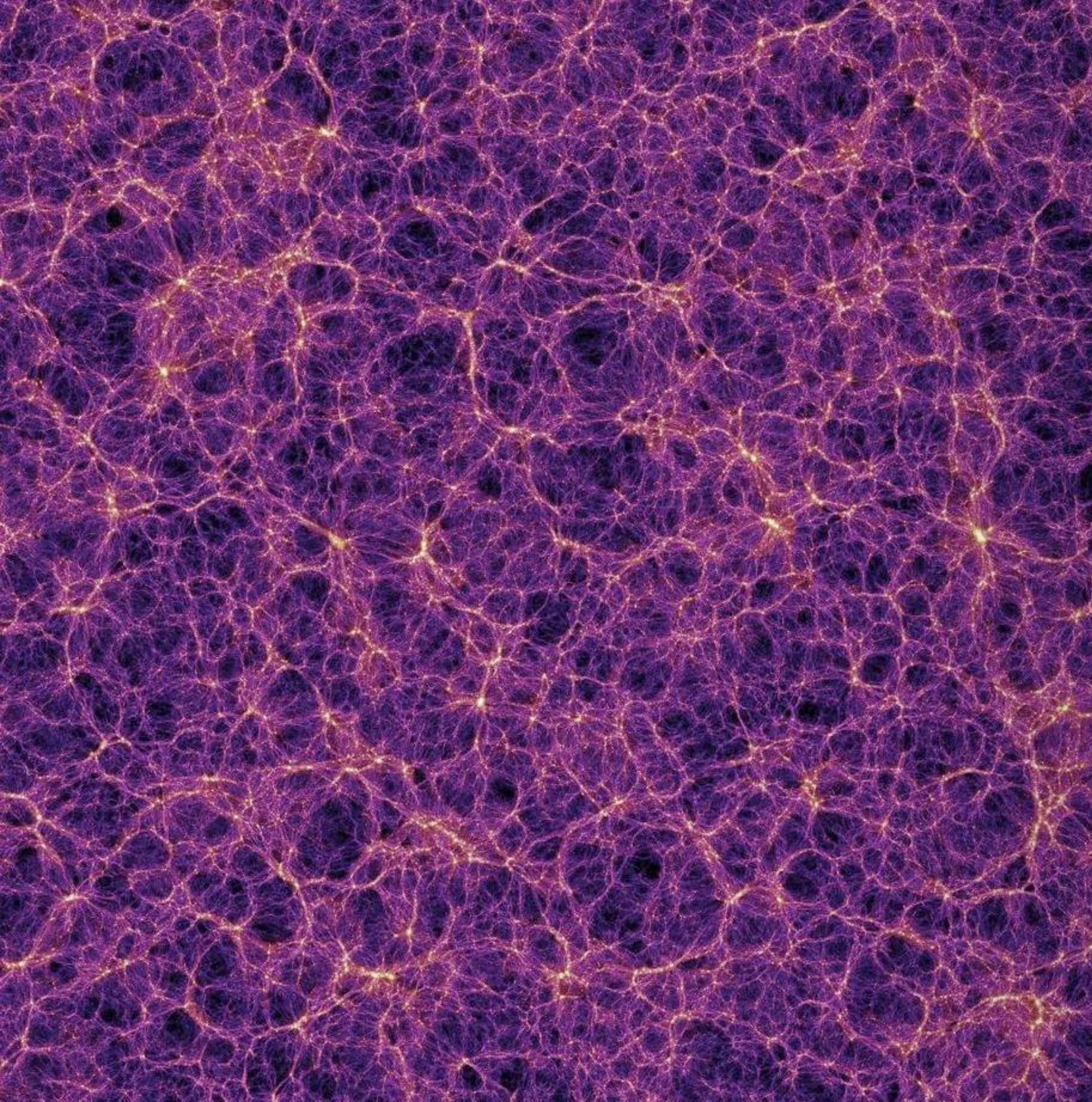
The ΛCDM model

Assumptions:

- The Universe: radiation, ordinary matter, cold (non-relativistic) dark matter (CDM) and Λ.
- Gravity described by GR on cosmological scales.
- The Cosmological Principle: the Universe is statistically homogeneous and isotropic (in large scales).
- Requires only 6 independent parameters (Ω_m, Ω_b, h, n_s, σ_8, τ)
- Primordial phase of cosmic inflation



Source: esa.int



ΛCDM challenges

Challenges:

- DM & DE lack of detection
- Some violations of the Cosmological Principle (homogeneity & isotropy)
- H_0 & S_8 tension
- Other problems…

Is there something wrong with the data?

Is there something wrong with the theory?

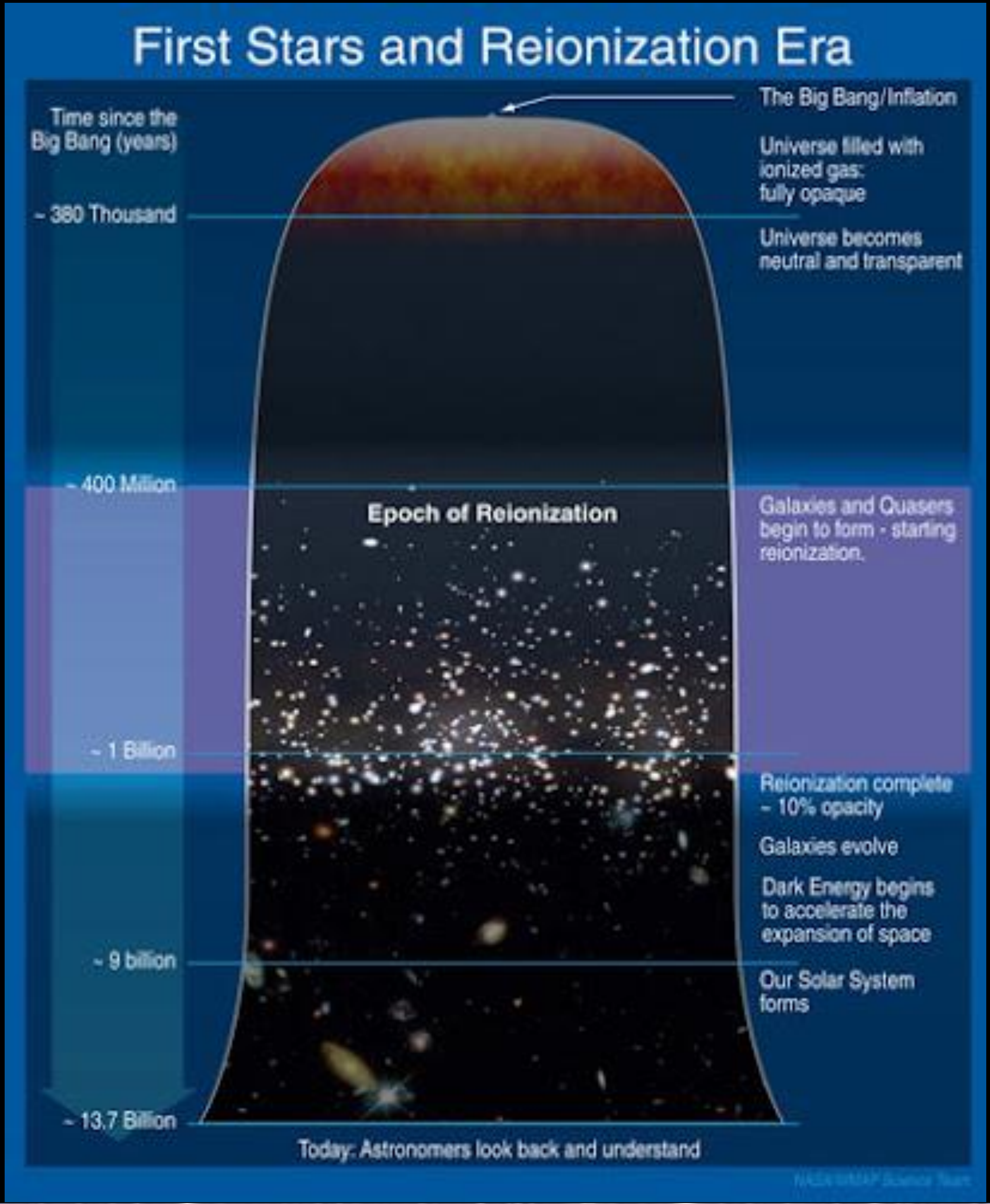
Beyond ΛCDM

Is GR the correct theory for the large cosmological scales?

- MG feature “screening mechanisms” that cause deviations from GR to “switch off” on small scales (show different predictions from GR on LS)
- Surveys like ***Euclid***, the Vera C. Rubin Observatory, DESI, are promising.
- Data exponentially increasing: we need new computational tools

AIM:

Test deviations from GR in a model-agnostic way (i.e. using Machine Learning techniques)



Source:philosophy-of-cosmology.ox

Growth of Matter Perturbations f

- Study the LSS through perturbation theory:

density: $\rho = \bar{\rho} + \delta\rho,$ pressure $P = \bar{P} + \delta P$

We study the eq:

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\rm eff}\,\rho\,\delta_m \approx 0 \qquad \delta_m \equiv \frac{\delta\rho}{\rho}$$

(evolution of the matter density perturbations).

With a solution (for LCDM, Geff = 1):

$$\delta_{\rm m}(a) = a \cdot {}_2F_1\left(\frac{1}{3}, 1; \frac{11}{6}; a^3\left(1 - \frac{1}{\Omega_{\rm m,0}}\right)\right) \longrightarrow \boxed{f = \frac{d\ln\delta_m}{d\ln a}}$$

For ΛCDM, μ =1

The growth fσ8

$$G_{\rm eff} = \mu\left(\frac{z}{1+z}\right) = 1 + g_a\left(\frac{z}{1+z}\right)^2 - g_a\left(\frac{z}{1+z}\right)^4$$

In galaxy surveys we observe the galaxy density fluctuations

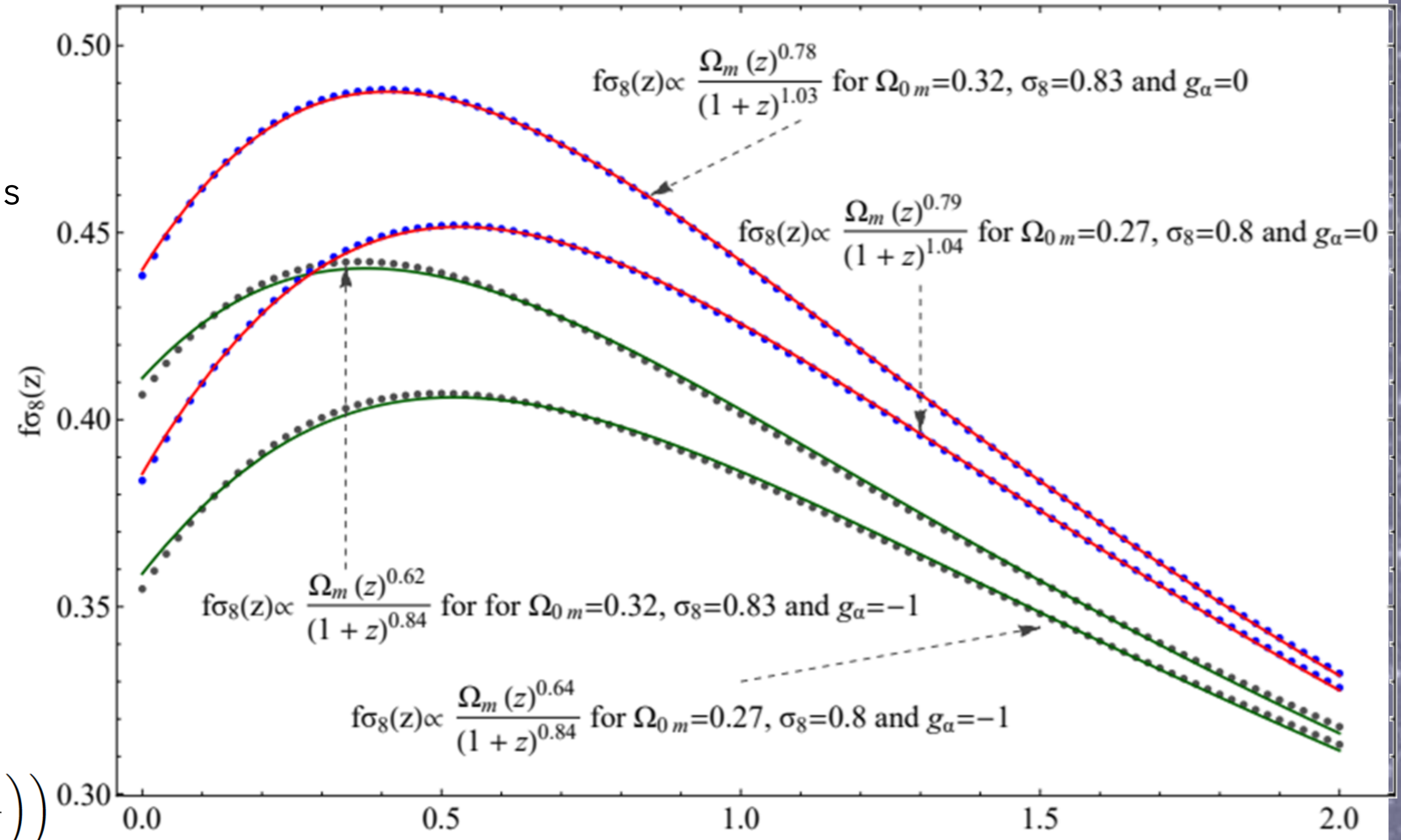
$$\delta_g = b\,\delta_m$$

The growth is measured in a bias independent way

$$f\sigma_8(z) \equiv f(z)\sigma_8(z)$$

$$f\sigma_8(a) = a\frac{\delta'_{\rm m}(a)}{\delta_{\rm m}(1)}\cdot\sigma_{8,0}.$$

$$\delta_{\rm m}(a) = a\cdot {}_2F_1\left(\frac{1}{3},1;\frac{11}{6};a^3\left(1-\frac{1}{\Omega_{\rm m,0}}\right)\right)$$



Kazantzidis, Perivolaropoulos (2018)

f(R) model – Hu Sawicki

$$f(R) = R - \frac{2\Lambda}{1 + \left(\frac{b\Lambda}{R}\right)}$$

Extension of GR: R → R+ f (R)

$$R = 6(\dot{H} + 2H^2).$$

Can be considered as a **small perturbation around the ΛCDM model.**

$$\lim_{b\to 0} f(R) = R - 2\Lambda$$

Interesting alternative to test GR at very large scales, because:

- Consistent with some CMB, LSS and SNIa observations
- Reproduces the ΛCDM expansion history and formation of structures

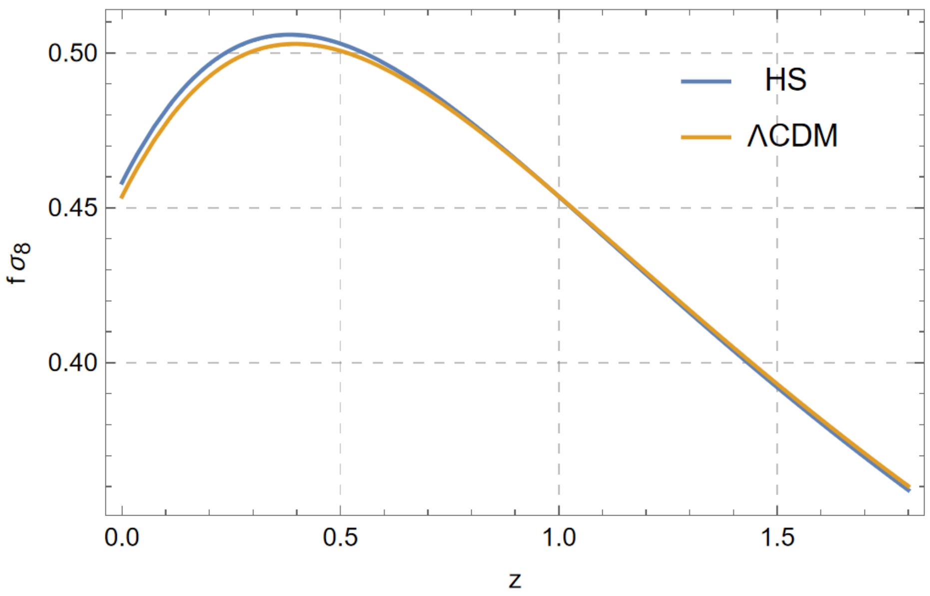
f(R) model – Hu Sawicki

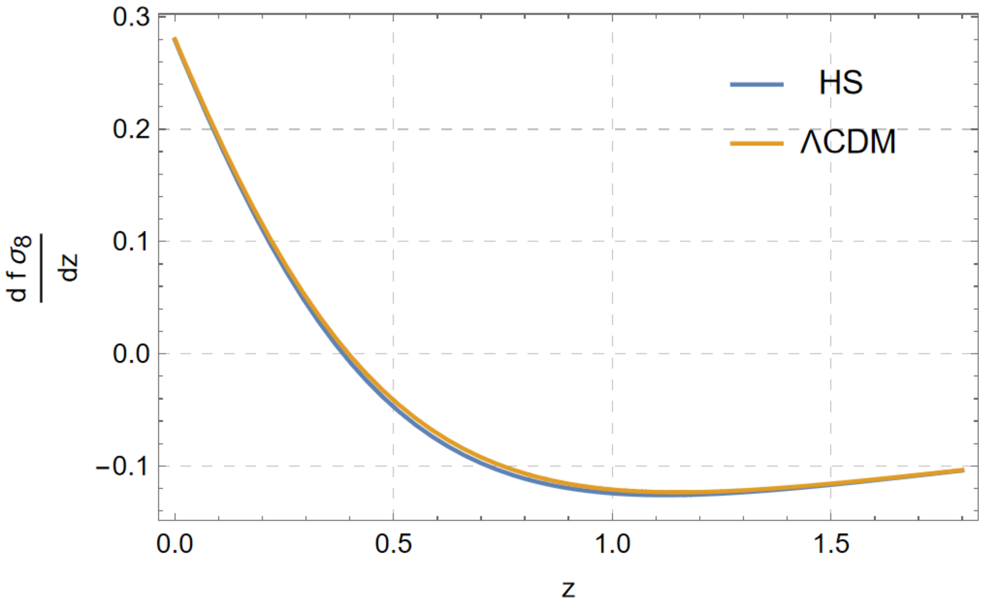
$$f(R) = R - \frac{2\Lambda}{1 + \left(\frac{b\Lambda}{R}\right)^n}$$

Analytic expresion of G_{eff} for the HS model

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\rm eff}\rho_m\delta_m = 0, \longrightarrow G_{\rm eff} = \frac{G}{F}\left[\frac{4}{3} - \frac{1}{3}\frac{M^2a^2}{k^2 + M^2a^2}\right], \quad \begin{cases} M^2 = \frac{F}{3F_R} \\ F = \frac{df(R)}{dR} \end{cases}$$

$$f\sigma_8(a) = a\frac{\delta_m'(a)}{\delta_m(1)} \cdot \sigma_{8,0}.$$





Dataset simulation strategy

$f\sigma_8$ values with their uncertainty. Cosmological parameters varied as:

ΛCDM

$\sigma_8 \in [0.7, 0.9]$

$\Omega_m \in [0.2, 0.4]$

$$G_{\rm eff} = 1$$

HS - f(R)

$\sigma_8 \in [0.7, 0.9]$, $\Omega_m \in [0.2, 0.4]$

$b \in [0.000001, 0.00005]$

$$G_{\rm eff} = \frac{G}{F}\left[\frac{4}{3} - \frac{1}{3}\frac{M^2a^2}{k^2 + M^2a^2}\right], \quad M^2 = \frac{F}{3F_{\rm R}}$$

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\rm eff}\,\rho\,\delta_m \approx 0 \qquad \delta_m \equiv \frac{\delta\rho}{\rho}$$

↓

$$f = \frac{d\ln\delta_m}{d\ln a}$$

↓

$$f\sigma_8(a) = a\frac{\delta'_{\rm m}(a)}{\delta_{\rm m}(1)} \cdot \sigma_{8,0}. \qquad a = \frac{1}{1+z}$$

↓

$z \in [0.05, 1.85]$

↓ +

$\sigma_{f\sigma_8}(a)$ (DESI-like Cij)

Model independent framework: ML

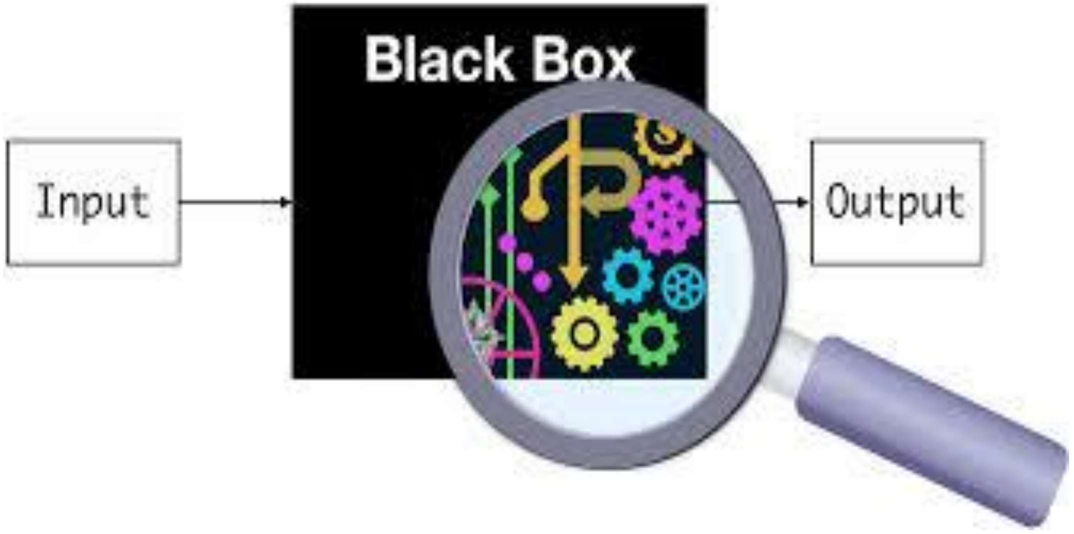
- Can we use cosmological observables to learn something from our theory of gravity?

Data → Theory

- Therefore, ML techniques are a good candidate to test gravity in a model-independent way.

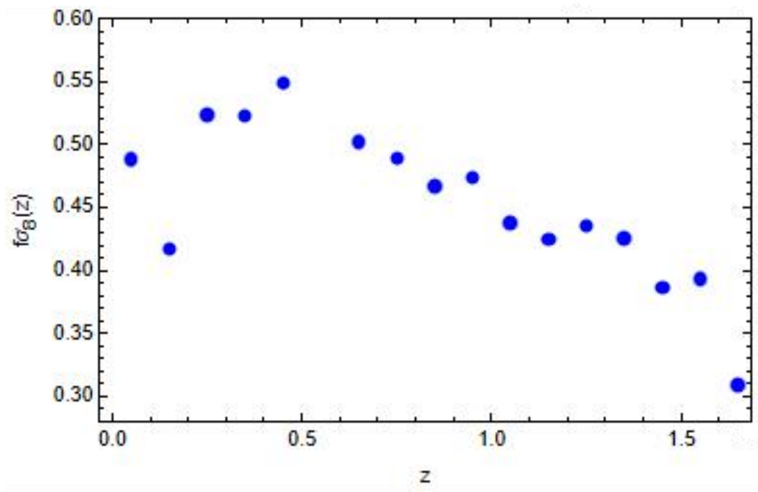
BUT… ML is still considered a “Black Box”

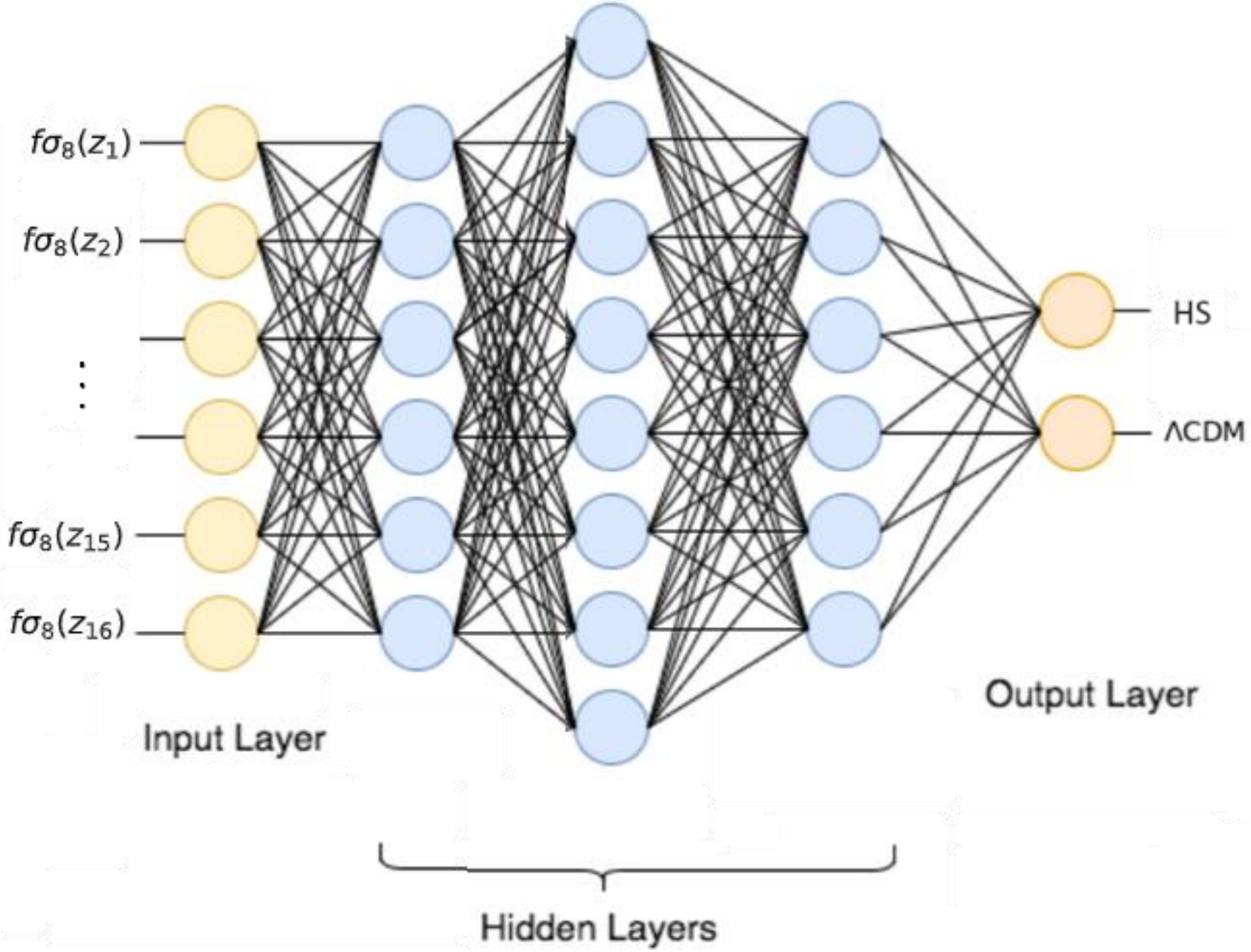
- We need to build trust in such techniques, as their incorporation in data analysis is speeding up.
- We need interpretable tools to understand what is the machine learning



Source: investopedia.com

Machine Learning strategy

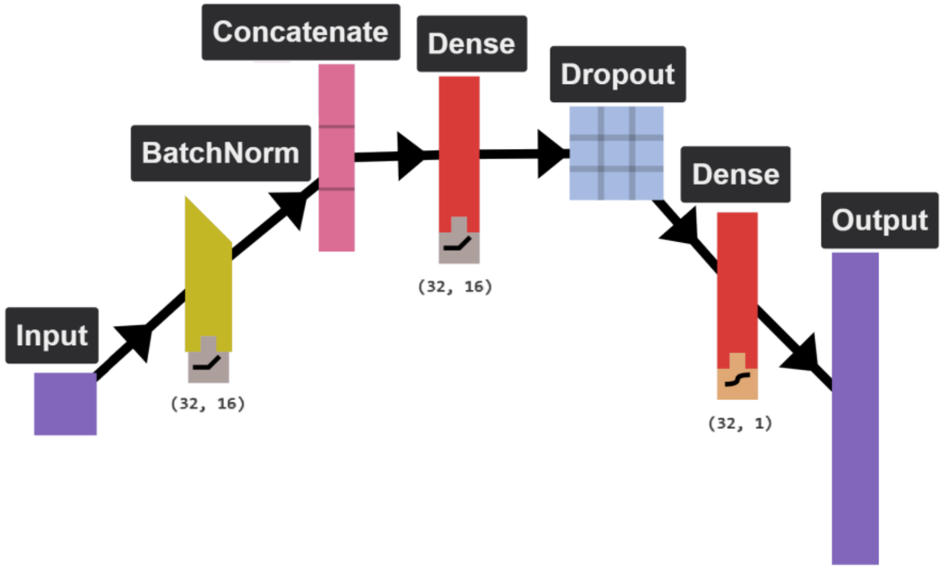




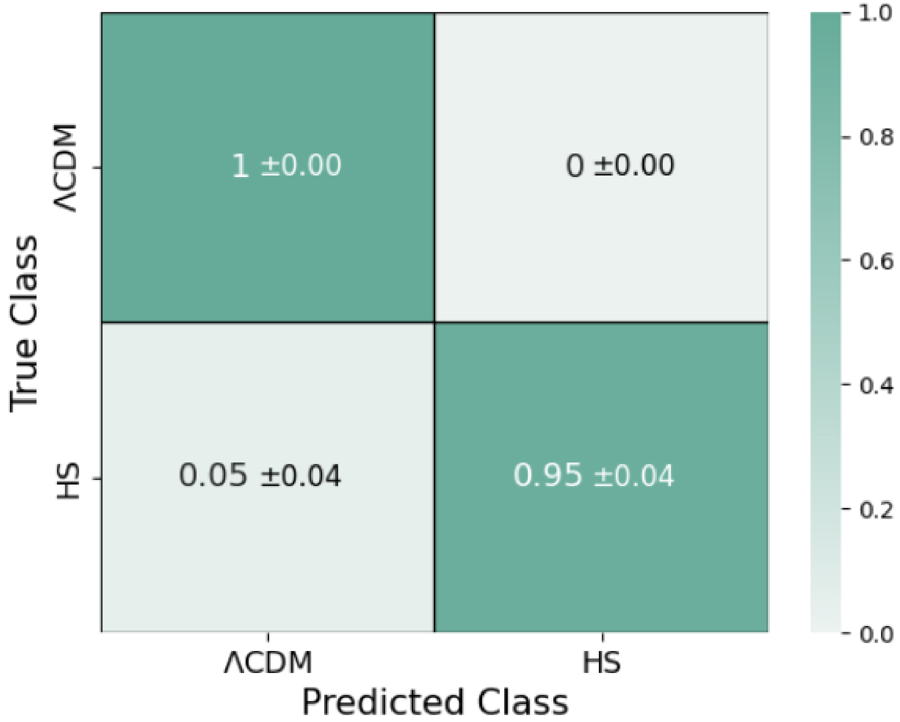
5000 realizations:
70% training + 30% testing
50% f(R) + 50% ΛCDM

Architecture and performance:

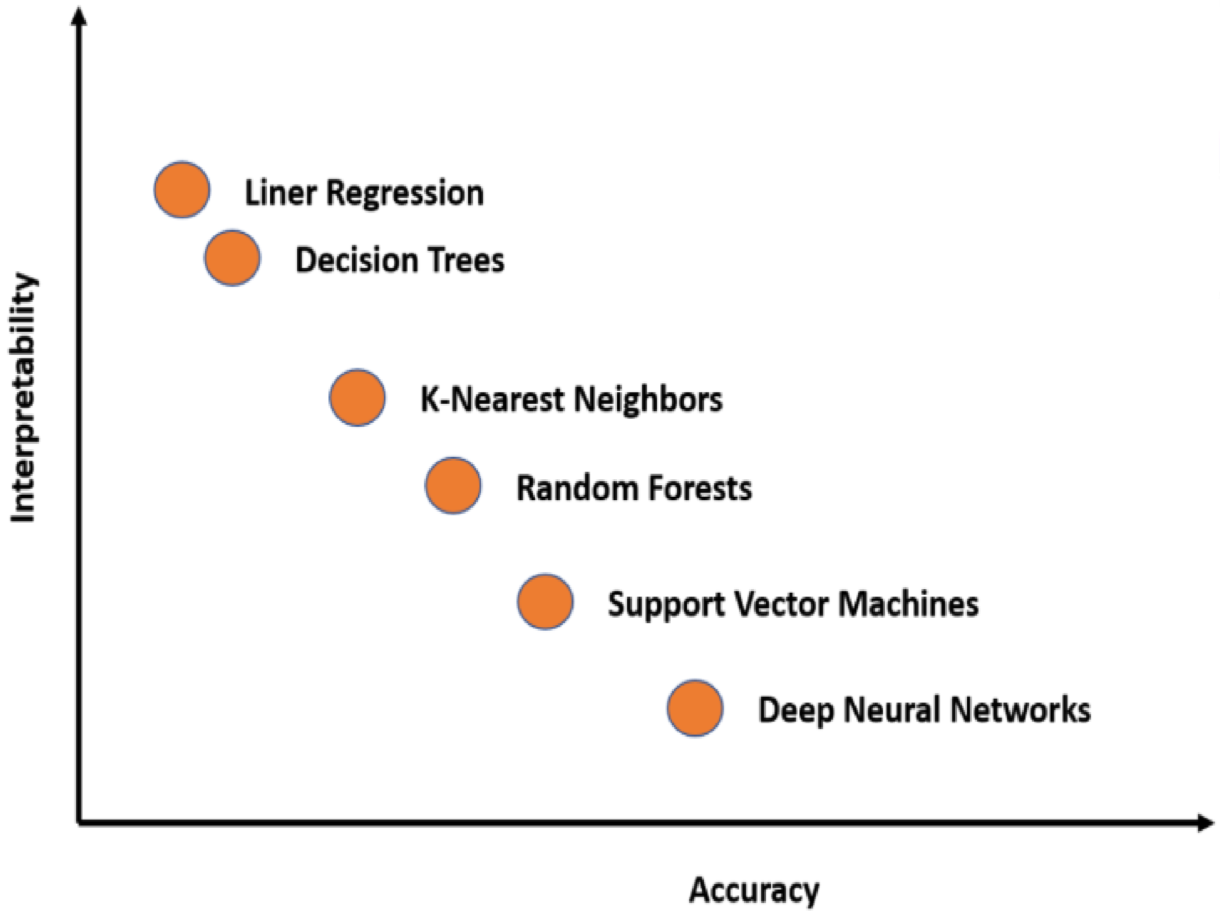
Model architecture to test deviations from ΛCDM

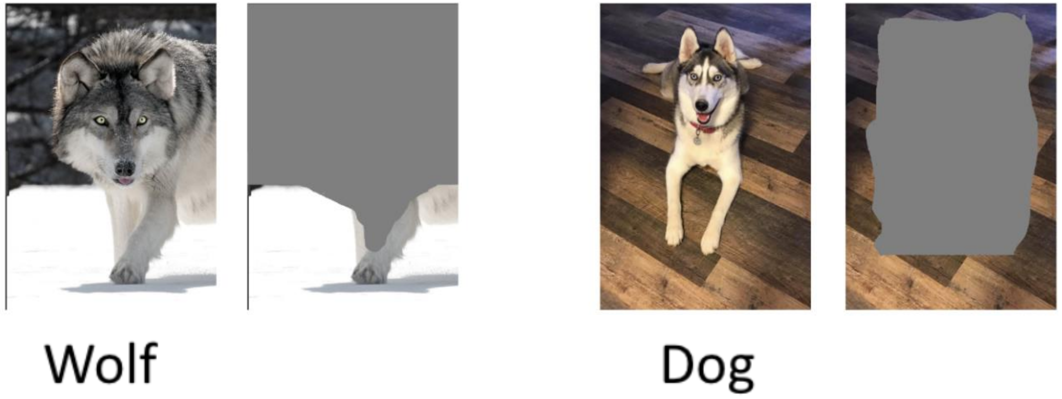


Confusion matrix of our model

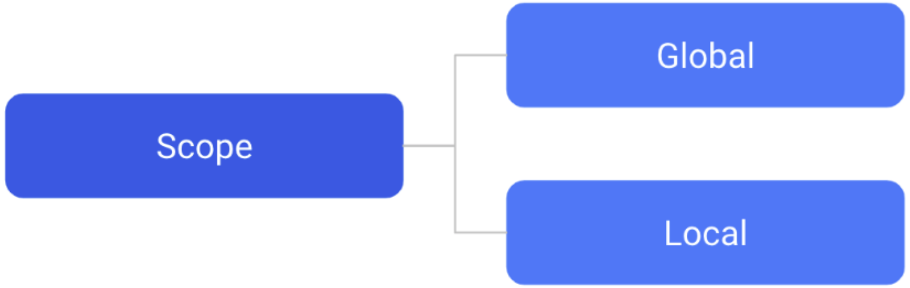


Interpretable Machine Learning



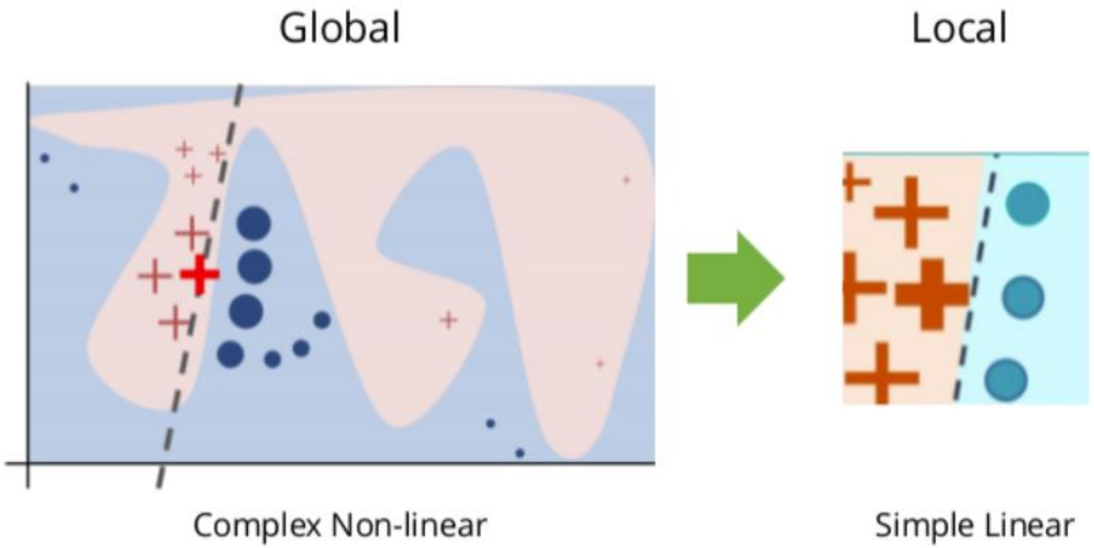


ML Interpretability approaches:

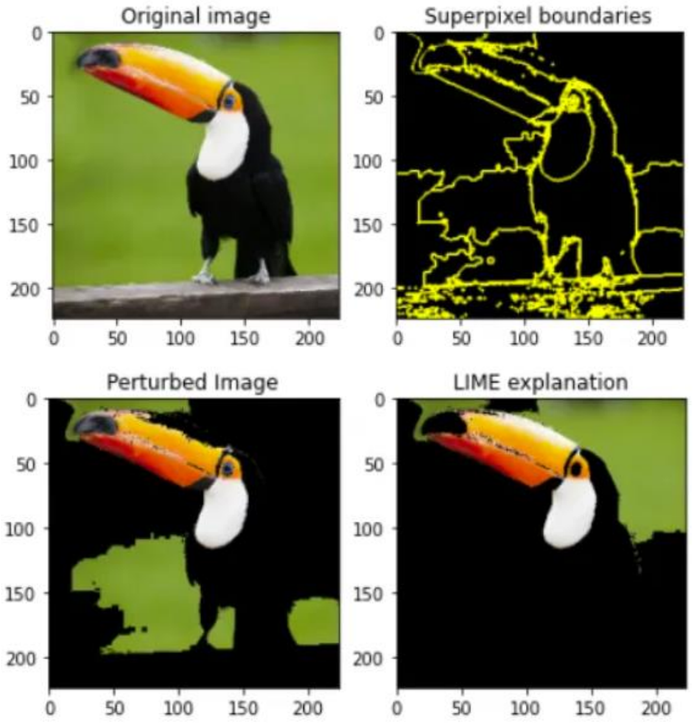


Source: Decoding the Black Box: An Introduction to Interpretable Machine Learning Models: analyticsvidhya.com/

LIME (Local Interpretable Model agnostic Explanations)



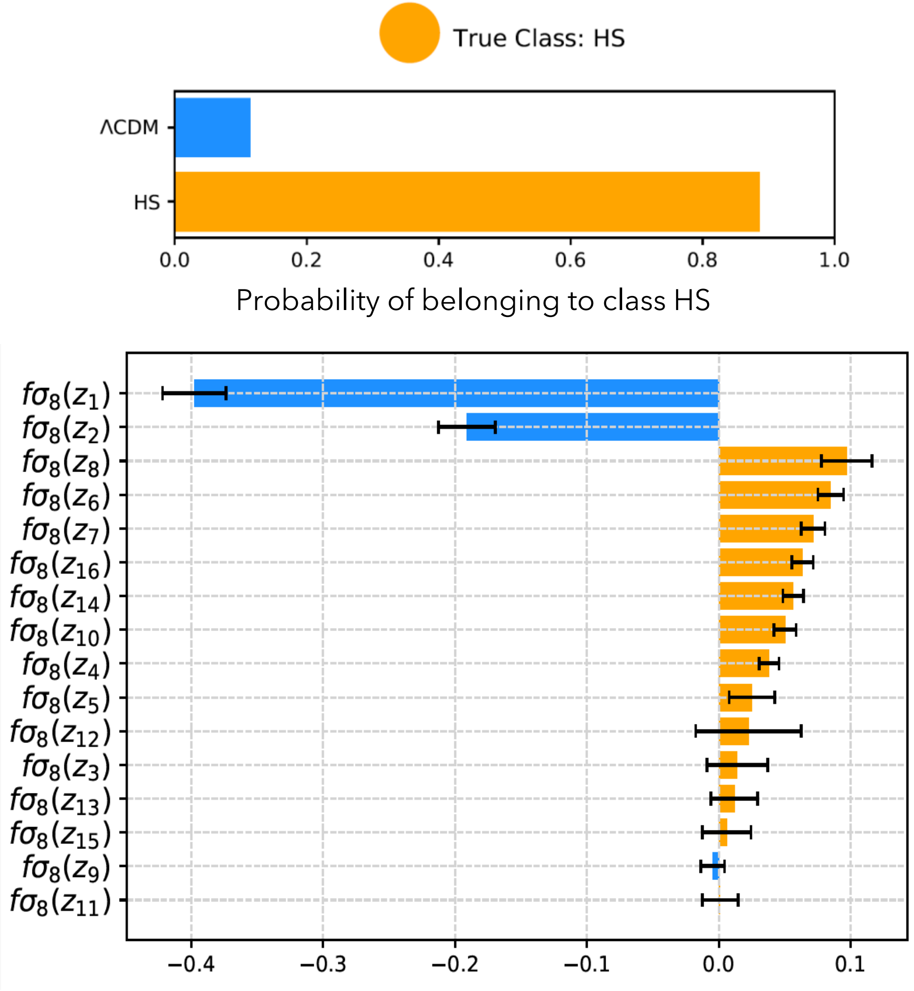
Ribeiro, Singh (2018)

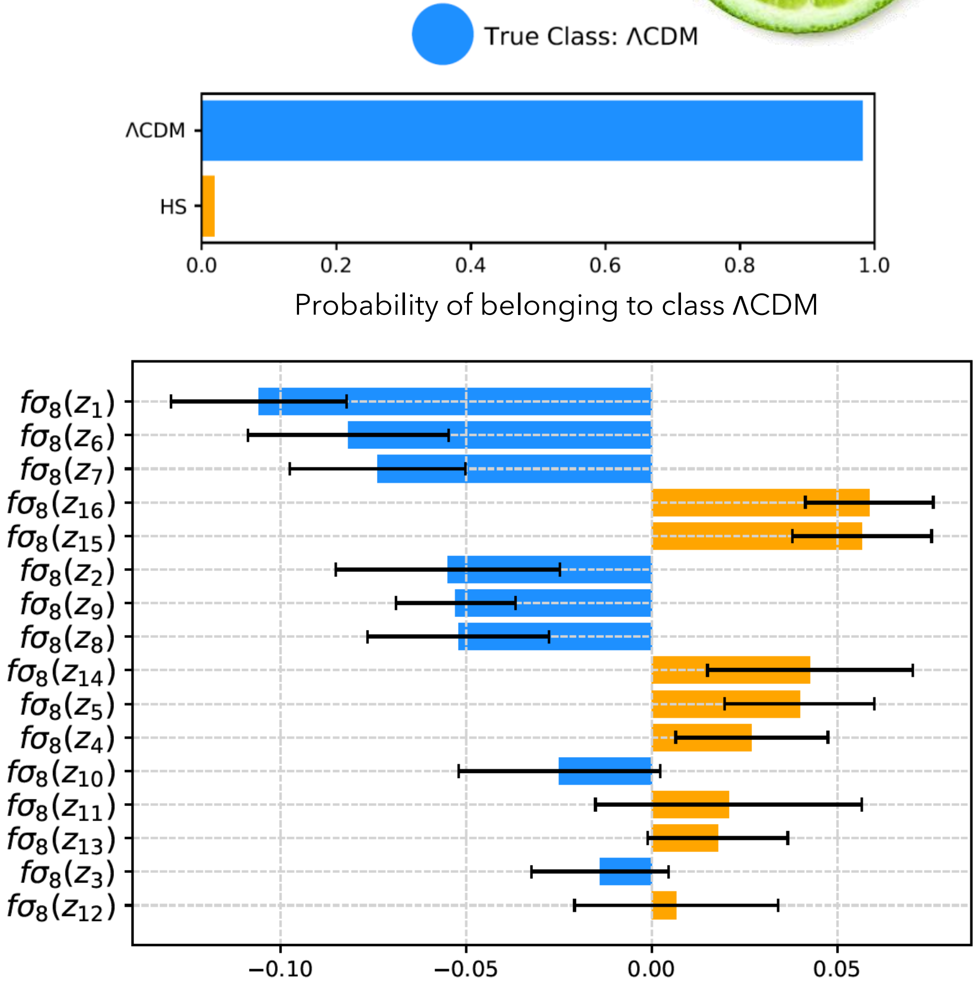


Multiple plots demonstrating a) the original input image, b) the boundaries of all superpixel areas, c) a sample of a perturbed image, and d) the most important super pixel areas to predict the 'toucan' class (Image by Author)

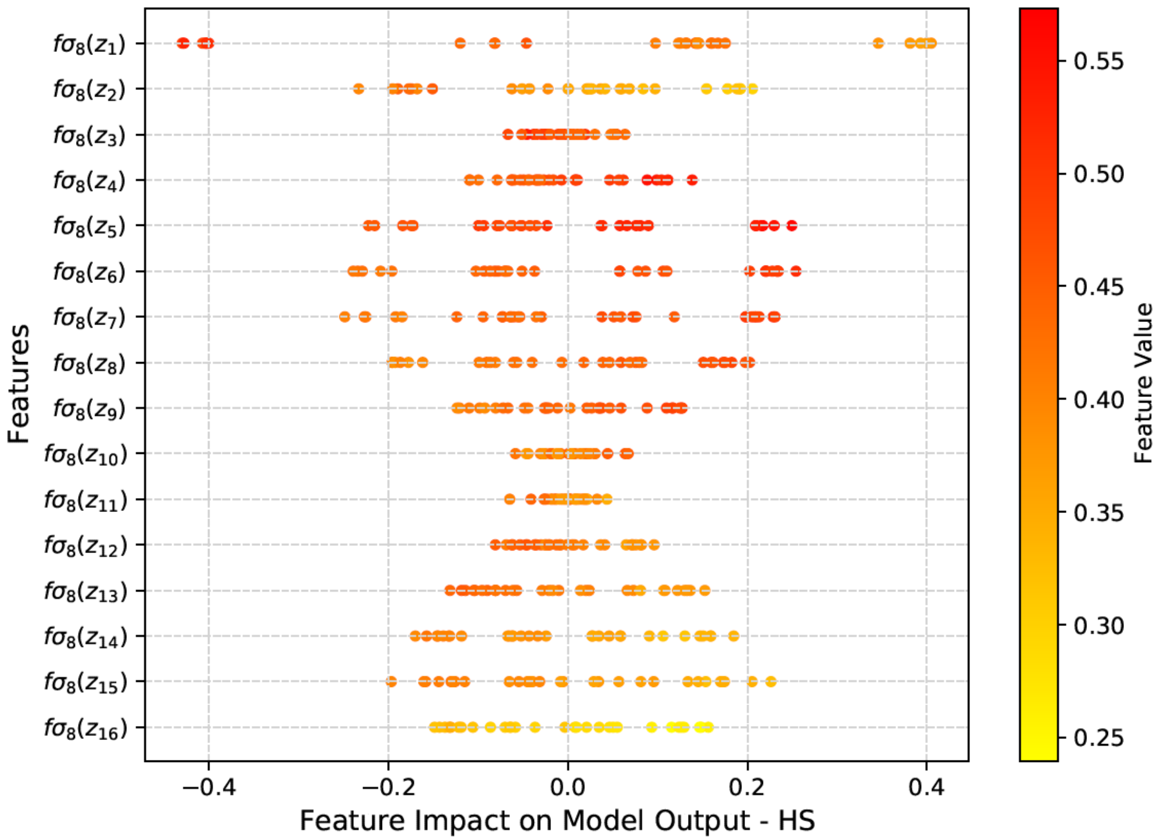
Source: Explainable AI, https://bigdatarepublic.nl/

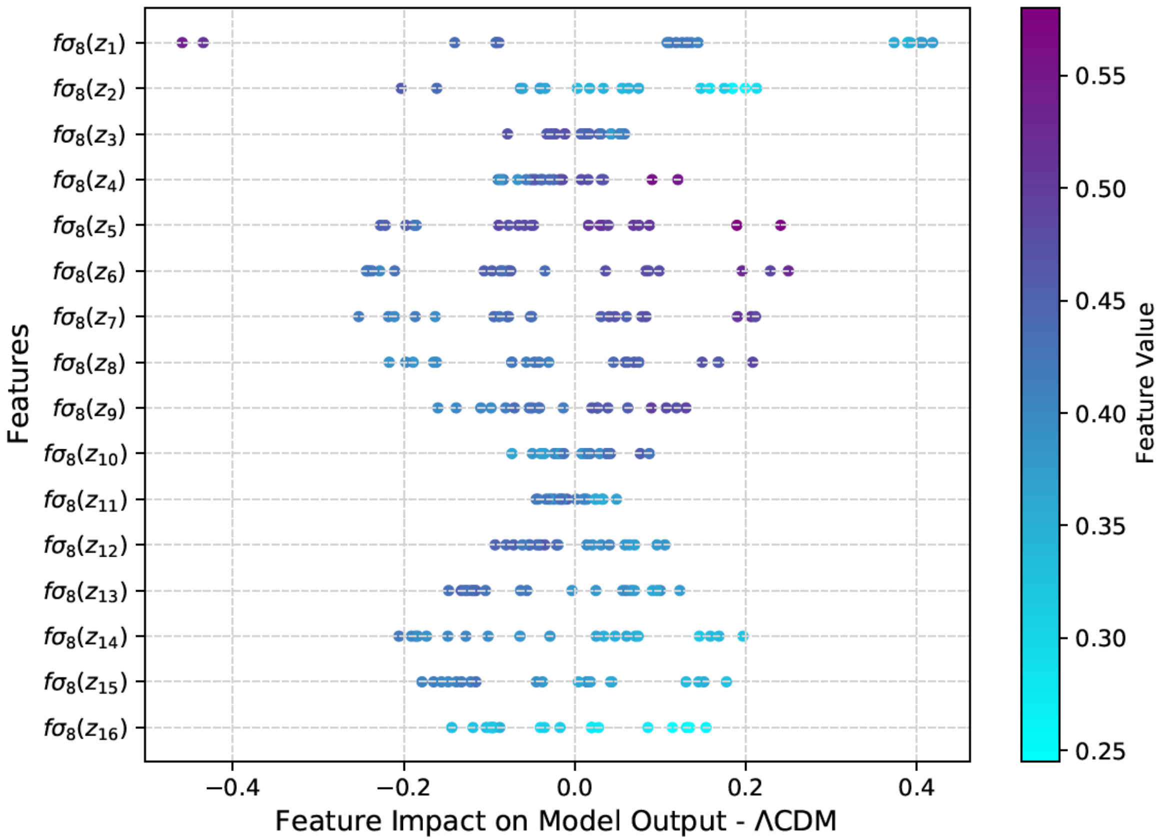
Feature Importance using LIME





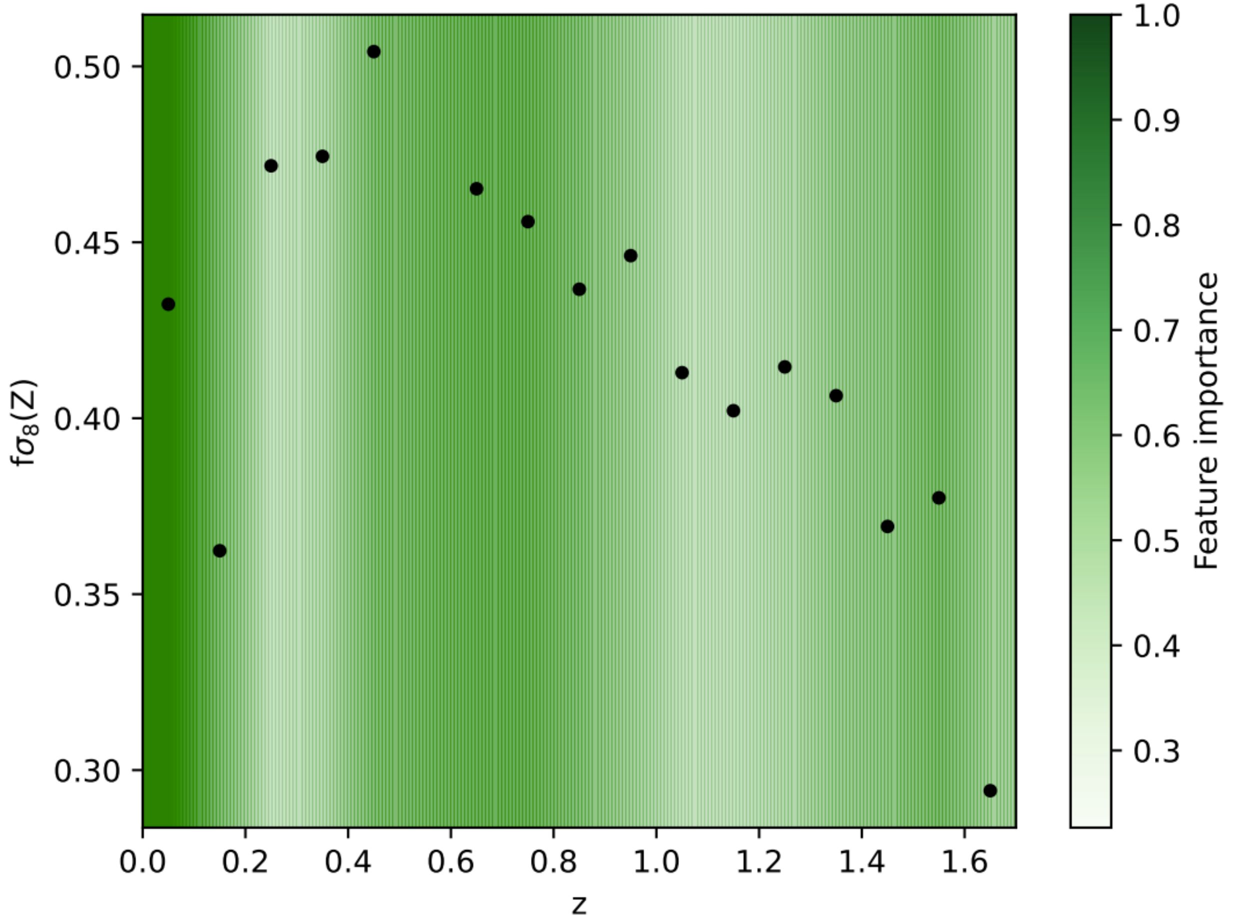
Distribution of LIME feature impact





Feature Importance and Redshift

One realization of $f\sigma_8$, as a function of the redshift z. The color shading is the feature importance of each z-bin (darker implies stronger feature importance), according to LIME.



Conclusions

We are building some trust in machine learning techniques with these interpretability tools.

LIME is able to extract the most relevant features that have an influence on the decision made by the NN. These features are: fs8(z1), fs8(z5), fs8(z6), fs8(z7) and fs8(z15) -> overall, seems that first features are the most relevant.

ML-based model agnostic approaches seem to be good for testing GR at large scales, the differences between both datasets are of the order of 10^{-5} but we need to understand at which point this performance breaks.

ΛCDM is maybe the most tested model, we still need to find out whether GR is the correct theory for gravity at large scales.

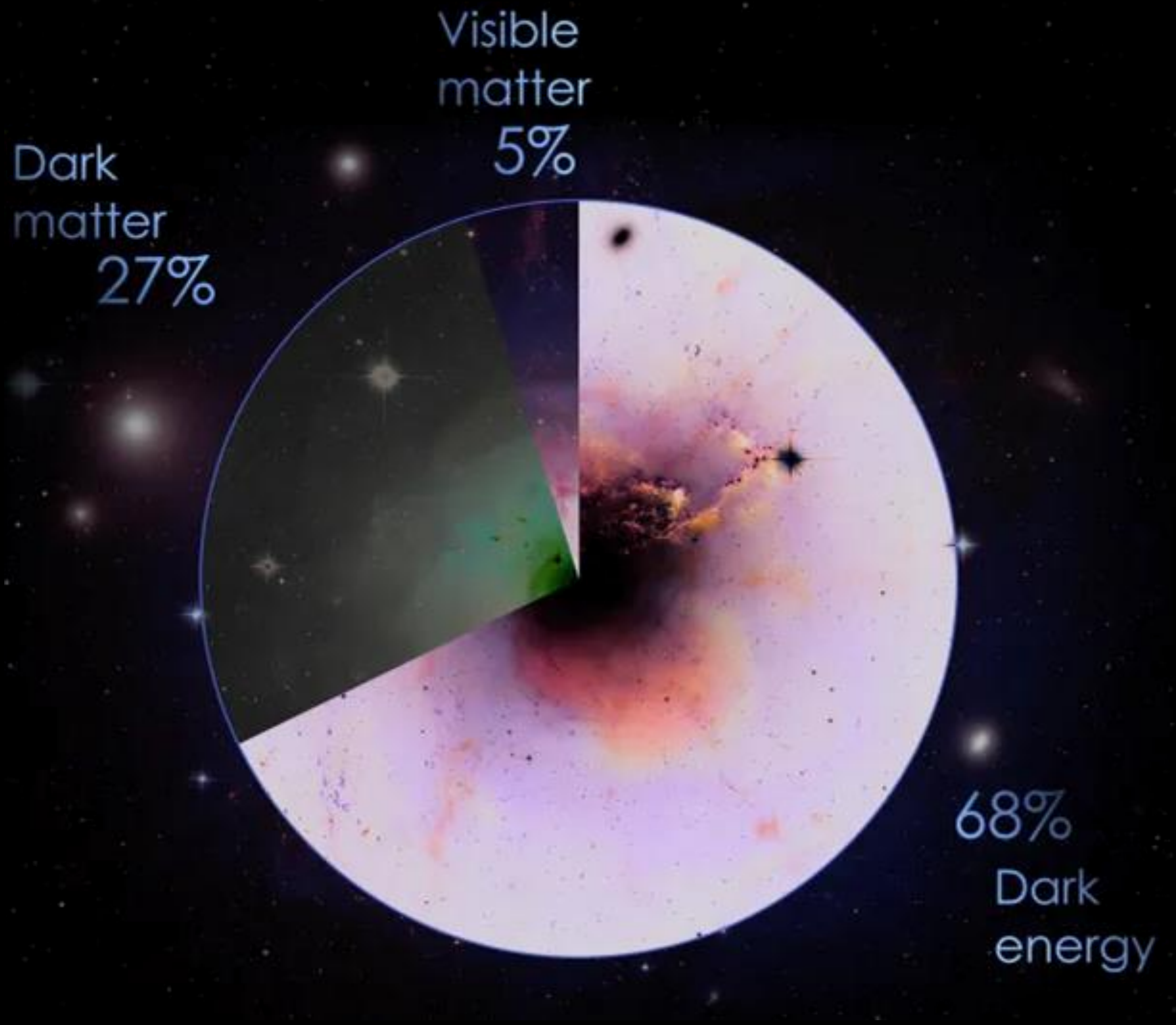
Thank you!

Backup

The ΛCDM model

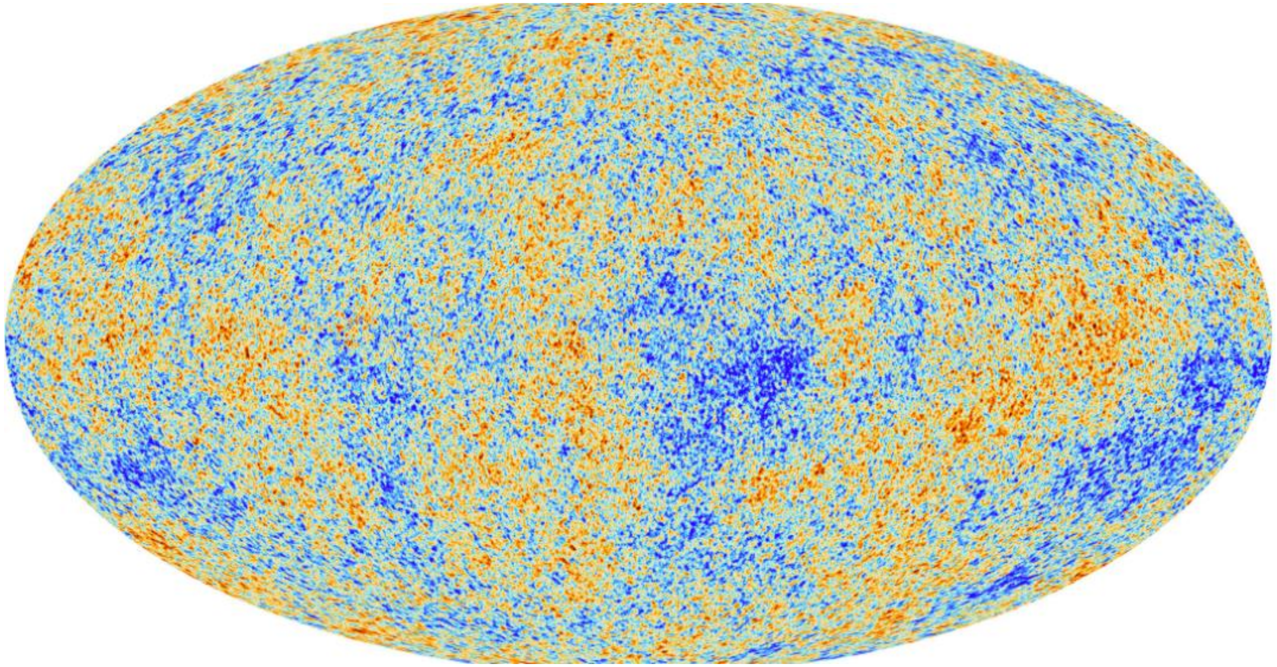
Parameter	TT,TE,EE+lowE+lensing 68% limits
$\Omega_b h^2$	0.02237 ± 0.00015
τ	0.0544 ± 0.0073
$\ln(10^{10} A_s)$	3.044 ± 0.014
n_s	0.9649 ± 0.0042
H_0 [km s^{-1} Mpc^{-1}]	67.36 ± 0.54
Ω_Λ	0.6847 ± 0.0073
Ω_m	0.3153 ± 0.0073
$\Omega_m h^2$	0.1430 ± 0.0011
$\Omega_m h^3$	0.09633 ± 0.00030
σ_8	0.8111 ± 0.0060
$S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$	0.832 ± 0.013

Planck collaboration 2018



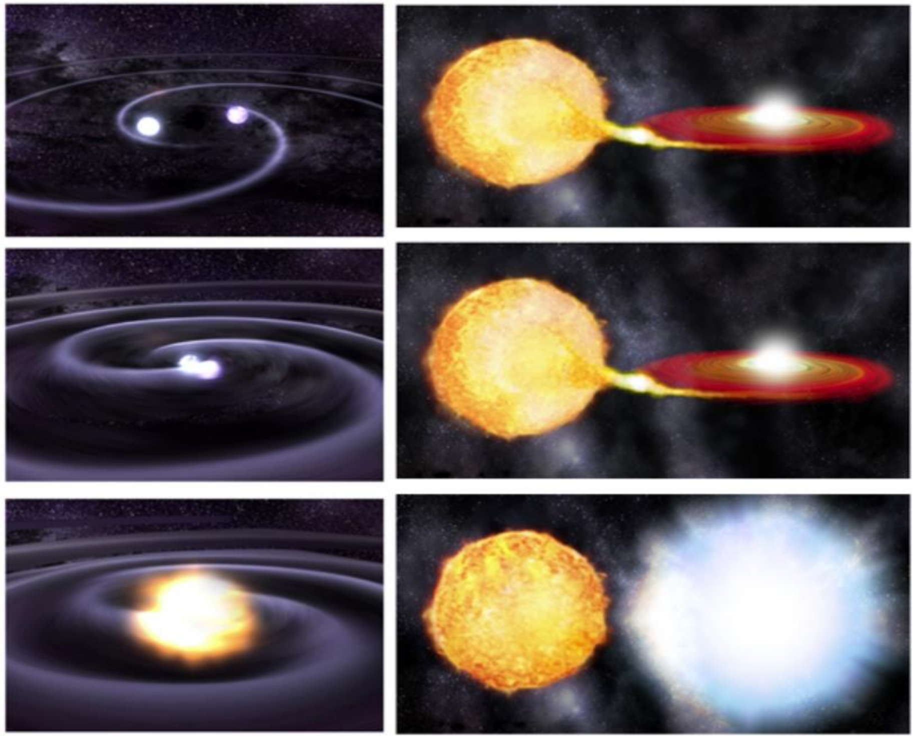
Source: NASA GSFC SVS

H_0 tension



H0 = 67.27 ± 0.60 km/s/Mpc in ΛCDM

Planck 2018, Aghanim et al., arXiv:1807.06209 [astro-ph.CO]



H0 = 74.03 ± 1.42 km/s/Mpc

Riess et al. arXiv:1903.07603 [astro-ph.CO]

H_0 tension

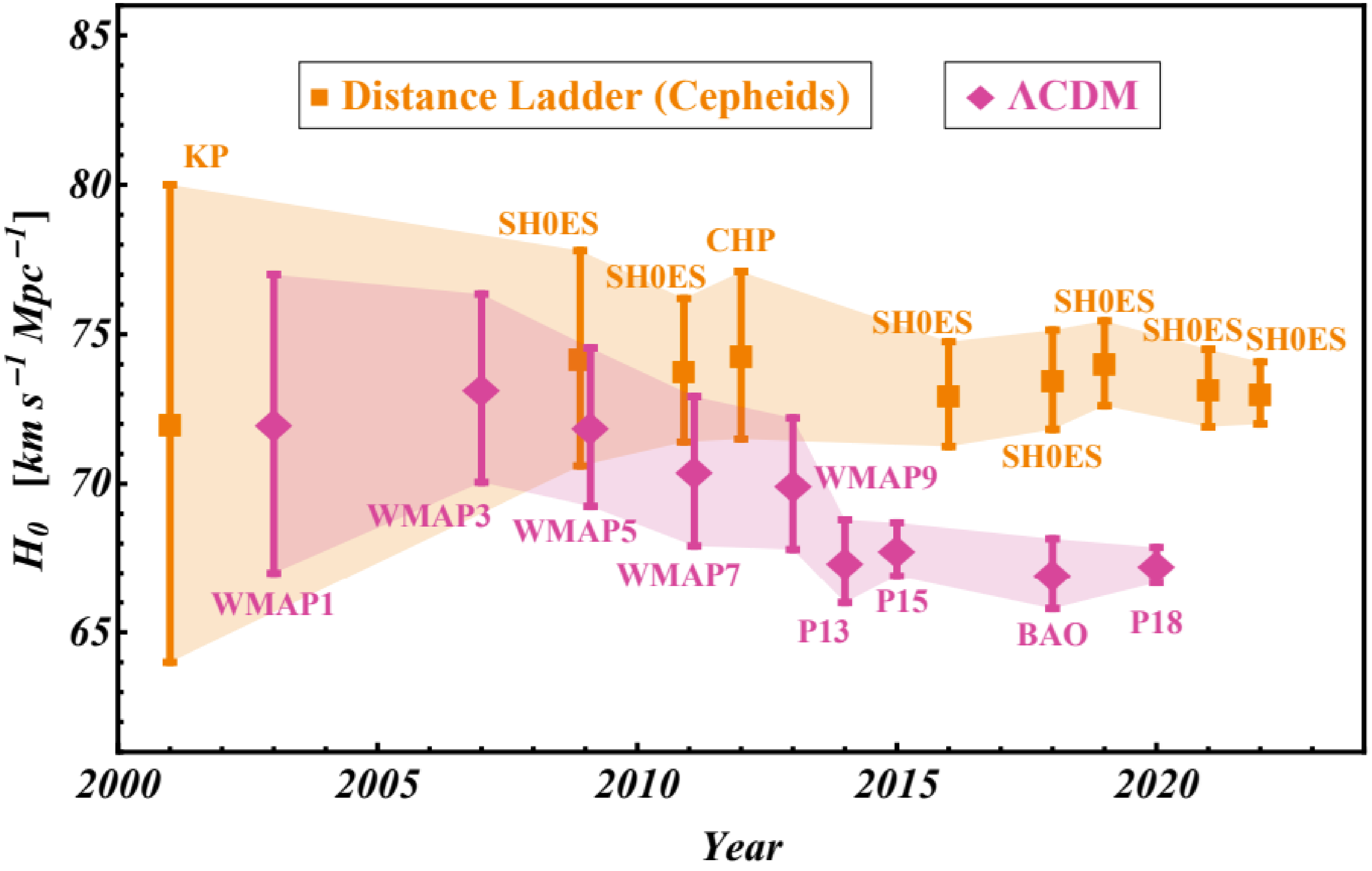
Tension between the value of H0 from:

CMB (early universe)

SNIa (late universe)

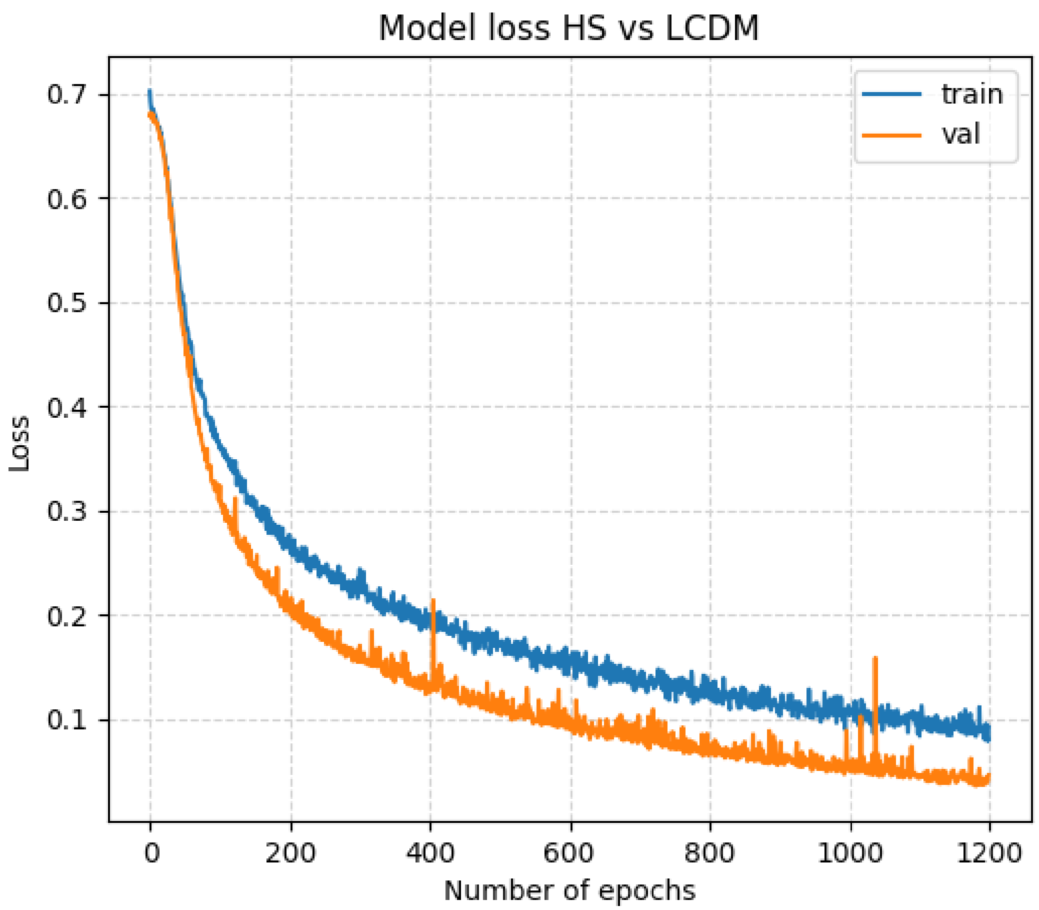
1. **Is there something wrong with the data?**
2. **Is there something wrong with the theory?**

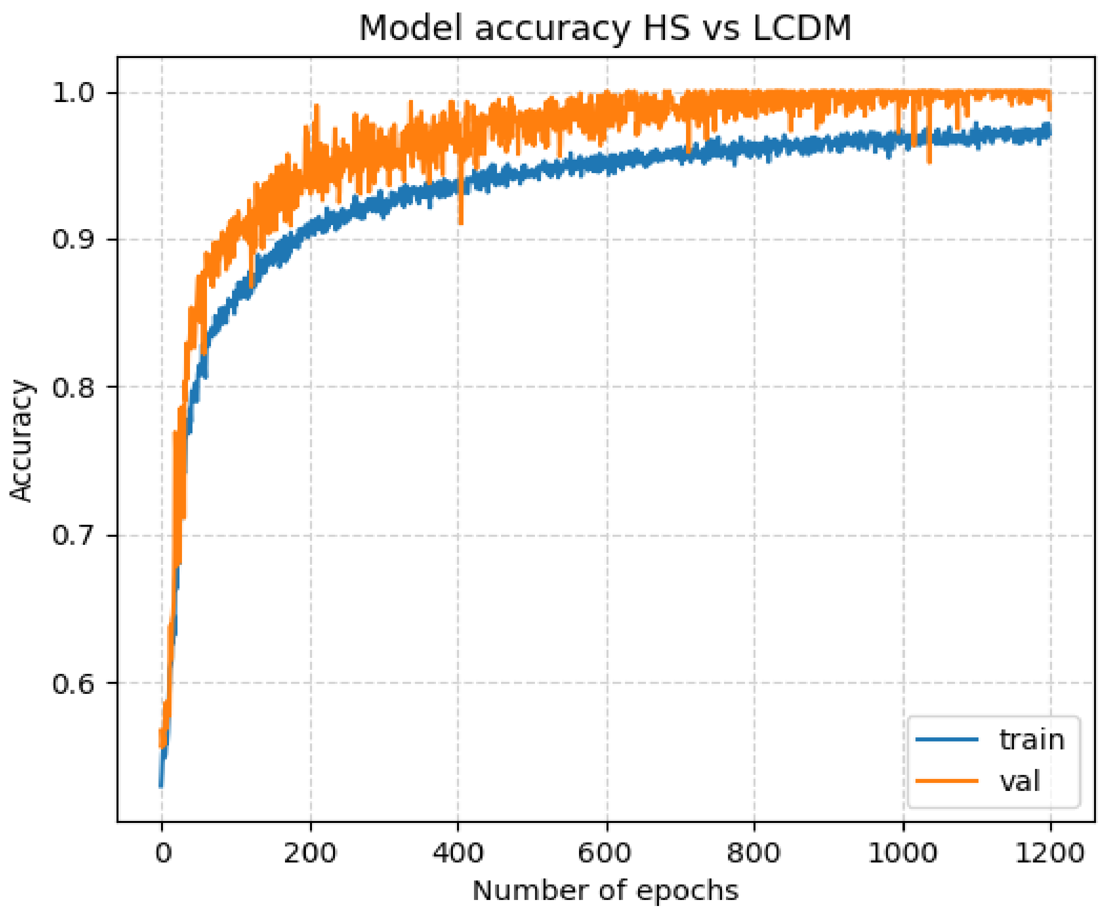
- Ways of taking data have been refined up to an incredible level precision.
- Model?



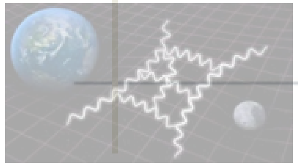
Perivolaropoulos, Skara (2022)

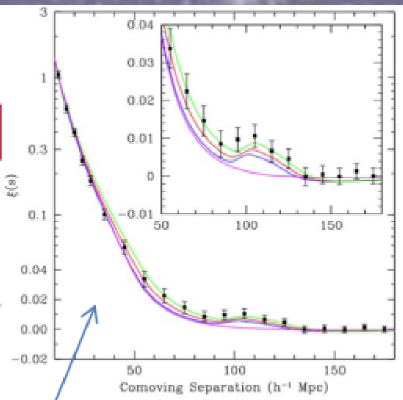
Learning Curves





Measuring H_0-H(z) with a standard early time calibrators





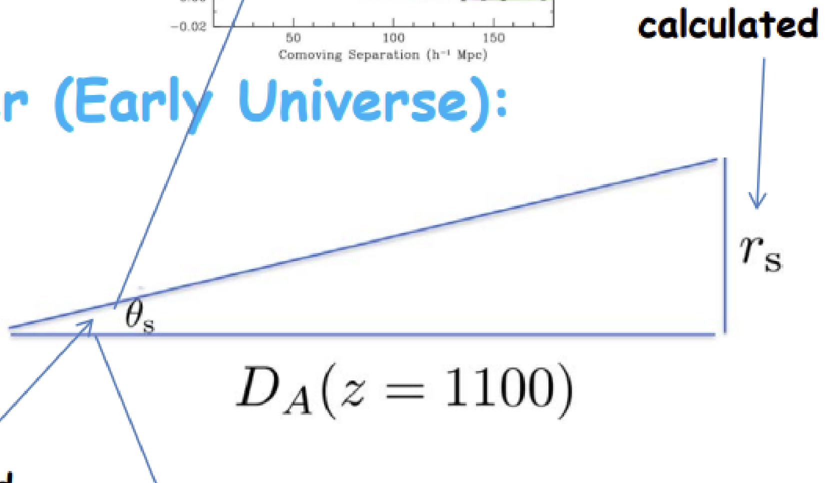
Sound Horizon at Recombination Standard Ruler (Early Universe):

$$r_s = \int_0^{t_d} c_s dt/a = \int_0^{a_d} c_s \frac{da}{a^2 H(a)}$$

Depends on ρ_b, ρ_γ and ρ_{CDM}

r_s=147.6 Mpc from Planck and BBN values of ρ_b, ρ_γ and ρ_{CDM}

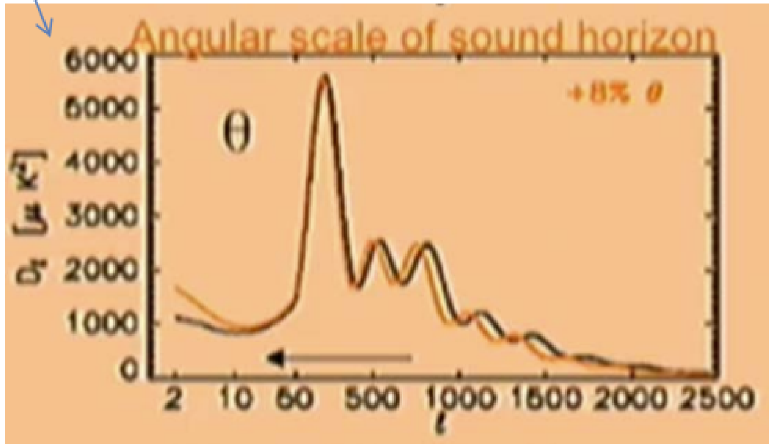
inferred



$$\theta_s = \frac{r_s}{D_A(z)}$$

comoving

$$E(z) = \left[\Omega_{0m}(1+z)^3 + (1-\Omega_{0m})\right]^{\frac{1}{2}}$$



Degeneracy between r_s and H_0 and E(z).

Source: Perivolaropoulos, The Tensions of ΛCDM